14.03 Fall 2000: Handout on Price Indices

The ideal cost of living index

The ideal cost of living index measures the relative cost of obtaining the same level of *utility* (not consumption) at different points of time. To simplify, consider two time periods, two goods, and one consumer. The ideal cost of living index for a consumer is:

1. $$I_{Ideal} = \frac{E(P_x^1, P_Y^1, U_0)}{E(P_x^0, P_Y^0, U_0)}$$

The problem with this Ideal Index is that it is only usable in theory. In practice, we do not observe the expenditure function (or Hicksian demand). We must approximate the ideal index using an observable alternative. Two common approximations are the Laspeyres index and the Passche index.

Laspeyres Index

The Laspeyres Index answers the question "What does it cost to buy the old (period 0) bundle at the new (period 1) prices?" The formula is:

2. $$I_L = \frac{P_x^1 X^0 + P_y^1 Y^0}{P_x^0 X^0 + P_y^0 Y^0}$$

How does this compare to the Ideal Index? Notice that the denominator is equal to the denominator of the Ideal Index by definition. However, you should be able to demonstrate to yourself that:

3. $$P_x^1 X^0 + P_y^1 Y^0 \geq E(P_x^1, P_Y^1, U_0)$$

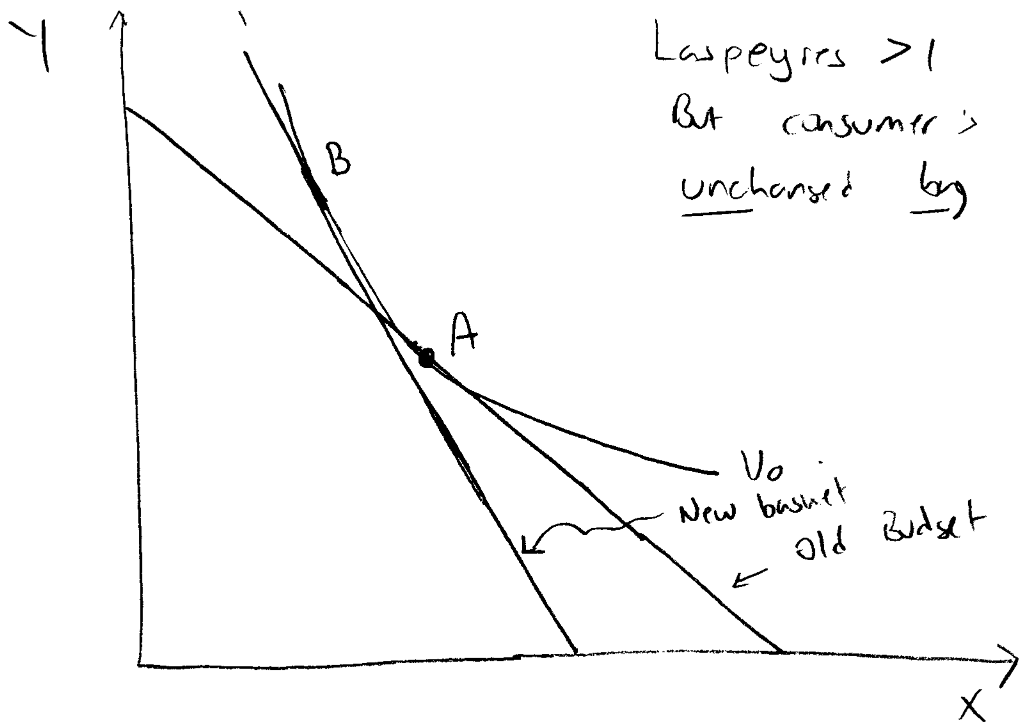
In other words, the numerator is too large relative to the ideal index. The Laspeyres index is *upward biased.* The reason: consuming the old bundle at the new prices will surely get you the old level of utility, but there is probably a cheaper way to achieve the old level of utility at the new prices. The reason is *substitution.* As prices change, consumers substitute towards good with falling prices and away from goods with rising prices. Substitution partly (not fully) insulates consumers from price increases. Hence, the Laspeyres index always provides an *upper bound* on the true increase in the cost of living. This is illustrated by the two figures that follow.

Figure 1 illustrates a case where Laspeyres >1 (i.e., Laspeyres says the Cost of Living has risen) but the consumer is *no worse off* (indifferent). In this case, the Ideal Index would be equal to 1.

