Introduction to Aircraft
Performance and Static Stability

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September 18, 2003
Today’s Topics

• Specific fuel consumption and Breguet range equation
• Transonic aerodynamic considerations
• Aircraft Performance
  – Aircraft turning
  – Energy analysis
  – Operating envelope
  – Deep dive of other performance topics for jet transport aircraft in Lectures 6 and 7
• Aircraft longitudinal static stability
Thrust Specific Fuel Consumption (TSFC)

• Definition:  \( TSFC = \frac{\text{lb of fuel burned}}{\text{(lb of thrust delivered)(hour)}} \)

• Measure of jet engine effectiveness at converting fuel to useable thrust

• Includes installation effects such as
  – bleed air for cabin, electric generator, etc..
  – Inlet effects can be included (organizational dependent)

• Typical numbers are in range of 0.3 to 0.9. Can be up to 1.5

• Terminology varies with time units used, and it is not all consistent.
  – TSFC uses hours
  – “c” is often used for TSFC
  – Another term used is \( c_t = \frac{\text{lb of fuel burned}}{\text{(lb of thrust delivered)(sec)}} \)
Breguet Range Equation

- Change in aircraft weight = fuel burned
  \[ dW = -c_t T dt \quad c_t = \text{TSFC}/3600 \quad T = \text{thrust} \]

- Solve for \( dt \) and multiply by \( V_\infty \) to get \( ds \)
  \[
  ds = V_\infty dt = \frac{-V_\infty dW}{c_t T} = \frac{-V_\infty W}{c_t T} dW = \frac{-V_\infty L}{c_t D} dW
  \]

- Set \( L/D, c_t, V_\infty \) constant and integrate
  \[
  R = \frac{3600}{\text{TSFC}} V_\infty \frac{L}{D} \ln\left(\frac{W_{\text{TO}}}{W_{\text{empty}}}\right)
  \]
Insights from Breguet Range Equation

\[ R = \frac{3600}{TSFC} V_\infty \frac{L}{D} \ln \frac{W}{W_{\text{TO}}} \]

\( \frac{3600}{TSFC} \) represents propulsion effects. Lower TSFC is better.

\( V_\infty \frac{L}{D} \) represents aerodynamic effect. L/D is aerodynamic efficiency.

\( V_\infty \frac{L}{D} = a_\infty M_\infty \frac{L}{D} \). \( a_\infty \) is constant above 36,000 ft. \( M_\infty \frac{L}{D} \) important.

\[ \ln \frac{W_{\text{TO}}}{W_{\text{empty}}} \] represents aircraft weight/structures effect on range.
Optimized L/D - Transport A/C

“Sweet spot” is in transonic range.

Ref: Shevell

Losses due to shock waves
Transonic Effects on Airfoil $C_d$, $C_1$

Region I. $M_\infty < M_{cr}$

Region II. $M_{cr} < M_\infty < M_{drag \ divergence}$

Region III. $M_\infty > M_{drag \ divergence}$
Strategies for Mitigating Transonic Effects

• Wing sweep
  – Developed by Germans. Discovered after WWII by Boeing
  – Incorporated in B-52

• Area Ruling, aka “coke bottling”
  – Developed by Dick Whitcomb at NASA Langley in 1954
    • Kucheman in Germany and Hayes at North American contributors
  – Incorporated in F-102

• Supercritical airfoils
  – Developed by Dick Whitcomb at NASA Langley in 1965
    • Percey at RAE had some early contributions
  – Incorporated in modern military and commercial aircraft
Basic Sweep Concept

- Consider Mach Number normal to leading edge
  \[ \sin \mu = \frac{1}{M_\infty} \]
  \[ \mu = \text{Mach angle, the direction disturbances travel in supersonic flow} \]

- For subsonic freestreams, \( M_n < M_\infty \) - Lower effective “freestream” Mach number delays onset of transonic drag rise.
- For supersonic freestreams
  - \( M_n < 1, \Lambda > \mu \) - Subsonic leading edge
  - \( M_n > 1, \Lambda < \mu \) - Supersonic leading edge
- Extensive analysis available, but this is gist of the concept
Wing Sweep Considerations $M_\infty > 1$

- **Subsonic leading edge**
  - Can have rounded subsonic type wing section
    - Thicker section
    - Upper surface suction
    - More lift and less drag

- **Supersonic leading edge**
  - Need supersonic type wing section
    - Thin section
    - Sharp leading edge
Competing Needs

• Subsonic Mach number
  – High Aspect Ratio for low induced drag
• Supersonic Mach number
  – Want high sweep for subsonic leading edge
• Possible solutions
  – Variable sweep wing - B-1
  – Double delta - US SST
  – Blended - Concorde
  – Optimize for supersonic - B-58
Supercritical Airfoil

Supercritical airfoil shape keeps upper surface velocity from getting too large.

