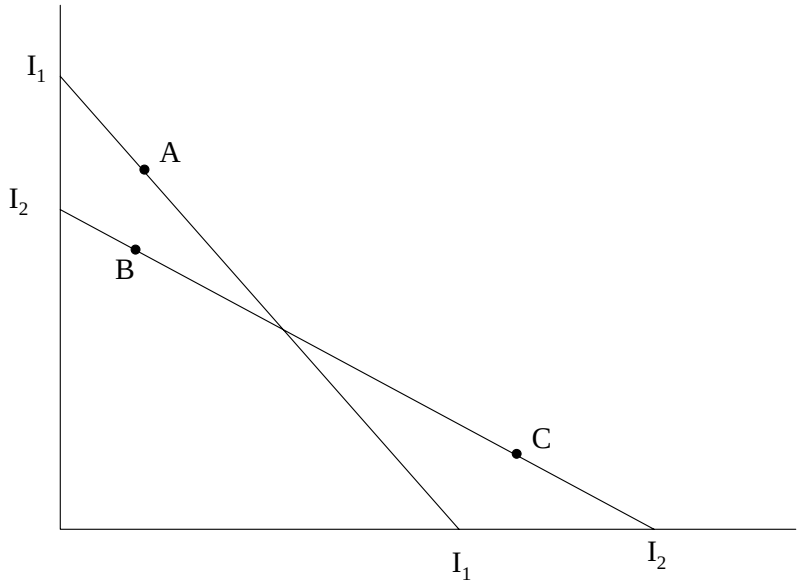
Lecture 8 - Revealed Preference and Consumer Welfare

14.03, Spring 2003

1 Revealed Preference and Consumer Welfare

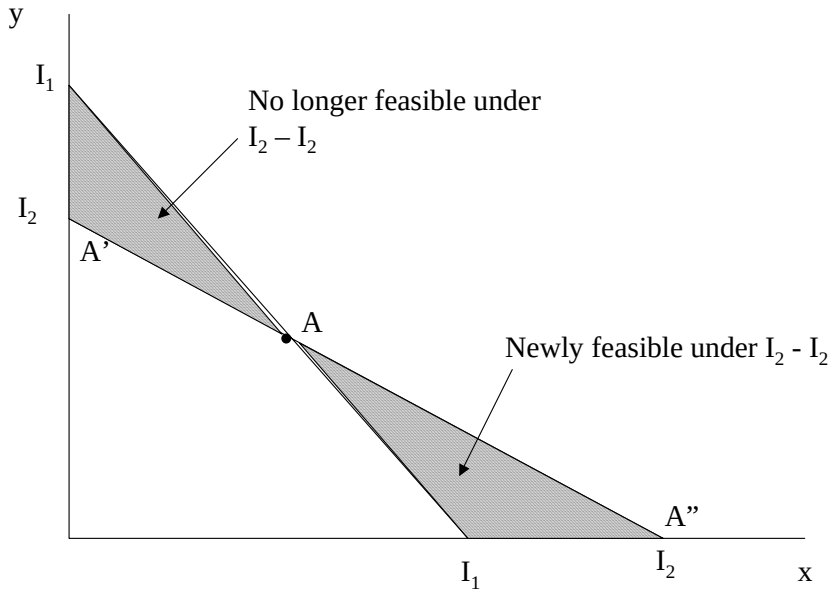
- We have so-far been applying an axiomatic approach to characterizing consumer choice (based on the six axioms given in Lecture 4).
- But there is an approach to assessing utility that requires even fewer assumptions and nevertheless gives strong results. This approach is called *Revealed Preference*.
- Consider Figure 8#1 where the consumer sequentially faces two budget sets, $I_1 - I_1$ and $I_2 - I_2$.Point A on $I_1 - I_1$ is initially chosen. This point is said to be "revealed preferred" to all other feasible points inside of the budget set.

