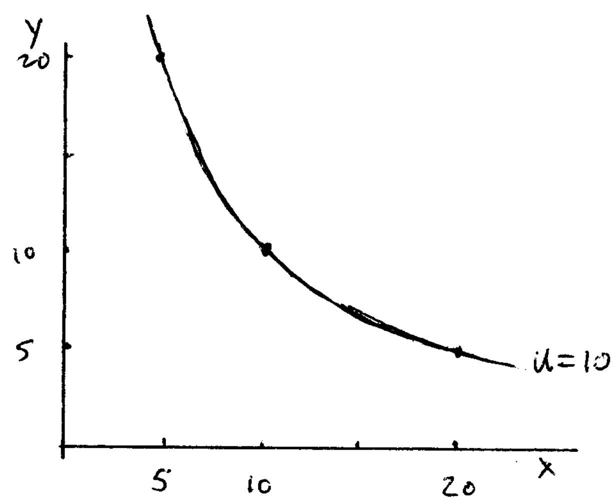
14.03 Problem Set 2 Solutions
Fall 2000

Theory:

Nicholson 3.2

a) The $U = 10$ indifference curve is (x, y) s.t. $\sqrt{xy} = 10 \Rightarrow y = 100/x$.



b) If $x = 5$, to be on the $U = 10$ indifference curve we must have $y = 20$. The marginal rate of substitution is $MU_x / MU_y = y/x$. Hence evaluated at this point the MRS is 4.

c) As shown in part b), the MRS is y / x, the ratio of the marginal utilities.

d) $U' = \log_{10} U = \log_{10} x^{0.5} y^{0.5} = 0.5\log_{10} x + 0.5\log_{10} y$. We want to show that when we set $U' = 1$ and solve for y we get $100 / x$.

$0.5\log_{10} x + 0.5\log_{10} y = 1$

$\log_{10} y = 2 - \log_{10} x$

$$y = 10^{2-\log_{10} x} = \frac{10^2}{10^{\log_{10} x}} = \frac{100}{x}$$

Since $MU_x = 1/2\ln(10)x$ and $MU_y = 1/2\ln(10)y$, the MRS is again y / x.

Nicholson 3.4

a) Margarine and butter are perfect substitutes

b) Peanut butter and jelly are perfect complements

c) One possible interpretation of this statement is that the marginal utility of other goods is increasing in the amount of Coke consumed: $\frac{\partial^2 U}{\partial x \partial C} > 0$.

d) The preferences between popcorn and another good exhibit increasing MRS. Note that it is not correct to say that popcorn exhibits increasing marginal utility (see problem 3.7).

e) Mosquitoes are a "bad": $\frac{\partial U}{\partial m} < 0$.

f) A perfect-complements interpretation of this statement is that U(no wine, sunshine) = U(wine, no sunshine) = U(no wine, no sunshine).

g) If for some strange reason we derive utility from watching people tango, then $U(L, F) = \min\{L, F\}$, where L is the number of leaders and F is the number of followers.

Nicholson 3.7

a) $U(x, y) = xy$

$$\frac{\partial U}{\partial x} = y, \frac{\partial^2 U}{\partial x^2} = 0$$

$$\frac{\partial U}{\partial y} = x, \frac{\partial^2 U}{\partial y^2} = 0$$

$\Rightarrow$ constant marginal utility

$$MRS = \frac{y}{x}$$

To check whether MRS is decreasing, we need to take into account the fact that y depends on x: $y = \frac{\overline{U}}{x}$, where $\overline{U}$ is an arbitrary utility level.

