14.03 Fall 1999 Problem Set #6. Due in Class #22

1. The government of Rothschildia (a small, densely populated island nation off of the coast of Cambridge) is considering implementing a *voluntary* national health insurance plan. Everyone in Rothschildia is risk averse with VNM utility function $U(W) = W^{0.5}$. Each citizen has a wealth of 1 Stiglitz (the local curency) but should he or she become ill, s/he must spend her entire wealth of 1 Stiglitz on health care. (Since the cure is immediate and complete, the only disutility of illness is this 1 Stiglitz cost.) The only respect in which Rothschildians differ from one another is that each has a different *ex ante* probability of becoming ill, p_i. Illness probability is distributed *uniformly* among citizens on the $[0,1]$ interval, meaning that *on average* one-half of the population becomes sick in a year, but that each person has a different probability, p_i, ranging from zero to one where all values of p_i are equally likely. If $p_i = 0$, person (i) is certain not to become ill, if $p_i = 1$, person (i) is certain to become ill, if $p_i = .5$, person (i) has a 50% probability of becoming ill, etc. Moreover, each citizen *knows* his own individual illness probability but the government only knows about the distribution of probabilities. Finally, for this problem, you should assume that each citizen lives only 1 year but that a new generation is born every year (each also knowing his or her p_i). The government, of course, persists from year to year.

   <u>You should also bear in mind the following facts about uniform probability distributions</u>:

   1. If a variable is distributed uniformly on the $[0,1]$ interval, the mean (i.e., expected) value of the variable greater than or equal to a given cutoff, L, is $(L+1)/2$. Similarly, the expected value below a given cutoff, U, is $(U+0)/2 = U/2$. Hence, the expected value of observations $\geq$ ½ is ¾, and the expected value of observations $\leq$ ½ is ¼.

   2. If you want to calculate the expected value of a *function of a random variable*, you must integrate that function over the probability distribution of the random variable which is specified by its 'probability density function' (PDF).

   The probability density function of a U[0,1] (where U stands for uniform) variable could not be simpler. It's:

   $f(x) = 1$

   Meaning that all values between 0 and 1 are equally likely (and, moreover, the sum of probabilities of all possible values – the integral of $f(x)$ on $[0,1]$ – is 1).

   So, for example, if $y = x^2$ and x is distributed $U[0,1]$, and you want to know the expectation of y, you would integrate the function y over the PDF of x:

   $$E(y) = E(x^2) = \int_0^1 (x^2 \cdot 1)dx = \frac{1}{3}$$

The problem:

A. A policy maker for the Board of Health sits down to design the new national health plan. She reasons that since half of all Rothschildians get sick each year, the government should offer an actuarially fair full insurance policy that charges a premium of ½ Stiglitz and pays a benefit of 1 Stiglitz to any enrollee who gets sick (of course, the enrollee pays the premium regardless of whether or not she becomes ill). Given this premium, calculate who chooses to enroll in the plan:

What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?

What is the average health of those who enroll in the plan?

Does the plan break even, make money, or lose money in year 1, and by how much per person on average?

B. In year 2, a different policy maker at the board of health (recall, the first has passed away) notes that something went wrong in the first year: the plan made/lost money (depending on your answer above). He reasons, "Clearly we set the premium too high/low in the first year. What we'll do is set the new premium to reflect our average cost from last year. This should straighten things out."

What is the new premium?

What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?

What is the average health of those who enroll in the plan?

Does the plan break even, make money, or lose money in year 2, and by how much (per person average)?

C. In year 3, a third policy maker observes that something is again amiss. The plan made/lost money *again* last year, although the intention was to break even. This policy maker suggests that the board fix the problem by setting the new premium at the average cost for year 2.

What is the new premium?

What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?

What is the average health of those who enroll in the plan?

Does the plan break even, make money, or lose money in year 3, and by how much (per person average)?

D. In year 4, a fourth policy maker notes that something went wrong yet again. This policy maker has taken 14.03, however, and says, "Alas, I see the error of our ways! Every time we change the premium, a different pool of citizens enrolls in the plan. I wonder if there is a premium we could set so that the pool of citizens who enrolls at that price costs us on average exactly that price. That way, we'd break even and provide insurance to all those who want to buy it." After a few strokes of the pen, she shouts, "Eureka! There is."

