Quick Start for Using Maple

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WHAT IS MAPLE?

To quote the introduction from the Maple Help menu: 'Maple 8 is a comprehensive computer system for advanced mathematics. It includes facilities for interactive algebra, calculus, discrete mathematics, graphics, numerical computation, and many other areas of mathematics. It also provides a unique environment for rapid development of mathematical programs using its vast library of built-in functions.' Basically Maple is a powerful symbolic mathematics engine. Although somewhat lacking in its ability for curve fitting, etc, it is very useful for things like integration and graphics.

This guide will hopefully allow you to pick up some syntax and enable you to begin to explore Maple's capabilities.

GETTING STARTED

To run the Maple software on Athena simply add the Maple locker then enter the command 'maple'.

```
>add maple
```

```
>maple
```

The Maple work environments are called worksheets. Each new worksheet is self contained and any variables or special functions you enable will only apply to that worksheet. All Maple inputs (expressions, commands, etc.) must be followed by a semicolon in order to be evaluated or executed. Some inputs will result in lengthy outputs displayed. An input ending with a colon will be computed, but no result will be displayed.

In order to enable different functions in Maple, particular packages must be loaded. For example, most graphics functions will be contained within the plots package. To load a package simply type 'with(package);' Some useful packages besides plots are linalg or Linear Algebra and stats or Statistics. Certain variable names have special functions in some packages so make sure any variables you name are not reserved. The Help menu is a useful resource for determining what each package will do and what kind of input it expects. The Help menu has a button link on the upper right hand side of the screen.

LOADING DATA

To load data you may have from a file into Maple you must first decide in which package you will be working. For an example look at some data taken from the Mossbauer experiment. First load the linalg package as well as the plots package. Next, the command to import a data file is ImportMatrix. Each command in Maple may have certain options to specify exactly how you want the command to be executed. Again, the Help menu is a good place to find an explanation of what each command options are available. Please see Figure 1 (The .eps files for all of these figures are in the /mit/8.13/maple directory).
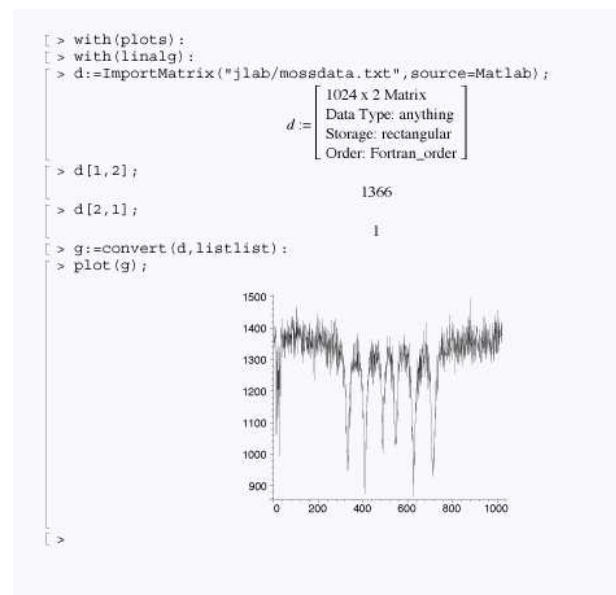

FIG. 1: Example of loading data from file and plotting in linalg package. Note some steps were taken to convert the raw text file to a usable format, namely turning the matrix into coordinate pairs. See Help menu for more on this.

This next example will get a similar result in the stats package. Again it depends on what you wish to use the data for as to which package is appropriate.

```
>   with(stats):
```

```
>   jj:=readdata("jlab/mossdata.txt",2):
```

```
[ > with(plots):
[ > with(linalg):
[ > f(x,y) := sin(x + y) * exp(x);
```
$$f(x,y) := \sin(x+y)\,\mathbf{e}^x$$
```
[ > plot3d(f(x,y),x=0..Pi, y=-Pi..Pi);
```

```
[ > g:=Int(f(x,y),x);
```
$$g := \int \sin(x+y)\,\mathbf{e}^x\,dx$$
```
[ > gg:=int(f(x,y),x);
```
$$gg := -\frac{1}{2}\mathbf{e}^x\cos(x+y)+\frac{1}{2}\sin(x+y)\,\mathbf{e}^x$$
```
[ > plot3d(gg,x=0..Pi,y=-Pi..Pi);
```

```
[ > diff(gg,x);
```
$$\sin(x+y)\,\mathbf{e}^x$$
```
[ >
```

FIG. 2: Example of integration and differentiation using Maple. Note the difference between the int and Int commands. See Maple Help menu for full details on options available.

