Philosophy of QM 24.111

Third lecture.

CORRELATIONS—WHAT ARE THEY?

Answer: Systematic similarities between the values of each of a pair of variables.

Examples:

- Height and weight in humans.
- Smoking and lung cancer.
- Readings of different measuring devices.

Moral: Correlations in nature are ubiquitous, and their study is essential to the practice of science.

CORRELATIONS—HOW CAN THEY BE EXPLAINED?

1. Logical connection.

2. Mere coincidence.

3. Pre-established harmony.

4. Direct causal connection.

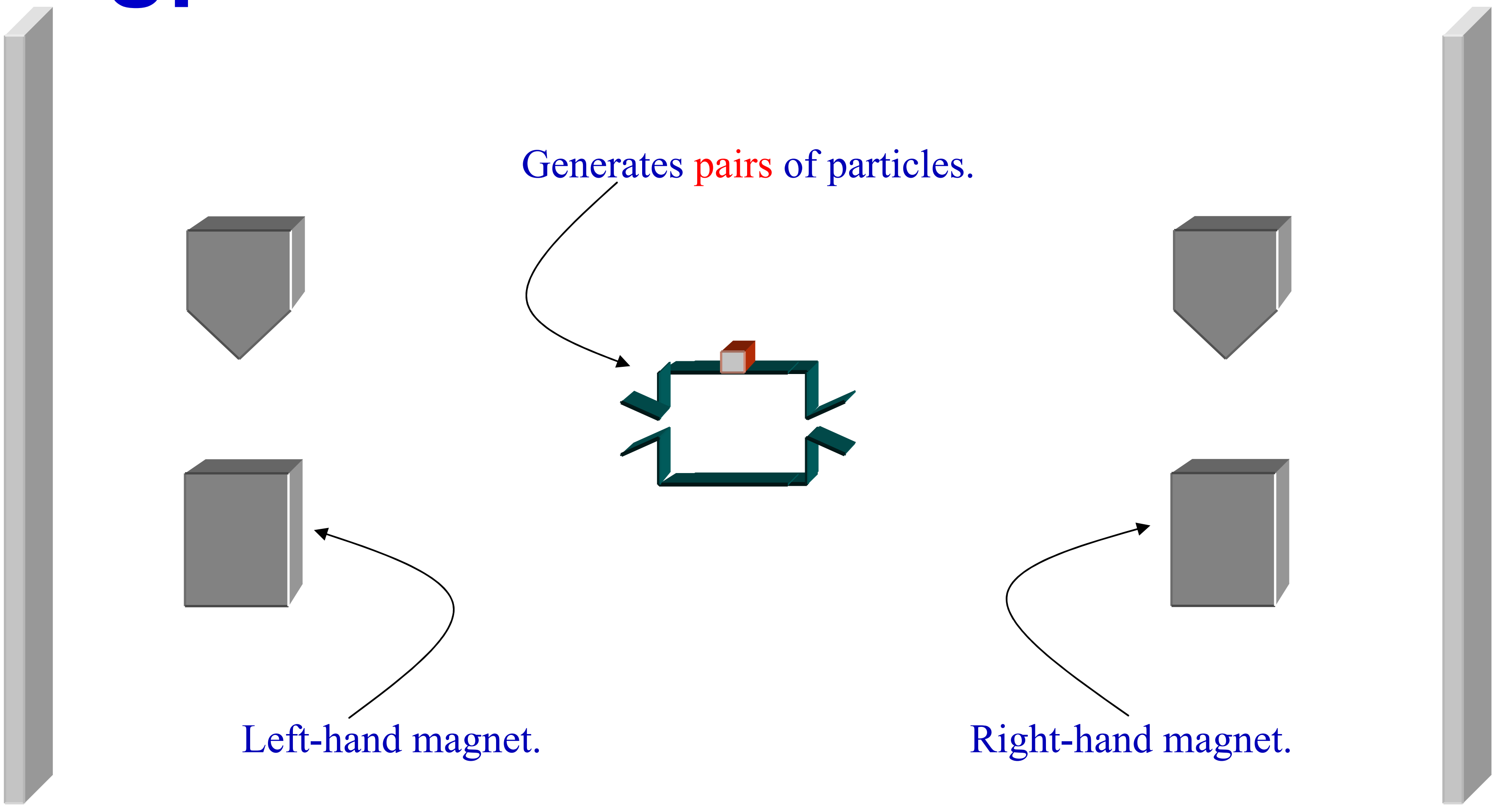
5. Common cause.

