Evolution of a Non-cohesive Granular Bed Subject to a Succession of Erosive Turbidity Currents

by

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ABSTRACT

An experiment was performed to study the evolution of a non-cohesive granular bed subject to a succession of erosive turbidity currents to evaluate the applicability of a proposed interface model that prescribes a relationship between the bed shear stress and the local bed elevation and local bed slope. This study considered only the relationship between the local bed elevation and the shear stress by using non-cohesive plastic particles as bed sediment that was laid down in a subaqueous straight channel.

The elevation of the plastic bed was measured before and after it was subjected to two erosive salt water turbidity currents using a high resolution displacement laser mapping system. These changes in elevation of the sediment bed are compared to determine whether a correlation exists between local bed elevation and bed erosion. The results from this experiment display a weak correlation between the erosion of the plastic bed and the initial local bed elevation when the initial elevation height is small relative to the thickness of the turbidity currents. However, a strong correlation exists when the initial elevation height is approximately 2% to 6% of the turbidity current thickness. The strength of this correlation also depends upon the window size of the analysis which is shown to be associated with the downstream variability in bed roughness.

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1.0 Introduction

A study of the evolution of subaqueous bed forms [*Jerolmack and Mohrig*, 2005] proposes a simple interface model that approximates the influence of shear stresses on the dynamics of a sand bed. This model ignores the characterization of the fluid flow and, instead, considers the relationship between the sediment flux and the shear stress. This approach has also been considered by others as a local growth model [*Barbasi and Stanley*, 1995; *Dodds and Rothman*, 2000]. The applicability of the interface model to the erosion of a non-cohesive sediment bed by a succession of erosive turbidity currents is investigated in the present analysis.

The model relates the shear stress at the sediment bed to the local elevation and slope of the bed by first considering the 1D conservation of mass equation:

$$\frac{\partial \eta}{\partial t} = -\frac{1}{(1-p)}\frac{\partial q_s}{\partial x} \tag{1}$$

where t is time, x is the downstream coordinate, p is porosity, q_s is the sediment flux, and η is the local bed elevation. A power law relationship links the flux of the sediment to the shear stress on the bed:

$$q_s = m\tau^n \tag{2}$$

where τ is the bed shear stress, m varies between 5.7 and 12 depending on the rate of sediment transport [*Wiberg and Smith*, 1989] and n varies from 1.5 [*Meyer-Peter and Muller*, 1948] to 2.5 [*Fernandez-Luque and van Beek*, 1976]. In essence, the fluid dynamics is parameterized as the shear stress. The shear stress is formulated as a function of the bed elevation and slope using

$$\tau(x) = \tau_b\left(1 + A\frac{\eta}{h} + B\frac{\partial \eta}{\partial x}\right) \tag{3}$$

where τ_b is the boundary shear stress, h is the distance between the average elevation of the bed and the water-air interface and A and B are constant coefficients. In equation 3, the first term is a background stress, the second term is the local elevation dependence and the last term is the local slope dependence.

In this study, the experiment is used to only address the model relationship between the shear stresses observed on a sediment bed and the local elevation (represented by the second term in equation 3) by examining the correlation between the bed elevation and the bed erosion. The tests for this experiment were executed by depositing a bed of non-cohesive plastic sediment over a straight channel and subjecting the bed to two erosive salt water turbidity currents. Changes of the bed were observed by measuring the local elevation of the initial bed prior to a flow and comparing it to the profiles of the bed after each erosive turbidity current flow.

The applicability of the interface model depends on the magnitude of the correlation between the erosion of the plastic bed and the initial elevation of the bed surface. A strong correlation between these two variables would support the hypothesis (equation 3) that the shear stress and erosion on a bed is closely associated with the initial local bed elevation. This would suggest that the erosion of a mobile sediment bed could be characterized by its local surface conditions and that neglecting the dynamics of the overriding flow and treating the evolution of the bed using a simple interface model may be valid.

The third term in equation 3 also needs to be analyzed using the data collected from these experiments to determine the dependence of the local bed slopes on the erosion of the bed. However, characterization of local slope is not as straightforward as that for elevation and time did not allow for this analysis. A combination of these two analyses is needed to fully test the accuracy of the proposed model and the predictions regarding the evolution of a mobile sediment bed.

2.0 Apparatus and Methods

The straight channel was built in sections, as shown in the plan view of Figure 1, with consistent width and depth dimensions throughout the entire length. A 15:1 mixture of sand and mortar was used for the physical composition of the channel (see Figure 2). After the channel was built, the tank was filled to a depth of 65 cm with fresh water, submerging the channel.

After the tank is filled with water, a Keyence LK-503 long range displacement laser was used to map the channel. By integrating the laser with a two-dimensional servo motor driven carriage, bathymetry maps were produced. This carriage moved the laser across and down the channel at a constant velocity while a computer collected the stored the bathymetry data. The submerged channel was mapped on a 0.2 x 0.2 cm grid. The mapping system was set to record 251 bathymetry points in the cross-stream direction (a single section) and 2079 sections downstream to create 521,829 total data points. The laser can measure over a total vertical range of 50 cm with a vertical resolution of approximately 150 microns. To map the initial plastic bed and the surface after each of the successive turbidity current flows, the same procedure is used. The bathymetry maps of the successive surfaces are then differenced to produce bed thickness maps.