SYMBOLIC INTEGRATION AND DIFFERENTIATION

Maple's capacity for symbolic manipulation is one of its strongest features. As such a moment spent on this

```
>   restart:
>   with(linalg):
>   g(r):= (r1/r-1)^(1/2);
```
$$g(r) := \sqrt{\frac{r1}{r}-1}$$
```
>   q:=Int(g(r),r=R..r1);
```
$$q := \int_{R}^{r1}\sqrt{\frac{r1}{r}-1}\,dr$$
```
    r=<r1 so r=r1*(sin(theta))^2
>   r=r1*(sin(theta))^2;
```
$$r = r1\sin(\theta)^2$$
```
>   q2:=subs(r=r1*(sin(theta))^2,q);
```
$$q2 := \int \sqrt{\frac{1}{\sin(\theta)^2}-1}\,dr1\sin(\theta)^2 = R..r1$$

topic is useful. For integration and differentiation the commands are int and diff respectively, and again there are many options available and again refer to the Help menu for the many details which are presented there. In Figure 2, there is an example of a very simple integration, differentiation, and graphical representation. This is meant merely to introduce the syntax required and on that note one important thing to keep in mind is that Maple can either evaluate the expression or it can display the actual integral symbolically. The int command, un-capitalized, will evaluate, and the Int command, capitalized, will display the integral. Maple can also evaluate such expressions numerically.

BINOMIAL EXPANSION

This next example demonstrates how to perform a binomial expansion in Maple.
```
>   sum('(binomial(4,n))*(x^n)' , n=0..100);
```
$$1 + 4x + 6x^2 + 4x^3 + x^4$$

the expression binomial(a,n) computes the binomial coefficient
```
>   expand((1+x)^4);
```
$$1 + 4x + 6x^2 + 4x^3 + x^4$$

EXAMPLE: BARRIER PENETRATION

This example is taken from French and Taylor's *An Introduction to Quantum Physics*. This example is taken from the section of this text on particle scattering and barrier penetration. Specifically, this example involves the computation of an integral involved in determining the penetrability of a barrier by an alpha particle. Hopefully this example will introduce commands and techniques which may be useful in computing other integrals you may encounter.

since d(r1*(sin(theta))^2) can be evaluated as:
```
>  diff(r1*sin(theta)^2,theta);
```
$$2\,r1\,\sin(\theta)\cos(\theta)$$

and since sin(theta)^2 is equal to 1/(csc(theta)^2) which we can confirm with trigsubs:
```
>  trigsubs((sin(theta)^2));
```
$$\left[\sin(\theta)^2,\ 1-\cos(\theta)^2,\ \frac{1}{2}-\frac{1}{2}\cos(2\,\theta),\ \sin(\theta)^2,\ \sin(\theta)^2,\ 4\sin(\frac{\theta}{2})^2\cos(\frac{\theta}{2})^2,\ \frac{1}{\csc(\theta)^2},\ \frac{1}{\csc(\theta)^2},\right.$$
$$\left.\frac{4\tan(\frac{\theta}{2})^2}{(1+\tan(\frac{\theta}{2})^2)^2},\ -\frac{1}{4}\,(e^{(\theta\,I)}-e^{(-I\,\theta)})^2\right]$$

we can now write the expressions as:
```
>  q4:=theta->((csc(theta))^2-1)^(1/2)*2*r1*sin(theta)*cos(theta);
```
$$q4:=\theta\rightarrow 2\,\sqrt{\csc(\theta)^2-1}\,r1\,\sin(\theta)\cos(\theta)$$

