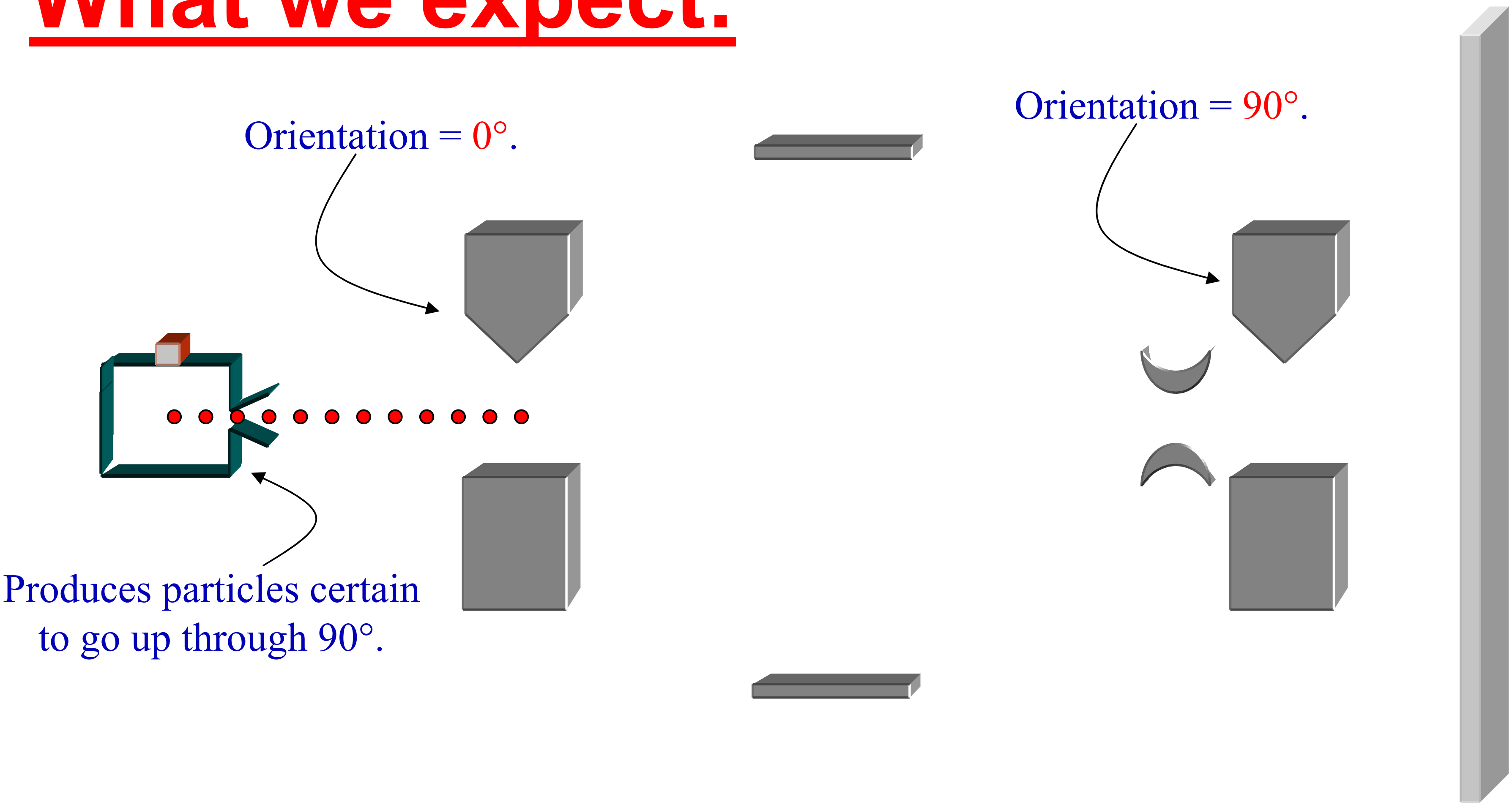
Philosophy of QM 24.111

Sixth lecture.

THE TWO-PATH EXPERIMENT—
What we expect:



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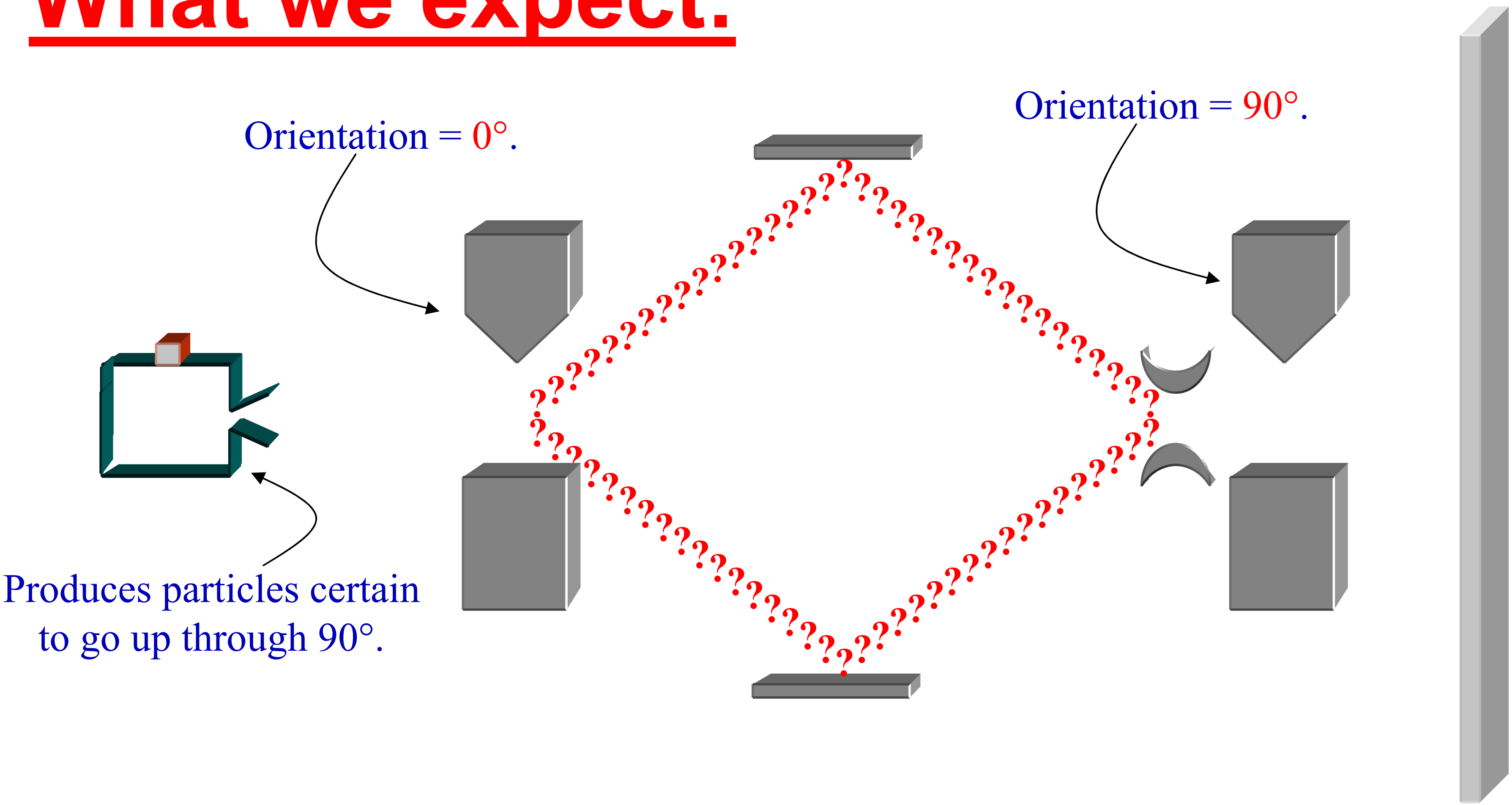