Then $\frac{dMRS}{dx} = \frac{d}{dx}\left(\frac{\overline{U}}{x^2}\right) = -\frac{2\overline{U}}{x^3} < 0 \Rightarrow$ diminishing MRS.

b) $U(x, y) = x^2 y^2$

$$\frac{\partial U}{\partial x} = 2xy^2, \frac{\partial^2 U}{\partial x^2} = 2y^2 > 0$$

$$\frac{\partial U}{\partial y} = 2x^2 y, \frac{\partial^2 U}{\partial y^2} = 2x^2 > 0$$

$\Rightarrow$ increasing marginal utility

$MRS = \frac{y}{x}, \frac{dMRS}{dx} = -\frac{2\hat{U}}{x^3} \Rightarrow$ diminishing MRS

c) $U(x, y) = \ln x + \ln y$

$$\frac{\partial U}{\partial x} = \frac{1}{x}, \frac{\partial^2 U}{\partial x^2} = -\frac{1}{x^2} < 0$$

$$\frac{\partial U}{\partial y} = \frac{1}{y}, \frac{\partial^2 U}{\partial y^2} = -\frac{1}{y^2} < 0$$

$\Rightarrow$ decreasing marginal utility

$MRS = \frac{y}{x}, \frac{dMRS}{dx} = -\frac{2\tilde{U}}{x^3} \Rightarrow$ diminishing MRS

Conclusion is that monotonic transformations of the utility function preserve the MRS, but not properties of marginal utility. Among other things, this implies that we cannot determine the shape of the indifference curves from marginal utility alone.

Nicholson 4.2

a) $\max W_F^{2/3} W_C^{1/3} s.t. p_F W_F + 4W_C = 300$

$L = W_F^{2/3} W_C^{1/3} + \lambda(300 - p_F W_F - 4W_C)$

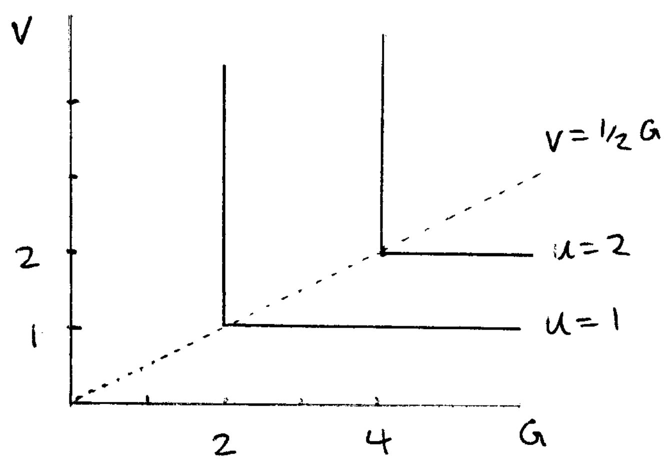
$$\partial L / \partial W_F = \frac{2W_C^{1/3}}{3W_F^{1/3}} - \lambda p_F = 0$$

$$\partial L / \partial W_C = \frac{W_F^{2/3}}{3W_C^{2/3}} - 4\lambda = 0$$

The FOCs imply that $W_C = p_F W_F / 8$. Substituting into the budget constraint yields $W_F = 200 / p_F$. Hence when p_F = 20, the wine connoisseur should buy 10 bottles of French wine and 25 bottles of California wine.

b) When p_F drops to 10, the connoisseur should buy 20 bottles of French wine and 25 bottles of California wine.

Nicholson 4.5



a) No matter what the prices, Mr. A will always consume the fixed proportions $V = G/2$. This can be proved by contradiction. Suppose $V > G/2$. Then utility is $G/2$. If Mr. A consumed δ less V, he could consume $\delta p_V / p_G$ more G and still satisfy his budget constraint. At this new consumption bundle his utility would be $(G + \delta p_V / p_G)/2 > G/2$. Hence the original bundle was not optimal. A similar argument applies for $V < G/2$. Since the proof holds for any positive values of p_V and p_G, we have shown that regardless of the prices of the two ingredients, Mr. A will never alter the way he mixes martinis.

b) We can solve for the demands by substituting the fixed proportion equation into the budget constraint.

