Optics Overview

What is light?

- Light is a form of **electromagnetic energy** – detected through its effects, e.g. heating of illuminated objects, conversion of light to current, mechanical pressure (“Maxwell force”) etc.

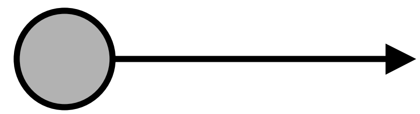
- Light energy is conveyed through particles: “photons”
  - ballistic behavior, e.g. shadows

- Light energy is conveyed through waves
  - wave behavior, e.g. interference, diffraction

- Quantum mechanics reconciles the two points of view, through the “wave/particle duality” assertion

Particle properties of light

Photon=elementary light particle



Mass=0
Speed $c=3\times10^{8}$ m/sec

According to Special Relativity, a mass-less particle travelling at light speed can still carry momentum!

Energy $E=h\nu$ ⟹ relates the dual particle & wave nature of light;

h=Planck's constant
$=6.6262\times10^{-34}$ J sec

ν is the temporal oscillation frequency of the light waves

Wave properties of light

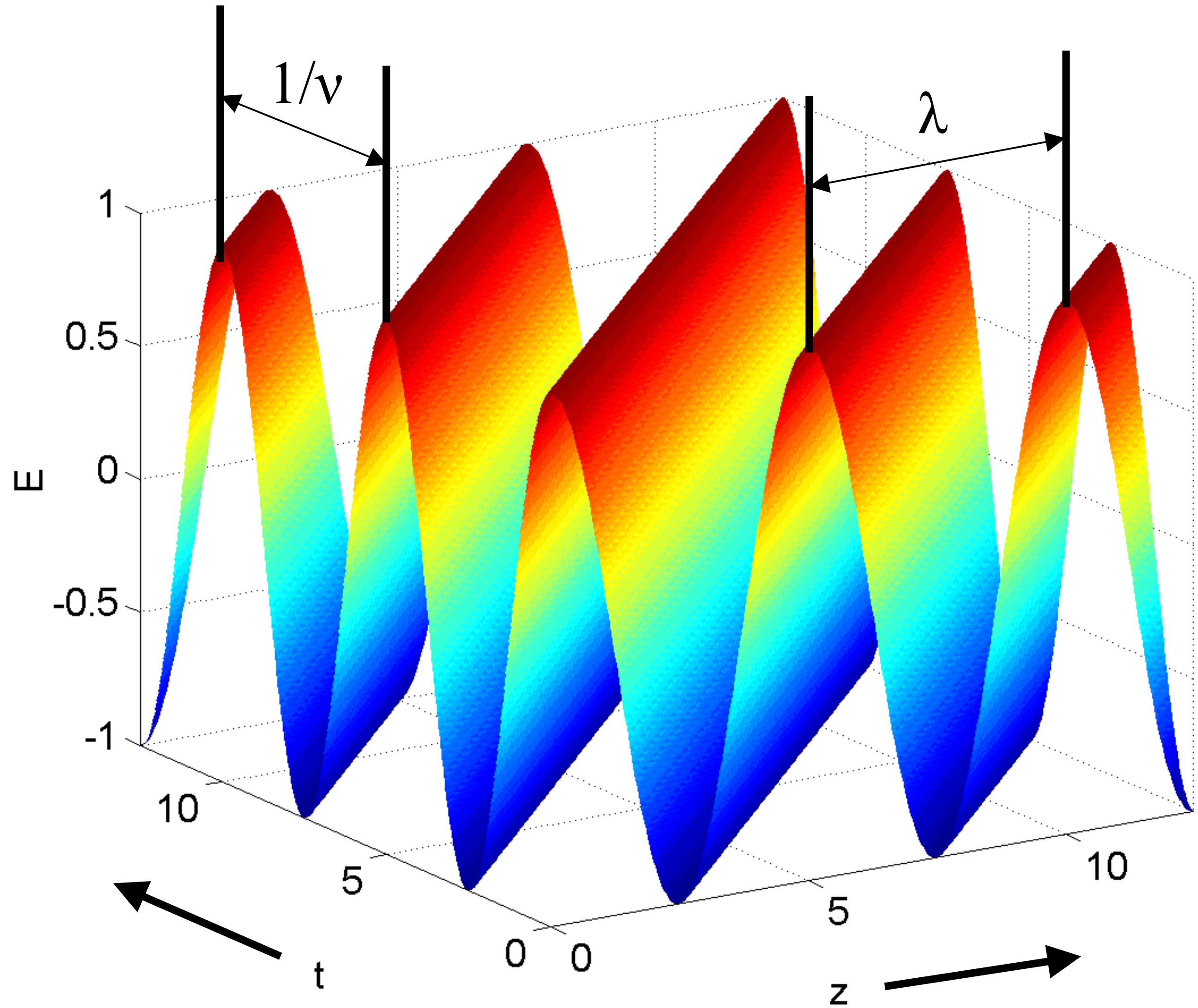
λ: **wavelength**
(spatial period)

k=2π/λ
wavenumber

ν: temporal
frequency

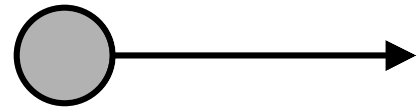
ω=2πν
angular frequency

E: electric
field



Wave/particle duality for light

Photon=elementary light particle



Mass=0
Speed $c=3\times10^{8}$ m/sec

Energy $E=h\nu$

h=Planck's constant
$=6.6262\times10^{-34}$ J sec

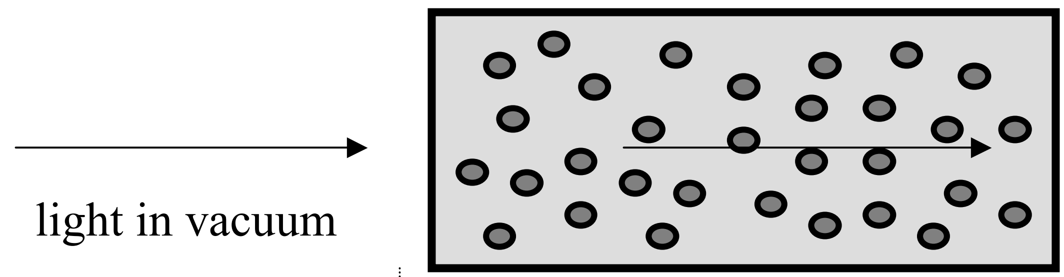
ν=frequency (sec^{-1})
λ=wavelength (m)

$$\boxed{c=\lambda\nu}$$

"Dispersion relation"

(holds in vacuum only)

Light in matter



light in vacuum

light in matter

Speed $c=3\times10^8$ m/sec

Absorption coefficient 0

Speed c/n

n : refractive index

(or index of refraction)

Absorption coefficient α

energy decay coefficient,

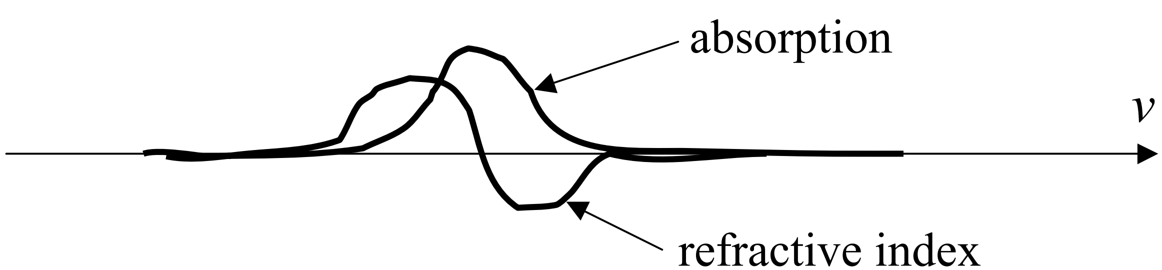
after distance L : $e^{-2\alpha L}$

E.g. vacuum $n=1$, air $n \approx 1$;

glass $n\approx1.5$; glass fiber has $\alpha \approx 0.25$dB/km=0.0288/km

Materials classification

- Dielectrics
  - typically electrical isolators (e.g. glass, plastics)
  - low absorption coefficient
  - arbitrary refractive index
- Metals
  - conductivity ⇒ large absorption coefficient
- Lots of exceptions and special cases (e.g. "artificial dielectrics")
- Absorption and refractive index are related through the Kramers–Kronig relationship (imposed by *causality*)



Overview of light sources

non-Laser

Thermal: polychromatic, spatially incoherent (e.g. light bulb)

Gas discharge: monochromatic, spatially incoherent (e.g. Na lamp)

Light emitting diodes (LEDs): monochromatic, spatially incoherent

Laser

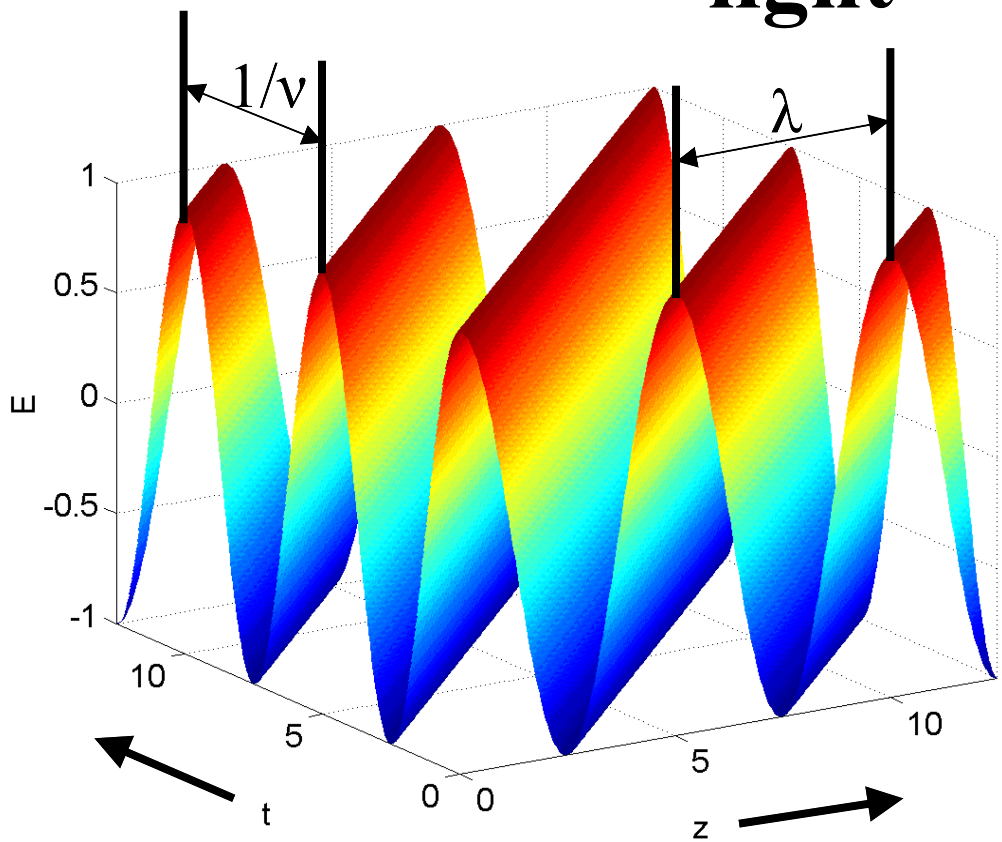
Continuous wave (or cw): strictly monochromatic, spatially coherent (e.g. HeNe, Ar^{+}, laser diodes)

Pulsed: quasi-monochromatic, spatially coherent (e.g. Q-switched, mode-locked)

Q-switched: ~nsec; mode-locked: ~psec to few fsec (pulse duration)

mono/poly-chromatic = single/multi color

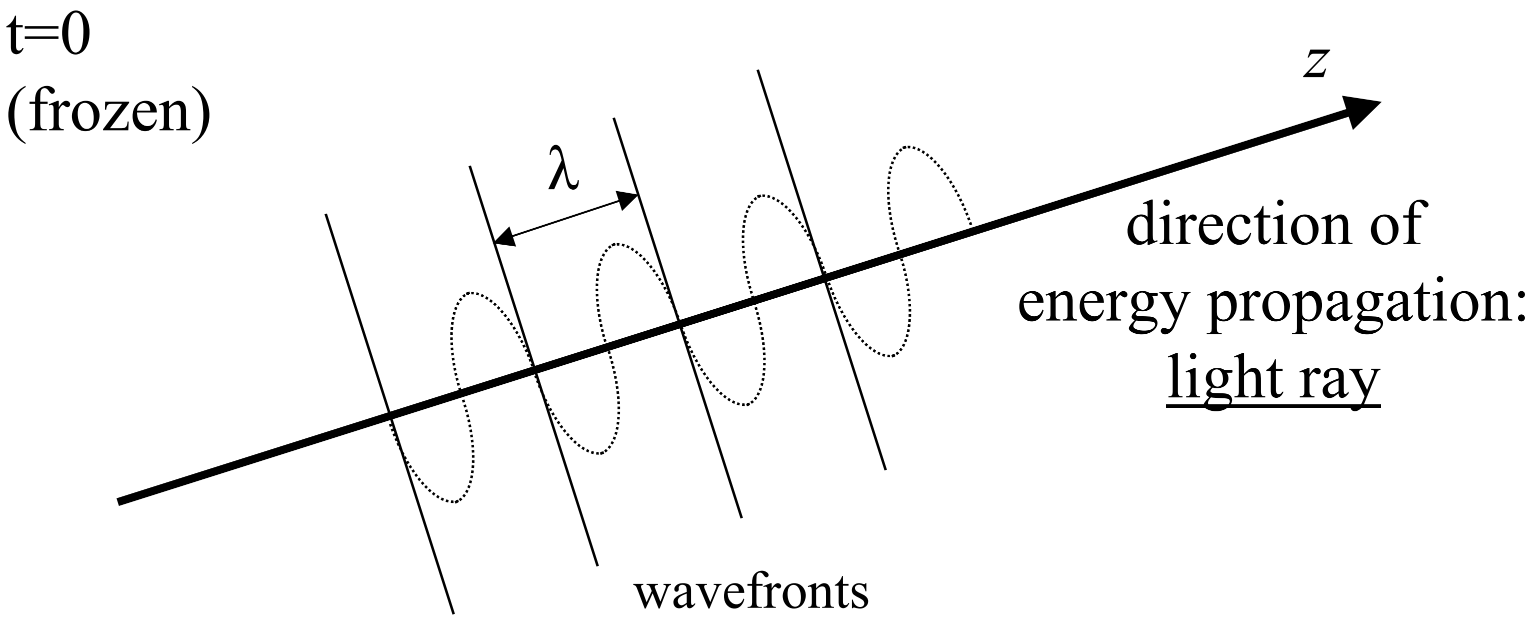
Monochromatic, spatially coherent light



- nice, regular sinusoid
- λ, ν well defined
- stabilized HeNe laser good approximation
- most other cw lasers rough approximation
- pulsed lasers & non-laser sources need more complicated description

