1. Complete the following statements:

a) If you took a long time-exposure photograph of a luminous particle moving with the flow in a river, the image would be a ( pathline, streakline, streamline ).

b) A ________________ is a curve in a flow field which has the property that the velocity vector of each particle on the curve is tangent to the curve.

c) There are a ( finite, variable, infinite ) number of streamlines in a flow field.

d) If $d\vec{R}$ is the infinitesimal displacement vector along a streamline and $\vec{V}$ is the velocity of a particle on the streamline, then $d\vec{R} \times \vec{V} = _______$. 

e) If you continuously inject dye at a fixed point into a flowing fluid in such a way that the dye particles essentially match the speed of the fluid as they enter the flow, the pattern of dye you see in a snapshot of the flow is a ( pathline, streakline, streamline ).

f) A streamline is an ( Eulerian, Lagrangian ) concept.

g) A pathline is an ( Eulerian, Lagrangian ) concept.

2. If you saw a plot of a flow in which two initially separate streamlines eventually intersected with each other at a point, what would that necessarily tell you about the fluid velocity at that point? Why?

3. We know that $\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$. Using vector identities, show that the following expressing is equivalent: $\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \left( \frac{\nabla \cdot \vec{V}^2}{2} \right) - (\nabla \times \vec{V}) \times \vec{V}$. Note that $V^2 = \vec{V} \cdot \vec{V}$.

Hint: See Recitation 1 handout.
4. In a certain river with a velocity field \( \bar{v}(x,t) \) with respect to the ground, the concentration of dissolved oxygen is given by the function \( f(x,t) \). A small fish is in the river and has a velocity \( \bar{U}(t) \) with respect to the ground.

The time rate of change of \( f \) experienced by the fish is:

The fish lays eggs that are very small and neutrally buoyant. The time rate of change of \( f \) experienced by the eggs is:

Some of the eggs eventually become trapped among rocks and remain there. The time rate of change of \( f \) experienced by the eggs is then:

5. In an incompressible flow where \( \dot{\rho} = \nabla \cdot \rho + \partial \rho/\partial t = 0 \), the density \( \rho \) in general

a) may vary in time at a particular point in the flow field
b) must be a constant at all points in the flow field
c) may vary in space but not time throughout the flow field

6. For a steady one-dimensional flow with a velocity magnitude \( u \),

a) the velocity gradient \( \partial u/\partial x \) must be zero at any point
b) the density \( \rho \) must be the same at all points in space
c) the rate of change of the density of a particle as it moves through the flow depends only on the spatial variation of the density