some more trig substitution:
```
>  trigsubs((csc(theta))^2);
```
$$\left[\csc(\theta)^2,\ 1+\cot(\theta)^2,\ \csc(\theta)^2,\ \csc(\theta)^2,\ \frac{1}{\sin(\theta)^2},\ \frac{1}{\sin(\theta)^2},\ \frac{1}{4}\frac{1}{\sin(\frac{\theta}{2})^2\cos(\frac{\theta}{2})^2},\right.$$
$$\left.\frac{1}{4}(1+\tan(\frac{\theta}{2})^2)^2\tan(\frac{\theta}{2})^2,\ -\frac{1}{4}\frac{1}{(e^{(\theta\,I)}-e^{(-I\,\theta)})^2}\right]$$
```
>  q5:=r1*cot(theta)*2*sin(theta)*cos(theta);
```
$$q5:=2\,r1\,\cot(\theta)\sin(\theta)\cos(\theta)$$

```
>  q6:=simplify(q5,trig);
```
$$q6:=2\,r1\,\cos(\theta)^2$$

So we have:
```
>  Int(g(r),r=R..r1)=Int(q6,theta=R..r1);
```
$$\int_{R}^{r1}\sqrt{\frac{r1}{r}-1}\,dr=\int_{R}^{r1}2\,r1\,\cos(\theta)^2\,d\theta$$
```
>  combine(simplify(int(q6,theta),trig));
```
$$\frac{1}{2}\,r1\,\sin(2\,\theta)+r1\,\theta$$

So we have:
```
>  Int(g(r),r=R..r1)=Eval(combine(simplify(int(q6,theta),trig)),r=R..r1) ;
```
$$\int_{R}^{r1}\sqrt{\frac{r1}{r}-1}\,dr=(\frac{1}{2}\,r1\,\sin(2\,\theta)+r1\,\theta)\Big|_{r\,=\,R..r1}$$

Now at r=r1:
```
>  r1=r1*(sin(theta))^2;
```
$$r1=r1\,\sin(\theta)^2$$

```
>  R1 := solve({r1 = r1*sin(theta)^2});
```
$$R1:=\{\theta=\theta,\ r1=0\},\ \{\theta=-\frac{\pi}{2},\ r1=r1\},\ \{\theta=\frac{\pi}{2},\ r1=r1\}$$

we know from above that when r=r1 then theta=Pi/2, want sin(2*theta)

```
>  th:=Pi/2;
```
$$th:=\frac{\pi}{2}$$

```
>  sin(2*th);
```
$$0$$

so we have at r=r1:
```
>   theta=Pi/2;sin(2*theta)=0;
```
$$\theta = \frac{\pi}{2}$$
$$\sin(2\,\theta) = 0$$

At r=R we have:
```
>   R=r1*(sin(theta))^2;
```
$$R = r1\,\sin(\theta)^2$$

```
>   R0 := solve({R = r1*sin(theta)^2},{theta});
```
$$R0 := \{\theta = \arcsin(\frac{\sqrt{r1\,R}}{r1})\},\ \{\theta = -\arcsin(\frac{\sqrt{r1\,R}}{r1})\}$$

```
>   th2:=arcsin(1/r1*(r1*R)^(1/2));
```
$$th2 := \arcsin(\frac{\sqrt{r1\,R}}{r1})$$

```
>   trigsubs(sin(2*theta));
```
$$[\sin(2\,\theta),\ -\sin(-2\,\theta),\ 2\cos(\theta)\sin(\theta),\ \frac{1}{\csc(2\,\theta)},\ -\frac{1}{\csc(-2\,\theta)},\ \frac{2\tan(\theta)}{1+\tan(\theta)^2},$$
$$\frac{-1}{2}I\,(e^{(2\,I\,\theta)} - e^{(-2\,I\,\theta)})]$$
```
>   2*sin(th2)*cos(th2);
```
$$\frac{2\,\sqrt{r1\,R}\,\sqrt{1 - \dfrac{R}{r1}}}{r1}$$

so at r=R we have:
```
>   theta=arcsin(1/r1*(r1*R)^(1/2));sin(2*theta)=2*sin(th2)*cos(th2);
```
$$\theta = \arcsin(\frac{\sqrt{r1\,R}}{r1})$$
$$\sin(2\,\theta) = \frac{2\,\sqrt{r1\,R}\,\sqrt{1 - \dfrac{R}{r1}}}{r1}$$