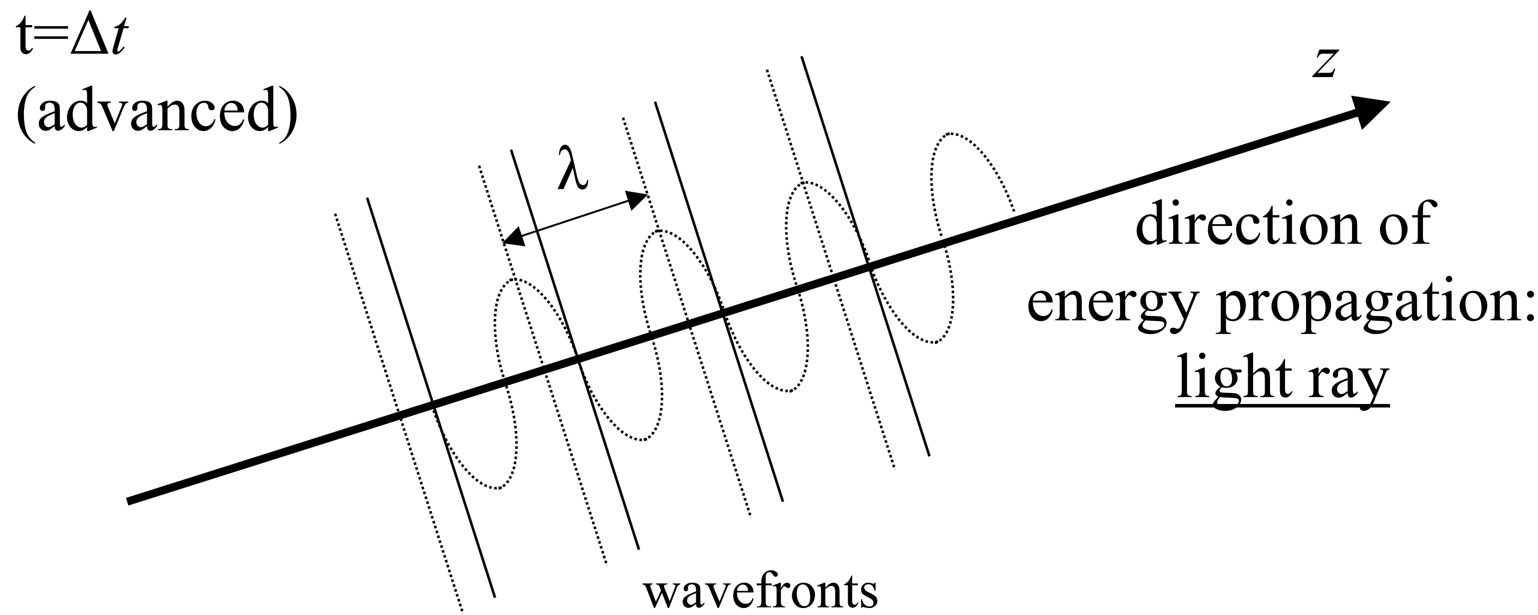
Incoherent: random, irregular waveform

The concept of a monochromatic "ray"



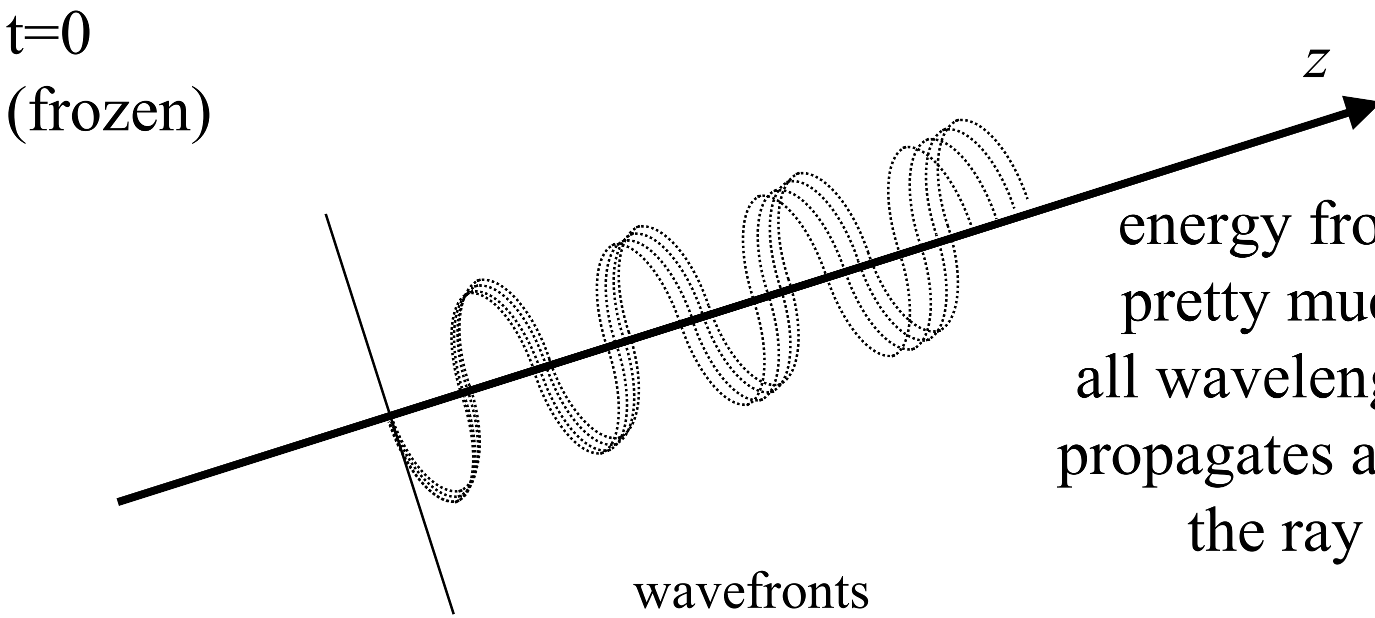
In homogeneous media,
light propagates in rectilinear paths

The concept of a monochromatic "ray"



In homogeneous media,
light propagates in rectilinear paths

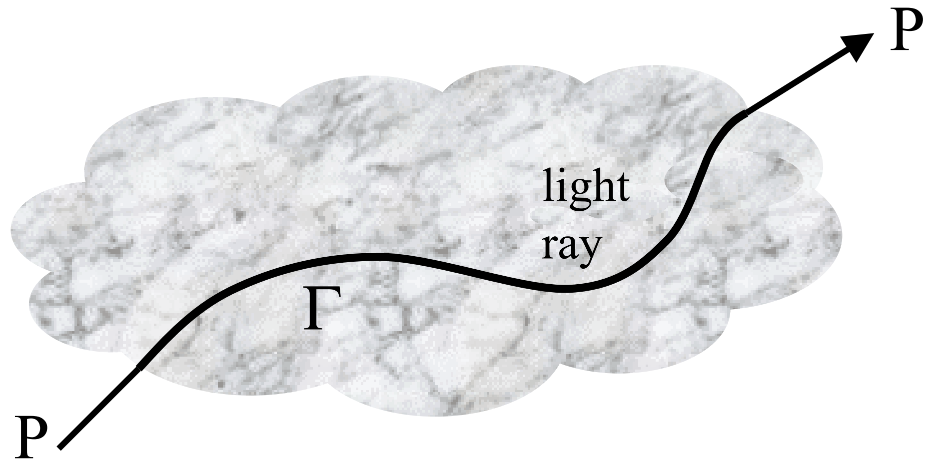
The concept of a polychromatic "ray"



energy from pretty much all wavelengths propagates along the ray

In homogeneous media,
light propagates in rectilinear paths

Fermat principle

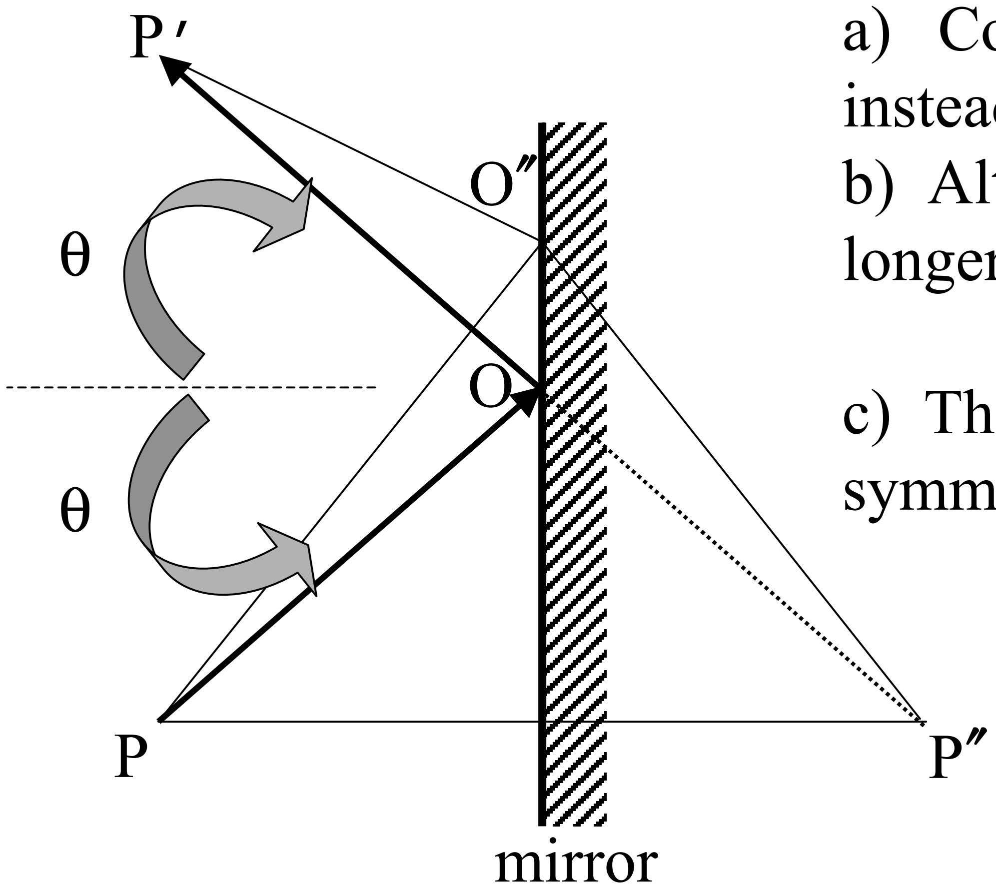


$$\int_{\Gamma} n(x, y, z)\, \mathrm{d}l$$

Γ is chosen to minimize this "path" integral, compared to alternative paths

(aka **minimum path** principle)
Consequences: law of reflection, law of refraction

The law of reflection

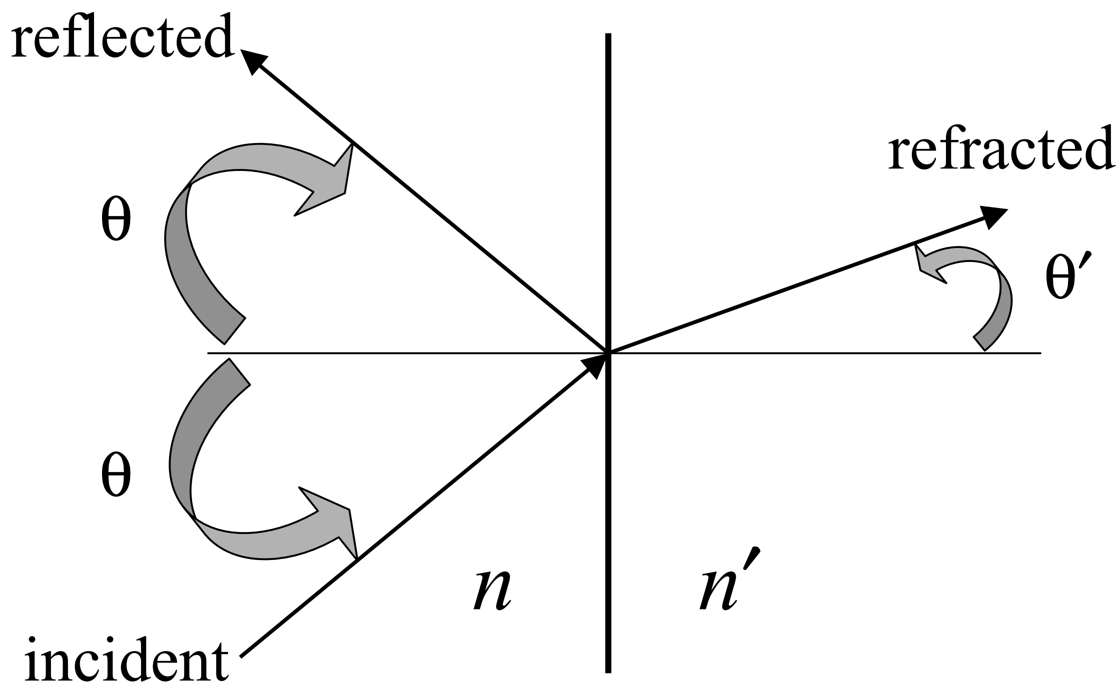


a) Consider virtual source P" instead of P

b) Alternative path P"O"P' is longer than P"OP'

c) Therefore, light follows the symmetric path POP'.

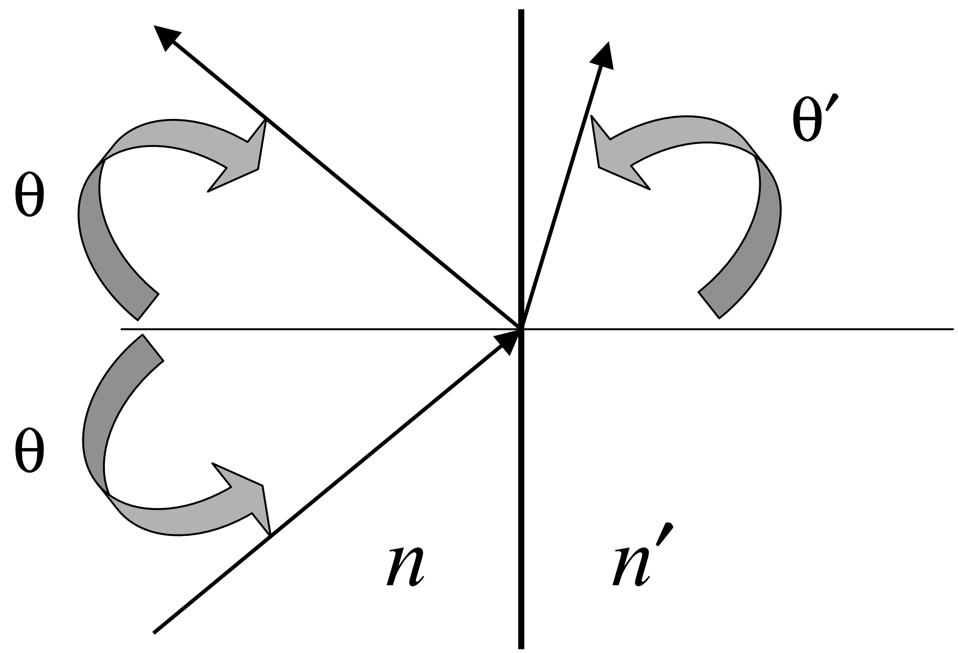
The law of refraction



$$n \sin\theta = n' \sin\theta'$$

Snell's Law of Refraction

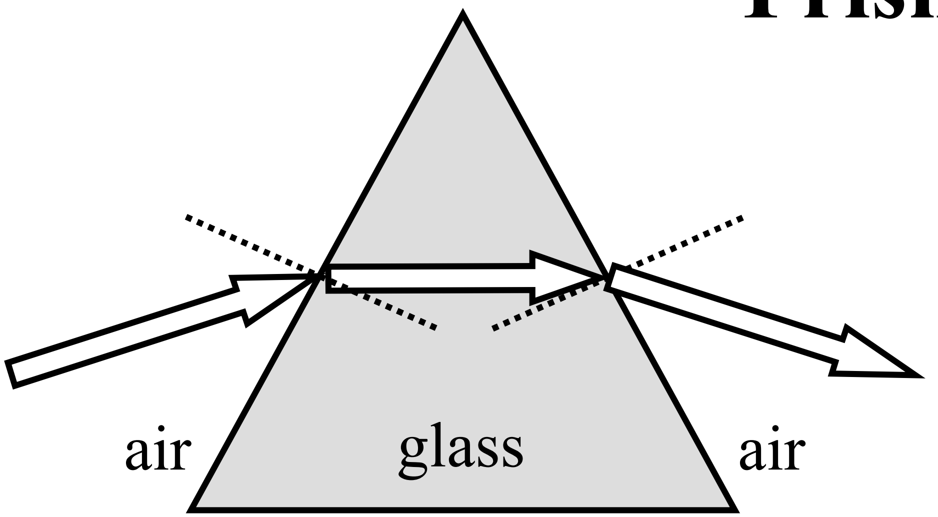
Total Internal Reflection (TIR)