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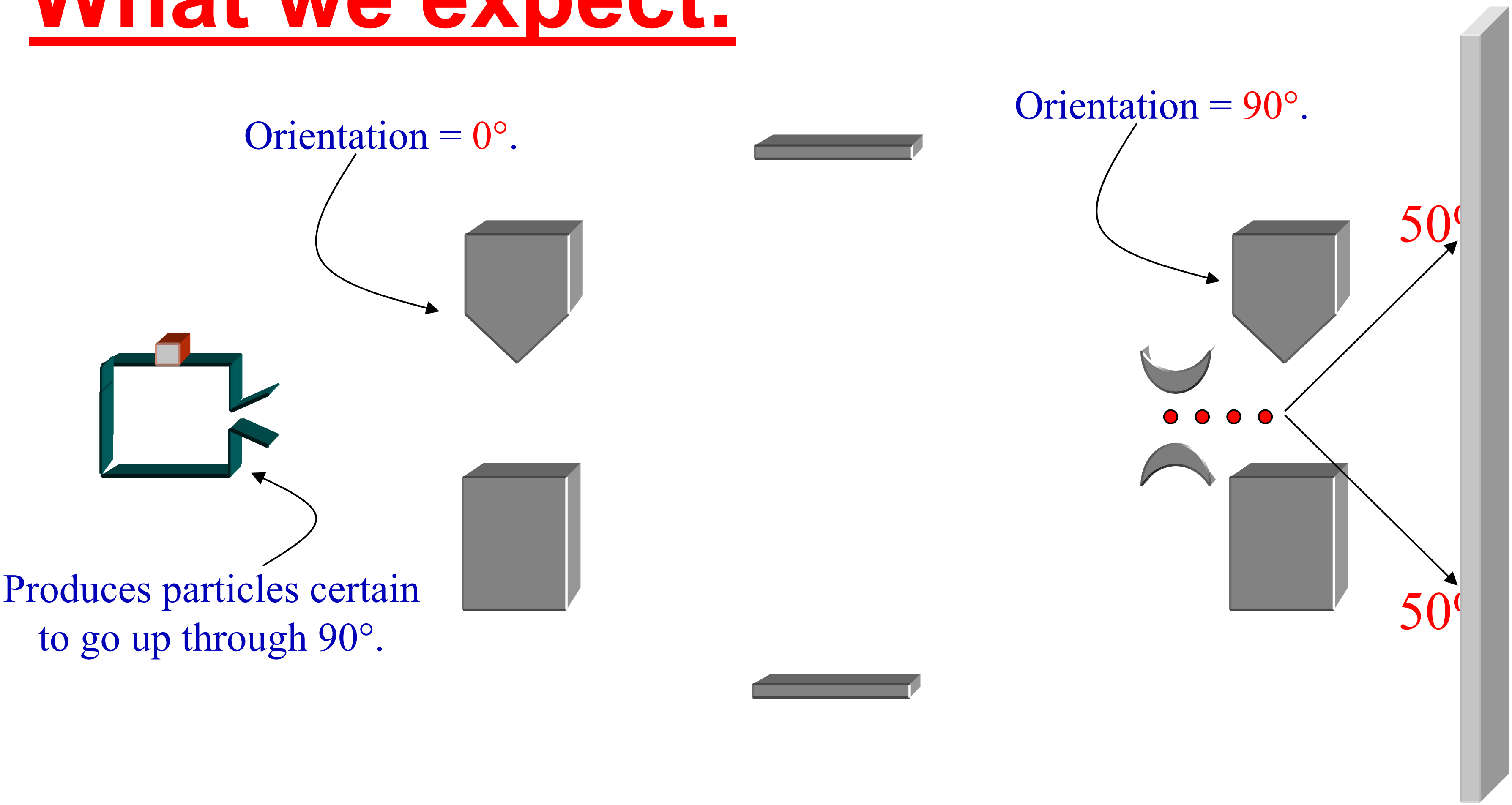
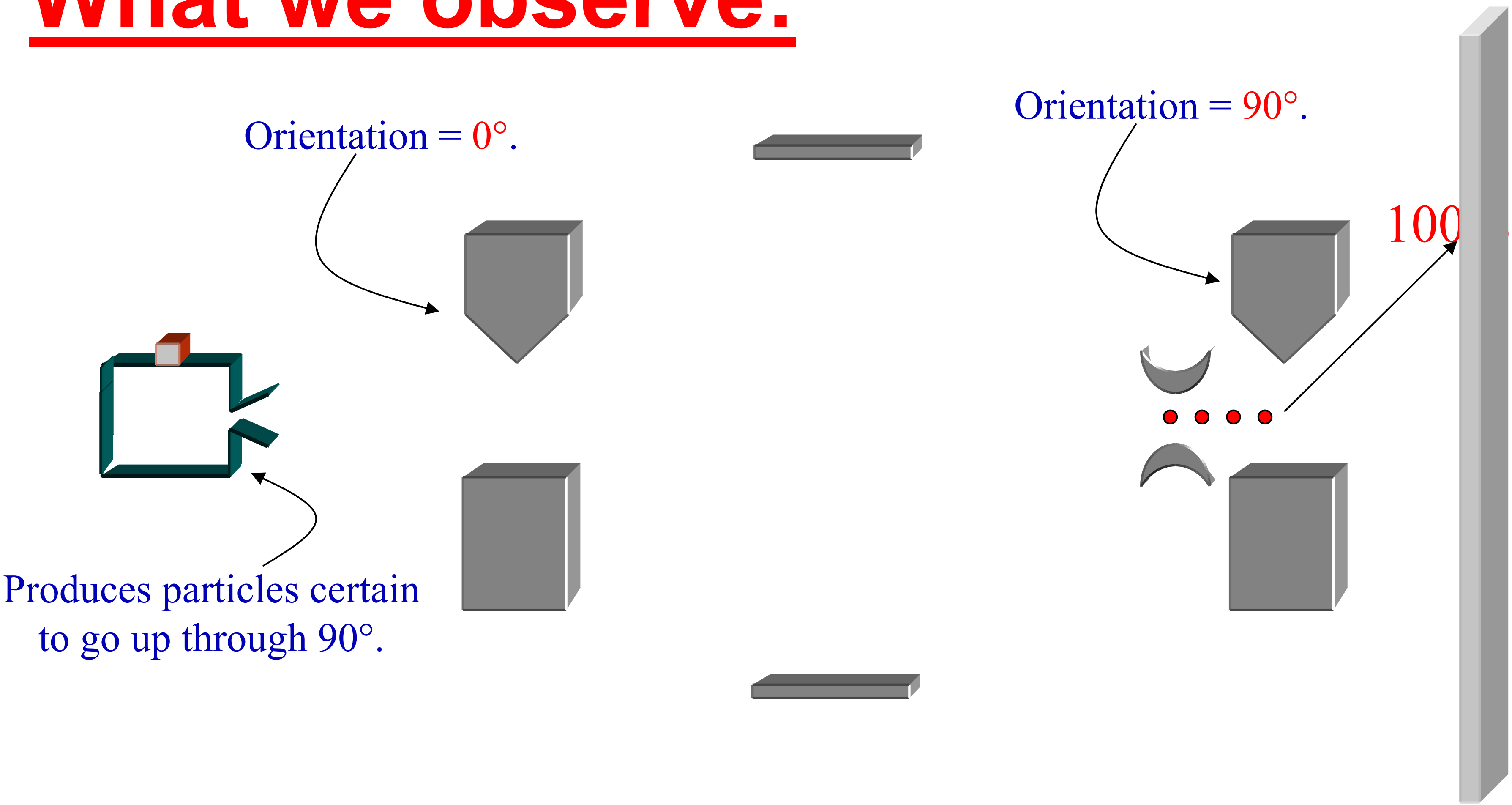
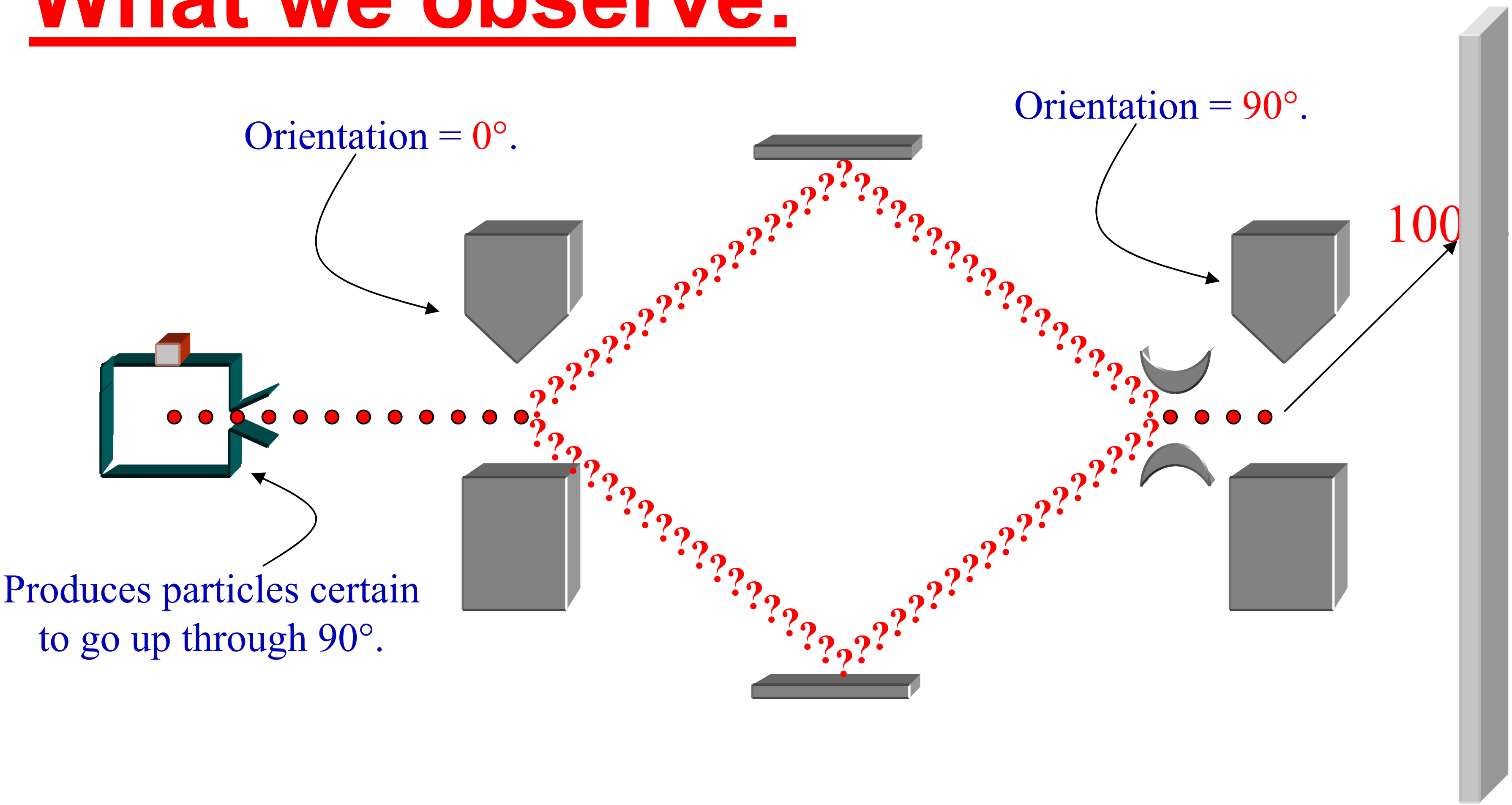