8#1



- Now the consumer is faced with $I_2 - I_2$.
  - If they choose point B, are they better or worse off? *Worse off*, because point A was revealed preferred to point B under the initial choice conditions.
  - If they choose point C, are they better or worse off? The axiomatic approach would tend to suggest they are better off because unless the indifference curve tangent to point A has an extremely shallow slope, point C would probably put them on a higher indifference curve.
  - But under Revealed Preference, the answer is ambiguous. The reason is that we do not have any revealed preference information on whether A is preferred to C or vice versa–these choices were never available simultaneously.
- Take a second example (See Figure 8#2).

8#2



Here the second budget set rotates through the originally chosen point A on the first budget set. We do not know the consumer's new choice. Is the consumer better off, worse off, or can't we say?

- We know that point A r.p. $(A, A']$
- We do not know–but it is possible–that a point on $(A, A'']$ is preferred to A.
- We say that the consumer is "weakly" better off.

Definition 1 *Weak Axiom of Revealed Preference: If A, B feasible and A chosen, then at any prices and income where A, B are feasible, the consumer will choose A over B.*

1. This axiom says two things:
2. People choose what they prefer.
3. Preferences are *consistent*. Therefore, a single observed choice reveals a stable preference.

There is also a stronger form of this axiom.

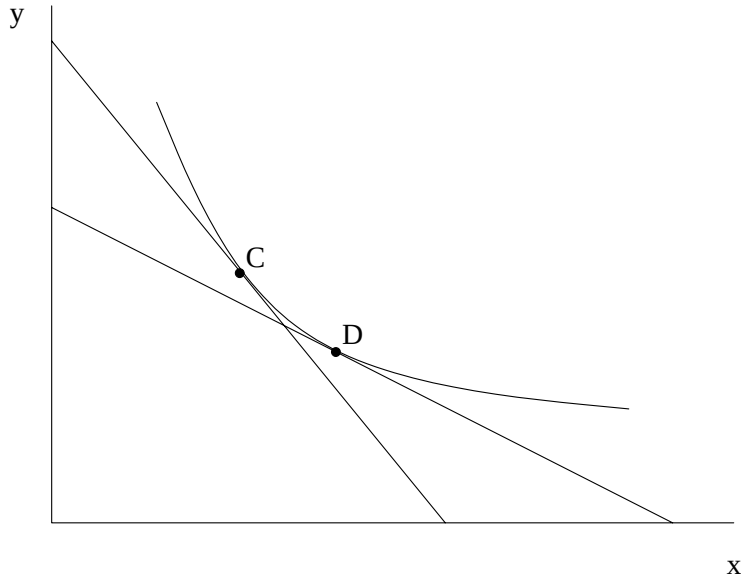
Definition 2 *Strong Axiom of Revealed Preference (SARP): If commodity bundle 0 is revealed preferred to bundle 1 and bundle 1 is r.p. to bundle 2 and bundle 2 is r.p. to bundle 3... and bundle $k-1$ is r.p. to bundle k, then bundle k cannot be r.p. to bundle 0.*

SARP is simply WARP with an added transitivity assumption. But this places much greater strictures on behavior.

1.1 The power of WARP

- The result that $\frac{\partial X}{\partial P_x}|_{u=u_0} < 0$ (i.e., the Hicksian demand curve is always downward sloping) depends on an untested assumption about diminishing MRS, giving rise to indifference curves that are bowed inward towards the origin.
- Can we obtain the same result using only WARP?
- Suppose two points C, D on intersecting budget sets are indifferent for consumer utility (see Figure 8#3).

$$C = (X_c, Y_c) \sim D = (X_d, Y_d).$$



- The crucial thing about this figure is that C, D do *not* lie in the same budget sets. When C is available, D is not and vice versa.
- Since $C \sim D$, is must be true by WARP that

$$\begin{aligned} P_x^c X_c + P_y^c Y_c &\leq P_x^d X_d + P_y^d Y_d \text{ when } C \text{ is chosen,} \\ P_x^d X_d + P_y^d Y_d &\leq P_x^c X_c + P_y^c Y_c \text{ when } D \text{ is chosen.} \end{aligned}$$

