

Uncertainty in Future Carbon Emissions: A Preliminary Exploration

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Abstract

In order to analyze competing policy approaches for addressing global climate change, a wide variety of economic-energy models are used to project future carbon emissions under various policy scenarios. Due to uncertainties about future economic growth and technological development, there is a great deal of uncertainty in emissions projections. This paper demonstrates the use of the Deterministic Equivalent Modeling Method, an efficient means for propagating uncertainty through large models, to investigate the probability distributions of carbon emissions from the MIT Emissions Prediction and Policy Analysis model. From the specific results of the uncertainty analysis, several conclusions with implications for climate policy are given, including the existence of a wider range of possible outcomes than suggested by differences between models, the fact that a “global emissions path through time” does not actually exist, and that the uncertainty in costs and effects of carbon reduction policies differ across regions.

1. Introduction

“Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of \$250 and/or six months in prison.”

Section 889, New York State Code of Criminal Procedure

Formulating a response to global climate change is one of the greatest environmental policy challenges facing the nations of the world. The United Nations Framework Convention on Climate Change (FCCC) has the overall objective “...to achieve...stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system.” (United Nations, 1992) While determining this safe level remains an elusive goal, the parties to the FCCC are currently negotiating a protocol to reduce greenhouse gas emissions over the next several decades.

The choices of how much to control greenhouse gas emissions and by what means to control them are economically, politically, and technically difficult decisions. One simple depiction of the analysis needed to develop a policy response to climate change is given in Figure 1. The emissions of greenhouse gases that result from economic development and technology choices over time are projected, although these projections are highly uncertain. For any emissions projection, the effect upon climate is then estimated, which is also uncertain. Any projected change in climate will result in some set of uncertain impacts, which can then be compared to the cost of any control strategy to limit greenhouse gas emissions, which is itself uncertain.

Climate change occurs over a very long time horizon, because many greenhouse gases, such as CO₂, are long-lived in the atmosphere once emitted, and because there is a delay in the climate system’s response to increased radiative forcing. Due to the long time scales of climate change and

the many uncertainties in both control costs and impacts, effective policy response needs to be flexible over time by being able to respond to changes in our understanding of the many uncertain aspects of the climate change problem.

A first step towards such a flexible analysis of long-term policy must begin by characterizing the uncertainty in key parameters that we know right now, even for simple once-only policy choices. The uncertainty in all components of the analysis framework (Figure 1) should be explored, but a reasonable starting place is to look at the uncertainty in emissions of greenhouse gases under some initial set of policies. A wide variety of economic-energy models are used to project emissions under alternative emission control strategies, as well as to project the economic costs of control. This paper demonstrates a method by which the uncertainty in emissions from one economic-energy model can be characterized. This method is generally applicable to any model. The type of information obtained from an analysis such as the one presented here can then feed into a larger analysis of policy choice as described above.

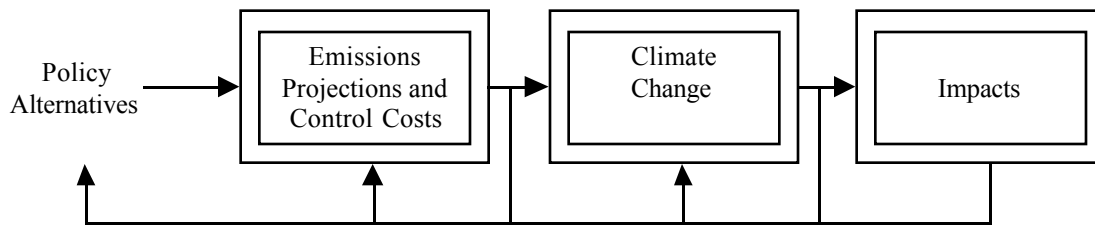


Figure 1. Simple Framework for Analysis of Climate Change Policy

Uncertainty in Models

In any modeling endeavor, there are two distinct types of uncertainty: parametric uncertainty and structural uncertainty. Parametric uncertainty consists of uncertainty in the value that a model parameter should take. Structural uncertainty concerns uncertainty in the form or structure of a model, and is more difficult to treat in a rigorous and consistent fashion. Alternative model structures can result in a wide range of outcomes (IPCC, 1996). However, even when restricting oneself to a single model specification, uncertainty in some of the data or parameter values can yield a wide range of likely outcomes. Any analysis exercise, therefore, will yield more robust results and insights through the application of uncertainty analysis, by which we mean the propagation of uncertainty in model parameters, expressed as probability distributions, to obtain the resulting probability distributions of model outcomes.

In this paper, we will apply parametric uncertainty analysis to an economic-energy model used to project future emissions. In economic-energy models, there are several sources of uncertainty, including:

1. Structural uncertainty: There are several general types of economic models, such as computable general equilibrium models or econometric models. Within a model as well, there are many possible structural specifications, including the number of production sectors or the nesting of a production function.
2. Data uncertainty: Are the base data correct? While economic-energy models are almost always calibrated to real world data, the quality and accuracy of these data is often less than desirable.
3. Forecast uncertainty: What will future trends be? Most economic models for climate analysis have very long time horizons, often over 100 years. Thus assumptions about future trends, such as productivity growth or the appearance of new technological options, are both crucial and highly uncertain.

The latter two sources of uncertainty can be explored by performing a parametric uncertainty analysis. The standard approach for parametric uncertainty propagation, Monte Carlo simulation, is often computationally expensive. This paper demonstrates an application of the Deterministic Equivalent Modeling Method (DEMM) (Tatang *et al.*, 1997), an efficient and practical technique for performing uncertainty analysis on large models. As an illustrative application, parametric uncertainty will be investigated in the Economic Prediction and Policy Analysis (EPPA) model¹ (Jacoby *et al.*, 1997; Yang *et al.*, 1996), one of many economic models used in climate change analysis. The resulting probability distributions of outcomes from the EPPA model will be used to demonstrate several insights for policy analysis obtainable from the application of DEMM for uncertainty analysis. The focus of this study will be on the uncertainty in CO₂ emissions resulting from uncertainty in three input parameters relating to energy efficiency, productivity growth, and carbon-free electricity alternatives.

Section 2 will describe in detail the uncertain parameters chosen for this analysis. Section 3 shows the resulting uncertainty in CO₂ emissions over time and under different policies. Since the uncertainty in outputs is dependent on the assumed uncertainty in inputs, Section 4 explores the sensitivity in emissions uncertainty to alternative assumptions about input uncertainty and about the existence and magnitude of correlation between uncertain inputs. Section 5 will outline next steps for research to build on this initial analysis. The main conclusions that result from this analysis are summarized in Section 6.

2. Uncertainty in Input Parameters

The first step in using DEMM or any other uncertainty analysis tool is to identify the uncertain inputs to be studied, and assess a probability distribution to describe the uncertainty. Since the primary purpose of this study is to demonstrate the application of the DEMM approach, short cuts have been taken; particularly with regard to the process of elicitation of parameter distributions. Planned future work in these aspects is summarized in Section 5. This section will describe how three uncertain parameters were chosen for analysis: AEEI, labor productivity growth, and

¹ The EPPA model is the economic-emissions component of the Global Systems Model of the MIT Joint Program on the Science and Policy of Global Change (Prinn *et al.*, 1997).

backstop price. Each parameter's function within the EPPA model will be described, and the probability distribution assessed for each will be given.

This step and those that follow in later sections are part of a general approach to treating uncertainty in models. Figure 2 outlines the general procedure for uncertainty analysis, which can be used in for any model exercise.

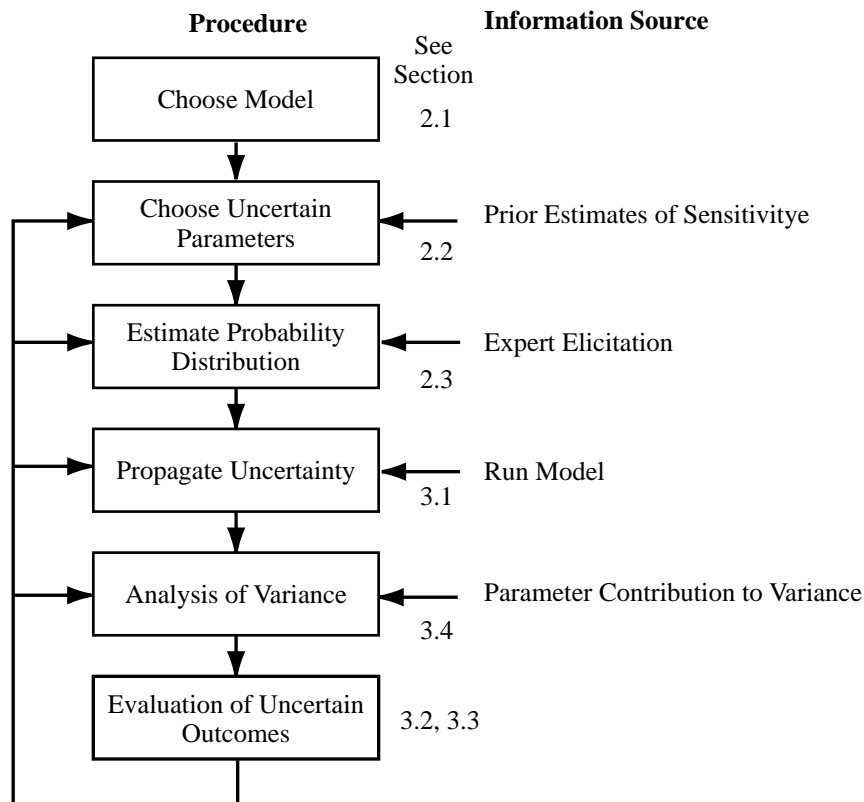


Figure 2. General Procedure for Uncertainty Analysis

2.1 The Economic Prediction and Policy Analysis Model

Before introducing the uncertain parameters chosen for this exercise, we briefly highlight the essential features of the EPPA model, Version 2.1.² EPPA is a recursive- dynamic, computable general equilibrium (CGE) model. It is derived from the GREEN model of the OECD (Burniaux *et al.*, 1992). EPPA is divided into twelve geopolitical-economic regions. Four regions represent the OECD: the United States (USA), the European Community (EEC), Japan (JPN), and the remainder of the OECD as of 1992 (OOE), which includes Canada, Australia, New Zealand, and

² The results in this paper use Version 2.1 of EPPA, which has a number of modifications from Version 1.7, used in (Jacoby *et al.*, 1997). The modifications include the addition of a third backstop technology (hydrogen fuels), different nesting of production functions between electric and non-electric energy, modified elasticities of substitution between factors, and the addition of a materials input to the backstop production functions.

most of Scandinavia. The remaining eight regions are Brazil (BRA), China (CHN), India (IND), the Former Soviet Union (FSU), Central and Eastern Europe (EET), the dynamic Asian economies (DAE)³, the energy exporting countries in the Middle East as well as Mexico and Indonesia (EEX), and the remaining countries (ROW).

Each region has eight production sectors (agriculture, energy intensive industries, other industries, crude oil, refined oil, gas, coal, and electricity), three future technology production sectors (carbon-free electric backstop, synthetic liquid fuel backstop from shale oil⁴, and hydrogen fuels), and four consumption sectors (food and beverage, transport and communication, energy, and other goods and services). The model accounts for trade between regions, and can track the imports and exports from each region. In each region, there is a representative consumer that maximizes its utility based on a bundle of the consumption goods.