6. Fundamental physical law.

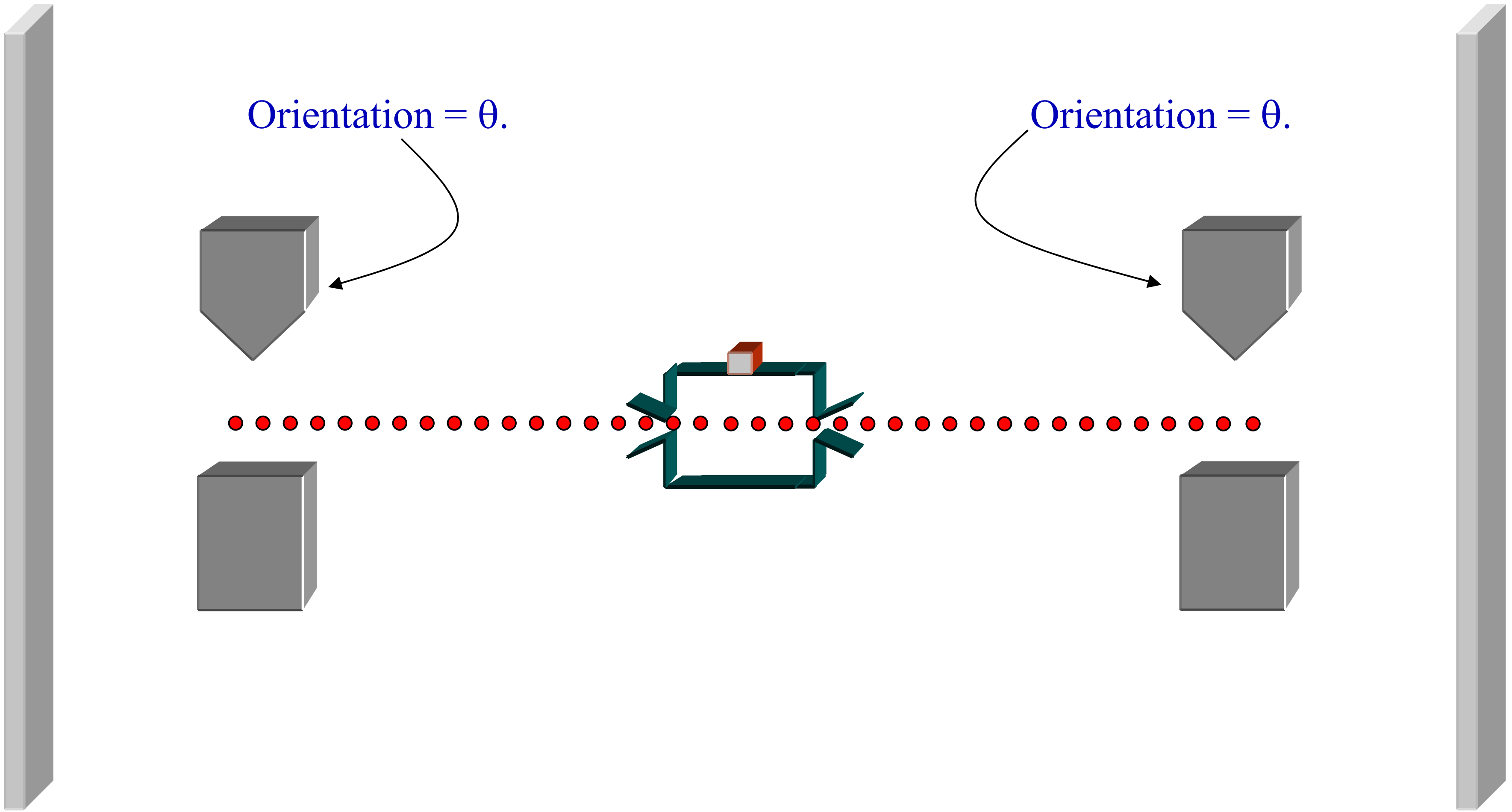
CORRELATIONS—HOW CAN THEY BE EXPLAINED?