THE TWO-PATH EXPERIMENT—
What we observe:



THE TWO-PATH EXPERIMENT—
What we observe:



TWO PROBLEMS:

Our examination of the two-path experiment left us with two different problems:

• What is the particle doing when we do not observe which path it follows? Does it somehow follow both paths? Neither path?

• How can we construct a theory that will give us the right prediction?

We will now focus on the *second* problem—
it is much easier than the first!

THE CENTRAL IDEA:

We will use vector spaces to represent

- the physical states of systems of particles;
- the experiments we can perform on these systems.

WARNING: In the literature, these experiments are almost always called "measurements". Be careful of this word's connotations!!!

SPIN MEASUREMENTS

First approximation (for the spin state of a single particle):

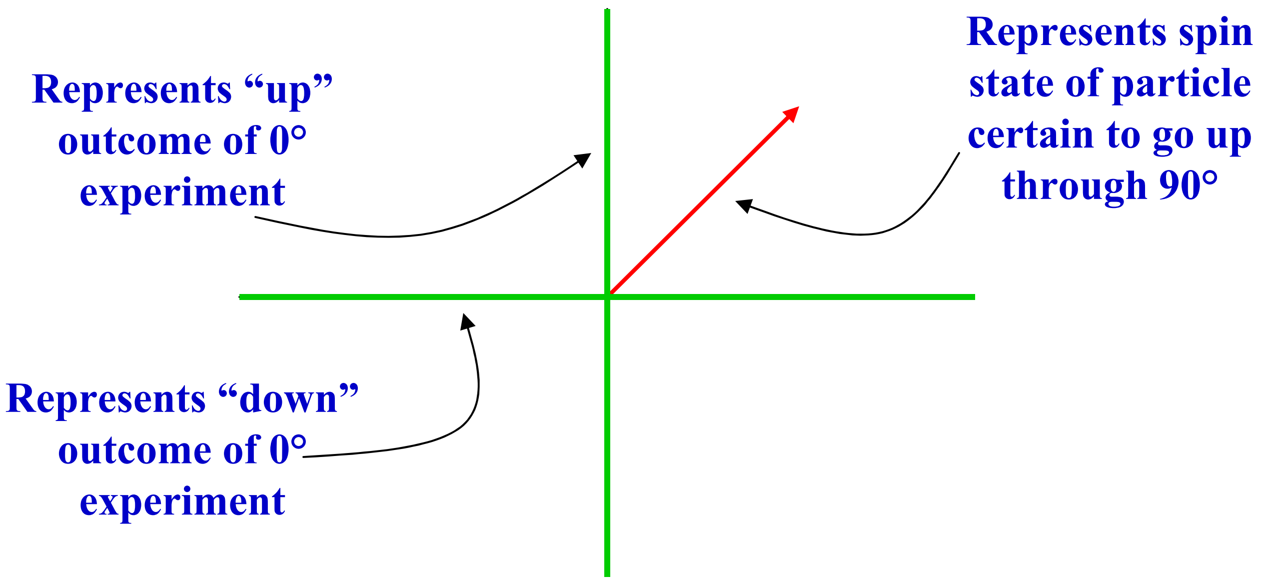
Vector space used to represent states and experiments is $\mathbf{R}^2$.

