Topic 5
Power Transmission Elements I

Topics:
• Transmissions
• Pulleys
• Winches
• Belts & Cables
• Wheels
• Steering & Suspensions
• Clutches & Differentials
• Cams
• Shafts
• Couplings
Pulleys (Sheaves) are a most fundamental power transmission element

- Mechanical advantage
- Capstans
- Efficiency
- Tracking
- Mounting
Pulleys: Capstans

- A capstan is typically a fixed, or controlled rotation, body-of-revolution which a cable wraps around
  - A capstan can also form the basis for a band brake, where a band is anchored to a structure, and then wraps around a shaft…
- A cable wrapped around a capstan by \( \theta \) radians with coefficient of friction \( \mu \) and being held with a force \( F_{\text{hold}} \), can resist the pull of a cable with many times higher force \( F_{\text{pull}} \)
  \[
  F_{\text{hold}} = F_{\text{pull}} e^{-\mu \theta}
  \]
- If a belt or cable runs around a fixed shaft, then there is a lot of friction between the belt and the shaft, and the efficiency is low:

| Angle of wrap (degrees, radians) \( \theta \) | 720 | 6.283 |
| Coefficient of friction \( \mu \) | 0.3 | 0.2 | 0.1 |
| Holding force \( F_{\text{hold}} \) | 1 |
| Pulling force that can be resisted \( F_{\text{pull}} \) | 3.52 |
Winches

- A winch is a motorized drum that controls cable tension
  - A single wrap of cable on the drum requires a long or large diameter drum
    - Effective drum diameter and winch force capability remain constant
  - Multiple wraps of cable on the drum allow for more cable in a smaller place
    - Drum diameter and winch force capability vary
  - A fairleader is a device to control the input/output of the cable so it winds on the drum in an orderly fashion
    - The simplest design just uses smooth rounded static features
    - Vertical and horizontal roller designs reduce cable wear
Belts & Cables

- Applications and Engineering of Belts & Cables
  - Linear motion
  - Rotary motion
  - Crawler tracks
  - Manufacturing & Assembly

\[
\begin{align*}
\text{Power}_{\text{out}} &= \text{Power}_{\text{in}} \\
\text{speed}_{\text{out}} \cdot \text{Torque}_{\text{out}} &= \text{speed}_{\text{in}} \cdot \text{Torque}_{\text{in}} \\
\Gamma_{\text{out}} &= \omega_{\text{in}} \cdot \Gamma_{\text{in}} / \omega_{\text{out}} \\
V_{\text{belt}} &= R_{\text{out}} \cdot \omega_{\text{out}} \\
V_{\text{belt}} &= R_{\text{in}} \cdot \omega_{\text{in}}
\end{align*}
\]
Belts & Cables: Stress, Tension, & Center Distance

- Belts and cables are very robust elements, but they require engineering of 3 basic details:
  - Stress from wrapping a belt or cable around a pulley
  - Tension in the belt or cable
  - Center distance between pulleys

Bandstress.xls
By Alex Slocum
Last modified 4/22/02 by Alex Slocum
Stress in a flat belt wrapped around a pulley
Enters numbers in BOLD, Results in RED

| Belt parameters | | | |
|-----------------|-----------------|
| Thickness, t (mm) | 0.10 |
| Width, w (mm) | 5 |
| Modulus, E (N/mm²) | 2.00E+05 |
| Poisson ratio, v | 0.29 |

| Forces | | | |
|--------|-----------------|
| Load to be carried, F (N) | 10 |
| Belt stress, sigF (N/mm²) | 20.0 |

| Pulley wrap stresses | | | |
|----------------------|-----------------|
| Pulley diameter, D (mm) | 50 |
| Stress | 437 |
| Motor torque required (N-mm) | 250 |

| Capstan effect | | | |
|----------------|-----------------|
| Coefficient of friction, µ | 0.2 |
| Wrap angle, q (degrees) | 180 |
| Required pre-tension, pT (N) | 5.3 |
| Belt stress, sigT (N/mm²) | 10.6 |

| Total stress | 467 |
| Total strain | 0.23% |

Check: Tension in the belt (pluck it like a guitar string)

| Measured frequency of lateral vibration (hz) | 150 |
| Free-length (mm) | 300 |
| Density (g/mm³) | 7 |
| Mass per unit length (g/mm) | 0.004 |
| Tension (N) | 7.1 |
Belts & Cables: *Linear Motion*

- Belts & Cables are a very effective way to convert rotary to linear motion

![Diagram of belt and pulleys]

