Solutions to Problem Set 3

Question 1

The indirect utility function is

$$V(p_x, p_y, I) = \ln I - a \ln p_x - b \ln p_y$$

A) Use Roy's Identity

$$d_x = -\frac{\frac{\partial V}{\partial P_x}}{\frac{\partial V}{\partial I}}$$

$$d_y = -\frac{\frac{\partial V}{\partial Py}}{\frac{\partial V}{\partial I}}$$

So that using the indirect utility function given above, we get

$$\frac{\partial V}{\partial P_x} = -\frac{a}{p_x} \quad \frac{\partial V}{\partial P_y} = -\frac{b}{p_y} \quad \frac{\partial V}{\partial I} = \frac{1}{I}$$

and the marshallian demands are

$$d_x = a\frac{I}{p_x} \qquad d_y = b\frac{I}{p_y}.$$

B) To recover the expenditure function use the identity

$$V(p_x, p_y, E(p_x, p_y, \bar{u})) = \bar{u}$$

so that using the indirect utility function given above, we get

$$\ln E(p_x, p_y, \bar{u}) - a \ln p_x - b \ln p_y = \bar{u}$$

and solving for $E(p_x, p_y, \bar{u})$ we obtain the expenditure function

$$E(p_x, p_y, \bar{u}) = e^{\bar{u}} p_x^a p_y^b$$

C) To obtain the hicksian demands use the Sheppard's Lemma

$$h_i = \frac{\partial E(p_x, p_y, \bar{u})}{\partial p_i}$$

where $i = x, y$.

So that, using the expenditure function that we found in part (B), we can obtain the hicksian demands

$$h_x = a e^{\bar{u}} p_x^{a-1} p_y^b$$

and

$$h_y(p_x, p_y, \bar{u}) = b e^{\bar{u}} p_x^a p_y^{b-1}.$$

Question 2

The utility function is

$$U(x, y) = -1/x - 1/y$$

and let $I = 18$, $p_x = 1$ and $p_y = 4$.

A) Use the solutions from P.S.2

$$d_x(p_x, p_y, I) = \frac{I}{p_x + (p_x p_y)^{1/2}} = \frac{18}{1+2} = 6$$

$$d_y(p_x, p_y, I) = \frac{I}{p_y + (p_x p_y)^{1/2}} = \frac{18}{4+2} = 3$$

B) Because of the cash transfer, income increases to $I' = 30$. The new marshallian demands are obtained by substitution of this value of income into the demand functions given above

$$d_x(p_x, p_y, I') = 10$$

$$d_y(p_x, p_y, I') = 5$$

C) An in-kind transfer of 12 units of X shifts out the budget line to a new budget constraint that overlaps with the cash transfer budget constraint for values of X larger than 12. Since the consumer cannot resell the in-kind transfer, she has to consume at least 12 units of X. Therefore at X=12 there is a kink in the budget line in the case of in-kind transfer (see picture).

We noted in part (B) that with a cash transfer the consumer would like to consume 10 units of X, which is less than the 12 units of X that he must at least consume in the case of in-kind transfer. Therefore she will choose to consume the minimum amount of X that she is allowed to, and to spend the rest on Y. The solutions are

$$d_x = 12$$

$$d_y = I/p_y = 18/4 = 4.5$$

These solutions correspond to the point where the indifference curve touches the in-kind transfer budget line on the kink (see picture).

D) By replacing the demands from part (C) into the utility function, we find that

$$U(x = 12, y = 4.5) = -\frac{11}{36}$$

E) The answer to the question is given by the expenditure function, which is the minimum expenditure required to achieve some utility level for given prices. In this case we can recover the expenditure function from P.S. 2

$$E(p_x, p_y, \bar{u}) = -\frac{p_x + p_y + 2(p_x p_y)^{1/2}}{\bar{u}}$$

where we replace $\bar{u} = -\frac{11}{36}, p_x = 1, p_y = 4$.
So that

$$E(1, 4, -\frac{11}{36}) = \frac{1 + 4 + 4}{\frac{11}{36}} = \frac{324}{11} \simeq 29.4$$

Notice that 29.4 is smaller than 30, which the income level after the cash transfer. This means that the minimum expenditure necessary to achieve the cash transfer utility (which is simply the initial income plus the cash transfer, i.e. 30), is larger than the minimum expenditure necessary to achieve the in-kind transfer utility (i.e. 29.4)

This in turn implies that the cash transfer gives higher utility than the in-kind transfer.

