Philosophy of QM 24.111

Eighth lecture.

Strategy for analysis

1. Break the experimental situation up into steps.

2. At each step, look for easy cases.

3. Describe hard cases as linear combinations of easy cases.

4. Make use of the linearity of Schrödinger's Equation to "transfer" the analysis of the easy cases over to the hard cases: if

$\Phi \rightarrow \Phi'$,

and

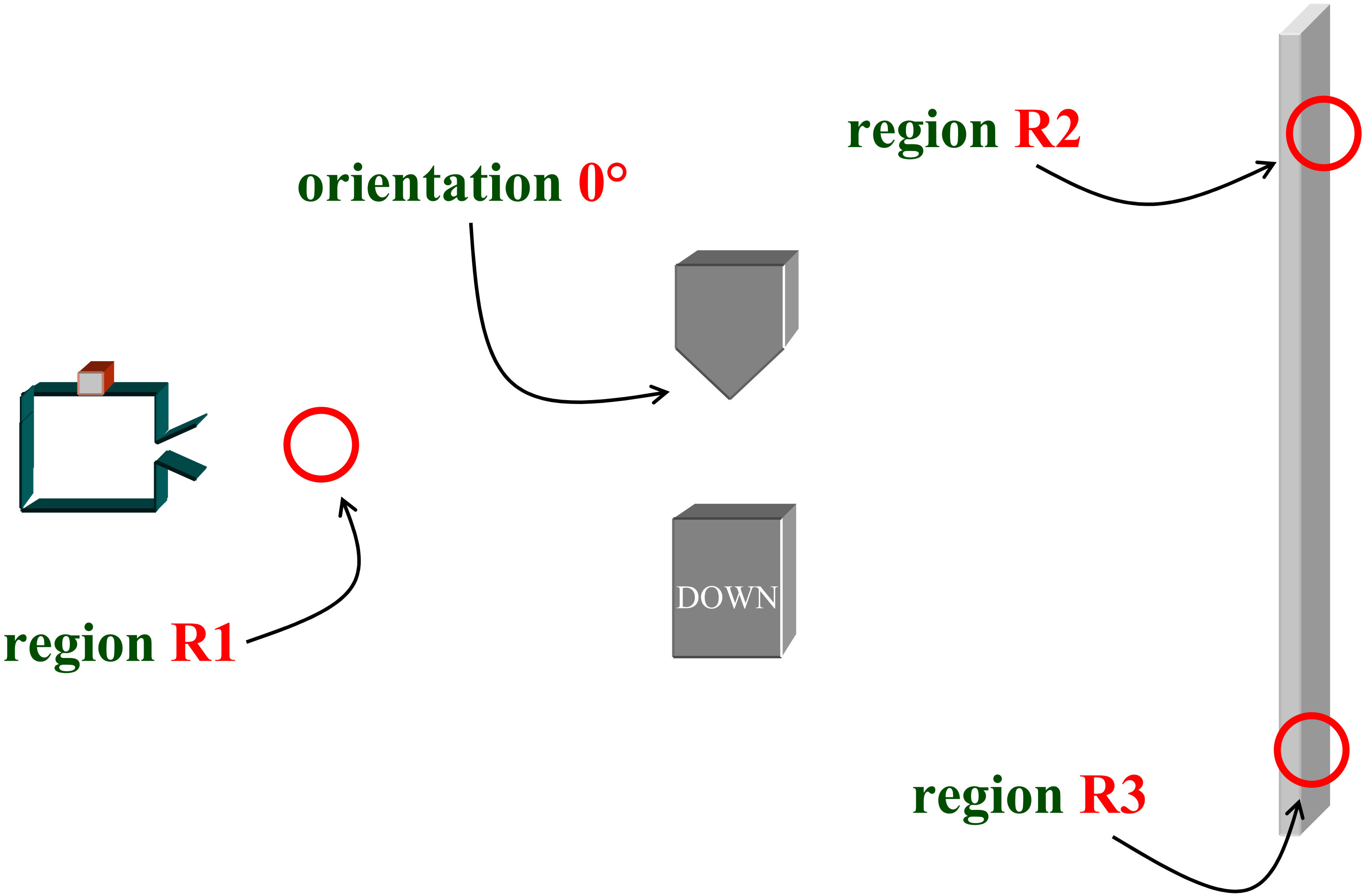
$\Psi \rightarrow \Psi'$,

then

$a\Phi + b\Psi \rightarrow a\Phi' + b\Psi'$.

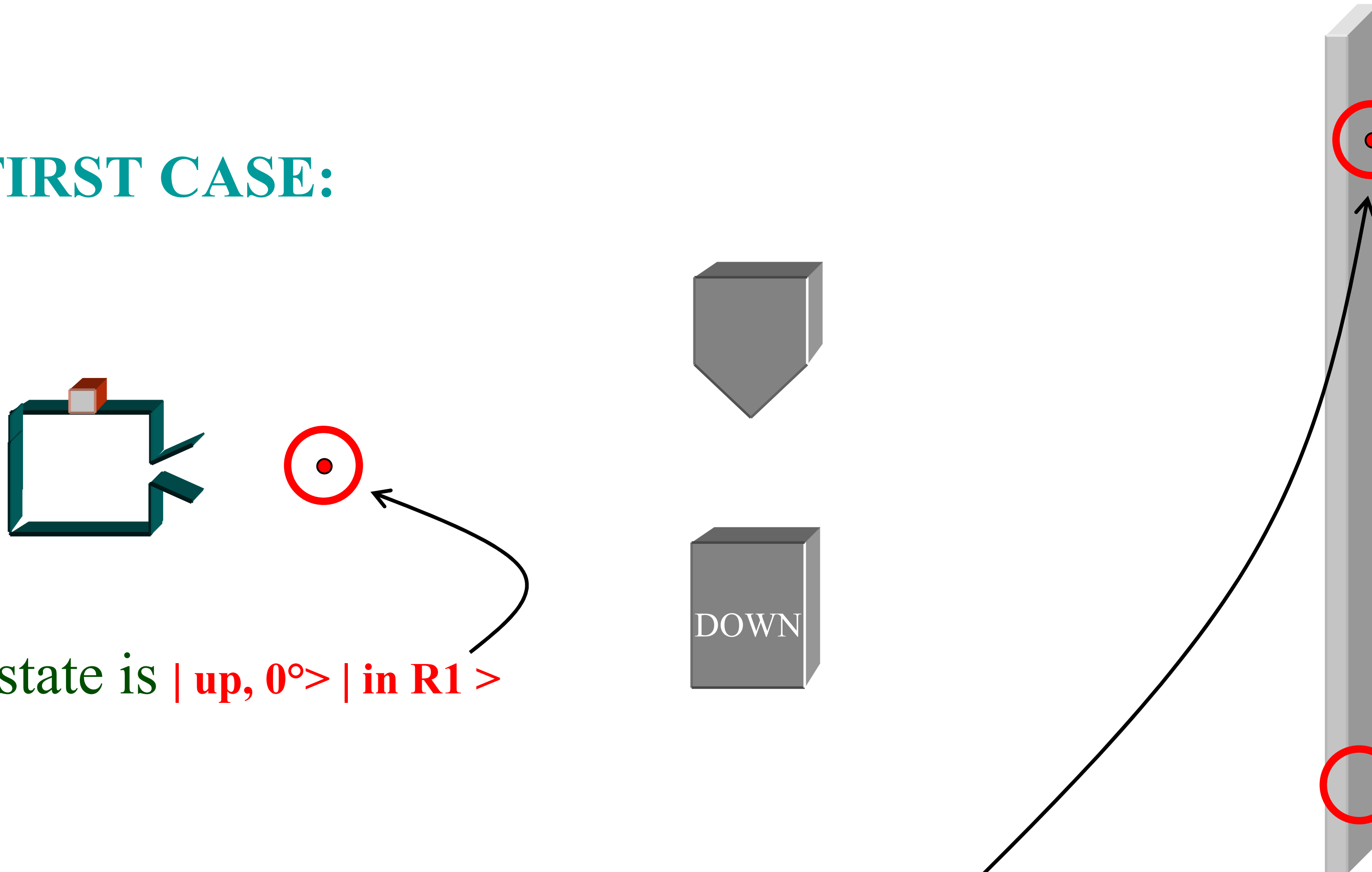
5. Make discreet use of the collapse postulate, as needed.

Spin measurements revisited



Spin measurements revisited

FIRST CASE:

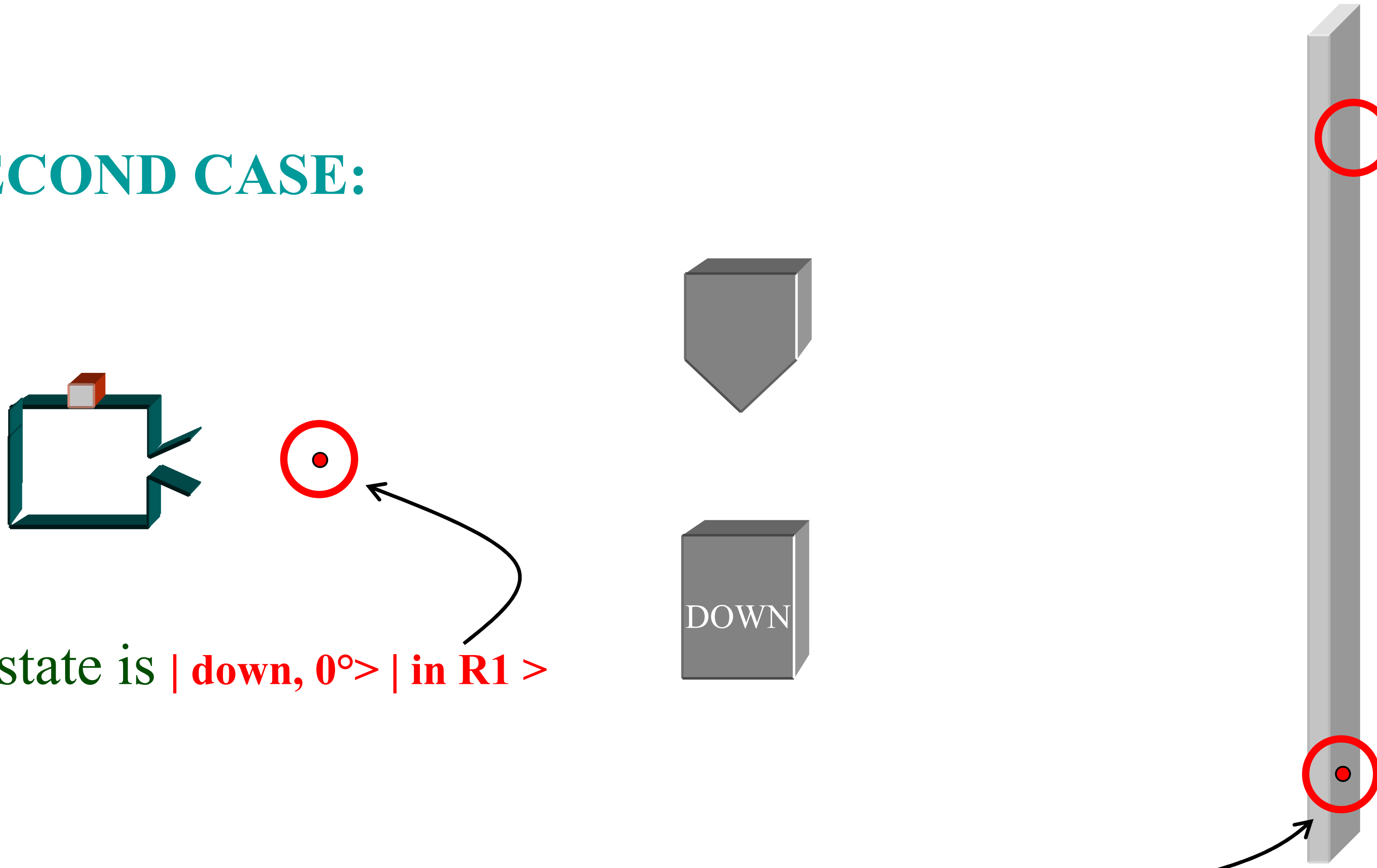


Initial state is **| up, 0°> | in R1 >**

Therefore final state must be **| up, 0°> | in R2 >**

Spin measurements revisited

SECOND CASE:

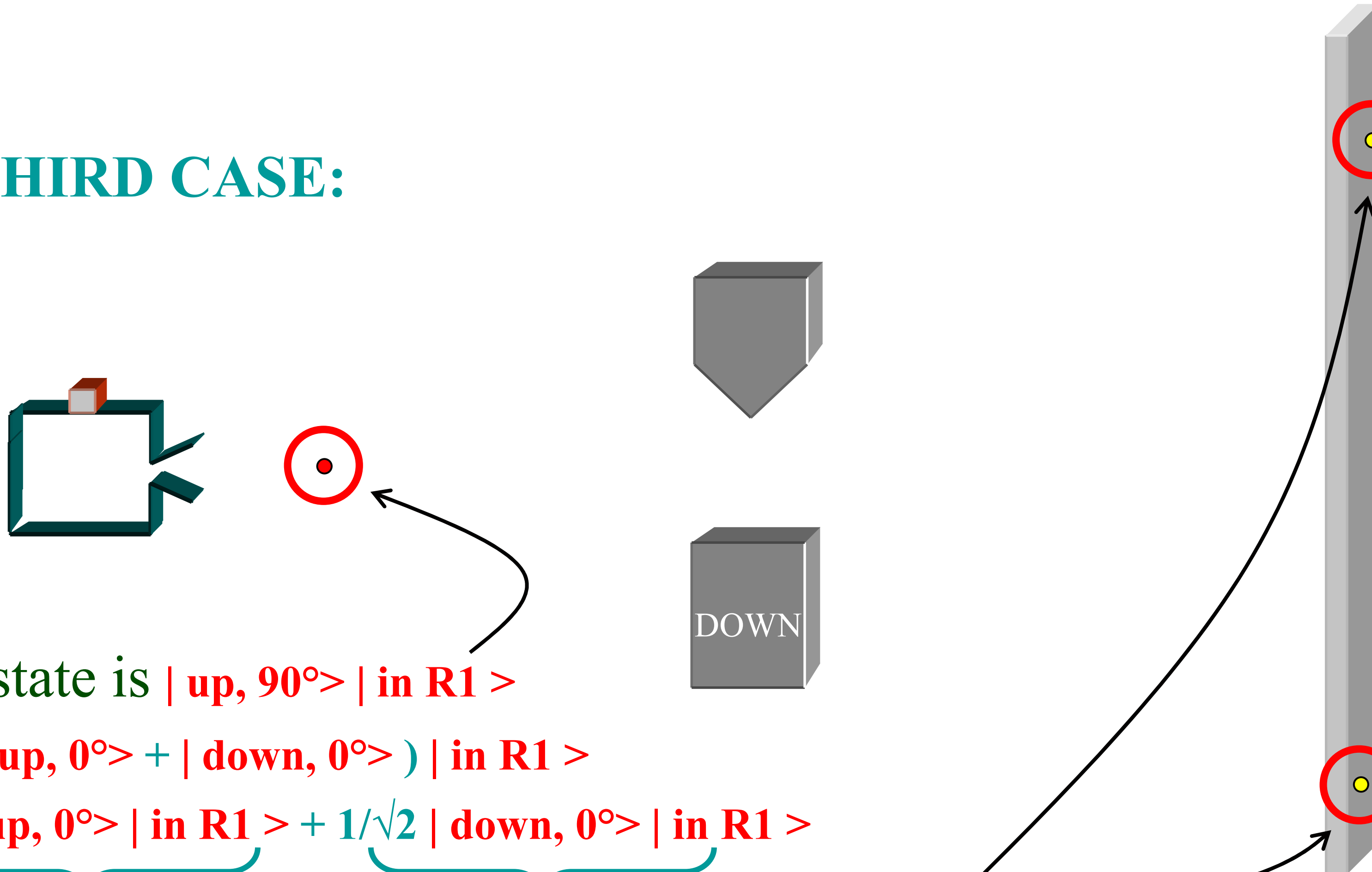


Initial state is **| down, 0°> | in R1 >**

Therefore final state must be **| down, 0°> | in R3 >**

Spin measurements revisited

THIRD CASE:



Initial state is **| up, 90°> | in R1 >**

= 1/√2 (| up, 0°> + | down, 0°>) | in R1 >

= 1/√2 | up, 0°> | in R1 > + 1/√2 | down, 0°> | in R1 >

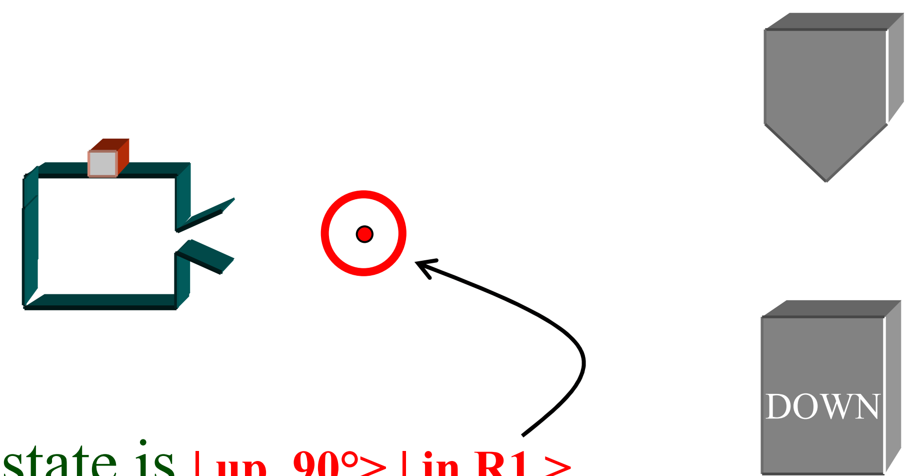
Therefore final **pre-measurement** state must be

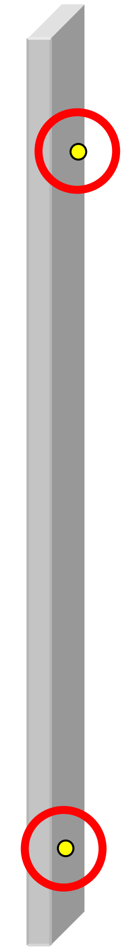
1/√2 | up, 0°> | in R2 > + 1/√2 | down, 0°> | in R3 >

Spin measurements revisited

What the hell sort of state is that???