Figure 2 illustrates a case where Laspeyres =1 but the consumer is actually better off. In this case, the Ideal Index would be < 1, the cost of living has fallen.

Note that there is *no case* where Laspeyres is =1 but the consumer is worse off. The reason: If Laspeyres = 1, the consumer *can afford* the old bundle at the new prices. Hence, the consumer cannot be worse off.

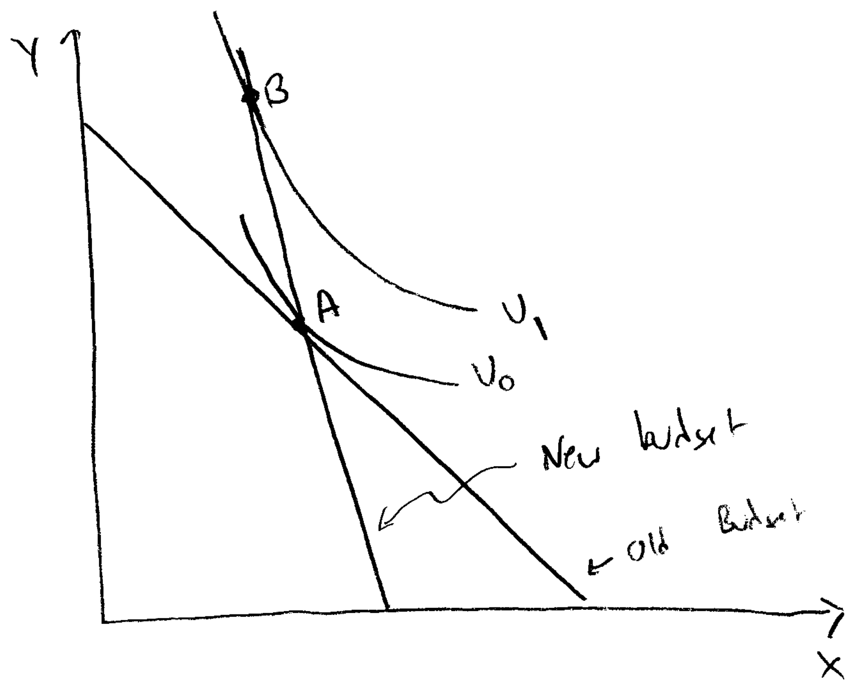
Figure 1



Laspeyres >1
But consumer's utility unchanged by price changes.

A - original bundle
B - New bundle
Observe: Both lie on same indifference curve
Q: How do we know Laspeyres >1 ?
A: B/c A lies outside feasible budset set at new prices.

Figure 2: Laspeyres = 1 but consumer is better off



A - Original bundle
B - New bundle

Observe: B lies on $U_1 > U_0$

Q: How do we know Laspeyres = 1 ?
A: B/C A lies exactly on new budget set - price of bundle A unchanged (b/c $P_X \uparrow$ and $P_Y \downarrow$).

Paasche Index

The Paasche Index answers the question "what does it cost to buy the new (period 1) bundle at the old (period 0) prices?" The formula is:

2. $$I_P = \frac{P_x^1 X^1 + P_y^1 Y^1}{P_x^0 X^1 + P_y^0 Y^1}$$

How does this compare to the Ideal Index? First, note that the *reference utility* is different for this index. We're now asking about the cost of achieving the new utility at the old prices whereas previously we were asking about the cost of achieving the old utility at the new prices. This is problematic if we're interested in holding utility constant at the *old* level (what the ideal index does).

Now look carefully at the numerator and denominator. The numerator is equal to the numerator of the Ideal Index by definition (in this case, fixing utility at U_1). However, the denominator will be greater than or equal to the denominator of the ideal index:

3. $$P_x^0 X^1 + P_y^0 Y^1 \geq E(P_x^0, P_Y^0, U_1)$$

The denominator will generally be *too large*, leading to a downward bias. Why? Because consuming the new bundle at the old prices is generally not the most efficient way to achieve U_1. Again because of substitution, the consumer is likely to find a less costly bundle that provides utility equivalent to U_1. Since the denominator will generally be too large (and never too small), the Paasche index always provides a *lower bound* on the true increase in the cost of living. You can see this in the two figures on the following page.