F) The government should give the consumer the amount of cash that would allow the consumer to achieve the utility $\bar{u} = -\frac{11}{36}$. This utility is achieved with a minimum expenditure of \$29.4. Since the consumer already has an income of \$18, the government must give her \$11.4.

G) The argument against converting food stamps into a cash transfer program is that the recipients of would make the "wrong" consumption choices. They would buy, e.g., drugs, alcohol and cigarettes instead of food. This argument assumes that the government knows better than the consumer what their good is, and it is therefore labelled as "Paternalism".

Question 3

For the Cobb-Douglas utility function

$$U(x,y) = (xy)^{1/2}$$

we have that

$$d_x(p_x, p_y, I) = \frac{I}{2p_x},\ d_y(p_x, p_y, I) = \frac{I}{2p_y}$$

$$h_x(p_x, p_y, \bar{u}) = \bar{u}\left(\frac{p_y}{p_x}\right)^{1/2},\ h_y(p_x, p_y, \bar{u}) = \bar{u}\left(\frac{p_x}{p_y}\right)^{1/2}$$

$$E(p_x, p_y, \bar{u}) = 2\bar{u}(p_x p_y)^{1/2}$$

A) Rebate required to keep consumer at original level of utility $u_0 = V(p_{x0}, p_{y0}, I_0)$ is

$$E(p_{x0} + t, p_{y0}, V(p_{x0}, p_{y0}, I_0)) - E(p_{x0}, p_{y0}, V(p_{x0}, p_{y0}, I_0))$$

Substituting the actual values we find that the rebate R is

$$R = \frac{2I_0(p_{x0}+t)^{1/2}p_{y0}^{1/2}}{2p_{x0}^{1/2}p_{y0}^{1/2}} - \frac{2I_0 p_{x0}^{1/2}p_{y0}^{1/2}}{2p_{x0}^{1/2}p_{y0}^{1/2}} = I_0\left[\left(\frac{p_{x0}+t}{p_{x0}}\right)^{1/2} - 1\right]$$

B) The tax revenue TR is the tax per unit multiplied by the units of X demanded at the price $p_{x0}+t$ and at the compensated income level $I_0 + R$

$$TR = td_x\left(p_{x0}+t, p_{y0}, I_0\left(\frac{p_{x0}+t}{p_{x0}}\right)^{1/2}\right) = \frac{I_0 t}{2p_{x0}^{1/2}(p_{x0}+t)^{1/2}}$$

C) The deadweight loss DWL is the difference between what the government pays as a rebate and what the government receives as tax revenue

$$\begin{aligned} DWL &= R - TR = I_0\left[\left(\frac{p_{x0}+t}{p_{x0}}\right)^{1/2} - 1\right] - \frac{I_0 t}{2p_{x0}^{1/2}(p_{x0}+t)^{1/2}} \\ &= I_0\left[\frac{2p_{x0}+t}{2p_{x0}^{1/2}(p_{x0}+t)^{1/2}} - 1\right] \end{aligned}$$

It can be easily shown that in this case the deadweight loss is strictly positive whenever the tax rate is strictly positive.

In general, the deadweight loss from taxation is the distortion introduced by a tax that does not equally affects the prices of all goods, i.e. a tax that changes the slope of the budget constraint. The utility of the consumer after the tax is lower than the utility after a revenue equivalent lump-sum tax, which does not induce substitution between goods.

The deadweight loss is always positive or zero. It is zero when there is no substitution between goods, i.e. when the goods are perfect complements.

In the picture the DWL is measured on the Y axis, since the price of Y does not change after the taxation. The tax revenue TR is the difference between the expenditure on the new bundle (B) at the after-tax prices and the cost of the same bundle at the pre-tax prices. The rebate R is the difference between the expenditure on the new bundle at the after-tax prices and the expenditure on the old bundle (A) at the pre-tax prices.

The DWL is the difference between R and TR, and you can see that it is positive whenever the substitution effect is not zero.