Never mind. It will quickly “collapse” into either





Initial state is | up, 90°> | in R1 >

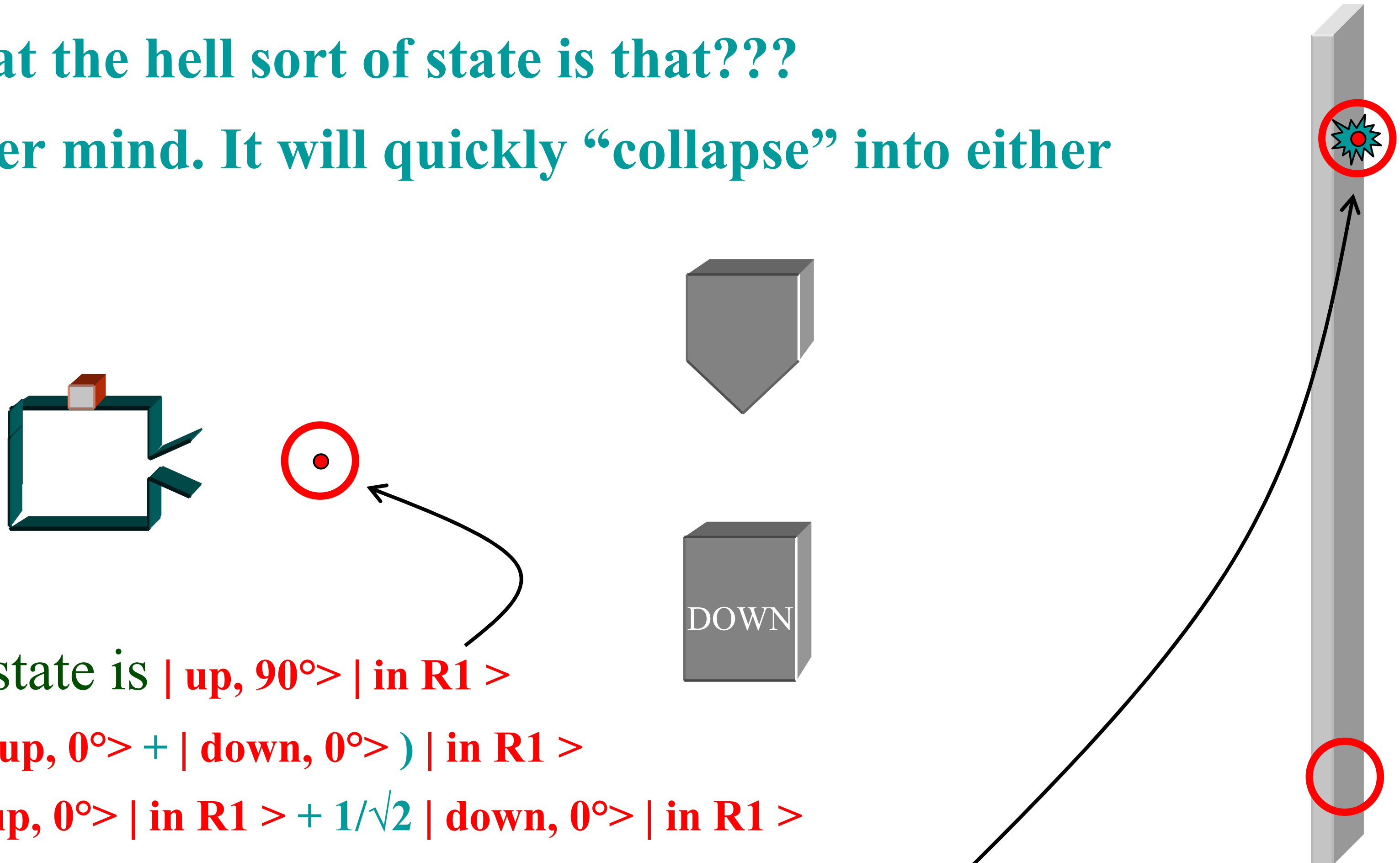
= 1/√2 (| up, 0°> + | down, 0°>) | in R1 >

= 1/√2 | up, 0°> | in R1 > + 1/√2 | down, 0°> | in R1 >

Spin measurements revisited

What the hell sort of state is that???

Never mind. It will quickly "collapse" into either



Initial state is **| up, 90°> | in R1 >**

= 1/√2 (| up, 0°> + | down, 0°>) | in R1 >

= 1/√2 | up, 0°> | in R1 > + 1/√2 | down, 0°> | in R1 >

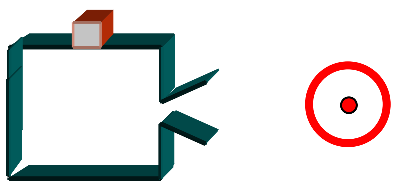
| up, 0°> | in R2 >
Prob = 1/2

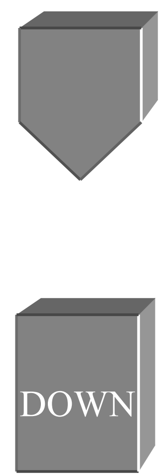
Spin measurements revisited

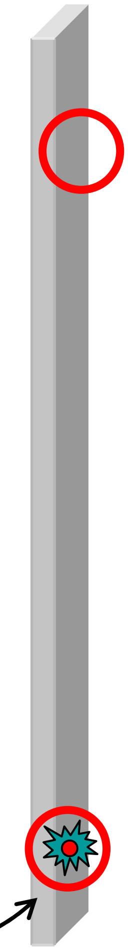
What the hell sort of state is that???

Never mind. It will quickly "collapse" into either

or







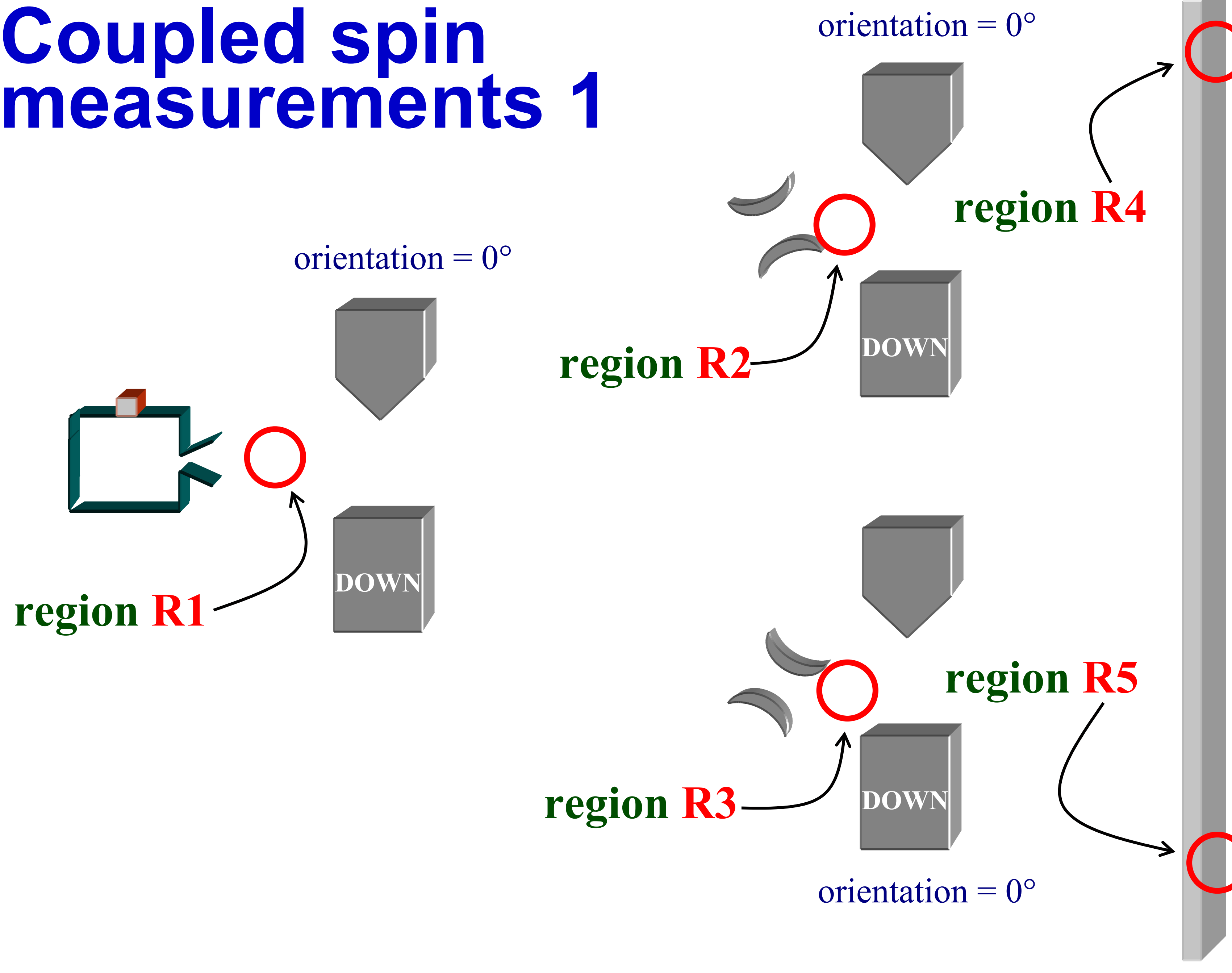
Initial state is **| up, 90°> | in R1 >**

= 1/√2 (| up, 0°> + | down, 0°>) | in R1 >

= 1/√2 | up, 0°> | in R1 > + 1/√2 | down, 0°> | in R1 >

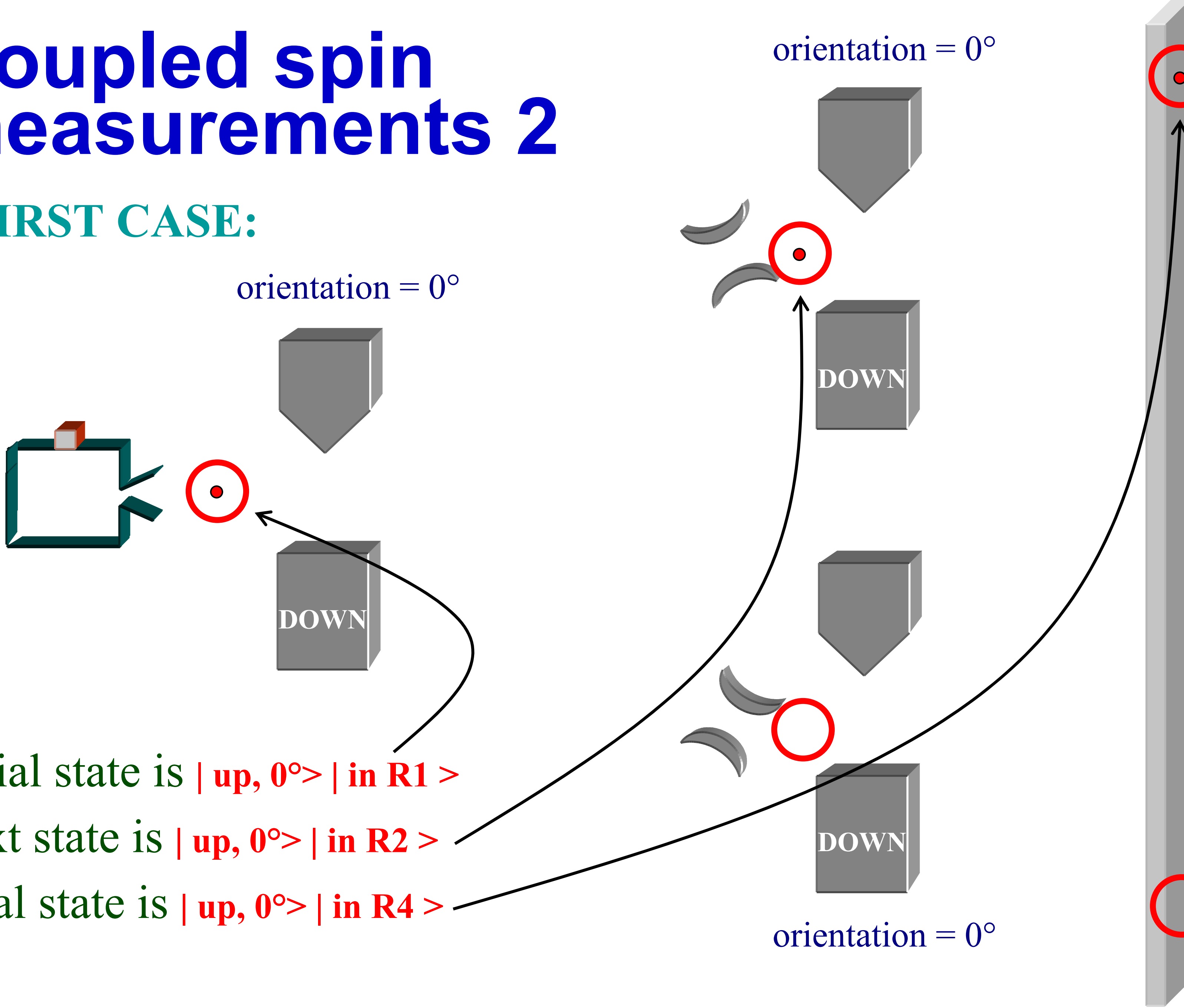
| down, 0°> | in R3 >
Prob = 1/2

Coupled spin measurements 1



Coupled spin measurements 2

FIRST CASE:



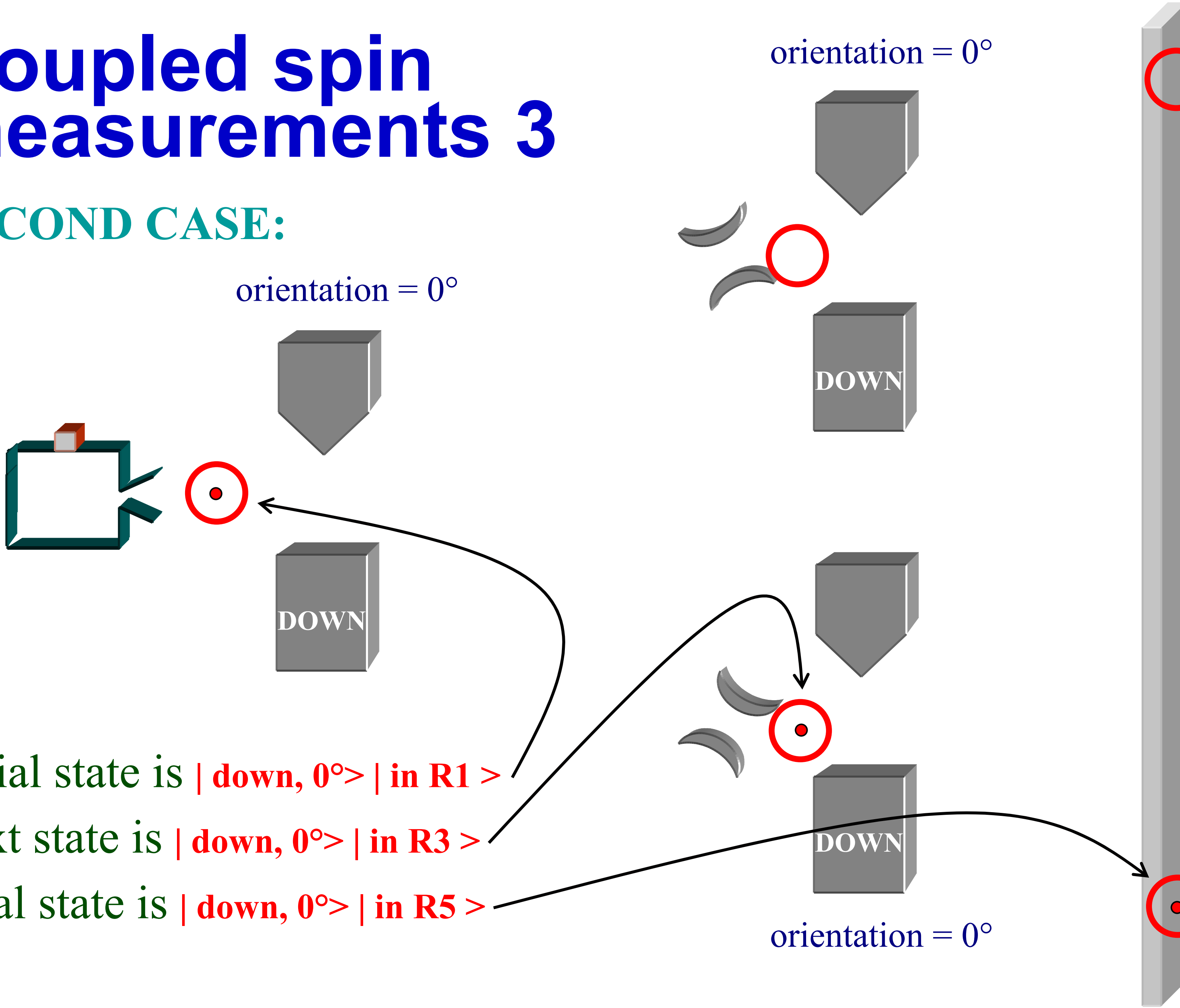
Initial state is | up, 0°> | in R1 >

Next state is | up, 0°> | in R2 >

Final state is | up, 0°> | in R4 >

Coupled spin measurements 3

SECOND CASE:



