Rutherford Scattering

MIT Department of Physics

This is an experiment on the scattering of alpha particles by atomic nuclei. You will shoot alpha particles, emitted by ^{241}Am, at thin metal foils and measure the scattering cross section of the target atoms as a function of the scattering angle, the alpha particle energy, and the nuclear charge.

PREPARATORY QUESTIONS

1. What is the closest possible distance of approach of a 5.5 MeV alpha particle to a gold nucleus, and how does that distance compa re to the radius of the nucleus?
2. Define the differential scattering cross section.
3. Sketch the expected curve of counting rate as a function of the scattering angle for the gold foil.
4. Describe how charged particles lose energy in traversing matter. How does the rate of energy loss depend on the velocity of a particle?
5. How much energy does an alpha particle emitted by ^{241}Am lose in traversing 1 cm of air at STP?
6. How does the silicon barrier-layer detector used in this experiment work? (see Melissinos, 1966).

WHAT YOU WILL MEASURE

The intensity of alpha particles scattered by thin metal foils as a function of the scattering angle for two elements of very different atomic number and for two different alpha-particle energies.

INTRODUCTION

Little was known about the structure of atoms when Geiger and Marsden began their experiments in 1909 at the Cavendish Laboratory on the scattering of alpha particles by thin metal foils. A decade earlier at the Cavendish J. J. Thomson had discovered the electron and determined the ratio of its charge to mass by measuring the deflections of electron beams (cathode rays) by electric and magnetic fields. In 1909 Millikan measured the charge of the electron in the oil drop experiment. Thus by 1909 both the charge and mass of the electron were known with considerable accuracy. Furthermore, Thomson's interpretation of X-ray scattering from carbon and other light elements had established that the number of electrons per atom of a given element was equal not to its atomic weight but to its atomic number as determined by its position in the periodic table. Since the mass of an electron is much less than the mass of the lightest atom, hydrogen, it was clear that most of the mass in any atom is associated with the positive charge. The central problem was to figure out how the positive and negative parts of an atom are held together in such a way as to produce optical emission spectra with the regularities expressed by the Balmer formula for hydrogen and the combination rules and series limits for the complex spectra of multi-electron atoms.

Thinking within the limitations of Newtonian mechanics and Maxwell's electromagnetic theory Thomson imagined the atom as a sphere of positive charge within which the electrons occupy certain positions of equilibrium, like raisins in a pudding. Set in motion, the electrons should vibrate harmonically, radiating electromagnetic energy with characteristic sharp frequencies that would be in the optical range if the radii of the atomic spheres were of the order of 10^{-8} cm. However, the "raisin pudding" model yielded no explanation of the numerical regularities of optical spectra, e.g. the Balmer formula for the hydrogen spectrum and the Ritz combination principle for spectra in general.

At this point Earnest Rutherford got the idea that the structure of atoms could be probed by observing the scattering of alpha particles, the positively charged emanation of radioactive substances that he had recently demonstrated were helium ions. According to the raisin pudding model an alpha particle traversing a thin gold film should suffer many small angle deflections as it passes close to or through the positive spheres of the gold atoms. Rutherford showed that the fraction of particles scattered in this way through an angle θ or greater should decrease exponentially according to the equation

$$F_\theta = exp(-\theta/\theta_m) \tag{1}$$

where θ_m is the mean multiple scattering angle. For a typical foil of gold leaf $\theta_m \tilde{} 1^\circ$. Thus at $\theta = 30^\circ$ one finds F_θ on the order of exp(-30) or 10^{-13}.

Rutherford's formula turned out to be correct for very small angles of scattering. Evidently there was substantial truth in the idea of multiple scattering. But in experiments initiated at Rutherford's direction, Geiger and Marsden (1909) found that 1 in 8000 alpha particles passing through a thin film of platinum was scattered through more than 90°! It was as though bullets fired at a bale of cotton could occasionally ricochet backward. Such an

observation might lead one to suspect rocks in the cotton.

At this point Rutherford (1911) advanced the hypothesis that the positive charge and most of the mass of an atom is concentrated in a "nucleus" with dimensions of the order of 10^{-12} cm (10,000 times smaller than the atom as a whole) with the electrons in some sort of configuration around it. Applying the principles of classical mechanics he calculated the trajectories of alpha particles passing near such nuclei, and derived an expression for differential scattering cross section which accounted accurately for the scattering data, thereby validating the hypothesis of the nuclear atom.