1. Logical connection.
2. Mere coincidence.
3. Pre-established harmony.
4. Direct causal connection.
5. Common cause.
6. Fundamental physical law.

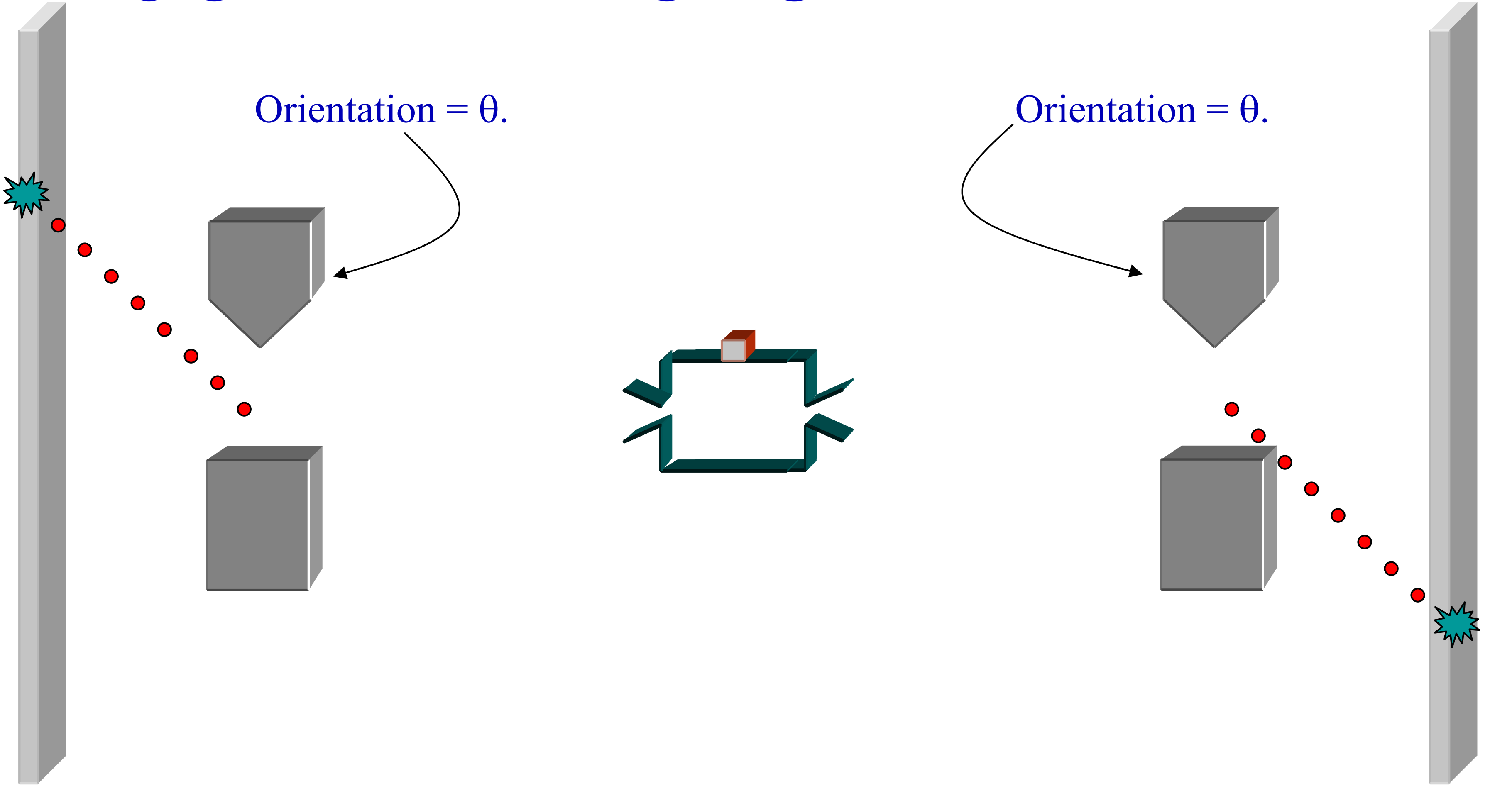
BELL'S INEQUALITIES: THE EXPERIMENTAL SET-UP



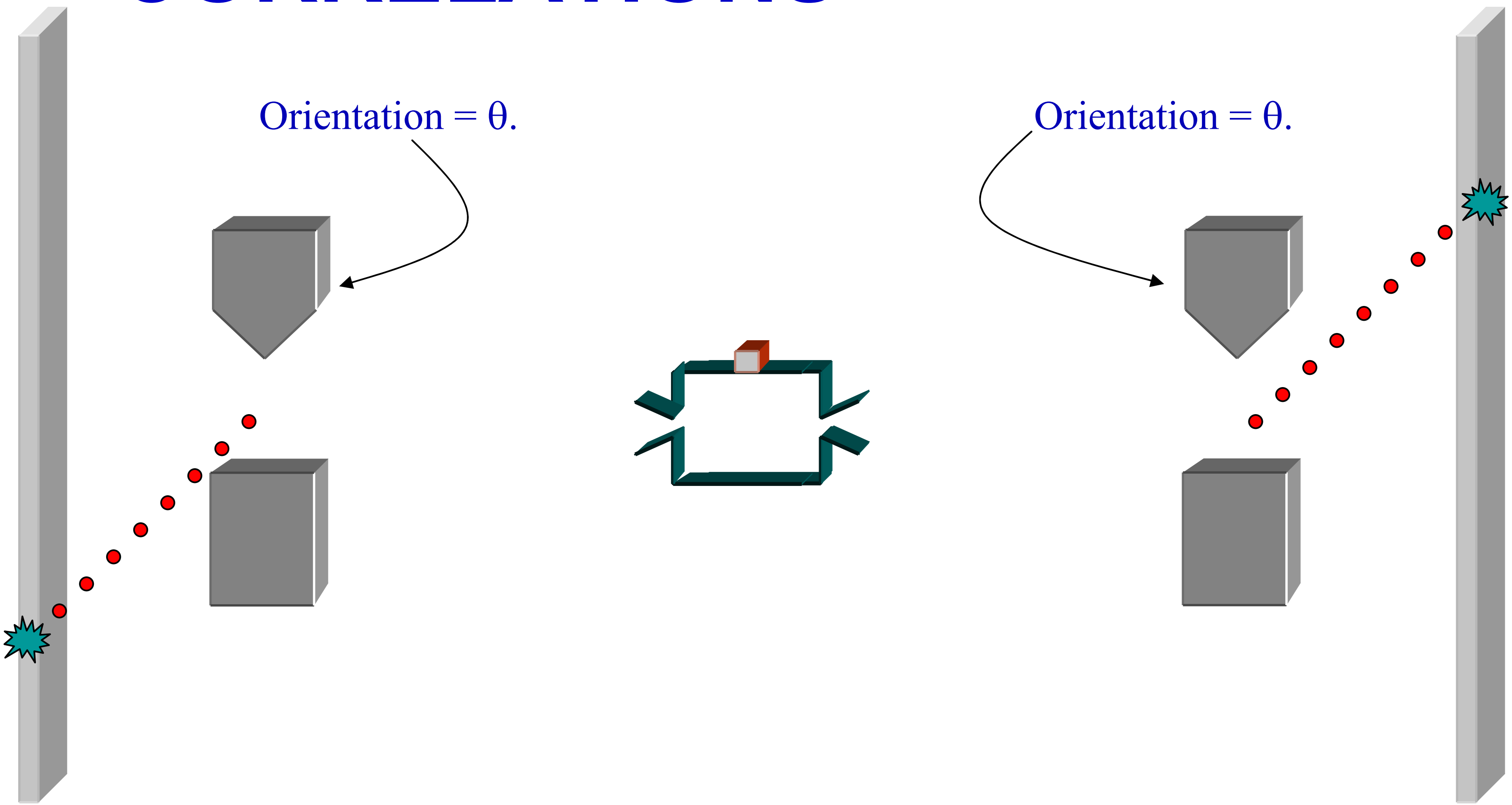
BELL'S INEQUALITIES: THE PERFECT CORRELATIONS



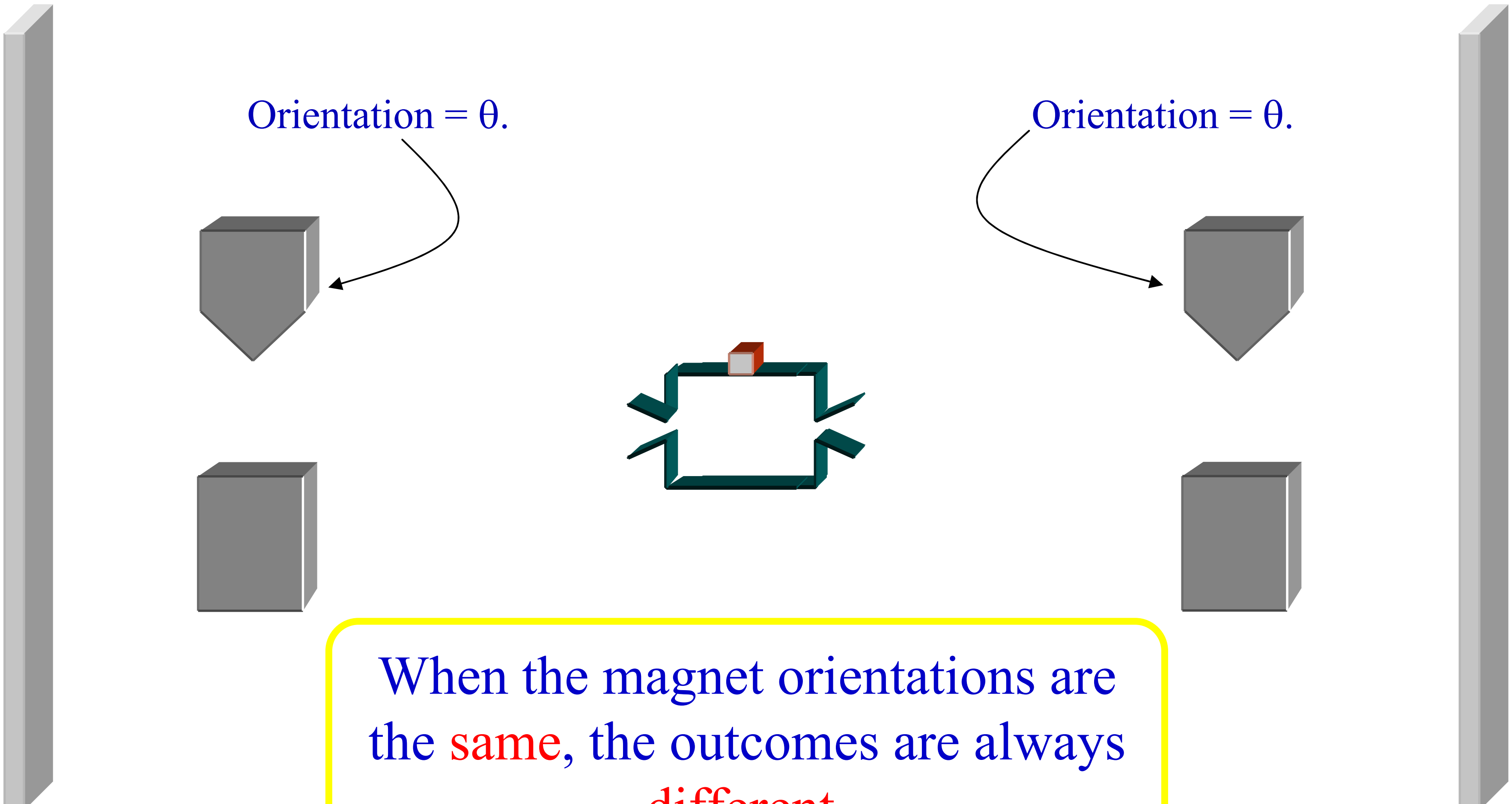
BELL'S INEQUALITIES: THE PERFECT CORRELATIONS



BELL'S INEQUALITIES: THE PERFECT CORRELATIONS



BELL'S INEQUALITIES: THE PERFECT CORRELATIONS



When the magnet orientations are the same, the outcomes are always different.

INTERLUDE: EINSTEIN VS. BOHR

Einstein says:

"Suppose we measure the right-hand particle first, and it goes up through a magnet with orientation θ. Then that outcome tells us what will happen if we perform the same measurement on the left-hand particle: it will go down. But nothing we did to the right-hand particle can possibly have affected the *physical state* of the left-hand particle. So that state must *already* have been