As with most CGE formulations, EPPA Version 2.1 is a static model. Based on the previous period solution and prescribed growth in some exogenous parameters, EPPA recursively solves for equilibrium in each time period in five year intervals, starting with the calibration year of 1985, and ending with 2100. For each time period, the model produces as output the amount produced by each sector in each region and the relative prices. By using carbon emission coefficients, the CO₂ emissions are also calculated. The cost of a policy to a region can be measured as the change in consumption between a no-policy scenario and a scenario in which a policy is implemented. EPPA can model a variety of policy instruments, including carbon taxes, energy taxes, and quantitative restrictions, with or without allowing tradable permit schemes. It is worth noting that when implementing a quantitative instrument without trading (*e.g.*, OECD regions stabilize at 1990 emissions levels), the restriction is a firm constraint within which an optimal solution is found; thus some real-world uncertainties such as failure to meet agreed targets are not modeled, but are important considerations nevertheless.

This paper will focus on characterizing uncertainty in CO₂ emissions from the EPPA model, both regional and global, in specific years and cumulative over all years, and differences between policies. The intention is to use these results to demonstrate an uncertainty analysis and the insights that result from it. The uncertainty in costs is equally crucial to policy analysis, but an analysis of cost will be left to a future paper.

2.2 Choosing the Parameters

The first step in conducting a parametric uncertainty analysis is to identify which parameters are to be treated as uncertain and which are to be left at fixed reference values. The formal approach to making this choice is to subject every parameter in the model to a sensitivity test, and then to choose the most sensitive parameters as the focus for the uncertainty analysis.⁵

Unfortunately, this method is difficult in practice on larger models, such as EPPA, which have hundreds of parameters, all of which are in some sense uncertain. One alternative, employed in this

³ *e.g.*, Taiwan, Singapore, Malaysia.

⁴ The shale oil backstop is assumed to be present only in USA, FSU, EEX, and OOE.

⁵ For an example of this procedure, see Nordhaus' uncertainty analysis of the DICE model (Nordhaus, 1994).

study, is to rely on the intuition and judgment of the model experts.⁶ In particular, one rule of thumb is to focus on parameters that represent the rate in exponential growth functions.

Based on expert judgment, three uncertain parameters were selected:

- Autonomous Energy Efficiency Improvement Rate (AEEI)
- Labor Productivity Growth Rate (LPG)
- Price of Carbon-Free Electric Backstop Technology (BS)

There are other parameters in EPPA that are also uncertain and that could have a comparable effect on uncertainty in emissions. Some other parameters that could be included are the population growth rate, elasticities of substitution, and the price of the shale and hydrogen backstops. The selection of three parameters presented here is designed to illustrate the ranges of uncertainty in emissions that can result and their implications for climate policy, and plans for a more complete exploration of inputs is summarized in Section 5.

2.3 Choosing the Probability Distribution Family

Having selected which parameters to study, the next step is to assess a subjective probability distribution, and first an appropriate family of distributions must be chosen. All of the three uncertain parameters are constrained to be at least non-negative and non-infinite, which eliminates any choice of distribution family with infinite tails. As a result, the beta family of distributions was chosen for its finite end-points and its flexibility in representing different distribution shapes.

A beta distribution is a probability distribution defined over the interval (0,1), although a linear transformation can be applied to cover any finite interval. The beta density function has two parameters α and β , and is defined as:

$$f(x/\alpha, \beta) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} & 0 < x < 1 \\ 0 & \textit{otherwise} \end{cases}$$

The formal assessment process entails the elicitation of several fractiles, such as the 0.05, 0.50, and 0.95 fractile values, which are then used to find appropriate values for the parameters. In this study however, the experts provided estimates of the mode (most likely value), the endpoints (the 0.00 and 1.00 fractiles), and chose a level of variance. The sensitivity of the uncertainty results to the variance assumption is given in Section 4. Again it should be stressed that the distributions shown here are not meant to precisely capture the uncertainty, but rather to demonstrate that conservative estimates of uncertainty in inputs can cause great variation in outcomes.

The remainder of this section will describe each parameter in more detail, and give the probability distribution that resulted from the assessment.

⁶ The experts were Professor R. Eckaus, Dr. A.D. Ellerman, Professor H. Jacoby, and Dr. Z. Yang.

2.4 AEEI

The autonomous energy efficiency improvement (AEEI) parameter reflects the long-run historical trend of a decrease in the energy input required per unit of output that is not explained by price changes. The notion of AEEI can be found in the Edmonds-Reilly model (Edmonds and Reilly, 1983), the Global 2100 model (Manne and Richels, 1990), and many other economic studies. In EPPA, AEEI is defined as an increase in the effective energy input into non-energy sectors beyond the physical energy input. For the non-energy production (*ne*), consumption (*c*), government (*gov*), and investment (*inv*) sectors, the energy input is calculated as:

$$E_j^e(t) = (1 + \gamma_{e,j})^t E_j(t) \quad j \in (ne, c, gov, inv)$$

where E_j^e is the effective energy input into sector j , and $\gamma_{e,j}$ is the AEEI rate.

EPPA reference runs (neglecting uncertainty) use a nominal value of 0.75% per year for all regions and all sectors. This value is taken as the mode of the distribution. The lowest possible value is 0.2% per year and the highest is 1.5% per year. The resulting distribution is a beta distribution with parameters $\alpha = 2.2$ and $\beta = 3.3$, and is shown in Figure 3.

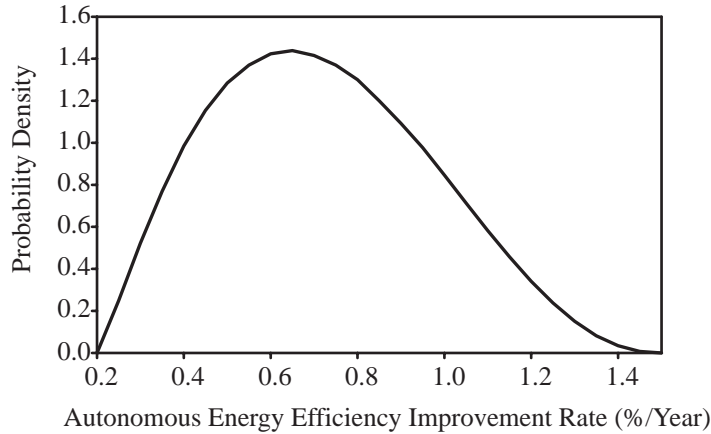


Figure 3. Probability Distribution for AEEI Parameter

2.5 Labor Productivity Growth

An important driver in the growth of the regional economies over time is the amount of “effective” labor supply available. The effective labor supply is the product of the population and the level of labor productivity. The labor productivity level is determined by:

$$Pl_r(t) = e^{\alpha_{l,r}(1 - e^{-\beta_{l,r}t})},$$

$$\alpha_{l,r} = \frac{\ln(1 + g_{0,r})}{1 - e^{-\beta_{l,r}}},$$

$$\beta_{l,r} = \ln \left[\frac{\ln(1 + g_{0,r})}{\ln(1 + g_{n,r})} \right] / Ho_z$$

where $g_{0,r}$ is the initial labor productivity growth rate for region r from calibration data, $g_{n,r}$ is the assumed labor productivity growth rate in the last time period (2100), and Ho_z is the time horizon, which is 115 years for 1985 to 2100. Thus, a driving assumption is the growth rate in 2100.

The reference assumption is that all OECD regions (USA, JPN, EEC, and OOE) as well as EEX all converge to a rate of 1.0% per annum by 2100, and that all other regions converge to a rate of 2.0% per annum. By making $g_{n,r}$ uncertain, the labor productivity growth rate will change in all periods, and have an impact on the effective supply of labor over time.

The mode of the distribution is again chosen to reproduce the reference assumption. The lowest possible assumed rates in 2100 are 0.5% and 1.0% for the OECD and non-OECD respectively; the highest possible rates are 1.5% and 3.0% respectively. This uncertainty is represented by a multiplicative factor applied to $g_{n,r}$ which ranges from 0.5 to 1.5 with a most likely value of 1.0. This is modeled by a beta distribution with parameters $\alpha = 1.5$, $\beta = 1.5$, as depicted in Figure 4.

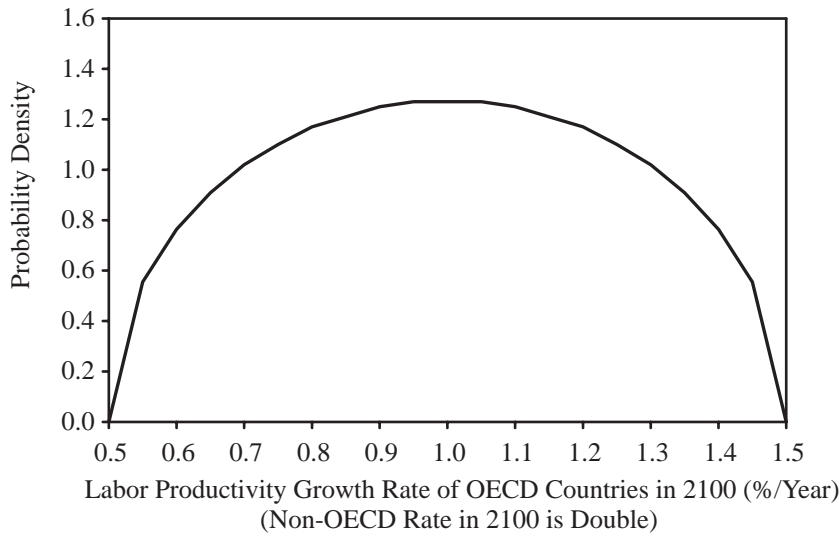


Figure 4. Probability Distribution for Labor Productivity Growth Rate

2.6 Uncertainty in Backstop Price

In addition to the eight conventional production sectors in EPPA, there are three future technology production sectors that may enter anytime after 2000: 1) a synthetic liquid fuel substitute for refined oil produced from shale, tar sands, or heavy oils, 2) a hydrogen fuel source, and 3) a carbon-free alternative technology which produces electricity. The hydrogen fuel source is very expensive and only enters the solution under carbon restrictions far more stringent than

anything studied here, and the carbon liquid technology was found in earlier studies to be less important as a source of uncertainty. Therefore we focus on the carbon-free electricity technology as an example of the effects of future technology assumptions.

The carbon-free electric backstop⁷ is not a particular technology, but rather a generic representative of a collection of technologies, including solar, advanced nuclear, and wind. It is modeled as being available in unlimited supply, requiring only capital, labor, and material inputs, the prices of which determine the cost of producing electricity from the backstop. The technology produces electricity that is perfectly substitutable for that produced by the conventional electricity sector.

The fundamental uncertainties about the backstop are when and how quickly this technology would become available. However, the 1985 cost is currently used as a proxy for characterizing this uncertainty. The lower the initial year price, the sooner the backstop will become economically desirable; higher prices cause the backstop to take longer to become available. The third uncertain parameter for this study was therefore chosen to be the initial year price for the carbon-free electric backstop.

The price of the carbon-free electric backstop is calibrated by the price in ¢/kWh of a unit of the output in the initial period (1985). This is used to set the relative quantities of the labor and capital inputs needed, given their 1985 prices. After the initial calibration, the price is determined from the changing prices of capital and labor, which evolve endogenously in the model. When the price of the backstop in a region falls below that of conventional electricity (the price of which tends to rise over time in the model as oil and gas are depleted), the backstop begins producing in the economy.