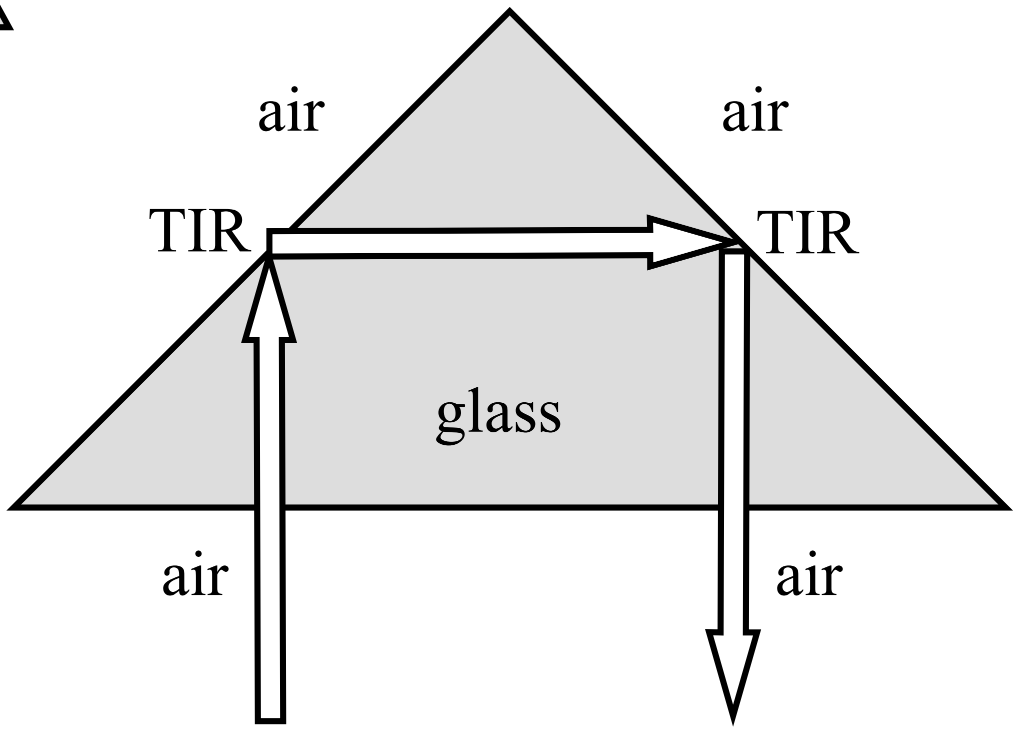


$$n > n' \Rightarrow \theta' \text{ becomes imaginary when } \theta > \theta_{\text{crit}} = \sin^{-1}\frac{n'}{n}$$

$\Rightarrow$ refracted beam disappears, all energy is reflected

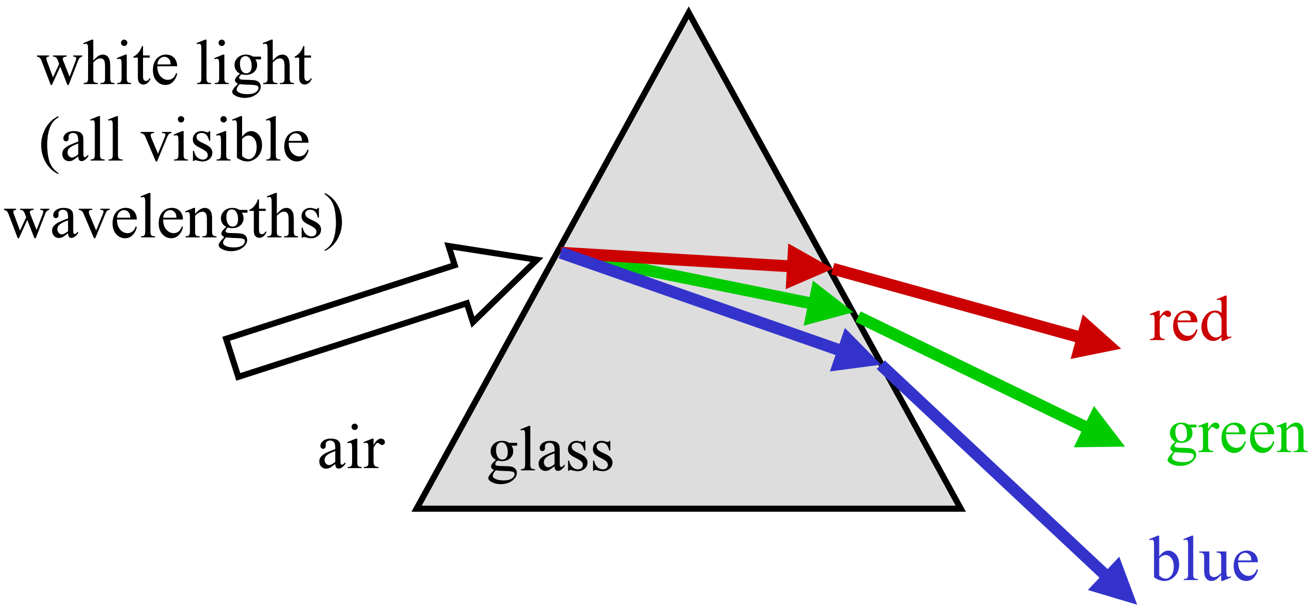
Prisms





Dispersion

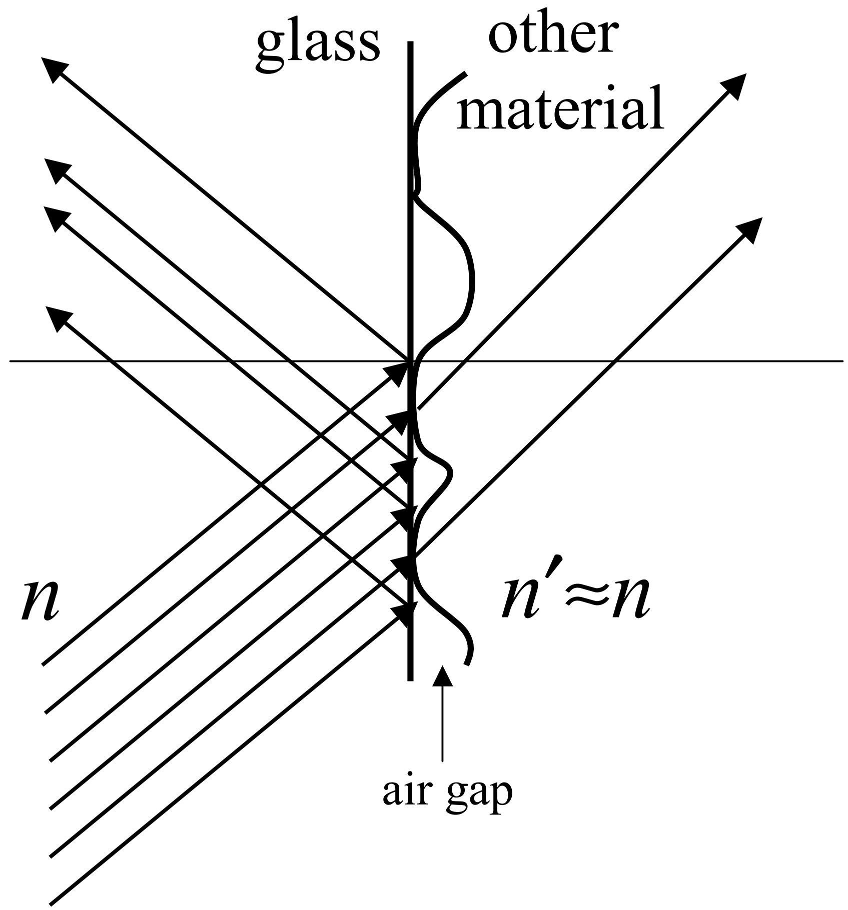
Refractive index n is function of the wavelength



Newton's prism

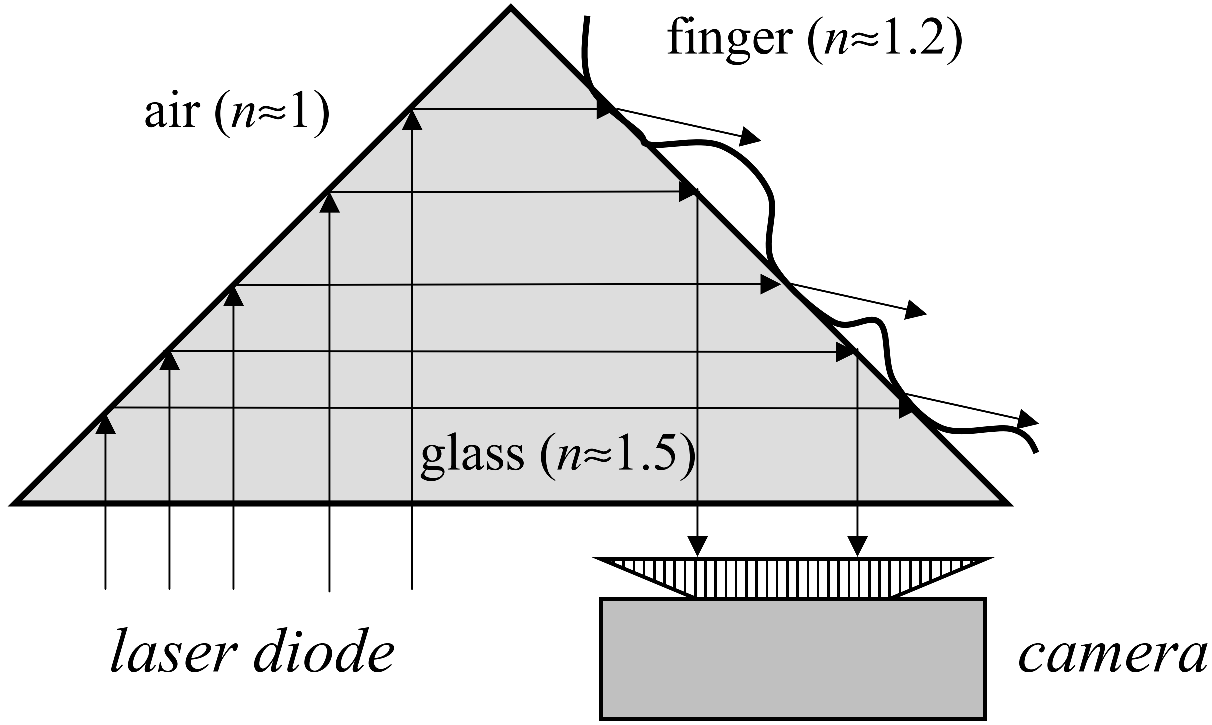
Frustrated Total Internal Reflection (FTIR)

Reflected rays are missing where index-matched surfaces touch ⇒ shadow is formed



Angle of incidence exceeds critical angle

Fingerprint sensor



$$\theta_{\text{crit}}(\text{glass} \mid \text{air}) \approx 42^{\text{o}}$$

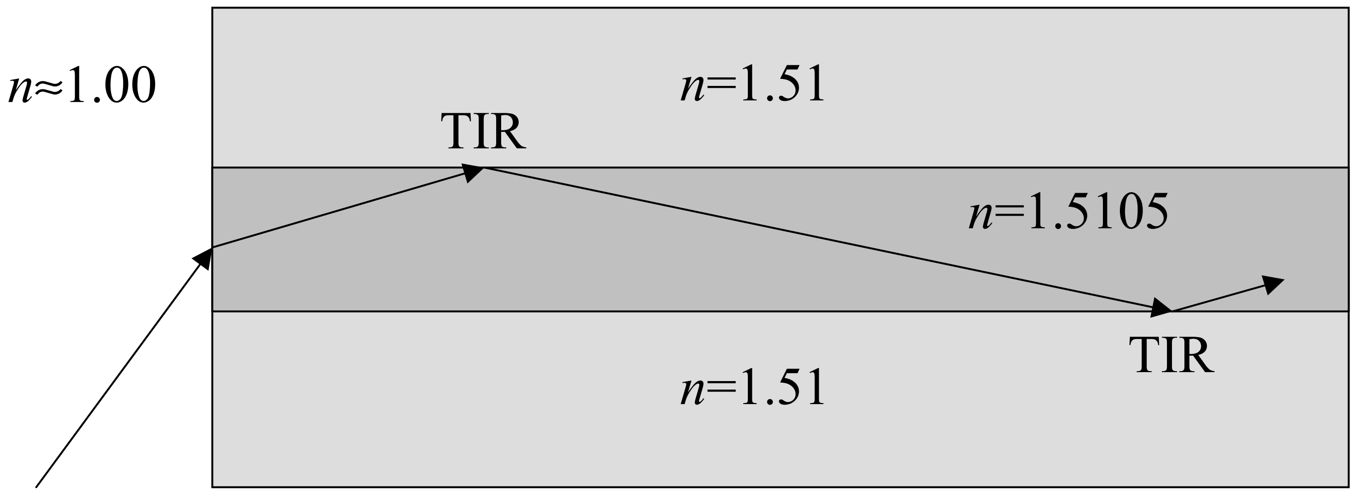
TIR occurs @45°

$$\theta_{\text{crit}}(\text{glass} \mid \text{finger}) \approx 53^{\text{o}}$$

TIR does not occur @45°

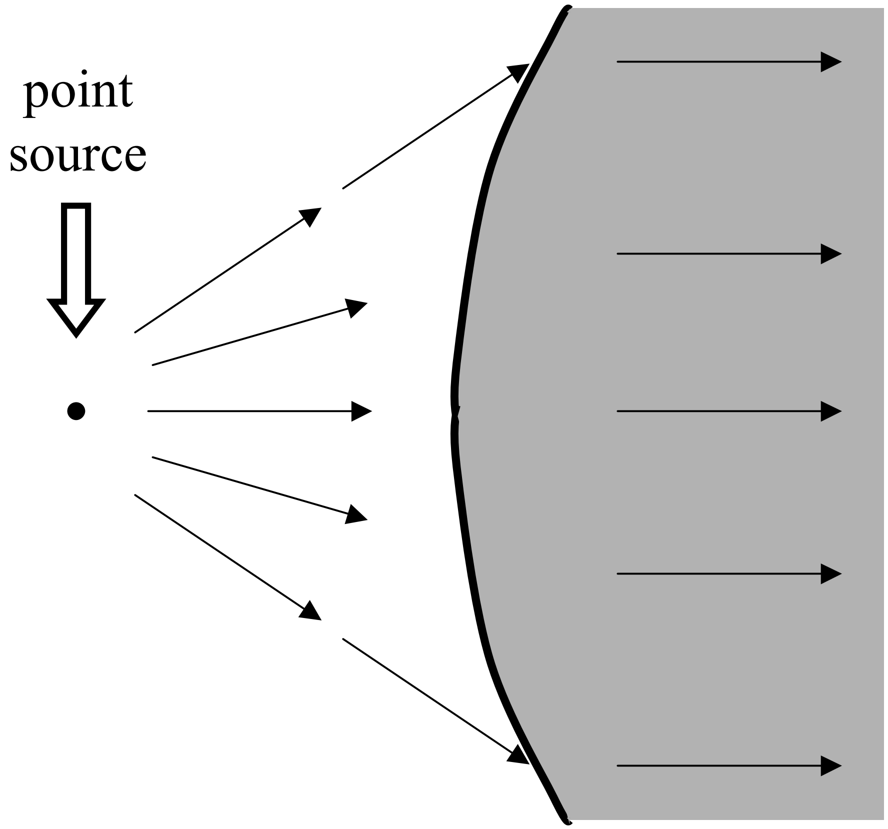
(FTIR)