Unit vectors represent different possible spin states.

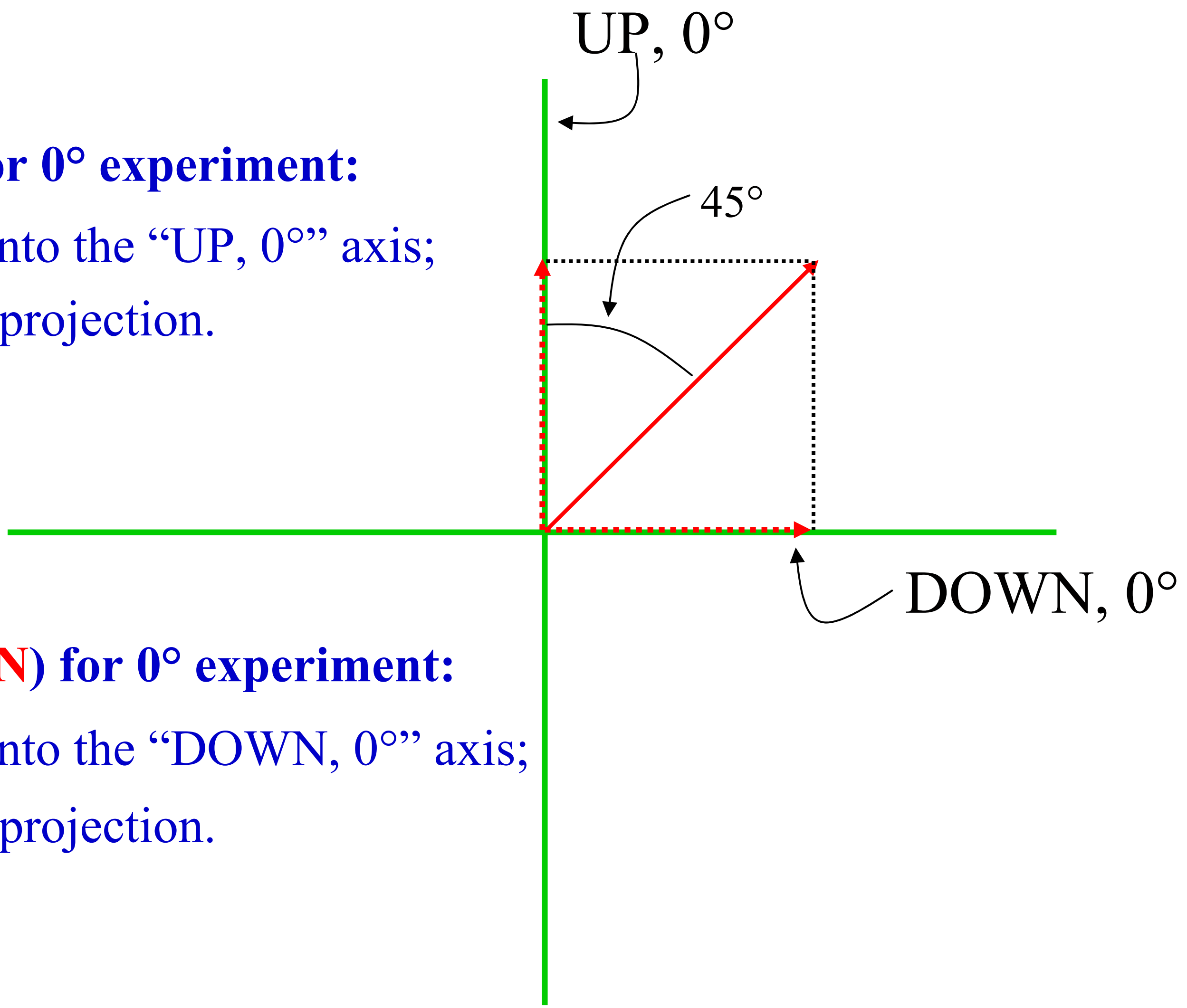
Orthogonal axes represent different possible experiments.

One axis corresponds to the "up" outcome and the other to the "down" outcome.

Example:



THE STATISTICAL ALGORITHM



To calculate Prob(UP) for 0° experiment:

Project the state-vector onto the “UP, 0°” axis;

Square the length of this projection.

Answer: $\cos^2(45°) = 1/2$.

To calculate Prob(DOWN) for 0° experiment:

Project the state-vector onto the “DOWN, 0°” axis;

Square the length of this projection.

Answer: $\sin^2(45°) = 1/2$.

Pythagoras’ theorem guarantees that these probabilities will sum to 1.

CAUTIOUS INSTRUMENTALISM

We will, for the time being, adopt a cautiously instrumentalist approach to this way of representing physical states. That is, all we take it to “mean”, when we say that the spin state of a particle is represented by such-and-such a vector, is that this vector can be “plugged into” the statistical algorithm so as to yield the correct probabilities for outcomes of spin measurements performed on that particle.

DERIVING THE COS^2 LAW

If a particle is certain to go up through a magnet with orientation θ_1, then its probability for going up through a magnet with orientation θ_2 is

$$COS^2\left(\frac{\theta_1 - \theta_2}{2}\right)$$

DERIVING THE COS^2 LAW

If Prob(UP,θ_1) = 1, then Prob(UP,θ_2) = f(θ_1,θ_2).

Assumptions:

1. f depends only on the angle difference $|\theta_1 - \theta_2|$.
2. f is the same with UP replaced by DOWN.
3. f(0) = 1 (of course); f(π) = 0.
4. f is monotonically decreasing.
5. f is continuous.
6. The values of f are determined in accordance with the statistical algorithm.

FIRST STEP:

If Prob(UP,θ_1) = 1, then Prob(UP,θ_2) = f(θ_1,θ_2).

Suppose Prob(UP,0) = 1.

Then Prob(UP,π) = 0. (by assumptions 1&3)

So Prob(DOWN,π) = 1. (either DOWN or UP must happen)

And Prob(UP,θ) = f(θ). (by 1)

And Prob(DOWN,θ) = f(π - θ). (by 1 and 2)

But Prob(UP,θ) + Prob(DOWN,θ) = 1.

$\therefore$ f(π - θ) = 1 - f(θ).

