6.9 Wave Forces on a Body

\[ U = \omega A \]
\[ Re = \frac{U\ell}{\nu} = \frac{\omega A\ell}{\nu} \]
\[ K_e = \frac{UT}{\ell} = \frac{A\omega T}{\ell} = 2\pi \frac{A}{\ell} \]

\[ C_F = \frac{F}{\rho g A \ell^2} = f \left( \frac{A}{\lambda}, \frac{\ell}{\lambda}, R_e, \frac{h}{\lambda}, \text{roughness, \ldots} \right) \]

- Wave steepness
- Diffraction parameter
6.9.1 **Types of Forces**

1. **Viscous forces** Form drag, viscous drag = \( f(R_e, K_e, \text{roughness}, \ldots) \).
   - *Form drag (\( C_D \))*
     Associated primarily with flow separation - normal stresses.

   ![Particle vel.](image1)

   ![wake](image2)

   - *Friction drag (\( C_F \))*
     Associated with skin friction \( \tau_w \), i.e., \( \vec{F} \sim \int \tau_w dS \).

   ![Friction drag](image3)
2. **Inertial forces** Froude-Krylov forces, diffraction forces, radiation forces.

Forces arising from potential flow wave theory,

\[
\vec{F} = \iint p \hat{n} dS, \quad \text{where } p = -\rho \left( \frac{\partial \phi}{\partial t} + gy + \frac{1}{2} |\nabla \phi|^2 \right)
\]

For linear theory, the velocity potential \(\phi\) and the pressure \(p\) can be decomposed to

\[
\phi = \phi_I + \phi_D + \phi_R
\]

\[
-\frac{p}{\rho} = \frac{\partial \phi_I}{\partial t} + \frac{\partial \phi_D}{\partial t} + \frac{\partial \phi_R}{\partial t} + gy
\]

(a) **Incident wave potential**

- **Froude-Krylov Force approximation** When \(\ell << \lambda\), the incident wave field is not significantly modified by the presence of the body, therefore ignore \(\phi_D\) and \(\phi_R\). Froude-Krylov approximation:

\[
\phi \approx \phi_I
\]

\[
p \approx -\rho \left( \frac{\partial \phi_I}{\partial t} + gy \right)
\]

\[
\Rightarrow \vec{F}_{FK} = \iint_{\text{body surface}} -\rho \left( \frac{\partial \phi_I}{\partial t} + gy \right) \hat{n} dS \quad \approx \quad \text{can calculate knowing (incident) wave kinematics (and body geometry)}
\]

- **Mathematical approximation** After applying the divergence theorem, the \(\vec{F}_{FK}\) can be rewritten as \(\vec{F}_{FK} = -\iint_{\text{body volume}} p_I \hat{n} dS = -\iiint_{\text{body volume}} \nabla p_I d\mathcal{V}\).

If the body dimensions are very small comparable to the wave length, we can assume that \(\nabla p_I\) is approximately constant through the body volume \(\mathcal{V}\) and 'pull' the \(\nabla p_I\) out of the integral. Thus, the \(\vec{F}_{FK}\) can be approximated as

\[
\vec{F}_{FK} \approx \left( -\nabla p_I \right) \left| \iiint_{\text{body volume}} d\mathcal{V} = \iiint_{\text{body volume}} \left( -\nabla p_I \right) \right| \text{at body center}
\]

The last relation is particularly useful for small bodies of non-trivial geometry - for 13.021, that is all bodies that do not have a rectangular cross section.
(b) Diffraction and Radiation Forces

(b.1) **Diffraction or scattering force** When $\ell \ll \lambda$, the wave field near the body will be affected even if the body is stationary, so that no-flux B.C. is satisfied.

$$\phi_t \rightarrow \phi_D$$

\[
\begin{align*}
\frac{\partial \phi}{\partial n} &= 0 = \frac{\partial}{\partial n} (\phi_t + \phi_D) \\
\text{or } \frac{\partial \phi_D}{\partial n} &= -\frac{\partial \phi_t}{\partial n} \leftarrow \text{given}
\end{align*}
\]

\[
\vec{F}_D = \int\int_{\text{body surface}} -\rho \left( \frac{\partial \phi_D}{\partial t} \right) \hat{n} dS
\]

(b.2) **Radiation Force - added mass and damping coefficient** Even in the absence of an incident wave, a body in motion creates waves and hence inertial wave forces.

\[
\frac{\partial \phi_t}{\partial n} = \hat{U} \cdot \hat{n}
\]

\[
\vec{F}_R = \int\int_{\text{body surface}} -\rho \left( \frac{\partial \phi_R}{\partial t} \right) \hat{n} dS = - m_{ij} \hat{U}_j - d_{ij} U_j
\]
6.9.2 Important parameters

\[
(1) \quad K_c = \frac{U_T}{\ell} = 2\pi \frac{A}{\ell} \quad \text{Interrelated through maximum wave steepness} \quad \frac{A}{\lambda} \leq 0.07
\]
\[
(2) \quad \text{diffraction parameter } \frac{\ell}{\lambda} \quad \left(\frac{A}{\ell}\right) \left(\frac{\ell}{\lambda}\right) \leq 0.07
\]

- If \( K_c \leq 1 \): no appreciable flow separation, viscous effect confined to boundary layer (hence small), solve problem via potential theory. In addition, depending on the value of the ratio \( \frac{\ell}{\lambda} \),
  
  - If \( \frac{\ell}{\lambda} << 1 \), ignore diffraction, wave effects in radiation problem (i.e., \( d_{ij} \approx 0, m_{ij} \approx m_{ij} \) infinite fluid added mass). F-K approximation might be used, calculate \( \vec{F}_{FK} \).
  
  - If \( \frac{\ell}{\lambda} >> 1/5 \), must consider wave diffraction, radiation \( \left(\frac{A}{\ell}\right) \leq \frac{0.07}{\ell/\lambda} \leq 0.035 \).

- If \( K_c >> 1 \): separation important, viscous forces can not be neglected. Further on if \( \frac{\ell}{\lambda} \leq \frac{0.07}{A/\ell} \) so \( \frac{\ell}{\lambda} << 1 \) ignore diffraction, i.e., the Froude-Krylov approximation is valid.

\[
F = \frac{1}{2} \rho \ell^2 U(t) |U(t)| C_D(R_e) 
\]

- Intermediate \( K_c \) - both viscous and inertial effects important, use Morrison’s formula.

\[
F = \frac{1}{2} \rho \ell^2 U(t)|U(t)| C_D(R_e) + \rho \ell^3 \dot{U} C_m(R_e, K_c)
\]
I. Use: \( C_D \) and \( F - K \) approximation.

II. Use: \( C_F \) and \( F - K \) approximation.

III. \( C_D \) is not important and \( F - K \) approximation is not valid.