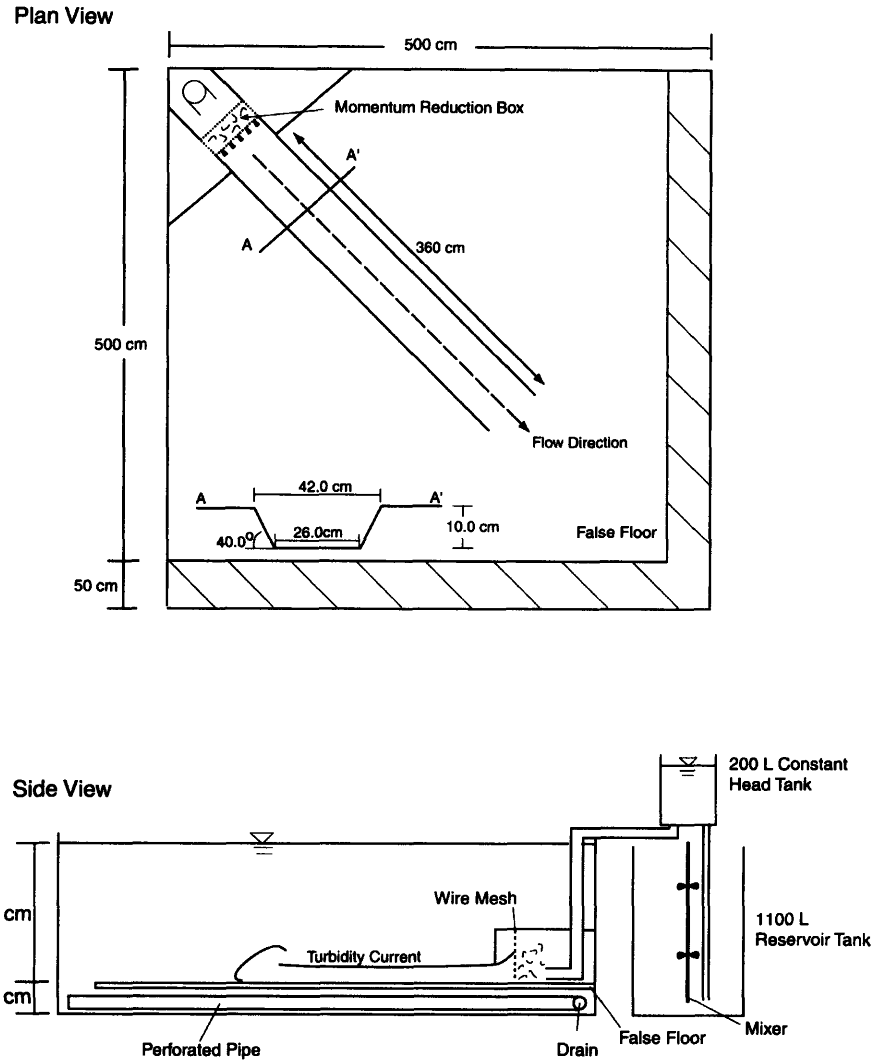

Figure 1: The plan and side views of the tank and channel system.

The initial sediment bed laid down in the channel was composed of non-cohesive clear cut plastic which has a specific gravity of 1.3. The grain size distribution ranges from 5 microns to 180 microns with a mean grain size of approximately 90 microns. To form the initial bed, an 1100L reservoir tank was used to create a mixture of fresh water, calcium salt and clear cut plastic. This mixture was then pumped to a separate 300L overhead tank which was used to create a constant head for the experiment duration. Once the tank had filled to 200L, the mixture was released at a low discharge into the momentum reduction box submerged inside the tank. As the mixture flowed through the wire mesh inside the box, the forward momentum was reduced, producing a turbidity current. This current was then driven through the channel due to the excess density provided by the addition of salt and clear cut plastic. The slow grain settling velocities of the plastic allowed it to be easily transported by the turbidity current down the entire channel. Once the current exited the straight channel it was allowed to fall over the edge of the false floor to avoid any reflections that could have been produced from the tank's sidewalls. A drainage system located underneath the false floor was used to drain the current from of the tank system. By continuously sending the turbidity current through the channel for a period of about six hours, a plastic sediment bed was laid down. The channel is then mapped using the laser system described above. The initial bed thickness varied in the downstream direction because more plastic was deposited near the source than near the channel exit, as shown in the upper plot of Figure 5.

Figure 2: Image of the channel after two turbidity current flows (the surrounding tank water has been drained). The current flow direction is from the bottom to the top of the image. The remolded plastic bed appears gray and the channel banks brown. The plastic bed was initially thicker towards the bottom of the image.

The two erosive salt flows were created in the same manner as that described to deposit the initial plastic bed, but without any plastic sediment. Measured amounts of fresh water and salt were mixed in the 1100L tank to create a fluid with a 3.0% by volume excess density relative to the clear water. The mixture was pumped into the constant head tank to a volume of 200L. The fluid mixture was then released into the momentum reduction box at a higher discharge than that used to lay down the initial bed. The current was driven down the channel by its excess density compared to that of the surrounding water. The input conditions were held constant for both turbidity current flows. Measurements showed that both turbidity currents had an average thickness of 10 cm and a maximum velocity of 15 cm/s. As a turbidity current exited the channel it fell over the false floor edge and was drained out of the tank using the drainage system. Each erosion flow was continuously run for a period of approximately nine minutes. The channel was then laser mapped to determine the change of plastic bed thickness along the entire channel length. A difference map following the first erosive turbidity current flow is shown in Figure 3. The difference map highlights the redistribution of sediment along the channel length.