Uses aft camber to generate lift.

Gives nose down pitching moment.
Today’s Performance Topics

• Turning analysis
  – Critical for high performance military a/c. Applicable to all.
  – Horizontal, pull-up, pull-down, pull-over, vertical
  – Universal M-ω turn rate chart, V-n diagram

• Energy analysis
  – Critical for high performance military a/c. Applicable to all.
  – Specific energy, specific excess power
  – M-h diagram, min time to climb

• Operating envelope

• Back up charts for fighter aircraft
  – M-ω diagram - “Doghouse” chart
  – Maneuver limits and some example
  – Extensive notes from Lockheed available. Ask me.
Horizontal Turn

W = L \cos \phi, \phi = \text{bank angle}

Level turn, no loss of altitude

F_r = (L^2 - W^2)^{1/2} = W(n^2-1)^{1/2}

Where n \equiv L/W = 1/ \cos \phi \text{ is the load factor measured in “} \text{g’s”}.

But F_r = (W/g)(V^2_{\infty}/R)

So radius of turn is

R = V^2_{\infty}/g(n^2-1)^{1/2}

And turn rate \( \omega = V_{\infty}/R \) is

\[ \omega = g(n^2-1)^{1/2} / V_{\infty} \]

Want high load factor, low velocity
Pull Up

\[ F_r = (L - W) = W(n-1) \]
\[ = (W/g)(V_\infty^2/R) \]
\[ R = V_\infty^2/g(n-1) \]
\[ \omega = g(n-1)/ V_\infty \]

Pull Over

\[ F_r = (L + W) = W(n+1) \]
\[ = (W/g)(V_\infty^2/R) \]
\[ R = V_\infty^2/g(n+1) \]
\[ \omega = g(n+1)/ V_\infty \]

Vertical

\[ F_r = L = Wn \]
\[ = (W/g)(V_\infty^2/R) \]
\[ R = V_\infty^2/gn \]
\[ \omega = gn/ V_\infty \]
Let $\dot{\gamma} = \omega$

**Pull Over**

$K_\omega = \frac{(n+1)}{(n^2 - 1)^{1/2}}$

**Vertical Maneuver**

$K_\omega = \frac{n}{(n^2 - 1)^{1/2}}$

**Pull Up**

$K_\omega = \frac{(n-1)}{(n^2 - 1)^{1/2}}$

For large $n$, $K_\omega \ll 1$ and for all maneuvers $\omega \approx gn/ V_\infty$

Similarly for turn radius, for large $n$, $R \approx V_\infty^2/gn$.

For large $\omega$ and small $R$, want large $n$ and small $V_\infty$. 
\[ \omega \approx \frac{g}{V_\infty} = \frac{g}{a_\infty M_\infty} \text{ so } \omega \sim \frac{1}{M_\infty} \text{ at const } h \text{ (altitude) & } n \]

Using \( R \approx \frac{V^2_\infty}{g} \), \( \omega \approx \frac{V_\infty}{R} = \frac{a_\infty M_\infty}{R} \). So \( \omega \sim M_\infty \) at const \( h \) & \( R \)

For high Mach numbers, the turn radius gets large
**R_{\text{min}} and \omega_{\text{max}}**

Using $V_\infty = (2L/\rho_\infty SC_L)^{1/2} = (2nW/\rho_\infty SC_L)^{1/2}$

$$R \approx V_\infty^2 / gn \quad \text{becomes} \quad R = 2(W/S)/ g\rho_\infty C_L$$

$W/S = \text{wing loading, an important performance parameter}$

And using $n = L/W = \rho_\infty V_\infty^2 SC_L/2W$

$$\omega \approx gn / V_\infty = g \rho_\infty V_\infty C_L / 2(W/S)$$

For each airplane, $W/S$ set by range, payload, $V_{\text{max}}$.

Then, for a given airplane

$$R_{\text{min}} = 2(W/S)/ g\rho_\infty C_{L,\text{max}}$$

$$\omega_{\text{max}} = g \rho_\infty V_\infty C_{L,\text{max}} / 2(W/S)$$

Higher $C_{L,\text{max}}$ gives superior turning performance.

