Basic MR image encoding
Physical Foundations of MRI

What is NMR?
   The basic signal we excite and detect.

Tricks of NMR
   The gradient and spin echo

How do we encode an image?
   slice select, frequency and phase encoding.

What are some problems (artifacts) relevant to our application.
Physical Foundations of MRI

NMR: 60 year old phenomena that generates the signal from water that we detect.

MRI: using NMR to generate an image

Three magnetic fields (generated by 3 coils)

1) static magnetic field $B_0$
2) RF field that excites the spins $B_1$
3) gradient fields that encode spatial info $G_x, G_y, G_z$
What is NMR?

NUCLEAR
MAGNETIC
RESONANCE

A magnet, a glass of water, and a radio wave source and detector....
protons

Earth's Field

compass

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Nuclei and Magnetic Fields

Not every nucleus lines up with applied magnetic field.

Why?
Direction of spins becomes randomized by thermal motion.

protons at 1.5 Tesla, at room temperature
net # aligned with field is 1 part in 100,000
Compass needles

The vector sum of all the nuclei can be viewed as a compass needle.

Points North. (aligns along the magnetic field lines of the external field (earth or MR magnet)

If displaced from North, it will wobble about north with a characteristic frequency (called Larmor freq.)
Compass needles

Earth’s Field

Freq = $\gamma B$

42.58 MHz/T

Main Field $B_0$

North
**EXCITATION :** Displacing the spins from Equilibrium (North)

**Problem:** It must be moving for us to detect it.

**Solution:** knock out of equilibrium so it oscillates

**How?**  
1) Tilt the magnet or compass suddenly

2) Drive the magnetization (compass needle) with a periodic magnetic field
Excitation: Resonance

Why does only one frequency efficiently tip protons?

Resonant driving force.
It’s like pushing a child on a swing in time with the natural oscillating frequency.
$z$ is "longitudinal" direction  
$x$-$y$ is "transverse" plane

Static Field

Applied RF Field

The RF pulse rotates Mo the about applied field
"Exciting" Magnetization

Magnetization processes about new axis (of oscillating RF B field) as long as resonant field is applied.

Total amount vector processes is called the "tip angle" of the excitation.
"Exciting" Magnetization tip angle

45°

90°
Detecting the NMR Signal

A moving bar magnet induces a Voltage in a coil of wire. (a generator…)

The RF coil design is the #1 determinant of the system SNR
Detecting the NMR: the noise

Noise comes from electrical losses in the resistance of the coil or electrical losses in the tissue.

For a resistor:
\[ P_{\text{noise}} = 4kTRB \]

- Noise is white.
- \( \gg \) Power \( \alpha \) bandwidth
- Noise is spatially uniform.
- \( R \) is dominated by the tissue.
- \( \gg \) big coil is bad.
Signal to Noise Ratio in MRI

Most important piece of hardware is the RF coil.

\[
\text{SNR} \propto \text{voxel volume} \quad \ (# \ of \ spins) \\
\text{SNR} \propto \text{SQRT( total time of data collection)} \\
\text{SNR is also dependent on the amount of signal you throw away to get contrast.}
\]
Review: the NMR Signal

RF

Voltage (Signal)

$B_0$

$\nu_o$

$V(t)$
Three Steps in MR:

0) Equilibrium (magnetization points along Bo)

1) RF Excitation (tip magn. away from equil.)

2) Precession induces signal, dephasing (timescale = T2, T2*).

3) Return to equilibrium (timescale = T1).
Magnetization vector during MR

- RF encode
- Voltage (Signal)
- Mz
- B1 RF Field (from RF excitation pulse)
- $\omega = \gamma B_1$

Wald, MGH-NMR
Three places in process to make a measurement (image)

0) Equilibrium (magnetization points along Bo)

1) RF Excitation (tip magn. away from equil.)

2) Precession induces signal, allow to dephase for time TE.

3) Return to equilibrium (timescale =T1).

proton density weighting

T2 or T2* weighting

T1 Weighting
T2*-Weighting

Wait time $\text{TE}$ after excitation before measuring $M$. Shorter T2* spins have dephased

initially

at $t = \text{TE}$

vector sum
T2* Dephasing

Just the tips of the vectors…
Transverse Magnetization vs Time (milliseconds)

- T2* = 200
- T2* = 60
T2 Weighting

Phantoms with four different T2 decay rates...

There is no contrast difference immediately after excitation, must wait (but not too long!).

Choose TE for max. inten. difference.
Dephasing: local field variations

homogeneous magnet.

inhomogeneous magnet.

