14.03 Fall 2000 Problem Set 4 Solutions

1. Nicholson 8.5

a. EU = 0.75ln(10000) + 0.25ln(9000) $\approx$ 9.184

b. By purchasing full insurance at a premium of 250, Ms. Fogg's wealth in each state becomes 9750. Hence her expected utility is ln(9750) $\approx$ 9.185, which is greater than her expected utility if she does not buy insurance.

c. The maximum amount that Ms. Fogg is willing to pay for full insurance is *P* s.t. ln(10000 – *P*) = 0.75ln(10000) + 0.25ln(9000)
By the properties of logs, ln(10000 – *P*) = $\ln\left(10000^{0.75}9000^{0.25}\right)$
$\Rightarrow P = 10000 - \left(10000^{0.75}9000^{0.25}\right) \approx 259.96$

2. Part 1: Both of the risky choices (B and D) have higher expected values than the certain choices (A and C). If Bill were risk neutral or risk loving, he would prefer B to A and D to C. The fact that he is indifferent between them implies that he is risk averse.

   Part 2: The expected utility of F is
$$EU(F) = .25u(400) + .25u(900) + .25u(800) + .25u(1500)$$
$$EU(F) = .5(.5u(400) + .5u(900)) + .5(.5u(800) + .5u(1500))$$
$$EU(F) = .5EU(D) + .5EU(B) = .5EU(C) + .5EU(A)$$
   Then note that a 50/50 gamble over C and A has expected value \$750. Since Bill is risk averse he will prefer \$750 with certainty to this gamble. Hence he prefers E to F.

3. If choices are consistent with expected utility maximization, then there exists some utility function *u*(.) such that the lottery that gives wealth $(w_1,...,w_n)$ with probabilities $(p_1,...,p_n)$ is preferred to the lottery $(w_1',...,w_n')$, $(p_1',...,p_n')$ iff $\sum_{i=1}^{n} p_i u(w_i) > \sum_{i=1}^{n} p_i' u(w_i')$.

   Thus A preferred to B implies that
   u(1m)>.1*u*(5m)+.89*u*(1m)+.01*u*(0), or .11*u*(1m)>.1*u*(5m)+.01*u*(0).
   And C preferred to D implies that
   .1*u*(5m)+.9*u*(0)>.11*u*(1)+.89*u*(0), or .1*u*(5m)+.01*u*(0)>.11*u*(1m),
   which is a contradiction. Hence choosing A over B and C over D is inconsistent with expected utility maximization.

4.
$$U(W) = 100W^{0.9}$$
$$U'(W) = 90W^{-0.1}$$
$$U''(W) = -9W^{-1.1}$$

a) $rr(W) = \frac{9W^{-0.1}}{90W^{-0.1}} = \frac{1}{10}$

b) willingness to pay for full insurance is *WTP* such that

$U(1-WTP) = 0.9U(1) + 0.1U(0.9)$

$100(1-WTP)^{0.9} = 0.9(100)(1)^{0.9} + 0.1(100)(0.9)^{0.9}$

$WTP = 1-(0.9+0.1(0.9)^{0.9})^{\frac{1}{0.9}} = 0.01004687$

or about $10.05.

c) loss of wealth in good state that is equivalent to death of kids is *k* such that

$100(1-k)^{0.9} = (1)^{0.9}$

$k = 1-(1/100)^{\frac{1}{0.9}} = 0.994005157$

or about $994.

d) Actuarially fair price for insurance policy that pays *k* in bad state is 0.01*k* ($\approx$ $9.94).
expected utility if do not purchase insurance is

$$0.99(100)(1)^{0.9} + 0.01(1)^{0.9} = 99.01$$

expected utility if purchase insurance is

$$0.99(100)(1-0.01k)^{0.9} + 0.01(1-0.01k+k)^{0.9} = 98.13242641$$

so the consumer will *not* purchase actuarially fair insurance in this case. The reason for this is that the consumer's utility depends on the state. Since the marginal utility of wealth is 100 times lower in the bad state, the benefit he gets when he pays the actuarially fair premium is insufficient for insurance to increase his expected utility.

Now, you might think that since I am risk-averse, I would choose to buy an actuarially fair insurance policy. But this is wrong. In order to be desirable for me, what insurance has to do is cause me to have more money under circumstances when my marginal utility of income is high at the expense of less money when my marginal utility of income is low. In other words, the function of insurance is to transfer wealth from states of the world in which the marginal utility of wealth is low to states of the world in which the marginal utility of income is high. However, the loss of my kids does *not* result in an increase in my marginal utility of income. Quite the contrary. Losing my kids reduces both my level of utility *and* the marginal effect of wealth on my utility level. Hence, it is not desirable for me to buy an actuarially fair policy that pays me money in the event of my kids' death.

