14.03 Fall 2000 Problem Set 7 Solutions

Theory:

1. If used cars sell for \$1,000 and non-defective cars have a value of \$6,000, then all cars in the used market must be defective. Hence the value of a defective car is \$1,000. Since consumers are risk neutral, the price of a new car is equal to the expected value of a new car: \$4000 = \$6000(1-d)+\$1000(d) $\Rightarrow$ d = 2/5.

2. If the wage is less than 50, no one will apply for the job. If the wage $\in$ [50,200), the composition of the applicant pool is (1G, 0E, 0S), and the expected utility increment due to TA quality is 0. If the wage $\in$ [200,400), the composition of the applicant pool is (½ G, ½ E, 0S), and the expected utility increment is ½ (0) + ½ (5) = 2.5. If the wage is greater than or equal to 400, the composition of the applicant pool is (1/3 G, 1/3 E, 1/3 S), and the expected utility increment is 1/3 (0) + 1/3 (5) + 1/3 (7) = 4.

A. Since increases in the price of classes decrease student utility, the breakeven constraint will bind, which means the per student price MIT charges for a class will equal the wage divided by 100. If W_0 is initial wealth and the wage is w, expected student utility is

$$(W_0 - w/100)\Pr(G \mid w) + (W_0 + 5 - w/100)\Pr(E \mid w) + (W_0 + 7 - w/100)\Pr(S \mid w)$$
$$= W_0 - (w/100) + E(Q \mid w)$$

Where $E(Q|w)$ is the expected utility increment due to TA quality. Hence the problem reduces to choosing w to maximize $E(Q|w) - w/100$. Furthermore, the only wages that we need to consider are 50, 200, and 400, since any other wages increase the cost of the class without increasing the expected quality of the TA.

w	$E(Q\|w)$	$w/100$	$E(Q\|w) - w/100$
50	0	0.5	-0.5
200	2.5	2	0.5
400	4	4	0

So MIT should offer a wage of \$200 and the price should be \$2. The composition of the applicant pool at this wage is (½ G, ½ E, 0S).

B. The best applicant pool that MIT can get is (1/3 G, 1/3 E, 1/3 S), which requires a wage of \$400 and a price of \$4.

C. If MIT can distinguish G TAs from the other two types, the Gs can be eliminated from the applicant pool. This means that when w = 200 the pool is (0G, 1 E, 0S) and $E(Q|200) = 5$, and when w = 400 the pool is (0G, ½ E, ½ S) and $E(Q|400) = 6$. The new values of $E(Q|w) - w/100$ are 3 (for w = 200) and 2 (for w = 400). Although MIT should continue to offer a wage of \$200, the expected utility for each student increases by an amount equivalent to \$2.50 (= 3 – 0.5). Thus MIT should be willing to pay \$250 per course for the right to use this test.

The advanced test can distinguish all three types. This means that when $w = 400$, the pool is (0G, 0E, 1S) and $E(Q|400) = 7$. The new value of $E(Q|w) - w/100$ for $w = 400$ is 3, which is the same as the maximum value attainable with the basic test. Hence the advanced test provides no additional value.

D. If MIT requires TAs to have a degree, TAs will decide whether to go to school based on the following considerations: If a G does not go to school she gets \$50 this semester and \$50 next semester, while if she does go to school she gets nothing this semester and next semester she gets w with probability 0.1 and \$50 with probability 0.9. Since there is no discounting, G's go to school iff $0.1w + 0.9(50) \geq 100 \Leftrightarrow w \geq 550$. Similarly, E's go to school iff $0.9w + 0.1(200) \geq 400 \Leftrightarrow w \geq 422\ {}^{2}\!/_{9}$, and S's go to school iff $w \geq 800$.