- Rearranging, we get:

$$\begin{aligned} P_x^c (X_c - X_d) + P_y^c (Y_c - Y_d) &\leq 0, \\ P_x^d (X_d - X_c) + P_y^d (Y_d - Y_c) &\leq 0. \end{aligned}$$

  which simply says that D would be more expensive than C under the prices that causes the consumer to choose C, and vice versa for the prices that cause the consumer to choose D.
- Combining, we get

$$(P_x^c - P_x^d)(X_c - X_d) + (P_y^c - P_y^d)(Y_c - Y_d) \leq 0. \tag{1}$$

- Now, consider a case where only P_x changes. In other words $P_c^y = P_d^y$. Using 1, this gives

$$(P_x^c - P_x^d)(X_c - X_d) \leq 0,$$

which in calculus terms is equivalent to:

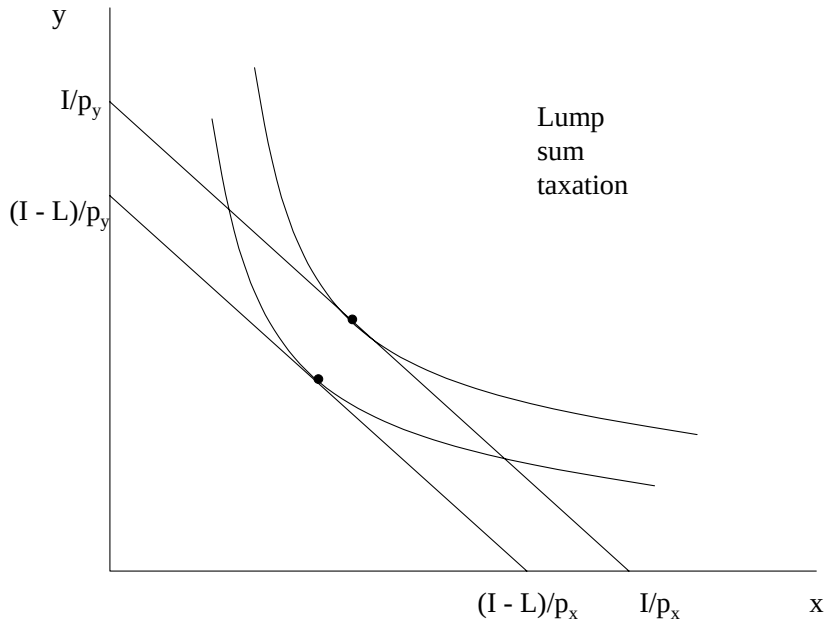
$$\frac{\partial X}{\partial P_x}|_{u=u_0} \leq 0.$$

(Remember that $C \sim D$, so we are holding utility constant.)

- *So, WARP is sufficient to establish weakly downward sloping Hicksian demand curves.*
- The *entire* idea of revealed preference is simply by using the weak notion of "choosing what you prefer," you get strong rationality properties, including:
  - Weakly downward sloping demand curves.
  - Only relative prices matter (as can be seen in the example above).
- We therefore don't have to make strong assumptions about diminishing MRS to get strong predictions about "rational" behavior.

1.2 Using WARP to evaluate the consequences of taxation

- For many reasons, governments need to tax:
  - Pay for public goods: Defense, law enforcement, regulatory agencies.
  - Transfer income–social insurance.
  - Correct externalities (pollution, 'sins').
- Are there better and worse ways to tax?
- Let's compare two types of taxes:
  - A lumpsum tax: reduces the consumer's budget by L.
  - A sales tax on a single good: charge tax t on X so that $P_x^t = P_x + t$.
- Obviously, consumer's are worse off for being taxed, but can we say anything stronger than that?
- See Figure 8#4.



- Note the algebra of the lump sum tax:

$$\begin{aligned} XP_x + YP_y &= I, \quad (2) \\ X_L P_x + Y_L P_y &= I - L, \quad (3) \\ (X - X_L)P_x + (Y - Y_L)P_y &= L \end{aligned}$$

- To compare the lumpsum tax with a revenue equivalent sales tax, we need "revenue equivalence" (i.e., same level of taxes collected).

- This requires:

$$t^* = d_x(P_x + t^*, P_y, I) = L.$$

  In words, the "revenue equivalent" sales tax generates the same total taxation as L by charging t^* on each X purchased.