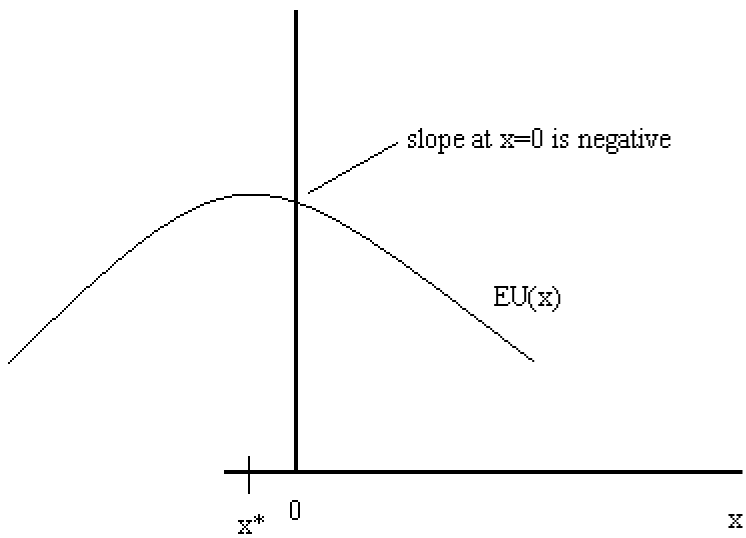
The subtlety of this problem, therefore, is that "I" have a *state-dependent utility function* where my marginal utility of income not only varies with wealth but also with states of the world. What you needed to notice to get this right is that because my marginal utility of wealth is dramatically lower in the state where I lose my kids, I would not want place much value on a policy that get me additional money in this state.

This result is actually quite general. You should not insure against "losses" per se. You should insure against "increases in the marginal utility of income." Many forms of losses (including all monetary losses for risk-averse individuals) do increase the marginal utility of income. But even for risk-averse individuals, losses that reduce their ability to "enjoy" income should not be insured against.

5.

a. Al's wealth in state i if he invests x dollars in the risky asset is $W_0 - x + (1 + r_i)x = W_0 + r_i x$. Hence his expected utility as a function of x is
$EU(x) = pU(W_0 + r_g x) + (1 - p)U(W_0 + r_b x)$

b. The first order condition is $\frac{\partial EU}{\partial x} = 0$, or
$r_g pU'(W_0 + r_g x) + r_b(1 - p)U'(W_0 + r_b x) = 0$.
Since Al is risk averse, $\frac{\partial EU}{\partial x}$ is declining in x. (You can prove this by showing that $\frac{\partial^2 EU}{\partial x^2} < 0$ if $U''(W) < 0$.) Since the FOC for x^* is $\left.\frac{\partial EU}{\partial x}\right|_{x=x^*} = 0$, x^* is less than or equal to zero if $\left.\frac{\partial EU}{\partial x}\right|_{x=0} \leq 0$.



$$\left.\frac{\partial EU}{\partial x}\right|_{x=0} = r_g pU'(W_0) + r_b(1 - p)U'(W_0) = (r_g p + r_b(1 - p))U'(W_0) \leq 0$$

Since $U'(W) > 0$, this condition reduces to $r_g p + r_b(1 - p) \leq 0$. Hence Al will not invest if the expected return on the risky asset is less than or equal to zero.

c. If $U(W) = -e^{-aW}$, then $U'(W) = ae^{-aW}$. Hence the FOC becomes

$$ar_g pe^{-a(W_0+r_g x)} + ar_b(1-p)e^{-a(W_0+r_v x)} = 0$$

$$ar_g pe^{-aW_0}e^{-ar_g x} + ar_b(1-p)e^{-aW_0}e^{-ar_b x} = 0$$

Dividing both sides by ae^{-aW_0} we get

$$r_g pe^{-ar_g x} + r_b(1-p)e^{-ar_b x} = 0$$

Since W_0 does not enter into this equation, the optimal choice of *x* does not depend on initial wealth.

Applications: Kane & Staiger article

1.

a. Figure VI shows that decreases in the distance to an abortion clinic *increase* the teen birth rate. A decline in distance of about 80 to 90 miles is associated with an increase in the teen birth rate of about 0.005 or 0.006.