The Rutherford scattering cross section *per target atom* (for a derivation see Melissinos p. 231) is

$$\frac{d\sigma}{d\Omega} = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{sin^4(\theta/2)} \qquad (2)$$

where θ is the scattering angle, Ze is the charge of the target nuclei, $Z'e$ is the charge of the alpha particles and E is their kinetic energy. Further excruciatingly tedious experiments by Geiger and Marsden confirmed the validity of the formula within the statistical errors of their measurements. Geiger hadn't invented the Geiger counter yet, and electronic detection methods were still 20 years in the future. They used a low power microscope to observe and count by eye the scintillations produced by the alpha particles when they impinged on a screen lightly coated with zinc sulfide dust.

Melissinos (1966) presents the Rutherford theory and discusses the interpretation of data from a scattering experiment that is quite similar to that in the Junior Lab with the exception of the specific detector and circuit arrangement. Here we will assume you have studied that section of his text and his discussion of solid state detectors, and we will confine our discussion to the features of the experimental setup and procedures that are peculiar to our setup.

APPARATUS

Figure 1 is a schematic drawing of the apparatus in the vacuum chamber. The source is ^{241}Am which emits alpha particles of various discrete energies, the most frequent of which are 5.486 MeV (86%), 5.443 MeV (12.7%), and 5.391 MeV (1.4%). All these decays lead to excited states of ^{237}Np. The half-life of ^{241}Am is 458 years. The source, deposited on a thin metal disk with the highest activity of ^{241}Am per unit area commercially available (˜1.5 millicuries per square inch) and sealed with an evaporated gold coating 1.5 microns thick, is covered by a metal washer with a 0.64 cm diameter hole and enclosed in a "howitzer" with a 0.64 cm diameter aperture in its snout. Under vacuum a collimated beam of alpha particles emerges from the snout (the range of 5.5 MeV alpha particles in air at atmospheric pressure is only ˜ 4 cm). The targets are mounted in a target holder which can be manipulated from outside the vacuum so as to bring a multi-layer gold leaf target, a titanium foil or no foil into the path between the source and the detector. The alpha particles are detected by a surface barrier silicon detector in the form of a 1.2 cm diameter cylindrical disk which acts as a solid state ionization chamber. A description of how such detectors work can be found in Melissinos (p. 208).

CAUTIONARY NOTE: DO NOT CHANGE THE BIAS VOLTAGE SUDDENLY. TURN IT UP OR DOWN SLOWLY - NOT FASTER THAN 20 VOLTS PER SECOND.

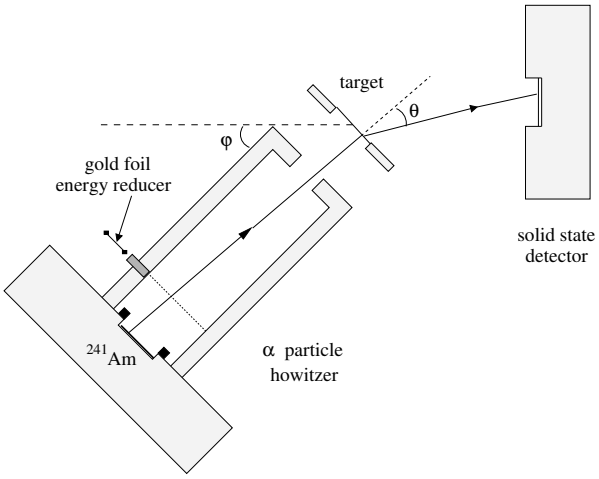

FIG. 1: Schematic diagram of the apparatus for measuring the scattering of alpha particles. Note the difference between the howitzer position angle ϕ and the scattering angle θ of a representative alpha-particle trajectory.