- The force $F$ in a belt with tension $T$ on a pulley of diameter $D$ that can be generated by the torque $\Gamma$ can be conservatively estimated by:
  - $F = \frac{2\Gamma}{D}$ for toothed belts, and $F = \frac{T\mu D}{2}$ for flat belts
  - A more exact model would consider the capstan effect
  - Play with *bandstress.xls*
  - The speed is simply
    - $V_{\text{linear}} = 2\omega_{\text{motor}} D$

- Belts run on pulleys
  - For flat belts, the pulleys must be crowned to prevent the belt from coming off the pulley due to pulley misalignment
  - Timing belt pulleys must also be carefully aligned to prevent premature failure
Belts: *Crawler Tracks*

- Treads only help when there is loose media or a surface into which they can dig
- Treads DO NOT help on smooth surfaces
  - Smooth surfaces often are covered with a dust layer, and sharp-groove treads can help
- Treads can be created by cutting angled slices from a rubber strip, and gluing them onto the belt surface
Belts & Cables: *Rotary Motion*

- Flat belts and cables (string drives) require higher tension to transmit torque
  - Conservatively, the belt needs to be held in tension equal to the desired torque divided by the coefficient of friction and the small pinion radius

- Vee-belts use the principle of self help:
  - Increased tension caused by power being transmitted, wedges the belt in a Vee-shaped pulley groove, so it can transmit more torque…

- *Synchronous Drives (timing or gear belts)* can transmit torque between shafts and also achieve a transmission ratio
  - They combine the positive timing action of gears with the flexibility, speed and low noise level of belts
    - For an in-depth discussion on synchronous drive design, see Stock Drive Products on-line tech library: [http://www.sdp-si.com/Sdptech_lib.htm](http://www.sdp-si.com/Sdptech_lib.htm)
Belts & Cables: *Manufacturing & Assembly*

- Pulleys can be grooved to provide a guide for a cable
- Two pulleys’ axes of rotation can never be perfectly parallel, so a flat belt will want to drift off (tracking)
  - Pulleys must be crowned (round profile) to keep a belt from walking off
    - On a concave surface, the side with more belt in contact will cause the belt to drift further to that side until it falls off
    - A flat pulley is at best neutrally stable
    - Great, OK, bad, & horrid pulleys:
  - Tension must be maintained either by proper pulley center distance and belt elasticity, or a mechanism to tension the pulleys or a pulley that pushes sideways on the belt
Wheels

- Traction & Controllability
- Size, Torque & Contact Pressure
- Manufacturing & Mounting
Wheels: *Traction & Controllability*

- Friction is independent of surface finish, unless there is physical engagement
  - Like gear teeth on a rack, sandpaper grips carpet, plastic…

Two-Wheel Drive vehicles are simple to build
- Against a 4-Wheel Drive or a Tracked vehicle, they lose

- The smaller the wheel:
  - The greater the traction force the motors can cause
  - The slower the vehicle per unit input to the controller!

This was a very successful 2WD vehicle with sandpaper covered rear driven wheels

A sandpaper covered platform enabled this machine to get to the other side every time!
Wheels: *Size, Torque and Contact Pressure*

- How big must a wheel be to make sure it can overcome obstacles?
  - Yet the smaller the wheel diameter, the less the torque to turn the shaft
    - Ideally, the wheel just slips when maximum torque is applied
      - This keeps you from stalling the motor and potentially burning it out
    - And the smaller the diameter, the higher the contact pressure, and the greater the wear
  - Hertz Contact stress theory can determine if wheels are too heavily loaded!

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Wheels: Manufacturing & Mounting

- A positive engagement is required to effectively transmit torque from a shaft to a wheel
  - A hexagon hole can be broached into pulleys and wheels
  - Square and hexagon shafts can be threaded!
  - A round shaft’s ends can be machined square to fit into a square-broached hole
- The wheel-on-shaft subassembly should overhang the gearbox as little as possible!
  - If the overhung shaft has a lot of compliance, an “outrigger” bearing can be added without over constraining the shaft, if it confines the shaft to its nominal central position
- Beware of system deformations and the fact that they can take up all the clearance in a system and cause it to bind

Too much overhang (remember Saint-Venant!). Consider the use of an outrigger bearing, or mount the wheel on a properly supported shaft and drive the shaft with the motor using a coupling
Wheels: *Steering & Suspensions*

- A *steering linkage* ensures that all the vehicle’s wheels axes of rotation meet at a single point, which is the instant center (center of the turning radius)
  - The Ackerman linkage is a 6-bar linkage developed for automotive steering
- A *suspension linkage* ensures that all wheels keep contact with uneven ground
  - The linkage allows vertical motion of the wheel, but maintains wheel alignment
    - Advanced suspensions may cant (tilt) wheels to help with high speed cornering
    - Recall page 3-26 and the concept of elastically averaged design
- Vehicles with crawler tracks (e.g., bulldozers) typically allow at least one of the Crawler tracks to pivot to help ensure the treads to maintain ground contact
How to transmit power to the wheels and enable them to move with respect to the vehicle to absorb shocks and move across uneven surfaces?