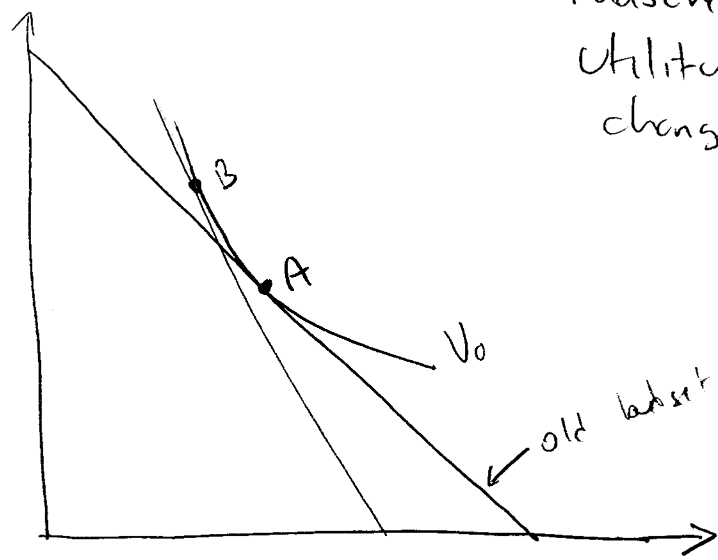
Figure 3 illustrates a case where Paasche <1 (i.e., the Cost of Living has fallen according Paasche) but the consumer is no better off. In this case, the Ideal Index would be = 1.

Figure 4 illustrates a case where Paasche =1 but the consumer is actually worse off. In this case, the Ideal Index would be > 1.

Note that there is *no case* where Paasche =1 but the consumer is better off. The reason: If Paasche = 1, the consumer *can* just afford the new bundle at the old prices, meaning that the new bundle was on the frontier of the old budget set. If the consumer would have chosen the new bundle at the old prices, then she is no worse off. But due to the substitution effect, this is very unlikely. Hence, if Paasche =1 and the consumer would not have chosen the new bundle at the old prices, then the consumer is worse off. You can also see this point in Figure 4 by applying the Weak Axiom of Revealed Preference.

Figure 3

Paasche < 1 but consumer's utility unaffected by price changes.



A - Original bundle

B - New bundle

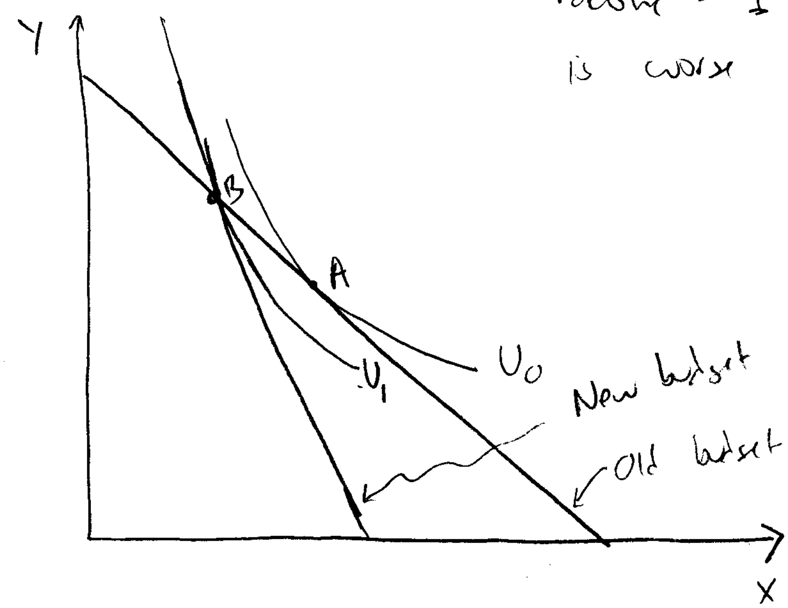
Observe: Both lie on same indifference curve

Q: How do we know that Paasche < 1?

A: B/c B lies outside feasible budget set at old prices. B was more expensive at old prices. According to Paasche, "cost of living" has fallen. In fact, consumer is not better off.

Figure 4

Paasche = 1 but consumer is worse off.



A - Original bundle
B - New bundle

Observe that B lies on old budget set but $U_1 < U_0$

Q: How do we know Paasche = 1?
A: B/c B lies exactly on the old budget set. The new bundle costs the same at both the old & new prices.

Note: Even w/o indifference curves, you can make the same point using the Weak Axiom of Revealed Preference.