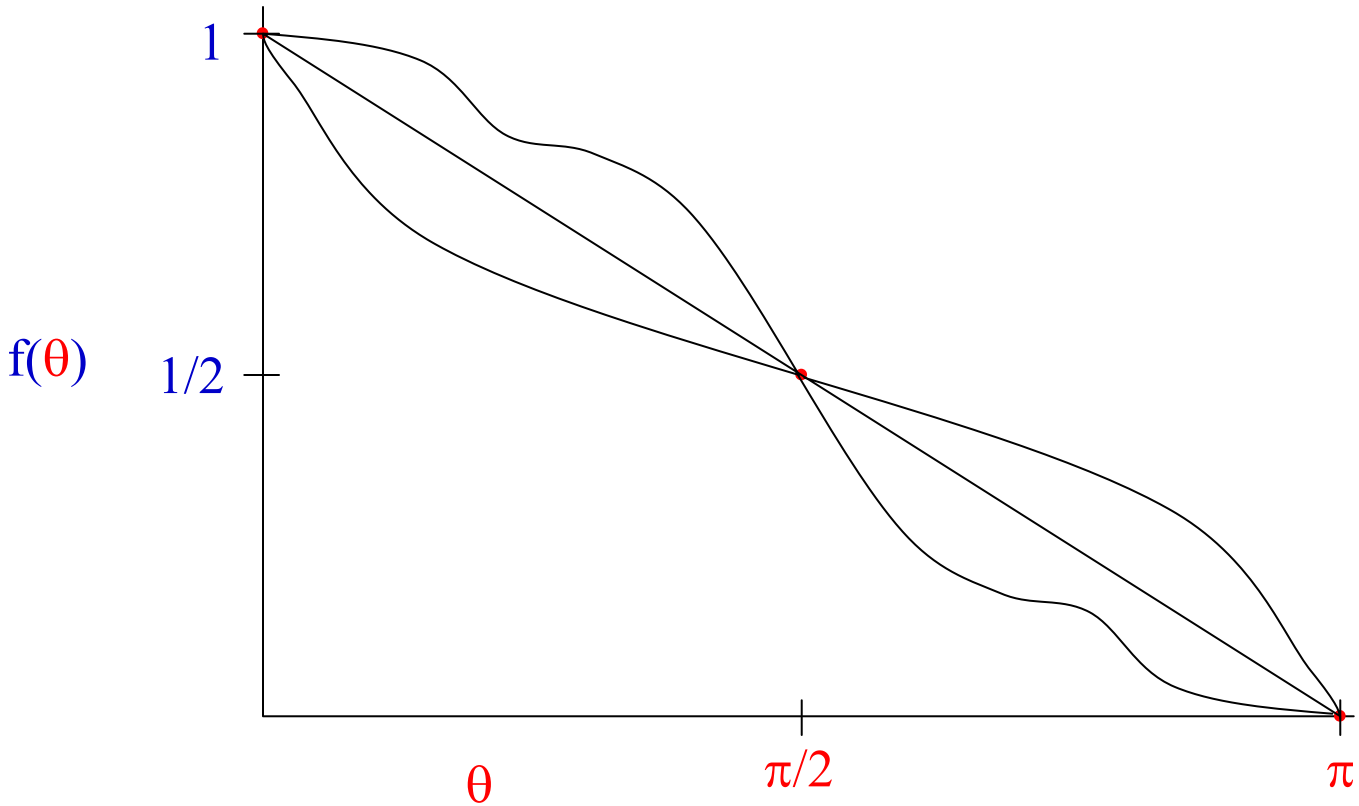
$\therefore$ f(π/2) = 1/2.

1. f depends only on $|\theta_1 - \theta_2|$.
2. f is the same with UP replaced by DOWN.
3. f(0) = 1; f(π) = 0.
4. f is monotonically decreasing.
5. f is continuous.

WHAT THIS SHOWS:

Assumptions 1 - 5 constrain f only this much:

f must be a continuous, monotonically decreasing function with the values $f(0) = 1$, $f(\pi/2) = 1/2$, $f(\pi) = 0$; and $f(\pi - \theta) = 1 - f(\theta)$.

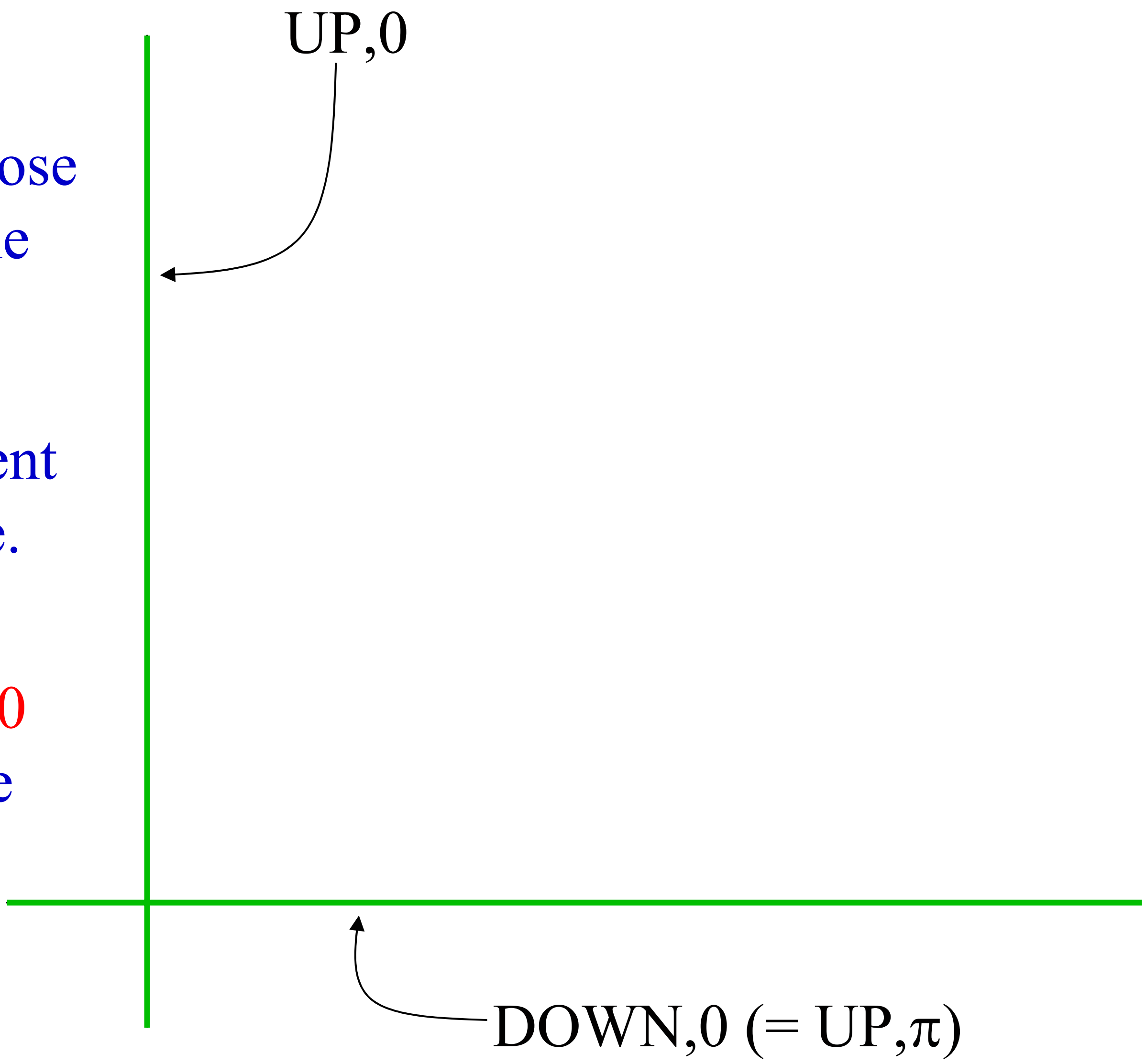


ADDING ASSUMPTION 6:

First, we arbitrarily choose an axis to represent the UP,0 outcome…

…and an axis to represent the DOWN,0 outcome.