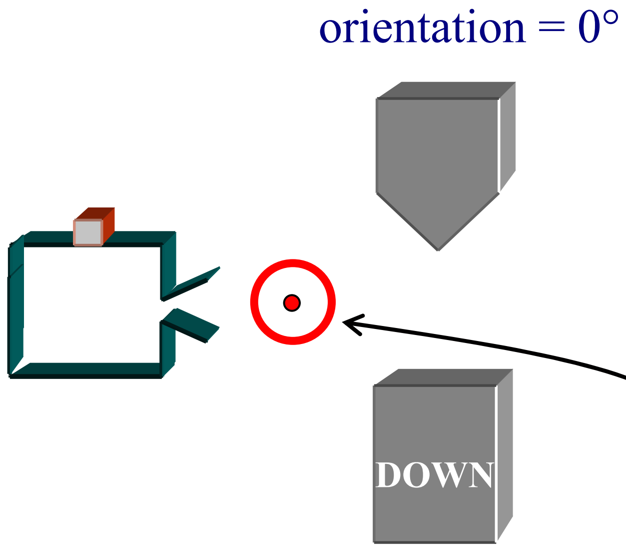
Initial state is **| down, 0°> | in R1 >**

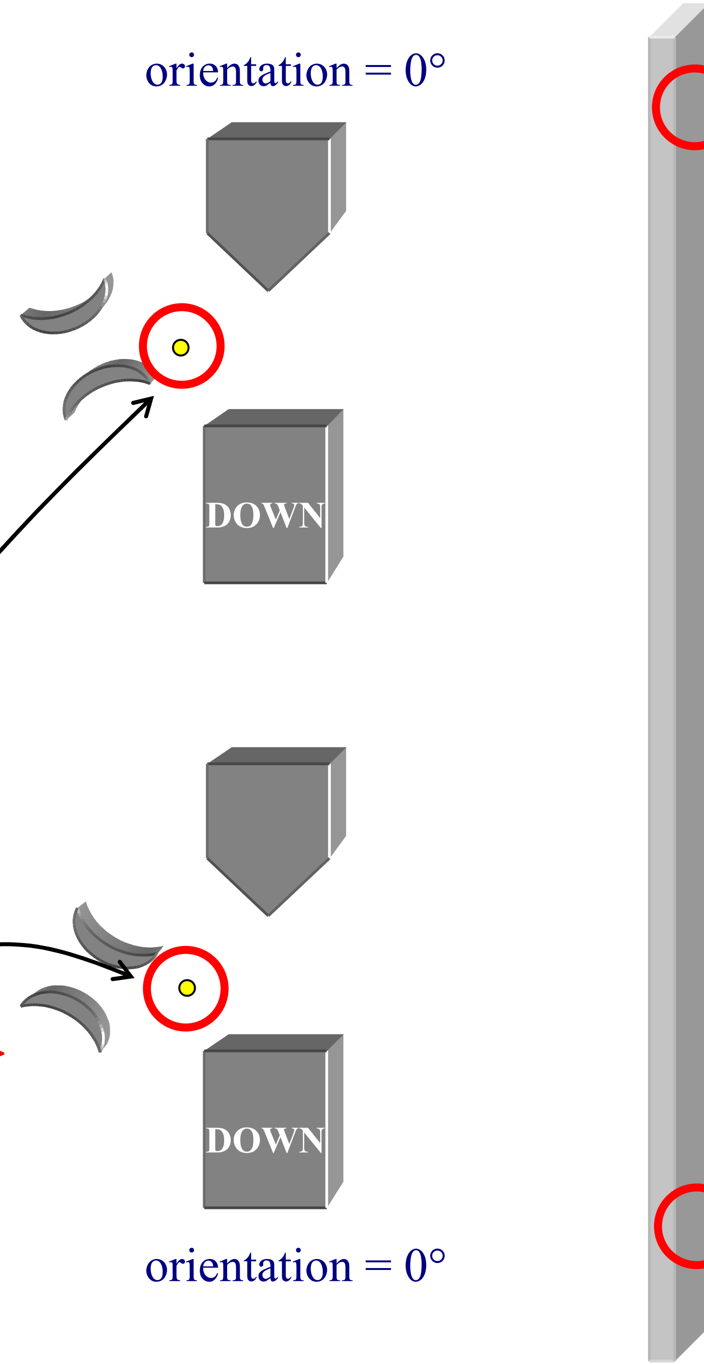
Next state is **| down, 0°> | in R3 >**

Final state is **| down, 0°> | in R5 >

Coupled spin measurements 4

THIRD CASE:





Initial state is $|\,\text{up}, 90°> |\,\text{in R1} >$

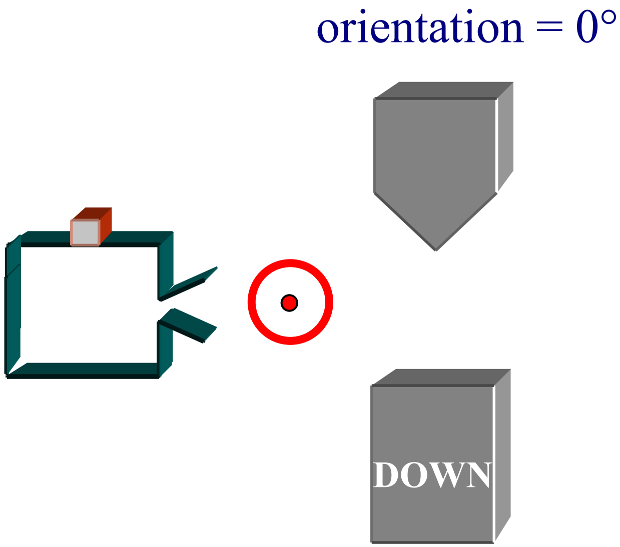
$= 1/\sqrt{2}\,|\,\text{up}, 0°> |\,\text{in R1} > + 1/\sqrt{2}\,|\,\text{down}, 0°> |\,\text{in R1} >$

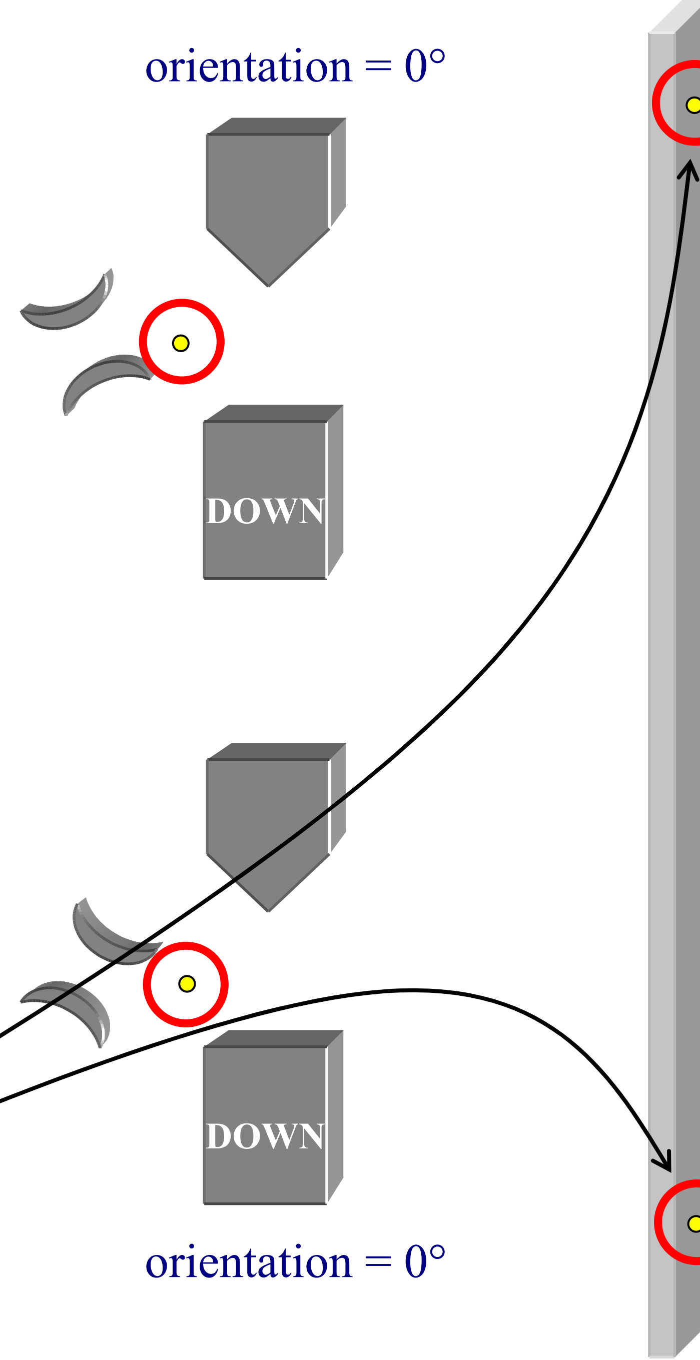
via Schrödinger

$1/\sqrt{2}\,|\,\text{up}, 0°> |\,\text{in R2} > + 1/\sqrt{2}\,|\,\text{down}, 0°> |\,\text{in R3} >$

Coupled spin measurements 5

THIRD CASE:





Then:

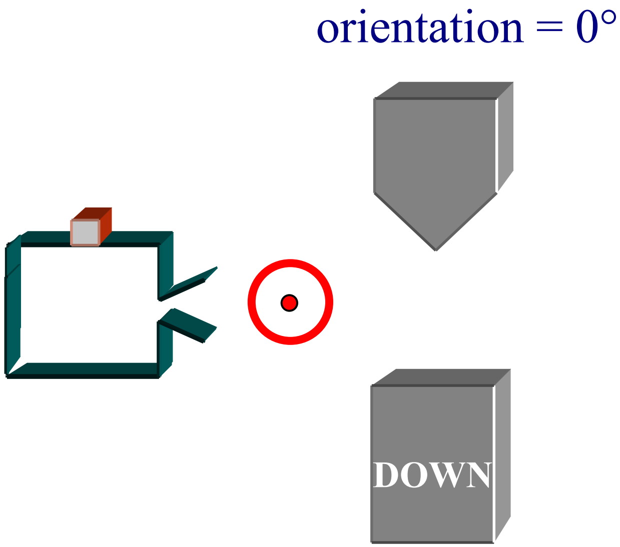
$$1/\sqrt{2}\ |\text{up}, 0°\rangle\ |\text{in R2}\rangle + 1/\sqrt{2}\ |\text{down}, 0°\rangle\ |\text{in R3}\rangle$$

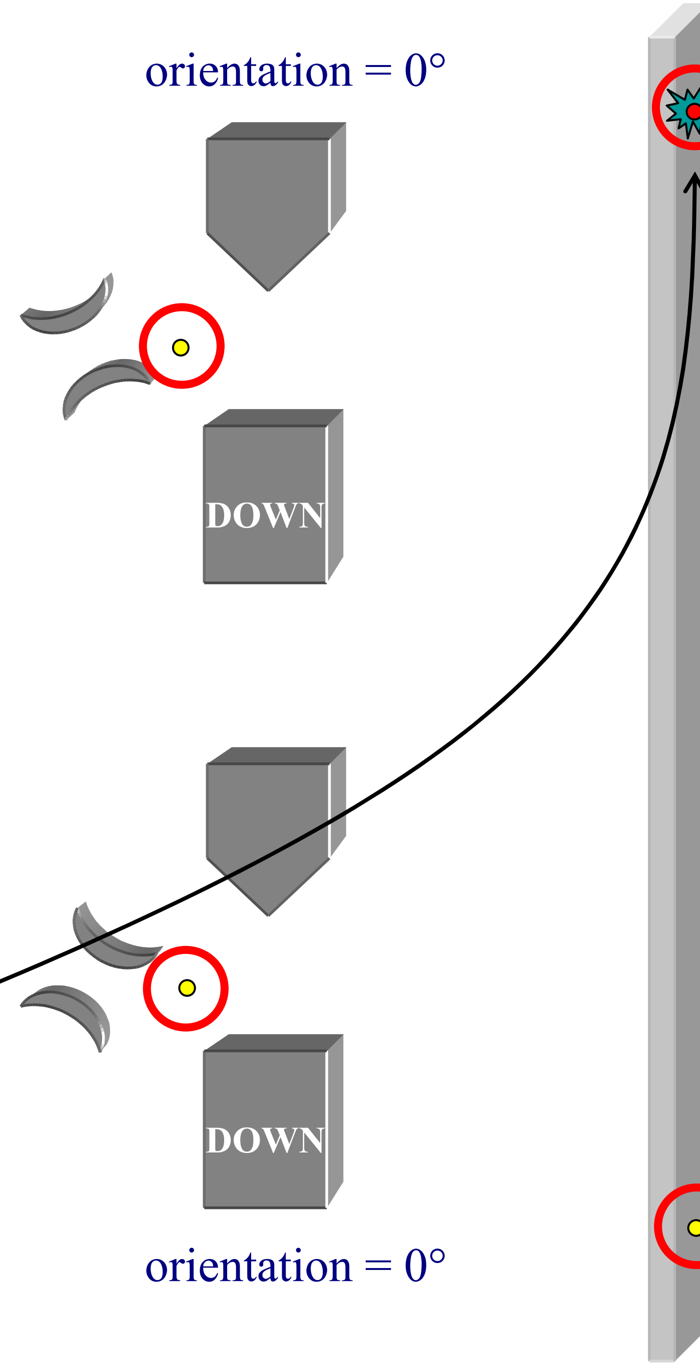
via Schrödinger ↓

$$1/\sqrt{2}\ |\text{up}, 0°\rangle\ |\text{in R4}\rangle + 1/\sqrt{2}\ |\text{down}, 0°\rangle\ |\text{in R5}\rangle$$

Coupled spin measurements 6

THIRD CASE:





Then:

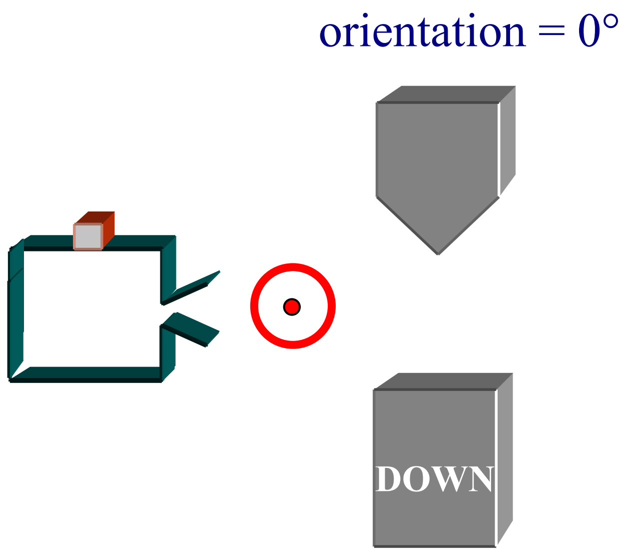
$1/\sqrt{2}$ | up, 0°> | in R4 > + $1/\sqrt{2}$ | down, 0°> | in R5 >

via "collapse"

| up, 0°> | in R4 >
(PROB = .5)

Coupled spin measurements 7

THIRD CASE:

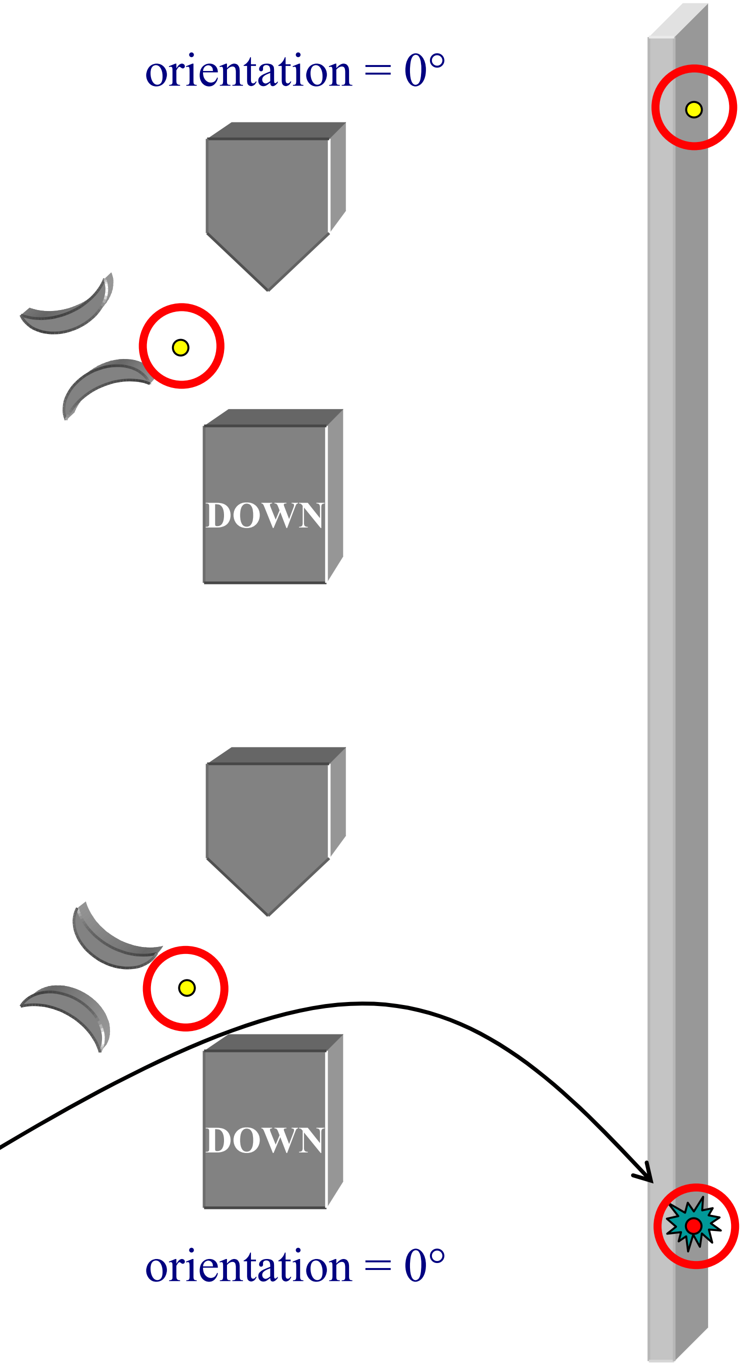


Then:

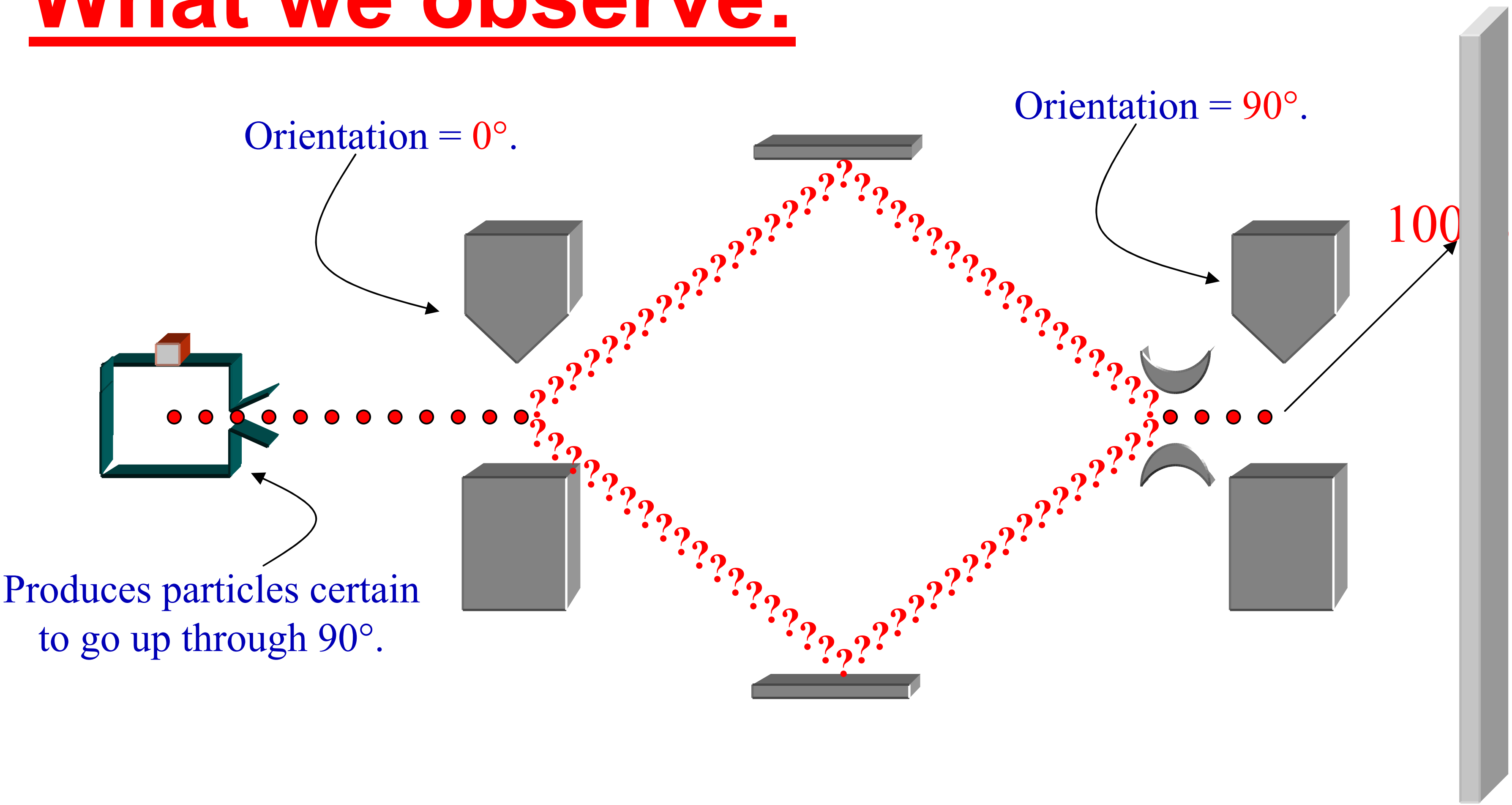
1/√2 | up, 0°> | in R4 > + 1/√2 | down, 0°> | in R5 >

via "collapse"

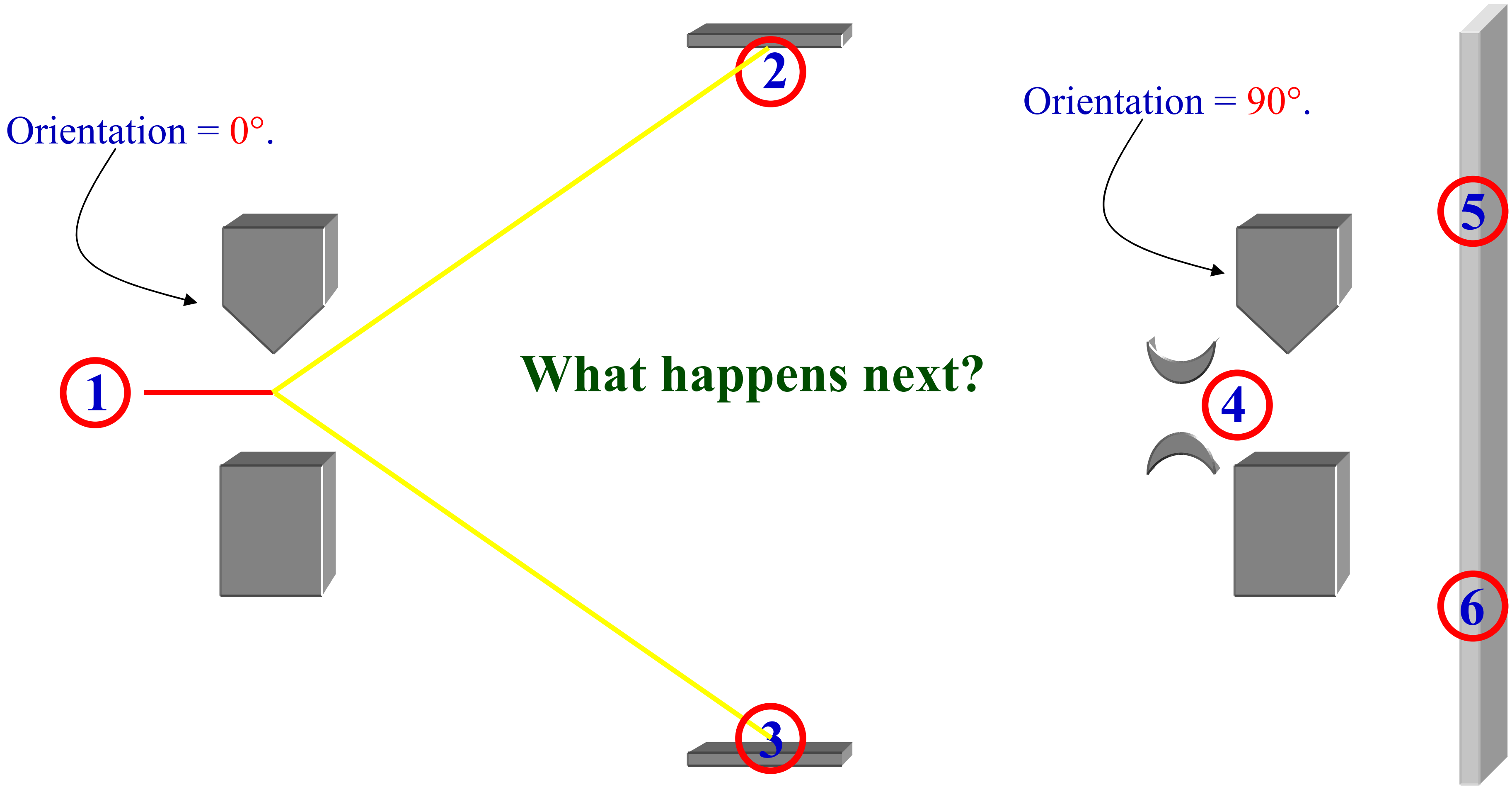
| up, 0°> | in R4 > **(PROB = .5)** **OR** **| down, 0°> | in R5 >** **(PROB = .5)**



THE TWO-PATH EXPERIMENT—
What we observe:

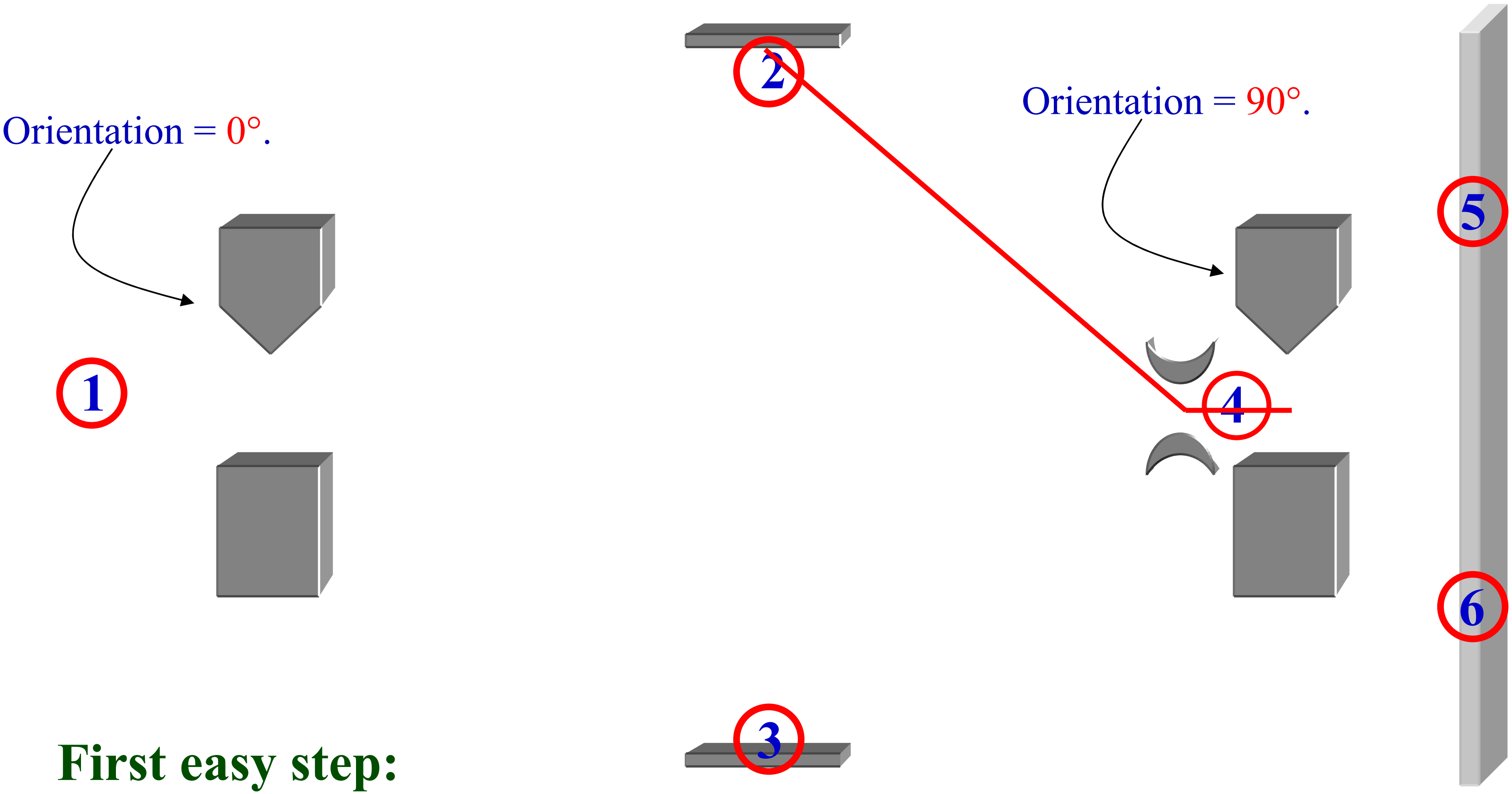


Two-path analyzed 1



| up, 90°> | in R1 > ⟶ $1/\sqrt{2}$ | up, 0°> | in R2 > + $1/\sqrt{2}$ | down, 0°> | in R3 >

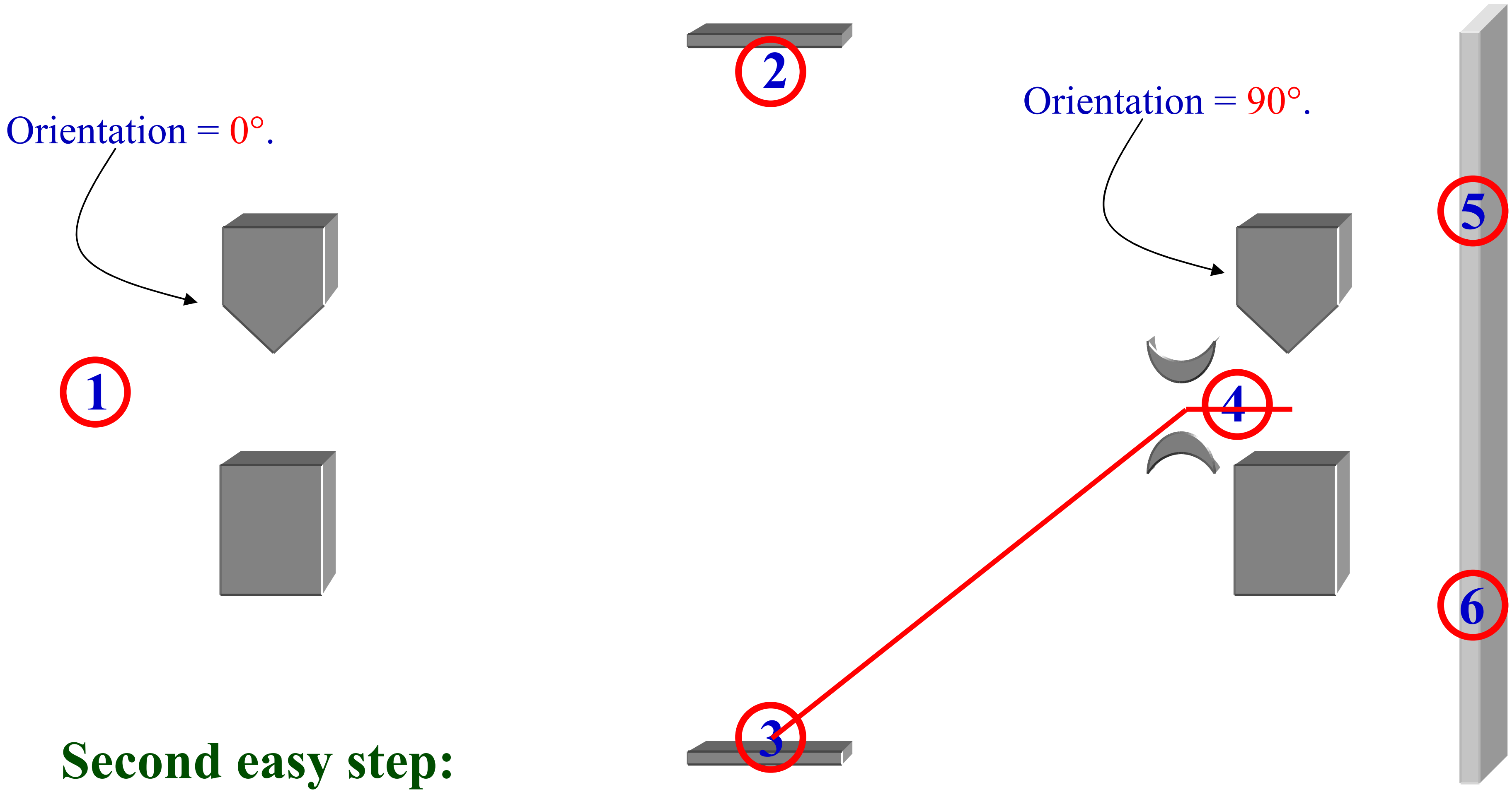
Two-path analyzed 2



First easy step:

| up, 0°> | in R2 > ⟶ **| up, 0°> | in R4 >**

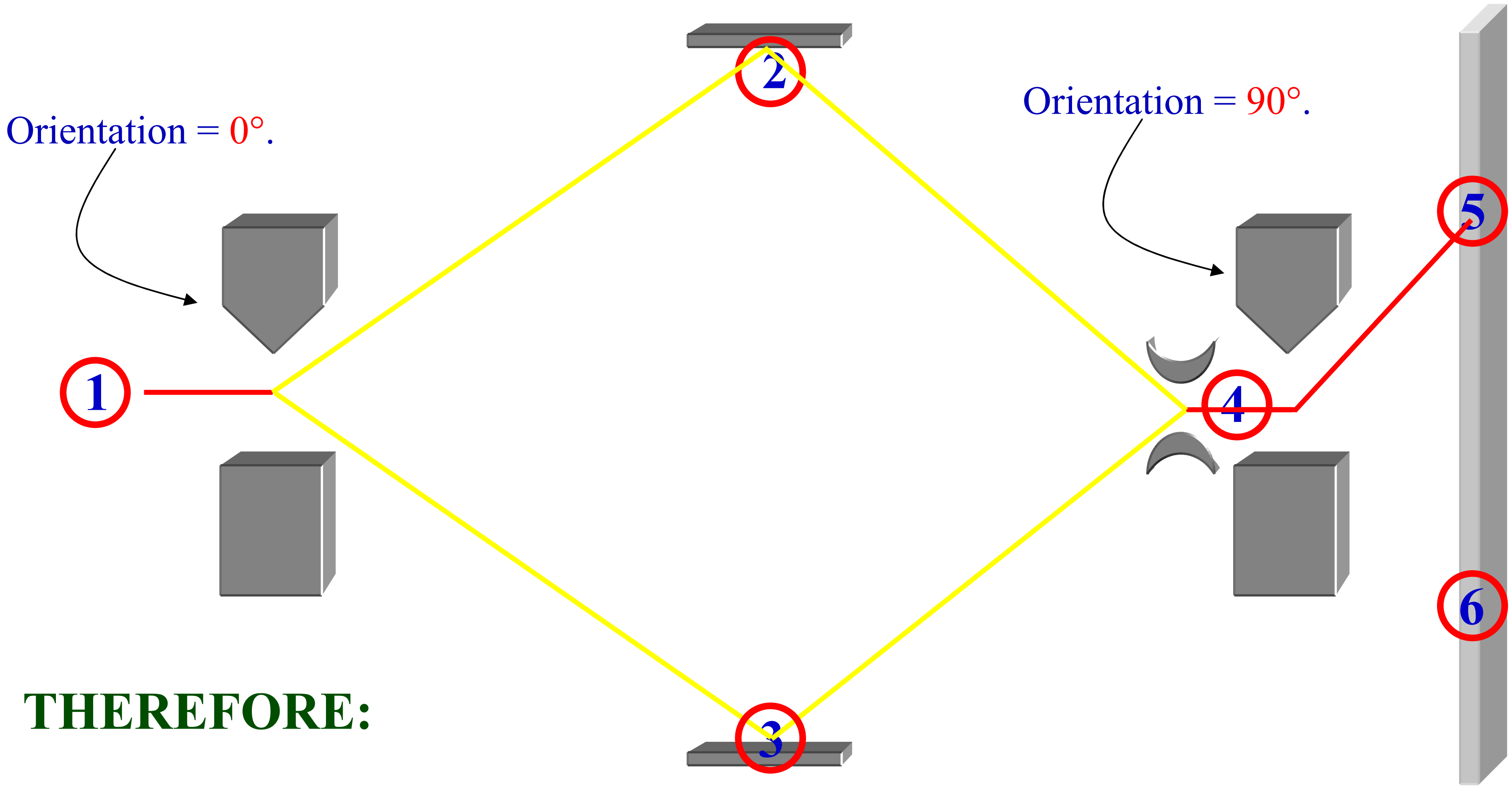
Two-path analyzed 3



Second easy step:

| down, 0°> | in R3 > ⟶ | down, 0°> | in R4 >

Two-path analyzed 4



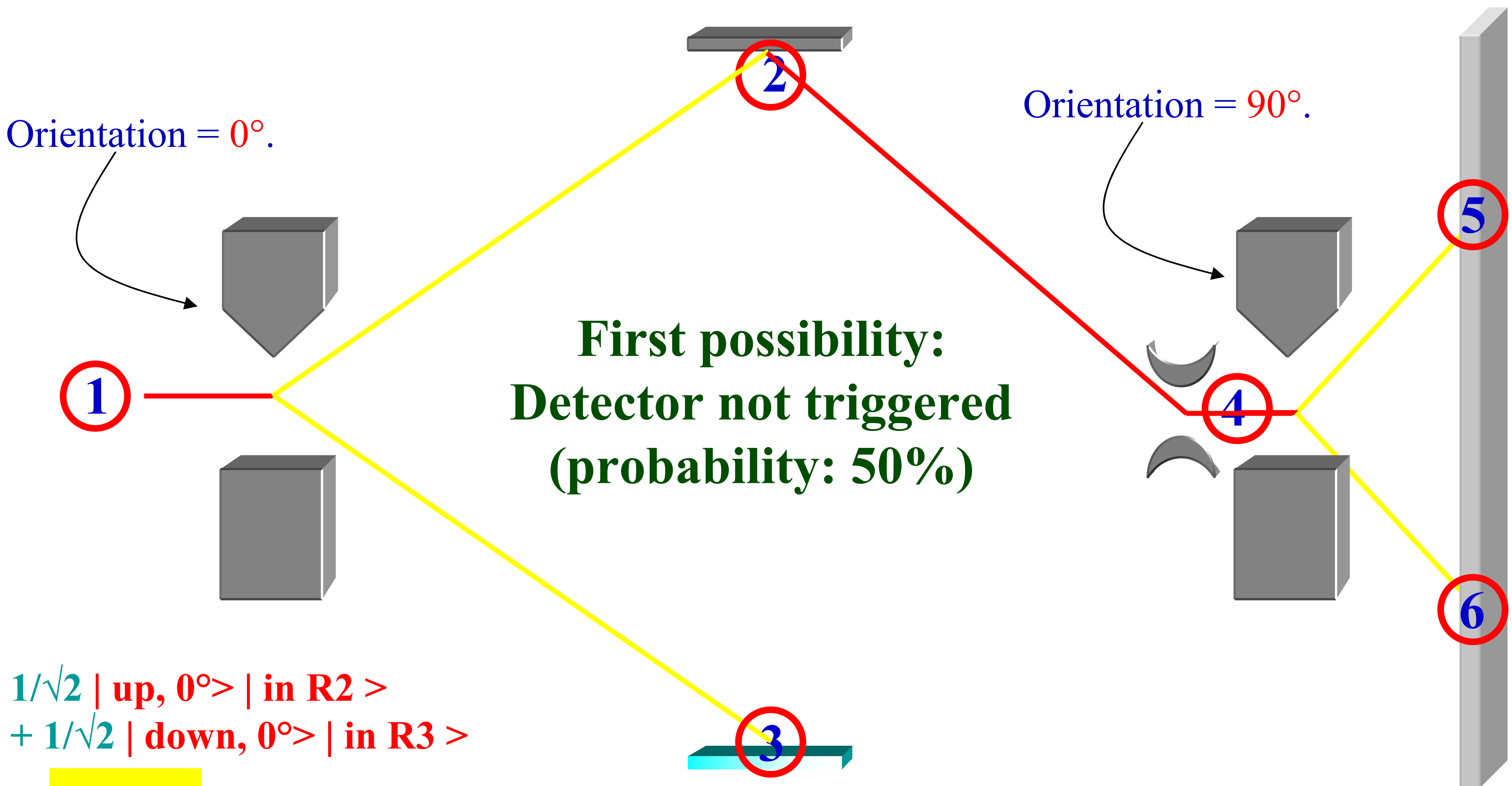
THEREFORE:

1/√2 | up, 0°> | in R2 > + 1/√2 | down, 0°> | in R3 >

⟶ 1/√2 | up, 0°> | in R4 > + 1/√2 | down, 0°> | in R4 >

= | up, 90°> | in R4 > ⟶ | up, 90°> | in R5 >

Two-path with detector 1



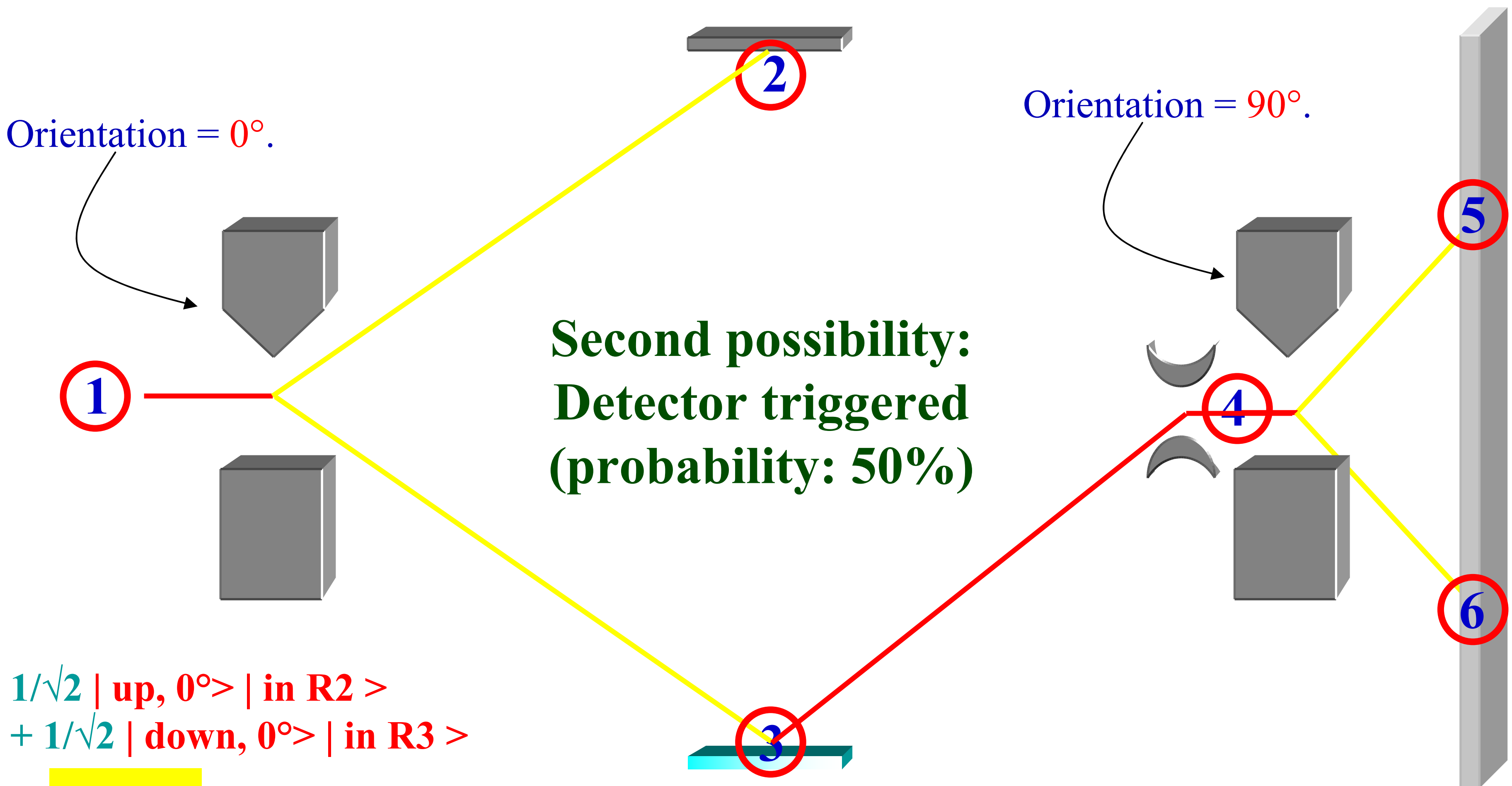
$1/\sqrt{2}$ | up, 0°> | in R2 >
+ $1/\sqrt{2}$ | down, 0°> | in R3 >

→ **collapse** → | up, 0°> | in R2 > → | up, 0°> | in R4 >

= $1/\sqrt{2}$ | up, 90°> | in R4 > + $1/\sqrt{2}$ | down, 90°> | in R4 >

→ $1/\sqrt{2}$ | up, 90°> | in R5 > + $1/\sqrt{2}$ | down, 90°> | in R6 >

Two-path with detector 2



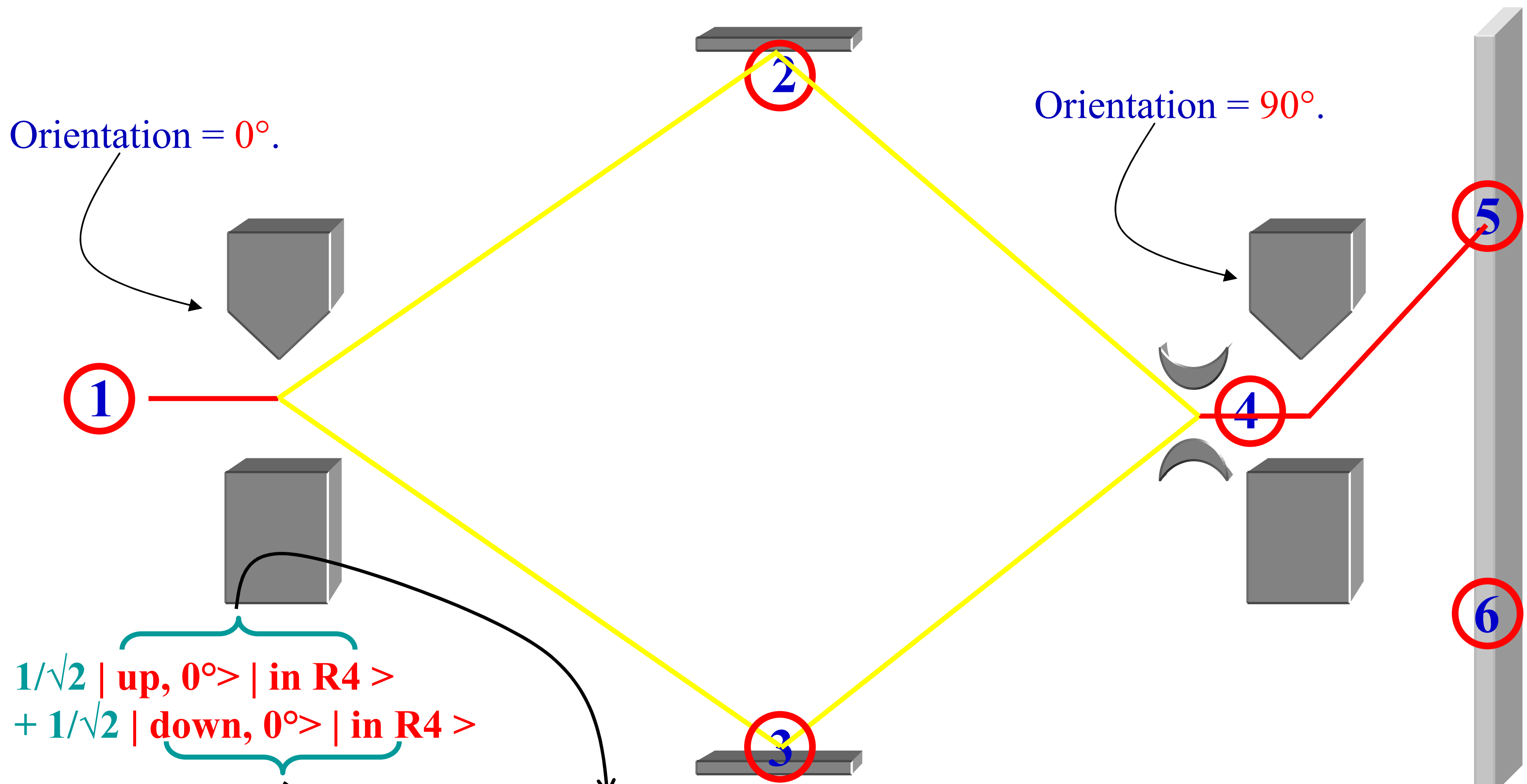
$1/\sqrt{2}$ | up, 0°> | in R2 >
$+ 1/\sqrt{2}$ | down, 0°> | in R3 >

→ **collapse** → | down, 0°> | in R3 > → | down, 0°> | in R4 >

$= 1/\sqrt{2}$ | up, 90°> | in R4 > - $1/\sqrt{2}$ | down, 90°> | in R4 >

→ $1/\sqrt{2}$ | up, 90°> | in R5 > - $1/\sqrt{2}$ | down, 90°> | in R6 >

Two-path revisited



$$1/\sqrt{2}\,|\,\text{up}, 0°\rangle\,|\,\text{in R4}\rangle + 1/\sqrt{2}\,|\,\text{down}, 0°\rangle\,|\,\text{in R4}\rangle$$

$$1/\sqrt{2}\{1/\sqrt{2}\,|\,\text{up}, 90°\rangle\,|\,\text{in R5}\rangle + 1/\sqrt{2}\,|\,\text{down}, 90°\rangle\,|\,\text{in R6}\rangle\}$$

$$+\,1/\sqrt{2}\{1/\sqrt{2}\,|\,\text{up}, 90°\rangle\,|\,\text{in R5}\rangle - 1/\sqrt{2}\,|\,\text{down}, 90°\rangle\,|\,\text{in R6}\rangle\}$$

CANCEL

A **warning** about "superposition"

Remember that

"SUPERPOSITION"

is a mathematical term. It means the same thing as

"LINEAR COMBINATION".

Thus,

$1/\sqrt{2}$ | up, 0°> | in R2 > + $1/\sqrt{2}$ | down, 0°> | in R3 >

is a superposition of

| up, 0°> | in R2 >

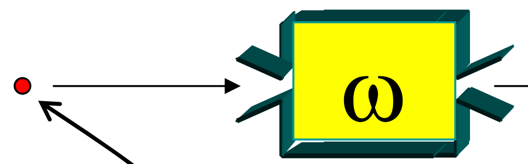
and

| down, 0°> | in R3 >.

It does not follow that the components | up, 0°> | in R2 > and | down, 0°> | in R3 > describe distinct **parts** of the particle (one part located in **R2**, the other located in **R3**.