$S(t)$

$T^{2*}$

$S(\nu)$

$\Delta \nu$

$Z$

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Aside: Magnetic field gradient

\[ G_x \equiv \frac{\partial B_z}{\partial x} \]

- Uniform magnet
- Field from gradient coils
- Total field

\[ B_0 \quad G_x \quad B_0 + G_x \]
A gradient causes a spread of frequencies

MR frequency of the protons in a given location is proportional to the local applied field.

\[ v = \gamma \mathbf{B}_{\text{TOT}} = \gamma (\mathbf{B}_0 + G_z z) \]
A gradient causes dephasing

I caused it, I can reverse it…

Gradient echo

\[ \nu = \gamma B_{\text{TOT}} = \gamma B_0 + G_z z \]

\[ \Delta \nu = \gamma \Delta B_{\text{TOT}} = \gamma G_z z \]

\[ \Delta \theta = \Delta \nu \tau = \gamma G_z z \tau \]

Gratuitous manipulation… (?)

What happens if the spin moves?
Less trivial manipulation…
the Spin Echo

Refocus the dephased signal without resorting to direct control of the B₀ field.
Spin Echo

Some dephasing can be refocused because its due to static fields.

Blue/Green arrows precesses faster due to local field inhomogeneity than red/orange arrow
Spin Echo

180° pulse only helps cancel static inhomogeneity.

The “runners” can have static speed distribution.

If a runner trips, he will not make it back in phase with the others.
T2 weighed image

Signal

Time (ms)

CSF

gray

white

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Part II

Image encoding
MR frequency of the protons in a given location is proportional to the local applied field.

\[ v = \gamma B_{TOT} = \gamma (B_o + G_z z) \]

\[ B_o \]
Step one: excite a slice

While the grad. is on, excite only band of frequencies.

Why?
Slice profile considerations

\[ A(\omega) \]

\[ \Delta \omega \]

\[ F(t) \]

\[ \Delta t \]

\[ \mathrm{FT} \]
Step two: encode spatial info. in-plane

- $B_0$ along $z$
- $B_{\text{TOT}} = B_0 + G_z$

"Frequency encoding"

- Signal with gradient
- Signal without gradient
‘Pulse sequence’ so far

RF

“slice select”

“freq. encode”
(read-out)

$G_z$

$G_x$

$S(t)$

Sample points

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“Phase encoding”

RF

“slice select”

G_z

“phase encode”

G_y

“freq. encode” (read-out)

G_x

S(t)
How does blipping on a grad. encode spatial info?

\[ \nu(y) = \gamma B_{\text{TOT}} = \gamma B_0 \Delta y G_y \]

\[ \theta(y) = \nu(y) \tau = \gamma B_0 \Delta y (G_y \tau) \]
How does blipping on a grad. encode spatial info?

After RF

After the blipped y gradient...

\[ \theta (y) = \nu(y) \tau = \gamma B_o \Delta y (G_y \tau) \]

position \( y_1 \)  

position 0  

position \( y_2 \)
The magnetization vector in the xy plane is wound into a helix directed along the y axis.

Phases are ‘locked in’ once the blip is over.
The bigger the gradient blip area, the tighter the helix.

\[ \theta(y) = v(y) \tau = \gamma B_o \Delta y (G_y \tau) \]
What have you measured?

Consider 2 samples:

- **uniform water**
  - no signal observed

- signal is as big as if no gradient

1 cm
Measurement intensity at a spatial frequency...

\[ k_x \]

\[ k_y \]

1/1.2mm = 1/Resolution
1/2.5mm
1/5mm
1/10 mm

10 mm
Fourier transform

\[ \text{FOV}_x = \text{matrix} \times \text{Res}_x \]

\[ \frac{1}{\text{FOV}_x} \]

\[ \frac{1}{\text{Res}_x} \]
Frequency encoding revisited
“Spin-warp” encoding

one excitation, one line of kspace...
“Spin-warp” encoding mathematics

The “image” is the spin density function: $\rho(x)$

Phase due to readout:

$$\theta(t) = \omega_0 t + \gamma G_x x t$$

Phase due to P.E.

$$\theta(t) = \omega_0 t + \gamma G_y y \tau$$

$$\Delta \theta(t) = \omega_0 t + \gamma G_x x t + \gamma G_y y \tau$$
“Spin-warp” encoding mathematics

Signal at time $t$ from location $(x,y)$

$$S(t) = \rho(x,y)e^{i\gamma G_x x t + i\gamma G_y y \tau}$$

The coil integrates over object:

$$S(t) = \int\int_{\text{object}} \rho(x,y)e^{i\gamma G_x x t + i\gamma G_y y \tau} \, dx \, dy$$

Substituting $k_x = -\gamma G_x t$ and $k_x = -\gamma G_x t$:

$$S(k_x, k_y) = \int\int_{\text{object}} \rho(x,y)e^{-ik_x x - ik_y y} \, dx \, dy$$
“Spin-warp” encoding mathematics

View signal as a matrix in $k_x$, $k_y$...

$$S(k_x, k_y) = \int \int \rho(x, y) e^{-i k_x x - i k_y y} \, dx \, dy$$

: 
Solve for $\rho(x, y)$

$$\rho(x, y) = FT^{-1} \left[ S(k_x, k_y) \right]$$

$$\rho(x, y) = \int \int S(k_x, k_y) e^{i k_x x + i k_y y} \, dk_x \, dk_y$$
Fourier transform

FOV_x = matrix * Res_x

1 / Res_x

1 / FOV_x
Kspace facts

Resolution is determined by the largest spatial freq sampled.

