Lecture 15 - General Equilibrium with Production

14.03 Spring 2003

1 General Equilibrium with Production

1.1 Motivation

- We have already discussed general equilibrium in a pure exchange economy, and seen the two most fundamental results of general equilibrium analysis–the first and second welfare theorems.
- Now we are going to add production to this model. In other words, consumers will not just be trading goods. Producers will be making goods as well–turning factors of production into final consumption goods. As it turns out, this adds some complexity.
- So why bother? Because we next want to talk about international trade and comparative advantage. We can't have an intelligent conversation on that topic until we understand how general equilibrium determines the patterns of production.

1.2 Setup of the problem

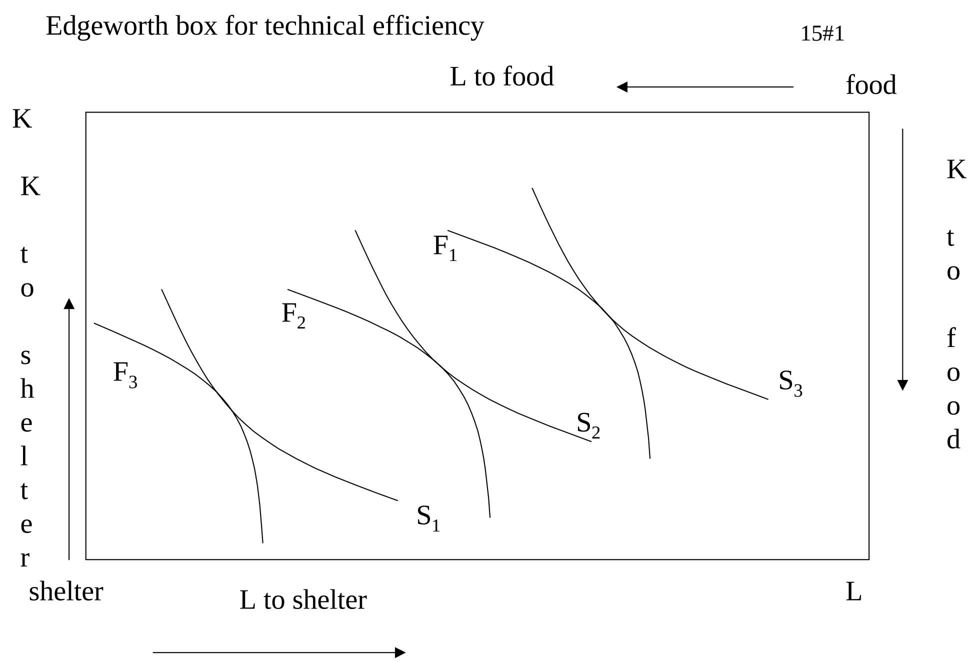
- In the pure exchange case, we had two consumers A, B and two goods F, S for food and shelter.
- Now let's add two factors of production, capital and labor K, L. These represent raw endowments that can be transformed into either of the two final consumption goods F, S.
- Let's also assume that food is labor intensive and shelter is capital intensive. What this means is that for any given price ratio P_f/P_s, the food industry will demand relatively more labor per unit output and the shelter industry will demand relatively more capital.
- Now, we want to see how the market will allocate the following three things:

1. How of the capital and labor factors are used to produce each good: Productive efficiency
2. How much each consumer consumes of each good (food, shelter): Allocative efficiency
3. How total resources are allocated to each sector: Product mix efficiency

- So, in the case of general equilibrium with production, there are three problems that must be solved simultaneously, not just one (allocative efficiency) as in the pure exchange case we discussed earlier.

1.3 The Edgeworth box for production

- Just like the Edgeworth box we drew for consumer exchange, we can construct a box for production.
- The axes of the box will reflect the allocations of K, L to production of F, S.
- In this case, the Southwest corner will represent the origin for shelter and the Northeast corner will represent the origin for food.
- See Figure 1.



- There is a very simple symmetry between the Edgeworth box for exchange among consumers and the Edgeworth box for production:

- Production isoquants are like indifference curves. They come from technology not tastes, but have similar properties.
- Contract curve traces out the set of Pareto efficient allocations, i.e., where the production isoquants for the two goods are tangent. But of course, production is occurring here rather than consumption. K, L are transformed into food, shelter.
- But rather than equating the ratio of marginal utilities across consumers, the points on this contract curve equate the Marginal Rate of Technical Substitution (MRTS) among goods.