The solid-state detector and the preamplifier to which it is attached are fragile. The preamplifier has a special low-noise field effect transistor (FET) which can be ruined by an excessive current pulse. A sudden change in the voltage across the input capacitance will drive a large current through the FET. The manufacturer warns that the bias voltage on the detector must be turned off before the preamplifier is disconnected from the system. Too high a bias voltage will damage the surface barrier detector. A satisfactory operating bias voltage on the detector is +60 volts (positive voltage only). There is a voltage limiter in the bias supply line which should keep the voltage under the maximum allowable value. As the bias is first applied, look at the pulse output from the amplifier to be sure that breakdown is not occurring (the bias is applied across a very small gap, generally less than a few microns). The detector is light sensitive: it should be covered when the bias voltage is on. Turn down the bias voltage before exposing the detector to light. The alpha particle source is well protected and secure (the radiation protection office examines it each year to be

sure that it is completely sealed). It is not a particularly strong source (˜100 μCi or microcuries). Nevertheless, as with any radioactive source, it must be treated with care. It is encased in the howitzer, and need never be disturbed. The howitzer should not be disassembled by anyone other than a member of the technical staff or a professor.

EXPERIMENT

The goal of this experiment is to observe the phenomena of Rutherford scattering and determine how the differential scattering cross section depends on the scattering angle, the Z of the target, and the energy of the alpha particles.

Your data from this experiment should consists of **at least** the results of the following operations:

1. Determine the essential characteristics of the experimental setup.
   (a) Measure the counting rate as a function of howitzer position angle with the open hole.
   (b) Measure the loss in energy of the alpha particles in traversing the three targets and determine the thicknesses of the targets from the data.
2. Measure the angular dependence of the scattering cross section of gold. Measure the counting rate with a gold target as a function of howitzer position angle out to the largest angles that counting statistics and time limitations allow, for the full energy alpha particle beam.
3. Determine the absolute differential cross section. Measure the intensity of the beam that emerges from the howitzer. Note that the howitzer is designed so that the entire emergent beam irradiates the target foils, i.e. the beam is sufficiently narrow to pass through the foils without intercepting the target holder. To make this measurement you must remove the target holder (very carefully) and move the howitzer close to the detector in order to catch the whole beam emerging from the snout.

At large scattering angles the counting rates are very low, so the accuracy of your data will be severely limited by the (Poisson) statistical accuracy you can achieve. Your best strategy will be to make, during the first session, a complete set of measurements with short integrating times (10-30 minutes) in each configuration you plan to use, and to carry out a preliminary analysis of the resulting data. Then you can make an observing plan in the light of what you have learned about the counting rates and the problems of analysis, allotting enough time to each configuration to obtain enough counts to insure good statistical accuracy. Obviously, you cannot afford to measure the counting rates at 2° intervals of the howitzer angle. You may decide, for example, to settle for measurements of good statistical accuracy of the angular dependence of the counting rate with one target only at howitzer angles of, say, 20°, 30°, 40°, and 60°, and measurements of the Z and E dependence at only one howitzer angle, say 20°.

EXPLORE THE OPERATION OF THE EQUIPMENT

If the setup is under vacuum turn the bias voltage slowly down to zero, close the vacuum valve between the pump and the chamber, and open the vacuum release valve, leaving the pump running. Place the black hood on a table where it can serve as a cushion for the plastic cover. Lift the cover off the steel cylinder and place it flat on the hood, being careful not to damage the underside of the plastic cover. Find out how you can adjust the relative positions of the howitzer and detector with respect to the target, and how you can turn, raise and lower the target with the control rod that protrudes from the bottom of the chamber. Note how you can simultaneously rotate the howitzer and the target about a vertical axis through the target by turning the knurled outer cylinder under the chamber. This feature enables you to maintain a fixed relative orientation of incident beam and target while you vary the scattering angles of the detected alpha particles.

CALIBRATE THE MEASUREMENT CHAIN

Move the howitzer support arm so that the howitzer points directly at the detector through the empty target hole. Pump the system down with the bias voltage off. A good way to apply the necessary pressure to initially 'seal' the o-ring is by weighting the plexiglass cover with 6-8 lead bricks available next to the experiment. On the way down you can watch the pulses from the unbiased detector appear and gradually grow in amplitude as the amount of air between the source and detector diminishes. The pressure should reach 200 microns in 10 to 15 minutes. If it doesn't, you probably have leak around the O-ring seal under the plastic cover. It can be remedied by the application of a little vacuum grease. When the pressure is below 200 microns leave the bias voltage supply switch on its lowest position and turn the continuous bias voltage control slowly up till the meter reads +60 volts. Adjust the gain so that the alpha-particle pulses have an amplitude of about 5 volts. Then observe the pulse size distribution with the MCA and readjust the gains so that the peak of the alpha-particle pulse-size distribu-

tion lies at a convenient position, say channel 600. You can assume that the amplitude of a detector pulse is proportional to the energy lost by the particle in the silicon detector. Thus a linear plot of the channel number versus energy, scaled to match the energy of the alpha-particles coming straight from the source, should be an accurate plot for the interpretation of the pulse sizes you will be measuring. For a quick-start guide on the operation of the multi-channel analyzer (Perkin-Elmer Trump-PCI), see the Junior Lab electronic-library.

