Problem 1: DC Generator

Operating at 2000 RPM and with no load, the situation is as shown in Figure 1. The equilibrium situation is that

\[ R_f I_f = E_a - R_a I_f \]

The trick is to characterize \( E_a \), which we may do using a piecewise-linear method:

- if \( 0 < I_f < I_1 \) \( E_a = G \Omega I_f \)
- if \( I_1 < I_f < I_2 \) \( E_a = E_1 + G_1 \Omega (I_f - I_1) \)
- if \( I_2 < I_f \) \( E_a = E_2 + G_2 \Omega (I_f - I_2) \)

Note that as \( \Omega \) changes, so do the breakpoints \( E_1 \) and \( E_2 \) and the slopes \( G \Omega, G_1 \Omega \) and \( G_2 \Omega \).

In the first part of the problem we are simply interested in seeing if there is a solution: if there is any value for which the field current line crosses the voltage characteristic. This will be the case if:

\[ G \Omega \geq R_f + R_a \]

Since

\[ G = \frac{200}{\Omega_0} \]

We find the speed at which the machine will just self-excite as

\[ \frac{\Omega_a}{\Omega_0} = \frac{R_f + R_a}{200} \]
This scaling works in RPM too, so the speed at which the thing will self-excite is:

\[ N_s = 2000 \times \frac{101}{200} = 1010 \text{RPM} \]

In each of the (piecewise) linear ranges, we can characterize operation in the following way:

\[ V = R_f I_f = E_n + R_n (I_f - I_n) - R_a (I_f + I_g) \]

where \( I_g \) is load current and \( R_n = G_n \Omega \) may be used because we are running this at constant speed.

This becomes, for each region:

\[ I_f = \frac{E_n - R_n I_n - R_a I_g}{R_a + R_f - R_n} \]

The no-load field current (and hence voltage) are found using this expression in the upper range with, obviously, zero load current. This evaluates to about 259.5 volts (which is consistent with the drawing in Figure 1.

To find the limits of the regions, this expression can be solved for the value of \( I_g \) which results in field current being the boundary current for that region, or:

\[ I_{gn} = \frac{E_n - (R_a + R_f) I_n}{R_a} \]

And, once field current is found within each region the terminal voltage is simply:

\[ V = E_n + R_n (I_f - I_n) - R_a I_g \]

The values of load current \( I_g \) that correspond to the two break points are 48 A (upper breakpoint) and 99 A (lower breakpoint). The full load voltage curve is shown in Figure 2. Note that the machine will not sustain load currents above 99 A; those will cause 'voltage collapse'.

To 'flat compound' the machine we must provide \( \Omega G_s = 1 \Omega \). At the operating point the incremental field characteristic is \( \Omega G_f = \frac{50}{3} \Omega \), so the number of series field turns required is \( N_s = 500 \times \frac{3}{50} = 30 \).

Simulation is straightforward. Using \( I_f \) as the single state variable,

\[ \frac{dI_f}{dt} = \frac{1}{L_f} (E_a - (R_a + R_f) I_f) \]

and \( E_a \) must satisfy the nonlinear relationship:

\[ \begin{align*}
  &\text{if} \quad I_f < 1 \quad E_a = 200 I_f \\
  &\text{if} \quad 1 < I_f < 2 \quad E_a = 200 + 50 (I_f - 1) \\
  &\text{if} \quad 2 < I_f \quad E_a = 250 + \frac{50}{3} (I_f - 2)
\end{align*} \]

The simulation is shown in Figure 3.
**Problem 2:** Compound Motor

With no series field winding, the machine is characterized by:

\[
I_a = \frac{V - G_f I_f \Omega}{R_a}
\]

\[
I_f = \frac{V}{R_f}
\]

\[
T = G_f I_f I_a
\]

With the machine in long shunt connection the equivalent expressions are:

\[
I_a = \frac{V - G_f I_f \Omega}{R_a + G_s \Omega}
\]

\[
I_f = \frac{V}{R_f}
\]

\[
T = (G_f I_f + G_s I_a) I_a
\]

Short shunt operation is one step more complicated (See Figure 4). We can write loop equations like this:

\[
V = (R_s + R_a + G_s \Omega) I_a + (R_s + G_f \Omega) I_f
\]

\[
0 = (R_a + G_s \Omega) I_a + (G_f \Omega - R_f) I_f
\]
It is just as easy to let MATLAB solve this linear system. The results are shown in Figures 5, 6 and 7.