- So, along a production isoquant:

$$
\begin{aligned}
S &= S(K, L), \\
dS|_{S=\overline{S}} &= \frac{\partial S}{\partial K} dK + \frac{\partial S}{\partial L} dL = 0, \\
-\frac{dK}{dL} &= \frac{\partial S / \partial L}{\partial S / \partial K} = MRTS_F.
\end{aligned}
$$

- And:

$$
\begin{aligned}
F &= F(K, L), \\
dF|_{F=\overline{F}} &= \frac{\partial F}{\partial K} dK + \frac{\partial F}{\partial L} dL = 0, \\
-\frac{dK}{dL} &= \frac{\partial F / \partial L}{\partial F / \partial K} = MRTS_F.
\end{aligned}
$$

- Hence, along the Contract Curve, we have

$$MRTS_F = MRTS_S.$$

- These points of tangency are called the Efficient Production Set, and the rate of technical substitution is equated among factors at this point.
- This means that the *marginal* productivity of K, L is equated at these points.
- What if instead

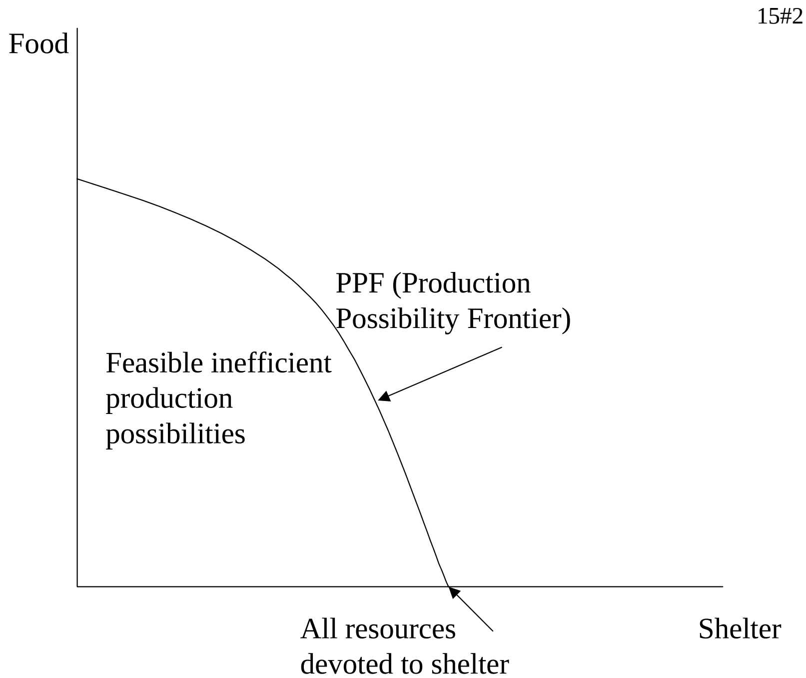
$$\frac{\partial F / \partial L}{\partial F / \partial K} > \frac{\partial S / \partial L}{\partial S / \partial K},$$

  that is, the marginal product of labor in the Food sector was higher than in the Shelter sector? Then more labor should be allocated to Food and more capital to Shelter until these marginal products are equated. It will be possible to raise output in either (or both) sectors without lowering output in the other simply by reallocating factors between them. In other words, there are 'gains from trade' among these industries.

1.3.1 Production Possibility Frontier (PPF)

- The points along the Efficient Production Set can also be drawn in final good (F, S) space rather than factor (K, L) space.
- The PPF represents the alternative combinations of two final goods that can be produced with fixed quantities of inputs.
- The frontier of this set corresponds to the technically efficient, feasible combinations of outputs.
- The points inside the set are feasible but are not technically efficient as defined above.
- Points outside of this set might be technically efficient and are certainly more desirable than the points on the frontier but are not feasible.
- Note that the slope of the PPF is *not* the $MRTS$ between K, L. (That is the slope of the production isoquants).
- Instead, the slope of the PPF is the $MRTS$ between F, S, which we call the $MRPT$, Marginal Rate of Product Transformation. It is the rate of technical transformation from one good to the other (at the margin).
- Of course, we don't actually transform food into shelter or vice versa. Rather, producers choose whether to transform K, L into food or shelter.
- What gives rise to the curvature of the PPF? Several possibilities:

  1. Diminishing returns in each sector: Efficiency falls as scale increases. This is not too appealing. Although one firm may have diminishing returns, this should not be true in aggregate.
  2. Specialized inputs: Some capital and labor is more suited to food or shelter production. This is somewhat appealing, but it actually implies that we are not specifying the problem precisely in the Edgeworth box above; we should instead draw as many distinct inputs as there are 'types' of labor and capital.
  3. Products have differing factor intensities: If as we have assumed, Shelter is more capital intensive than Food, then the marginal productivity of the Shelter sector will decline as we begin to give it all the labor, and vice versa for Food as we allocate it all of the capital. So, even if there is not diminishing returns in each sector, we will get diminishing returns as we force a sector to use a comparatively less technically productive mix of inputs.



1.3.2 Example: PPF for food and shelter

- For simplicity, assume that Food and Shelter use only labor in this example:

$$F = {L_F}^{.5},$$
$$S = .5{L_s}^{.5}$$

- Budget constraint:

$$L_F + L_s = 100.$$

- Implies

$$F^2 + 4S^2 = 100.$$

- To obtain the PPF, totally differentiate:

$$2FdF + 8SdS = 0,$$
$$-\frac{dS}{dF} = \frac{F}{4S}.$$

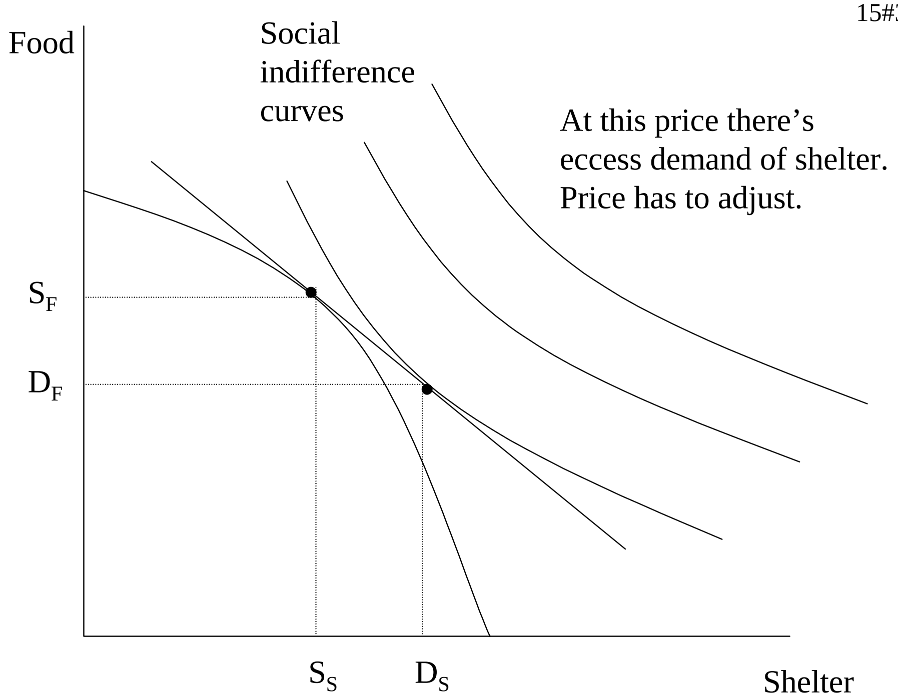
- Note that the absolute value of this expression increases as F rises – the marginal product of labor in the S sector rises as more labor is allocated to the F sector, and vice versa as labor is allocated to the F sector.

- In this case, the curvature of the PPF comes from diminishing returns in each sector (b/c the exponent of each production function is less than unity).
- So note that the following points are all technically efficient allocations on the PPF:

$$\begin{aligned} F &= 10, S = 0 \\ F &= 0, S = 5 \\ F &= 5, S = 4.3 \end{aligned}$$

1.4 Determination of equilibrium prices

- Now that we have a PPF, we need to determine how equilibrium goods prices are determined.
- The PPF shows all of the efficient combinations of *supply* of goods.
- What we need next is demand for goods. And this will come from consumer preferences.
- So, assume an aggregate utility function, which can be represented using a set of 'community indifference curves.' These curves reflect the aggregation of individual preferences reflecting societal willingness to trade off among goods.
- See Figure 3.