ADJUST THE POSITION OF THE TARGET

A plastic ruler is mounted under the platform to provide a means for measuring the position of the rod that controls the vertical position of the target holder. Accurate vertical positioning of the target holder is essential to assure that the particle beam passes cleanly through one or another of the holes in the aluminum sheet that supports the foils, i.e. without touching the edge of the hole. Before starting your measurements you must determine the narrow range of acceptable vertical positions for each of the four holes. It is wise to move the snout of the howitzer to within ˜1 cm of the target holder so that the diverging beam spreads as little as possible before it passes through the hole in the target mount.

MEASURE THE COUNTING RATE AS A FUNCTION OF THE POSITION ANGLE WITH NO SCATTERING FOIL

With the target holder in the open hole position, measure the counting rate as a function of the howitzer position angle from ˜-20° to ˜+20° relative to the nominal center position. Plot the data as you proceed. Determine the exact pointer readings of the center position and at the positions of zero counting rate intercepts on either side of the center.

MEASURE THE EFFECTS ON THE PULSE-SIZE SPECTRUM OF PASSAGE THROUGH THE TARGET FOIL

Using the preset accumulation time feature of the MCA accumulate a size spectrum of the pulses produced by unscattered alpha particles with the howitzer position angle set at the center position. To characterize the width of the distribution, place the start and stop cursors on either edge of the distribution at the positions where the counting rate is half the maximum value, and note the channel numbers. Measure the channel number of the peak counting rate. Compare the results with no foil, the two gold foils, and the titanium foil, and figure out the most probable energy lost by the alpha particles in traversing each of the films. Describe and explain the changes in the shape of the size spectrum when a target foil is in the beam.

Determine the thicknesses of the target by reference to the range-energy data available on the web at [1]. Be sure to use the "projected range" data and not the CSDA range data. To interpolate this table plot the tabulated range versus energy. Call c1 and c2 the median channels of the pulse before and after passage through a target. You can assume with some confidence that the median channel number is proportional to the particle energy. From your range energy plot read the range of the incident alpha particles (E0=5.48 MeV) and the range of particles with energy (c2/c1)E0. The difference in range is the thickness of the target in $mg \cdot cm^{-2}$.

PLAN AND MAKE YOUR SCATTERING MEASUREMENTS.

You will probably want to attain at least ±10% (>100 counts) statistical accuracy in each of your measured rates. If you occupy the first lab sessions getting acquainted with the experiment, then you will have a total of about 9 hours in the next three sessions to get your definitive scattering data plus a possible overnight run at a very large scattering angle to observe the amazing phenomenon of atomic bullets ricocheting nearly straight back. Clearly you cannot afford to creep along the curve of rate versus position angle at one-degree intervals. Rather, you must take large steps in position angle to define the general shape of the curve, and then fill in as time permits to refine your data.

To measure the energy dependence of the cross section you can reduce the incident energy by inserting into the slot in the howitzer the titanium foil in the holder provided. You should make your measurement of the energy dependence of the differential cross section at a position angle large enough (say 20°) to reduce the complications caused by multiple small-angle Rutherford scattering.

Your analysis will be simpler if you maintain the same geometrical relation between the howitzer and the scattering foil. This is feasible for measurements at position angles from 0° to ˜60°. If you want to measure scattering at very large angles you will have to do it by "reflection", i.e. with the howitzer and detector on the same side of the target, and accept the tricky task of analyzing the data in such a way as to make a comparison with the smaller angle data meaningful.

ANALYSIS AND INTERPRETATION

1. On log-log graph paper plot the gold-foil counting rates against $sin^{-4}(\phi/2)$, where ϕ is the howitzer

position angle, and compare the result with the theoretical expectation based on the Rutherford formula.