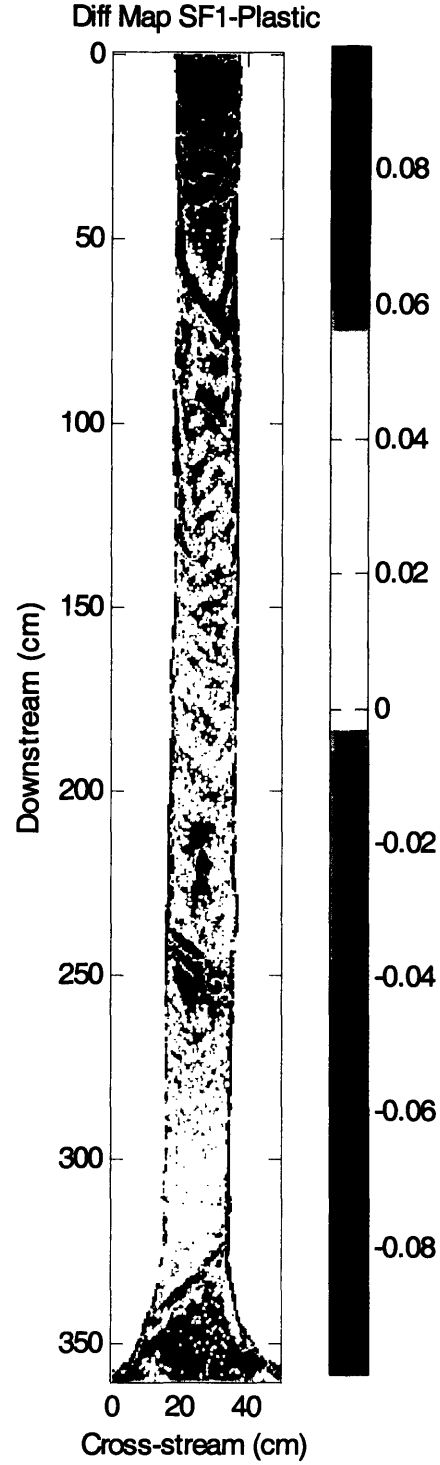

Figure 3: Difference map following the first erosive turbidity current flow. Positive values represent an increase in deposit thickness and negative values represent a decrease in deposit thickness. Note the strong negative values over the first 40 cm. Turbidity current flow was from top to bottom.

Suspended sediment samples were also collected for each turbidity current using a siphoning procedure. A vertical array of 12 gravity driven siphons was positioned 225 cm downstream in the channel (see Figure 3 for location). The siphons were spaced 0.5 cm apart with the lowest siphon approximately 0.5 cm above the plastic bed. The siphons were opened for approximately two minutes after the turbidity current front had traveled passed the siphons. Fluid from each siphon was collected in separate containers. The sediment present in each container was separated from the salt water component using filter paper. After allowing the filter paper to dry, the captured sediment was analyzed using a Horiba Laser Particle Size Analyzer. Figure 4 shows the vertical grain size distribution from the first erosive turbidity current.

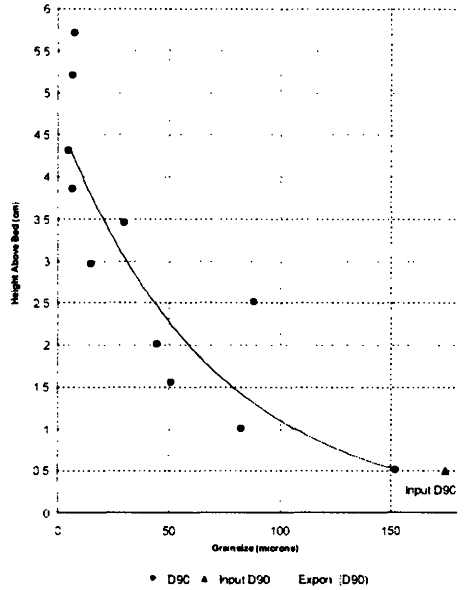

Figure 4: Grain size distribution of the clear cut plastic from the first erosive turbidity current.

The points labeled D90 represent the diameter of the grain size fraction larger than 90% of the deposited grains at the respective height above the bed in the erosive turbidity current. D90 from the mixing reservoir was also analyzed for comparison and is labeled Input D90. The profile shows that D90 is approximately constant at about 4 cm height above the plastic bed and that the coarse fraction of the input grain sizes is not suspended at 0.5 cm height above the bed since D90 there is less than the input D90. The presence of sediment in the samples confirms that plastic was eroded from the bed, vertically mixed and transported downstream.

3.0 Results and Analysis

The following analysis focuses on a single downstream section located along the approximate channel centerline at 24.8 cm (see Figure 3). Figure 5 shows the downstream thickness profile of the initial plastic bed, the thickness profile following the first erosive turbidity current flow, and the difference of these two profiles respectively. The upper plot in Figure 5 shows that the initial bed thickness varied in the downstream direction because more plastic was deposited near the source than near the channel exit. The maximum thickness is approximately 0.5 cm. The profile following the first erosive turbidity current flow shows that the first 40 cm of the bed experiences the greatest change in elevation and surface roughness (Figure 5, middle plot). The difference plot (Figure 5, lower plot) shows that this section of the initial bed undergoes considerable erosion relative to the remaining downstream section since the differences are largely less than zero. Roughness in this section develops as a series of sediment elevations that fluctuate between 0 cm and 0.5 cm. Rough areas also developed within the profile between 50 cm and 100 cm and between 200 cm and 250 cm. The amplitude of the roughness in both of these sections is much less than that in the first 40 cm. The downstream slope present along the initial plastic bed profile between 40 cm and 360 cm (Figure 5, upper plot) is preserved after the first turbidity current has passed. This is illustrated in the difference plot (Figure 5, lower plot) because the change in thickness is approximately zero over this range.

