14.03 Fall 2000 Problem Set 6 Solutions

1. Expected utility for consumer with health *p* is $(1-p)U(1)+pU(1-1)=1-p$. Expected utility for this consumer if she purchases full insurance at premium *r* is $(1-p)U(1-r)+pU(1-r-1+1)=U(1-r)=\sqrt{1-r}$. So a consumer purchases insurance iff $\sqrt{1-r}\geq 1-p \Leftrightarrow p\geq 1-\sqrt{1-r}$. Define p_c to be the probability of sickness for the most healthy person to buy insurance, i.e. $p_c=1-\sqrt{1-r}$. Then the average health of those who enroll when the premium is *r* is $(1+p_c)/2=(2-\sqrt{1-r})/2$. The average profit for the insurance plan is given by average revenue (equal to the premium) minus average cost (equal to average health), or $r-(1+p_c)/2=(2r-2+\sqrt{1-r})/2$.

A. premium = ½ $\Rightarrow p_c=(2-\sqrt{2})/2$
most healthy enrollee: $p=(2-\sqrt{2})/2$
least healthy enrollee: *p* = 1
average health of enrollees = $(4-\sqrt{2})/4$
average profit = $(\sqrt{2}-2)/4<0$, plan loses money.

B. premium = $(4-\sqrt{2})/4 \Rightarrow p_c=(2-\sqrt[4]{2})/2$
most healthy enrollee: $p=(2-\sqrt[4]{2})/2$
least healthy enrollee: *p* = 1
average health of enrollees = $(4-\sqrt[4]{2})/4$
average profit = $(\sqrt[4]{2}-\sqrt{2})/4<0$, plan loses money.

C. premium = $(4-\sqrt[4]{2})/4 \Rightarrow p_c=(2-\sqrt[8]{2})/2$
most healthy enrollee: $p=(2-\sqrt[8]{2})/2$
least healthy enrollee: *p* = 1
average health of enrollees = $(4-\sqrt[8]{2})/4$
average profit = $(\sqrt[8]{2}-\sqrt[4]{2})/4<0$, plan loses money.

D.,E. The premium such that the pool of citizens who enroll at that premium cost on average exactly that premium is *r* such that $r=(2-\sqrt{1-r})/2$.

$$\sqrt{1-r}=2-2r$$
$$1-r=4-8r+4r^2$$
$$0=3-7r+4r^2$$
$$0=(4r-3)(r-1)$$

The relevant root gives us a premium of ¾.
Most healthy enrollee: *p* = ½

Least healthy enrollee: p = 1
Average health of enrollees = ¾

F. 1) If there is no health plan, expected utility is 1-p, so average expected utility is ½.
2) Under the current (voluntary) break-even plan, r = ¾ and p_c = 1/2. The expected utility for enrollees is U(1-3/4) = ½. The expected utility for non-enrollees is 1-p, and since p<1/2 for non-enrollees, the average expected utility for non-enrollees is ¾. The average expected utility for all consumers (which is equal to the average expected utility for enrollees times the probability of being an enrollee, plus the average expected utility for non-enrollees times the probability of being an non-enrollee) is (1/2)(1/2)+(3/4)(1/2) = 5/8.
3) The mandatory break-even plan would set the premium equal to the average cost when all citizens enroll, which is ½. This yields an average expected utility of $U(1-1/2) = \sqrt{2}/2$.

If you want to maximize average expected utility, you should recommend the mandatory break-even plan.

G. Mandatory insurance increases average expected utility because it eliminates the adverse selection problem. Since low-risk citizens can no longer opt out, the cost of providing insurance to everyone goes down. In fact, one of the strongest arguments for public insurance programs like national health insurance is that they can prevent adverse selection from spoiling (or reducing the social efficiency) of the insurance market by requiring people to enroll. Since private insurance providers cannot require people to enroll, it may often be the case that governments can improve net (or average) social welfare by requiring everyone to buy insurance. In fact, governments do this quite frequently by using taxes (compulsory) to pay for social insurance plans like flood and earthquake insurance. As we have discussed in class, private markets for flood and earthquake insurance do not exist.

However, you should observe that the mandatory plan is not a strict Pareto improvement as compared to the voluntary break-even plan since some healthy citizens are made worse off under the mandatory plan. At the same time, not everyone who is compelled to buy the plan is made worse off. In fact, under the mandatory plan, a person with illness probability .292 or higher is strictly better off under the social insurance plan, whereas someone with an illness probability of <.292 is better off without having to pay the ½ Stiglitz insurance premium. Notice however that under the private break-even insurance plan, no one with illness probability below ½ would buy the plan. Hence, people in the region ½ > p >.292 are made better off by the mandated plan whereas people in with p<.292 are made worse off. (It is actually easy to come up with examples where *everyone* is made better off by mandated insurance.)