To make an accurate comparison between the data plot and the predictions of the Rutherford theory you should take account of the spread in the angular response of the apparatus. The inescapable fact of any scattering measurement is that events with a range of scattering angles contribute to the counting rate at any given position angle of the detector. Ideally one would like that range to be very small so that a plot of counting rate against position angle would be, in effect, a plot of counting rate against scattering angle. But then the counting rate would be impractically small. Thus, in designing a scattering experiment one must strike a compromise between angular resolution and counting rate. To achieve acceptable counting rates in the present experiment it was necessary to design it with a broad angular acceptance. Consequently, your plot of counting rate against position angle should be compared to a convolution of the Rutherford cross section with the angular response function of the apparatus. To see how this can be done, call $g(\theta, \phi)$ the angular response function such that $g(\theta, \phi)d\theta$ is the probability that a particle scattered at an angle between θ and $\theta + d\theta$ will be detected when the howitzer is at position angle ϕ. Then the expected the counting rate at ϕ is

$$C(\phi) = C_0 \int_0^{\pi} g(\phi, \theta) sin^{-4}(\theta/2) d\theta \tag{3}$$

where C_0 is a constant that includes the solid angle subtended by the detector at the point of scattering. For a crude approximation one might represent g by a triangular function defined by

$$g(\phi, \theta) = (1 - \tfrac{|\theta - \phi|}{\theta_0}), \quad |\theta - \phi| < \theta_0 \tag{4}$$

$$= \quad 0, \qquad |\theta - \phi| > \theta_0 \tag{5}$$

where θ_0 is the half-width of the base of a triangular function. The value of θ_0 can be estimated by analysis of Figure 2 which is a scale drawing of the howitzer, target holder and detector. One might set $2\theta_0$ equal to the difference between the extreme angles of scattering that detected particles can undergo when ϕ=0. The convolution should yield a curve of counting rate versus position angle that conforms more closely to the data than the function $sin^{-4}(\phi/2)$ of the Rutherford cross section.

A little thought will convince you that it is a formidable job of geometrical analysis to construct an exact analytical expression for g. A more practical approach is to write a "Monte Carlo" program that simulates the experiment by following individual particles through the system, choosing positions and directions of emission and scattering with random numbers according to appropriate probability distributions among which is the Rutherford scattering probability distribution to be tested. If you are an experienced programmer you may want to try this approach, which is also not a trivial job. You can probably get advice on how to proceed from one of your instructors.

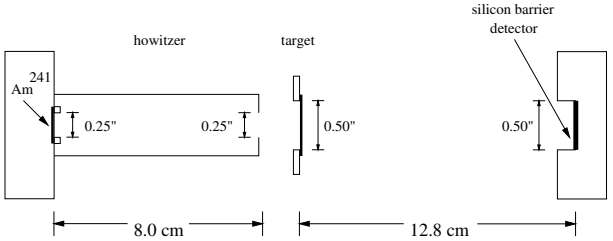

FIG. 2: Dimensioned drawing of the alpha-particle howitzer, target foil, and detector geometry

2. Determine the absolute differential scattering cross section per gold atom at an angle (say 30°) large enough to reduce substantially the problems of interpretation caused by multiple scattering.

[1] Berger, M.J., Coursey, J.S., and Zucker, M.A., (1999). *ESTAR, PSTAR, and ASTAR: Computer Programs for Calculating Stopping-Power and Range Tables for Electrons, Protons, and Helium Ions* (version 1.21), [Online]. Available: `http://physics.nist.gov/PhysRefData/Star/Text` [2002, September 3]. National Institute of Standards and Technology, Gaithersburg, MD

[2] P. R. Bevington & D. K. Robinson 1992, *Data Reduction and Error Analysis for the Physical Sciences*, 2nd Edition, McGraw Hill.

[3] S. Gasiorowicz 1974, *Quantum Physics*, Wiley, p. 417.

[4] A. C. Melissinos 1966, *Experiments in Modern Physics*, Academic Press, chapter 6.2.

[5] Northcliffe & Shilling, 1970, "Range and Stopping-Power Tables for Heavy Ions", *Nuclear Data Tables* Part A, 7

[6] E. Rutherford 1911, "The Scattering of α and β Particles by Matter and the Structure of the Atom", *Philosophical Magazine*, **21**, 669.

[7] E. Serge 1977, *Nuclei and Particles*, Benjamin, chapter 2.

SUGGESTED THEORETICAL TOPICS

1. The Rutherford scattering cross section.

2. Energy loss of charged particles in matter.

3. Multiple Coulomb scattering.

4. Silicon barrier detector.