Question 4

$$\begin{aligned} p_{x0} &= 1, p_{y0} = 2 \\ p_{x1} &= 2, p_{y1} = 3 \end{aligned}$$

A)

$$\text{Ideal Index } = \frac{E(p_{x1}, p_{y1}, u_0)}{E(p_{x0}, p_{y0}, u_0)} = \frac{E(2, 3, u_0)}{E(1, 2, u_0)} = \frac{2u_0\sqrt{2}\sqrt{3}}{2u_0\sqrt{1}\sqrt{2}} = 1.73$$

So the increase in cost of living is 73% according to the ideal index.

B)

$$\text{Laspeyres Index} = \frac{x_1 p_{x0} + y_1 p_{y0}}{E(p_{x0}, p_{y0}, u_0)} = \frac{x_0 p_{x1} + y_0 p_{y1}}{x_0 p_{x0} + y_0 p_{y0}}$$

where

$$x_1 = d_x(p_{x1}, p_{y1}, I) = \frac{I}{2}$$

and

$$y_1 = d_y(p_{x1}, p_{y1}, I) = \frac{I}{4}$$

so that

$$\text{Laspeyres Index} = \frac{2(I/2) + 3(I/4)}{I/2 + 2(I/4)} = 7/4 = 1.75$$

The cost of living increases by 75% according to the Laspeyres index.

Note that this index overstates the increase in the cost of living as given by the ideal index.

$$\text{Paasche Index} = \frac{x_1 p_{x1} + y_1 p_{y1}}{E(p_{x1}, p_{y1}, u_1)} = \frac{x_1 p_{x1} + y_1 p_{y1}}{x_1 p_{x0} + y_1 p_{y0}} = 12/7 = 1.71$$

The cost of living increases by 71% according to the Paasche index.

Note that this index understates the increase in the cost of living as given by the ideal index computed at the utility level u_1, which is the utility computed at the optimal choices at the new level of prices.

C) The reference utility in the graph for the Lasp. Index is the utility u_0 computed at the original consumption bundle (x_0, y_0) and at the original prices (p_{x0}, p_{y0}).

The reference utility in the graph for the Paasche Index is the utility u_1, i.e. the utility at the new consumption bundle (x_1, y_1) and at the new prices .

Question 5

The CPI assumes that the cheaper prices available at discount stores are completely offset by lower quality service. If this is not true then there is a

positive outlet bias in the CPI, meaning that the CPI overstates inflation due to neglecting the lower prices available at discount outlets. Rapid growth in market shares for discount stores would indicate that they appear to be offering consumers a more preferred price/quality tradeoff, which would be evidence against the assumption in the CPI.

Question 6

The editorial is essentially arguing that declines in the price of goods that consumers cannot afford cause inflation to be understated, not overstated. This argument does not make sense because it ignores the fact that actual consumption patterns are used to form the market basket in the CPI. If the "average family" really cannot afford beef, then beef will have a very low or even zero weight in the market basket, and a decline in the price of beef will have little effect on the CPI.

More realistically, if families can afford some beef but eat less of it when the price of beef rises relative to chicken, then they have shifted consumption from an expensive good to a relatively less expensive substitute. These families are clearly worse off for the price rise, a point with which the Boskin Commission would agree. However, the extent of their monetary harm is not the original quantity of beef times the change in price - that is what the Laspeyres index calculates. Rather, it is a smaller number since consumers clearly have other substitutes available (e.g., chicken). In fact, only in the case where beef had no substitutes whatsoever (i.e., demand was perfectly inelastic) would the Laspeyres index correctly estimate the change in the cost of living for the rise in beef prices.

Question 7

Commissioner Abraham appears to be confused about the substitution effect. The substitution bias in the CPI depends on the assumption that consumers switch away from the original good and towards its substitutes as it becomes relatively more expensive (or, equivalently, buy less of a substitute as the original good becomes cheaper). Clearly, a washing machine is not a substitute for a dryer; it is a complement. Hence, the Commissioner's comment does not appear relevant. The real question is whether consumers can partially substitute for doing laundry as the price of washing machines rise. For those of you who have recently moved from living at home (where washing laundry is probably free) to living in dorms (where washing laundry is costly in time as well as money), we assume that you can introspect to discover some of the partial substitutes for washing your clothes.