- The equilibrium price ratio $\frac{P_F}{P_S}$ will equate demand and supply for both goods.
- At this point, the community indifference curve is tangent to the PPF, and these two convex sets are separated by the price ratio.
- With many goods, these points of tangency would be divided by a separating hyperplane representing the price vector. The existence of this equilibrium price vector could be proved using the Separating Hyperplane theorem.
- Note also that the existence of the equilibrium depends upon:
    1. No transaction costs
    2. Full information
    3. No externalities
    4. Preferences satisfy utility axioms $A1 - A6$ as we discussed at the beginning of the semester
    5. No market power (firms and consumers are small relative to the market)

1.5 Outcomes if these conditions satisfied

1. Each consumer takes price as given and maximizes utility subject to her budget constraint.

2. Each producer takes prices as given and maximizes profits given these prices.

3. The "Law of One Price" prevails – Each good sells at a single price regardless of whom is buying or selling it. So the identity of sellers and buyers is immaterial.

4. The market clears (as seen by the tangency of the PPF and the community indifference curve).

- But who cares? Other than clearing the market, what useful properties does general equilibrium price setting (with production) offer?

- To answer this question, return to the 3 dimensions of efficiency from the introduction.

  1. Productive ('technical') efficiency
  2. Allocative efficiency
  3. Product mix efficiency

1.5.1 Productive (Technical) Efficiency

- Productive efficiency requires that we cannot produce more food without producing less shelter.

- Q: What is the marginal condition that expresses this idea?

- As above:

$$\begin{aligned} \frac{\partial F/\partial L}{\partial F/\partial K} &= \frac{\partial S/\partial L}{\partial S/\partial K}, \\ MRTS_F &= MRTS_S. \end{aligned}$$

- What guarantees that this condition will hold in equilibrium?

- *Law of one price.* Since both sectors face same factor prices for K, L, we have:

$$\frac{\partial F/\partial L}{\partial F/\partial K} = \frac{\partial S/\partial L}{\partial S/\partial K} = \frac{P_L}{P_K}$$

  Each sector equates the ratio of marginal productivities of factors to the price ratio and so these ratios are equated across sectors.

- Therefore, productive efficiency is satisfied in equilibrium.

1.5.2 Allocative Efficiency

- On the consumer side, what is the analogous requirement to Efficient Production?
- Efficient Consumption $\leftrightarrow$ Allocative efficiency:

$$\begin{aligned} MRS_A &= MRS_B, \\ \left(\frac{\partial u/\partial F}{\partial u/\partial S}\right)_A &= \left(\frac{\partial u/\partial F}{\partial u/\partial S}\right)_B \end{aligned}$$

- What guarantees that this will hold?
- Also the price ratio, in this case the relative prices of *final goods* (F, S) rather than the prices of factors K, L :

$$MRS_A = MRS_B = \frac{P_F}{P_S}.$$

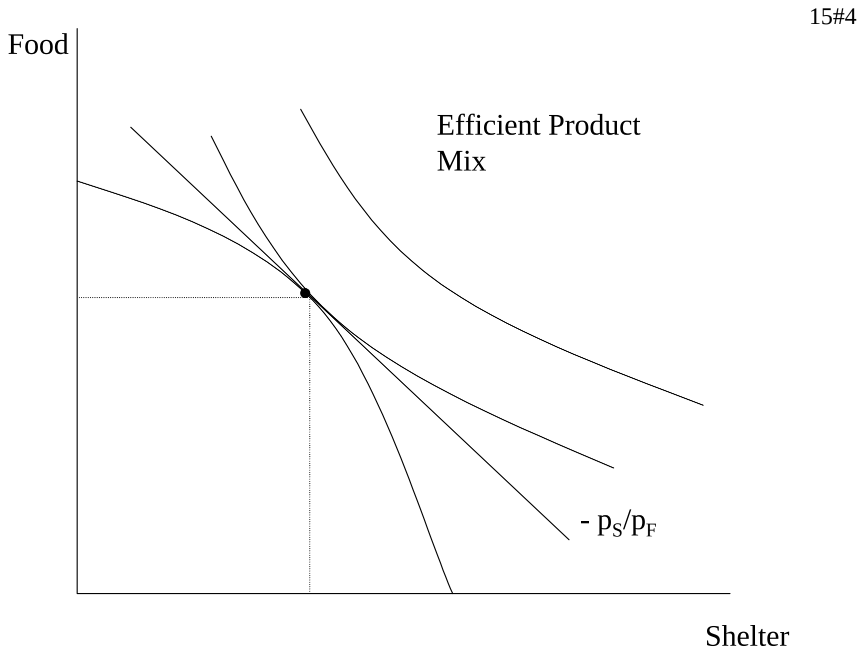
- And we saw in the Edgeworth box that this will be satisfied.
- So, the market equilibrium also satisfies Allocative Efficiency.
- Note that mapping between efficient consumption and efficient production:

Problem	Scarce inputs	Outputs	Efficiency conditions
Efficient consumption	F, S	$u_A = u_A(F_A, F_B)$ $u_A = u_A(F_A, F_B)$	$MRS_A^{FS} = MRS_B^{FS}$ $(u^F/u^S)_A = (u^F/u^S)_B$
Efficient production	K, L	$S = S(K_S, L_S)$ $F = S(K_F, L_F)$	$MRTS_A^{FS} = MRTS_B^{FS}$ $\frac{\partial F/\partial K}{\partial F/\partial L} = \frac{\partial S/\partial K}{\partial S/\partial L}$

1.5.3 Efficient Mix/Variety

- There remains a third problem.
- We could have produces F, S produced efficiently using K, L *and* F, S efficiently consumed by consumers A, B but still something would be missing. What is that thing?
- We could have produced a suboptimal mix of goods – too much food and too little shelter or vice versa.
- What is the market condition that guarantees that this does not occur?
- It is the tangency between the community indifference curve and the PPF.

- As shown in Figure 4, although points A, B are technically efficient, only C represents an efficient mix of F, S.



- So along the PPF:

$$MRPT_{FS} = -\frac{dF}{dS}$$

- For a given price ratio:

$$MRPT_{FS} = -\frac{dF}{dS} = \frac{P_F}{P_S},$$

meaning for a given price ratio, the marginal rate of product transformation must be equal to the price ratio.

- Why? Assume $P_F/P_S > 1$, so it costs more than 1 unit of shelter to buy one unit of food. Then it needs to be the case that at the technical level that to produce more than one unit of food, it would require more than 1 unit of shelter (or, more accurately, the K, L required to produce that shelter). If this were not the case, we should be making more food and less shelter.

- Similarly, along the community indifference curve, it will also be the case that

$$MRS_{FS} = \frac{\partial U/\partial F}{\partial U/\partial S} = \frac{P_F}{P_S}.$$

- So, given the equilibrium price vectors, we'll have

$$MRS_{FS} = MRPT_{FS} = \frac{P_F}{P_S}.$$

1.5.4 The Link Between Factor and Goods Markets

- We've now established the equilibrium conditions for efficient consumption, efficient production and efficient mix.
- But we are still missing one thing. We also need to show that there is an appropriate linkage between the factor and goods markets Without this, how do we know that the price vector that clears the goods market is compatible with the price vector that clears the factor market?
- This is actually straight-forward to show. What we require is that

$$\frac{\partial u/\partial F}{\partial u/\partial S} = \frac{\partial S/\partial K}{\partial F/\partial K} = \frac{\partial S/\partial L}{\partial F/\partial L} \tag{1}$$

- This equation says that the marginal rate of substitution in consumption must be equated with the marginal rate of transformation. Notice the inversion here, however.
- If the marginal utility of F is relatively high (relative to S), then we want the marginal product of K in production of F to be relatively low (relative to the marginal product of K in S). The reason is that if we get a lot of utility from consumption of F at the margin, we are willing to allocate relatively more K to its production.
- A simple example should clarify:
  - Say I can produce two goods using Acme baking mix: pancakes and playdough.
  - Assume further that there is diminishing marginal returns to each, so that the more pancakes I make, the more Acme baking mix it takes at the margin and similarly for playdough (of course, this is not realistic).
  - Assume finally that I greatly prefer eating pancakes to playdough on average, but that I have diminishing marginal utility of consumption of each.
  - It is therefore efficient for me allocate Acme mix to pancakes (rather than playdough) until there is very low marginal productivity of Acme in pancakes and relatively high marginal productivity in playdough. In other words, since I value pancakes more at the margin, I'm happy to use most of Acme for pancakes, even though the marginal physical productivity of Acme is low in this use.