such as to guarantee that the left-hand particle would go up through a magnet with orientation θ. But the quantum-mechanical representation of its state tells us nothing of the sort. So that representation is incomplete."

Bohr replies:

"The very possibility of meaningfully applying a physical concept is conditioned by the measurement we choose to perform."

HOW CAN THESE CORRELATIONS BE EXPLAINED?

Logical connection?

—But there is no definitional link between “up on the left” and “down on the right”.

HOW CAN THESE CORRELATIONS BE EXPLAINED?

Logical connection?

—But there is no definitional link between “up on the left” and “down on the right”.

Mere coincidence?

—Nonsense: the correlations are too robust for this to be believable.

HOW CAN THESE CORRELATIONS BE EXPLAINED?

Logical connection?

—But there is no definitional link between “up on the left” and “down on the right”.

Mere coincidence?

—Nonsense: the correlations are too robust for this to be believable.

Pre-established harmony?

—Only if nature conspired to make the processes that determine the magnet orientation match up smoothly with the (completely distinct) processes that generate the particle pairs—and that’s crazy.

HOW CAN THESE CORRELATIONS BE EXPLAINED?

Logical connection?

—But there is no definitional link between "up on the left" and "down on the right".

Mere coincidence?

—Nonsense: the correlations are too robust for this to be believable.

Pre-established harmony?

—Only if nature conspired to make the processes that determine the magnet orientation match up smoothly with the (completely distinct) processes that generate the particle pairs—and that's crazy.

Direct causal connection?

—But the correlations are unaffected either by distance or by the interpositions of shielding devices; we can also make the measurements space-like separated.

HOW CAN THESE CORRELATIONS BE EXPLAINED?

Logical connection?

—But there is no definitional link between “up on the left” and “down on the right”.

Mere coincidence?

—Nonsense: the correlations are too robust for this to be believable.

Pre-established harmony?

—Only if nature conspired to make the processes that determine the magnet orientation match up smoothly with the (completely distinct) processes that generate the particle pairs—and that’s crazy.

Direct causal connection?

—But the correlations are unaffected either by distance or by the interpositions of shielding devices; we can also make the measurements space-like separated.

Fundamental physical law?

—But we aren’t dealing here with a restriction on *states*.

HOW CAN THESE CORRELATIONS BE EXPLAINED?

Logical connection?

—But there is no definitional link between “up on the left” and “down on the right”.

Mere coincidence?

—Nonsense: the correlations are too robust for this to be believable.

Pre-established harmony?

—Only if nature conspired to make the processes that determine the magnet orientation match up smoothly with the (completely distinct) processes that generate the particle pairs—and that’s crazy.

Direct causal connection?