So if MIT requires a degree and offers $w = 422\ {}^{2}\!/_{9}$, the applicant pool is (0G, 1E, 0S), and $E(Q|w) - w/100 = 5 - 4\,{}^{2}\!/_{9} = {}^{7}\!/_{9}$. Now note that it could never be optimal for MIT to offer a higher wage. At $w = 550$ the expected utility increment is lower (since now there are some G's in the applicant pool) while the price is higher. At $w = 800$ the expected utility increment minus the price is necessarily less than ${}^{7}\!/_{9}$, since the price is 8 and the expected utility increment never exceeds 7. Hence if MIT decides to require a degree, it should offer $w = 422\ {}^{2}\!/_{9}$. To determine whether it is optimal to require a degree we need to compare ${}^{7}\!/_{9}$ to the maximum value of $E(Q|w) - w/100$ attained in part A, which is ½. Hence MIT should require TAs to have a degree, it should offer a wage of \$422.22, and the resulting applicant pool will all be Excellent TAs.

Nicholson 10.3

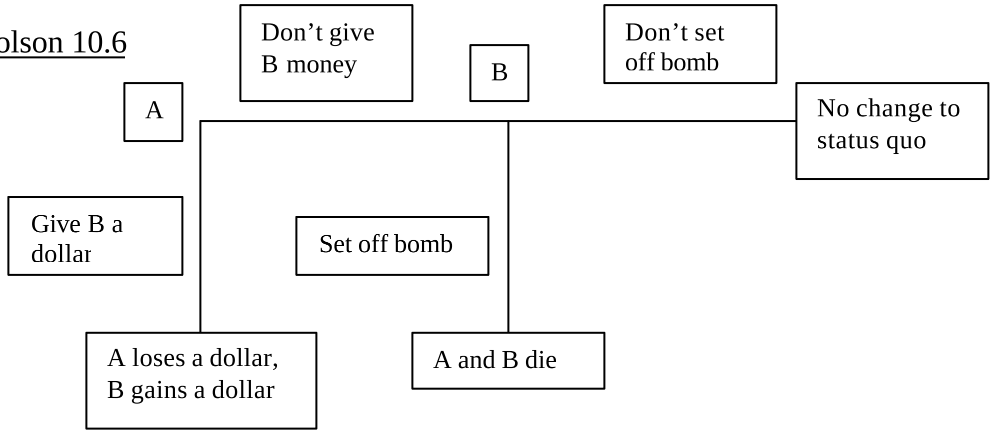
	Stag	Hare
Stag	2,2	0,1
Hare	1,0	1,1

a. As you can see from the best responses (which are underlined in the table above), the pure strategy NE are (Stag, Stag) and (Hare, Hare). There is also a mixed strategy NE, which we find by using the indifference property. If one player plays Stag with probability p and Hare with probability $(1-p)$, the other player's payoffs are $2p$ for Stag and 1 for Hare. Hence the mixed strategy NE has each player playing each strategy with probability ½.

b. If one player plays Stag with probability p, it is optimal for the other player to play Hare if $p \leq$ ½, and Stag if $p \geq$ ½. If $p =$ ½, any strategy is optimal for the other player.

c. If there are n players who play Stag with probability p_i, it is optimal for player B to play Stag if $\prod_{i \neq B} p_i \geq$ ½. If $p_i = p_j = p \forall i \neq j \neq B$, this reduces to $p^{n-1} \geq$ ½. Cooperation is less likely in the sense that the threshold value of p that is required for Stag to be optimal is higher.

Nicholson 10.6



B's threat to set off the bomb is not subgame perfect if B prefers the status quo to dying. Since A then has no reason to give B any money, it is in B's interest to convince A that B is crazy.

Nicholson 10.7

Child moves first and chooses r to maximize $U_A(Y_A(r)+L)$.
Parent moves second and chooses L to maximize $U_B(Y_B - L) + l\,U_A(Y_A + L)$.
$Y_A'(r) > 0, Y_B'(r) < 0, l > 0$

The first order condition for the parent's maximization problem is
$-U_B'(Y_B(r) - L(r)) + l\,U_A(Y_A(r) + L(r)) = 0$ (note that L is a function of r).

Since the child moves first, the child will take into account the effect of r on the parent's subsequent choice of L. The child's maximization problem is
$\max_r U_A(Y_A(r) + L(r))$, and the first order condition is $(Y_A'(r) + L'(r))U_A'(\cdot) = 0$.
Thus the child will choose r such that $Y_A'(r) + L'(r) = 0$.