Optical waveguide

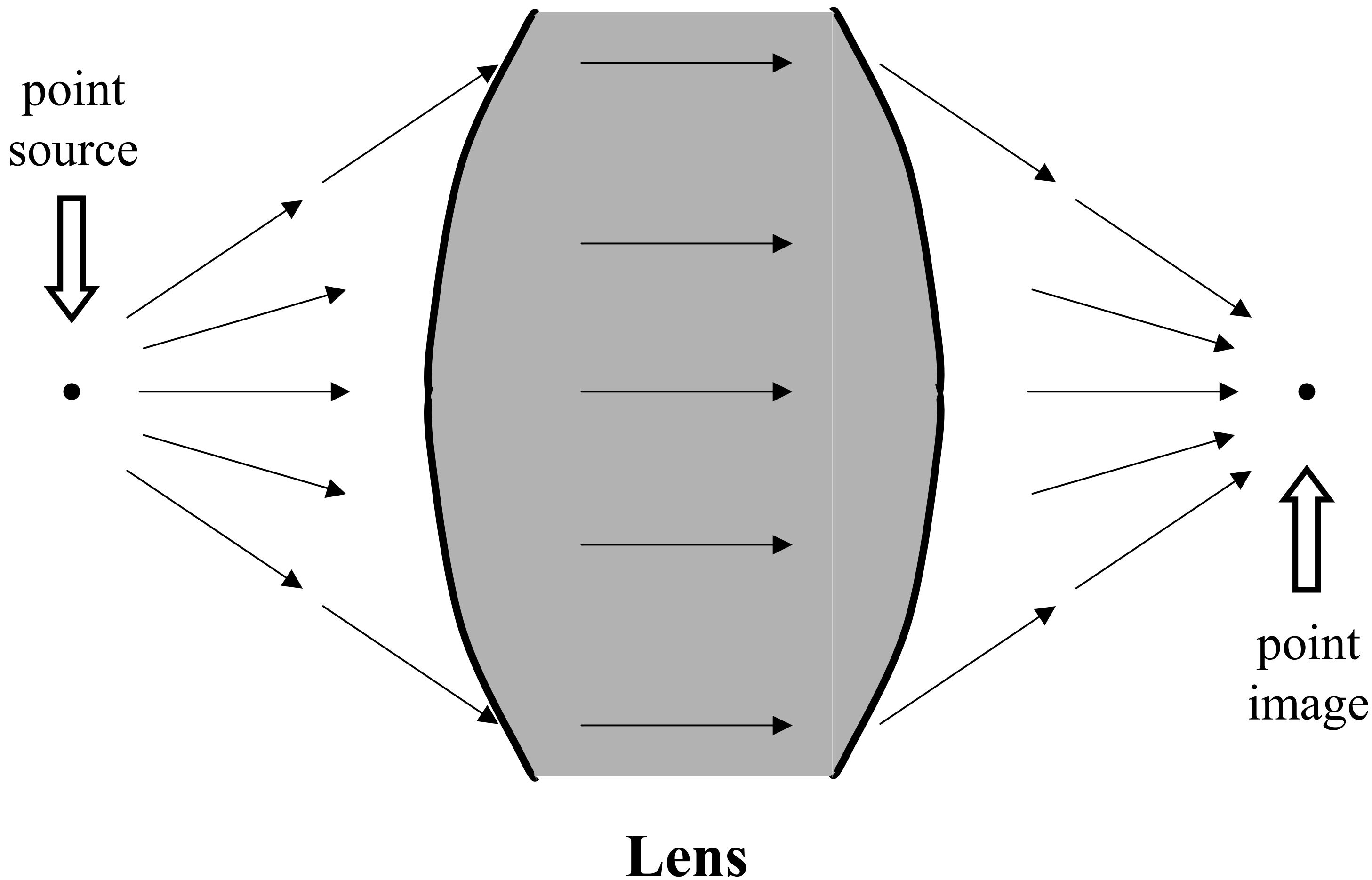


- Planar version: *integrated optics*
- Cylindrically symmetric version: *fiber optics*
- Permit the creation of "light chips" and "light cables," respectively, where light is guided around with few restrictions
- Materials research has yielded glasses with very low losses (<0.25dB/km)
- Basis for optical telecommunications and some imaging (e.g. endoscopes) and sensing (e.g. pressure) systems

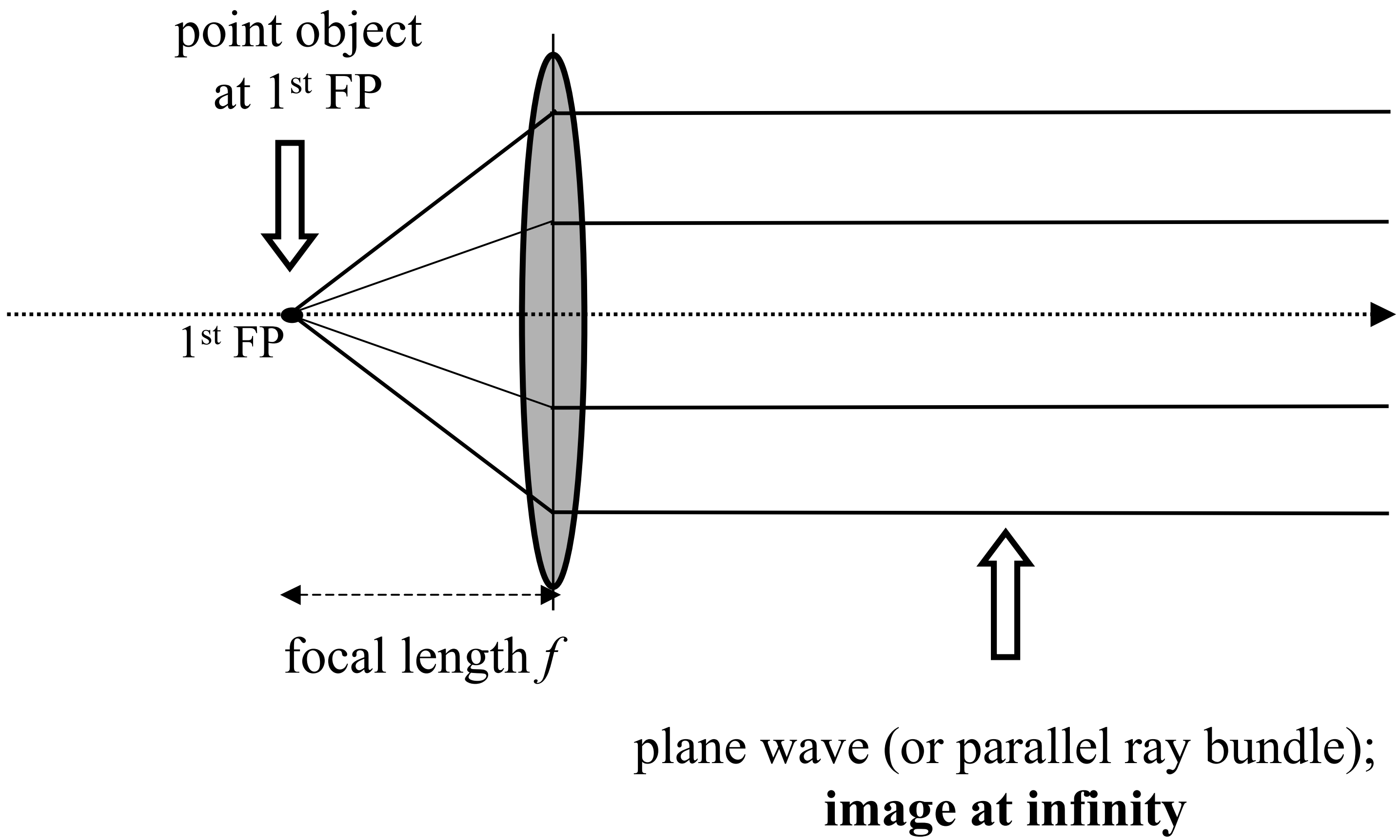
Refraction at a spherical surface



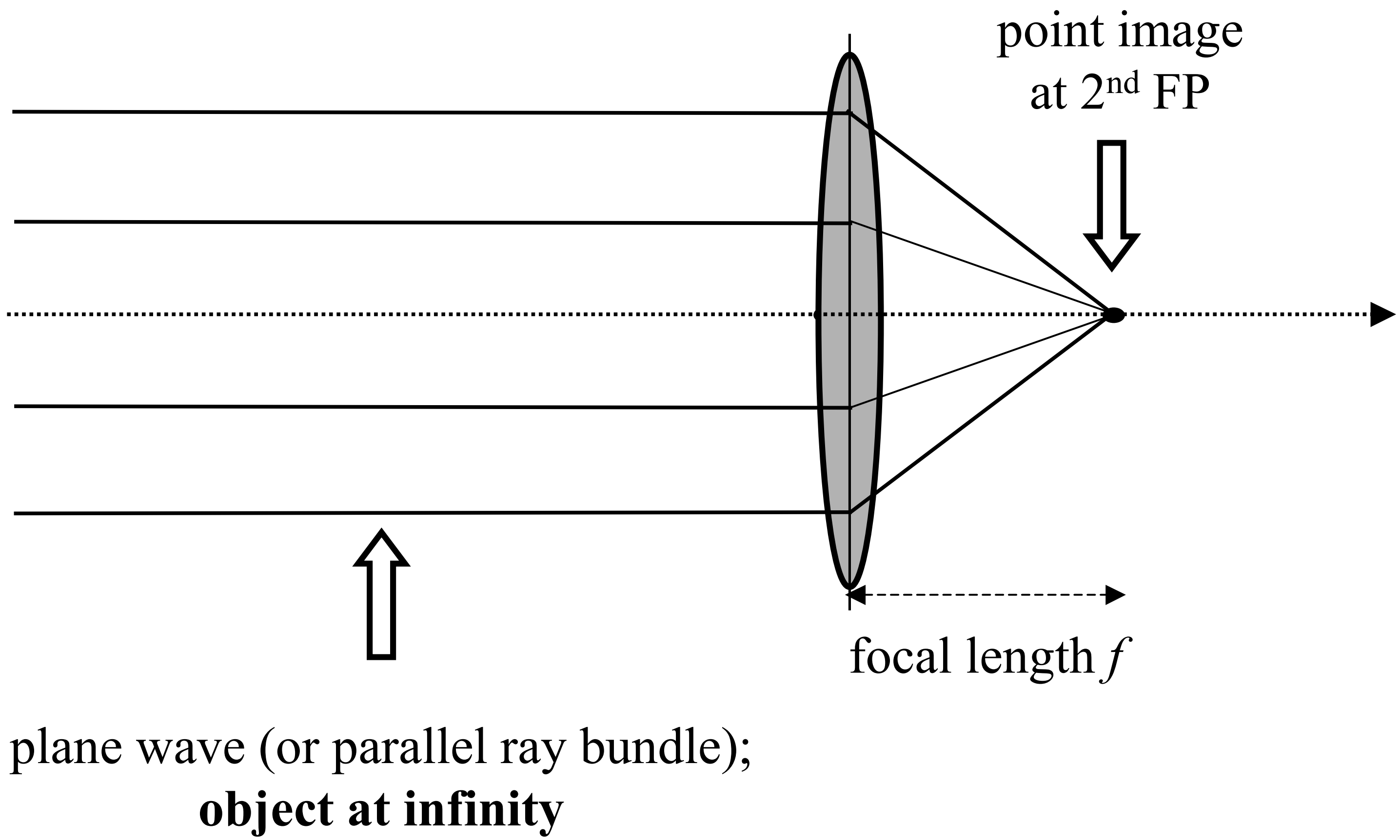
Imaging a point source



Model for a thin lens



Model for a thin lens



Types of lenses

positive
($f > 0$)

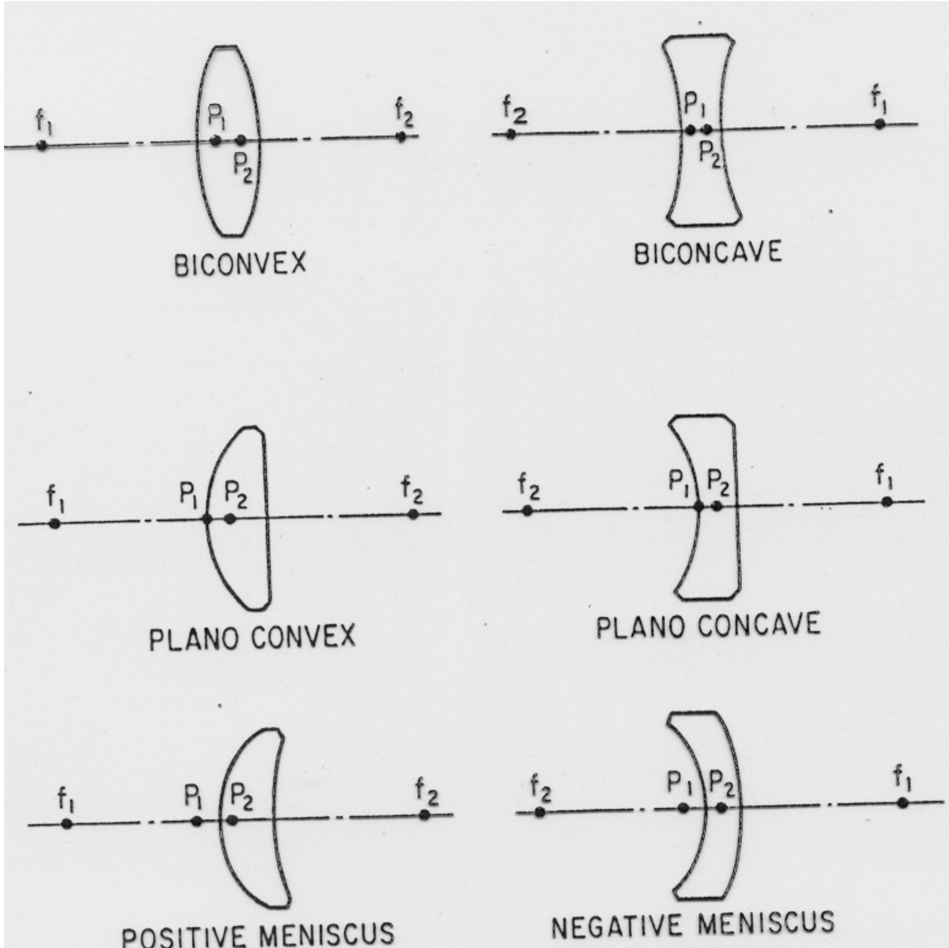
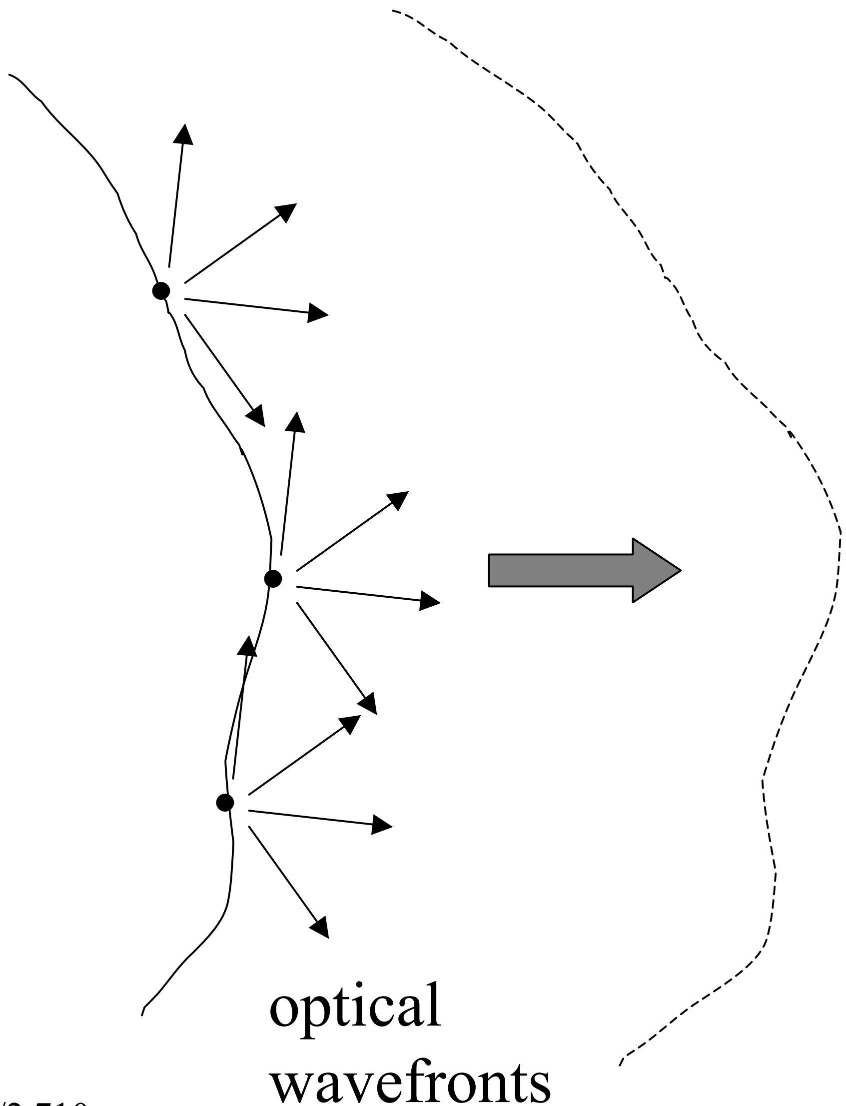