The nominal 1985 price for the backstop used is 15¢/kWh. Retaining this as a most likely value, the extreme endpoints for the distribution were assessed as 3¢/kWh at the low end and 27¢/kWh at the high end. These values are chosen for their effect on the time required for the backstop to become available, and do not reflect the true prices in 1985. The probability density function was fit to a beta with the same shape and variance as the PDF of labor productivity growth. The resulting distribution is shown in Figure 5. The three uncertain parameters are summarized in Table 1, which shows their mean, standard deviation, and relative error (defined as standard deviation divided by the mean).

Table 1. Mean and Standard Deviation of Uncertain Parameters

Parameter	Mean (Reference)	Standard Deviation	Relative Error
AEEI	0.75	0.185	25%
LPG	1.00	0.189	19%
BS	15.00	4.540	30%

⁷ “Backstop” is an economic term used to mean an alternative substitute for a scarce resource. The alternative is originally too costly to use, but as the conventional resource becomes more expensive, the alternative is eventually substituted, which prevents the price of the conventional resource from rising infinitely.

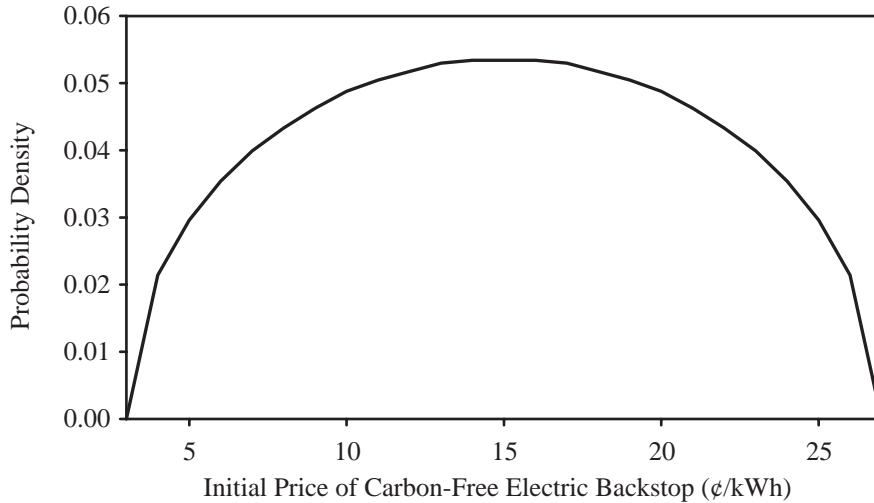


Figure 5. Probability Distribution for Price of Carbon-Free Electric Backstop

3. Uncertainty in CO₂ Emissions

Having identified uncertain outputs of interest (CO₂ emissions), and having represented the experts' belief about uncertainty in input parameters (AEEI, LPG, BS), the task now is to propagate the parametric uncertainty through the model and derive the distribution for the output variables.

3.1 The Deterministic Equivalent Modeling Method

There are a number of methods for propagating parametric uncertainty through models. The method most commonly used in practice is Monte Carlo simulation (Morgan and Henrion, 1990). The procedure for performing a “crude” Monte Carlo simulation is to randomly choose a sample value for each parameter from its probability distribution, run the model with this set of sample parameter values, and record the output(s) of interest. The procedure is then repeated with another independent random drawing of sample input values. Given a large number of simulations, the histogram of the output(s) can approximate with arbitrarily small error the “true” probability density function of the model output (Lewis and Orav, 1989).

One problem with this crude Monte Carlo approach is that the number of model runs needed to obtain a good approximation of the output distribution is usually quite large, often on the order of thousands of runs. For larger models, the computational cost can be prohibitive. There are many different variants to the Monte Carlo method that significantly reduce the number of simulations needed to obtain a good approximation. One widely used variant is the stratified sampling approach of Latin Hypercube Sampling (Iman, Davenport, and Zeigler, 1980). Latin Hypercube Sampling (LHS) breaks the probability distribution for each input parameter into n segments of equal probability. Then n sample values for each parameter are obtained either by choosing either a random value from each segment, or by choosing the median of each segment. A sample is generated by choosing one of the n values for each parameter at random and without replacement.

Repeating this procedure n times yields n sample sets. LHS is more efficient than brute-force Monte Carlo for covering the parameter space to extract estimates of mean and variance, but it does have limitations (Morgan and Henrion, 1990). For large numbers of input parameters or for highly non-linear models, LHS may still require on the order of hundreds of runs.

One other approach, the one used in this study, is the Deterministic Equivalent Modeling Method (DEMM) (Tatang *et al.*, 1997; Webster *et al.*, 1996). DEMM seeks to characterize the probabilistic response of the uncertain model output as an expansion of orthogonal polynomials. If such an expansion can be found which accurately mimics the probabilistic behavior of the model, then a Monte Carlo procedure can easily be used to derive the approximate probability density of the uncertain output.

Although any numerical computer model is itself deterministic, by positing uncertainty in a model parameter, the model's outputs become uncertain and thus can be thought of as a random variable. One useful representation for a random variable is an expansion of some family of orthogonal polynomials:

$$y = B_0 + B_1(\xi) + B_2(\xi) + \dots + B_N(\xi)$$

Any family of orthogonal polynomials can be used, including Legendre, Laguerre, or Hermite. This expansion is sometimes referred to as a polynomial chaos expansion (Weiner, 1938).

For example, suppose random variable y is some function of an underlying Gaussian distribution. Then one might represent random variable y as a truncated expansion of the Hermite polynomials, which are polynomials of the unit Gaussian random variable ξ :

$$y = a_1 + a_2\xi + a_3(\xi^2 - 1) + a_4(\xi^3 - 3\xi) \quad [1]$$

This expansion is 3rd order, and requires that we find values of the coefficients a_1, \dots, a_4 to represent the random variable y .

Since a model output y is some function of its uncertain input parameter x , we can use information about the density of x to choose basis functions for the expansion. We need not necessarily be limited to traditionally used orthogonal families such as Hermite polynomials. For the general case, we can derive the set of orthogonal polynomials weighted by the density function of the parameter, according to the definition of orthogonal polynomials:

$$\int_x P(x)H_i(x)H_j(x)dx = C_i\delta_{ij} \quad \text{where} \quad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad [2]$$

$H_i(x)$ and $H_j(x)$ are orthogonal polynomial functions of x of order i and j , $P(x)$ is some weighting function, and C_i is some constant.⁸ In other words, the integral of the product of two orthogonal polynomials of different order is always 0. By using the probability density function of an input as the weighting function $P(x)$, a set of orthogonal polynomials can be derived recursively.⁹

⁸ This constant is usually 1, and thus omitted, when the polynomials are normalized.

⁹ The zeroth order polynomial is always assumed to equal one.

Once we have derived the basis functions for the expansion, we need a way to choose the weighting coefficients. There is a class of methods designed for solving this problem known as the methods of weighted residuals (MWR) (Villadsen and Michelsen, 1978). The residual at any realization x_j of the random variable x , for some approximation $\hat{y}(x)$ of the function $y(x)$ is simply the difference:

$$R_N(\bar{a}, x_j) = y(x_j) - \hat{y}(a, x_j)$$

where $R_N(\bar{a}, x_j)$ is the residual for an N -term expansion with weighting coefficients $\bar{a} = [a_1, a_2, \dots, a_N]$. In general, MWR solves for N coefficients by solving the N relations:

$$\int_0^1 R_N(a, x) W_j(x) dx = 0, \quad j = 1, 2, \dots, N \quad [3]$$

Alternative schemes for MWR differ by the choice of the form of the weighting function $W_j(x)$.

Commonly used schemes include the least squares method, which chooses $W_j(x)$ to be $\frac{\partial R_N}{\partial a_j}$, or

Galerkin's method, which chooses $W_j(x)$ to be the derivatives of the approximation $\frac{\partial y_N}{\partial a_j}$.

The difficulty with these schemes is that they require the explicit analytical form of the model in order to solve for the weighting coefficients.

Because our goal is to approximate the uncertainty in a model output for any model, however complex, a method that allows the model to be treated as a "black-box" is preferable. This leads us to choose the collocation method, which consists of the choice of weighting function as the dirac delta function:

$$W_j(x) = \delta(x - x_j), \quad j = 1, 2, \dots, N$$

Since the integral of a function multiplied by a delta function is just the function evaluated at that point, solving Eq. [3] is equivalent to solving:

$$R_N(\bar{a}, x_j) = 0, \quad j = 1, 2, \dots, N \quad [4]$$

In other words, we simply solve for the set of a_j such that the approximation is exactly equal to the model at N points, and thus only require the model solution at N points and not the explicit model equations.

The final step in determining the polynomial chaos expansion to approximate the random variable is to choose the points x_j at which we evaluate the "true" model $y(x)$, in order to then solve for the a_i using Eq. [4]. For this step, we borrow from the well-known technique of Gaussian Quadrature, which uses the summation of orthogonal polynomials multiplied by weighting coefficients to approximate the solution of an integral. In Gaussian Quadrature, the optimal choice

of abscissas at which to evaluate the function being integrated are the N roots of orthogonal polynomial $p_N(x)$ (Press *et al.*, 1992). Similarly in DEMM, to solve for the N coefficients in the expansion

$$a_0 + a_1 p_1(x) + \dots + a_{N-1} p_{N-1}(x),$$

we use the residual evaluated at the N roots of $p_N(x)$.

In this paper, the goal is to characterize the uncertainty in carbon dioxide emissions. Therefore, the emissions in any time period is considered to be a random variable, which DEMM seeks to approximate with a polynomial chaos expansion. As an example, a second-order approximation for CO₂ emissions in 2100 is:

$$\begin{aligned} \text{CO}_2(2100) = & a_0 + a_1 AEEI_1 + a_2 LPG_1 + a_3 BS_1 + \\ & a_4 AEEI_2 + a_5 LPG_2 + a_6 BS_2 + \\ & a_7 AEEI_1 LPG_1 + a_8 LPG_1 BS_1 + a_9 AEEI_1 BS_1 \end{aligned} \quad [5]$$

where the a_i are constant weighting coefficients and $AEEI_j$ is the j^{th} order orthogonal polynomial derived from the AEEI probability density function using Eq. [2]. In order to find the approximation, DEMM needs only to derive the orthogonal polynomials from the input distributions, and then solve for the coefficients. To solve for the coefficients in the above example, we would need 10 runs of the model at different parameter sets of (AEEI, LPG, BS).

DEMM cannot find a sufficiently accurate approximation in every case. In particular, discontinuities in the response surface result in poor approximations. The approximation should always be checked against model results at values of the uncertain inputs other than those used to solve for the coefficients. An optimal choice of points to check the approximation against the model are based on the roots of the next higher orthogonal polynomial than the one used to find points to solve at. The reasons for this choice are that the roots of the next higher order always interleave the lower order roots (Press *et al.*, 1992), and that if the expansion of order N is poor, we already have the values needed to find the expansion of order $N+1$.

Once the expansion for the probabilistic model response is solved and found to be reasonably accurate, the approximate probability density of the model can be derived by applying crude Monte Carlo simulation to this expansion. Whereas the original model may be too computationally intensive to use crude Monte Carlo or even a stratified sampling scheme such as LHS, DEMM can approximate the probability density of a model output by using a small number runs of the model to find the approximation. Compared with a single realization of the model, the computational cost of performing crude Monte Carlo simulation on a polynomial chaos expansion can be considered negligible.