- To see this, note:

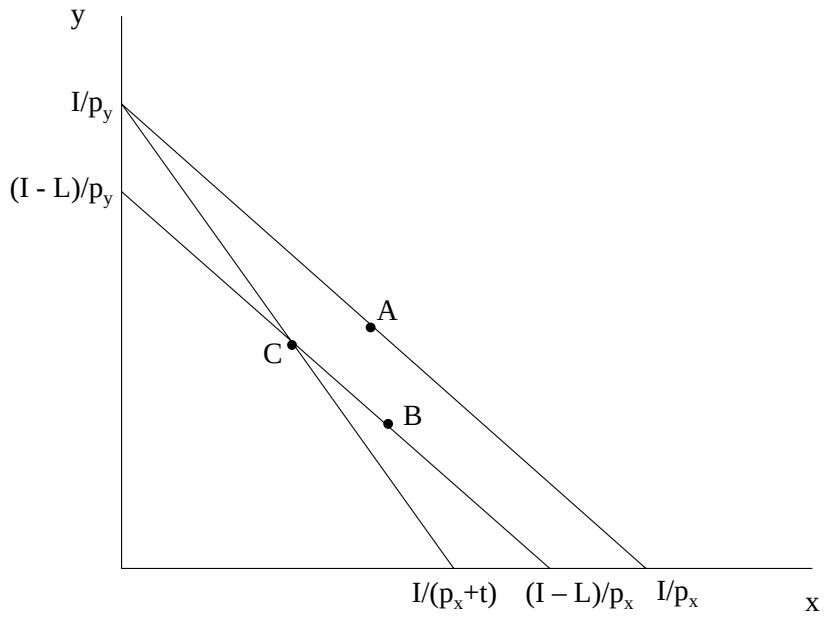
$$\begin{aligned} X_t(P_x + t) + Y_t P_y &= I, \quad (4) \\ I - X_t P_x - Y P_y &= X_t t. \end{aligned}$$

- So, for revenue equivalence, we need $X_t t = L$.

- We know from 2 that the budget set that characterizes the lumpsum tax is given by $X_L P_x + Y_L P_y = I - L$.

- So, the revenue equivalent tax *must* also put the consumer on this budget set. Hence,
$$I - X_t P_x - Y P_y = X_L P_x + Y_L P_y.$$
- Graphically, the revenue equivalent tax is the tax that causes the consumer to consume on the Lumpsum budget set. See Figure 8#5.

8#5



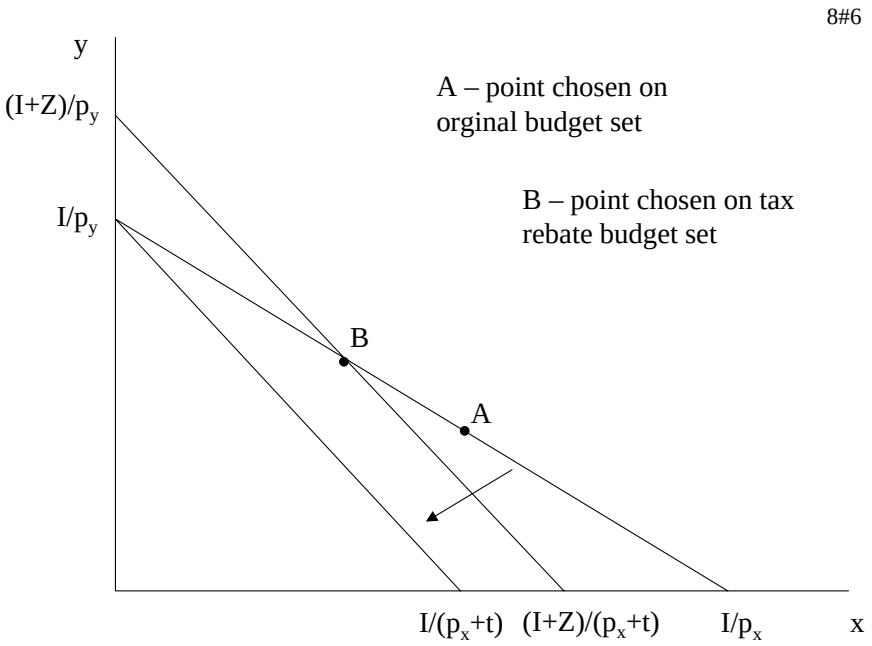
- Q: Since the tax puts the consumer back on the lumpsum budget set, does this imply that she is just as well off under either tax scheme?
- A: No, precisely the opposite. The consumer is strictly worse off under the sales tax than the lumpsum tax.
- Why: Because by shifting the price ratio, the tax has caused the consumer to choose a point on the lumpsum budget set that is not the most preferred point on this set. *Tax has distorted the choice.*
- This is a Revealed Preference argument.
- A powerful and general result: If you must tax, you harm consumers less by simply taking a chunk of their budget than by distorting prices *and* ultimately taxing an identical share of their budget.
- Q: Drawing on the axiomatic approach to consumer utility, what is the exact distortion? In the lumpsum case, it remains true that
$$\frac{U_x}{U_y} = \frac{P_x}{P_y}.$$