Figure 2.12 The location of the focal points and principal points for several shapes of converging and diverging elements.

negative
($f < 0$)

from *Modern Optical Engineering* by W. Smith

Huygens principle

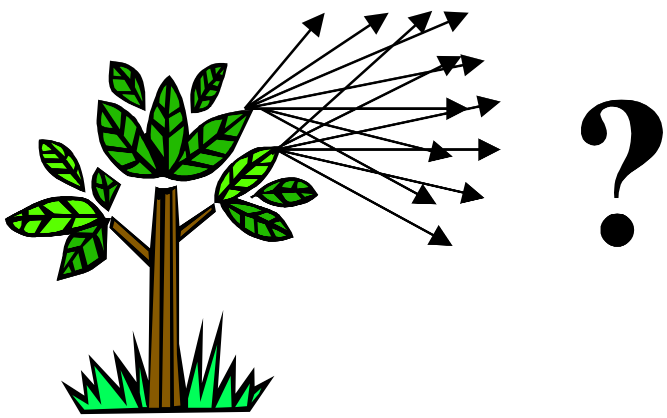


Each point on the wavefront acts as a secondary light source emitting a spherical wave

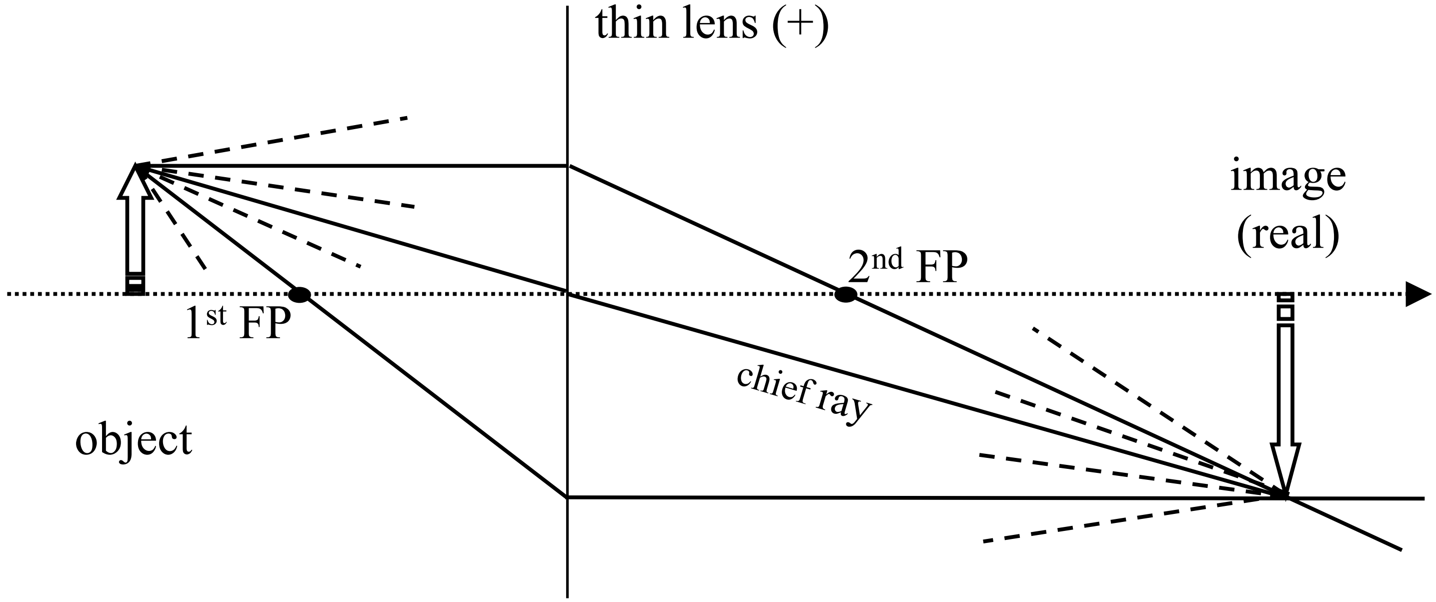
The wavefront after a short propagation distance is the result of superimposing all these spherical wavelets

Why imaging systems are needed

- Each point in an object scatters the incident illumination into a spherical wave, according to the Huygens principle.
- A few microns away from the object surface, the rays emanating from all object points become entangled, delocalizing object details.
- To relocalize object details, a method must be found to reassign ("focus") all the rays that emanated from a single point object into another point in space (the "image.")
- The latter function is the topic of the discipline of Optical Imaging.

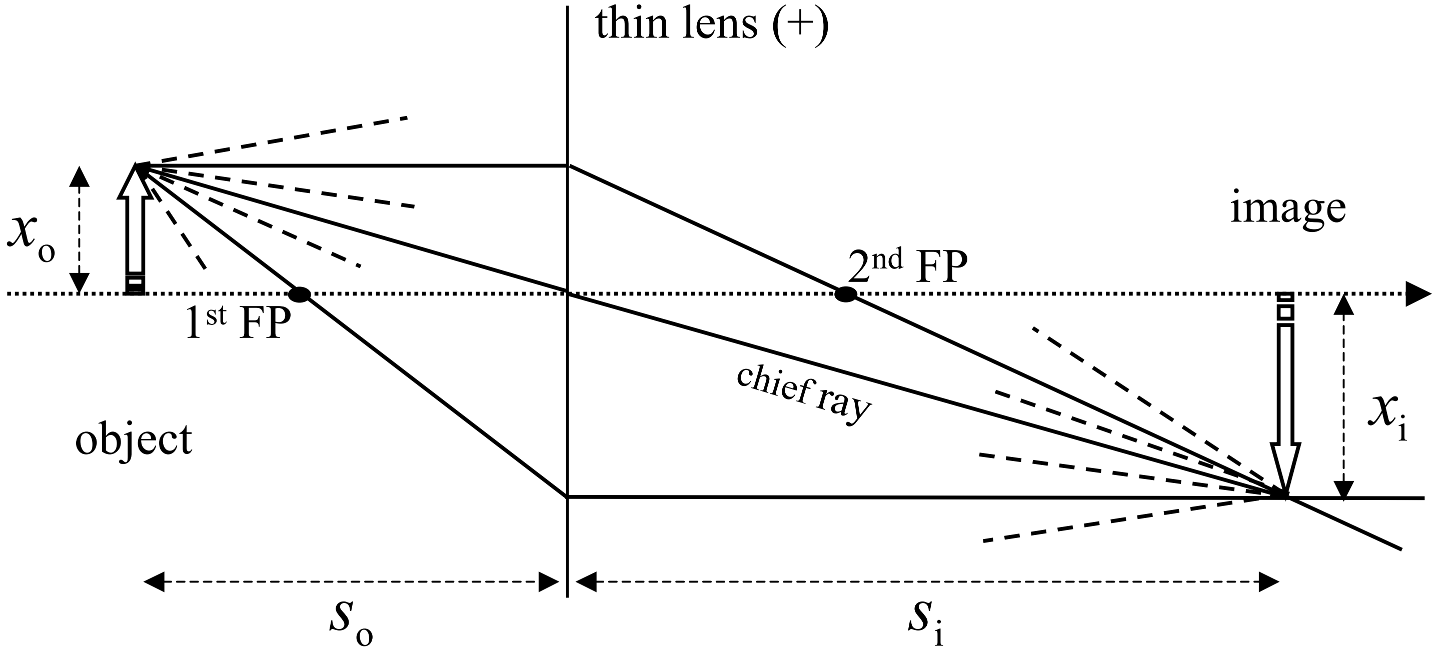


Imaging condition: ray-tracing



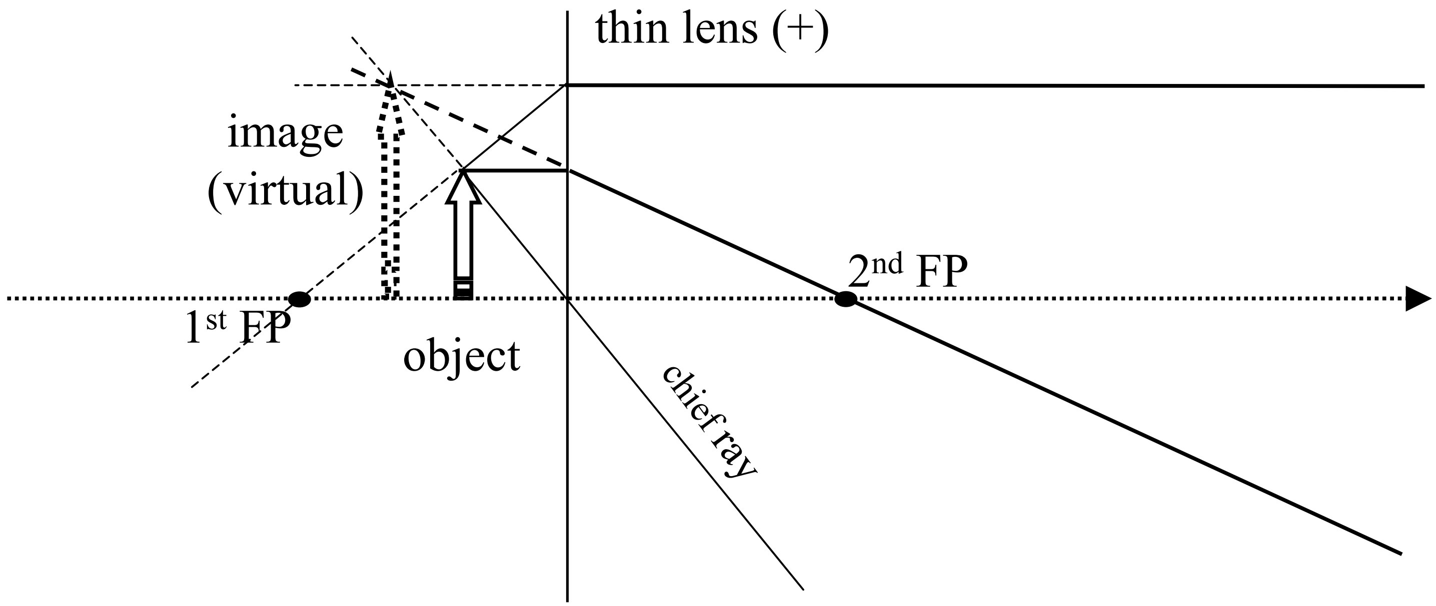
- Image point is located at the common intersection of *all* rays which emanate from the corresponding object point
- The two rays passing through the two focal points and the chief ray can be ray-traced directly
- The real image is **inverted** and can be **magnified** or **demagnified**

Imaging condition: ray-tracing



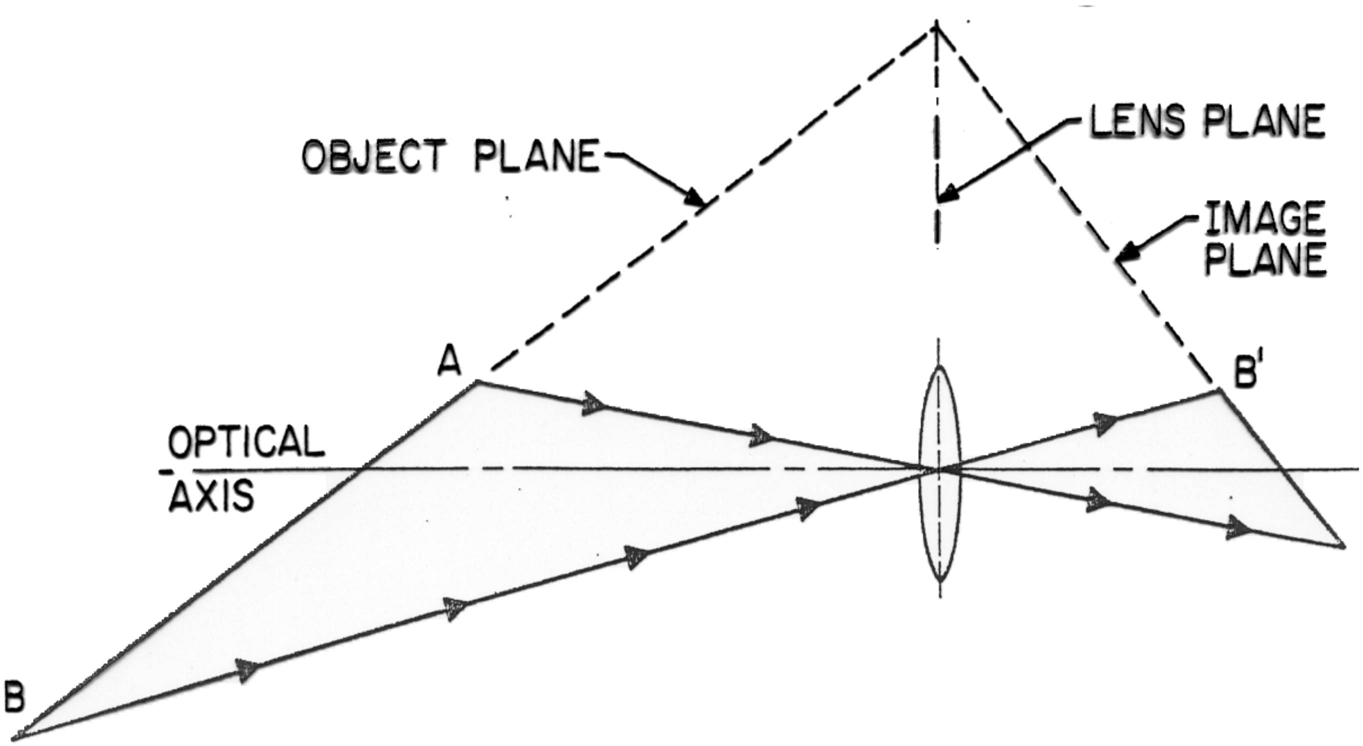
Lens Law	Lateral magnification	Angular magnification	Energy conservation
$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$	$M_x=\frac{x_i}{x_o}=-\frac{s_o}{s_i}$	$M_a=-\frac{s_i}{s_o}$	$M_x M_a=1$