To figure out what $L'(r)$ is, we need to go back to the parent's FOC. Differentiate with respect to r to get $-(Y_B'(r) - L'(r))U_B''(\cdot) + l\,(Y_A'(r) + L'(r))U_A''(\cdot) = 0$. Since the second term is zero by the child's FOC, this implies that $L'(r) = Y_B'(r)$. Hence the child will choose r such that $Y_A'(r) + Y_B'(r) = 0$, which is exactly the FOC for the problem $\max_r Y_A(r) + Y_B(r)$.

Nicholson 10.8

	Chicken	Not C
Chicken	2,2	1,3
Not C	3,1	0,0

a. There are three NE in this game. The pure strategy NE are (NC,C) and (C,NC). The mixed strategy NE is for each player to mix with probability ½.

b. The threat to not chicken out is not credible. If the other player did not chicken out, you would be better off playing chicken and getting payoff 1 than not chickening out and getting payoff 0.

c. If one player could credibly commit to not chickening out, he could guarantee himself a payoff of 3, which is the highest payoff available. Hence such a commitment would be desirable.

Nicholson 10.9

	L	M	R
U	5,5	2,6	0,7
M	6,2	3,3	0,0
D	7,0	0,0	1,1

a. The pure strategy NE are (M,M) and (D,R).

b. If the game is played twice, any strategy profile that results in any combination of the static NE being played at each stage is a SPE. If there is no discounting, there is also an SPE in which (U,L) is played in the first stage and (M,M) is played in the second stage. The strategy profile that supports this SPE is as follows: play (U,L) in the first period. If (U,L) is played in the first period, play (M,M) in the second period, otherwise play (D,R). Since both (M,M) and (D,R) are NE of the last period subgame, we only need to check that neither player has a profitable deviation in the first period. By deviating in period 1, a player can get 7 instead of 5, but by doing so she will get 1 instead of 3 in the last period. Hence as long as there is no discounting this deviation is not profitable.

c. To support (U,L) in the infinitely repeated game, consider the following strategy profile: Begin by playing (U,L), and continue playing (U,L) as long as it is the only strategy profile that has been played in the past. If anything else is played, play (D,R) forever. (We could also use (M,M) as the punishment phase, but since the payoffs at (D,R) are lower, using (D,R) allows us to support cooperation for a wider range of discount factors.)

Since the punishment phase is a NE of the stage game, to check whether this is an SPE that supports (U,L), we only need to check that the benefit from deviating in the cooperation phase does not exceed the cost of deviating. The optimal deviation from (U,L) gives a payoff of 7, and since the payoff to each player at (U,L) is 5, the benefit from deviating is 2. The cost of deviating is that starting in the next period you get 1 forever instead of 5 forever. If the discount factor is δ, this cost is $4\delta/(1-\delta)$. The benefit from deviating will not exceed the cost if $2 \le 4d\ /(1-d) \Leftrightarrow d\ \ge 1/3$.

Hence (U,L) is sustainable if the discount factor is at least 1/3.

Nicholson 10.10

First-price sealed-bid auction, with ties decided by coin flip. Player A's valuation = \$600, player B's valuation = \$500, and valuations are common knowledge.

a. There are three categories of strategies: bidding more than your valuation, bidding your valuation, and bidding less than your valuation. If you bid more than your valuation, your payoff is negative if you win and zero if you lose. If you bid your valuation, your payoff is zero no matter what your opponent does. If you bid less than your valuation, your payoff is positive if you win and zero if you lose. Hence bidding greater than or equal to your valuation is *weakly* dominated by bidding less than your valuation. The dominance is weak because bidding less than your valuation gives you the same payoff as bidding greater than or equal to your valuation if your opponent's bid is high enough.

b. Whether or not a NE exists in this game depends on the nature of the players' strategy sets. If there is a minimum bidding increment (e.g., all bids must be integer multiples of 1 cent or 1 dollar), then a NE exists. But if there is not a minimum bidding increment, then no NE exists.