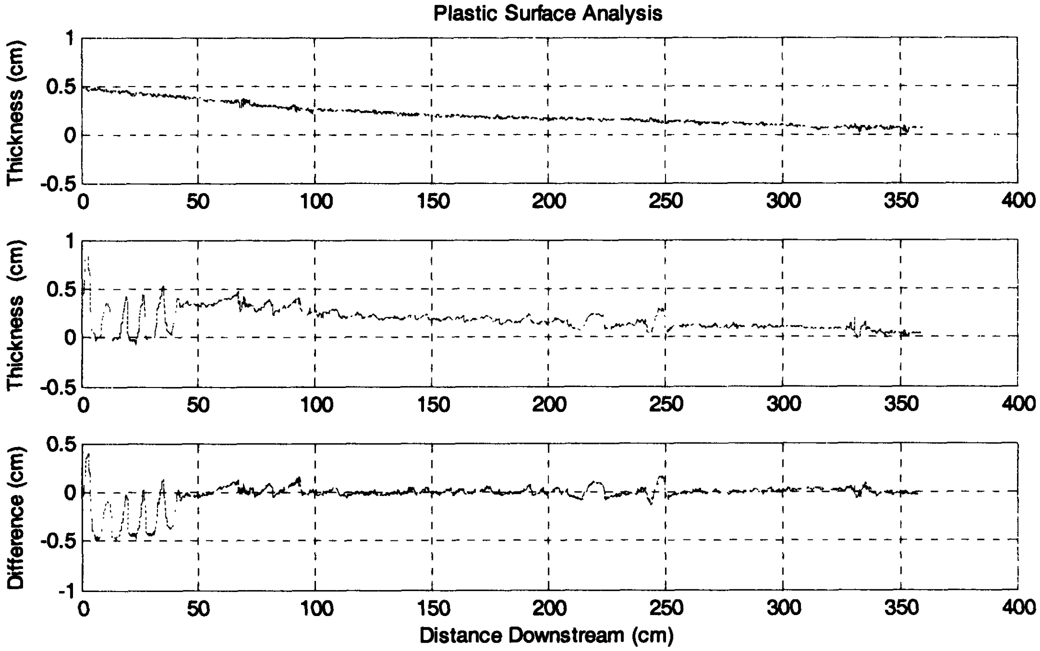

Figure 5: From upper to lower: downstream profile of the plastic bed thickness; profile of bed thickness after the first erosive turbidity current flow; and the difference between the two profiles respectively. Downstream profiles are located along the approximate channel centerline at 24.8 cm (see Figure 3).

Local elevations along the profile were determined by dividing the profile into sub-windows and applying a linear fit to the local data. In this way, relative (to the linear fit) local elevations are defined that can be compared to the change in elevations or erosion to determine whether a correlation exists. Data deviations initially below this curve fit are set equal to zero. Data deviations above the curve fit are subject to a minimum thickness cutoff condition such that points less than the minimum thickness are also set equal to zero. The remaining data points are the positive local elevations used for the analysis. These elevations are tagged and stored so that the corresponding points in the difference profile can be compared. Changes in elevation of the bed (difference profiles) were determined by subtracting successive profiles (e.g. Figure 5, lower plot).

Figure 6 shows the correlation (R^2) as a function of window (downstream length) size for the initial plastic bed elevations and the associated elevation changes after the first erosive turbidity current flow. For a minimum thickness cutoff of 100 microns, the optimal window size was found to be 354 cm, or 1770 points, which is nearly the entire length of the channel. Although there is an optimal window size, the corresponding correlation for this peak value is still low. Varying the size of the window used to analyze the profile does not significantly affect the degree of correlation. The small correlation value, approximately 0.3, associated with this optimal window size suggests that only a weak linear correlation exists between the two profiles. By using a window size of 354 cm, the maximum correlation between the elevation change and the local elevation is shown in Figure 7.

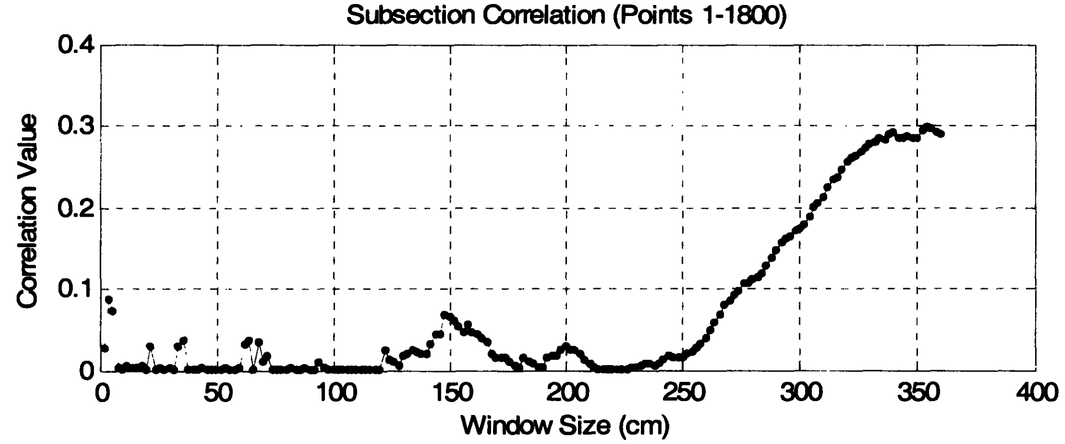

Figure 6: Correlation (R^2) as a function of window size for the initial plastic bed elevations and the associated elevation changes after the first erosive turbidity current flow.