How to transmit power to the wheels and enable them to steer?

Wheels: Steering & Suspensions
Wheels: *Clutches & Differentials*

- Clutches and differentials allow for differential velocities between rotating shafts
  - A classic example is when a car makes a turn, all the wheels have the same instant center, but the radius from each wheel to the instant center is different
    - Thus each wheel travels a different distance along an arc in the same time, and a *differential* is needed to enable the speed difference to occur
      - However, if one wheel is on ice, and another wheel is on dry pavement, the wheel on ice will spin
  - Limited slip differentials typically use spring-loaded clutch plates to ensure some torque is always transmitted to both wheels
  - Other designs use a centrifugal force to engage locking features
  - New designs use electromagnetic clutches to optimize torque transfer to the wheels
  - Clutches can be activated mechanically or electrically to control the torque transmitted between shafts, while allowing differential velocity to occur (see page 6-17)

Electric clutch allows limited slip across the differential, or the differential to lock forcing both shafts to be driven. See: http://automotive.eaton.com/product/traction_stability/intellitraxx.html
**Cams**

- A *cam* is a rotating shape whose angular motion is converted into output motion by a *cam follower* which rides on the cam surface
  - Rolling elements provide the highest degree of efficiency, but they take up more space
  - Sliding elements are very compact, and can be efficient if the speed is high enough to maintain oil film lubrication
- A *cam follower* is a rolling or sliding contact element that follows the contour of a surface and transmits the motion to a mechanism
- A *cam profile* can be designed to create corresponding *acceleration*, *velocity*, *position*, and *dwell* profiles in a mechanism (e.g., an engine valve)
  - Specialized cam design software can create virtually any type of cam and provide all required engineering data
  - See url for example [http://www.camnetics.com/](http://www.camnetics.com/)

![Diagram of cam and cam follower](image)

Radial follower  Offset follower  Swinging follower  Roller  Flat face  Spherical  Knife edge
Shafts

- Shafts transmit rotary and linear power via motors, leadscrews, pistons…
  - Shafts are one of the most common machine elements
- The primary design issues are:
  - Loading conditions and stress concentrations
  - Mounting
  - Component Attachment

pneumatic or hydraulic cylinder

Leadscrew driven carriage

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Shafts: *Axial Loads*

<table>
<thead>
<tr>
<th>Shaft loaded by axial force $F$</th>
<th>Stress</th>
<th>Stress concentration factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td>$D$</td>
<td>$r$</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td>$D$</td>
<td>$r$</td>
</tr>
<tr>
<td><strong>Case 3</strong></td>
<td>$D$</td>
<td>$d$</td>
</tr>
<tr>
<td><strong>Case 4</strong></td>
<td>$D$</td>
<td>$r$</td>
</tr>
</tbody>
</table>

$B = D/4$, $t = D/8$
### Shafts: Torsional loads

<table>
<thead>
<tr>
<th>Shaft loaded by Torque $\Gamma$</th>
<th>Stress</th>
<th>Stress concentration factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$\tau = \frac{K_t 16\Gamma}{\pi d^3}$</td>
<td>$K_t = 1 + \frac{(r/d)^{-0.3} - (D/d)^{-0.2}}{13 + 0.3/(D/d - 1)}$</td>
</tr>
<tr>
<td>D $\begin{array}{c} r \ d \end{array}$</td>
<td>$\tau = \frac{K_t 16\Gamma}{\pi d^3}$</td>
<td>$K_t = 1 + \frac{(r/d)^{-0.609} - (D/d)^{-0.146}}{5 + 3.73/(D/d - 2r - 1)^{0.252}}$</td>
</tr>
<tr>
<td>Case 2</td>
<td>$\tau = \frac{K_t \Gamma}{\pi D^3 / 16 - d D^2 / 6}$</td>
<td>$K_t = 1 + 1.47(d/D)^{-0.197}$</td>
</tr>
</tbody>
</table>
| D $\begin{array}{c} \text{Corner r} \\ \text{Width b} \\ \text{Depth t} \end{array}$ | $\tau = \frac{K_t 16\Gamma}{\pi D^3}$ | $K_t \approx 1.5$  
$B=D/4, \ t=D/8$ |
Shafts: *Bending Loads*