Question 8

The price of cell phone service fell between 1983 and 1998. So cell phone service was introduced in the CPI only when its price was very low. If it had been introduced earlier, the CPI would have captured the drop in the price and the estimate of the increase in the cost of living would have been lower. So the CPI is biased upwards.

Another reason of upward bias when new goods are introduced is related to consumer surplus. For the goods on the market most consumers would be willing

to pay more than the market price (which is the evaluation of the marginal consumer). The difference between the willingness to pay and the market price is the consumer surplus (see Nicholson ch. 5 if you want a formal treatment). But since new goods enter the CPI at the market price, the value that most consumers attribute to these goods is not accounted, and this fact causes an upward bias in the CPI (see the Notes on Price Indeces posted on the web, for a more extensive discussion).

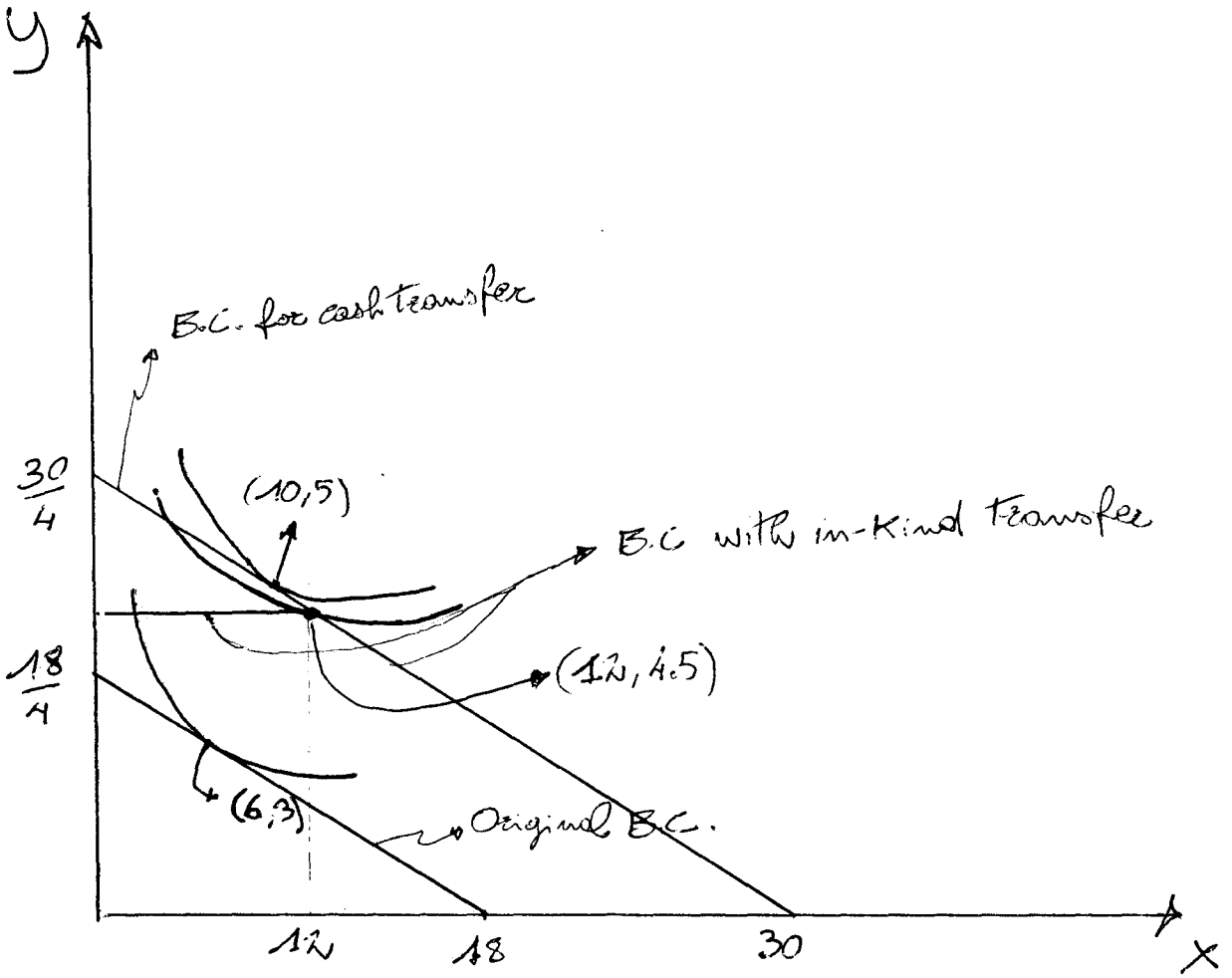
Question 9

A) The argument is based on the comparison between cash transfers and in-kind transfers. Gifts are in-kind transfer that cannot be resold. The gift receiver will end up on the kink of the budget constraint. That level of utility could be achieved with a cash transfer smaller than the cost of the gifts.