**Problem 3: Losses** For each of the two harmonics it is straightforward to find the complex amplitude of magnetic field:

\[
H_{yn} = \frac{j}{nk} \frac{K_n}{1 + \frac{j\omega_n\sigma}{\mu_0 n^2 k^2 g}}
\]

since current in the sheet is:

\[
K_{2sn} = \frac{\omega_n\sigma\mu_0}{nk} H_{yn}
\]

we can now write the sheet current for that harmonic as:
\[ K_{zn} = -\frac{R_n}{1 + jR_n}K_n \]

where
\[ R_n = \frac{\omega_n\mu_0\sigma_s}{n^2k^2g} \]

Total current in the sheet is the sum of fifth and seventh harmonics:
\[ K_z = \text{Re} \left\{ K_{z5}e^{j(6\omega t+5kx)} + K_{z7}e^{j(6\omega t+7kx)} \right\} \]

Getting the average over time but not space we find:
\[ P_s = \frac{1}{2\sigma_s} \text{abs} \left( K_{z5}e^{j5kx} + K_{z7}e^{j7kx} \right) \]

Note that frequencies for both harmonics are \( 6\omega \). These calculations are carried out in a script and the results are shown in Figure 8.

**Problem 4: More Losses** To start, note that below the sheet the complex amplitudes of the fields must be simply:
\[ S_b = \frac{H_y}{H_x} = j \]

Next, see that any current in that sheet must be related to the flux density through the sheet:
\[ K_x = -\frac{\omega}{k}\mu_0\sigma_s H_y = -j\frac{\omega}{k}\mu_0\sigma_s H_{xb} \]

Then, since \( x \)-directed field above the sheet is equal to \( x \)-directed field below the sheet minus the \( z \)-directed surface current:

\[ H_{xa} = H_{xb} \left(1 + j\frac{\omega}{k}\mu_0\sigma_s\right) \]

and then, since \( H_y = jH_{xb} \), we have the ratio of fields at the top of the sheet:

\[ S_0 = \frac{H_y}{H_x} = \frac{j}{1 + j\frac{\omega}{k}\mu_0\sigma_s} \]

We can now transform this field ratio to the surface of the stator, using the form of expression shown in the problem statement. There is a bit of algebra to be done, but that same field ratio is found to be, in a straightforward way:

\[ S_s = j\frac{S_0 \cosh kg + j \sinh kg}{S_0 \sinh kg + j \cosh kg} \]

Now we can turn this into a surface impedance by using:

\[ Z_s = \frac{E_s}{-H_x} = \frac{\omega}{k}\mu_0 S_s \]
Figure 7: Series Field in Short Shunt Connection

Note that we are using a surface impedance which is the ratio of z-directed electric field to z-directed current in the region below the stator.

Loss density is found by using, for the current driven case:

\[ P_c = \frac{1}{2} |K_z|^2 \text{Re} \{ Z_s \} \]

In the case of a constrained flux, the stator provides the equivalent of a fixed electric field:

\[ E_{zs} = -\frac{\omega}{k} B_y \]

And the loss expression becomes:

\[ P_s = \frac{1}{2} \left( \frac{\omega}{k} B_y \right)^2 \text{Re} \left\{ \frac{1}{Z_s} \right\} \]

The results are calculated in a script and are shown in Figure 9.
Figure 8: Loss vs. position

Figure 9: Loss vs. conductivity
% 6.685 Problem Set 7, Problem 1

% Getting DC Generator Output Voltage
% Parameters
Ra = 1;
Rf = 100;
E1 = 200;
E2 = 250;
I1 = 1;
I2 = 2;
R1 = 50;
R2 = 50/3;

If0 = (E2-R2*I2)/(Ra+Rf-R2)
V0 = E2 + R2 * (If0-I2)

Ig1 = (E1-(Ra+Rf)*I1)/Ra
Ig2 = (E2-(Ra+Rf)*I2)/Ra

I_g2 = 0:Ig2/100:Ig2;
I_f2 = (E2-R2*I2 - Ra .* I_g2) ./ (Rf+Ra-R2);
V_2 = E2 + R2 .* (I_f2 - I2);

Id = Ig1-Ig2;
dI = Id/200;
I_g1 = Ig2+dI:dI:Ig1;
I_f1 = (E1-R1*I1 - Ra .* I_g1) ./ (Rf+Ra-R1);
V_1 = E1 + R1 .* (I_f1 - I1);

I_g = [I_g2 I_g1];
V_g = [V_2 V_1];

figure(1)
plot(I_g, V_g)
title('Problem Set 7, Problem 1')
ylabel('Output Voltage')
xlabel('Output Current')
grid on
%% Simulation for Problem 7.1
I_f0 = 6/200;
t0 = 0:.001:.1;
[t, I_f] = ode23('dcmsim', t0, I_f0);

Ea = zeros(length(t));
for i = 1:length(t)
    if I_f(i)<1,
        Ea(i) = 200 * I_f(i);
    elseif I_f(i) < 2,
        Ea(i) = 200 + 50 *(I_f(i)-1);
    else
        Ea(i) = 250 + (50/3) * (I_f(i)-2);
    end
end
V = (100/101) .* Ea;

figure(1)
clf
subplot 211
plot(t, I_f)
title('Voltage Buildup')
ylabel('Field Current')
grid on
subplot 212
plot(t, V)
ylabel('Terminal Voltage')
xlabel('Time, Seconds')
grid on