The weak correlation may have been due to a lack of roughness on the initial plastic bed surface. This is represented in Figure 7 by the relatively small local elevations that correspond to a significant concentration of data points which show almost no change in elevation. Figure 7 and the middle plot in Figure 5 indicate that the substantial amount of roughness that was generated after the first erosive turbidity current flow is essentially uncorrelated with the initial elevations. Alternatively, the length of time that the bed was subjected to the erosive turbidity current may have also limited the correlation between the two profiles. Extending the period of time that an erosive turbidity current flow contacts the bed may increase the correlation.

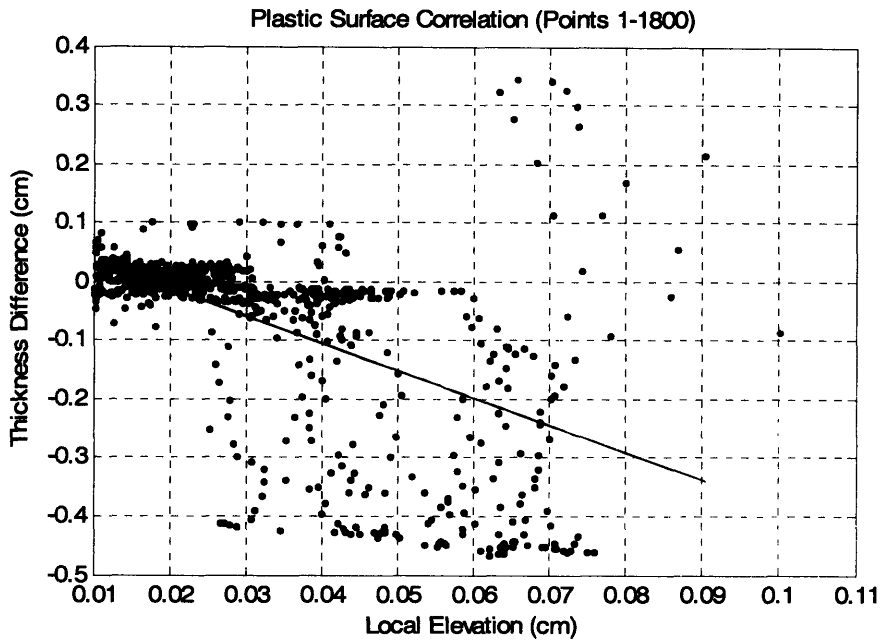

Figure 7: The correlation (R^2) between the change in bed elevation after the first erosive turbidity current flow and the local elevation of the initial plastic bed. The blue line is a linear fit to the data.

A similar analysis was performed to determine the correlation between the first and second erosive turbidity current flow beds. Figure 8 shows the downstream thickness profile of the plastic bed after the first erosive turbidity current flow, the bed profile after the second erosive turbidity current flow, and the difference of these two profiles. The upper plot in Figure 8 is the same as the middle plot in Figure 5. As stated previously, the first 40 cm of the bed experiences the greatest change in elevation and surface roughness. The middle plot shows the profile of the plastic bed after the second erosive turbidity current flow. The maximum roughness along the first 40 cm of the profile that developed after the first erosive turbidity current flow is still present after the second turbidity current flow. However, the roughness elements have lost elevation and seem to have propagated further downstream. Also, the roughness region has expanded downstream from 40 cm to approximately 75 cm. The downstream slope remains relatively unchanged between 100 cm and 360 cm and from that of the initial plastic bed (Figure 5, upper plot). The difference profile (Figure 5, lower plot) shows that the first 75 cm of the plastic bed experienced the most significant amount of change. The peak elevation difference between the two profiles is approximately -0.8 cm (erosion).