Therefore:
```
>   Int(g(r),r=R..r1)=(th+(1/2)*sin(2*th))-(th2+(1/2)*2*sin(th2)*cos(th2) );
```
$$\int_R^{r1} \sqrt{\frac{r1}{r} - 1}\,dr = \frac{\pi}{2} - \arcsin(\frac{\sqrt{r1\,R}}{r1}) - \frac{\sqrt{r1\,R}\,\sqrt{1 - \dfrac{R}{r1}}}{r1}$$

EXAMPLE: MULTIVARIABLE INTEGRATION

The next example shows how Maple can calculate a fairly complicated integral over several different vari-ables. Note how the Int (or int) command can be nested. In fact, using this method is easier than trying to integrate an expression one variable at a time.
```
>   restart;
>   with(linalg):
```
here is an example of a more complex integral
```
>   "<d(t)>"=(1/N)*Int(Int(Int(Int(R*(cos(phi))^2,yy=-30..30),xx=-20..20) ,y=-20..20),x=-30..30);
```
$$"{<}d(t){>}" = \frac{1}{N}\int_{-30}^{30}\int_{-20}^{20}\int_{-20}^{20}\int_{-30}^{30} R\cos(\phi)^2\,dyy\,dxx\,dy\,dx$$

where we take R to be:
```
>   R:=((x-xx)^2+(y-yy)^2+(d(t))^2)^(1/2);
```
$$R := \sqrt{(x - xx)^2 + (y - yy)^2 + d(t)^2}$$

and let d(t)=35.7cm
```
>   d(1):=35.7;
```
$$d(1) := 35.7$$

```
>   R:=((x-xx)^2+(y-yy)^2+(d(1))^2)^(1/2);
```
$$R := \sqrt{(x-xx)^2 + (y-yy)^2 + 1274.49}$$

and
```
>   (cos(phi))^2=((d(t))^2)/((x-xx)^2+(y-yy)^2+(d(t))^2);
```
$$\cos(\phi)^2 = \frac{\mathrm{d}(t)^2}{(x-xx)^2 + (y-yy)^2 + \mathrm{d}(t)^2}$$

```
>   N=Int(Int(Int(Int((cos(phi))^2,yy=-30..30),xx=-20..20),y=-20..20),x=- 30..30);
```
$$N = \int_{-30}^{30}\int_{-20}^{20}\int_{-20}^{20}\int_{-30}^{30} \cos(\phi)^2 \, dyy \, dxx \, dy \, dx$$

evaluate (cos(phi))^2 at d(t):
```
>   csphi:=((d(1))^2)/((x-xx)^2+(y-yy)^2+(d(1))^2);
```
$$csphi := \frac{1274.49}{(x-xx)^2 + (y-yy)^2 + 1274.49}$$

combine terms R and (cos(phi))^2 at d(t):
```
>   (R*csphi);
```
$$\frac{1274.49}{\sqrt{(x-xx)^2 + (y-yy)^2 + 1274.49}}$$

integrate over limits given:
```
>   ii:=evalf(Int(Int(Int(Int(R*csphi,yy=-30..30),xx=-20..20),y=-20..20), x=-30..30)):Int(Int(Int(Int(R*(cos
20..20),x=-30..30)=ii;
```
$$\int_{-30}^{30}\int_{-20}^{20}\int_{-20}^{20}\int_{-30}^{30} \sqrt{(x-xx)^2 + (y-yy)^2 + 1274.49}\cos(\phi)^2 \, dyy \, dxx \, dy \, dx = 0.1644305666\,10^9$$

need to evaluate 1/N term
```
>   NN:=1/(evalf(Int(Int(Int(Int(csphi,yy=-30..30),xx=-20..20),y=-20..20) ,x=-30..30)));
```
$$NN := 0.2656831317\,10^{-6}$$