—But the correlations are unaffected either by distance or by the interpositions of shielding devices; we can also make the measurements space-like separated.

Fundamental physical law?

—But we aren’t dealing here with a restriction on *states*.

THE OBVIOUS ANSWER: THERE IS A COMMON CAUSE

Specifically, we can explain the correlations by means of the following **deterministic hidden-variables hypothesis**:

When a pair of particles is created, each particle in the pair is endowed with properties that determine, for each possible magnet orientation θ, which way it will go through a magnet with that orientation. These properties are 'assigned' in such a way that the particles will invariably go in opposite directions, if the magnet orientations are the same.

THE PLOT THICKENS

What if we make the magnet orientations *different?*

To find out, let us choose orientations 0° and +120° for the left-hand magnet, and orientations 0° and -120° for the right-hand magnet. Then there will be eight possible property configurations, each with a corresponding probability of being generated:

left particle	right particle	probability
(U, 0°),(U, +120°)	(D, 0°),(U, -120°)	p_1
(U, 0°),(U, +120°)	(D, 0°),(D, -120°)	p_2
(U, 0°),(D, +120°)	(D, 0°),(U, -120°)	p_3
(U, 0°),(D, +120°)	(D, 0°),(D, -120°)	p_4
(D, 0°),(U, +120°)	(U, 0°),(U, -120°)	p_5
(D, 0°),(U, +120°)	(U, 0°),(D, -120°)	p_6
(D, 0°),(D, +120°)	(U, 0°),(U, -120°)	p_7
(D, 0°),(D, +120°)	(U, 0°),(D, -120°)	p_8

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 = 1.$$

What is the probability that the outcomes will be different, if the magnet orientations are different?

A simplified quantum-mechanical derivation:

Consider just the situation where the left-hand magnet has orientation +120° and the right-hand magnet has orientation 0°. Suppose that the right-hand particle goes through its magnet first. Then there are two cases:

Case 1: It goes up. Then the left-hand particle is certain to go down through a magnet with orientation 0°. So, by the cos-squared law, its probability for going down through a magnet with orientation +120° is 1/4.

Case 2: It goes down. Then the left-hand particle is certain to go up through a magnet with orientation 0°. So, by the cos-squared law, its probability for going up through a magnet with orientation +120° is 1/4.

Either way, the probability of opposite results is 1/4.

What probabilities does our hidden-variables hypothesis yield?

First case: θ_{left} = +120° and θ_{right} = 0°.

left particle	right particle	probability
(U, 0°),(U, +120°)	(D, 0°),(U, -120°)	p_1
(U, 0°),(U, +120°)	(D, 0°),(D, -120°)	p_2
(U, 0°),(D, +120°)	(D, 0°),(U, -120°)	p_3
(U, 0°),(D, +120°)	(D, 0°),(D, -120°)	p_4
(D, 0°),(U, +120°)	(U, 0°),(U, -120°)	p_5
(D, 0°),(U, +120°)	(U, 0°),(D, -120°)	p_6
(D, 0°),(D, +120°)	(U, 0°),(U, -120°)	p_7
(D, 0°),(D, +120°)	(U, 0°),(D, -120°)	p_8

Probability of opposite outcomes = $p_1 + p_2 + p_7 + p_8$

What probabilities does our hidden-variables hypothesis yield?

Second case: θ_{left} = 0° and θ_{right} = -120°.

left particle	right particle	probability
(U, 0°),(U, +120°)	(D, 0°),(U, -120°)	p_1
(U, 0°),(U, +120°)	(D, 0°),(D, -120°)	p_2
(U, 0°),(D, +120°)	(D, 0°),(U, -120°)	p_3
(U, 0°),(D, +120°)	(D, 0°),(D, -120°)	p_4
(D, 0°),(U, +120°)	(U, 0°),(U, -120°)	p_5
(D, 0°),(U, +120°)	(U, 0°),(D, -120°)	p_6
(D, 0°),(D, +120°)	(U, 0°),(U, -120°)	p_7
(D, 0°),(D, +120°)	(U, 0°),(D, -120°)	p_8

Probability of opposite outcomes = $p_2 + p_4 + p_5 + p_7$

What probabilities does our hidden-variables hypothesis yield?