But does $n_{CL,\text{max}} = \rho_\infty V_\infty^2 C_{L,\text{max}} / 2(W/S)$ exceed structural limits?
Each airplane has a V-n diagram.

Source: Anderson
Summary on Turning

- Want large structural load factor $n$
- Want large $C_{L,\text{MAX}}$
- Want small $V_\infty$
- Shortest turn radius, maximum turn rate is "Corner Velocity"

- Question, does the aircraft have the power to execute these maneuvers?
Specific Energy and Excess Power

Total aircraft energy = $PE + KE$

$$E_{tot} = mgh + mV^2/2$$

Specific energy = $(PE + KE)/W$

$$H_e = h + V^2/2g \text{ “energy height”}$$

Excess Power = $(T-D)V$

Specific excess power*

$$= (TV-DV)/W$$

$$= dH_e/dt$$

$$P_s = dh/dt + V/g \, dV/dt$$

$P_s$ may be used to change altitude, or accelerate, or both

* Called specific power in Lockheed Martin notes.
Excess Power

Power Required

\[ P_R = D V_\infty = q_\infty S (C_{D,0} + C_L^2 / \pi A R e) V_\infty \]
\[ = q_\infty S C_{D,0} V_\infty + q_\infty S V_\infty C_L^2 / \pi A R e \]
\[ = \rho_\infty S C_{D,0} V_\infty^3 / 2 + 2n^2 W^2 / \rho_\infty S V_\infty \pi A R e \]

Parasite power required
Induced power required

Power Available

\[ P_A = T V_\infty \]

and Thrust is approximately constant with velocity, but varies linearly with density.

Excess power depends upon velocity, altitude and load factor.
Altitude Effects on Excess Power

\[ P_R = DV_\infty = (nW/L) DV_\infty \]

\[ = nWV_\infty C_D/C_L \]

From \( L = \rho_\infty SV^2_\infty C_L/2 = nW \), get

\[ V_\infty = (2nW/ \rho_\infty SC_L)^{1/2} \]

Substitute in \( P_R \) to get

\[ P_R = (2n^3W^3C^2_D/ \rho_\infty SC^3_L)^{1/2} \]

So can scale between sea level “0” and altitude “alt” assuming \( C_D, C_L \) const.

\[ V_{alt} = V_0(\rho_0/\rho_{alt})^{1/2}, \quad P_{R,alt} = P_{R,0}(\rho_0/\rho_{alt})^{1/2} \]

Thrust scales with density, so

\[ P_{A,alt} = P_{A,0}(\rho_{alt}/\rho_0) \]
Summary of Power Characteristics

- $H_e =$ specific energy represents “state” of aircraft. Units are in feet.
  - Curves are universal
- $P_s = (T/W-D/W)V =$ specific excess power
  - Represents ability of aircraft to change energy state.
  - Curves depend upon aircraft (thrust and drag)
  - Maybe used to climb and/or accelerate
  - Function of altitude
  - Function of load factor
- “Military pilots fly with $P_s$ diagrams in the cockpit”, Anderson
### A/C Performance Summary

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<td>Radius of action*.</td>
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<td>E (hrs) = R (miles)/V(mph), where R = Breguet Range</td>
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<td>100 fpm climb</td>
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Lectures 6 and 7 for commercial and military transport

* Radius of action comprised of outbound leg, on target leg, and return.
Stability and Control

- Performance topics deal with forces and translational motion needed to fulfill the aircraft mission.
- Stability and control topics deal with moments and rotational motion needed for the aircraft to remain controllable.
S&C Definitions

- **L’** - rolling moment
- **Lateral motion/stability**

- **M** - pitching moment
- **Longitudinal motion/control**

- **N** - rolling moment
- **Directional motion/control**

Moment coefficient: \( C_M = \frac{M}{q_\infty Sc} \)
Aircraft Moments

- Aerodynamic center (ac): forces and moments can be completely specified by the lift and drag acting through the ac plus a moment about the ac
  - $C_{M,ac}$ is the aircraft pitching moment at $L = 0$ around any point
- Contributions to pitching moment about cg, $C_{M,cg}$ come from
  - Lift and $C_{M,ac}$
  - Thrust and drag - will neglect due to small vertical separation from cg
  - Lift on tail
- Airplane is “trimmed” when $C_{M,cg} = 0$
Absolute Angle of Attack

- Stability and control analysis simplified by using the absolute angle of attack which is 0 at $C_L = 0$.