   To keep the strategy space of reasonable size, let's assume that players must bid in multiples of \$25. Then the relevant portion of the game looks like this:

	\$475	\$500	\$525	\$550	\$575	\$600	\$625
\$475	62.5,12.5	0,0	0,-25	0,-50	0,-75	0,-100	0,-125
\$500	100,0	50,0	0,-25	0,-50	0,-75	0,-100	0,-125
\$525	75,0	75,0	37.5,-12.5	0,-50	0,-75	0,-100	0,-125
\$550	50,0	50,0	50,0	25,-25	0,-75	0,-100	0,-125
\$575	25,0	25,0	25,0	25,0	12.5,-37.5	0,-100	0,-125
\$600	0,0	0,0	0,0	0,0	0,0	0,-50	0,-125
\$625	-25,0	-25,0	-25,0	-25,0	-25,0	-25,0	-12.5,-62.5

   The NE is not unique. For this bid increment, there are 4 NE: (\$500,\$475), (\$525,\$500), (\$550,\$525), and (\$575,\$550). A similar line of reasoning would show that if players bid in multiples of \$1, there would be 100 NE: (\$(500+n),\$(500+n-1)) for n = 0 to 99.

   A few things to notice about this problem:

   1) In all of the NE except one, player B is playing a weakly dominated strategy. Hence this example shows that weakly dominated strategies, unlike strictly dominated strategies, can be played with positive probability in a NE.

2) To see why no equilibrium exists when there is not a minimum bidding increment, consider what A's best response is if B bids \$500. If A bids \$500, her payoff is \$50. By bidding a small amount $\varepsilon>0$ more than \$500, she can increase her payoff to $\$(100-\varepsilon)$. However, this is not a best response, because bidding $\varepsilon/2$ more than \$500 would strictly increase her payoff to $\$(100-\varepsilon/2)$. Since this is true for any $\varepsilon>0$, A's best response does not exist and there is no equilibrium. If the object went to A with probability 1 in the event of a tie the existence problem would be solved, because then A's best response to a bid of \$500 would be to bid \$500.

c. If each player does not know the other's valuation, then this becomes a game of incomplete information. The equilibrium will depend on the beliefs each player has about the other player's valuation.

Application: Tyler, Murnane, and Willett (TMW) study

1. Here's the basic experimental design you might have suggested that makes use of all of the elements in the question.

   Using the AFQT test, you identify, for example, 1,000 people of approximately equal ability, none of whom has graduated from high school.

   You track their earnings for one or two years.

   You then grant 500 of them a GED ('treatment group') and forbid the other 500 ('control group') from obtaining a GED (remember, you're the czar).

   You then track their earnings for another one or two years.

   You perform a difference-in-differences analysis to determine whether the treatment group had greater earnings gains over these two years than did the control group.

   If the treatment group did gain more, this indicates that the GED *does* function as a labor market signal. We know this because both the treatment and control groups have approximately identical ability. And of course assigning a GED did not increase the human capital of the treatment group. So it must be the case that the greater earnings gains of the treatment group are due to the signaling value of the GED.

   If you find no earnings difference for treatments versus controls, however, this would indicate that the GED has little or no signaling value.

2. The necessary conditions for a separating equilibrium are that the net benefits of GED acquisition are negative for low-productivity dropouts and positive for high-productivity dropouts. The key equations are W(GED) – C(GED|low productivity) < W(no GED) and W(GED) – C(GED|high productivity)>W(no GED).

The answer to the second part of the question is *yes*, a signaling equilibrium could still exist even if the GED is free, pleasant, and requires no preparation. The important key condition is that the GED is more psychically costly for low ability people to pass than for high ability people to pass. In the limiting case where the GED measures innate ability exclusively, the psychic cost for low ability people would be in some sense infinite (i.e., they could never pass). Hence, what really matters is that on some dimension of cost, high ability people find it 'cheaper' to obtain a GED than do low ability people.

3. No, the TMW study does not demonstrate that the human capital model is incorrect, because it studies the effect of a GED on wages holding human capital (as measured by the test score) constant.

4. If the Department of Education grants a GED to *all* high school dropouts, then the GED no longer signals productivity. All dropouts will have a GED and will be paid according to the productivity of the average dropout, which is necessarily less than the productivity of high-productivity dropouts. Hence this policy will not raise the earnings of dropouts by $1,500.