14.03 Fall 2000 Optional Problem Set 8 Solutions

1.

a) The pure strategy Nash Equilibria are (Up, C) and (Down, A).

b) There is only one mixed strategy Nash Equilibrium: Player 1 plays Up with probability 1/2 and Player 2 plays A with prob. 2/3, B with prob. 0, C with prob.1/3, and D with prob. 0. To find this equilibrium follow these steps. First, notice that D is strictly dominated by C for Player 2, so that Player 2 will never play D in a mixed strategy equilibrium, since he can do better by playing C instead of D. Next find the probability p of Player 1 playing Up that makes Player 2 indifferent between playing A and playing C. You will find that p=1/2. Then notice that for p=1/2 the expected payoff for Player 2 of playing either A or C is 3, which is bigger than 2.5 the expected payoff of playing B, so that B is left out of this mixed strategy equilibrium. Finally, find the probability q with which Player 2 has to randomize between A and C that makes Player 1 indifferent between playing Up and Down. You will find that q=2/3. Which establishes the result given at the beginning. You can convince yourself that there are no other mixed strategy equilibria by noticing that there is no randomization over Player 1's actions that makes Player 2 happy to leave out either A or C, since the payoff of what is left out is always bigger than the payoff of what is included.

2.

a) The normal form for this game is given below. The game has two pure strategy Nash Equilibria: (Out, Fight) and (In, Accommodate).

		Firm I	
		Fight	Accommodate
Firm E	Out	0, 2	0, 2
	In	-3, -1	2, 1

b) To find the SPNE of the game use the Extensive form and proceed by backward induction. If Firm E enters, Firm I decides to accommodate, because 1 is larger than –1. So, Firm E compares the payoff of entering, which is 2 (since Firm I would accommodate), with the payoff of staying out, which is 0, and it decides to enter. Therefore the SPNE of the game is (In, Accommodate). Notice that only one of the two Nash equilibria that you found in part (a) is a SPNE. The other one, (Out, Fight), is not subgame perfect because it is based on Firm I's threat that if Firm E enters, Firm I will fight. This threat is not credible, since once Firm E is in, it is in Firm E's best interest to accommodate. In other words, Fight is not a NE of the game starting after Firm E's entrance.

3. Two firms compete for a prize worth R. If firm i spends L_i, its probability of winning is given by $L_i/(L_i + L_{3-i})$.

   A. NE given by (L_1^*, L_2^*) such that $L_i^* = \arg\max_{L_i} p_i(L_i, L_{3-i}^*), i = 1,2$, where $p_i(L_i, L_{3-i}) = (L_i/(L_i + L_{3-i}))R - L_i$.

   The FOC for firm i is $R/(L_i + L_{3-i}) - RL_i/(L_i + L_{3-i})^2 - 1 = 0, i = 1,2$. Solving this system of equations gives us $(L_1^*, L_2^*) = (R/4, R/4)$.

   B. Although both firms would be better off if they spent zero on lobbying, this agreement is not self-enforceable. If the other firm spends zero, you are better off spending a small amount and getting the prize for sure. Since each firm understands the incentive to deviate, they will end up at the NE levels found in part A, $(R/4, R/4)$.

4.

$TY(B) = B(360 - B)$

A. $dTY/dB = 360 - 2B = 0 \Rightarrow B = 180$

$TY(180) = 32{,}400$

B. $TY_N(B_N, B_S) = B_N(360 - B_N - B_S)$

$dTY_N/dB_N = 360 - 2B_N - B_S = 0$

Symmetry $\Rightarrow B_N = B_S = B^{NC}$

$360 - 3B^{NC} = 0 \Rightarrow B^{NC} = 120$

$TY_N(120{,}120) = 14{,}400$

C. If the game is repeated three times, the subgame perfect equilibrium is for each country to send 120 boats in each year.

D. The treaty specifies that the countries will use a grim trigger strategy: play 90 boats each as long as nobody has cheated in the past, but play 120 boats each forever as soon as someone cheats.
Solving for the optimal deviation when your opponent sends 90 boats:

$TY_N(B_N, 90) = B_N(360 - B_N - 90)$

$dTY_N/dB_N = 270 - 2B_N = 0 \Rightarrow B_N = 135$

$TY_N(135{,}90) = 18{,}225$

PDV(cheating) = $18{,}225 + 14{,}400d/(1-d)$

PDV(cooperating) = $16{,}200/(1-d)$

The treaty will hold if PDV(cooperating) $\geq$ PDV(cheating) $\Leftrightarrow d \geq 9/17$.

E. If South sends the Super Trawler, North's payoff is $TY_N(B_N, 150) = B_N(360 - B_N - 150)$. Maximizing this with respect to B_N tells us that North's best response is to send 105 boats. If South does not fish, North's best response is to send 180 boats.

If South sends the Super Trawler, North's best response is to send 105 boats. If North sends 105 boats, South gets 15,750 by sending the Super Trawler, and nothing if it does not fish. Hence sending the Super Trawler is a best response to 105, and we have shown that (South sends Super Trawler, North sends 105 boats) is a Nash equilibrium.