The rotation box 1

We can build a box
with the following feature:

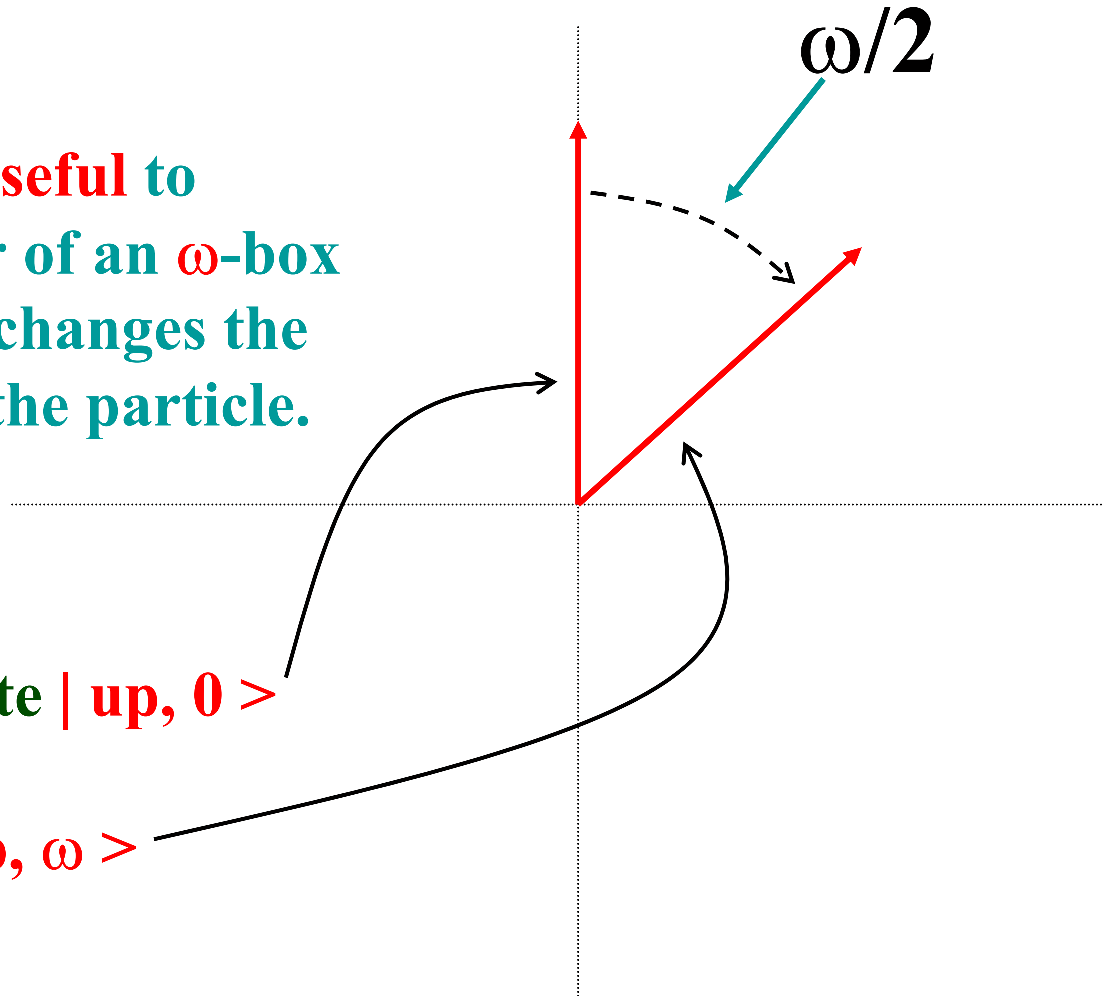


Initial spin state of particle:
$| \text{up}, \theta >$

Final spin state of particle:
$| \text{up}, \theta + \omega >$

The rotation box 2

It will be **extremely useful** to describe the behavior of an ω-box by describing how it changes the spin-state vector for the particle.

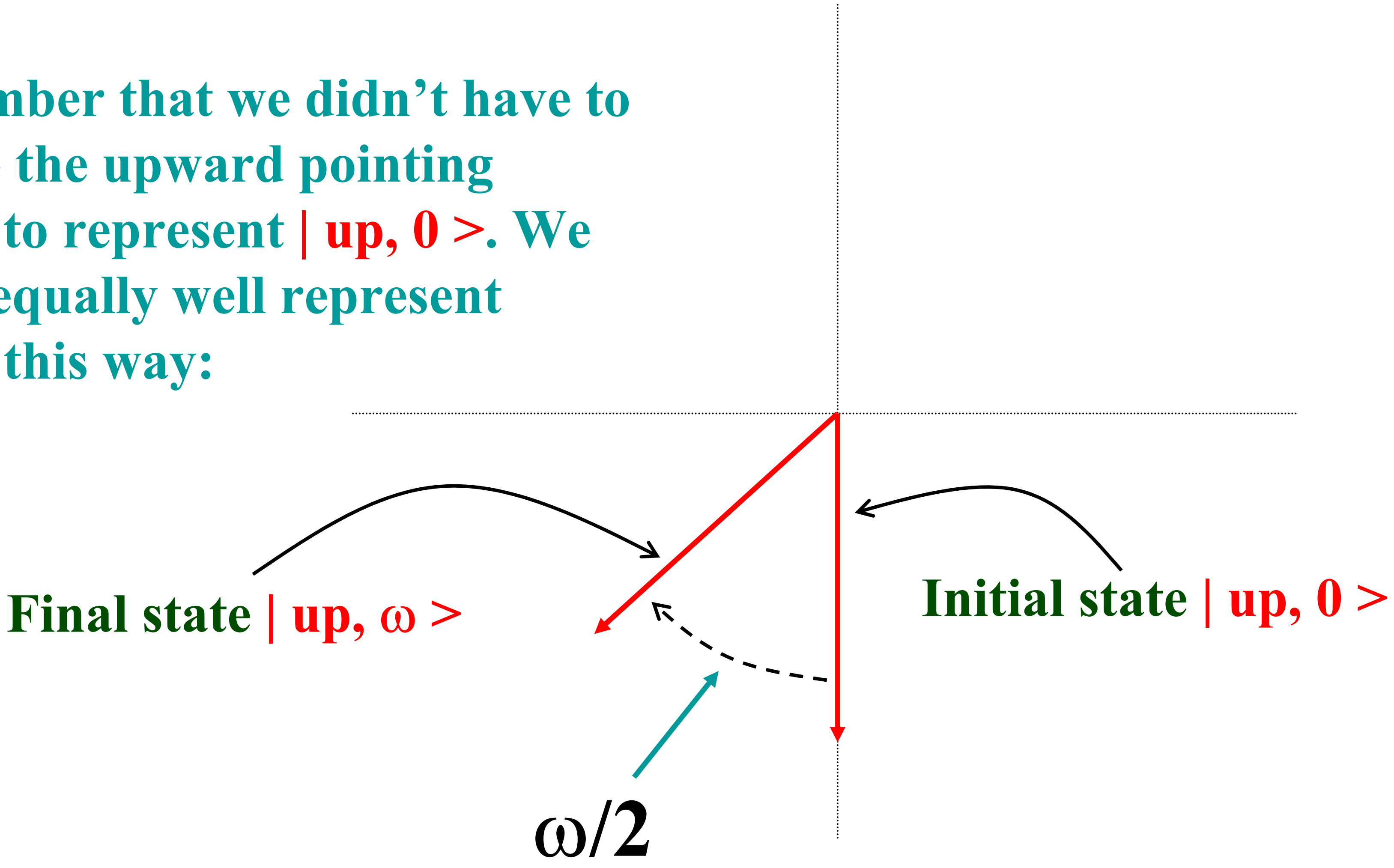


EX: Initial state | up, 0 >

Final state | up, ω >

The rotation box 3

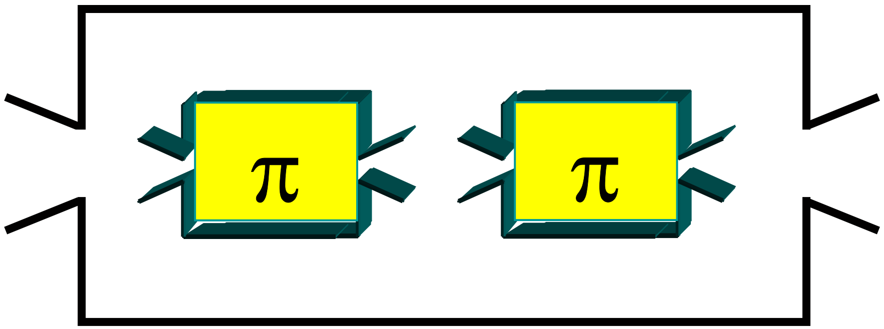
Remember that we didn't have to choose the upward pointing arrow to represent | up, 0 >. We could equally well represent things this way:



The "total-of-nothing" box 1

Now let's build a more complicated box

whose insides look like this:



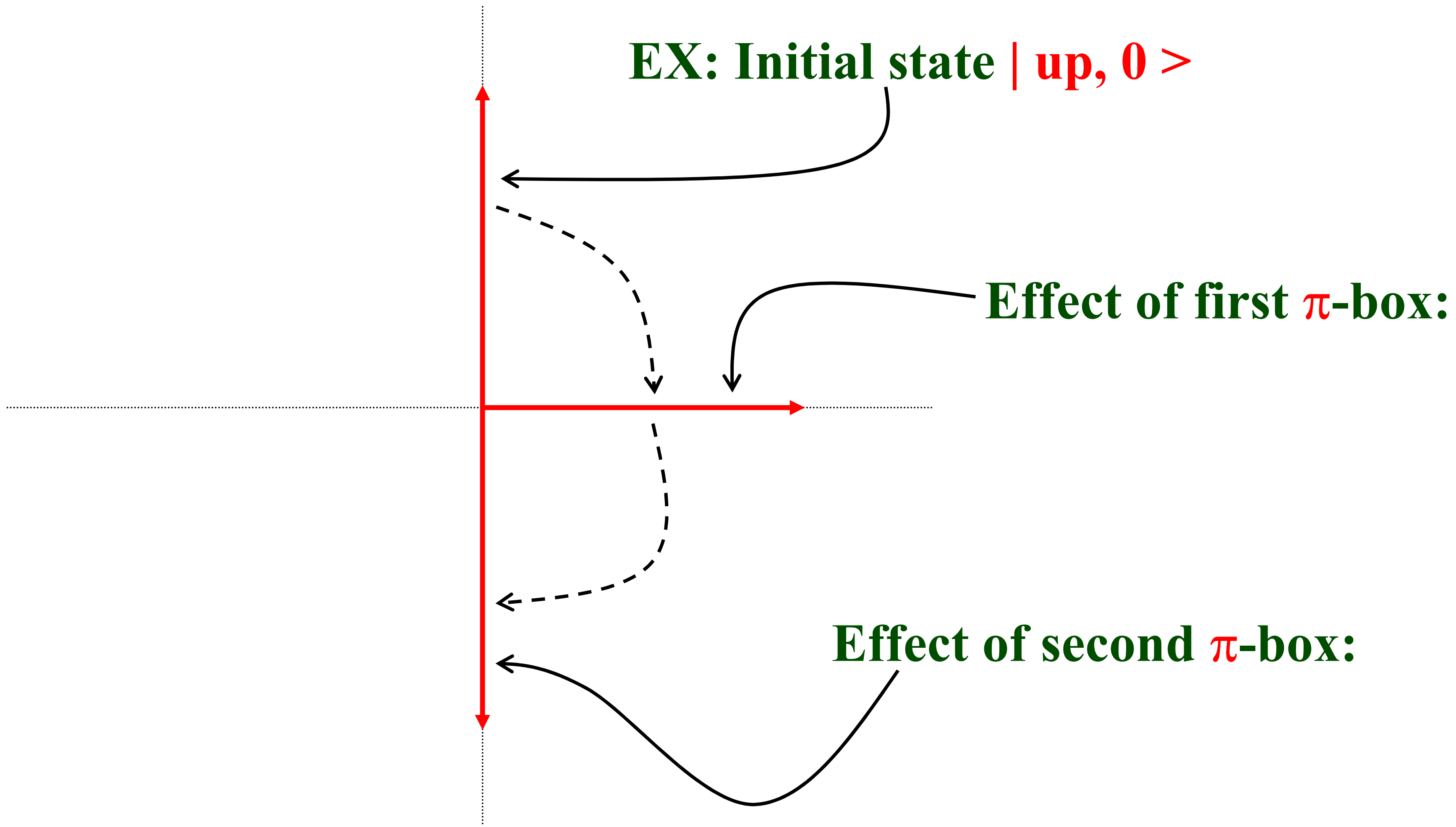
Total-of-nothing box

What will this box do to the spin state of a particle sent through it?

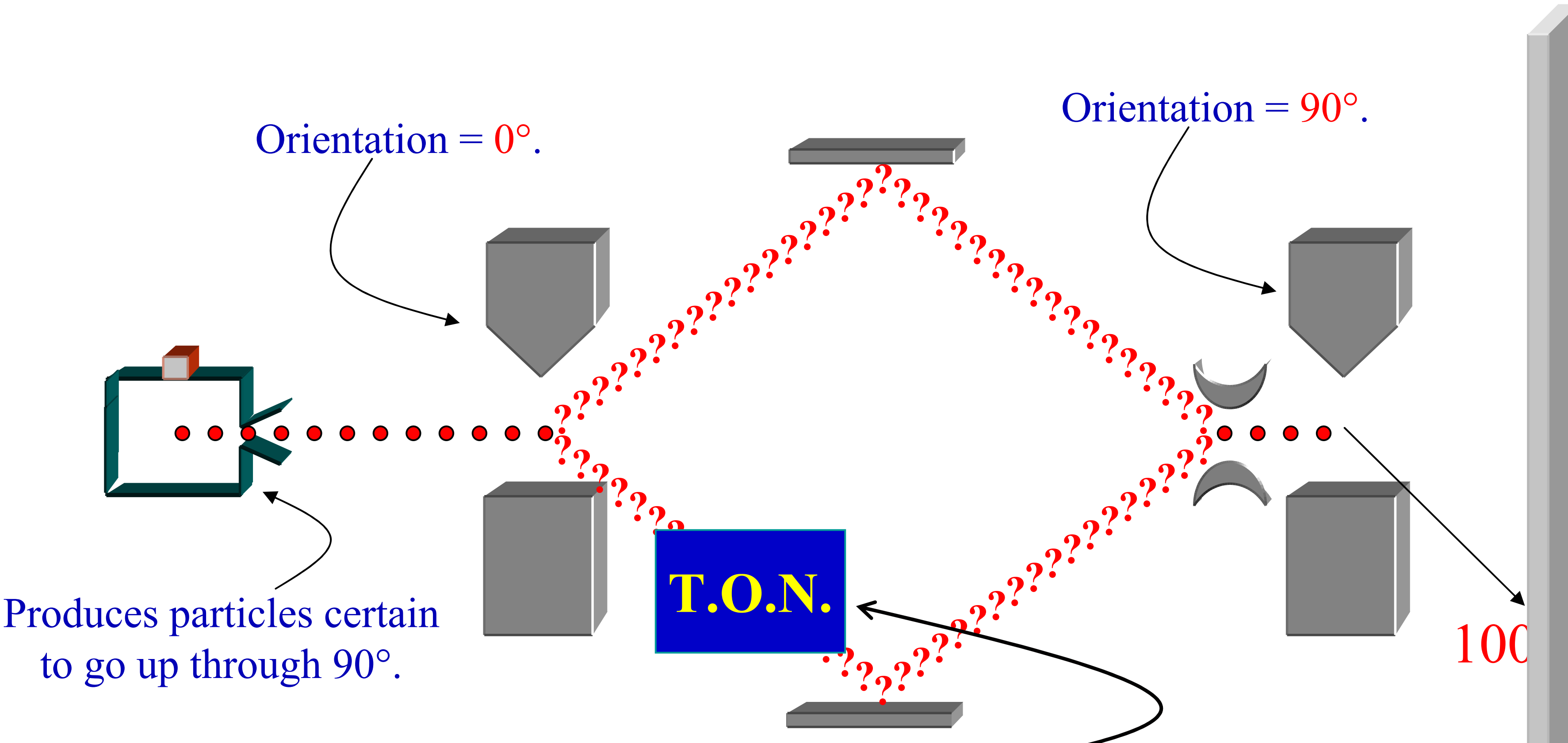
Answer: NOTHING.

The “total-of-nothing” box 2

To see why, observe how the spin-state vector changes:



A twist on the two-path 1

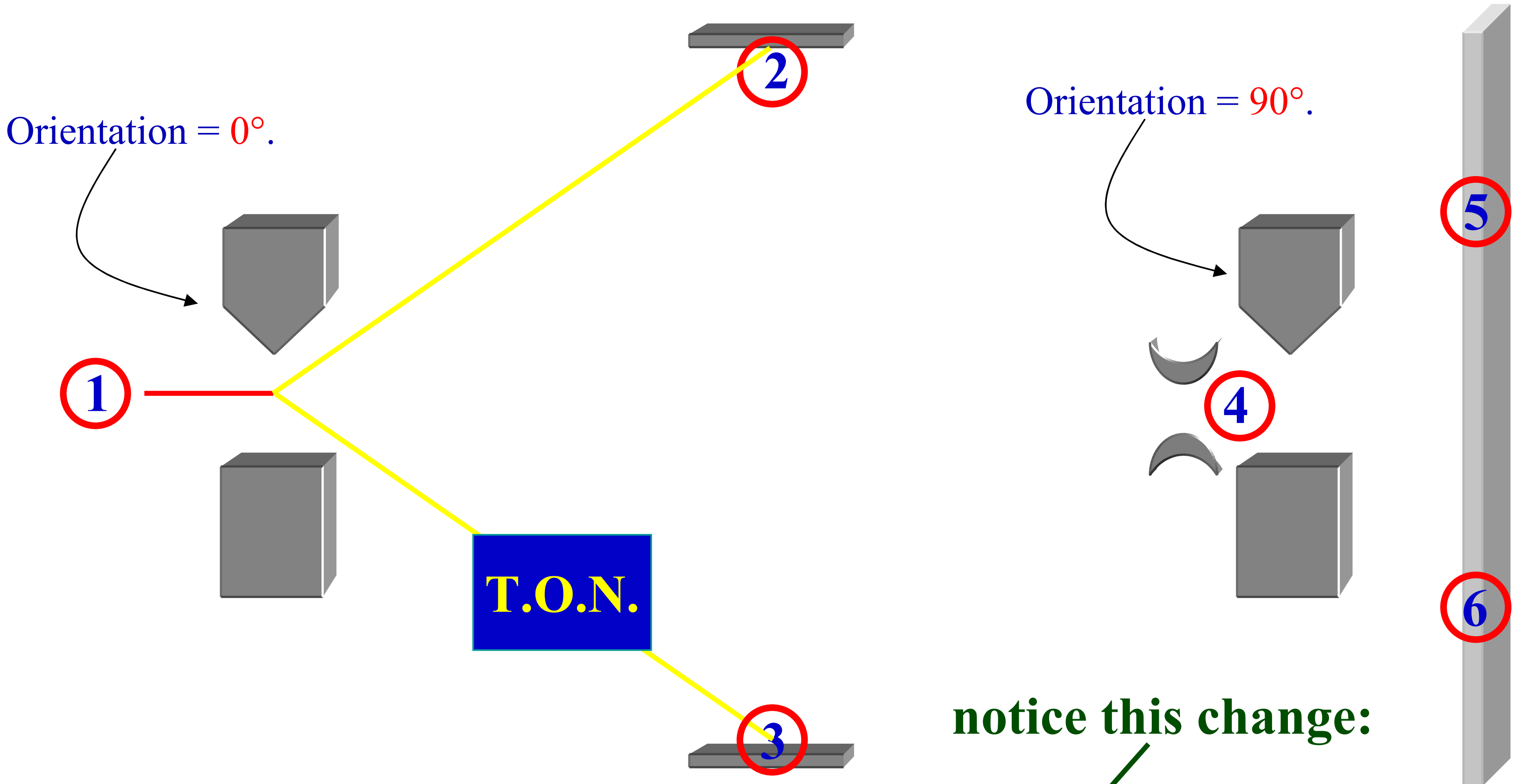


Stick a total-of-nothing box here:

What we observe is this:

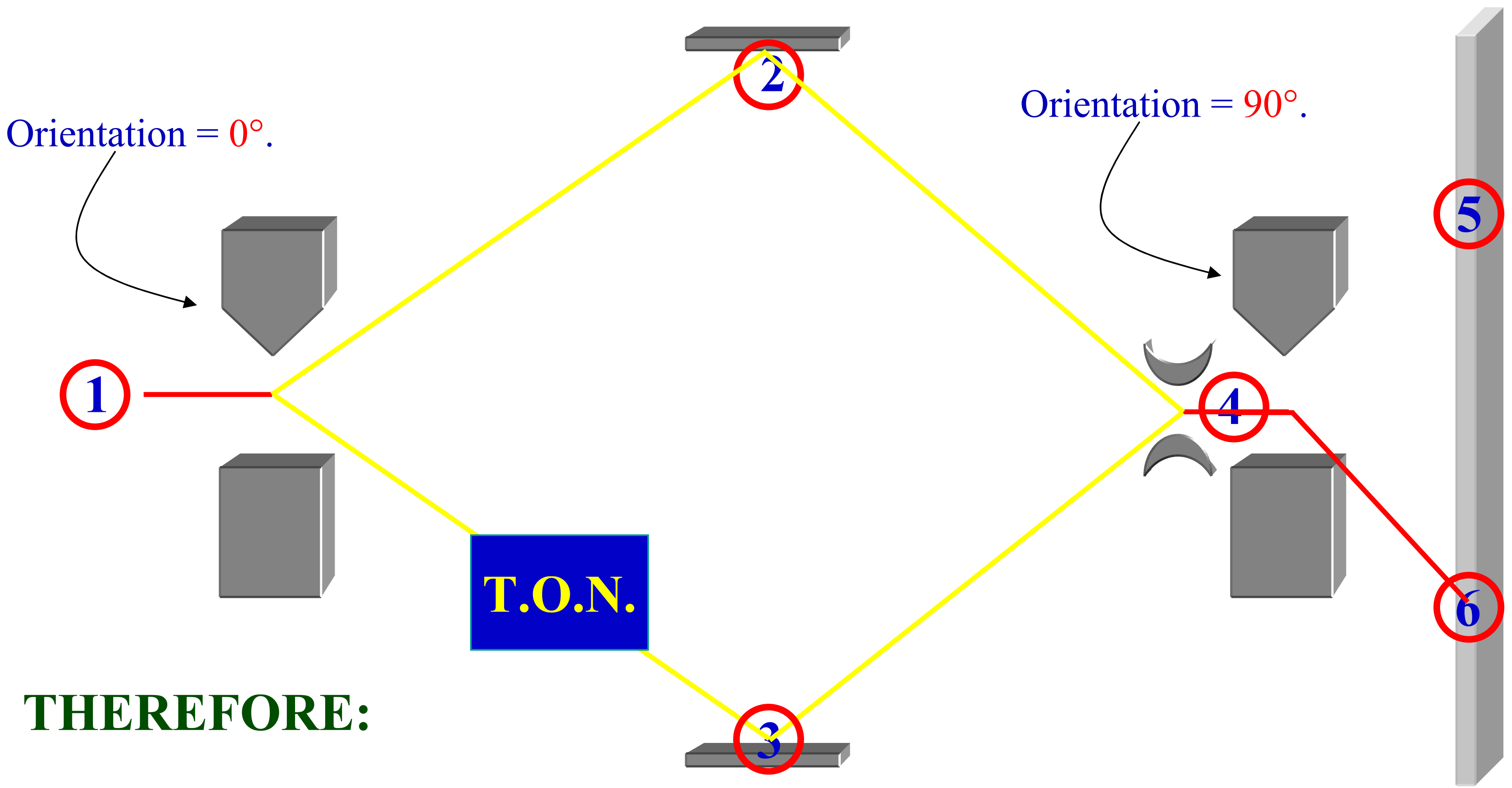
What is going on???

A twist on the two-path 2



| up, 90°> | in R1 > ⟶ $1/\sqrt{2}$ | up, 0°> | in R2 > - $1/\sqrt{2}$ | down, 0°> | in R3 >

A twist on the two-path 3



THEREFORE:

$1/\sqrt{2}$ | up, 0°> | in R2 > - $1/\sqrt{2}$ | down, 0°> | in R3 >

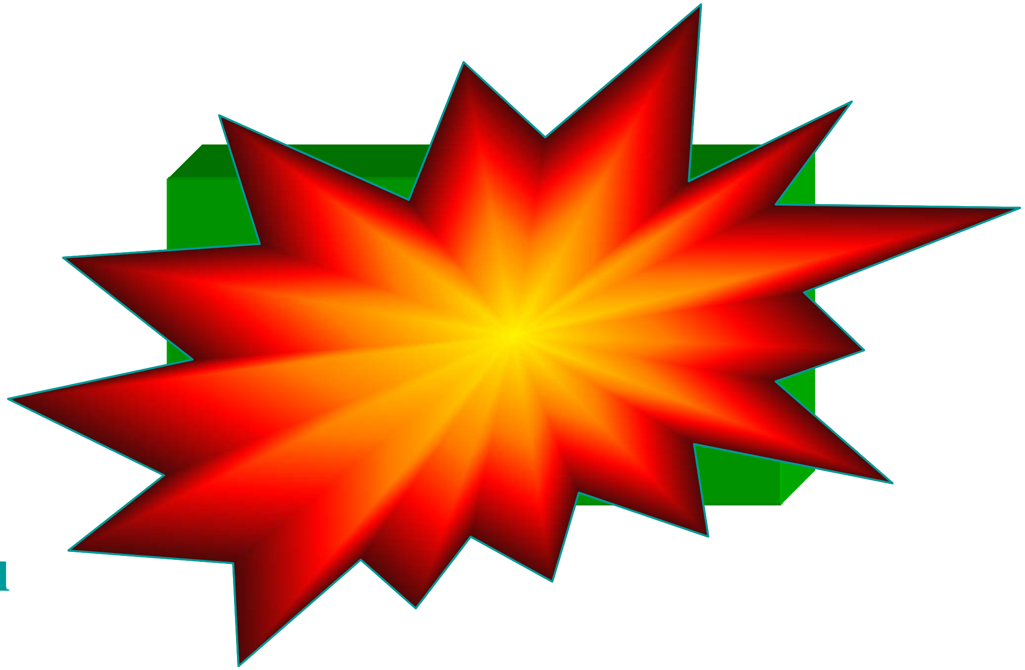
⟶ $1/\sqrt{2}$ | up, 0°> | in R4 > - $1/\sqrt{2}$ | down, 0°> | in R4 >

= | down, 90°> | in R4 > ⟶ | down, 90°> | in R6 >

Quantum bomb-detection 1

Here is a box.

Inside it there might be a

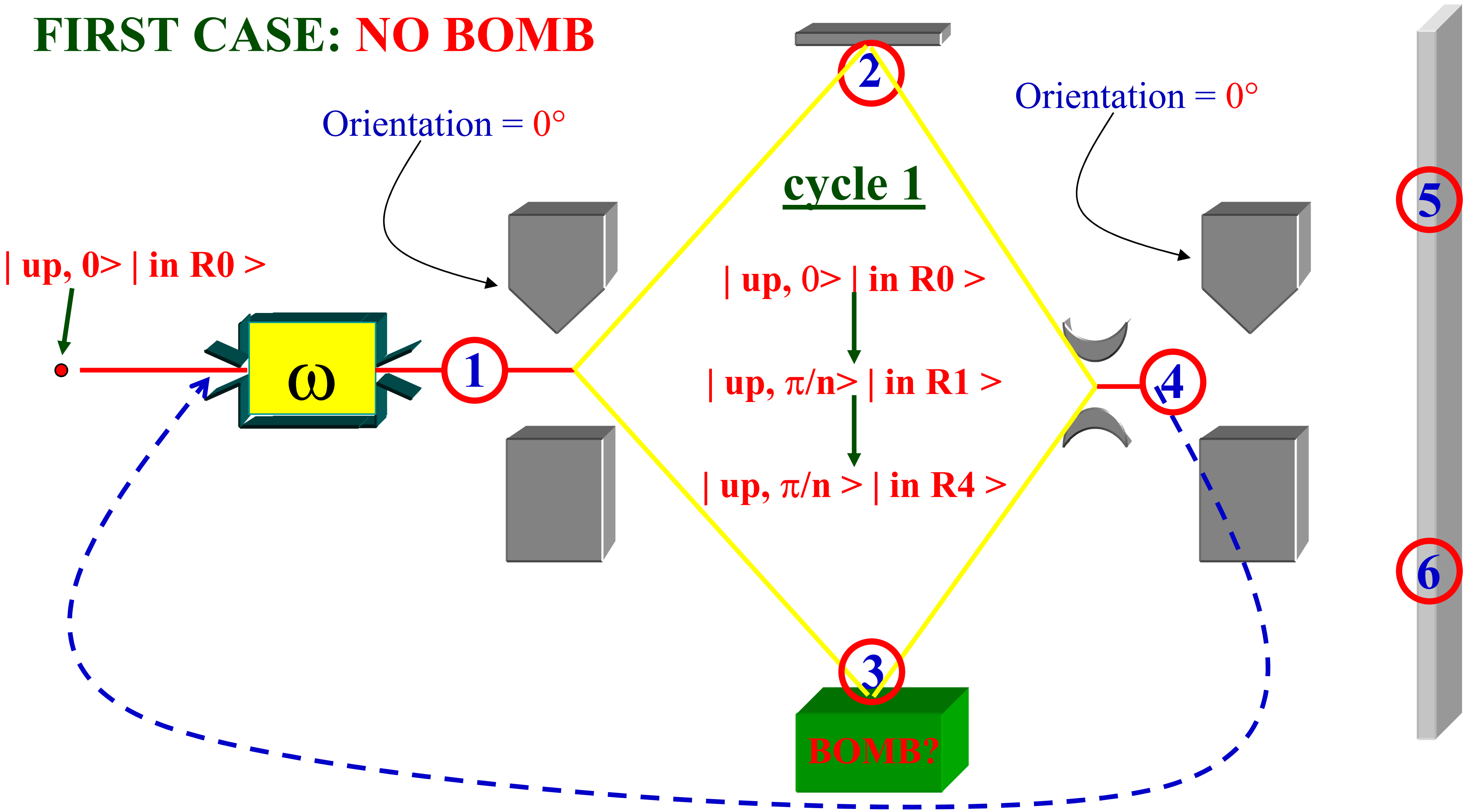


If there **is** a bomb, and you

- open the box;
- jiggle the box;
- send but a single photon through the box;
- try to find out whether there is a bomb by **any** other "classical" means;

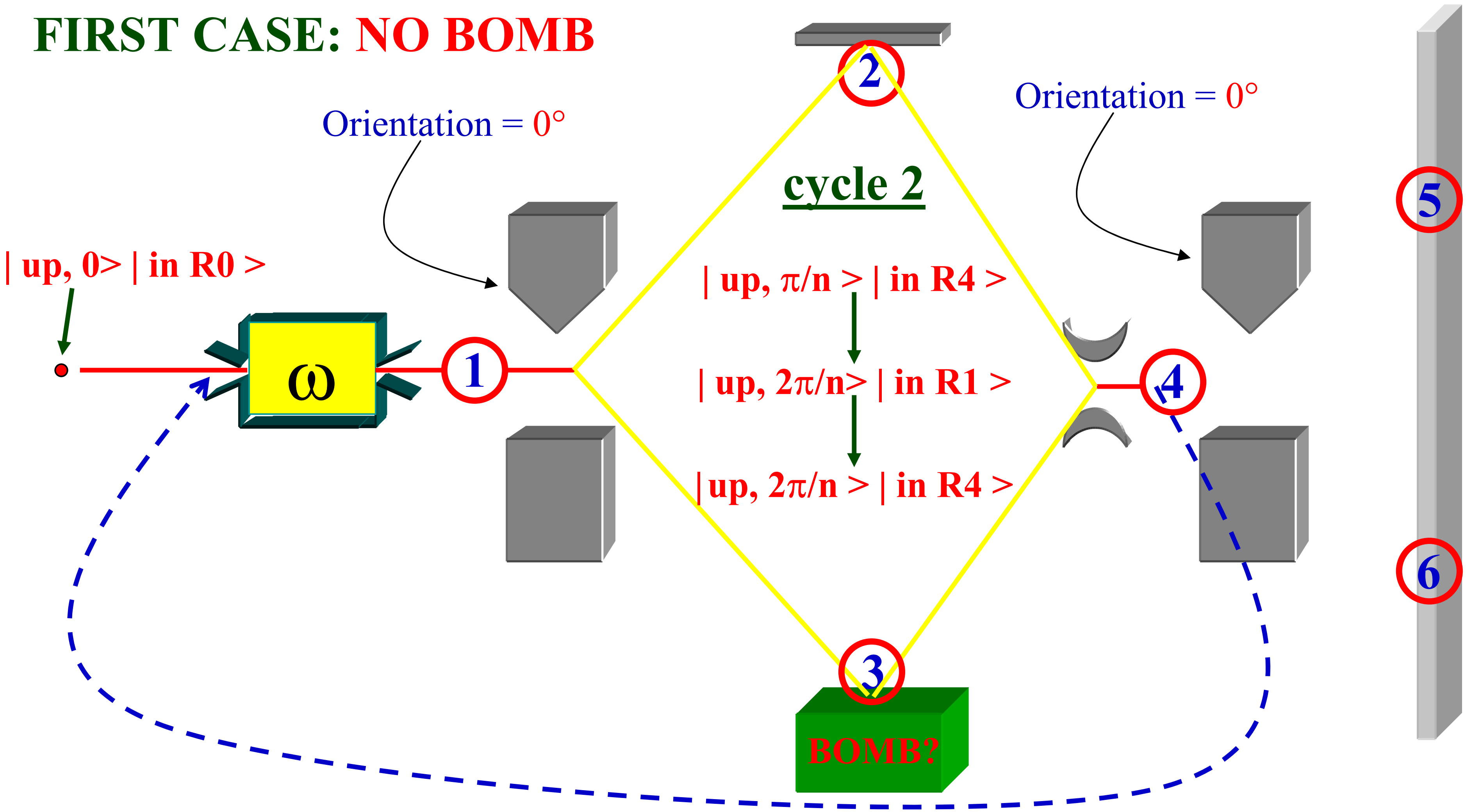
YOU WILL SET THE BOMB OFF.