-------------
function dI_f = dcmsim(t, I_f)
%% simulation script for Problem 7.1, voltage buildup
Ra = 1;
Rf = 100;
L = 1;
if I_f<1,
    Ea = 200 * I_f;
elseif I_f < 2,
    Ea = 200 + 50 *(I_f-1);
else
    Ea = 250 + (50/3) * (I_f-2);
end
dI_f = (1/L)*(Ea-(Ra+Rf)*I_f);
\end{verbatim}
\clearpage
\begin{verbatim}
\% Compound Motor:

Rf = 300;
Rs = 2;
Ra = .25;
Nf = 500;
Ns = 20;
Nz = 1000;
omt = 2*pi*Nz/60;
Vt = 600;
If0 = Vt/Rf;

Gf = Vt/(omt*If0);
Gs = Gf*Ns/Nf;
N = 0:5:Nz;
om = (pi/30).*N;

\% Part 1: No Shunt
If = Vt/Rf;
Ia = (Vt - Gf*If .* om) ./ Ra;

T = Gf * If .* Ia;
figure(2)
subplot(211)
plot(N, T);
title('6.685 PS7, Problem 2 No Shunt')
ylabel('Torque, N-m');
subplot(212)
plot(N, Ia)
ylabel('Current Ia, A')
xlabel('Speed, RPM')

\% Part 2: Long Shunt
If = Vt/Rf;
Ia = (Vt - Gf*If .* om) ./ (Ra + Rs + Gs .* om);

T = (Gf .* If + Gs .* Ia) .* Ia;
figure(3)
subplot(211)
plot(N, T);
title('6.685 PS7, Problem 2 Long Shunt')
\end{verbatim}
ylabel('Torque, N-m');
subplot(212)
plot(N, Ia+If)
ylabel('Current Ia+If, A')
xlabel('Speed, RPM')

% Part 3: Short Shunt

for i = 1:length(N);
    M = [Rs+Ra+Gs*om(i) Rs+Gf*om(i);Ra+Gs*om(i) Gf*om(i)-Rf];
    I = M\[Vt;0];
    Ia(i) = I(1);
    If(i) = I(2);
end

T = (Gf .* If + Gs .* Ia) .* Ia;
figure(4)
subplot(211)
plot(N, T);
title('6.685 Ps5, Problem 3 Short Shunt')
ylabel('Torque, N-m');
subplot(212)
plot(N, Ia+If)
ylabel('Current Ia+If, A')
xlabel('Speed, RPM')
Problem Set 7, Problem 3

Parameters:

\[ \lambda = 1; \] % fundamental wavelength
\[ g = .002; \]
\[ \sigma_s = 2.5e4; \]
\[ K_5 = 320; \]
\[ K_7 = 230; \]
\[ f = 60; \]
\[ \mu_0 = \pi \times 4e-7; \]
\[ \omega = 2 \pi \times f; \]
\[ k = 2 \pi / \lambda; \]
\[ R_5 = 6 \omega \mu_0 \sigma_s / (25 k^2 g); \]
\[ R_7 = 6 \omega \mu_0 \sigma_s / (49 k^2 g); \]

\[ K_{s5} = K_5 R_5 / (1 + j R_5); \]
\[ K_{s7} = K_7 R_7 / (1 + j R_7); \]

\[ x = 0: \lambda / 200: \lambda; \]

\[ P_d = (1 / (2 \sigma_s)) \times abs(K_{s5} \times \exp(j \times 5 k \times x) + K_{s7} \times \exp(j \times 7 k \times x))^2; \]

figure(5)
plot(x, P_d)
% PS7, Problem 4

Ks = 80000;  % amplitude of current source drive
Bs = .1;     % and of alternate flux source

lambda = .05; % wavelength
G = .005;     % gap
f=1000;      % frequency
rsig = [0:.001:1]; % range of relative conductivities
sigs = .001*6e7.* rsig; % surface conductivities
muzero = pi*4e-7;

k = 2*pi/lambda;  % here is wavenumber
om = 2*pi .* f;  % and frequency in radians/second

% first get surface coefficient at top of sheet
S0 = j ./ (1 + j*(muzero .* sigs ./ k) .* om);
% then the same at the stator surface
S = j .* (S0 .* cosh(k*g) + j*sinh(k*g)) ./ (S0 .* sinh(k*g) + j*cosh(k*g));
% now the surface impedance
Zs = (muzero .* om ./ k) .* S;
% Here is loss when driven by a current source
P_c = .5*Ks^2.* real(Zs);
% and here is loss when driven by a 'flux source'
P_v = .5 .* ((om ./ k) .* Bs) .* 2 .* real(1 ./ Zs);

figure(6)
plot(rsig, P_c, rsig, P_v)
title('Problem Set 7.4')
ylabel('Loss, w/m^2')
xlabel('Conductivity relative to copper')