Imaging condition: ray-tracing



- The ray bundle emanating from the system is divergent; the virtual image is located at the intersection of the backwards-extended rays
- The virtual image is **erect** and is **magnified**
- When using a <u>negative</u> lens, the image is always virtual, erect, and demagnified

Tilted object: the Scheimpflug condition



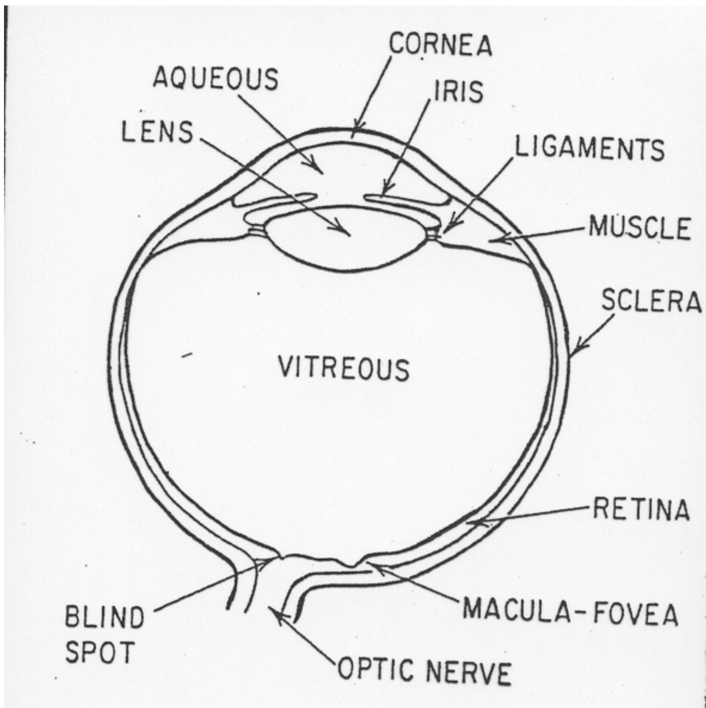
The object plane and the image plane intersect at the plane of the thin lens.

Lens-based imaging

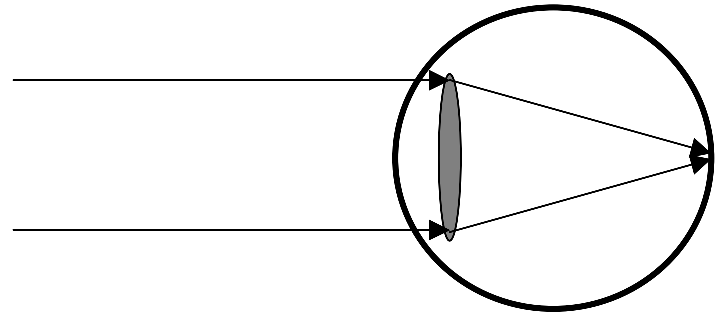
- Human eye
- Photographic camera
- Magnifier
- Microscope
- Telescope

The human eye

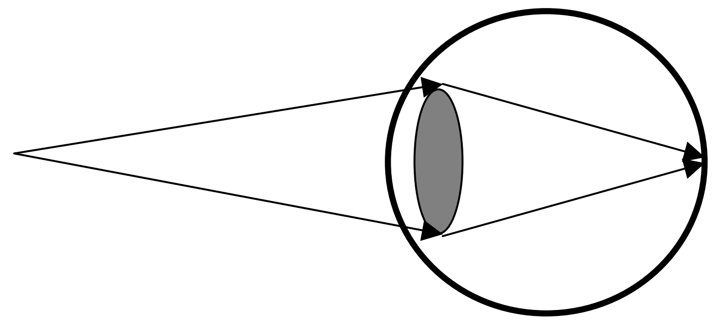
Anatomy



Near point (comfortable viewing)
~25cm from the cornea



Remote object (unaccommodated eye)



Near object (accommodated eye)

Eye defects and their correction

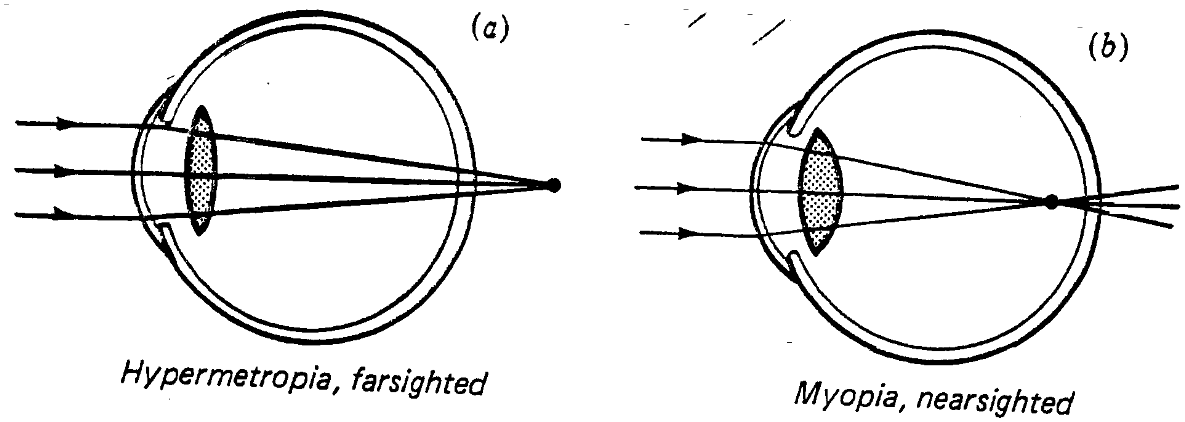


FIGURE 10K
Typical eye defects, largely present in the adult population.

from *Fundamentals of Optics* by F. Jenkins & H. White

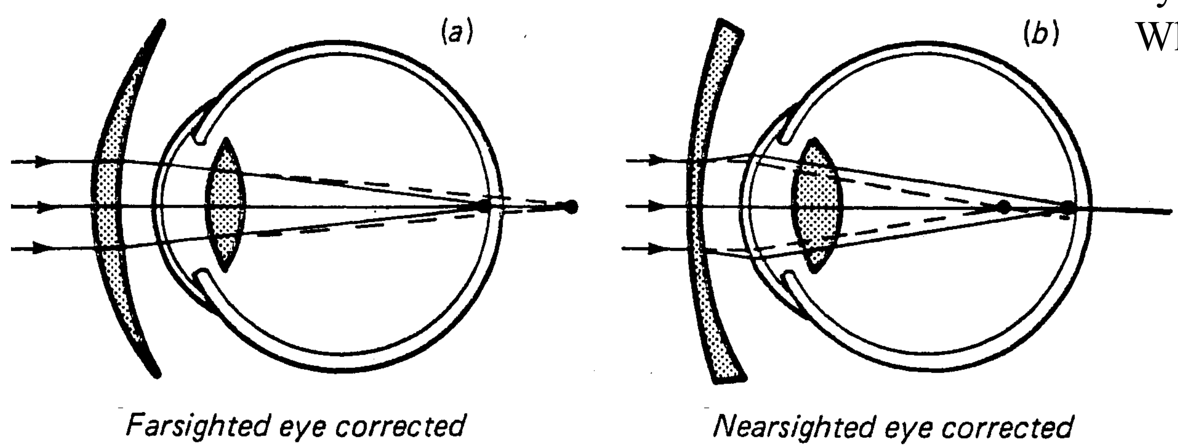
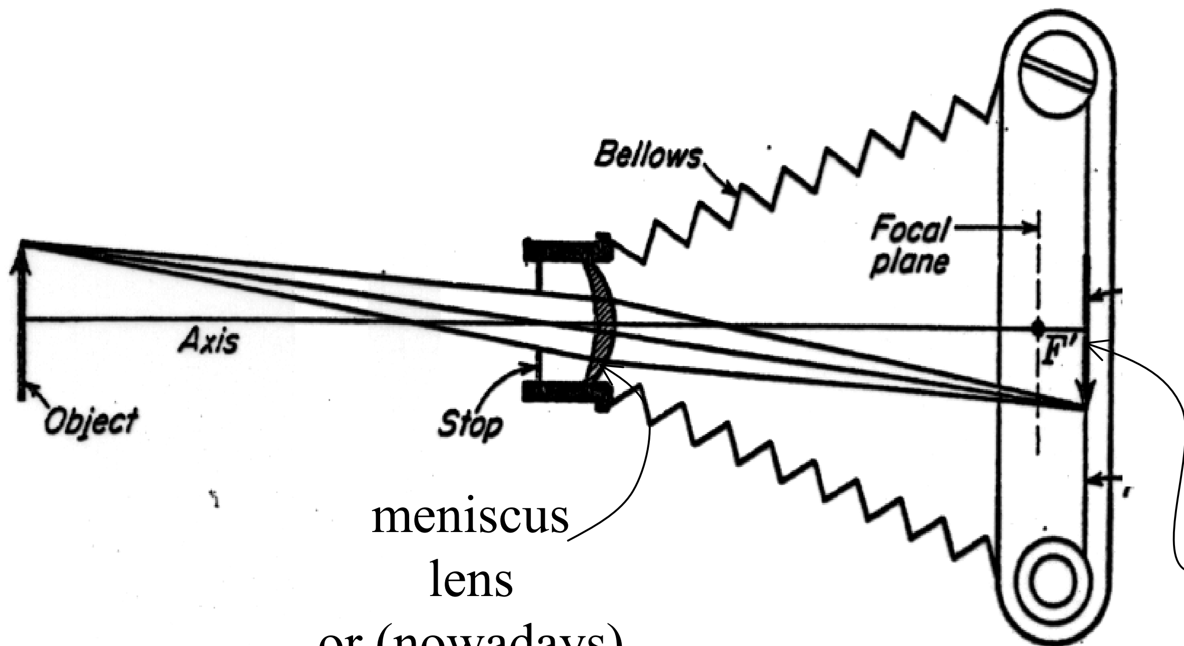


FIGURE 10L
Typical eye defects can be corrected by spectacle lenses.

The photographic camera



meniscus
lens
or (nowadays)
zoom lens

Film
or
detector array (CCD or CMOS)

"digital imaging" ⟹ detector array (CCD or CMOS)

The magnifier

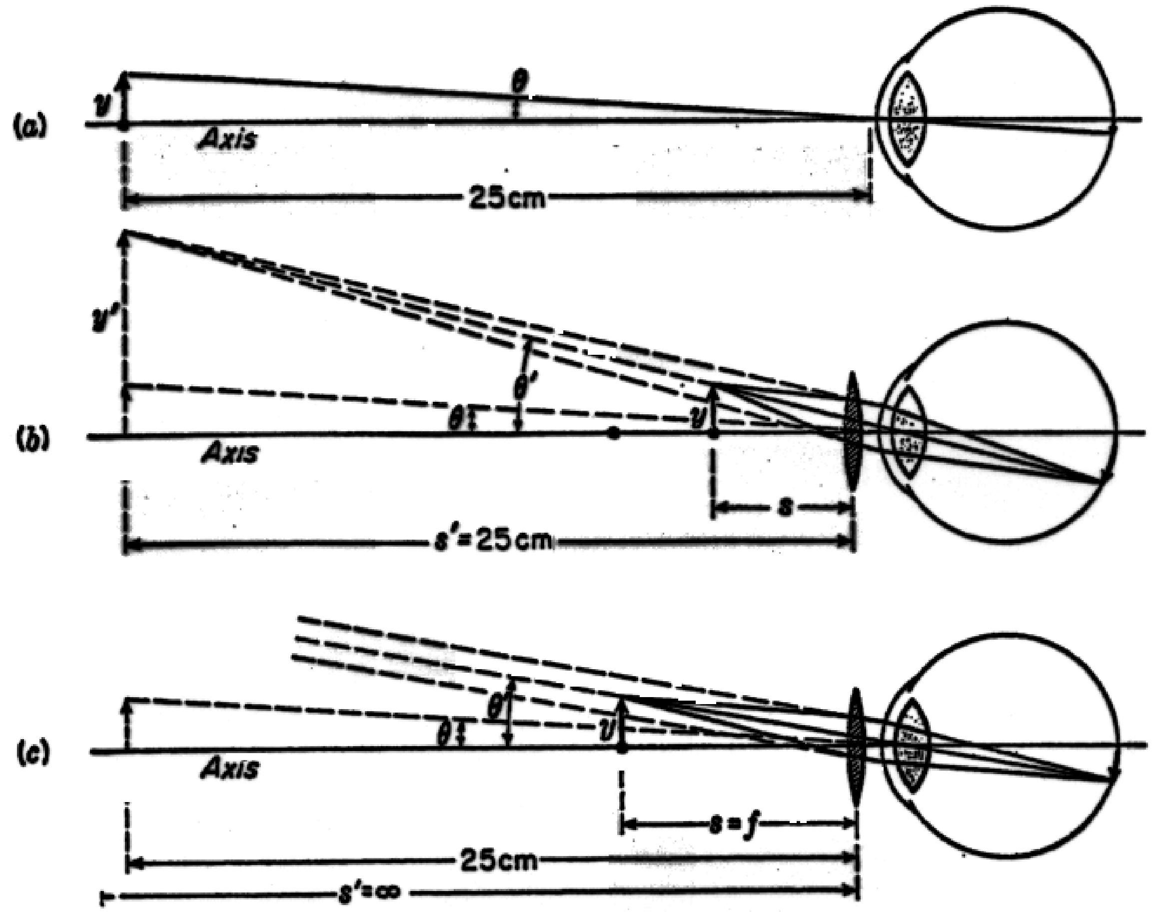
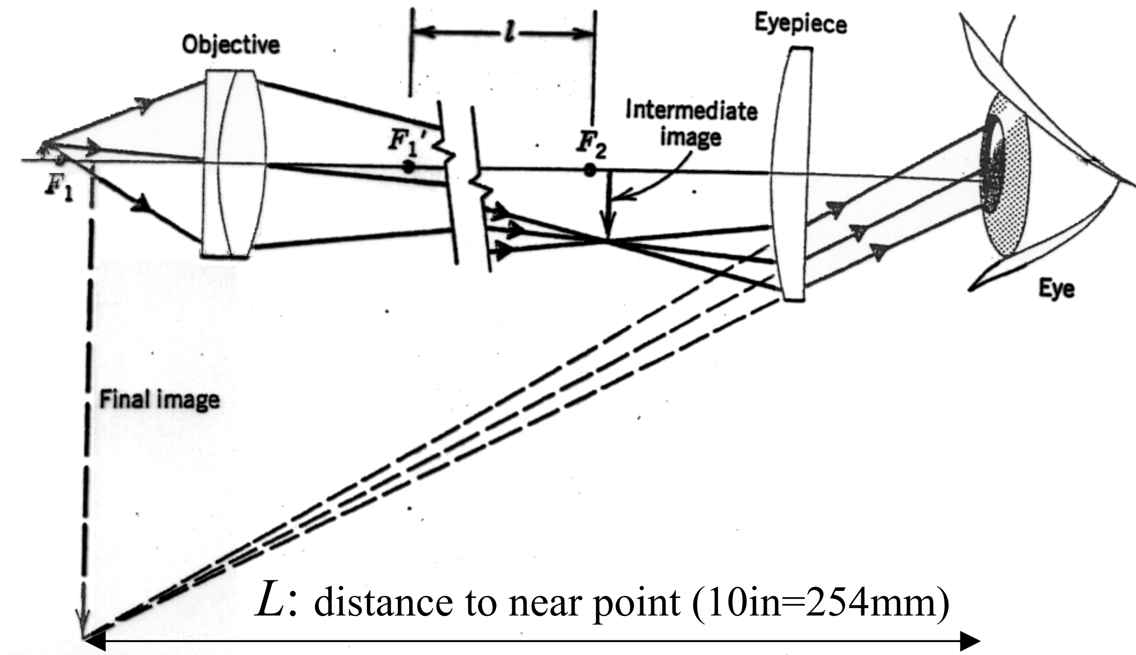