Third case: θ_{left} = +120° and θ_{right} = -120°.

left particle	right particle	probability
(U, 0°),(U, +120°)	(D, 0°),(U, -120°)	p_1
(U, 0°),(U, +120°)	(D, 0°),(D, -120°)	p_2
(U, 0°),(D, +120°)	(D, 0°),(U, -120°)	p_3
(U, 0°),(D, +120°)	(D, 0°),(D, -120°)	p_4
(D, 0°),(U, +120°)	(U, 0°),(U, -120°)	p_5
(D, 0°),(U, +120°)	(U, 0°),(D, -120°)	p_6
(D, 0°),(D, +120°)	(U, 0°),(U, -120°)	p_7
(D, 0°),(D, +120°)	(U, 0°),(D, -120°)	p_8

Probability of opposite outcomes = $p_2 + p_3 + p_6 + p_7$

DISASTER STRIKES!!

Compare the predictions of quantum mechanics with the predictions of our hidden-variables hypothesis:

θ_{left}	θ_{right}	probability of opposite outcomes QM	HV
+120°	0°	.25	$p_1 + p_2 + p_7 + p_8$
0°	-120°	.25	$p_2 + p_4 + p_5 + p_7$
+120°	-120°	.25	$p_2 + p_3 + p_6 + p_7$

Add the QM column: The sum is .75.

Add the HV column: The sum is $p_1 + p_2 + p_7 + p_8 + p_2 + p_4 + p_5 + p_7 + p_2 + p_3 + p_6 + p_7$

$= p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + 2p_2 + 2p_7 = 1 + 2p_2 + 2p_7.$

So HV contradicts QM—and when we run the experiment, QM wins.

CORRELATIONS EXPLAINED BY LOGICAL CONNECTIONS:

Example: Flip a coin many times. Two variables characterize each toss: a variable H which has value 1 if the coin lands heads, and value 0 if it lands tails; and a variable T which has value 0 if the coin lands heads, and value 1 if it lands tails. Observe that these variables are perfectly correlated, in that on each toss, one has value 1 if and only if the other has value 0!

CORRELATIONS EXPLAINED BY MERE COINCIDENCE:

Example: Flip one million coins, twenty times each. Since there are only slightly over one million sequences of outcomes for each coin to exhibit, the probability is very high that at least two of the coins will land the same way each time—that is, their outcomes will be perfectly correlated.

CORRELATIONS EXPLAINED BY PRE-ESTABLISHED HARMONY:

Periodic processes that happen, by chance, to have the same periodicity will be correlated. For example, suppose that there is a species of insect that spawns exactly once every four years—in, as it happens, early November. Then there will be a perfect correlation between the behavior of these insects and the presidential elections.

CORRELATIONS EXPLAINED BY DIRECT CAUSAL CONNECTION:

Any time one type of event typically causes another, a correlation will emerge. Smoking and lung cancer provide an obvious example: there is a fairly firmly established correlation between smoking when young and contracting lung cancer later in life—presumably because the first causes the second.

CORRELATIONS EXPLAINED BY COMMON CAUSE:

Correlations can be explained by a particular kind of *indirect* causal connection: When one type of event typically has two characteristic effects, these effects will be correlated with each other. For example, there is a fairly well-established correlation between smoking *now* and contracting lung cancer *now*—not because smoking right now has any chance of immediately giving you lung cancer, but because it is a sign that you have smoked a lot *previously*—which does have a decent chance of giving you lung cancer.

CORRELATIONS EXPLAINED BY FUNDAMENTAL PHYSICAL LAW:

Consider atoms. There is a tight correlation between the number of electrons an atom contains and the average energy of these electrons. Our best explanation of this correlation goes (roughly) like this: It is physically impossible for two electrons in an atom to occupy the same quantum-mechanical state, and when lower-energy states get filled up only higher energy states remain.