14.03 Fall 2000 Problem Set 2 Due in Class #7

Theory:

- 1) Nicholson 3.2 (Note that: $\partial \log_{base a} x/\partial x = 1/(x \ln a)$, 3.4, 3.7 (Hint: observe how these three functions are related), 4.2, 4.5 (Note: use low-tech math and common sense here), 5.2 and 5.9.
- 2) Let U(x, y) = -1/x 1/y. Suppose that prices are P_x and P_y , and income is I.

(A) Calculate the utility-maximizing choices of x and y, that is, the Marshallian demand functions $d_x(P_x, P_y, I)$ and $d_y(P_x, P_y, I)$.

(B) Calculate "indirect utility," i.e., the utility at the optimal choices, $V(P_x, P_y, I)$.

(C) For a given utility level U_0 , solve the dual expenditure-minimization problem, and compute the optimal choices of *X* and *Y*, i.e., the "compensated demand functions" $h_x(P_x, P_y, U_o)$ and $h_y(P_x, P_y, U_o)$.

(D) Calculate the minimum expenditure function $E(P_x, P_y, U_0)$. Show that the expenditure and indirect utility functions you have calculated are inverses of one another, i.e., show that $E(p_x, p_y, V(p_x, p_y, I)) = I$ and $V(p_x, p_y, E(p_x, p_y, U_0)) = U_0$.

(E) Consider the Slutsky equation for consumer theory (see equations 5.22 and 5.26 in Nicholson). State a verbal interpretation of the Slutsky equation.

(F) Verify directly that the Slutsky equation holds in this problem.

Application: Dwyer and Lindsay (Irish Potato) article

- 3) Draw a diagram that illustrates the surprising properties of a Giffen good. Your diagram should illustrate a consumer's 2-good utility maximization problem for two different budget sets, where P_x is higher under the second constraint (and x is the Giffen good).
- 4) Consider Figure 1 of Dwyer and Lindsay. (A) Explain why the top diagram is consistent with your diagram from the previous question. (B) Why is the lower diagram a better description of the Irish potato famine? (C) What is the evidence on what happened to potato prices during this period? Is this consistent with potatoes being a Giffen good?

- 5) Dwyer and Lindsay state that "Inferiority is necessary for a good to be Giffen." (A) Demonstrate mathematically why this statement must be true. (B) Why is it unlikely that potatoes were inferior during the Irish potato famine?
- 6) Dwyer and Lindsay also claim that "For a good to be Giffen, some normal good must be displaced by the inferior good as the price rise lowers real income." (A) Prove this statement.(B) Why is it also unlikely that this statement was applicable to potatoes during the Irish potato famine?
- 7) Someone who has read Dwyer and Lindsay says to you, "I don't see the big deal about Giffen goods – they're everywhere. Take gasoline, for example. Practically every summer, the price of gas increases and people buy more of it. Looks to me like a Giffen good." (A) Evaluate the logic of this assertion. (B) Can you think of a plausible 'natural experiment' (as in the Card & Krueger paper) that would allow you to test whether gasoline is a Giffen good? (C) What would you expect to find regarding the demand for gasoline?