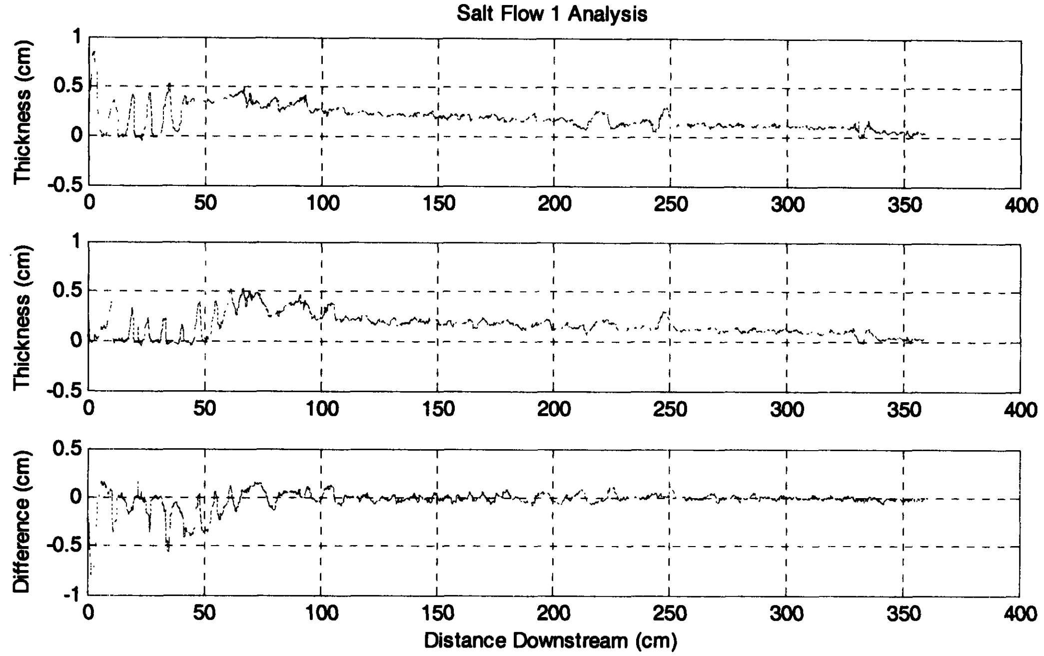

Figure 8: From upper to lower: downstream profile of the bed thickness after the first erosive turbidity current flow; profile of bed thickness after the second erosive turbidity current flow; and the difference between the two profiles respectively. Downstream profiles are located along the approximate channel centerline at 24.8 cm (see Figure 3).

Figure 9 shows the correlation (R^2) as a function of window size for the bed elevations after the first erosive turbidity current flow and the associated elevation changes after the second erosive turbidity current flow. For a minimum thickness cutoff of 600 microns, the optimal window size was found to be 40 cm, or 200 points. Interestingly, the optimal window size is the same size as the region of the bed that experienced the greatest change in elevation and surface roughness (Figures 5 and 8). In contrast to the analysis of the initial plastic bed, varying the window size does significantly affect the degree of correlation. Also, multiple widow sizes have larger correlation values than those for the initial plastic bed. The maximum correlation value is greater than 0.8, suggesting that at this scale, a relatively strong linear correlation exists between the two profiles. By using a window size of 40 cm, the maximum correlation between the elevation change and the local elevation for the entire profile (1800 points) is shown in Figure 10.

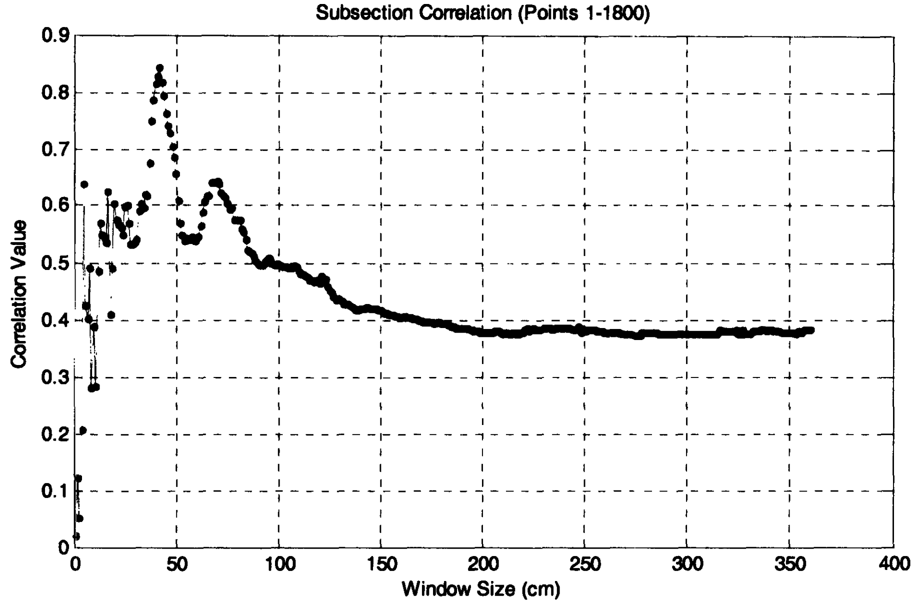

Figure 9: Correlation (R^2) as a function of window size for the bed elevations after the first erosive turbidity current flow and the associated elevation changes after the second erosive turbidity current flow. The maximum correlation occurs for a window size of 40 cm.

The correlation plot suggests that as the local elevation increases, the magnitude of the thickness difference also increases. This implies that locally, more sediment is eroded from higher relative elevations. The correlation shows that approximately 84% of the data can be fit by a linear curve. The plot also shows that there is more scatter for smaller elevation values; elevations smaller than 0.2 cm are widely distributed about the linear curve fit. This indicates that smaller elevations present along the profile do not significantly contribute to the correlation and that these elevations change little after the second erosive turbidity current flow. In contrast, elevation values greater than 0.2 cm contribute most to the curve fit.

To explore the contribution of the first 40 cm to the magnitude of the correlation, the analysis was again carried out by comparing the first 40 cm (points 1-200) to the remaining 320 cm (points 201-1800). Figure 11 shows the contribution to the correlation from the first 40 cm. The correlation is approximately 77%. This is only 7% lower than the correlation from the entire profile. This is a relatively insignificant amount of variation between the two correlations considering that approximately 89% of the profile is not being included. What is absent from Figure 11 are the smaller elevation values (compare Figures 10 and 11). This suggests that the remaining 320 cm of the profile contributes the majority of smaller elevations and has little influence on the correlation value.