$$Gp_G + (G/2)p_V = I \Rightarrow G = \frac{2I}{2p_G + p_V}, V = \frac{I}{2p_G + p_V}$$

c) Calculate the indirect utility functions by substituting the demand functions into the utility function.

$$U(G,V) = \min\{G/2, V\} = \min\{I/(2p_G + p_V), I/(2p_G + p_V)\}$$

$$V(p_G, p_V, I) = \frac{I}{2p_G + p_V}$$

d) Since utility is equal to the number of martinis, the minimum expenditure necessary to attain utility level M is M times the price of one martini, which is $2p_G + p_V$.

$$E(p_G, p_V, M) = M(2p_G + p_V)$$

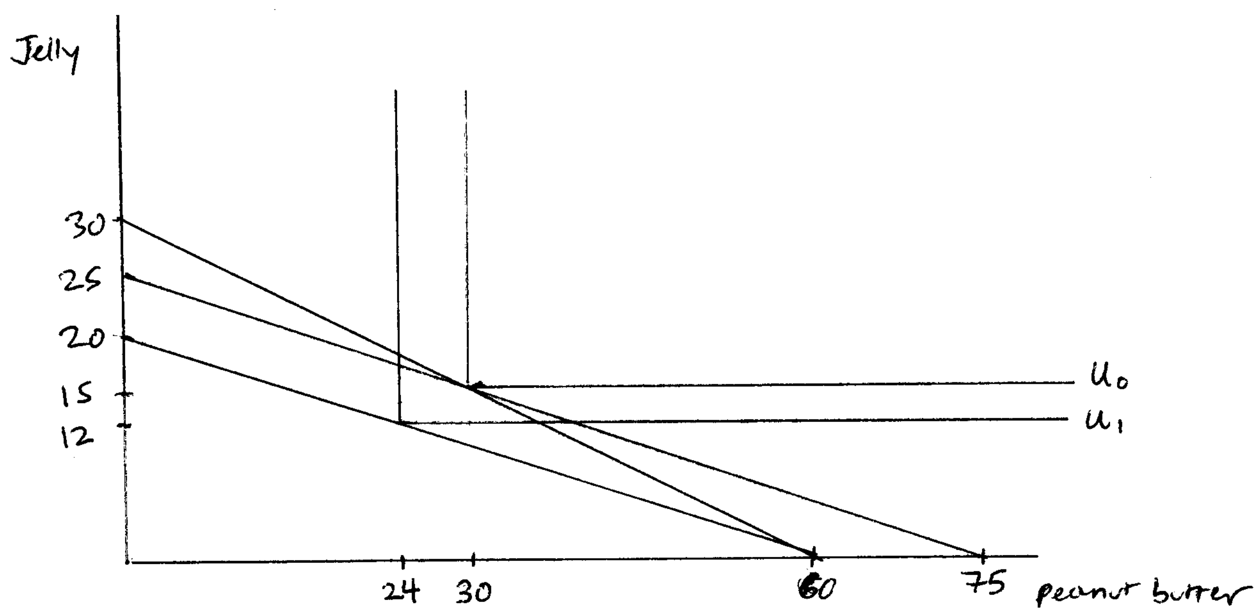
Nicholson 5.2

a) David will spend his entire allowance on sandwiches. Since the price of a sandwich is $0.20, he will buy 15 sandwiches (30 ounces of peanut butter and 15 ounces of jelly).

b) If the price of jelly goes up to $0.15, the price of a sandwich is $0.25. David will buy 12 sandwiches (24 ounces of peanut butter and 12 ounces of jelly).

c) In order to be as well off as he was before the price change, David needs to be able to afford 15 sandwiches. This means he needs a total allowance of $3.75, or an increase of $0.75.

d) The original budget constraint has endpoints at (0,30) and (60,0). When the price of jelly increases, the budget constraint pivots inward so that the jelly endpoint is at (0,20). Compensating David for the price increase shifts out this new budget constraint so that the endpoints are at (0,25) and (75,0).



e) Since peanut butter and jelly are consumed in fixed proportions we can think of sandwiches as being a single composite commodity. Holding income fixed at 3, the demand for sandwiches as a function of the price of sandwiches is $Q = 3/P$.

f) Since peanut butter and jelly are perfect complements, there is no substitution effect when the price of one of the goods changes. The decline in jelly demand when the price of jelly rises is entirely an income effect.