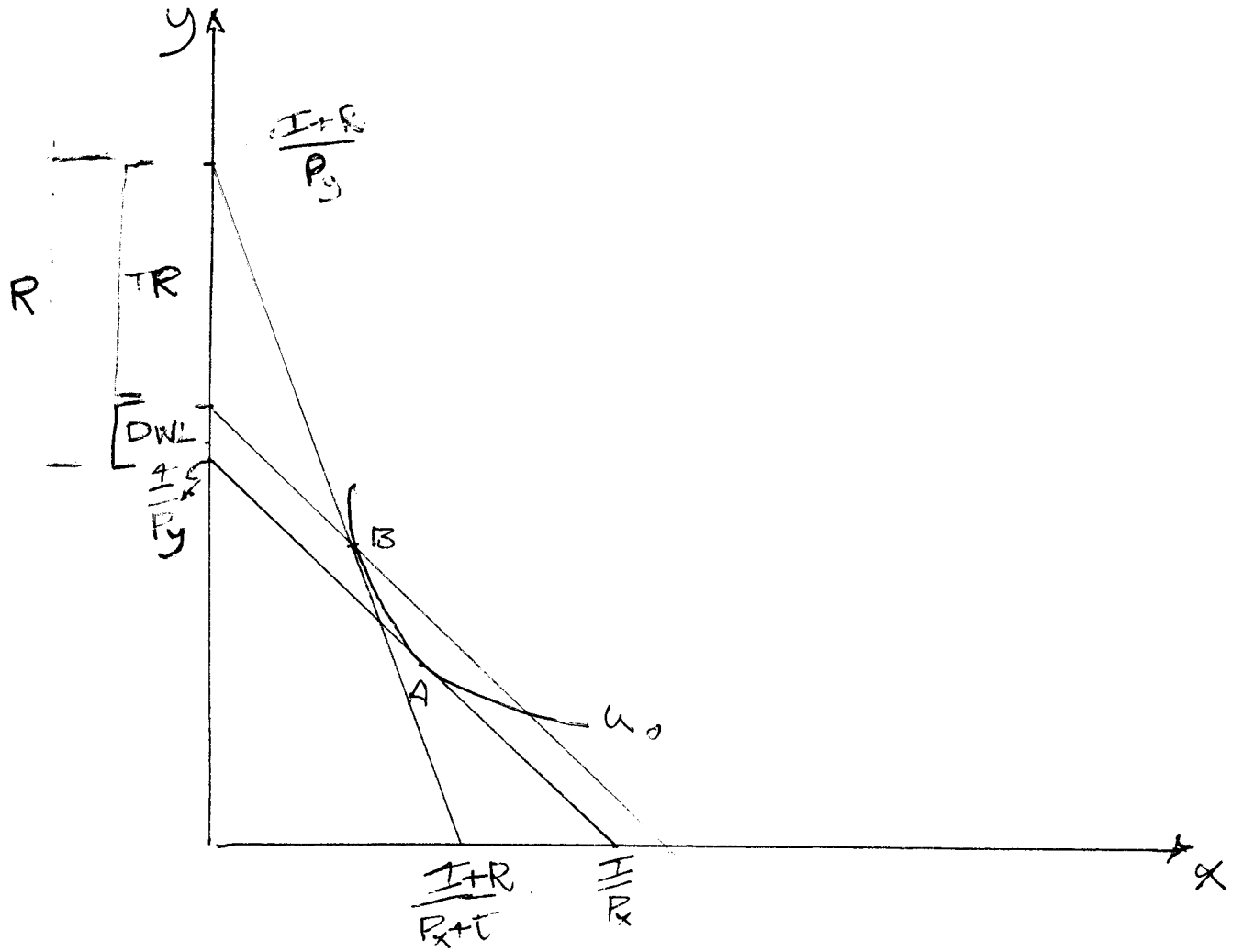
B) Since giving cash is always an option, a revealed preferences argument tells us that gift givers prefer buying non-cash gifts to simply giving cash. If the weight that the giver attributes to the receiver's utility is sufficiently low, the giver may still want to buy a gift rather than giving cash, because he values his higher utility more than the lower receiver's utility.