FIGURE 10I
The angle subtended by (*a*) an object at the near point to the naked eye, (*b*) the virtual image of an object inside the focal point, (*c*) the virtual image of an object at the focal point.

from *Optics*
by M. Klein &
T. Furtak

The compound microscope



from *Optics* by M. Klein & T. Furtak

Objective magnification $M_O \approx -\frac{l}{f_O}$

Eyepiece magnification $M_E \approx \frac{L}{f_E}$

Combined magnification

$$M = M_E M_O$$

The telescope

(afocal instrument – angular magnifier)

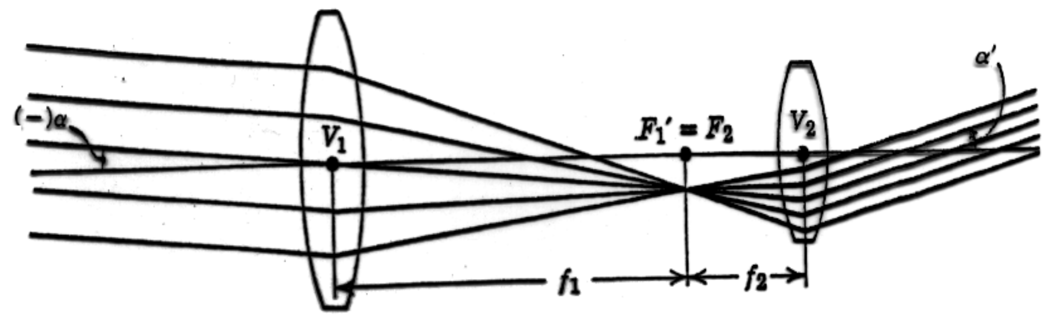


Fig. 3.40 Astronomical telescope.

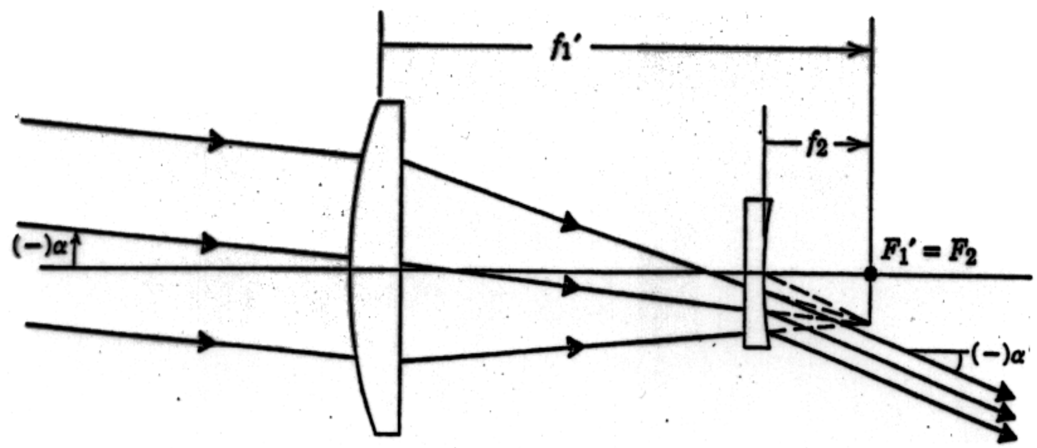
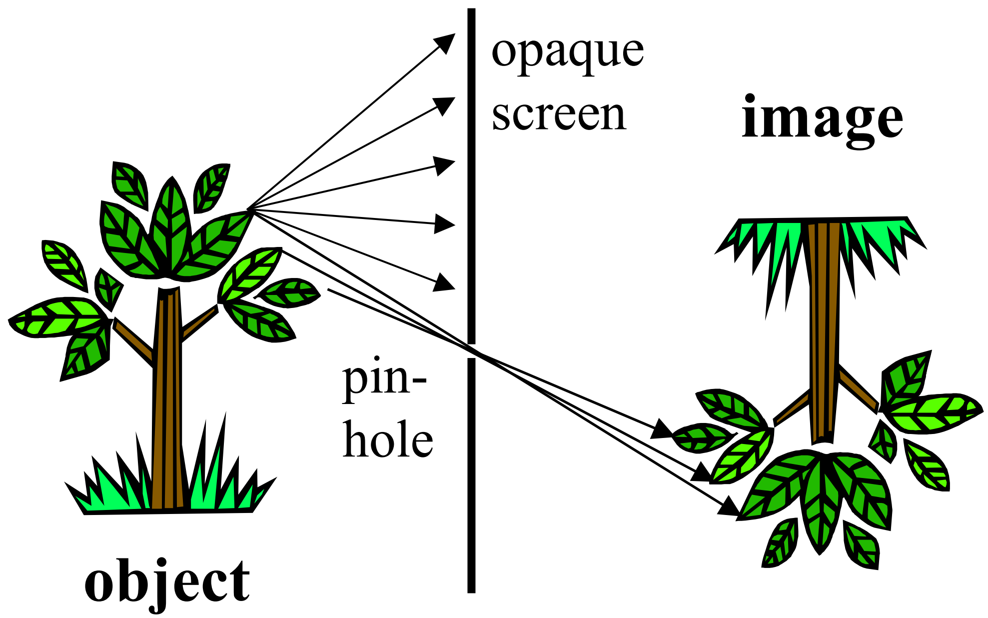


Fig. 3.41 Galilean telescope (fashioned after the first telescope).

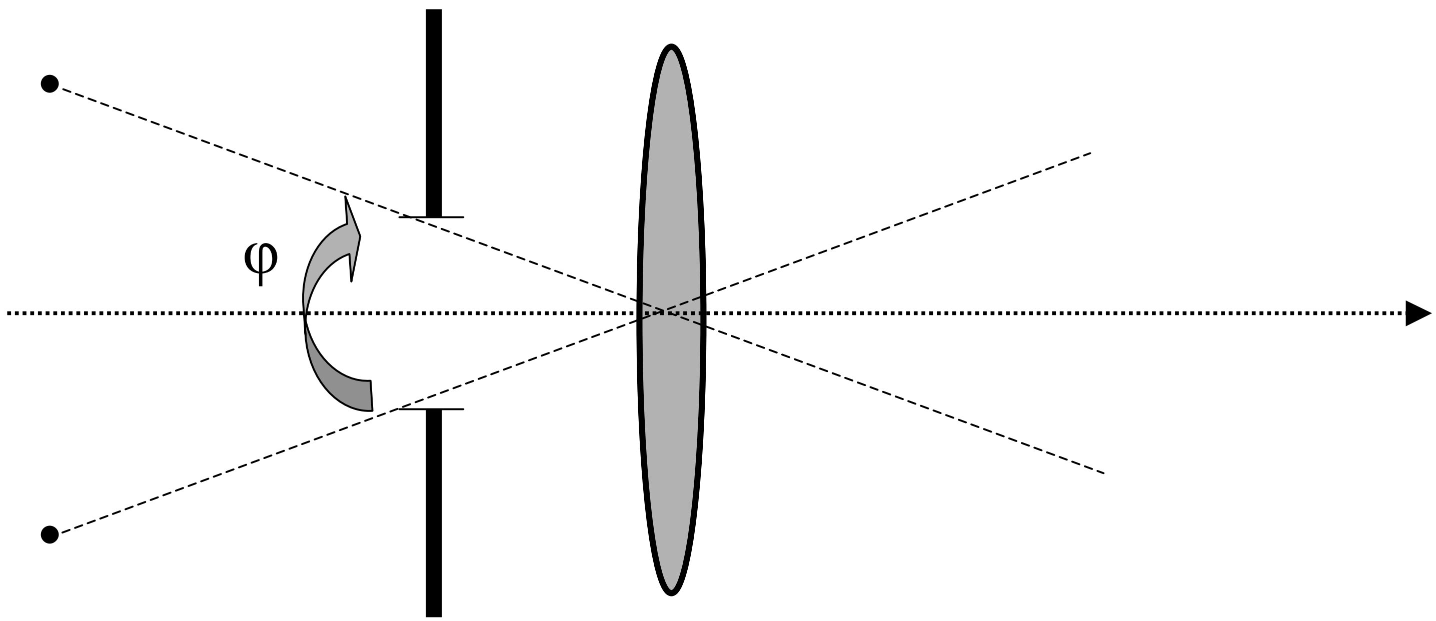
from *Optics*
by M. Klein &
T. Furtak

The pinhole camera



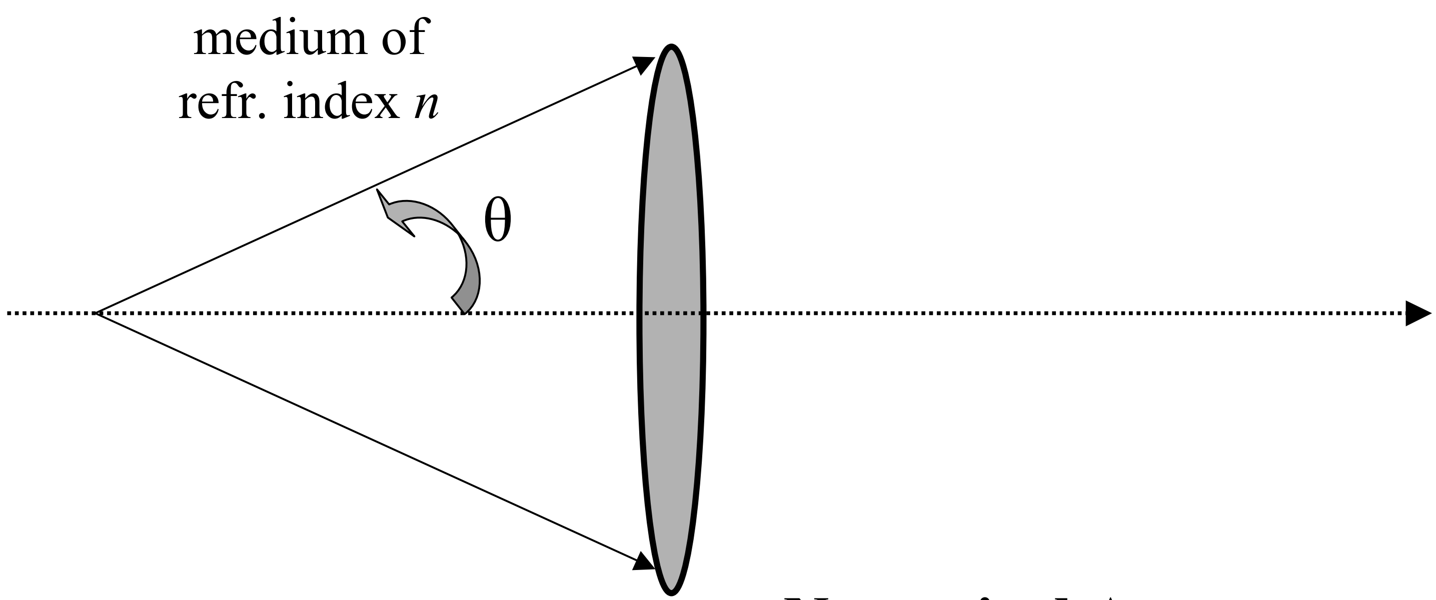
• The pinhole camera blocks all but one ray per object point from reaching the image space ⇒ an image is formed (*i.e.*, each point in image space corresponds to a single point from the object space).
• Unfortunately, most of the light is wasted in this instrument.
• Besides, light diffracts if it has to go through small pinholes as we will see later; diffraction introduces undesirable artifacts in the image.

Field of View (FoV)



FoV=angle that the *chief ray* from an object can subtend towards the imaging system

Numerical Aperture

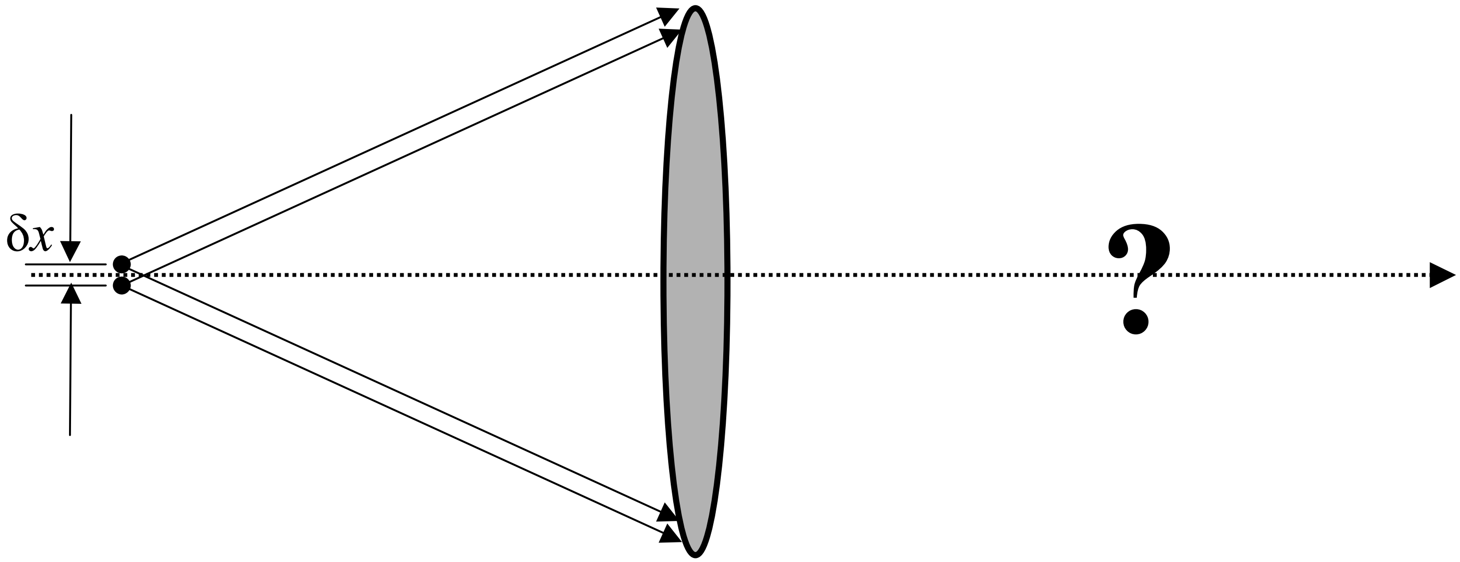


θ: half-angle subtended by the imaging system from an *axial* object

Numerical Aperture
(NA) = $n \sin\theta$

Speed (f/#)=1/2(NA)
pronounced f-number, e.g.
f/8 means (f/#)=8.

Resolution



How far can two distinct point objects be before their images cease to be distinguishable?