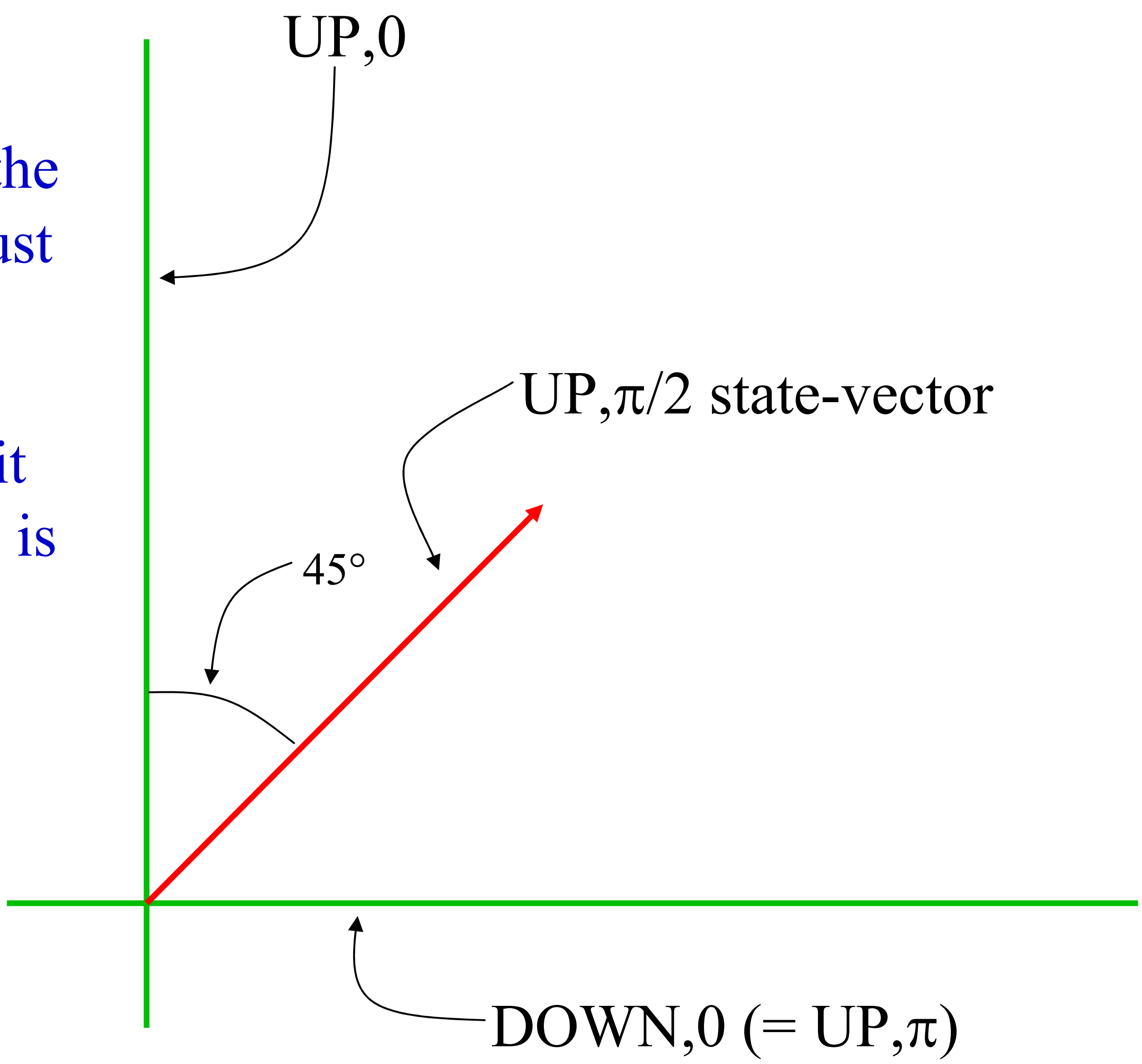
Note that the DOWN,0 axis is the same as the UP,π axis.



ADDING ASSUMPTION 6:

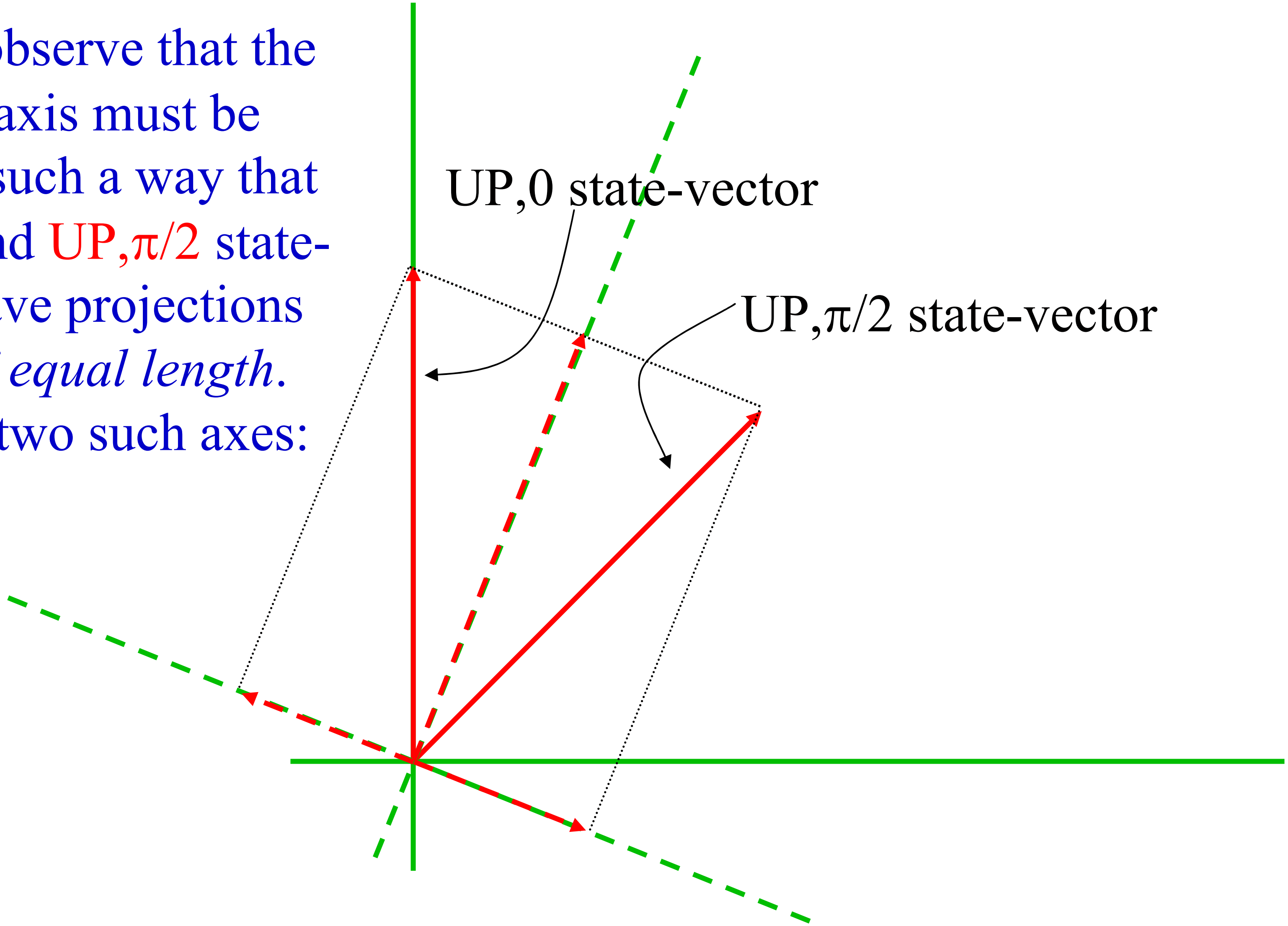
Next, we observe that the UP,π/2 state-vector must *bisect* these axes:

Notice that the angle it makes to the UP,0 axis is π/4.