combine these terms:
```
>   (1/N)*Int(Int(Int(Int(R*(cos(phi))^2,yy=-30..30),xx=-20..20),y=-20..2 0),x=-30..30)=NN*ii;
```
$$\frac{1}{N} \int_{-30}^{30}\int_{-20}^{20}\int_{-20}^{20}\int_{-30}^{30} \sqrt{(x-xx)^2 + (y-yy)^2 + 1274.49}\cos(\phi)^2 \, dyy \, dxx \, dy \, dx = 43.68642788$$

thus we have a value of approximately 43.7cm for <d(t)> when d(t)=35.7cm

APPENDIX A: USING MAPLE OUTPUT IN WRITTEN SUMMARIES

Maple gives the option of exporting you worksheet as a latex or .tex file. It will produce a file with the proper latex formatting to represent what you see while viewing your work in Maple. All plots will be exported individually as .eps files. If the name of the file you are exporting is **file.tex** then the name of the associated .eps files will be **file01.eps**, **file02.eps**, etc. for each of the plots included in your worksheet.

Similar to the set of macros, etc need for the Physics Review Letter format (specifically revtex4), so does Maple require a certain library of commands. This set of macros are defined in the file **maple2e.sty** and can be found in the /mit/8.13/maple locker.

Make sure you are aware as to what location Maple is exporting the latex file as well as the .eps files as they all need to be in the same directory in order to compile the document.

As an example, the following is the result of an export

as latex by Maple:
$$\mathrm{d}(t) = \frac{1}{N} \int_{-30}^{30}\int_{-20}^{20}\int_{-20}^{20}\int_{-30}^{30} R\cos(\phi)^2 \, dyy \, dxx \, dy \, dx$$

INSERTING WIDE MAPLE TEXT

Sometimes an equation or another line of Maple text is too wide to fit into the two column format. It is possible using the

```
begin{widetext}}
>   restart;
>   with(linalg):
```

```
Warning, the protected names norm and trace have been redefined and
unprotected
   here is an example of a more complex integral
>   "<d(t)>"=(1/N)*Int(Int(Int(Int(R*(cos(phi))^2,yy=-30..30),xx=-20..20) ,y=-20..20),x=-30..30);
```

command in Latex to interrupt the two column format. Latex will then bracket off part of the document and insert the wide Maple text. This technique was used it formatting the earlier Maple examples on barrier penetration and multivariable integration.

This formatting change will put the text before the inserted wide Maple text in two column format **above** the Maple and will continue below it in two column format, starting again on the left side off the page and wrapping to the right side after reaching the bottom.

The figure included (Figure 3) was inserted using the usual revtex4 **includegraphics** command, but the .eps figure was created by Maple in the export as Latex process. Maple assumes it is exporting the figures in Landscape mode, resulting in a rotated image. To correct this use the following command in Maple (Note that the height and width can be modified:

interface(plotoptions='portrait,height=40,width=40');

It is also possible to fit the Maple text into the two column format, but it is somewhat more time consuming. There is a latex command within Maple that will return latex code for the input given. For example >latex(sin(x)); will give: \sin \left(x \right) In this way you can directly input Maple as latex into your document as was done here:

$$\int_R^{r1} \sqrt{\frac{r1}{r} - 1}\, dr = 1/2\,\pi + 1234567 - \arcsin\left(\frac{\sqrt{r1\,R}}{r1}\right) - \sqrt{r1\,R}\sqrt{1 - \frac{R}{r1}}\, r1^{-1}$$
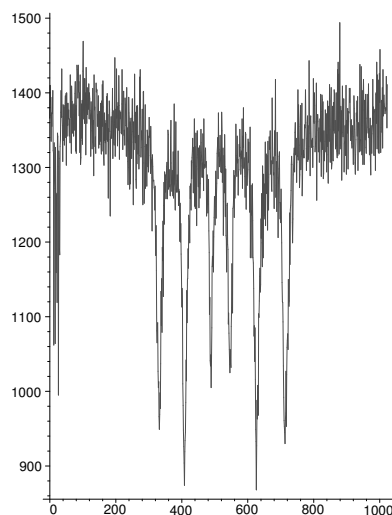

FIG. 3: This is an .eps file created by Maple, but inserted traditionally.