The approximations for uncertain outputs used in the remainder of this paper are each of third-order with all possible cross-terms. Note that each output variable will always have the same polynomial terms (derived from the same input uncertainties) and that only the coefficients vary for each output. Since each approximation has 20 terms, only 20 runs of the EPPA model were needed

to solve for the approximations. To check the accuracy of the approximations, 23 additional runs were performed, using the parameter value sets that would be needed to solve a fourth-order approximation. On average the errors measured as the normalized sum of squared deviations were less than 1%. Having solved for accurate approximations, all probability density functions for output variables were found by performing 10,000 Monte Carlo simulations on the approximations.

In short, DEMM allows the propagation of the input uncertainties from Section 2, and provides a quick and efficient means of estimating the probability density functions of many output variables of EPPA. We can now explore the resulting uncertainty in emissions and their implications.

3.2 Uncertainty in Emissions with No Policy

In trying to characterize the uncertainty in CO₂ emissions, we begin with emissions in the reference or no policy case. Any policy will be evaluated with respect to this reference case. Therefore, it is important to remind ourselves of the uncertainty in this reference itself.

There are several ways to think about emissions and their uncertainties: emissions from one region *vs.* global emissions, and emissions in any one time period *vs.* cumulative emissions over time. In this subsection, we consider global emissions in each time period, and return to cumulative emissions and regional emissions later. Figure 6 shows the probability distribution for global CO₂ emissions in the model periods 2020, 2050, and 2100. Several characteristics are apparent from this picture. First, the mean value of emissions is increasing over time. The variance of the distributions is also increasing, so that the range of uncertainty of the level of CO₂ emissions by 2100 is quite large. From the figure, it appears that there is little basis for distinguishing between emissions in 2100 of 16 GtC to 23 GtC (*i.e.*, emissions from 16 to 23 are of roughly equal probability).

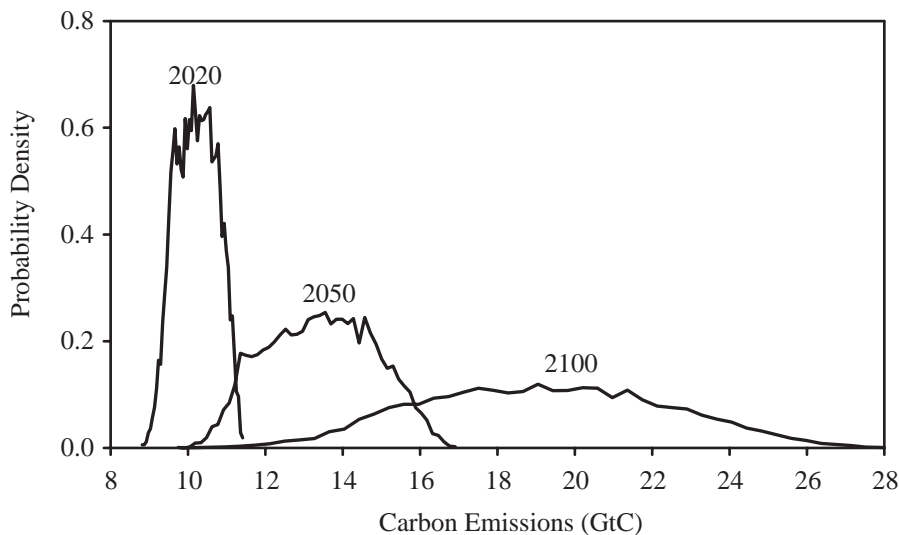


Figure 6. Probability Distribution for Global CO₂ Emissions in 2020, 2050, and 2100

The mean (middle line) and the 90% confidence interval (upper and lower lines) for global CO₂ emissions in each time period are shown in Figure 7. The increasing mean and variance are also illustrated here. The implications of uncertainty are important when evaluating the performance of a policy, but even in the absence of a policy the uncertainty in emissions forecasts has implications for interpreting results. What is often called a “baseline” for any model is in fact only one time series of values from a series of distributions of possible values. It is in this context that policy analysis with models should be viewed.

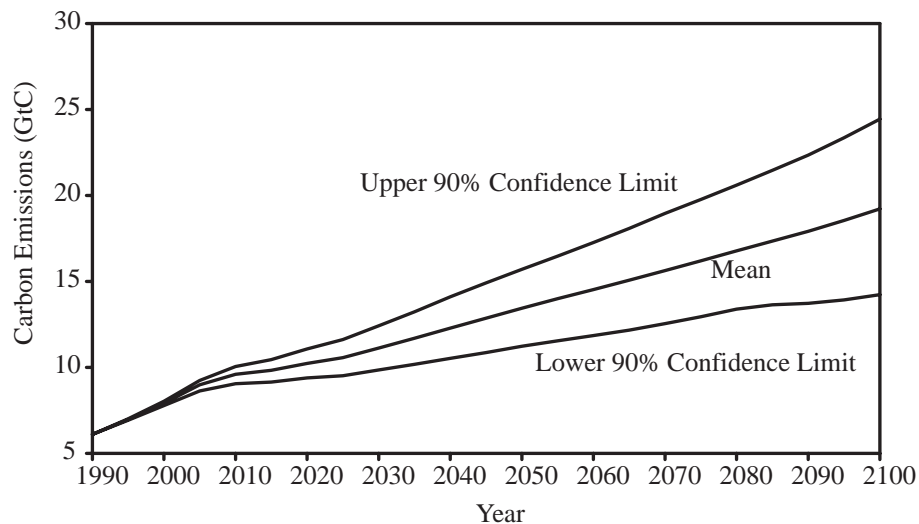


Figure 7. 90% Confidence Limits for Global CO₂ Emissions

As a part of the analysis, we want to be sure to understand the way in which the uncertain output is affected by each uncertain input. An added advantage to using DEMM is that the response surfaces of the model outputs have been approximated and can be viewed directly to understand the model’s behavior under uncertainty. Figures 8 and 9 show the response surface for global CO₂ emissions in the year 2050; the contour curves represent different levels of global emissions in GtC, and the ‘+’ indicates the emissions under the mean parameter values. Figure 8 shows emissions as a function of AEEI and labor productivity growth rate (with the backstop price fixed at 15¢/kWh). As the rate of productivity growth is increased, emissions increase; higher levels of economic activity result in higher energy usage and therefore higher emissions. As AEEI increases, emissions decrease. This is because higher rates of energy efficiency improvement cause less energy to be used, and thus leads to lower emissions.

Figure 9 shows emissions as a function of labor productivity growth and the initial year backstop price (with AEEI fixed at 0.75% per year). Over the central values for the backstop price, emissions decrease with lower prices. This is because a lower initial year price results in the carbon-free electric backstop being adopted sooner and more widely. However, for very low and

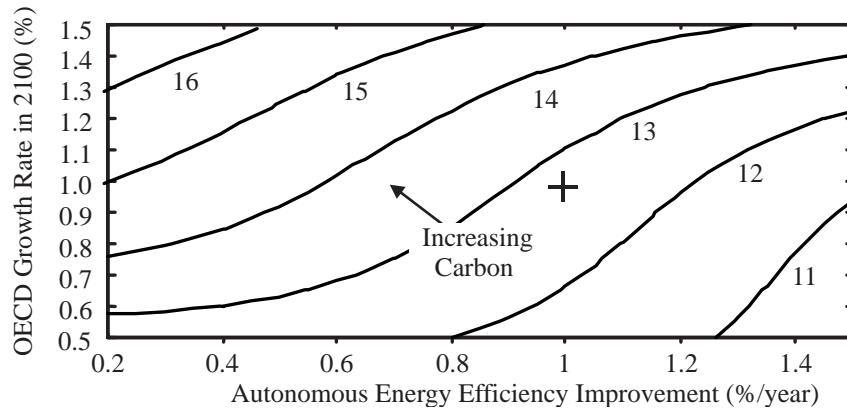


Figure 8. Contours for Global CO₂ Emissions in 2050: Economic Growth vs. Energy Efficiency (Curves Represent Global Emissions in GtC)

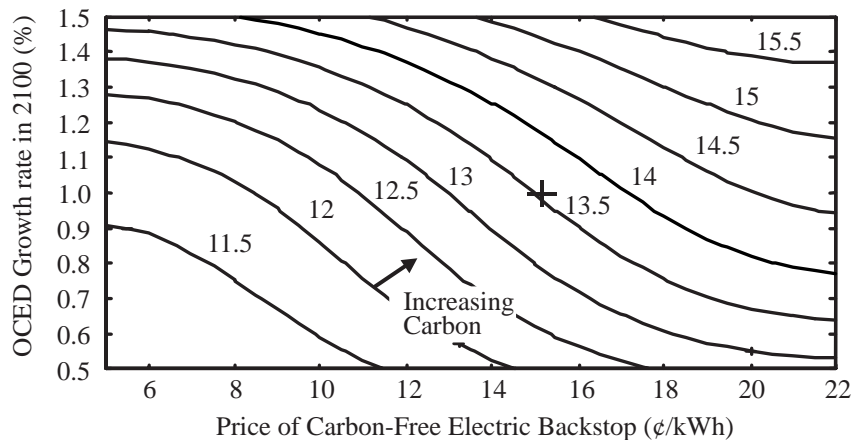


Figure 9. Contours for Global CO₂ Emissions in 2050: Economic Growth vs. Backstop Price (Curves Represent Global Emissions in GtC)

very high prices, emissions are relatively insensitive to changes in the initial year price. Such behavior is not surprising from a backstop, since there is likely to be a threshold price above which the backstop does not become affordable, and further increases in price will not change this result.

3.3 Uncertainty in the Effect of a Policy

EPPA and other economic-energy models are used to evaluate alternative policy proposals. To demonstrate the treatment of uncertainty in this type of analysis, we will work with a single policy proposal, the protocol proposed by the Alliance of Small Island States (AOSIS). The AOSIS protocol requires that all Annex I countries reduce to 20% below 1990 emission levels by 2005 (United Nations, 1997). The policy implemented here is a slightly altered and idealized version of the AOSIS protocol for the purpose of illustration, and has been used for several deterministic policy analyses with EPPA (Jacoby *et al.*, 1997). This version of AOSIS applies only to the four OECD regions (USA, EEC, JPN, and OOE), requiring that emissions be reduced to 20% below

1990 levels by 2010, and held constant at that level thereafter. No emissions constraints are applied to any other regions.

Although this analysis focuses only on an economic model and does not use a climate model to further analyze the impacts, a first measure of the benefits of a policy is the amount of *total cumulative* carbon emitted. This is because carbon dioxide is a stock pollutant with a relatively long lifetime (e-folding time) in the atmosphere, between 60 and 100 years (Wigley, 1993). Climate impacts depend more on the total emissions over time than on the precise timing of the emissions. Figure 10 shows the distribution for cumulative carbon for the case of no policy and the AOSIS protocol. Clearly the mean of the distribution for emissions under AOSIS is lower than the mean under reference, so cumulative emissions have been reduced. Also, because the OECD regions are constrained in the AOSIS case, the variance of cumulative emissions is lower. In fact, the variance in emissions in the AOSIS case is entirely due to the non-OECD regions. A more precise view of the probabilistic impact of AOSIS is the distribution of CO₂ reduction due to AOSIS, shown in Figure 11.