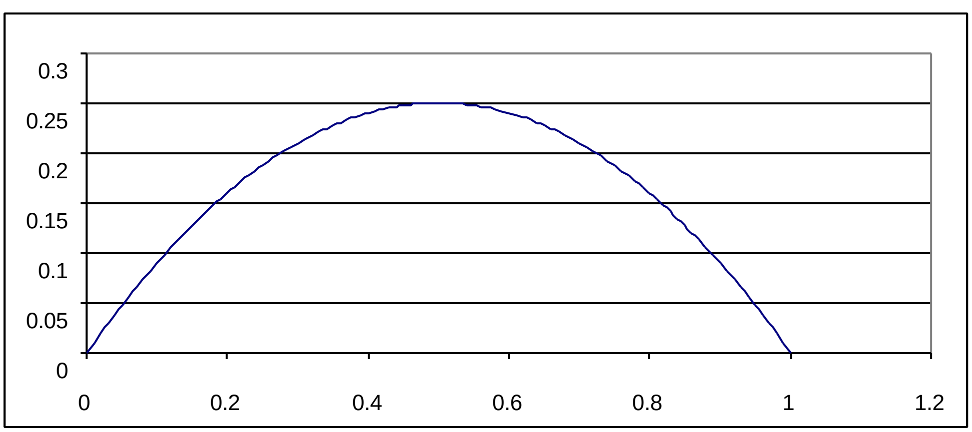
You also need to check that there is not a NE in which South does not fish (although this may seem obvious, note that it would not be true if the Super Trawler were equal to 180 boats). If South does not fish, North's best response is to send 180 boats. If North sends 180 boats, South gets 4,500 by sending the Super Trawler, and nothing if it does not fish. Hence not fishing is not a best response to 180, so this is not a Nash equilibrium.

South sends more boats and gets more fish in this equilibrium than in Part B, while North sends fewer boats and gets fewer fish. By building the Super Trawler and burning its regular fleet, South has made a credible commitment to send more boats.

Application: Network Externalities

1. Network externality model

A. The indifferent consumer is $\hat{x}$ such that $n(1-\hat{x}) - p = 0$ (1)

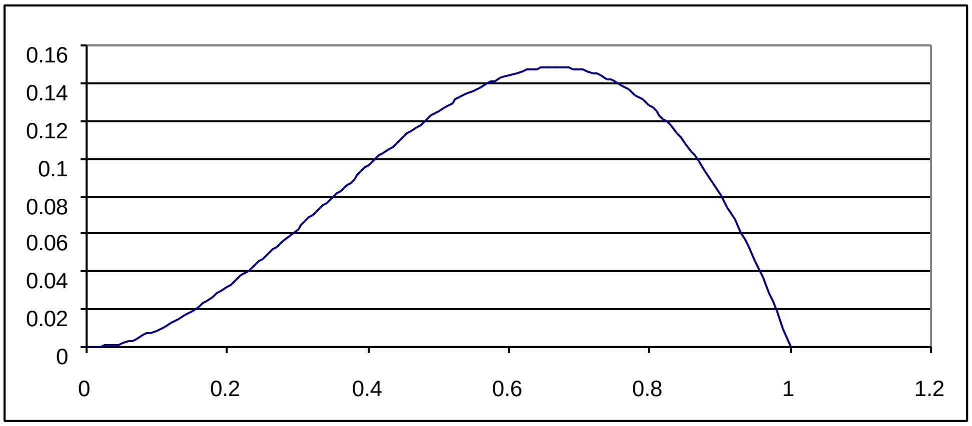


B. The demand curve is given by $p = \hat{x}(1-\hat{x})$ (2)

C. As is evident from the graph, for any $p_0 \in [0,1/4)$ there exists both a $\hat{x}_0^L < ½$ and a $\hat{x}_0^H > ½$ that are consistent with p_0. However, $\hat{x}_0^H$ is a stable equilibrium while $\hat{x}_0^L$ is not. To see why the low value is unstable, suppose the initial situation is an equilibrium with price equal to p_0 and demand equal to $\hat{x}_0^L$. Given that $\hat{x}_0^L$ consumers are already using the product, even a slight reduction in price will lead all consumers between $\hat{x}_0^L$ and $\hat{x}_0^H$ to purchase OS. Hence $\hat{x}_0^L$ can be thought of the critical mass associated with price p_0.

D. If the marginal cost of OS is zero, the price of OS in a competitive industry is zero. All potential users would purchase OS at that price.

E. MacroSoft's profits: $p(\hat{x}) = \hat{x}^2(1-\hat{x})$ (3)



The FOC for the profit-maximization problem is $2\hat{x} - 3\hat{x}^2 = 0$, so $\hat{x}^*$ is either 0 or 2/3. It is clear that the correct choice is $\hat{x}^*$ = 2/3. Profits are equal to 4/27, and the number of users is lower than the number of users in the competitive case.

F. Since the number of users is lower under monopoly than under competition, in this model MacroSoft does not exploit its monopoly power to build market share. Instead, MacroSoft exploits its monopoly power by increasing the price and restricting output, which is bad for consumers.

G. If the initial number of users of the new product is zero, setting the price that yields $\hat{x}^*$ in long-run equilibrium does not work because no single user would be willing to buy the new product at that price. Instead, MacroStart should start at a lower price, possibly zero. Once the number of users reaches the critical mass associated with $\hat{x}^*$, MacroSoft can increase the price to the long-run equilibrium level.

2. The charge that Microsoft has slowly increased the price of Windows over time is relevant because (as part G shows) it is the behavior we would expect from a monopolist over a product with network externalities.

3. Examples of inferior standards that are nonetheless widely used: QWERTY, VHS and the English-American system of measurement.
   Examples of standards for which there is neither a right nor a wrong, but for which it is crucial that everyone follow the same standard: side of the road to drive on, language.