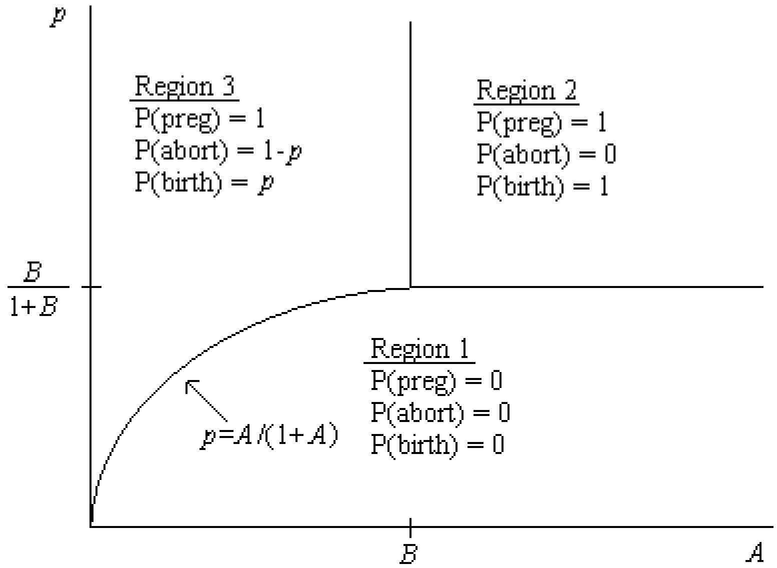
b. If the pregnancy rate were exogenous then it would be inconsistent for an increase in the cost of abortion to both reduce the abortion rate and reduce the birth rate. However, the Kane and Staiger model, which allows the pregnancy decision to depend on the cost of abortion, can explain both results.

In the KS model, teens face the following decision tree:

1. Decide whether to get pregnant. If no, get utility 0. If yes, go to step 2.
2. With probability *p* get married and get utility 1. With probability (1 - *p*) don't get married and go to step 3.
3. Decide whether to have an abortion. If no, get utility –*B*. If yes, get utility –*A*.

Hence a teen's decision to get pregnant depends on her values of *A*, *B*, and *p*. If *A* > *B*, she will not have an abortion, and her expected utility from getting pregnant is *p*(1) + (1 – *p*)(-*B*). Since the utility of not getting pregnant is 0, she will get pregnant if is *p*(1) + (1 – *p*)(-*B*) > 0 $\Leftrightarrow$ $p > \frac{B}{1+B}$. Similarly, if *A* < *B* she will get pregnant if $p > \frac{A}{1+A}$.

The outcomes as a function of *A*, *B*, and *p* can be summarized in the figure on the next page.



Note that an individual's values for *A* and *p* place her at a particular point in this figure, and her value of *B* determines the location of the boundaries of the regions. The aggregate birth rate and abortion rate depends on the distribution of *A*, *B*, and *p* in the population. To determine the effect of an increase in *A* on these rates, note that an increase in *A* is equivalent to shifting individuals to the right in the diagram. Hence an increase in *A* will shift some teens from Region 3 to Region 2, and other teens from Region 3 to Region 1. Since the probability of abortion in Region 3 is 1 – *p*, while it is zero in the other two regions, the effect on the abortion rate is unambiguous: an increase in *A* reduces the abortion rate.

The effect of an increase in *A* on the birth rate is not as clear cut. The probability of birth decreases from *p* to 0 for teens who move from Region 3 to Region 1, but it increases from *p* to 1 for teens who move from Region 3 to Region 2. In order for the birth rate to fall as *A* increases, we need for the fall in the birth rate due to teens moving into Region 1 to more than offset the increase in the birth rate from teens moving into Region 2. This is likely to be the case if there are enough teens with high values of *B*, because then an increase in the cost of an abortion will, on average, cause more teens to decide not to get pregnant. (To see this graphically, note that increasing *B* makes Region 1 larger relative to Region 2).

c. An analytical response to this argument is that in the context of this model an increase in *A* decreases the expected utility of some teens. Teens in Regions 1 and 2 are unaffected by the change in *A*. The increase does not move them to another region, and the expected utility in those regions does not depend on *A*. However, the expected utility for teens in Region 3 declines when *A* increases. Their expected utility is $p + (1 - p)(-A)$. If they remain in Region 3 when *A* increases their expected utility obviously falls, and if they move to Regions 1 or 2 they are worse off by revealed preference.