C) Table 3 tells us that the ones who give more cash are grandparents. Assuming that grandparents are also the ones whose non-cash gifts are the farthest from the receiver's tastes, we may think that the loss of receiver's utility from getting a non-cash gift outweighs the pleasure of giving a non-cash gift for grandparents. So that grandparents decide to give cash.

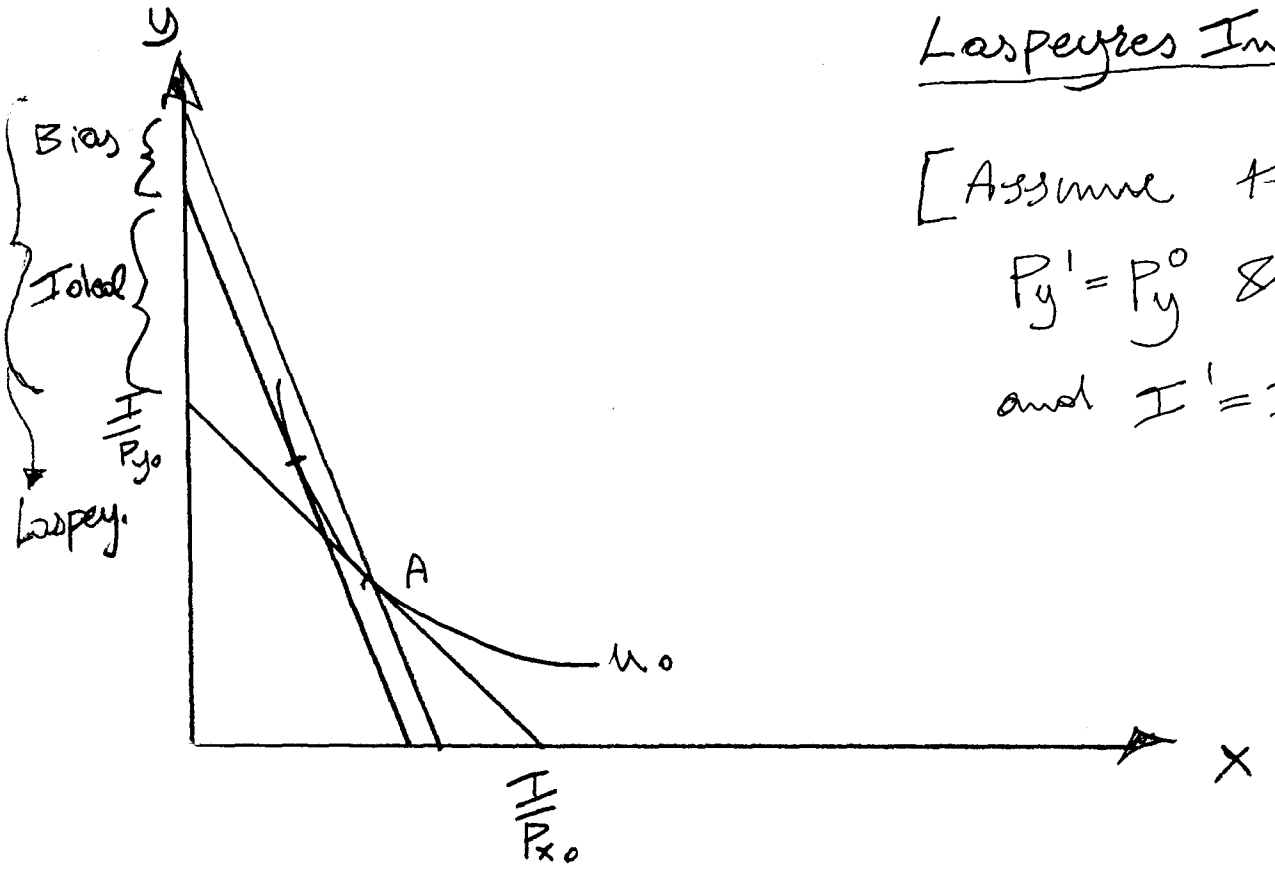
PICTURE FOR QUESTION 2



PICTURE FOR QUESTION 3

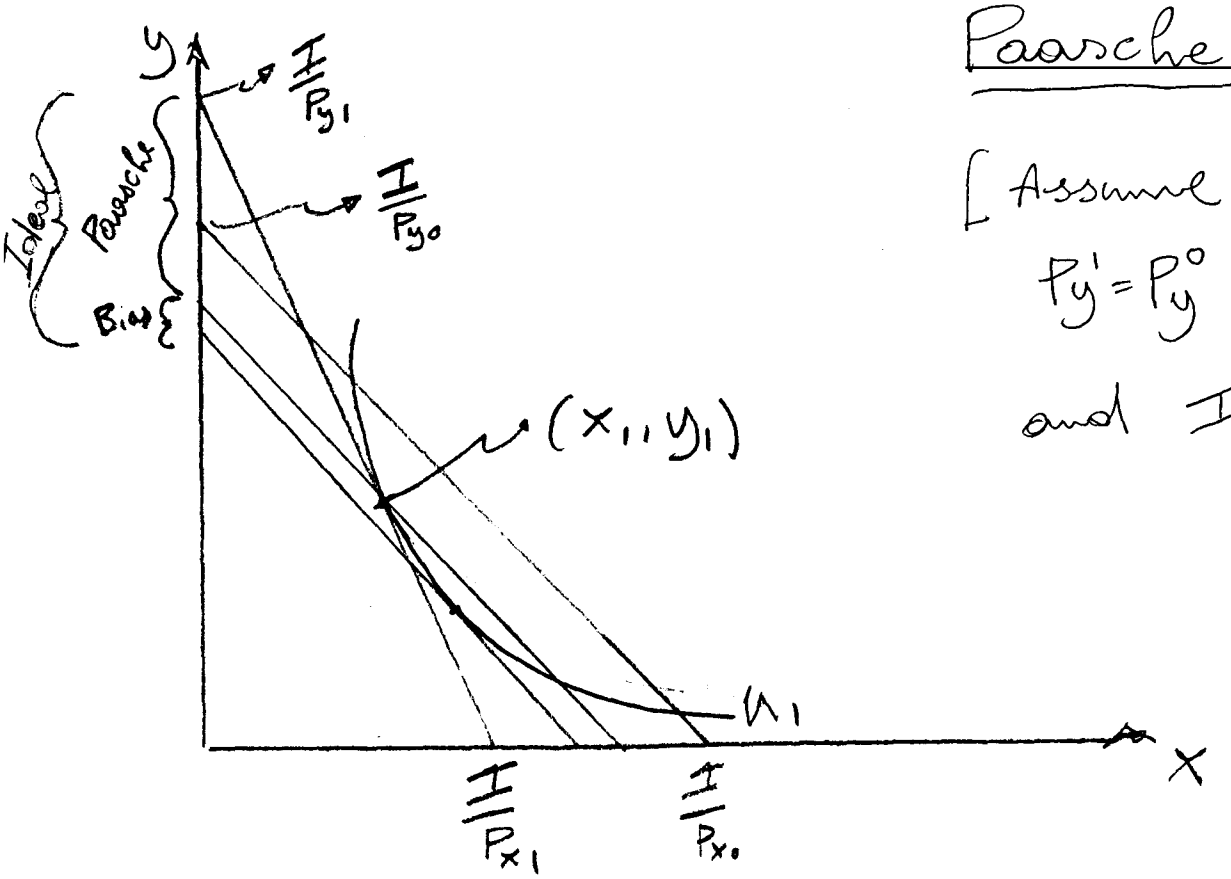


PICTURES FOR QUESTION 4



Laspeyres Index

[Assume that $P_y' = P_y^0$ & $P_x' > P_x^0$ and $I' = I^0$]



Paasche Index

[Assume that $P_y' = P_y^0$, $P_x' > P_x^0$ and $I' = I^0$]