- $\alpha_a = \alpha + \alpha_{L=0}$

Lift coefficient vs geometric angle of attack, $\alpha$

Lift coefficient vs absolute angle of attack, $\alpha_a$
Criteria for Longitudinal Static Stability

$C_{M,0}$ must be positive

$\frac{dC_{M,cg}}{d\alpha_a}$ must be negative

Implied that $\alpha_e$ is within flight range of angle of attack for the airplane, i.e. aircraft can be trimmed
Moment Around cg

\[ M_{cg} = M_{ac_{wb}} + L_{wb}(hc - h_{ac}) - l_t L_t \]

Divide by \( q_\infty Sc \) and note that \( C_{L,t} = \frac{L_t}{q_\infty S_t} \)

\[ C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}(h - h_{ac}) - \frac{l_t S_t}{cS} C_{L,t} , \text{ or} \]

\[ C_{M,cg} = C_{M,ac_{wb}} + C_{L_{wb}}(h - h_{ac}) - V_H C_{L,t} , \text{ where } V_H = \frac{l_t S_t}{cS} \]
\[ C_{M,\text{cg}} = C_{M,\text{ac}} + C_{L\text{ wb}}(h - h_{ac}) - VH C_{L,t} \]

\[ C_{L\text{ wb}} = \frac{dC_{L\text{ wb}}}{d\alpha} \alpha_{a,\text{wb}} = a_{wb} \alpha_{a,\text{wb}} \]

\[ C_{l,t} = a_t \alpha_t = a_t(\alpha_{wb} - i_t - \varepsilon) \]

where \( \varepsilon \) is the downwash at the tail due to the lift on the wing

\[ \varepsilon = \varepsilon_0 + \left( \frac{\partial \varepsilon}{\partial \alpha} \right) \alpha_{a,\text{wb}} \]

\[ C_{L,t} = a_t \alpha_{a,\text{wb}} \left( 1 - \frac{\partial \varepsilon}{\partial \alpha} \right) - a_t(i_t + \varepsilon_0) \]

At this point, the convention is drop the \( \text{wb} \) on \( a_{wb} \)

\[ C_{M,\text{cg}} = C_{M,\text{ac}} + a \alpha \left[ (h - h_{ac}) - VH \frac{a_t}{a} \left( 1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \right] + VH a_t(i_t + \varepsilon_0) \]
Eqs for Longitudinal Static Stability

\[ C_{M,cg} = C_{M,ac_{wb}} + a_\alpha \left[ \left( h - h_{ac} \right) - V_H \frac{a_t}{a} \left( 1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \right] + V_H a_t (i_t + \varepsilon_0) \]

\[ C_{M,0} = \left( C_{M,cg} \right)_{l=0} = C_{M,ac_{wb}} + V_H a_t (i_t + \varepsilon_0) \]

\[ \frac{\partial C_{M,cg}}{\partial \alpha_a} = a \left[ \left( h - h_{ac} \right) - V_H \frac{a_t}{a} \left( 1 - \frac{\partial \varepsilon}{\partial \alpha} \right) \right] \]

- \( C_{M,ac_{wb}} < 0, V_H > 0, \alpha_t > 0 \Rightarrow i_t > 0 \) for \( C_{M,0} > 0 \)
  - Tail must be angled down to generate negative lift to trim airplane

- Major effect of cg location (h) and tail parameter \( V_H = \frac{(lS)}{(cs)} \) in determining longitudinal static stability
Neutral Point and Static Margin

\[
\frac{\partial C_{M, cg}}{\partial \alpha_a} = a \left[ (h - h_{ac}) - V_H \frac{a_t}{a} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right]
\]

- The slope of the moment curve will vary with \( h \), the location of cg.
- If the slope is zero, the aircraft has neutral longitudinal static stability.
- Let this location be denote by \( h_n = h_{ac} + V_H \frac{a_t}{a} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) \)

\[
\frac{\partial C_{M, cg}}{\partial \alpha_a} = a \left( h - h_n \right) = -a \left( h_n - h \right) = -a \times \text{static margin}
\]

- For a given airplane, the neutral point is at a fixed location.
- For longitudinal static stability, the position of the center of gravity must always be forward of the neutral point.
- The larger the static margin, the more stable the airplane
Longitudinal Static Stability

Aerodynamic center location moves aft for supersonic flight

- cg shifts with fuel burn,
- stores separation,
- configuration changes

- “Balancing” is a significant design requirement
- Amount of static stability affects handling qualities
- Fly-by-wire controls required for statically unstable aircraft
Today’s References

• Lockheed Martin Notes on “Fighter Performance”


  
  – Note: There are extensive cost and weight estimation relationships in Raymer for military aircraft.