Nicholson 5.9

We can show that SARP is not satisfied by looking at years 2 and 3. In year 2, the cost of the bundle that is chosen is 6(4)+6(2) = 36. But the year 3 bundle is affordable at year 2 prices, since 7(4)+3(2) = 34 < 36. Hence the fact that the year 2 bundle is chosen when the year 3 bundle is affordable means that the year 2 bundle is revealed preferred to the year 3 bundle.

Now consider year 3. The cost of the year 3 bundle at year 3 prices is 7(5)+3(1) = 38. But the year 2 bundle is affordable at these prices, because 6(5)+6(1) = 36 < 38. So the fact that the year 3 bundle is chosen when the year 2 bundle is affordable means that the year 3 bundle is revealed preferred to the year 2 bundle. These two revealed preference statements are a violation of SARP.

2) $\max U(x,y) = -1/x - 1/y$ s.t. $xp_x + yp_y = I$

a) $L(x,y,\lambda) = -x^{-1} - y^{-1} + \lambda(I - xp_x - yp_y)$

$$\begin{aligned} \partial L/\partial x &= x^{-2} - \lambda p_x = 0 \\ \partial L/\partial y &= y^{-2} - \lambda p_y = 0 \end{aligned} \Rightarrow x = yp_y^{1/2}p_x^{-1/2}$$

Substitute into the budget constraint to get

$d_x(p_x,p_y,I) = I/(p_x + p_x^{1/2}p_y^{1/2}), d_y(p_y,p_x,I) = I/(p_y + p_x^{1/2}p_y^{1/2})$

b) $V(p_x,p_y,I) = U(d_x(p_x,p_y,I), d_y(p_y,p_x,I)) = \dfrac{-(p_x + p_y + 2p_x^{1/2}p_y^{1/2})}{I}$

c) min $xp_x + yp_y$ s.t. $-1/x - 1/y = U_0$

$L(x,y,\lambda) = xp_x + yp_y + \lambda(U_0 + x^{-1} + y^{-1})$

$\partial L/\partial x = p_x - \lambda x^{-2} = 0$

$\partial L/\partial y = p_y - \lambda y^{-2} = 0$

Use the FOCs to solve for x in terms of y, then substitute into the utility constraint to get

$$h_x(p_x,p_y,U_0) = \frac{-(p_x^{1/2} + p_y^{1/2})}{p_x^{1/2}U_0}, h_y(p_y,p_x,U_0) = \frac{-(p_x^{1/2} + p_y^{1/2})}{p_y^{1/2}U_0}$$

d) $E(p_x,p_y,U_0) = p_x h_x + p_y h_y = \dfrac{-(p_x + p_y + 2p_x^{1/2}p_y^{1/2})}{U_0}$

e) The Slutsky equation says that the responsiveness of demand to price changes holding income constant can be decomposed into two components:
   i) the substitution effect – the responsiveness of demand to price changes holding utility constant
   ii) the income effect – the responsiveness of demand to changes in income, multiplied by the initial level of demand

f) We want to verify that $\dfrac{\partial d_x}{\partial p_x} = \dfrac{\partial h_x}{\partial p_x} - x\dfrac{\partial d_x}{\partial I}$.