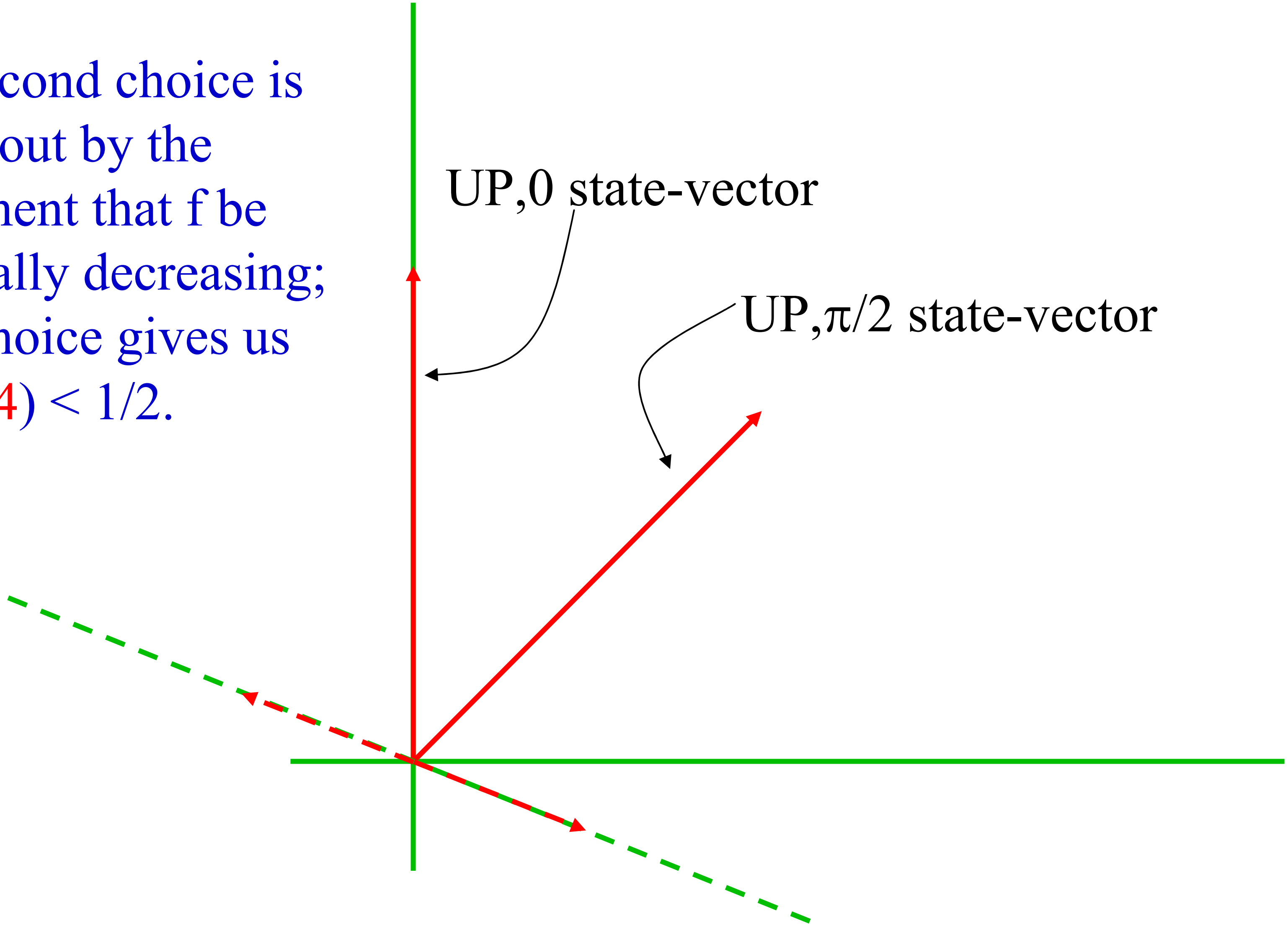
ADDING ASSUMPTION 6:

Next, we observe that the UP,π/4 axis must be chosen in such a way that the UP,0 and UP,π/2 state-vectors have projections onto it of *equal length*. There are two such axes:



ADDING ASSUMPTION 6:

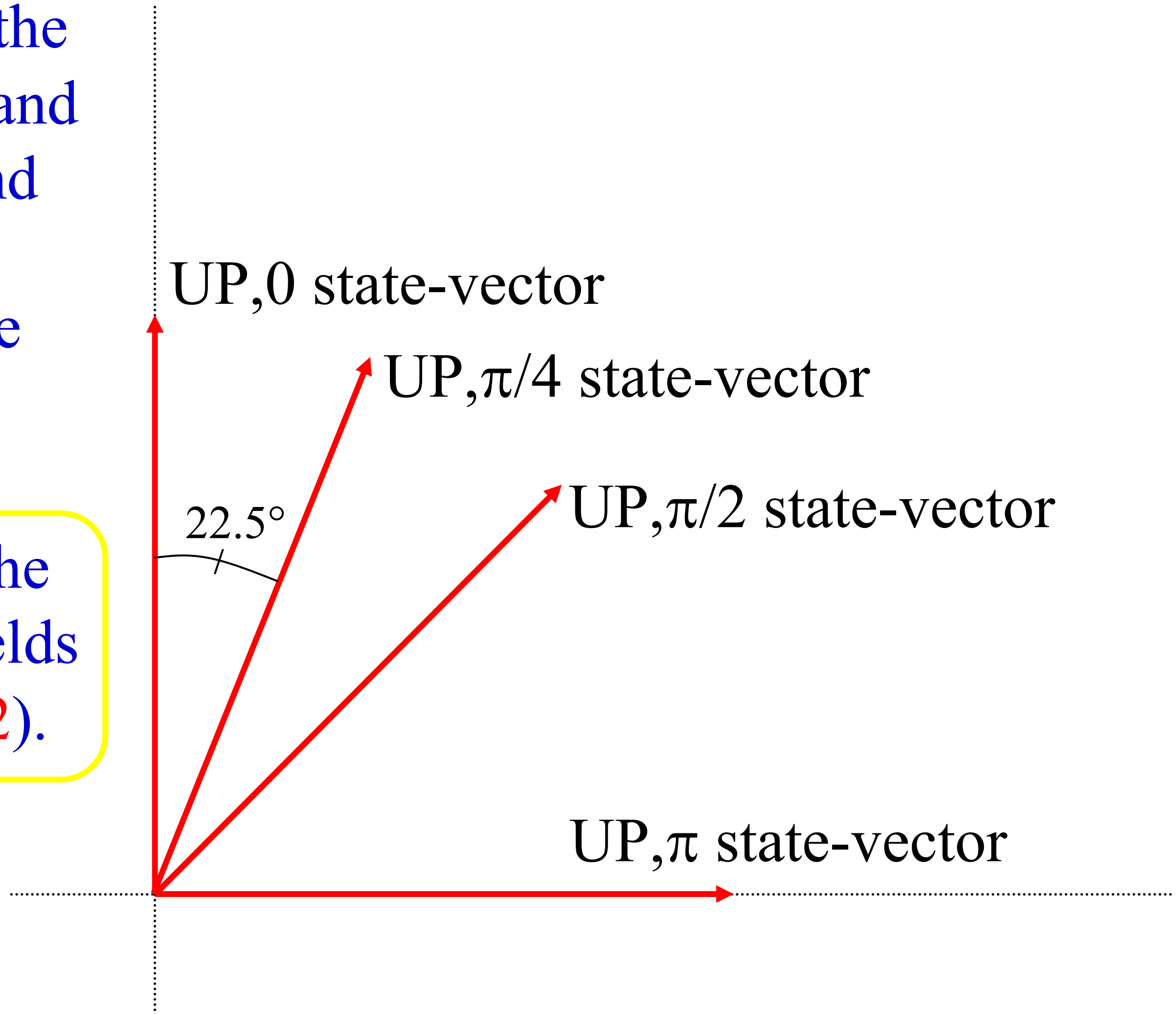
But the second choice is ruled out by the requirement that f be monotonically decreasing; for this choice gives us $f(\pi/4) < 1/2$.