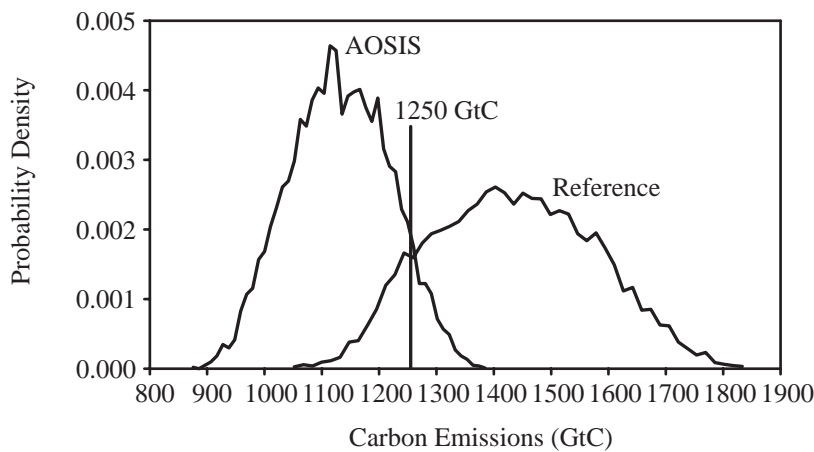


Figure 10. Cumulative CO₂ Emissions 1990–2100: No Policy vs. AOSIS

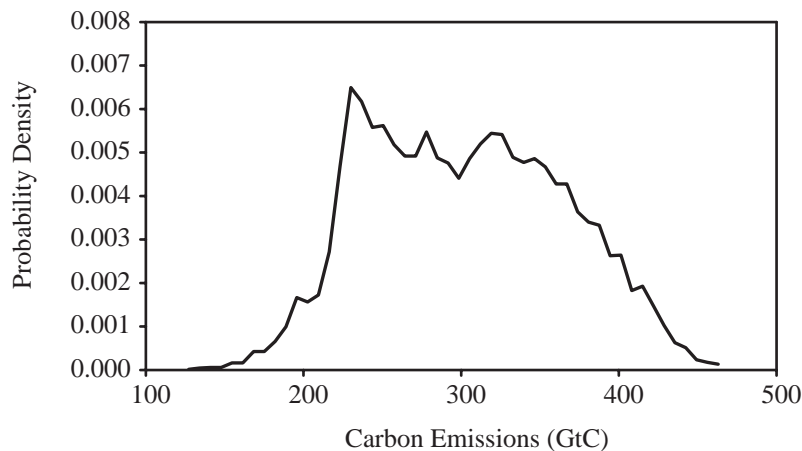


Figure 11. Reduction in Cumulative CO₂ Emissions 1990–2100 from AOSIS

While EPPA indicates that imposing the AOSIS protocol will reduce cumulative emissions regardless of the level of the uncertain parameters, there is another way to view these results. Suppose that through some process, the parties of the FCCC decide that it is desirable to have cumulative emissions over the next century be no more than 1250 GtC. Notice in Figure 10 that there is some small but not insignificant probability in the no policy case that the target may be achieved. At the same time, there is also some non-negligible probability that even under AOSIS, emissions will exceed the target.

This is also illustrated in Figure 12, which shows the 90% confidence interval for emissions in each time period for both the no policy and the AOSIS cases. In the earlier years when the variance in outcomes is less, the ranges of outcomes of the two policies are distinct. However, for more than 40 years into the future, the ranges overlap.

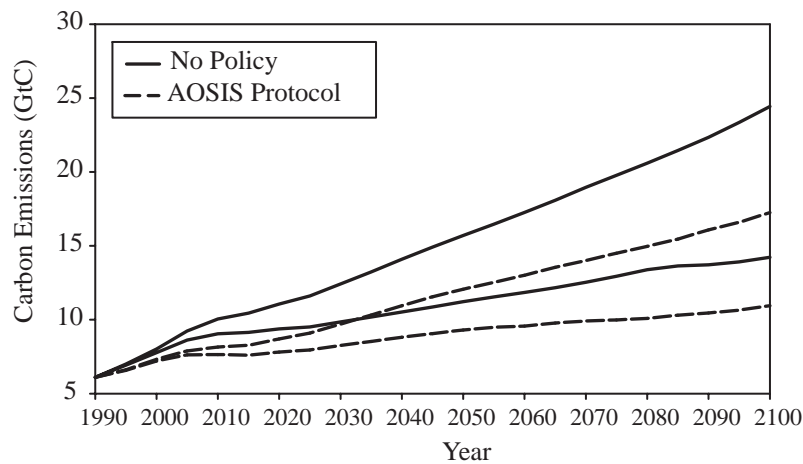


Figure 12. 90% Confidence Limits for CO₂ Emissions: No Policy vs. AOSIS

Depending on the rates of labor productivity and energy efficiency and the rate of penetration of alternative electricity technologies, there are worlds where AOSIS is imposed that have the same emissions or higher than worlds in which no policy was imposed. The influential work of Wigley, Richels, and Edmonds (Wigley, 1996) and subsequent analysis by others have focused attention on the question of how to choose a concentrations (and thus emissions) path through time. The overlap of resulting emissions (and concentrations), as shown in Figure 12, calls into question the validity and usefulness of approaches to climate policy of “choosing the appropriate path” of global emissions. This simple analysis of EPPA suggests that, due to economic variables beyond our control or ability to predict, we cannot choose a precise path of emissions through time, at least not very far into the future.

3.4 Regional Differences

Many economic-energy models used for climate policy analysis have a relatively small number of aggregated regions. The EPPA model is comparatively disaggregated with twelve regions, and

thus affords investigation into regional differences in emissions patterns. For the purpose here, it also allows a look at the performance of the different economies under uncertainty.

The distributions shown in Figure 13 illustrate the uncertainty in the percentage reduction in CO₂ emissions in the year 2050 that occurs in each of the OECD regions under the AOSIS protocol.¹⁰ Note that not only are the means different between regions, but the variance and shape of the distributions are all quite different. There are many possible reasons why each region's response is so different. One explanation is that the ability to substitute carbon-free electricity for other energy sources may vary between regions. A detailed analysis of the comparative effects is beyond the scope of this paper.

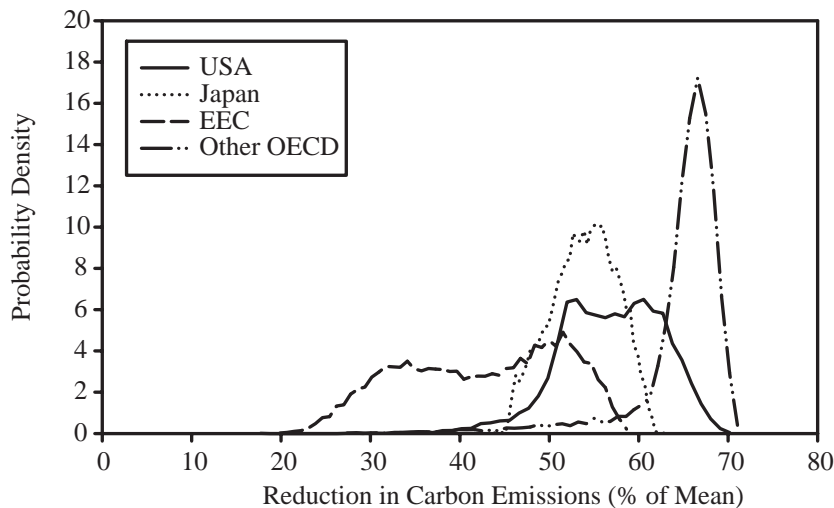


Figure 13. Reduction in OECD CO₂ Emissions in 2050 from AOSIS

Instead, we merely point out that in a multi-party negotiation process where uncertainty is large, such as the FCCC process, it is important to recognize that different economic and political structures will respond differently to the same uncertainties. Improved understanding of these differences can lead to more reasonable proposals from the beginning.

Another useful role for uncertainty analysis is to indicate when a result of a deterministic policy analysis is truly robust. As an example, we refer to the analysis of leakage performed with the EPPA model (Jacoby *et al.*, 1997). “Leakage” may occur when some regions enact a policy of carbon reductions (*e.g.*, the OECD countries), while others remain unconstrained (*e.g.*, developing countries). Emissions from the unconstrained regions may actually increase, partially offsetting the gains from the reductions. The main cause of leakage is a shift in relative prices that make certain high-emitting industries less competitive in the constrained regions and more competitive in the unconstrained regions. Thus, some of the “dirty” industries (for example, steel mills) will shift to developing countries as a result of the carbon reduction policy.

¹⁰ The curves in Figure 11 should not be construed as distributions of costs. These distributions illustrate the relative effectiveness of a policy in reducing emissions in different regions.

However, one might ask whether the demonstration of leakage in Jacoby *et al.* (1997) is robust to the uncertainty in EPPA. The answer is that the phenomenon is fairly robust for most regions under uncertainty, but not for all. Figure 14 shows two representative examples. The Dynamic Asian Economies (DAE) show a clear likelihood of increased emissions under AOSIS, most likely between 2 and 10%. On the other hand, emissions from Brazil (BRA) are so uncertain that one cannot draw any conclusions as to whether carbon will increase or decrease. Taking 2050 as a typical year, Figure 15 shows the probabilities of the AOSIS protocol causing an increase in carbon emissions for the unconstrained. Since all regions but Brazil have an 80% chance or better of increasing if AOSIS is imposed, leakage appears to be a fairly robust phenomenon under uncertainty. However, if all regions had displayed the behavior of BRA under uncertainty, the demonstration of leakage would have been questionable. This is an example of a robustness test that should ideally be applied to any major conclusions from deterministic model experiments.

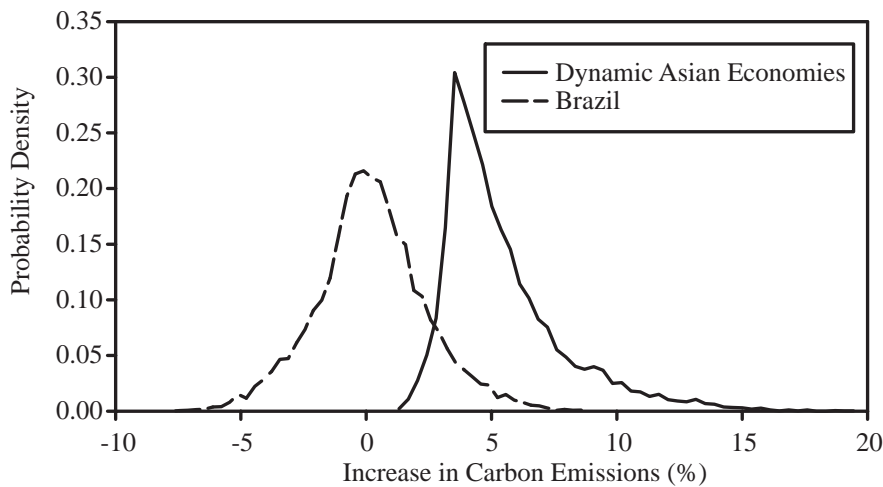


Figure 14. Increase in DAE and BRA Carbon Emissions from AOSIS

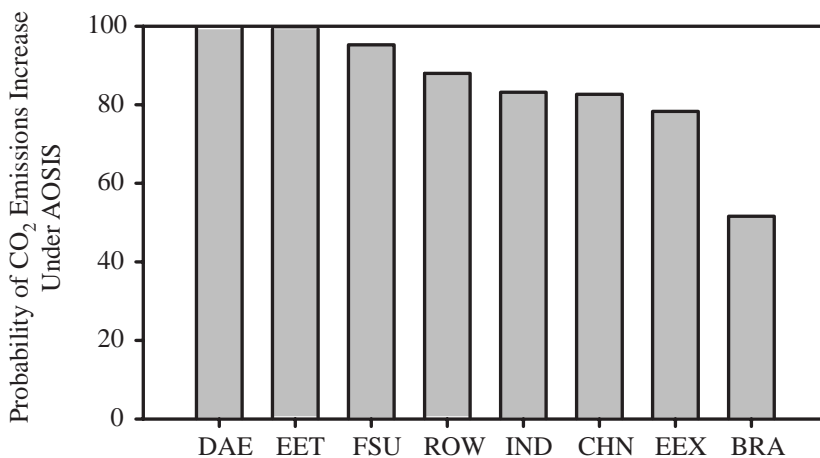


Figure 15. Probability of Increase in CO₂ Emissions due to AOSIS

3.4 Drivers of Uncertainty

One of the most important contributions of uncertainty analysis in policy models is the identification of the driving uncertainties, the uncertain parameters that cause the most variance in the outputs of interest. This is useful for two reasons. The first is that identifying the important uncertainties helps with the modeling effort itself. Modeling is an iterative process in which simple representations are constructed to help give us insight into a problem and its possible solutions. By identifying the aspects of the model that cause variance in the outputs, we can refine our understanding and representation of those important aspects, and neglect or further simplify the less important parameters or processes.