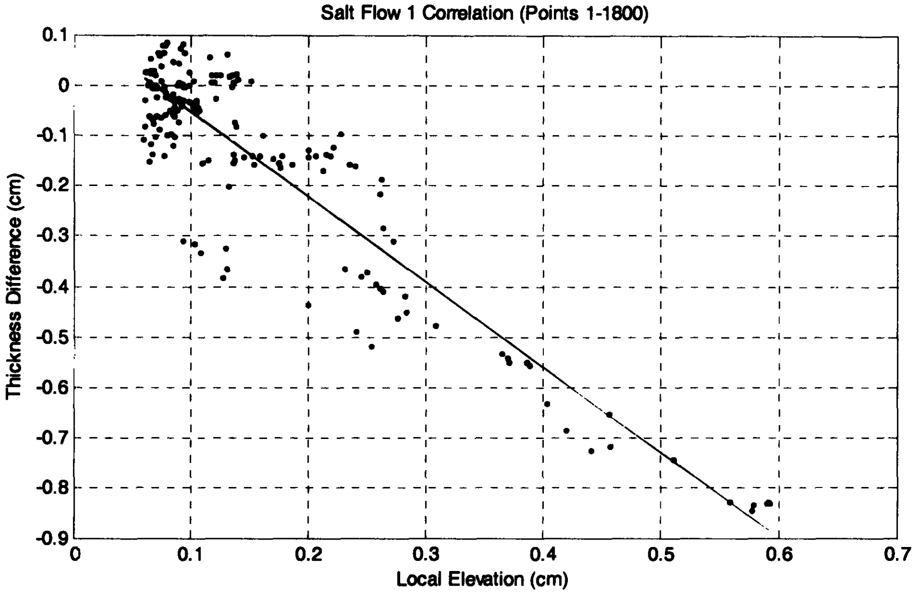

Figure 10: The correlation (R^2) between the change in bed elevation after the second erosive turbidity current flow and the local bed elevation after the first erosive turbidity current flow for the complete profile. The correlation widow is 40 cm. The blue line is a linear fit to the data.

Figure 12 shows the contribution to the correlation from the remaining 320 cm of the profile. The maximum correlation value is 1.4%. The majority of the bed in this section does not experience a significant amount of erosion, and the lack of significant correlation is similar to that found for the initial plastic bed. As shown in Figure 8, the last 320 cm is relatively smooth and its change in thickness is small compared to the first 40 cm of the channel. The maximum local elevation in Figure 12 is only 1.4 mm and the largest difference in thickness is approximately 0.1 cm, whereas the maximum local elevation in the first 40 cm is almost 6 mm and the largest thickness difference is approximately 0.9 cm (Figure 11). Figures 10-12 suggest that larger local elevations tend to erode more than relatively smaller local elevations. Figure 10 also suggests that elevations greater than about 0.2 cm are most important for the correlation. The strong correlation with elevations greater than 0.2 cm suggests that the time a bed is subjected to an erosive turbidity current flow is likely not an important factor in determining the correlation since the duration of each flow was similar.

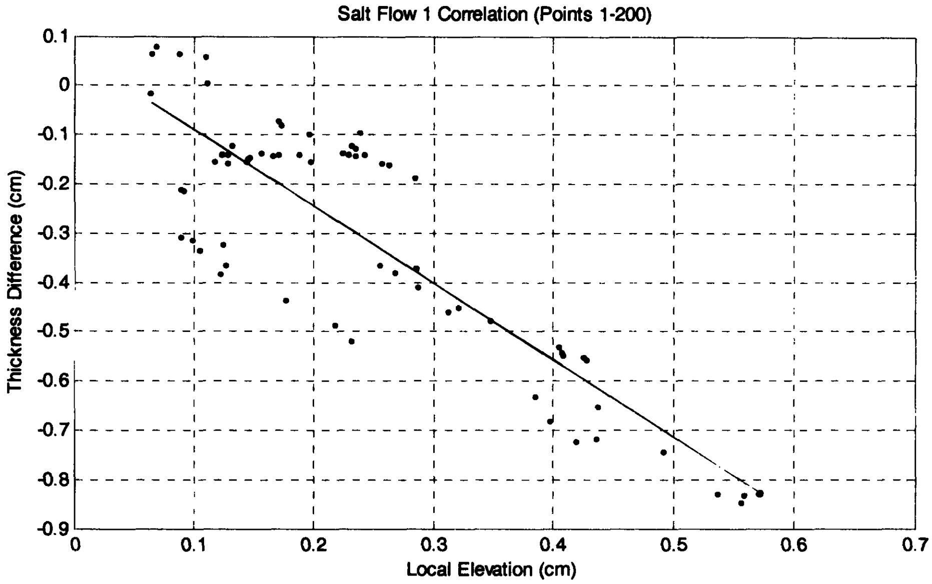

Figure 11: The correlation (R^2) between the change in bed elevation after the second erosive turbidity current flow and the local bed elevation after the first erosive turbidity current flow for the first 40 cm of the profile. The correlation widow is 40 cm. The blue line is a linear fit to the data.