<table>
<thead>
<tr>
<th>Case</th>
<th>Shaft loaded by Bending moment M</th>
<th>Stress</th>
<th>Stress concentration factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>![Shaft D] r, d</td>
<td>( \sigma = \frac{K \cdot 32M}{\pi d^3} )</td>
<td>( K_t = 1 + \frac{(r / d)^{-0.73 - 0.42(D/d - 1)}}{5 + 4.38 / (D / d - 1)^{0.16}} )</td>
</tr>
<tr>
<td>2</td>
<td>![Shaft D] r, d</td>
<td>( \sigma = \frac{K \cdot 32M}{\pi d^3} )</td>
<td>( K_t = 1 + \frac{(r / d)^{-0.59 - 0.184(d/(d-2r)-1)}}{5 + 0.081/(d/(d-2r)-1)} )</td>
</tr>
<tr>
<td>3</td>
<td>![Shaft D] d</td>
<td>( \sigma = \frac{K \cdot \Gamma}{\pi D^3 / 32 - d D^2 / 6} )</td>
<td>( K_t \approx 1 + 0.65 / (d / D)^{0.275} )</td>
</tr>
<tr>
<td>4</td>
<td>![Shaft D] r, Width b, Depth t</td>
<td>( \sigma = \frac{K \cdot 32M}{\pi D^3} )</td>
<td>( K_t = 1 + \frac{(r / d)^{-0.66}}{11.14} )</td>
</tr>
</tbody>
</table>

For Corner r, B=D/4, t=D/8

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Shafts: *Mounting & Stability*

- *Abbe* and *Saint-Venant’s Principles* and *Exact Constraint Design* must be carefully considered when designing the mounting of shafts.
- Design the system to accommodate misalignment:
  - Allow for clearance between bearing and shaft to accommodate misalignment.
  - Use flexible couplings or self-aligning (spherical) bearings.
  - Use designs that automatically accommodate misalignment.
  - Line-bore holes at the same time through mounting plates.
Shafts: *Component Attachment*

- The primary functional requirements for attaching a component to a shaft (in order of goodness) are:
  - Prevent the component from slipping
    - Press-fit: Best torque transmission, but can be difficult to properly manufacture
    - A variation is when the shaft and hub are ground to have a mating lobed fit
    - Spline: Excellent torque transmission, but can be expensive
    - Circumferential clamp: Very good torque transmission, low stress concentration, modest price
    - Keyway: Very good torque transmission, modest stress concentration, low cost
    - Pinned shaft: Good torque transmission, modest stress concentration, low cost
    - Setscrew: Your worst nightmare!
  - Minimize stress concentration
    - Raised diameter with radiused corner
      - Requires a lot of material to be removed from a long shaft
      - Rarely required in simple machines
Shafts: *Press-Fits*

- Press-fits are potentially the most reliable and strongest form of connection between components:
  - The spreadsheet *press_fit.xls* includes the effects of tolerance, differential thermal expansion, and expansion due to high speed rotation (of solid members)