The second important reason for finding the driving uncertainties is that they often can guide policy choice directly. One example, given in Abbaspour *et al.* (1996), is how an uncertainty analysis of a model of a tunnel to be dug gives information on how many measurements of hydraulic conductivity to take to yield a robust design. Since data measurements are expensive, the uncertainty analysis guides the design of the tunnel to better meet its objectives of lower-cost and minimum water uptake.

In the uncertainty analysis presented in this paper, what does an analysis of the drivers of uncertainty tell us about policy choices and about desirable refinements of the model? Figure 16 shows the relative contribution to the variance in CO₂ emissions in 2020, in 2050, and cumulative over the entire century. Note that in the near term, indicated by the 2020 and 2050 pictures, uncertainty in the cost of the carbon-free electricity alternative is the primary cause of uncertainty in the amount of CO₂ that will be emitted. Furthermore, the uncertainty in the backstop cost and the rate of energy efficiency improvement together account for the vast majority of uncertainty in what emissions will be. Over the entire time horizon, the labor productivity growth will account for roughly one-quarter of the variance in emissions, but the remaining three-quarters is caused by the two parameters that represent technological improvements.

What does this mean for policy choices in the short-term? Common approaches to uncertainty in climate change policy consider that uncertainty will be reduced over time (*e.g.*, Manne and

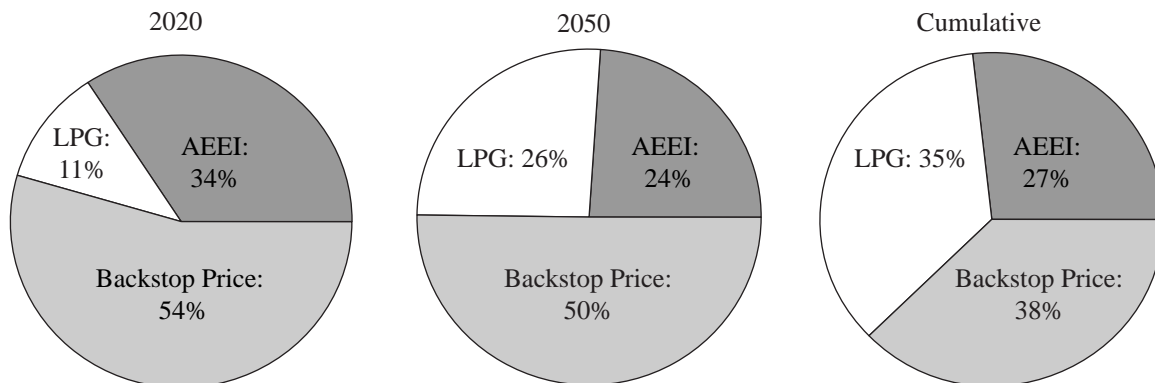


Figure 16. Relative Contribution to Uncertainty in CO₂ Emissions in 2020, 2050, and Cumulative 1990–2100

Richels, 1992; Nordhaus and Popp, 1996). In fact, this author contends that uncertainties in future technological developments will never be reduced to predictability, and that the long-term nature of climate change results in decisions at every point in time look far forward into the future. But this does not mean that there is nothing that can be done. Rather, policy can target these most influential uncertainties and attempt to shift the likely outcomes to those more desirable. In the case of climate change, focusing research and development to encourage energy efficiency improvements and develop new carbon-free energy alternatives sooner will both lower emissions and lower the costs of further emissions reductions in the future. This prescription is consistent with other recent policy recommendations, such as that by Jacoby *et al.* (1997), that investment in new technology is one clearly useful step that needs to be taken now in order to produce technological options for the future.

This analysis also suggests a course of action for further research. Although we know that technological innovation is an important lever for policy, the mechanisms through which such innovation can be encouraged is little understood (Jacoby *et al.*, 1997). Immediate policy dilemmas include whether such innovation can be encouraged more efficiently through price effects that would result from an emissions cap or a carbon tax, or whether in the near-term to simply invest more government and/or private research and development funds. Models such as EPPA that treat energy efficiency improvement and developments of technology alternatives as exogenous and unresponsive to policy choices are not helpful for the pressing questions. More thorough investigations into how these mechanisms for innovation might work and what the tradeoffs are is crucial.

This exploratory study only addresses three uncertain parameters, largely because of the intuition of the modelers and previous experimentation. As discussed further below, these conclusions need to be tested by formally including other uncertain parameters in comprehensive uncertainty analysis. Also, the relative importance of these economic uncertainties as compared with uncertainty in the climate science in affecting downstream outcomes must be investigated. Both of these steps are currently underway in the Integrated Global Systems Model of which EPPA is a component. Moreover, to account for structural uncertainties in models, it would be desirable if a larger set of integrated assessment models were subjected to similar uncertainty analyses.

4. Sensitivity to Assumptions about Input Distributions

4.1 Assumptions about Variance of Input Distributions

When performing a deterministic experiment with a model, it is accepted practice to subject the model to a sensitivity analysis. Some parameters are set to higher or lower values than in the original runs, and the experiment is repeated to observe how the results change. In a way, uncertainty analysis is no different from a deterministic model experiment, except that a parameter may have an assumed probability distribution of values instead of a single discrete value. Even the

results of an uncertainty analysis should be subjected to a form of sensitivity analysis if possible. Fortunately, the DEMM approach makes this sort of check relatively simple: the functional approximations of uncertain outputs can be subjected to a Monte Carlo procedure drawing from alternative input distributions.

Here we present two sensitivity tests, of the cumulative global carbon emissions under no policy and of the reduction in cumulative carbon under AOSIS. In addition to the standard assumptions described in Section 2, we test two alternative sets of assumptions about the parameter distributions. One alternative is to assume a uniform distribution over the same range of values for each parameter. Since the future trends represented by these parameters are so difficult to predict, this is not an unreasonable limiting case. On the other hand, we also perform Monte Carlo with beta distributions over the same range, but with half the variance of the standard assumption.¹¹ The question is whether halving the input uncertainty will reduce the output uncertainty by half, by more than half, or by less than half. These three alternative assumptions are shown for the backstop price uncertainty in Figure 17.

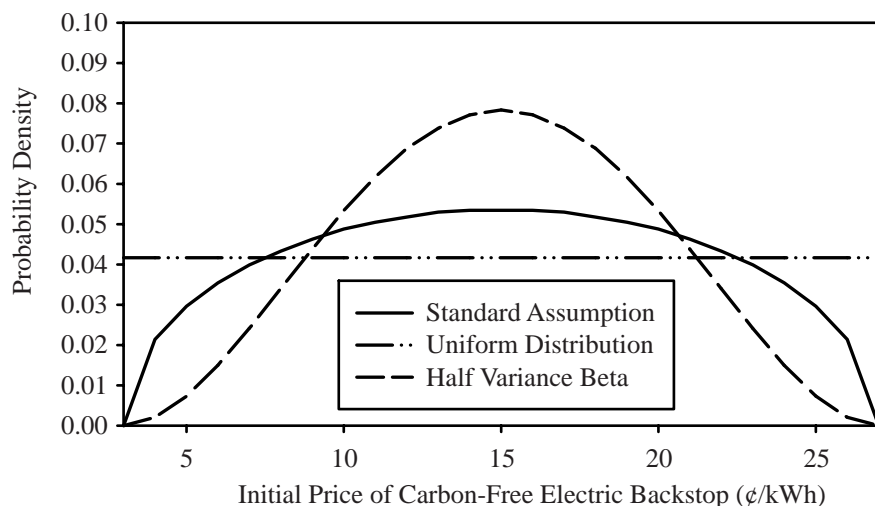


Figure 17. Alternative Assumptions about Input Uncertainties

The resulting distributions for each case are shown in Figure 18 for cumulative emissions and in Figure 19 for cumulative reductions from AOSIS. Interestingly, the first salient feature of these two figures is that the overall result does not change dramatically. The mean of each distribution is essentially unchanged, and the change in variance is small for the alternative assumptions (Figure 20). Most importantly, the insights generated from the distributions are fairly robust to different assumptions about the variance in the input parameters.

¹¹ The parameters for the half-variance beta are (4.8, 6.2) for AEEI and (3.0, 3.0) for LPG and BS.

In Figure 18, the distribution that results from the uniform assumption increases the probability of lower emissions worlds, but high emissions worlds do not become significantly more probable. This is another result of the threshold effects of a high initial backstop price. Increasing the probability of higher prices will not cause higher emissions if the backstop does not penetrate anyway; increasing the probability of lower prices *will* result in earlier penetration and decrease overall emissions. The half-variance assumption does reduce the variance in cumulative emissions, but not as much as might be expected. The changes in variance due to different input assumptions are easier to see in Figure 20, which shows the relative errors¹² for each distribution along with the relative errors of the standard input assumptions.

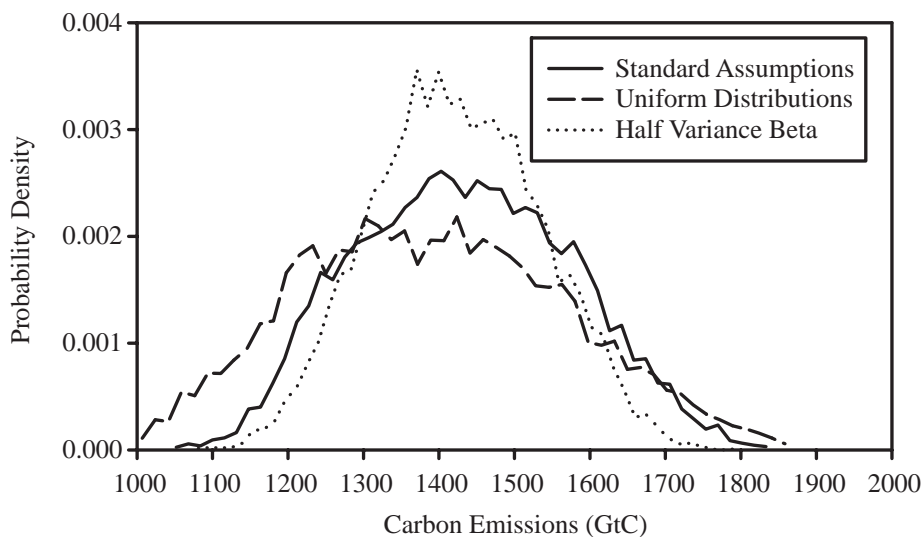


Figure 18. Distributions of Cumulative Global CO₂ Emissions with No Policy

¹² The relative error of a random variable is defined as the standard deviation divided by the mean, thus giving a normalized measure of spread for comparison.