E. What is that premium? Hint: you can either solve this problem analytically (i.e., on paper with a simple equation) or with a spreadsheet by repeating the steps you used for A, B, and C until you get a convergent solution.

F. In year 5, an economist (also schooled in 14.03) from the Rothschildian Board of Social Welfare visits the Board of Health and says, "I see that you've worked out the national health plan premium so that it no longer makes/loses money every year. That's a step forward. However, I'm concerned that your break-even program is not actually maximizing average social welfare. What I'd like you to do is calculate average well-being (utility) under three different policy options: 1) No health plan; 2) Your current break-even plan; 3) A third *mandatory plan for all citizens* that also breaks even. Then report back to me."

Please perform these calculations for the three plans.

Which health plan do you recommend based upon your calculations?

G. Explain substantively why your preferred policy option yields higher average social utility than the other two health insurance plans.

2. Imagine that employers in the mining industry are reluctant to hire workers of poor health due to the arduous working conditions in the mine. It takes a half-year of mining by a worker to cover her $1,000 annual salary, and unhealthy workers often quit before this time is up. Moreover, mining is so tough that after one year of work, even the healthiest person leaves mining.

   Assume that health (h) is distributed uniformly on the [0,1] interval where a person with health 0 becomes ill immediately on entering the mine, a person with health 1 works a whole year before becoming ill, and a person with health 0.5 works for half a year before becoming ill, etc. Hence, the firm's revenue per worker is $2,000h, where h is the fraction of the year that a worker stays in mining before becoming ill.

   Once a person becomes ill, she leaves the mine for good (but immediately recovers and hence suffers no additional disutility). Each person knows his health before applying. The disutility of applying for the job is $3 (because everyone hates filling out applications). People who don't work in the mine earn $500 in the Saloon sector (which provides fine drink and dining to miners).

   A. In the early days, mining companies could not tell the health of applicants apart. While the job application requested health information (on a zero-one scale), there was no way to verify this information and the Fairness in Mining Law clearly states that employers cannot refuse to pay the $1,000 salary of workers who become ill on the job.

What is the mean health reported by workers on their job applications?
What is the mean health of workers who are hired?
What are average profits per worker?

B. In 1970, a manicurist working for Consolidated Amalgamated Mining Corporation invents a screening test that can precisely determine applicants' health by shining a light on their fingernails (which, of course, are used heavily in scraping ore from mineshafts). This test is costless to the company and almost completely painless, yielding a disutility to the test taker of only $1. CAMC immediately requires it of all applicants.

What is the health of the least healthy worker who applies for the job?
What is the health of the least healthy worker hired?
What is the average health of workers hired?
What are average profits per worker?

C. In 1980, after a damning story on 60 Minutes, Congress passes the Non-Discrimination in Mining Act that stipulates that companies cannot require their applicants to take fingernail health-screening tests. CAMC's stock price falls precipitously as investors anticipate a return to the "bad old days" when half of the company's employees became ill within half a year. Unfazed, CAMC's Personnel Director suggests the following policy: the company will offer a $2 bonus to any applicant who volunteers for the health-screening test. "This should solve the problem," she says. The CEO replies, "That's a strange idea. People earn $500 *more* per year working for us than working for the saloon up the street. Why would anyone want to reveal their health status, possibly costing them a high paying job, for a mere $2 – less than the $4 disutility of applying and taking the test?" However, after further conversation, the CEO is persuaded of the wisdom of the Personnel Director's policy and the company implements the policy.

What is the health of the least healthy worker who applies for a job?
What is the health of the least healthy worker hired?
What is the average health of workers hired?
What are average profits per worker?

D. Provide a substantive explanation for your conclusion to part C. How does this result relate to Akerlof's lemons model?

[Hint: It may be helpful to consider what would have happened if the company had *not* offered any money to test takers. How does the behavior of test-takers affect the employment outcomes of non-test-takers?]