FOV = matrix * resolution

If the object is real, half the information in kspace matrix is redundant. We only need to record half of it.
kspace

Image space (magnitude)  kspace (magnitude)
kspace artifacts: spike

One “white pixel” in kspace from a electric spark
K-space artifacts: Symmetric N/2 ghost

Even numbered lines got $\exp(i\phi)$

Odd numbered lines got $\exp(-i\phi)$

$\phi = 12$ degrees
kspace artifacts: subject motion

Yellow = position 1
Orange = moved 2 pixels

Movement in real space = linear phase shift across kspace.

=>$ \text{Orange points have linear phase } \theta = a k_y$
Fast Imaging

“Dost thou love life? Then do not squander time, for that’s the stuff life is made of.”

- Benjamin Franklin
Requirements for brain mapping

Considerations:

• Signal increase = 0 to 5% (small)
• Motion artifact on conventional image is 0.5% - 3%
• Need to see changes on timescale of hemodynamic changes (seconds)

Requirement: Fast, “single shot” imaging, image in 80ms, set of slices every 1-3 seconds.
What’s the difference?

conventional MRI

- RF
- "slice select" \( G_z \)
- \( G_y \)
- "freq. enc" (read-out) \( G_x \)
- \( S(t) \)

- \( k_y \)
- \( k_x \)

echoplanar imaging

- RF
- \( G_z \)
- \( G_y \)
- \( G_x \)
- \( S(t) \)

- \( T2^* \)
- \( k_y \)
- \( k_x \)

etc...
“Echo-planar” encoding

one excitation, many lines of kspace...
“Echo-planar” encoding

Observations:

• Adjacent points along kx are taken with short $\Delta t$ (=$5\text{ us}$). (high bandwidth)

• Adjacent points along ky are taken with long $\Delta t$ (=$500\text{us}$). (low bandwidth)

• A given line is read quickly, but the total encode time is longer than conventional Imaging.

• Adjacent lines are traversed in opposite directions.
Enemy #1 of EPI: local susceptibility gradients

Orbitofrontal susceptibility region

Lateral temporal susceptibility region

$B_0$ field maps in the head
EPI: Local susceptibility gradients

Local susceptibility gradients have 2 effects:

1) Local dephasing of the signal (signal loss) mainly from thru plane gradients

2) Local geometric distortions, mainly from local in-plane gradients.
Signal from whole slice comes from adding together the MR vectors. When in phase, add constructively, SNR increases like slice thickness.
Signal from whole slice comes from adding together the MR vectors, which get out of phase when the magnetic field is not uniform.
Local susceptibility gradients: thru-plane dephasing

Bad for thick slice above frontal sinus...
Local gradients: geometric distortion

Local gradient alters the helix of phase we have so carefully wound.

Phase error accumulates over entire kspace. (conventional imaging phase is reset every line)

>> faster encoding is better.

Readout points are taken close together (~5us)

Phase encode points are taken farther apart (~500us)

>> distortion occurs in P.E. direction.
Local gradients: geometric distortion

Two sets of EPI:
1) encode in 32ms
2) encode in 23ms
Characterization of grad. performance

- length of readout train for given resolution (requires fast slew and high grad amplitude)

\[ G_y(t) \]
\[ G_x(t) \]
\[ S(t) \] (no grads)

'echo spacing' (esp) = 512 us for 1.5T, readout length = 32 ms
= 366us for 3T, readout length = 23 ms
EPI problems: N/2 ghost

Asymmetry in alternate lines gives N/2 image ghost.

Asymmetry from: Eddy currents receiver filter receiver timing head coil tuning.

object

N/2 ghost

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EPI problems: frequency offset

If one object has a different NMR frequency (e.g. fat and water) it gets shifted in PE direction. (why?)

True location

Echoplanar image
EPI and Spirals

\[ \mathbf{k}_x, \mathbf{k}_y, \mathbf{G}_x, \mathbf{G}_y \]
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<thead>
<tr>
<th></th>
<th><strong>EPI</strong></th>
<th><strong>Spirals</strong></th>
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<tr>
<td>Eddy currents:</td>
<td>ghosts</td>
<td>blurring</td>
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<tr>
<td>Susceptibility:</td>
<td>distortion, dephasing</td>
<td>blurring dephasing</td>
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<td>( k = 0 ) is sampled:</td>
<td>1/2 through</td>
<td>1st</td>
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<td>Corners of kspace:</td>
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<tr>
<td>Gradient demands:</td>
<td>very high</td>
<td>pretty high</td>
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