1. Consider an approximate velocity profile within a laminar boundary layer over a wall given by:

\[
\frac{u(y; x)}{U(x)} = \begin{cases} 
  a(x)\eta + b(x)\eta^2 & y < \delta(x) \\
  1 & y \geq \delta(x)
\end{cases}
\]  

where \( \eta \equiv y / \delta(x) \), and \( a(x) \) and \( b(x) \) are constants at any given \( x \), and \( \delta(x) \) is a measure of the boundary layer thickness at \( x \).

(a) By applying one boundary condition at \( y = \delta \) which matches the boundary layer flow with the outer flow \( U(x) \), obtain an algebraic relationship between \( a \) and \( b \).

(b) Now write both \( a(x) \) and \( b(x) \) in terms of a new parameter \( \Lambda(x) \) such that the relationship between \( a \) and \( b \) found in (a) is maintained. In other words, find \( a(\Lambda) \) and \( b(\Lambda) \).

(c) Write the velocity profile (1) in terms of \( \Lambda \) instead of \( a \) and \( b \).

(d) Using the assumed velocity profile and the \( x \)-momentum boundary layer equation

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2}
\]
evaluated at \( y = 0 \), obtain an expression for \( \Lambda \) in terms of \( U \), \( \delta \), and \( \nu \). Also obtain an expression for the pressure gradient \( dp / dx \) in terms of \( \rho \), \( U \), \( \Lambda \), \( \delta \), and \( \nu \).

(Hint: Polhausen approach, but a different profile than the Lecture 17 notes - be careful)

(e) For what value of \( \Lambda \) is \( dp / dx \) zero? The pressure gradient is favorable when \( \Lambda \) is [greater than, less than] the value \( \ldots \).
(f) For the velocity profile corresponding to \( \frac{dp}{dx} = \frac{dU}{dx} = 0 \), compute the (i) displacement thickness \( \delta^* \), (ii) momentum thickness \( \theta \), and (iii) wall shear stress \( \tau_0 \) at any location \( x \) in terms of \( \delta(x) \). Let \( U(x) = U_0 = \text{constant} \) in this case.

Assume \( U = U_0 \) from this point on. This corresponds to uniform flow over a flat plate.

(g) Derive a differential equation for \( \delta(x) \) using the von Karman momentum integral equation.

(h) Solve the differential equation in (g) for \( \delta(x) \) with the boundary condition \( \delta(0) = 0 \).

How do \( \delta'(x) / x \) and \( \theta / x \) depend on the local Reynolds number \( \text{Re}_x \)? How does this compare with the Blasius flat plate laminar boundary layer result?

2. Supplementary Problem Va22.

3. Supplementary Problem Va 24.