$$\frac{\partial d_x}{\partial p_x} = -I(p_x + p_x^{1/2} p_y^{1/2})^{-2}(1+0.5p_x^{-1/2} p_y^{1/2}) = \frac{-I}{\left(p_x + p_x^{1/2} p_y^{1/2}\right)^2} - \frac{Ip_x^{-1/2} p_y^{1/2}}{2\left(p_x + p_x^{1/2} p_y^{1/2}\right)^2}$$

$$\frac{\partial d_x}{\partial I} = \frac{1}{p_x + p_x^{1/2} p_y^{1/2}} \Rightarrow x\frac{\partial d_x}{\partial I} = \frac{I}{\left(p_x + p_x^{1/2} p_y^{1/2}\right)^2}$$

So now we need to show that $\frac{\partial h_x}{\partial p_x} = \frac{-Ip_x^{-1/2} p_y^{1/2}}{2\left(p_x + p_x^{1/2} p_y^{1/2}\right)^2}$.

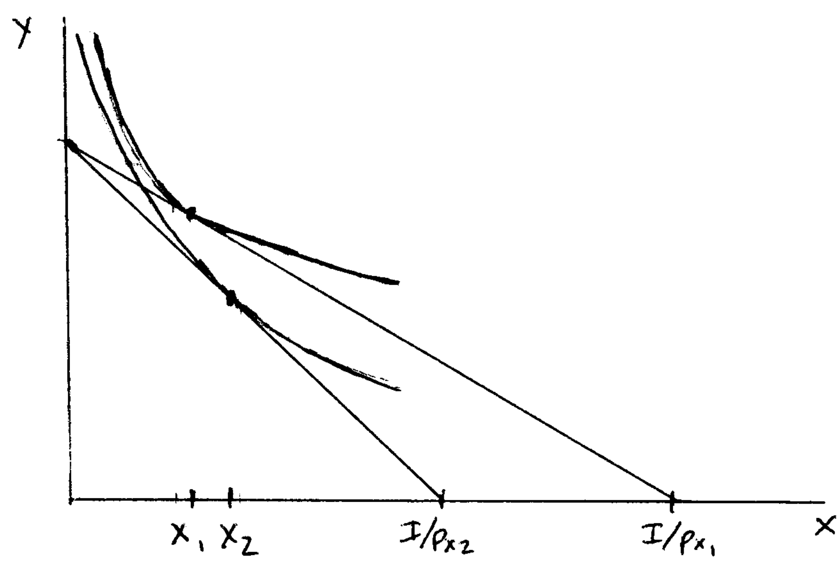
$$\frac{\partial h_x}{\partial p_x} = \frac{p_x^{-3/2} p_y^{1/2}}{2U_0}$$

Now substitute in for U_0 using the indirect utility function:

$$\frac{\partial h_x}{\partial p_x} = \frac{-Ip_x^{-3/2} p_y^{1/2}}{2\left(p_x + p_y + 2p_x^{1/2} p_y^{1/2}\right)} = \frac{-Ip_x^{-1/2} p_y^{1/2}}{2\left(p_x^2 + p_x p_y + 2p_x^{3/2} p_y^{1/2}\right)} = \frac{-Ip_x^{-1/2} p_y^{1/2}}{2\left(p_x + p_x^{1/2} p_y^{1/2}\right)^2}$$

Application: Dwyer and Lindsey (Irish Potato) article

3) Giffen good illustration: $p_{x2} > p_{x1} \Rightarrow d_x(p_{x2}, p_y, I) > d_x(p_{x1}, p_y, I)$



4)

a) The top picture in Figure 1 illustrates an increase in quantity demanded in response to an exogenous increase in price, which is also what the diagram in question 3) shows.

b) The bottom picture in Figure 1 illustrates a negative supply shock with inelastic supply. Since the potato blight destroyed potatoes in Ireland and there were few imports, this is a better description of the potato famine.

c) Both the people and the relief agencies shifted consumption away from potatoes to other foods, even though the price of substitute foods increased during this period. This evidence suggests that the price of potatoes rose in response to the supply shock, which is inconsistent with potatoes being a Giffen good.