- Whereas in the revenue equivalent taxation case, the consumer's 'optimal choice' will satisfy
$$\frac{U_x}{U_y} = \frac{P_x + t}{P_y}.$$
- Hence, they will under-consume X and overconsume Y. Their consumption choices do not reflect the 'real' cost of goods provided in the market – they are distorted by the tax.
- Question: What if the same proportional tax were applied to all goods?

1.3 Proof of distortionary impact of non-neutral taxation of goods

- Consider (implausibly) a tax that is *fully rebated* to the consumer:
$$td_x(P_x + t, P_y, I + Z) = Z. \tag{5}$$
- This tax is revenue neutral for consumer; rebated exactly the amount paid in taxes (Z).
- Hence, only effect is to alter the price ratio faced by consumer.
- You can check that the consumer spends the original budget I by writing:
$$(P_x + t) \cdot d_x(P_x + t, P_y, I + Z) + P_y \cdot d_y(P_x + t, P_y, I + Z) = I + Z.$$
- Subtracting 5 from both sides, we get
$$P_x \cdot d_x(P_x + t, P_y, I + Z) + P_y \cdot d_y(P_x + t, P_y, I + Z) = I.$$
Hence, the consumer is on the original budget set.
- But it is essentially a zero probability event that:
$$\begin{aligned} d_x(P_x + t, P_y, I + Z) &= d_x(P_x, P_y, I), \\ d_y(P_x + t, P_y, I + Z) &= d_y(P_x, P_y, I). \end{aligned}$$
- Unless there is no behavioral response to a change in prices, the consumer will be consuming on a different point on the original budget set I under the 'taxed' price ratio.
- And in this case, the consumer is worse off by Revealed Preference.
- Hence, the distortion induced by non-neutral taxation is that it causes the consumer to pick a non-preferred point on the 'true' (non-distorted) budget set.
- Note that this argument does not depend on any axioms of utility theory other than WARP. The essential point is:

- We know that the consumer will *have to* pick a point on the original budget set for the tax to be revenue equivalent.
- But rather than allow the consumer to simply face that budget set and choose the preferred point, we are distorting their behavior by shifting the slope while ultimately placing them back somewhere on the same line.
- If they choose any other point than the preferred point on the undistorted budget set, they are unambiguously worse off.



2 Application of carte blanche principle: "The Dead Weight Loss of Christmas"

- Needed: Figure I, Table 1, Table 3.
- Theory says that in-kind transfers are wasteful (in utility terms) relative to giving cash. How can we test this notion?
- Idea: Ask Yale undergraduates about their Christmas gifts
- 86 and 58 students per survey. Captives of Econ 101.
- This is what Joel Waldfogel did.

Q1: "Apart from sentimental value, if you did not have these items, how much would you be willing to pay to obtain them?"

Q2: "Amount of cash such that you are indifferent between the gift and the cash, not counting the sentimental value of the gift?"

- Deadweight loss: "Potential benefits squandered."
- Percentage 'yield:' 66 to 87 percent.
- Notice Table 3:
  - Yield is lower for more distant relatives.
  - Tendency to give cash is higher for more distant relatives.
  - Interpretations?
- Significance of specifically excluding "sentimental value?"
- Other thoughts?