Finally, observe that since average (and hence total) social welfare is greater under the mandatory plan than under the break-even plan, it must be the case that those helped by the mandatory plan could in theory compensate those hurt by the mandatory plan *and still be better off*. Hence, the plan represents a *potential Pareto improvement*, although not a strict Pareto improvement. This test of "potential Pareto improvements" is called the Kaldor Compensation test and it is used frequently for evaluating policy interventions that help some citizens while making others worse off. The mandatory insurance plan above passes the Kaldor criterion.

2. A. Since health status is non-verifiable, everybody will report $h = 1$. The mining company breaks even when it hires all applicants, so the average health of hired workers is ½, and average profit per worker is zero.

B. The firm will only hire applicants with $h \geq ½$, since it loses money by hiring workers with $h < ½$. Knowing this, workers with $h < ½$ will not apply, since it is costly for them to apply and they know they will be rejected. The health of the least healthy worker who applies is ½, which is also the health of the least healthy worker hired. The average health of hired workers is ¾, and average profit per worker is \$500.

C. If none of the applicants took the test, the situation would be the same as in part A: everybody would apply, get hired (getting a net payoff of \$997 (the salary minus the cost of applying)), and the firm would make zero profits. However, this cannot be an equilibrium, because workers with $h \geq ½$ can increase their payoff by volunteering to take the test. If applicants with $h \geq ½$ volunteer to take the test, they will still be hired and they will get \$998 (the salary minus the cost of applying minus the cost of taking the test plus the bonus for taking the test). Hence any applicant that does not volunteer to take the test signals to the firm that his h is less than ½, and will not be hired. Thus in equilibrium, workers with $h \geq ½$ apply, volunteer to take the test, and are hired, while workers with $h < ½$ do not apply. The average health of hired workers is ¾, and average profit per worker is \$498 (since the firm pays a \$2 bonus to each worker).

D. The subtlety of this question is that workers who choose to take the test *also provide information about the non-test takers.* Because the firm offers \$2 to anyone who volunteers for the test (greater than the \$1 disutility of taking it), people who know they will pass the test (and therefore get the job) always volunteer to take the test. These are people with $h \geq ½$. People with $h < ½$, however, would not want to volunteer for the test since the firm will not hire them after their health is known. But of course the firm understands that anyone who doesn't volunteer to take the test must have $h < ½$. Hence, the firm will not hire anyone who does not volunteer for the test. As a result, people with $h < ½$ do not even bother to apply. Only people with $h \geq ½$ apply for the job; all take the test, and all are hired.

This may result may seem absurd. Why should the firm pay people to take a test that all test-takers will pass? To answer this question, imagine instead that the firm made the test voluntary but *did not pay* \$2 to volunteers. Since the test causes a disutility of \$1, no one would volunteer for the test regardless of his or health. But in this case, the firm would have no way of distinguishing healthy from unhealthy workers and would be back in the same zero profit situation as (A). Hence, by offering \$2 to volunteers, the firm causes healthy people to voluntarily reveal their health status (through testing). And by so doing, the healthy people also reveal who is unhealthy – anyone who doesn't volunteer for the test.

This result is an example of the "full disclosure" principle. This principle says that if some individuals stand to benefit by revealing a favorable value of some trait (e.g., their health), others will be forced to disclose their less favorable values. In the case above, people with $h < ½$ were implicitly forced to reveal their health (by not applying) when people with $h \geq ½$ voluntarily revealed their own health. The "full disclosure" principle is closely related to the

Akerlof Lemons model as follows. In the Lemons model, sellers of low quality cars create negative externalities for sellers of high quality cars by *failing to disclose* that their cars are low quality. You can quickly reverse this result to get the “full disclosure” principle, however, by imagining that a company introduces a low cost car inspection that credibly demonstrates the quality of a car. Sellers of high quality cars will voluntarily obtain this inspection and sellers of low quality cars will not. But of course buyers will understand that sellers who don’t want an inspection must have low quality cars. Low quality car owners are therefore also forced implicitly to reveal their car quality. Thus, reversing the Lemons model, sellers of high quality cars create a negative externality for low quality sellers by *voluntarily disclosing* that their cars are of high quality. In short, the Lemons model and the full disclosure principle are inverses of one another.

Note that an important condition for moving from a Lemons equilibrium to a full disclosure equilibrium is that there exists a credible means for people to reveal their hidden information (e.g., about health, car quality). In the problem above, the health screening test performed this function. In the current example, the auto inspection does the same thing. Without a credible test, each person will claim to be of high quality or good health and hence no information is revealed.