Sources of Bias in the U.S. Consumer Price Index (CPI)

The CPI is a Laspeyres Index. It is thought to suffer from at least four sources of bias that we discussed in class.

1. Substitution bias.

This is the bias induced by tracking the change in the cost of a *fixed basket of goods* and is exactly what is illustrated by Figures 1 and 2. Because consumers substitute among goods when price changes – and because the Laspeyres index implicitly assumes that they do not – it overstates the change in the cost of living when relative prices among goods in the fixed basket shift.

Substitution bias is an inherent problem with the Laspeyres index. The other three biases discussed below are not inherent in theory but may be at least as important in practice.

2. Outlet bias

If an identical good is sold at multiple stores ('outlets') at different prices, the CPI (or, more precisely, the Bureau of Labor Statistics which administers the CPI) assumes that all of the price differences are due to quality of service or convenience. This might be a reasonable assumption; the quality of service at Walmart is certainly lower than at Bloomingdales. However, over the last 20 years, the market share of discount outlets like Walmart, Costco, and Sam's Club has grown enormously. This suggests that even accounting for quality differentials, discount outlets are offering greater value. Consumers have substituted their purchases from full price stores to discount outlets, accordingly. Because the CPI does not take into account *changes in the share of purchases* made at full price versus discount outlets, it will tend to overstate the cost of buying a fixed basket of goods.

3. Quality change bias

Over time, most products improve in performance, reliability, features and convenience. This creates a substantial problem for measuring the cost of a 'fixed basket' because many products in the CPI basket improve continually, often without any price change. Typically, but not exclusively, BLS has assumed that if the price of a good hasn't changed, the product has also not changed. But if the product has improved without the price increasing, then the real cost of the product has fallen. (Note that quality can also fall when the price remains unchanged.) Making appropriate quality change adjustments is an extremely difficult problem. Because quality is continually improving for many important products in ways that are difficult to quantify, economists generally believe that the CPI falls to fully account for quality improvement, resulting in upward bias.

4. New goods bias

Many new products enter the market at very high prices and then fall in price dramatically as they reach critical mass: mobile phones, anti-lock brakes, Internet access, new medical technologies, computers. These new goods induce two sources of bias in the CPI.

One source is simply timing. Because the CPI basket is only updated every decade or so, many new goods do not enter the CPI until their price has already fallen a long way. Hence, the typical price of a mobile phone fell from, say, \$800 to \$200 between 1985 and 1995 and then fell from \$200 to \$100 between 1995 and 2000. It's likely that mobile phones did not enter the CPI until the mid 1990s. Hence, most of the price decline (which reduced the cost of living) was entirely missed by the CPI. This will cause upward bias in the CPI.

A second problem is more subtle and could be more important. The value of a good in terms of consumer utility is not its price but the maximum amount the consumer would be willing to pay for the good (the area under the Hicksian demand curve). The difference between what a consumer would be willing to pay and the market price is called consumer surplus. Most goods generate consumer surplus because the utility benefit of the good is greater than its price (not for the marginal buyer, of course). When a new good is introduced, such as a drug that increases the life expectancy of a person dying from pancreatic cancer, how can the CPI measure the utility benefit this generates? One answer might be the price. If people pay \$100 per pill, then they must be getting *at least* \$100 in benefits. But it's likely that many people would be willing to pay \$200 per pill or \$500 per pill. In other words, the price of a new good only provides a *lower bound* on the value that consumers place on the good. Because it does not account for the consumer surplus generated by new goods, the CPI will be biased upwards.

You may ask, why isn't this a problem for 'old goods?' For example, you only pay 99 cents for a tube of toothpaste, but presumably you'd pay much more if you had to. This would have been a problem for the CPI when toothpaste was introduced. But once toothpaste is part of the fixed market basket, we only need the CPI to measure the *additional consumer surplus* (or harm) created by price changes. If toothpaste falls from 99 cents to 49 cents per tube, the *additional consumer surplus* generated is at most 50 cents per tube. So, for most goods (items that have been part of the market basket for many decades) price changes tell us a good deal about actual changes in the cost of living. However, because new goods can create vast consumer surplus that far exceeds their price (medical breakthroughs are the best example), the CPI is very unlikely to accurately capture the accompanying falls in the cost of living, that is, the cost of obtaining a fixed level of utility.