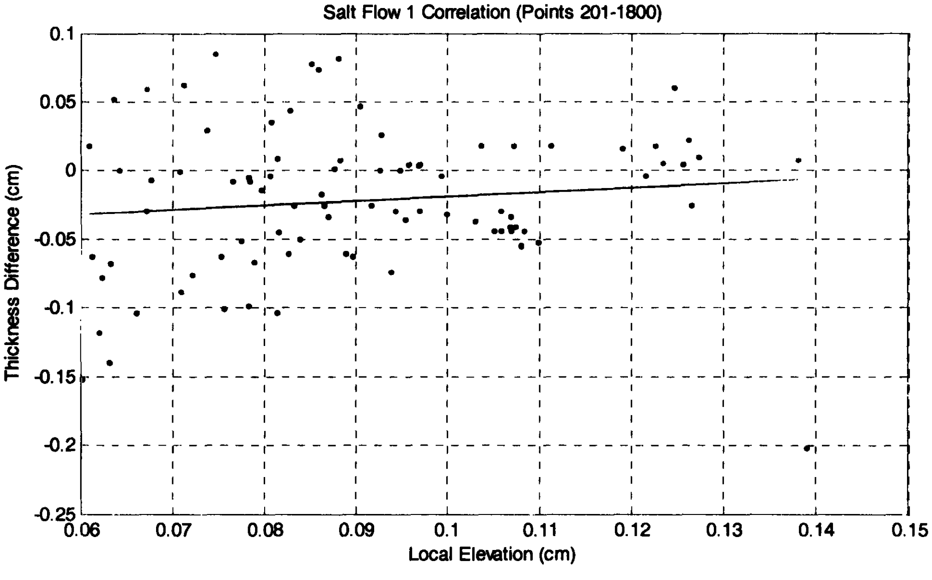

Figure 12: The correlation (R^2) between the change in bed elevation after the second erosive turbidity current flow and the local bed elevation after the first erosive turbidity current flow for points greater than 40 cm. The correlation widow is 40 cm. The blue line is a linear fit to the data.

4.0 Conclusions

This study examined the applicability of a proposed interface model which prescribes a relationship between the bed shear stress, the local bed elevation and the local bed slope by analyzing the evolution of a sediment bed profile during a succession of erosive turbidity current flows. By subjecting a bed of non-cohesive plastic sediment to two erosive turbidity currents, the changes and trends in local elevation were compared and contrasted. Correlations between successive bed profiles at a single downstream profile were computed to determine the strength of the relationship between the change in thickness of a bed and the initial local elevation. A dependence of the erosion of a bed on the local bed elevation suggests that a simple interface model may be valid.

Correlation (R^2) of the initial plastic bed elevations and the associated elevation changes after the first erosive turbidity current flow as a function of window (downstream length) size shows that the size of the window does not significantly affect the degree of correlation. A small correlation value, approximately 0.3, was calculated, suggesting that only a weak linear correlation exists between the two profiles. The weak correlation may have been due to a lack of roughness on the initial plastic bed surface or due to the limited time that the bed was subjected to the erosive turbidity current flow.

Correlation (R^2) of the bed after the first erosive turbidity current flow and the associated elevation changes after the second erosive turbidity current flow as a function of window size for the bed elevations shows that varying the window size does significantly affect the degree of correlation. The optimal widow size is 40 cm, with a maximum correlation value greater than 0.8, suggesting that at this scale, a relatively strong linear correlation exists between the two profiles. The optimal window size is the same size as the region of the bed that experienced the greatest change in elevation and surface roughness. The correlation shows that approximately 84% of the data can be fit by a linear curve using the optimal window size over the entire profile. Elevations less than about 0.2 cm show more scatter and are widely distributed about the linear curve fit. In contrast, elevations greater than 0.2 cm contribute most to the curve fit.

By separately analyzing the first 40 cm, the contribution to the magnitude of the correlation can be explored. This section contained the most significant roughness and the associated correlation is approximately 77%. The correlation value over the first 40 cm is only 7% lower than that for the entire profile. The correlation difference is small even though approximately 89% of the profile is not being considered. Most of the contributions to the linear curve fit come from elevations greater than 0.2 cm.

For the remaining 320 cm of the profile, the maximum correlation value is only 1.4%. This section of the profile does not significantly contribute to the correlation from the entire profile. The maximum local elevation is only 1.4 mm with the maximum thickness difference of approximately 0.1 cm, whereas the maximum local elevation in the first 40 cm is almost 6 mm and the maximum thickness difference of approximately 0.9 cm. This section was observed to contain a low degree of roughness compared to the first 40 cm of the profile, and the lack of significant correlation is similar to that found for the initial plastic bed. The analysis suggests that elevations greater than about 0.2 cm are most important in the correlation. The strong correlation with elevations greater than 0.2 cm also suggests that the time a bed is subjected to an erosive turbidity current flow is likely not an important factor in determining the correlation.

The analysis of the correlation of the bed after the first erosive turbidity current flow and the associated elevation changes after the second erosive turbidity current flow showed that an elevation range of 0.2 cm to 0.6 cm contribute most to the correlation. Using this range, a relationship between the height of the erosive turbidity current and the local elevations most likely to be eroded can be determined. The ratio of the local elevation to the turbidity current height over the elevation range is 2% to 6% respectively. This suggests that a minimum elevation height relative to the height of the turbidity current must be established for significant erosion to occur. This also suggests that it minimum elevation may be necessary when considering the applicability of the interface model. To determine the validity of this observation, a separate experiment using a variety of turbidity current heights should be performed.

The observations presented in this study support an assumption of the proposed interface model which states that the initial elevation of a sediment bed may affect the shear stress present on the bed and therefore the erosion of the bed. A similar correlation study to determine the erosion dependence on local bed slope should also be conducted to complete the analysis.

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