5) "Inferiority is necessary for a good to be Giffen."

a) This follows directly from the Slutsky equation. Since $\frac{\partial h_x}{\partial p_x} \leq 0$ and $x \geq 0$, the only way for $\frac{\partial d_x}{\partial p_x}$ to be greater than zero is if $\frac{\partial d_x}{\partial I} < 0$, i.e., if the good is inferior.

b) It is unlikely that potatoes were inferior during the famine, because incomes were at starvation levels and potatoes were a cheap source of food. Hence any increases in income would likely have been spent on potatoes.

6) "For a good to be Giffen, some normal good must be displaced by the inferior good as the price rise lowers real income."

a) By nonsatiation, $p_x d_x(p_x, p_y, I) + p_y d_y(p_x, p_y, I) \equiv I$

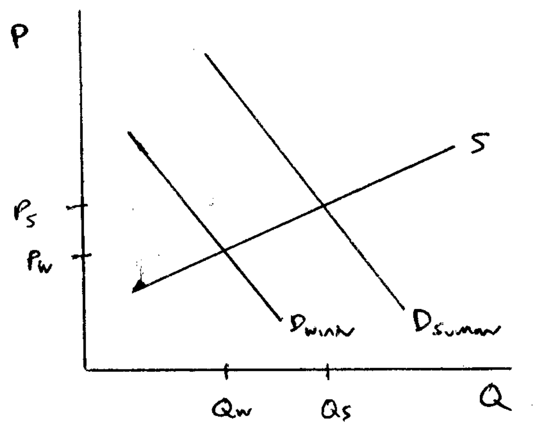
Differentiate both sides with respect to I to get $p_x \frac{\partial d_x}{\partial I} + p_y \frac{\partial d_y}{\partial I} = 1$

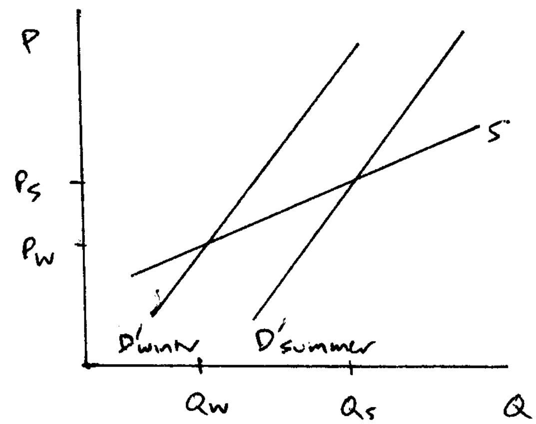
Since prices are nonnegative and inferiority $\left(\frac{\partial d_x}{\partial I} < 0 \right)$ is a necessary condition for x to be a Giffen good, the equation will hold only if $\frac{\partial d_y}{\partial I} > 0$. Intuitively, all goods can't be Giffen because otherwise increases in income would not be spent, violating nonsatiation.

b) Since the Irish diet consisted almost exclusively of potatoes there was no other good to be displaced. We might expect meat to be a normal good, but during this time the Irish ate little meat – the pig on the farm was not eaten, but rather the "gentleman who paid the rent".

7)

a) This assertion ignores the fact that the demand for gasoline shifts out in the summer because people drive more. Price and quantity move together because demand shifts out along an upward-sloping supply curve. Nothing can be inferred about the slope of the demand curve, as these diagrams illustrate.





b) Find a state that is going to change its state gasoline tax, and find a control state that has similar supply and demand conditions but whose state gas tax is not changing. Measure the quantity of gasoline sold in both states both before and after the tax change. The difference-in-differences estimator of the effect of the price change on demand is $\left(q_1^{treatment} - q_0^{treatment}\right) - \left(q_1^{control} - q_0^{control}\right)$.

c) Since gasoline is likely to be a normal good, we would expect to see quantity demanded fall as price increases.