<table>
<thead>
<tr>
<th>Loads</th>
<th>Required interference parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque to be transmitted, Torque (N-mm)</td>
<td>100 Minimum required interface pressure, rPI (N/mm²) 0.8017</td>
</tr>
<tr>
<td>Axial force to be transmitted, Force (N)</td>
<td>100 Add Poisson diametral interference, ddp (mm) 6.92E-06</td>
</tr>
<tr>
<td>Coefficient of friction, µ µµ</td>
<td>0.1 Add differential thermal expansion diameter, ted (mm) 7.50E-04</td>
</tr>
<tr>
<td>Operating temperature, ø (°C)</td>
<td>25 Add outer body rotating radial displacement, robd (mm) 1.80E-06</td>
</tr>
<tr>
<td>Stress concentration factor at interface edge, scf</td>
<td>2 Subtract inner body rotating radial displacement, ribd (mm) 4.17E-07</td>
</tr>
<tr>
<td>Rotation speed, omega (rpm)</td>
<td>100 Total additional interference amount to be added to ibod, addi (mm) 0.0008</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outer body input parameters</th>
<th>Interference fit calculations (assumes addi has been added to ibod)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter, obod (mm)</td>
<td>20 Maximum interference, maxdelta (mm) 0.0078</td>
</tr>
<tr>
<td>Interface diameter, obid (mm)</td>
<td>15 Maximum resulting interface pressure, maxIP (N/mm²) 26.36</td>
</tr>
<tr>
<td>Plus tolerance, obptol (mm)</td>
<td>0 Minimum interference, mandelta (mm) 0.0038</td>
</tr>
<tr>
<td>Minus tolerance, obmtol (mm)</td>
<td>0.002 Minimum resulting interface pressure, minIP (N/mm²) 12.77</td>
</tr>
<tr>
<td>Modulus of elasticity, obe (N/mm²)</td>
<td>200000 Minimum safety margin (min obtained pressure/required pressure) 15.9</td>
</tr>
<tr>
<td>Yield strength, obsy (N/mm²)</td>
<td>300 Outer body material stresses at maximum interface pressure</td>
</tr>
<tr>
<td>Poisson's ratio, obn</td>
<td>0.29 Radial displacement of inner surface (mm) 0.0038</td>
</tr>
<tr>
<td>Coefficient of thermal expansion, obcte (1/°C)</td>
<td>2.00E-05 Max radial press-fit stress is at ID, obsr (N/mm²) -26.36</td>
</tr>
<tr>
<td>Density, obrho (g/mm³)</td>
<td>7.9 Max circumferential press-fit stress is at ID, obsc(N/mm²) 94.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inner body input parameters</th>
<th>Axial stress from applied axial Force, obsz (N/mm²) 2.29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engagement length, L (mm)</td>
<td>30 Max shear stress (at ID) from applied Torque, obtau (N/mm²) 43.65</td>
</tr>
<tr>
<td>Outside diameter, ibod (mm)</td>
<td>15.003 Max circumferential centrifugal stress is at ID, obcc(N/mm²) 0.08</td>
</tr>
<tr>
<td>Plus tolerance, ibptol (mm)</td>
<td>0.002 Von Mises stress at ID, obvm (N/mm²) 132.74</td>
</tr>
<tr>
<td>Minus tolerance, ibmtol (mm)</td>
<td>0 Resulting safety factor (Yield stress)/(scf*Von Mises stress) 1.13</td>
</tr>
<tr>
<td>Inside diameter, ibid (mm)</td>
<td>5 Inner body material stresses at maximum interface pressure</td>
</tr>
</tbody>
</table>

| Modulus of elasticity, ibe (N/mm²) | 200000 Radial displacement of outer surface (mm) -0.0009 |
| Yield strength, ibsy (N/mm²)       | 300 Max radial press-fit stress is at OD, ibsr (N/mm²) -26.36 |
| Poisson's ratio, ibn                | 0.29 Max circumferential press-fit stress is at OD, ibsc(N/mm²) -32.94 |
| Coefficient of thermal expansion, ibcte (1/°C) | 1.00E-05 Axial stress from applied axial Force, ibsz (N/mm²) 2.00 |
| Density, ibrho (g/mm³)             | 7.9 Max shear stress (at OD) from applied Torque, ibtau (N/mm²) 38.19 |

<table>
<thead>
<tr>
<th>Shrink-fit design</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired assembly clearance at deltaT, ddt (mm)</td>
<td>0.005 Von Mises stress at OD, ibvm (N/mm²) 73.54</td>
</tr>
<tr>
<td>Required differential temperature if heating outer body, robd (°C)</td>
<td>43 Resulting safety factor (Yield stress)/(scf*Von Mises stress) 2.04</td>
</tr>
<tr>
<td>Required differential temperature if cooling inner body, ribdt (°C)</td>
<td>85</td>
</tr>
</tbody>
</table>
• No two moving components can be perfectly aligned
  – To prevent over constraint from destroying components and robbing your system of power, use couplings
• Identify the degrees of freedom desired, the accuracy (repeatability) needed, and the load capacity and stiffness required in the direction of force or torque transmission
  – Select either a sliding, flexural, or rolling element coupling
Couplings: Cheap & Easy Example

- What about in a robot design contest where two shafts may be linearly misaligned axially, vertically and horizontally, and angularly misaligned?
  - How can you design a simple one-piece coupling to enable one shaft to transmit torque to the other shaft?
    - Can the coupling be made from plastic tube to reduce shock loads?
    - Would O-rings be useful to nominally center it?

Critical: Note the clearance between the coupling bore and both shafts!