Quantum bomb-detection 2



- set $\omega = \pi/n$
- cycle particle through **n times** before second magnet

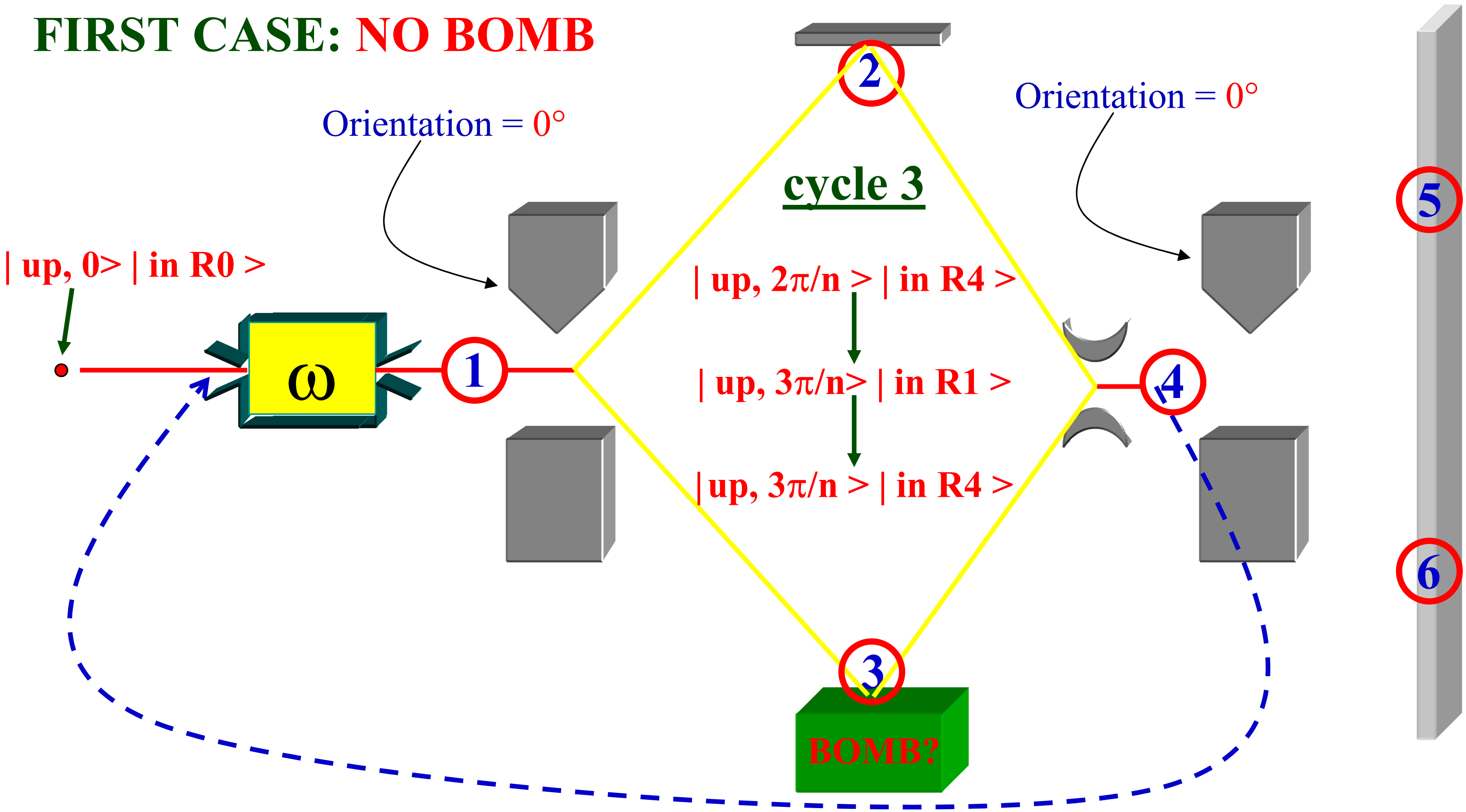
Quantum bomb-detection 3



- set $\omega = \pi/n$
- cycle particle through **n times** before second magnet

Quantum bomb-detection 4

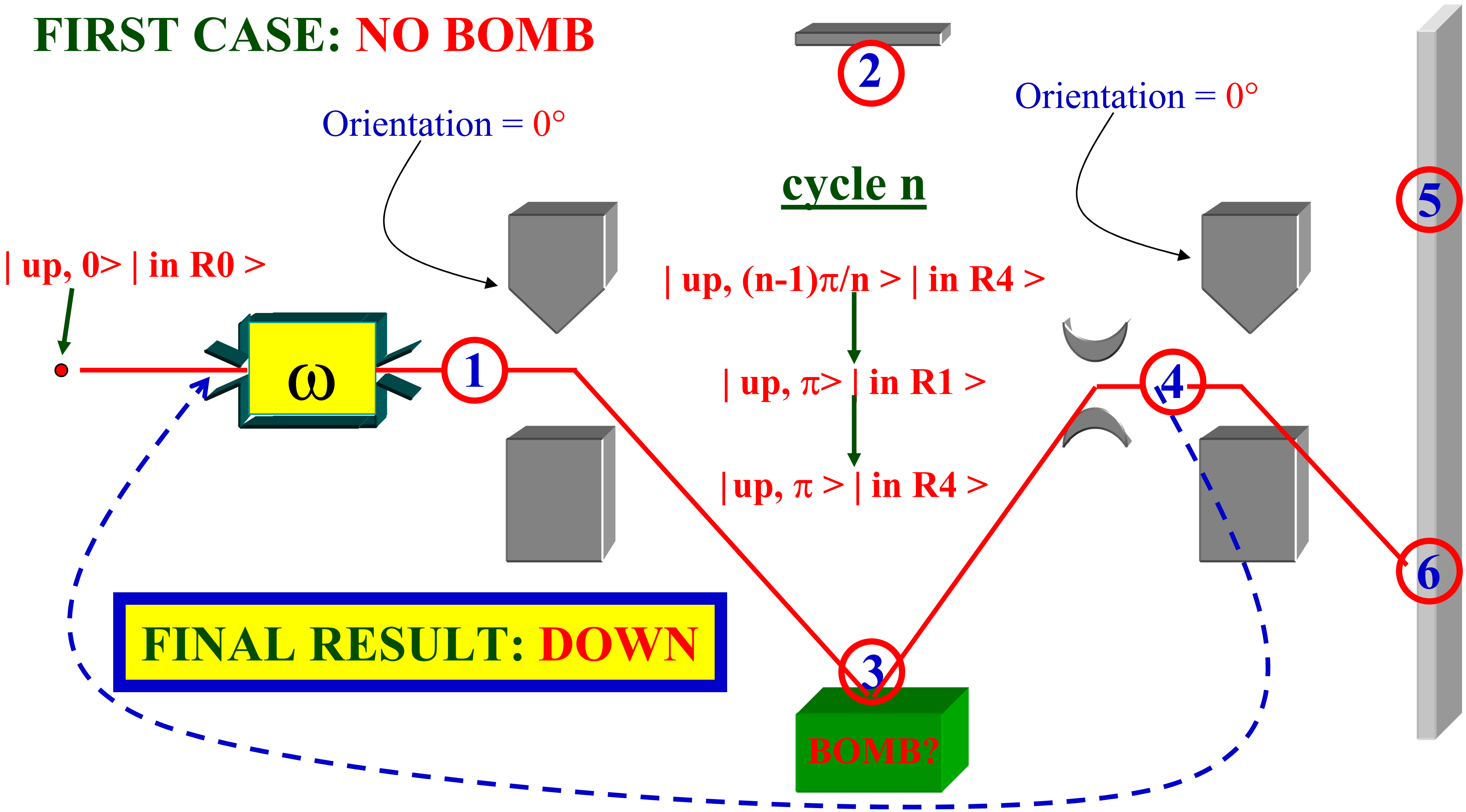
FIRST CASE: NO BOMB



- set $\omega = \pi/n$
- cycle particle through **n times** before second magnet

Quantum bomb-detection 5

FIRST CASE: NO BOMB

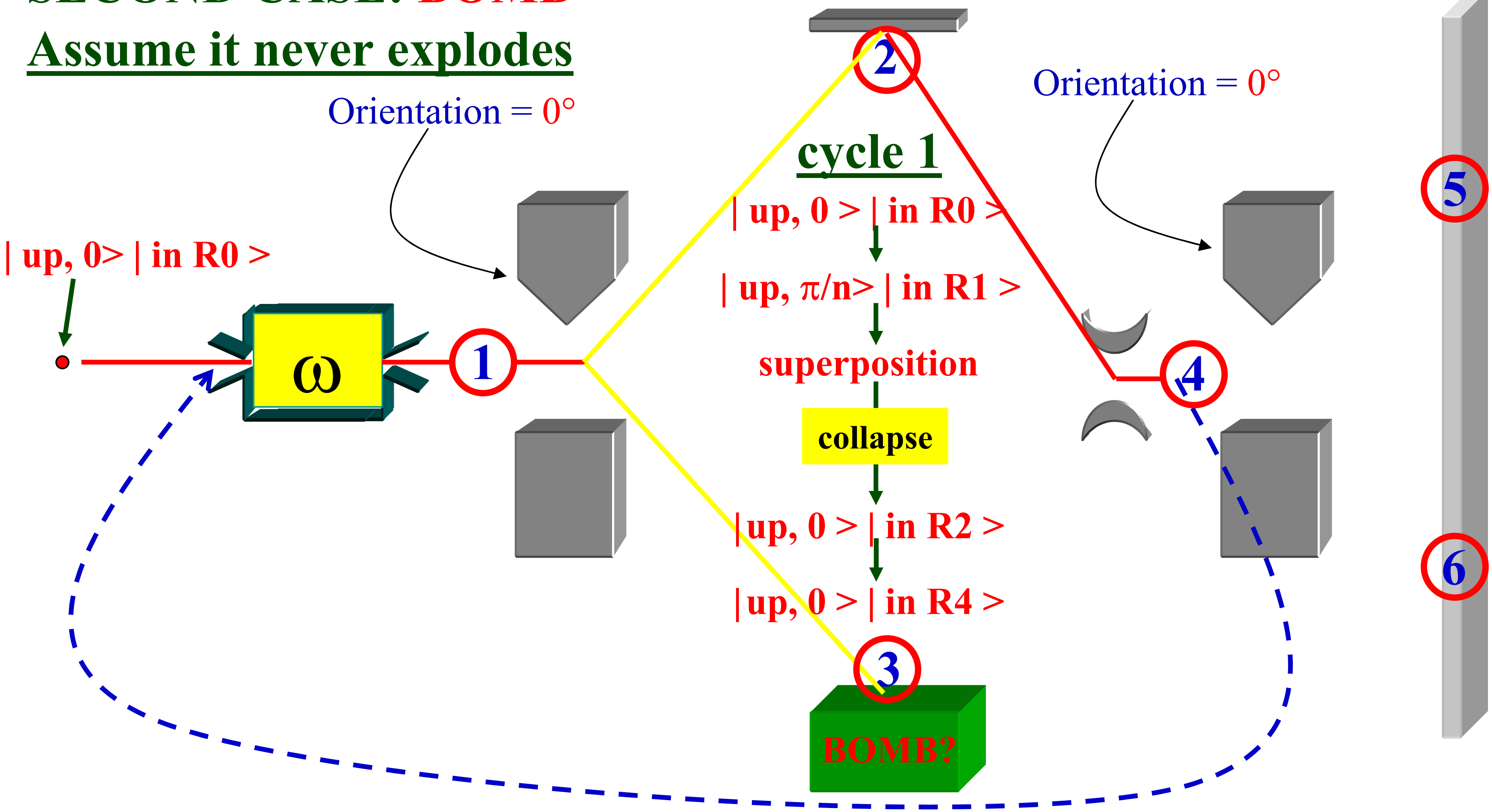


- set $\omega = \pi/n$
- cycle particle through **n times** before second magnet

Quantum bomb-detection 6

SECOND CASE: BOMB

Assume it never explodes

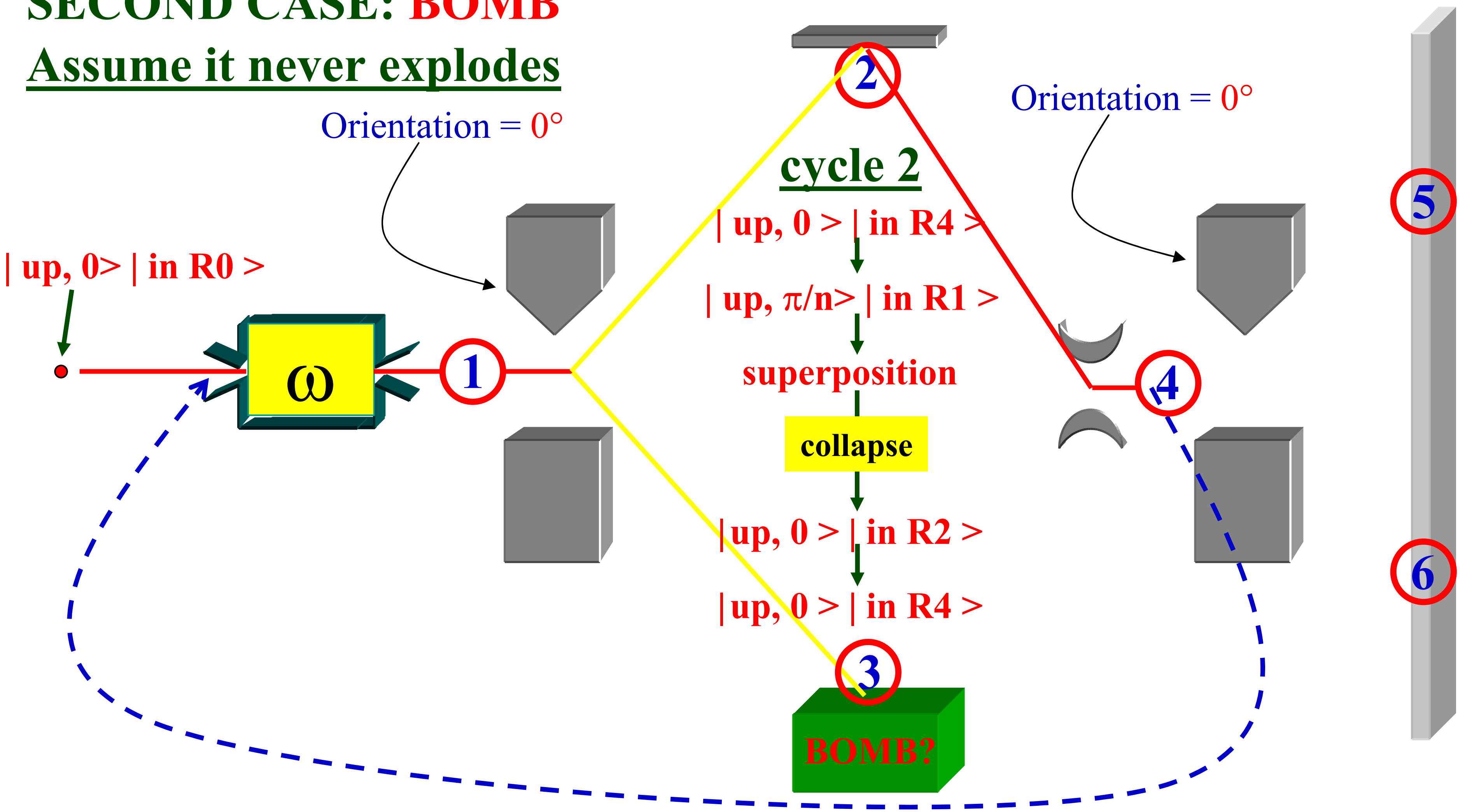


superposition = cos(π/2n)| up, 0>| in R2> + sin(π/2n)| down, 0>| in R3>

Quantum bomb-detection 7

SECOND CASE: BOMB

Assume it never explodes

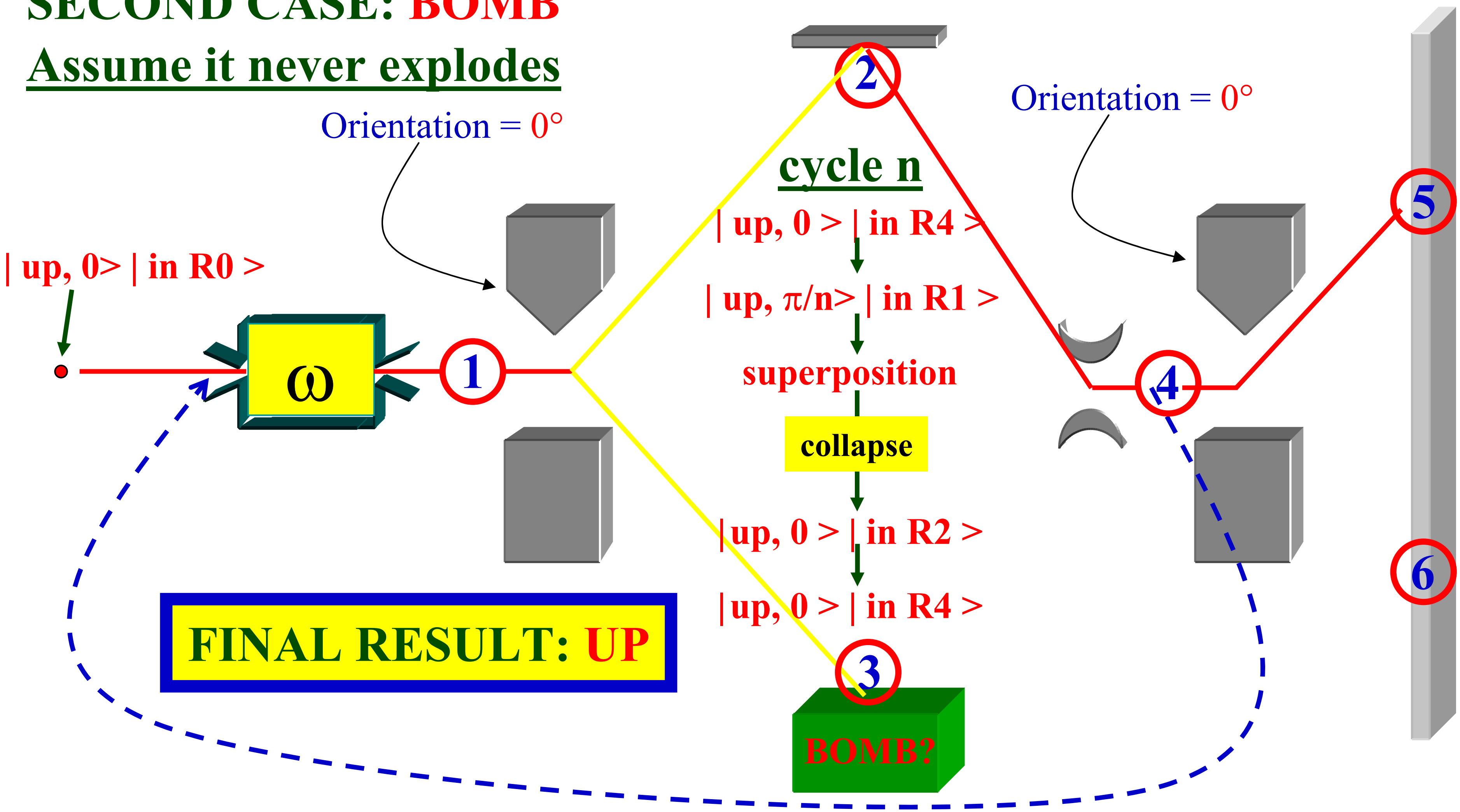


superposition = $\cos(\pi/2n)$| up, 0>| in R2> + $\sin(\pi/2n)$| down, 0>| in R3>

Quantum bomb-detection 8

SECOND CASE: BOMB

Assume it never explodes



superposition = $\cos(\pi/2n)$| up, 0>| in R2> + $\sin(\pi/2n)$| down, 0>| in R3>

Quantum bomb-detection 9

If there is a bomb, what is the probability that it never explodes?

ANSWER: Just the probability of getting n "up" outcomes in a row.

The probability of getting one "up" outcome, given that the spin state of the measured particle is $| \text{up}, \pi/n >$, is $\cos^2(\pi/2n)$.

So the probability of getting n "up" outcomes is

$$\cos^{2n}(\pi/2n)$$

This number goes to 1, in the limit as $n \Rightarrow \infty$.

So quantum bomb-detection works!!!

Quantum bomb-detection 10