- How do we know this marginal condition above will be satisfied?
- From the tangency condition between the community indifference curve and the PPF, we know that

$$MRS_{FS} = MRPT_{FS}.$$

- Consider shifting one unit of K from production of F to production of S.
- Using the PPF, we have:

$$\begin{aligned} -\frac{dF}{dS} &= \frac{\partial S/\partial K}{\partial F/\partial K} = MRPT_{FS}, \text{ and} \\ -\frac{dF}{dS} &= \frac{\partial S/\partial L}{\partial F/\partial L} = MRPT_{FS}, \end{aligned}$$

- But from the tangency condition between the community indifference curve and the PPF, we know that

$$MRS_{FS} = \frac{\partial u/\partial F}{\partial u/\partial S} = MRPT_{FS}.$$

- This implies that

$$\begin{aligned} \frac{\partial u/\partial F}{\partial u/\partial S} &= \frac{\partial S/\partial K}{\partial F/\partial K} \text{ and} \\ \frac{\partial u/\partial F}{\partial u/\partial S} &= \frac{\partial S/\partial L}{\partial F/\partial L} \end{aligned}$$

1.5.5 The exact link between goods and factor prices

- A question raised in class is what is the exact numerical link between the prices of goods and the prices of factors.
- One can demonstrate that:

$$\begin{aligned} P_K &= (\partial F/\partial K)P_F = (\partial S/\partial K)P_s, \\ P_L &= (\partial F/\partial L)P_F = (\partial S/\partial L)P_s, \\ \frac{P_K}{P_L} &= \frac{(\partial F/\partial K)P_F}{(\partial S/\partial L)P_s} = \frac{(\partial S/\partial K)P_s}{(\partial F/\partial L)P_F} \end{aligned}$$

- Why does this have to be true? In equilibrium, we know that

$$\frac{P_F}{P_s} = \frac{\partial U/\partial F}{\partial U/\partial S}.$$

But knowing this fact pins down the price of capital and labor through their marginal contributions to production of F and S. In particular:

$$\begin{aligned} P_K &= (\partial U/\partial F)\cdot(\partial F/\partial K) = P_F\,(\partial F/\partial K)\,, \\ P_L &= (\partial U/\partial F)\cdot(\partial F/\partial L) = P_F\,(\partial F/\partial L)\,. \end{aligned}$$

And you can write the analogous equations for P_K, P_L in the production of S rather than F.

- These equations indicate that P_K, P_L reflect the marginal contribution of $K\,, L$ to consumer utility through their production of consumer goods F, S. So a shorthand version of these equations is

$$\frac{P_K}{P_L} = \frac{\partial U/\partial K}{\partial U/\partial L}.$$

- Note that consumers do not obtain utility from capital and labor. But the prices of K, L reflect the 'indirect utility' generated by capital and labor through their transformation into F and S, which are consumed.
- Aren't you glad you asked?

1.6 Summary

- So, we have showed that a free market in equilibrium will simultaneously satisfy all of our efficiency criteria above:
  1. Technical efficiency
  2. Allocative efficiency
  3. Product Mix efficiency
- So, for given factor endowments K, L, given preferences u_A, u_b, and given technologies $F = F(K, L)$, $S = S(K, L)$:
  1. The market will *all by itself* find $F^*, S^*, \frac{P_F}{P_s}^*, \frac{P_k^*}{P_s}, \frac{P_L^*}{P_s}$. Note that of the four possible prices, only three are needed since, by Walras' law, the fourth follows from the third. The choice of P_s as the 'numeraire' is a normalization; any of the four prices can serve this purpose.
  2. Goods and factor markets will both clear with neither excess demand nor excess supply.
  3. All gains from trade among consumers and reallocation of resources across goods will be exhausted.
  4. In other words, the equilibrium will be Pareto efficient.
- As we have discussed before, these conditions are only perfectly satisfied under stringent conditions:

  - Perfectly competitive markets
  - Full information
  - No transaction costs
  - No externalities

- It is not realistic to think that markets achieve this ideal at all times, or perhaps ever.

- But it is nevertheless remarkable to realize that a free market economy can potentially solve this set of fundamental efficiency problems without any centralized decision making.

- And it should suggest that free market economies have considerable potential to generate many desirable outcomes.

- In later class lectures, we'll focus on two of four imperfections above: externalities and imperfect information. In 14.01, you discussed imperfectly competitive markets (e.g., monopoly, monopsony). Transaction costs are actually also closely related to externalities, as we'll discuss after the exam.

- Before we study these imperfections, we want to examine an important application of General Equilibrium modeling: International trade and comparative advantage.