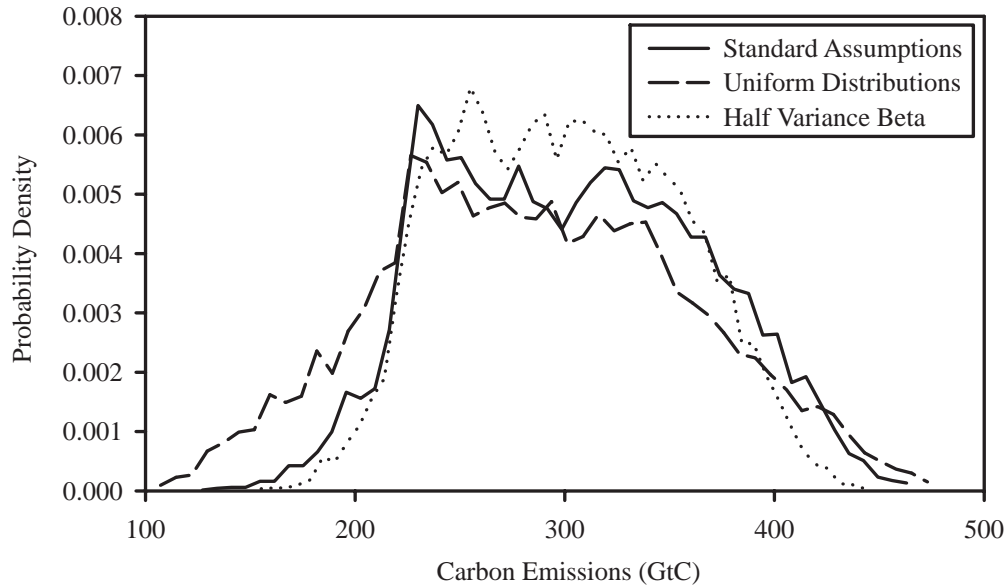


Figure 19. Distributions of Cumulative Global CO₂ Reductions under AOSIS

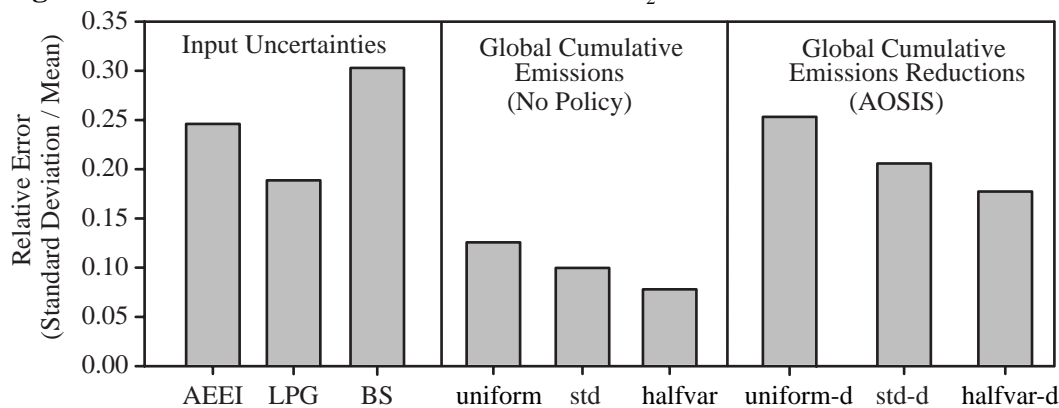


Figure 20. Relative Errors of the Input Parameters, Cumulative Global CO₂, and Cumulative Global Reductions under AOSIS

4.2 Sensitivity to Assumptions about Correlation

Another crucial assumption in an uncertainty analysis is whether correlations between input parameters are accounted for. Most studies of uncertainty in economic models for climate policy assume independence between all parameters (for an analysis that does address correlation, see Tschang and Dowlatabadi, 1995). The issue of correlation is discussed explicitly by Manne and Richels (1994) with regard to the relationship between AEEI and productivity growth, which are assumed to be independent because the model structure accounts for this relationship.

However, the assumption of independence, if not justified, can be problematic. Recall that if independence is assumed, a sample run with a very low value for productivity growth will have the same likelihood of a high rate of AEEI as a sample with high productivity growth. Intuition suggests that a world in which output is increasing at a faster rate is more likely to have rapid

improvements in energy efficiency than a world with much slower growth in output. Similarly, a rapidly increasing rate of output in general would suggest a greater likelihood that lower priced alternative energy substitutes will exist. Thus for the three parameters used here, an exploration of their correlation and how results change as a consequence would seem to be in order.

One main difficulty with treating correlation is its assessment. When two parameters x and y are independent, it means that the conditional distribution of y given some value for x is always simply the marginal distribution for y . However, if x and y are not independent, the conditional distribution for y given a value x_1 for x will be different than the conditional for y given another value x_2 for x . Thus, the standard approach consists of assessing the marginal distribution of one parameter and several conditional distributions of a second parameter at different values of the first (Morgan and Henrion, 1990). The difficulty with this approach is the large number of assessments needed from the experts. Since people consistently have biases in making judgments about probability (Tversky and Kahneman, 1974), large numbers of assessments introduce more potential for errors or internal inconsistency.

An alternative to assessing joint distributions as marginal and conditionals, is to assess marginals and a correlation coefficient. The correlation coefficient is a simple measure of association that is easier for people to think about and make qualitative judgments. However, as traditionally used, the correlation coefficient is only well defined for inducing correlation between two normal distributions, or with a transformation, lognormal distributions.

However, recent work by Clemen and Reilly (1997) has identified an alternative approach to inducing correlation between arbitrary marginal distributions. Their approach relies on a function called a copula, which can be used to encode the dependency in a joint distribution. In general, one can write the joint density function $f(x_1, \dots, x_n)$ as:

$$f(x_1, \dots, x_n) = f_1(x_1) \times \dots \times f_n(x_n) c[F_1(x_1), \dots, F_n(x_n)]$$

where $f_i(x_i)$ is the marginal density for the distribution function $F_i(x_i)$, and c is called the copula density. The details of the derivation of the copula will not be given here. Operationally, one assesses the rank correlation matrix between the marginal distributions, and then uses the formula for a copula. Given an independently drawn sample for one parameter (*e.g.*, for labor productivity), the rank correlation coefficient and the copula can be used to quickly derive the conditional distributions from which to draw a sample for other parameters¹³ (*e.g.*, for AEEI and backstop price).

As an initial exploration into the implications of including correlation, the subjective rank correlations between the three parameters were assessed by one of the EPPA experts. Applying a number of different assessment techniques (Clemen and Reilly, 1997), the rank correlation

¹³ Formally, DEMM assumes independence between the uncertain parameters used to derive the approximations of uncertain outputs. However, once the approximation is derived, if it is accurate over the entire sample space of the uncertain inputs, it is reasonable to perform Monte Carlo simulations using correlated sampling. On the other hand, if the approximation is less accurate in some regions of the sample space, performing correlated sampling may not be appropriate.

between AEEI and labor productivity was assessed as 0.4, the correlation of backstop price and productivity was -0.6 (anti-correlated), and no correlation (0) between AEEI and backstop price. Using these rank correlation coefficients and the copula method, an alternative Monte Carlo was performed again on cumulative emissions under no policy, and on cumulative reductions under AOSIS.

The effect of correlation on cumulative emissions in the no policy case is shown in Figure 21. Note that the distribution of cumulative carbon in the correlated case has lower variance. This is a consequence of associating higher AEEI, which tends to lower emissions, with higher productivity growth, which tends to increase emissions. These two effects work in opposite directions, and by introducing the correlation, the uncertainty in emissions is reduced. In addition, the association of higher productivity growth and lower backstop price also reduces variability in emissions due to the opposing effects of the two parameters. While the effect of correlation in these parameters on emissions uncertainty is to reduce the uncertainty, correlation between different parameters could just as easily increase uncertainty.

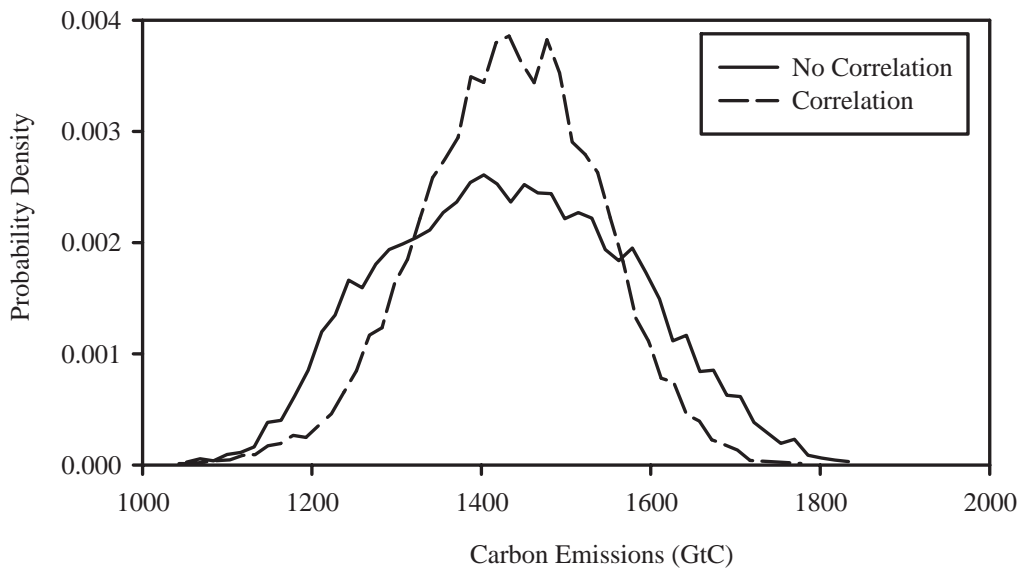


Figure 21. Distributions of Cumulative Global CO₂ Emissions: Correlation vs. No Correlation

Correlation has much less of an effect on the uncertainty in cumulative emissions reductions due to the AOSIS protocol, as seen in Figure 22, although here too variance is reduced in the correlated case. Figure 23 gives the relative error of the distributions in Figures 21 and 22. The means of the distributions of cumulative emissions and of cumulative reductions essentially do not change as a result of imposing correlation. Again, this is not true in general; for some other uncertainties the mean of a distribution may shift as a result of imposing correlation.

