Note: If you are finding insufficient mathematical guidance in Nicholson's Chapter 2, please see Binger and Hoffman, <u>Microeconomics with Calculus</u>, Chapters 1 – 3 on reserve at Dewey. These chapters are an excellent reference.

<u>Theory:</u>

1) Take the function: $y = f(x; a) = a^2 - 2x^2 - ax$. a) Find the first order condition for a maximum/minimum. b) Is this a maximum or a minimum? c) Solve for x^*. d) Explicitly solve for $\dfrac{dx^*}{da}$ and $\dfrac{dy^*}{da}$. e) Now re-write the first order condition and use the implicit function theorem to find $\dfrac{dx^*}{da}$. f) Use the envelope theorem to find $\dfrac{dy^*}{da}$ and explain briefly why this theorem allows you to simplify your calculation from (d). g) What is $\left.\dfrac{\partial y}{\partial x}\right|_{x=x^*}$? Would your answer differ depending on whether x^* was a maximum or minimum for this problem?

2) Let a consumer's utility be given by $U(x, y, z) = \ln(x) + \ln(y) + \ln(2 + z)$. The consumer has a budget constraint of $x + 3y + 2z = 18$. a) What are the optimal choices for this consumer? b) What would the choices be if the budget constraint were increased from 18 to 19? c) What is the consumer's utility in each case? d) How do you interpret the Lagrange multiplier in (b)? e) Show how you could use the Lagrange multiplier (and the envelope theorem for constrained problems) to estimate the change in consumer utility caused by increasing her budget from 18 to 19 without having to resolve for *x*, *y*, and *z* in (c).

3) Take the problem $\max z = \dfrac{xy}{x + y}$ subject to $x + 4y = 90$. a) Set up the Lagrangian and find the first order conditions b) Solve explicitly for x, y, z and λ, the Lagrangian multiplier. c) Set up the dual problem of (a), i.e., minimize the primal constraint function subject to the primal objective function being equal to the value of z^* obtained in (b). d) Solve explicitly

for $x, y,$ and λ^D, the Lagrangian from the dual. e) What is the relationship between λ and λ^D, and can you provide an economic explanation for this?

Application: Minimum wage articles

1) John Kennan describes a 1915 Bureau of Labor Statistics report that studies a natural experiment affecting retail stores in Oregon. Base your answers to the following questions on Kennan's description of the report. (a) What is the policy change that the report studies? (b) What are the experimental groups? (c) What is the control group? (d) Why is the control group necessary? (e) Are there any problems with this control group? (f) What are the results, and what do they suggest about the effects of minimum wages on employment?

2) Richard Freeman breaks the world into the BR's and SR's. Applying this logic to a change in the minimum wage in a *competitive* model of labor supply and demand, (a) give an example of a labor market where you think that the BR's are most likely to be correct, and (b) give an example where the SR's are most likely to be correct. What is your reasoning in each case? Illustrate your answers with diagrams that show clearly what features of supply and demand curves make the diagrams correspond to BR's and SR's.

3) Card and Krueger (p. 773) emphasize that one strength of their analysis is that the implementation of the New Jersey minimum wage occurred during a recession. (a) Consider the case where the minimum wage was implemented right before an economic expansion instead. Draw a diagram that clearly shows how an economic expansion affecting both New Jersey and Pennsylvania might have invalidated the results of the 'natural experiment.' (b) Someone points out to you that, according to Table 3 of C&K, fast food employment fell in PA but it barely budged in NJ. Hence, they say, if we just look at NJ, it seems pretty clear that there is no support for a monopsony scenario. How would you respond to this criticism within the 'natural experiment' framework. Draw a carefully labeled diagram that illustrates your point.

4) A NJ legislator, after hearing of the C&K results says to her colleagues, "This is good news: the minimum wage increase raised fast food employment. We should pass further increases to get more teenagers jobs." Taking her initial premise as correct – the minimum wage increase did raise employment – analyze her conclusion with a diagram that shows under what circumstances she would be right and what circumstances she would be wrong.

5) Richard Freeman distinguishes between the short and long run impacts of minimum wage increases, implying that the long run impacts are likely to be larger (and more negative). List one or two *specific* economic reasons why the long run response would be larger. Consider that labor is not the only factor in fast food restaurants' production functions.