ADDING ASSUMPTION 6:

So we have now fixed the UP,0, UP,π/4, UP,π/2, and UP,π state-vectors (and likewise the UP and DOWN axes for these four directions).

For these directions, the statistical algorithm yields Prob(UP,θ) = $\cos^2(\theta/2)$.



ADDING ASSUMPTION 6:

The very same reasoning can now be applied to the directions

$\theta = 3\pi/4$

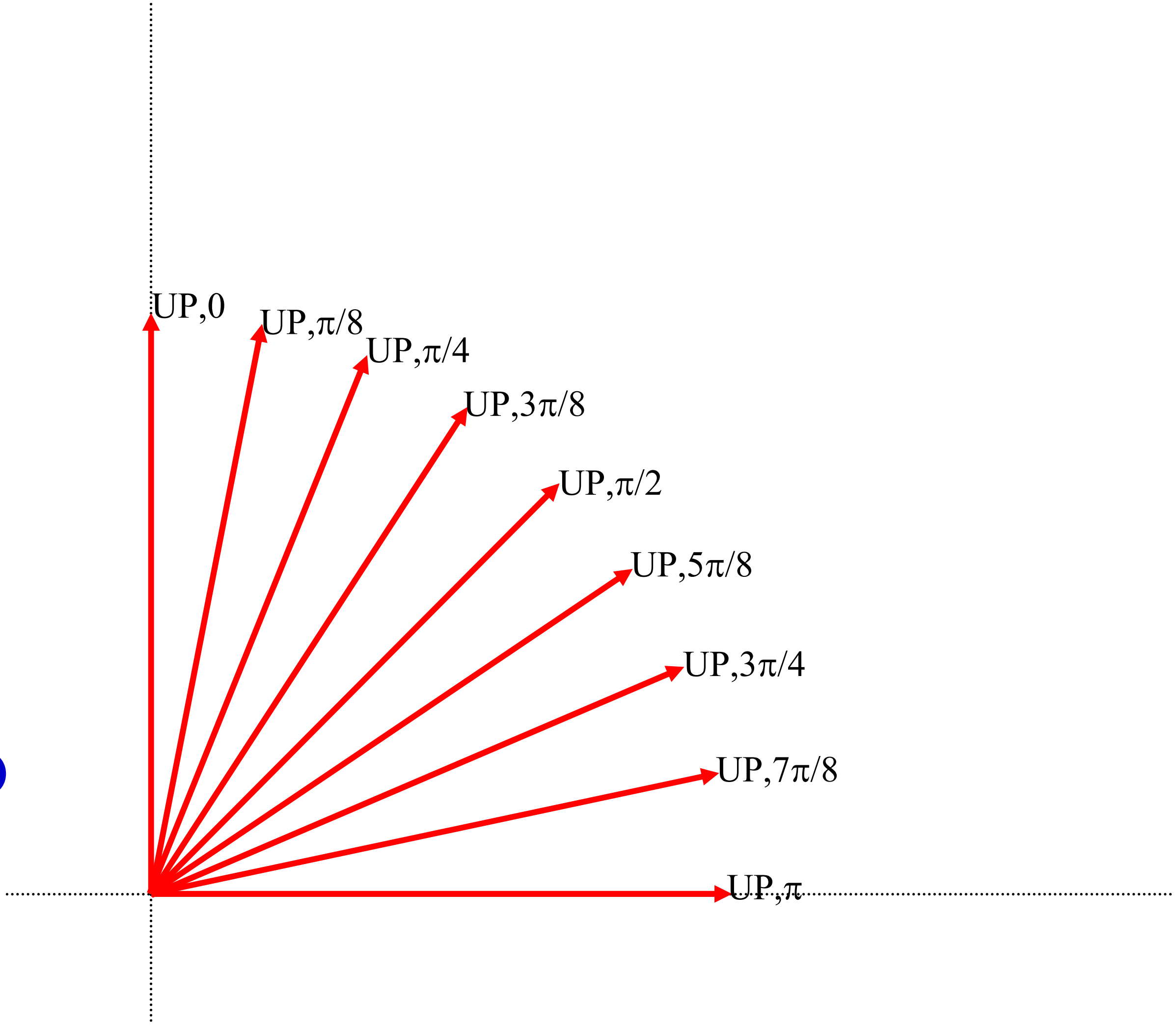
$\theta = \pi/8$

$\theta = 3\pi/8$

$\theta = 5\pi/8$

$\theta = 7\pi/8$

…etc. For each $\theta = k\pi/2^n$, the UP,θ state-vector lies at an angle of $\theta/2$ to the UP,0 axis; hence $f(\theta) = \cos^2(\theta/2)$. Since f is continuous, it follows that for <u>every</u> θ, $f(\theta) = \cos^2(\theta/2)$—i.e., we have derived the $\cos^2$-law.

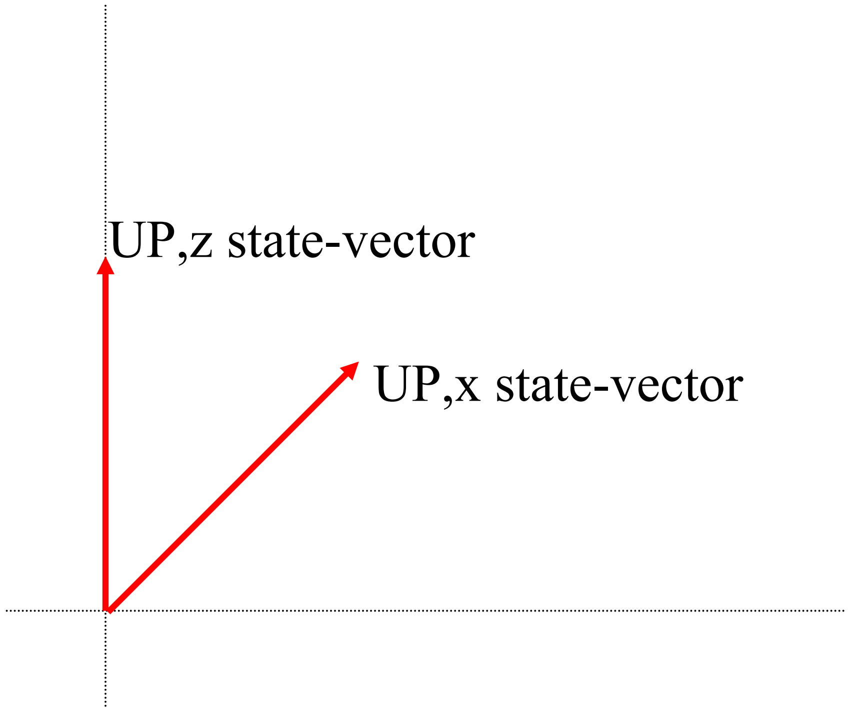


GETTING FANCIER: COMPLEX NUMBERS

It is in fact possible to measure the spin of a particle in *any* direction—not just directions confined to the x-z plane. So there is, for example, a possible spin state in which the particle is certain to go UP if spin along the y-direction is measured.

For such a particle, Prob(UP,x) = 1/2 = Prob(UP,z). What is its state-vector?

Where can the UP,y state-vector fit?



GETTING FANCIER: COMPLEX NUMBERS

Answer: It can't. In order to represent the spin state of such a particle, we need to employ not the vector space R^2 (the vector space of pairs of real numbers), but the vector space C^2 (the vector space of pairs of *complex* numbers).

We will mostly ignore this complication throughout the course.