

**14.03 Problem Set #4 Fall 2000**  
**Due in Class #14**

**Theory**

1. Nicholson problem 8.5
2. Bill is a Von-Neumann Morgenstern expected utility maximizer with a well-behaved, continuously differentiable utility function (i.e., no kinks or inflection points). Bill is presented with the following choices:
  - A. \$1,000 for sure
  - B. 50% chance of \$800  
50% chance of \$1,500
  - C. \$500 for sure
  - D. 50% chance of \$400  
50% chance of \$900

Bill is indifferent between A and B and is also indifferent between C and D. (Note: this does not imply that he is indifferent between A and C or B and D.)

Part 1. Is Bill risk neutral, risk averse, risk loving, or can't you tell? Explain.

He is now faced with the following choice:

- E. \$750 for sure
- F. 25% chance of \$400  
25% chance of \$900  
25% chance of \$800  
25% chance of \$1,500

Part 2. Will Bill choose E or F, or is he indifferent between them, or is not possible to tell? (You must prove your answer)

3. Consider these four choices:
  - A. \$1,000,000 for sure
  - B. .10 chance of \$5,000,000  
.89 chance of \$1,000,000  
.01 chance of \$0

C. .10 chance of \$5,000,000  
.90 chance of \$0

D. .11 chance of \$1,000,000  
.89 chance of \$0

Before reading any further, choose which you prefer between A and B, and then choose which you would prefer between C and D. (The choice you make will not affect your grade.)

It is commonly observed that people prefer A to B, and prefer C to D. Show that this pair of choices is inconsistent with expected utility maximization.

4. Suppose my utility function is given by:

$$U = 100W^{0.9}$$

where  $W$  is my wealth in thousands. My current wealth is \$1,000.

a. Characterize my relative risk aversion.

b. Suppose I face a 0.1 chance of losing \$100. Calculate my willingness to pay for an insurance policy that would pay me \$100 in the event of such a loss.

c. My utility is greatly augmented by my children. If they were to die, it would be reduced to:

$$U = W^{0.9}$$

Calculate the loss of wealth that would be equivalent, in terms of its utility impact, to the death of my children.

d. Now suppose that there is a .01 chance that they will die suddenly. What would be the actuarially fair price for this insurance policy that would pay the monetary equivalent you calculated in (c) in the event of their death? Would I purchase such a policy? (Ignore any effect on my wealth that might result if they were sick or injured). Provide a substantive explanation for your conclusion.

5. Al, a risk-averse expected-utility maximizer with utility over wealth given by  $U(W)$ , has  $W_0$  dollars that he can either put underneath his mattress or invest in a risky asset. With probability  $p$  the rate of return on the asset is  $r_g > 0$ , and with probability  $1 - p$  the rate of return on the asset is  $r_b < 0$ . (Note that if you invest  $x$  dollars in an asset with rate or return  $r$ , you end up with  $(1 + r)x$  dollars.)

a. What is Al's expected utility he invests  $x$  in the risky asset?

- b. Write down the first order condition for the optimal choice of  $x$ . Under what condition will Al choose not to invest at all in the risky asset? (Hint: evaluate the derivative of expected utility w.r.t.  $x$  at  $x = 0$ .) Interpret this condition.
- c. Now suppose that Al's utility for wealth takes the form  $U(W) = -e^{-aW}$ . Show that the amount Al invests does not depend on whether Al heads a "working family" or is a member of the "wealthiest one percent", i.e., show that the optimal choice of  $x$  does not depend on  $W_0$ .

### **Applications: Kane & Staiger article**

Suppose that an economist working for Planned Parenthood wishes to study the effects of abortion restrictions on the *abortion rate*, that is the number of abortions as a percentage of the number of pregnancies. The economist finds that increases in the average distance to an abortion clinic *reduce* the abortion rate for counties in the U.S. You then present this economist with Figure VI of Kane and Staiger's article.

6. Interpret Figure VI. What is the estimated effect of a decrease in the average distance to a clinic on the teenage *birth rate* (that is, the number of births as a percentage of the number of teenagers)?
7. There might seem to be an inconsistency with Kane and Staiger's findings and the findings of the Planned Parenthood economist. Carefully explain a model of decision-making by teenagers that would produce both predictions.
8. An economist working for Operation Rescue who is familiar with the Kane and Staiger article states in testimony before Congress, "Research by two Harvard economists demonstrates that restricting access to abortion reduces the teen birth rate. Since we know that teen motherhood is bad for teens, banning teen abortion altogether would make teenagers better off." Provide an analytical response to this argument. (It may be helpful to refer to the Kane and Staiger model).