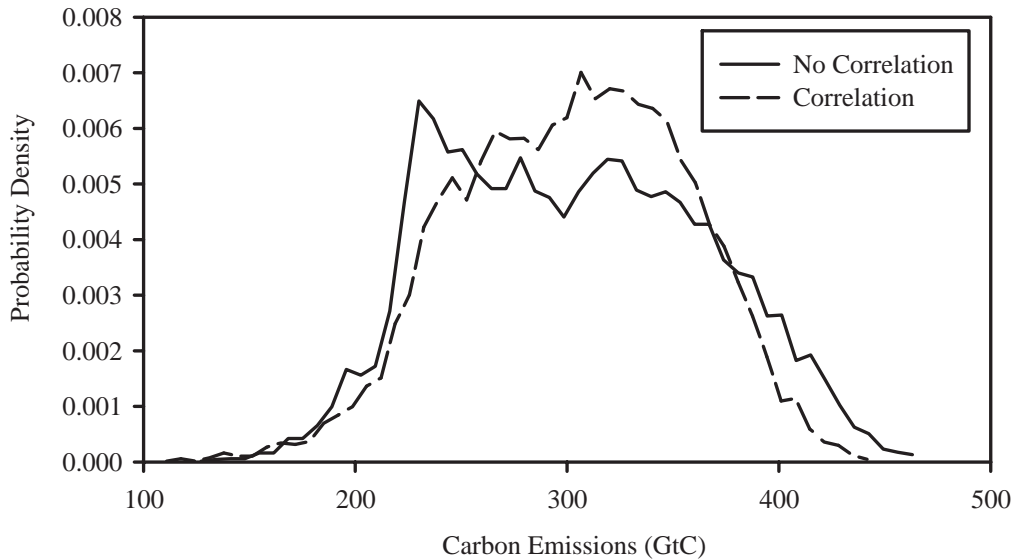


Figure 22. Distributions of Cumulative Global CO₂ Reductions: Correlation vs. No Correlation

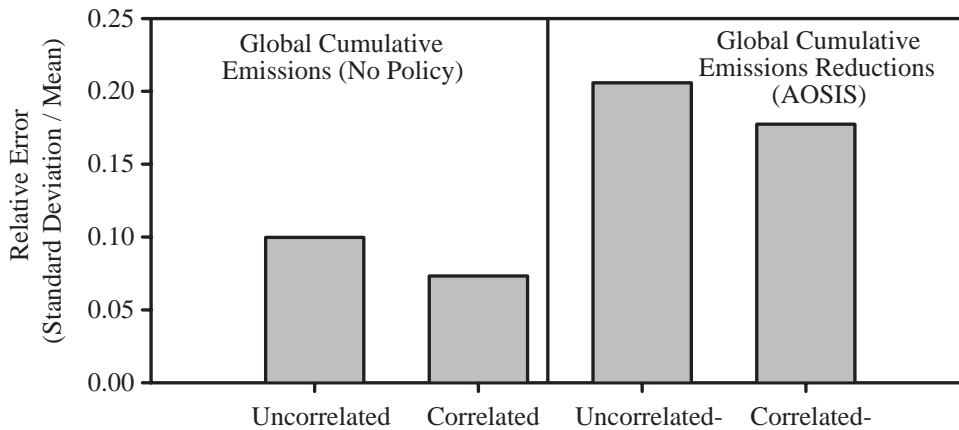


Figure 23. Relative Errors of Cumulative Global CO₂ and Cumulative Global Reductions: Correlation vs. No Correlation

Probability distributions of outcomes such as those presented here can also be used ultimately in decision-analytic framing of climate policy. In such an analysis, the variance of distributions and the probability of extreme events are major determinants. Uncertainty analysis, one part of a larger decision making framework, therefore must take assumptions such as correlation carefully into account.

5. Next Steps for Research

The purpose of this study has been both to demonstrate the application of the Deterministic Equivalent Modeling Method to a parametric uncertainty analysis and to provide a preliminary

exploration of the uncertainty in emissions predictions from the EPPA model. In the procedure described in Figure 2, this paper consists of one pass through the process, and the information obtained should be used to guide further iterations of uncertainty analysis. Future areas of research into uncertainty in EPPA include:

- Redo the probability assessments of the uncertain parameters, since they all appear to be important. A proper expert elicitation and distribution fitting is needed before results should be viewed as anything more than a demonstration.
- Propagate other uncertain parameters. There are several other uncertain parameters that may have important contributions to the uncertainty in outcomes. Other parameters to be studied include population growth, uncertainties in other future technologies, and elasticities of substitution. Uncertainty analysis with more parameters will allow more insight into the key drivers of uncertainty similar to the analysis in Section 3.4.
- Explore the uncertainty under other emissions reduction policies. The relative performance of different amounts of reduction and of different instruments for reduction (tax *vs.* tradable permit) under uncertainty is important as part of the overall climate policy analysis process.
- Quantify the uncertainty in other outcomes, especially the costs of implementing alternative policies.

All of this work on the economic side will still only contribute to one aspect of the overall analysis task to address the choice of a policy response to climate change. In the framework given in Figure 1, this all relates to the first box for emissions and cost predictions, as well as the initial choice of policies to study. To fully characterize the decision problem of climate policy under uncertainty requires investigation of other areas, including:

- Analysis of uncertainty in predicting climate change from a given emissions forecast;
- Analysis of uncertainty in impacts (ecological, health, agricultural, *etc.*) from changes in climate;
- Quantify overall uncertainty in costs and impacts under different policies that result from all economic and scientific uncertainties combined.
- Framing the analysis as a sequential decision problem, allowing the policy response to adapt over time as information changes.

6. Discussion and Conclusions

While uncertainty is at the heart of the debate over the response to climate change, relatively few policy analyses treat uncertainty explicitly. Although such treatment has traditionally been expensive, the advent of methods such as the Deterministic Equivalent Modeling Method and the copula approach to inducing correlation make uncertainty analyses quite tractable. The range of insights and information from an uncertainty analysis is considerable, and helps to frame results from deterministic model experiments.

The analysis presented in this paper is preliminary, focusing only on a small number of uncertain parameters, a small number of uncertain outcomes, and a small set of policies. Nevertheless, several useful insights result:

- There is a large range of likely outcomes, even in the reference or no policy case.
- Any policy case results in a wide band of possible emissions paths over time, and the bands from different policies often overlap. Policy proposals that are based on the notion of meeting a global path of emissions through time may be incorrectly framed, because they neglect the inherent stochastic nature of the outcomes. A more useful framing would be to think about the respective probabilities of meeting some goal.
- The costs and effectiveness of a policy in one nation in the presence of uncertainty may differ from the costs and effectiveness in another nation, as a result of differing economic and political conditions.
- Some results of deterministic model experiments may not always be robust under uncertainty, and should be checked. An example of this sort of test is given above, where the phenomenon of leakage is demonstrated to be robust, but not for all regions.
- The uncertainties regarding the cost of carbon-free energy alternatives and the rate of increasing energy efficiency improvements cause a much larger variance in the resulting CO₂ emissions. This suggests that a prudent policy approach include targeting the encouragement of new technology and reductions in energy use. Since the processes that prompt such technological innovation are poorly understood, this is a critical area for further research.
- The parameters studied here all concern future trends, which will always be uncertain. Given the large uncertainty in outcomes that result, the idea of flexible policies that can adapt over time appears to be a crucial element of the solution.

Ultimately, the most important lesson from the type of analysis presented in this paper is a reminder of how to view results of any model exercise: models can help to identify particular trends that result under different alternatives (policies), but they can *not* tell us what will happen, at least not within a wide range of outcomes. Models are tools that can help us think through complex decision problems, and uncertainty analysis of models is a way to improve the robustness of the insights derived and can indicate priorities for further study.

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References

- Burniaux, J., G. Nicoletti and J. Oliveira-Martins (1992). "GREEN: A Global Model for Quantifying the Cost of Policies to Curb CO₂ Emissions," *OECD Economic Studies*, **19**, Paris, Winter.
- Clemen, and J. Reilly (1997). "Correlations and Copulas for Decision and Risk Analysis," DRAFT, March.
- Edmonds, J., and J. Reilly (1983). "A Long-term Global Energy-Economic Model of Carbon Dioxide Release from Fossil Fuel Use," *Energy Economics*, April: 74-88.
- IPCC (1996). *Climate Change 1995—Economic and Social Dimensions of Climate Change*, Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change, J.P. Bruce, H. Lee and E.F. Haites (eds.), Cambridge University Press, Cambridge, UK.
- Jacoby, H.D., R.S. Eckaus, A.D. Ellerman, R.G. Prinn, D.M. Reiner and Z. Yang (1997). "CO₂ Emissions Limits: Economic Adjustments and the Distribution of Burdens," *The Energy Journal*, **18**(3): 31-58.
- Lewis, P.A.W., and E.J. Orav (1989). *Simulation Methodology for Statisticians, Operations Analysts, and Engineers, Vol. I*. Wadsworth & Brooks, Pacific Grove, CA.
- Manne, A.S., and R.G. Richels (1994). "The Costs of Stabilizing Global CO₂ Emissions: A Probabilistic Analysis Based on Expert Judgements," *The Energy Journal*, **15**(1): 31-56.
- Manne, A.S., and R.G. Richels (1990). "CO₂ Emissions Limits: An Economic Cost Analysis for the USA," *The Energy Journal*, **11**(2): 51-74.
- Morgan, M.G., and M. Henrion (1990). *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*, Cambridge University Press: New York.
- Nordhaus, W.D. (1994). *Managing the Global Commons*, MIT Press, Cambridge, MA.
- Press, W.H., S.A. Teukolsky, W.T. Vetterling and B.P. Flannery (1992). *Numerical Recipes in C*, Cambridge University Press: New York.
- Prinn, R., et. al. (1997). "Integrated Global System Model for Climate Policy Analysis: Feedbacks and Sensitivity Studies," submitted to *Climatic Change*, March.
- Tatang, M.A., W. Pan, R.G. Prinn and G.J. McRae (1997). "An Efficient Method for Parametric Uncertainty Analysis of Numerical Geophysical Models," *Journal of Geophysical Research*, **102**(D18): 21,925-21,932.
- Tschang, F.T., and H. Dowlatabadi (1995). "A Bayesian Technique for Refining the Uncertainty in Global Energy Model Forecasts," *International Journal of Forecasting*, **11**(1): 43-61.
- Tversky A., and D. Kahneman (1974). "Judgment under Uncertainty: Heuristics and Biases," *Science*, **185** (September): 1124-1131.
- United Nations (1992). "Framework Convention on Climate Change," *International Legal Materials*, **31**, pp. 849-873.
- United Nations (1997). "Framework Compilation of Proposals from Parties for the Elements of a Protocol or Another Legal Agreement," Framework Convention on Climate Change, Ad Hoc Group on the Berlin Mandate, FCCC/AGBM/1977/2. 31 January.
- Villadsen, J., and M.L. Michelsen (1978). *Solution of Differential Equation Models by Polynomial Approximation*, Prentice-Hall: Englewood Cliffs.
- Webster, M., M.A. Tatang and G.J. McRae (1996). *Application of the Probabilistic Collocation Method for an Uncertainty Analysis of a Simple Ocean Model*, MIT Joint Program on the Science and Policy of Global Change, Report No. 4, Cambridge, MA.

- Wigley, T.M.L., R. Richels and J.A. Edmonds (1996). "Economic and Environmental Choices in the Stabilization of Atmospheric CO₂ Concentrations," *Nature*, **379** (18 January): 240-243.
- Wigley, T.M.L. (1993). "Balancing the Carbon Budget. Implications for Projections of Future Carbon Dioxide Concentration Changes," *Tellus*, **45B**: 409-425.
- Yang, Z., R.S. Eckaus, A.D. Ellerman and H.D. Jacoby (1996). *The MIT Emissions Prediction and Policy Analysis (EPPA) Model*, Joint Program on the Science and Policy of Global Change, Report No. 6, MIT, Cambridge, MA.