1.011 Project Evaluation

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Lecture 30 -- Choosing Mutually Exclusive Alternatives, Review

After class, a student raised a question with regards to choosing between mutually exclusive alternatives. As a result of the ensuing discussion, the following case was developed to illustrate the concepts more clearly.

Estimation of Internal Rate of Return

Internal Rate of Return is a discount rate such that the net present value of the project go to zero. For a simple project (such as construction of a building) that generates a steady stream of fixed income, the IRR can be estimated simply by dividing the annuity by the initial investment. In other words, if I invest $100 and the interest payment is $10 per year, the IRR is 10%.

Estimation of Present Worth of Annuity

Present Worth of Annuity is the equivalent cash value of a stream of future cash flows. This is dependent on the discount rate (i.e. risk of that cash flow not actually appearing), and the number of interest periods. This is normally obtained using the capital worth formula:

$$(P/A) = \frac{(1+i)^N - 1}{i(1+i)^N}$$

As N approaches infinity, the term $((1+i)^N - 1) / (1+i)^N$ tends towards one; thus, the capital worth formula becomes $(P/A) = 1/i$. In practice, for large projects that have a high discount rate (about 10%) and an asset life of more than about 30 years, one can obtain the present worth of annuity simply by dividing the discount rate.

In the discussion that follows, we shall make use of the above approximations.

Incremental Project Evaluation

Incremental Project Evaluation Example

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MARR (Discount Rate) 10%

Case	Investmer	Income	IRR	ΔIRR	PV Rev	NPV	Accept?	Prefer?
Base	1	0.11	11.0%		1.1	0.10	Yes	
1	1.4	0.12	8.6%		1.2	-0.20		
2A	1.5	0.15	10.0%	8.0%	1.5	0.00		
2B	1.5	0.155	10.3%	9.0%	1.55	0.05		
2C	1.5	0.163	10.9%	10.6%	1.63	0.13	Yes	Yes
2D	1.6	0.17	10.6%	10.0%	1.7	0.10	Yes	

In this question, you are asked to evaluate a series of alternatives, some of which are mutually exclusive. The standard method of assessing such project is to evaluate the internal rate of return, and choose only

those projects that have IRR that is larger than the minimum acceptable rate of return (MARR). When evaluating incremental projects, only projects that have an incremental portion that generate an incremental IRR larger than the MARR should be selected. If only IRR is used, too small a project may be selected. Selecting the project with the highest Net Present Value (NPV) will always identify the preferred project.

The base alternative in this example generates an internal rate of return of 11% (you can estimate this simply by dividing $0.11 by $1), which is larger than the MARR of 10%, thus the project is acceptable. You can also estimate the project's NPV by dividing $0.11 by the discount rate of 10%, then subtracting the initial investment of $1. Thus:

$$\$0.11/0.10 - \$1 = \$0.10$$

Since NPV is positive, this project is acceptable. On the other hand, alterative 1 is not acceptable, since IRR is less than MARR. Using the NPV method, we also find that NPV is negative.

Alternatives 2A, 2B, 2C and 2D are mutually exclusive. This means if we should choose project 2A, we would not be able to carry out projects 2B, 2C, or 2D. The classic example is what to do with the ground-floor public space in a building:

Case	
Base	10-floor building, no frills
1	10-floor building, Simmons Hall style
2A	15-floor building, no frills
2B	15-floor building, first floor gym
2C	15-floor building, first floor shops
2D	15-floor building, first floor pool

In the proposed 15-floor building, the first floor could only be used for a single purpose: either residential, a gym, shops, or a pool. However, alternative 1 is not mutually exclusive with the others, since it is possible to build a Simmons-style 15-floor building independently of what use is made of the first floor space.

Thus, in evaluating the alternatives by the IRR method, we should choose the project with the largest incremental IRR over the largest acceptable case so far (i.e., the base alternative, in this case). Since alternative 1 is not acceptable, we calculate the incremental IRR based on the base alternative. The incremental IRR is calculated by dividing the incremental revenue by the incremental investment. If the incremental IRR is larger than the MARR, the alternative is acceptable.

Note that although alternative 2A and 2B have an IRR that is larger than the MARR, the incremental portion has an IRR less than the MARR, thus these are not acceptable alternatives. Alternatives 2C and 2D have an IRR that is still smaller than the base alternative, but they are acceptable alternatives because the incremental IRR is larger than the MARR.

Using the NPV method, for independent projects, we should invest in every project where the NPV is greater than zero. However, with mutually exclusive projects, we should choose to invest in the project that generates the highest NPV. Alternatives 2A and 2B generates a lower NPV than the base case, thus we would choose the base alternative over 2A and 2B. Alternative 2C generates the highest NPV, thus is the preferred project. Alternative 2D generates the same NPV as the base alternative, we are therefore indifferent between the base alternative and 2D. This is because the incremental portion of the project breaks-even exactly (incremental IRR = MARR).

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