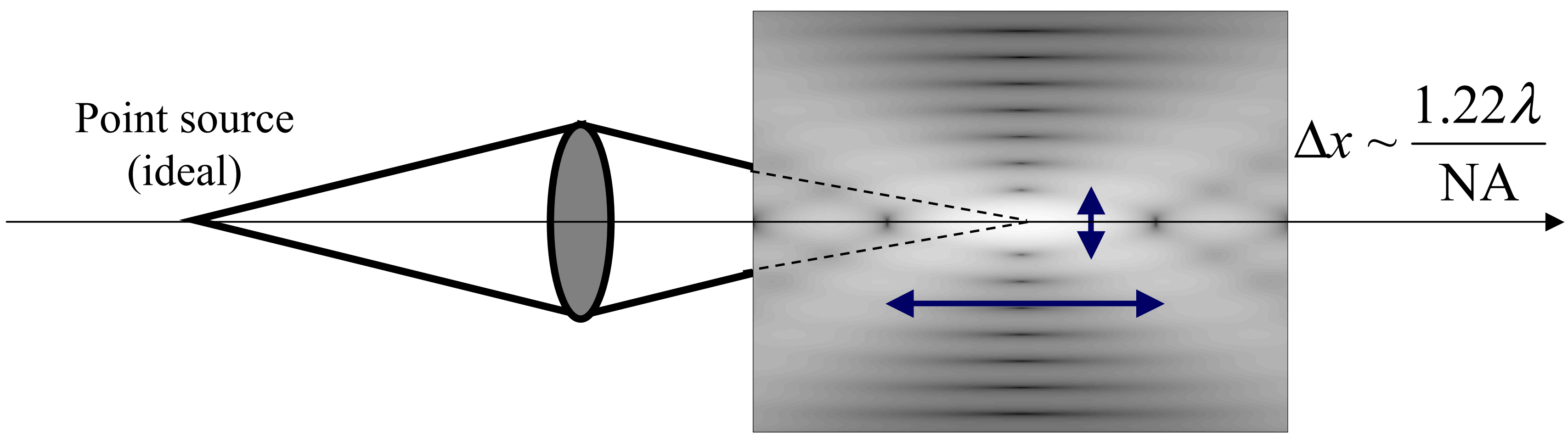
Factors limiting resolution in an imaging system

- Diffraction
- Aberrations
- Noise
  - electronic noise (thermal, Poisson) in cameras
  - multiplicative noise in photographic film
  - stray light
  - speckle noise (coherent imaging systems only)
- Sampling at the image plane
  - camera pixel size
  - photographic film grain size

(Diffraction, Aberrations, Noise →) Intricately related; assessment of image quality depends on the degree that the "inverse problem" is solvable (i.e. its *condition*) 2.717 sp02 for details

Point-Spread Function

Light distribution near the Gaussian (geometric) focus **= PSF** (rotationally symmetric wrt optical axis)

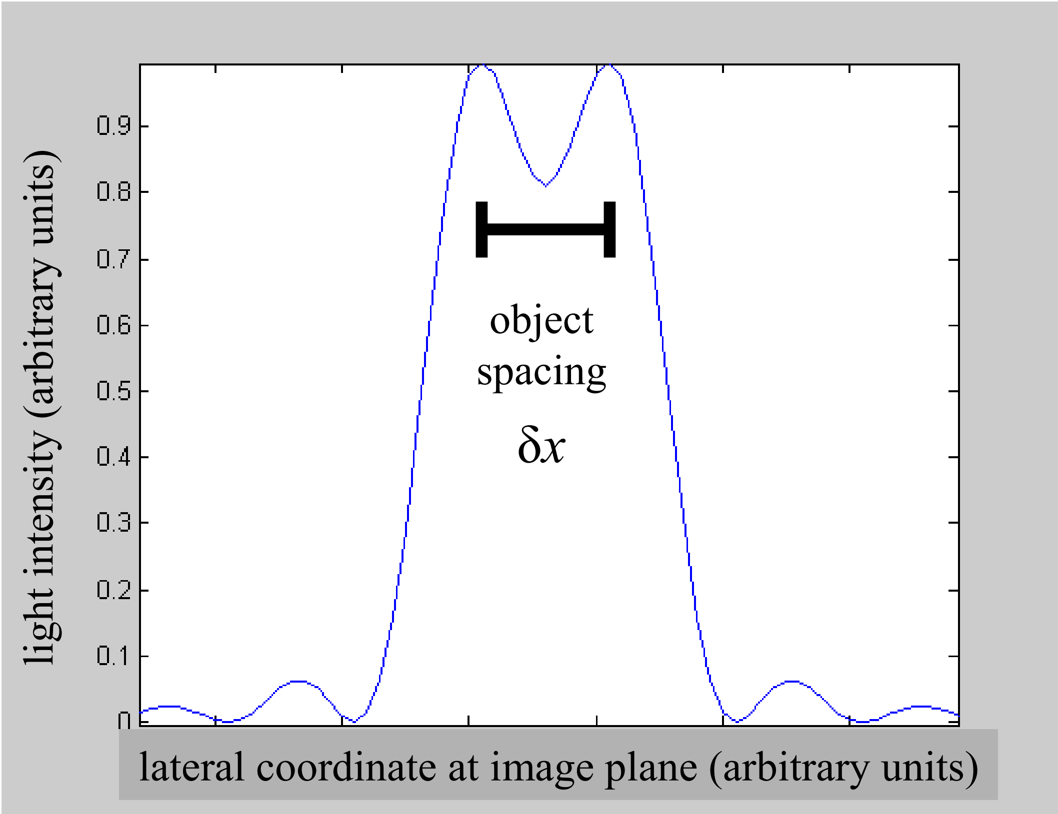


$$\Delta x \sim \frac{1.22\lambda}{\mathrm{NA}}$$

$$\Delta z \sim \frac{2\lambda}{\mathrm{NA}^2}$$

The finite extent of the PSF causes blur in the image

Diffraction limited resolution



Point objects "just resolvable" when

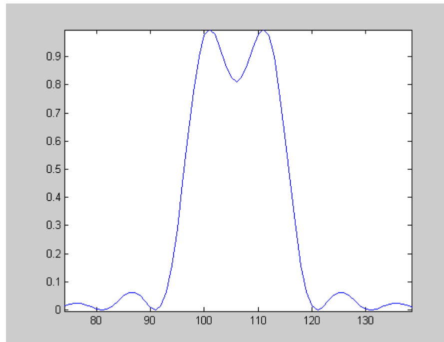
$$\delta x \approx \frac{1.22\lambda}{(\mathrm{NA})}$$

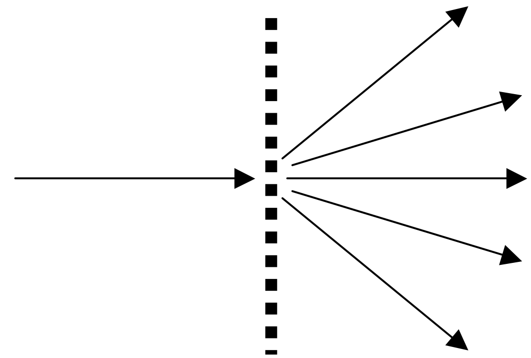
Rayleigh resolution criterion

Wave nature of light

- Diffraction

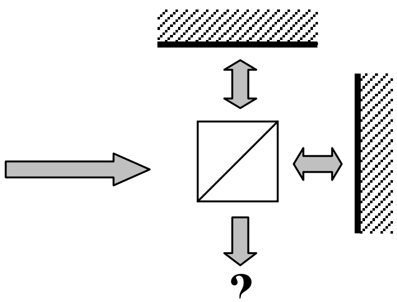
broadening of point images



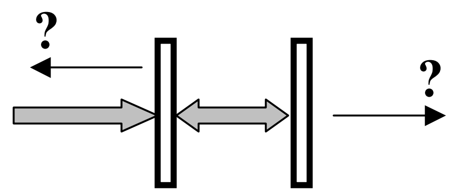


diffraction grating

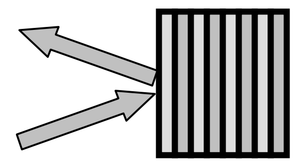
- Inteference



Michelson interferometer



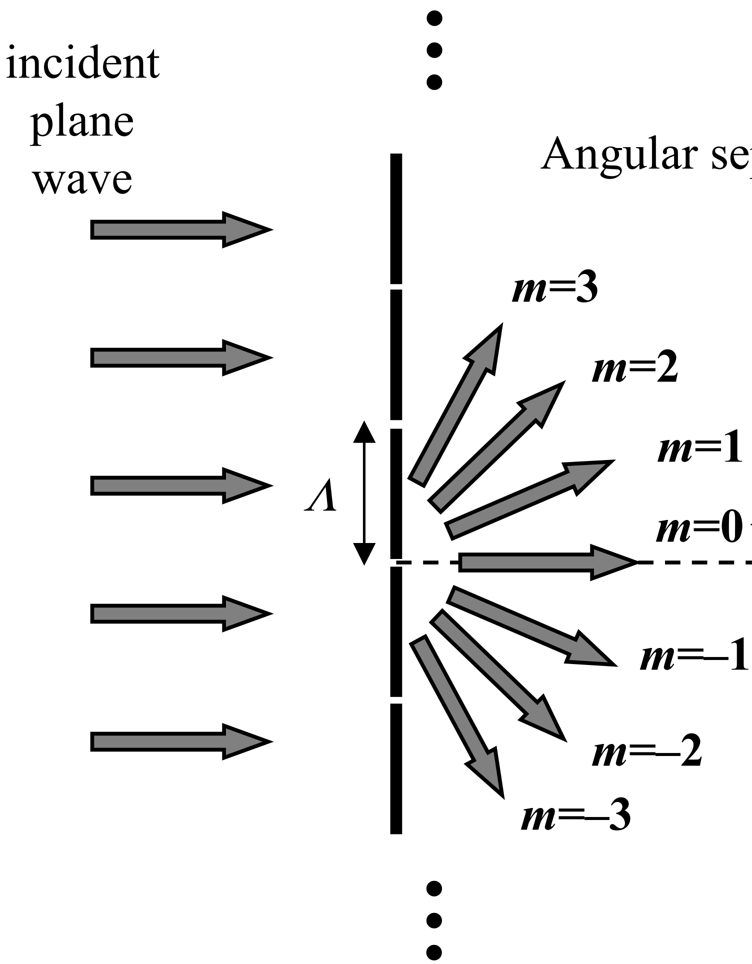
Fabry-Perot interferometer



Interference filter
(or dielectric mirror)

- Polarization: polaroids, dichroics, liquid crystals, ...

Diffraction grating



"straight-through" order or DC term

Grating spatial frequency: **1/Λ**

Angular separation between diffracted orders: **$\Delta\theta \approx 1/\Lambda$**

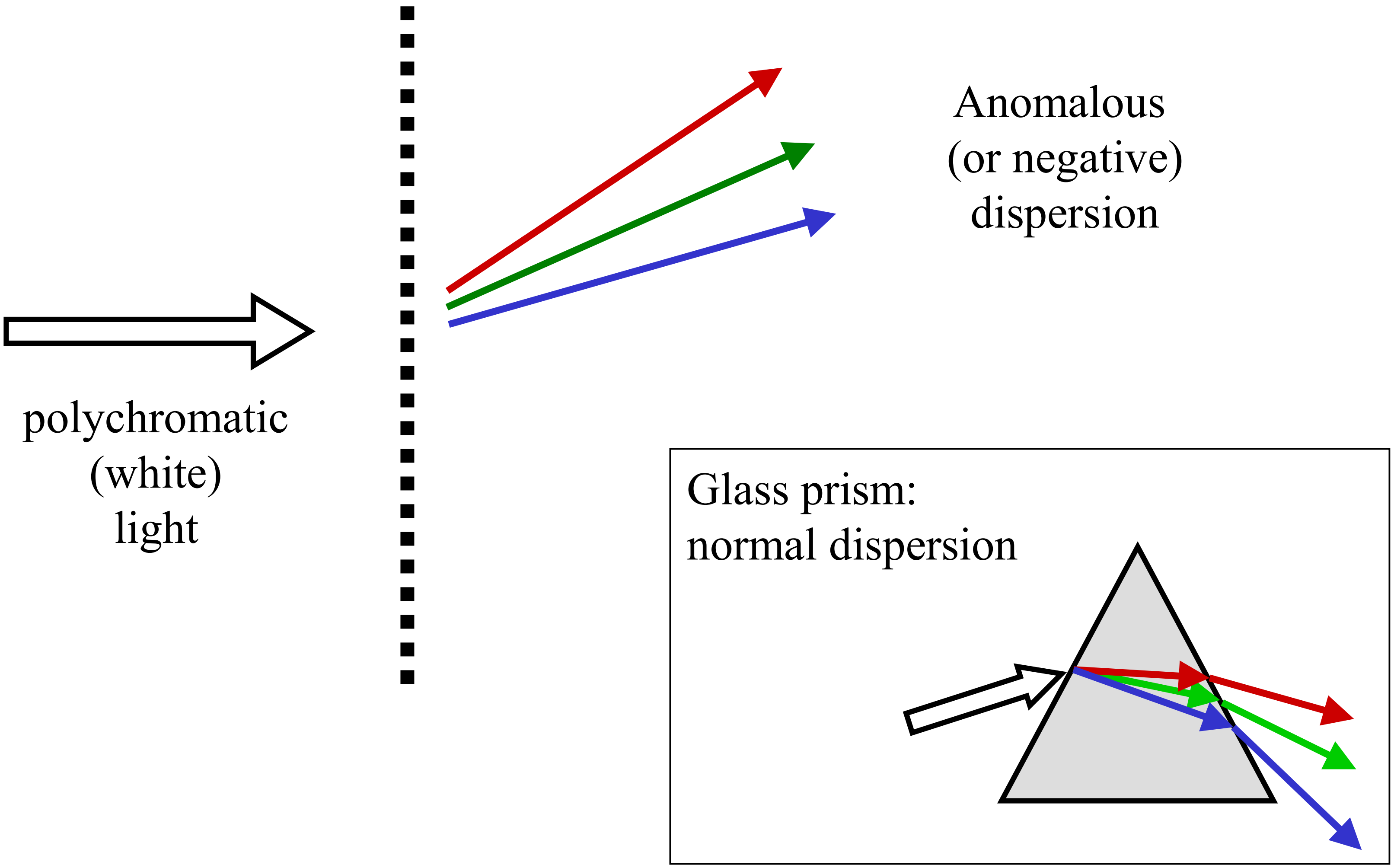
Condition for constructive interference:

$$2\pi \frac{\Lambda}{\lambda} = 2\pi m \qquad (m \text{ integer})$$

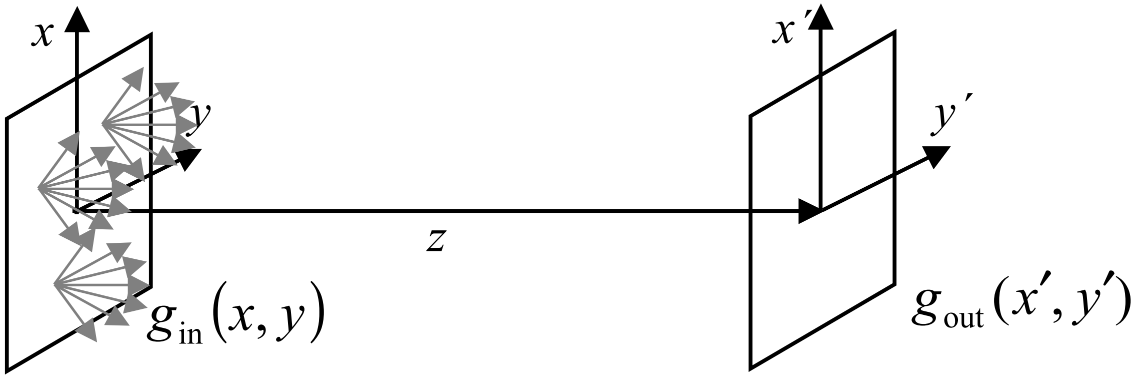
$$\Leftrightarrow \sin\theta = m\frac{\lambda}{\Lambda}$$

diffraction order

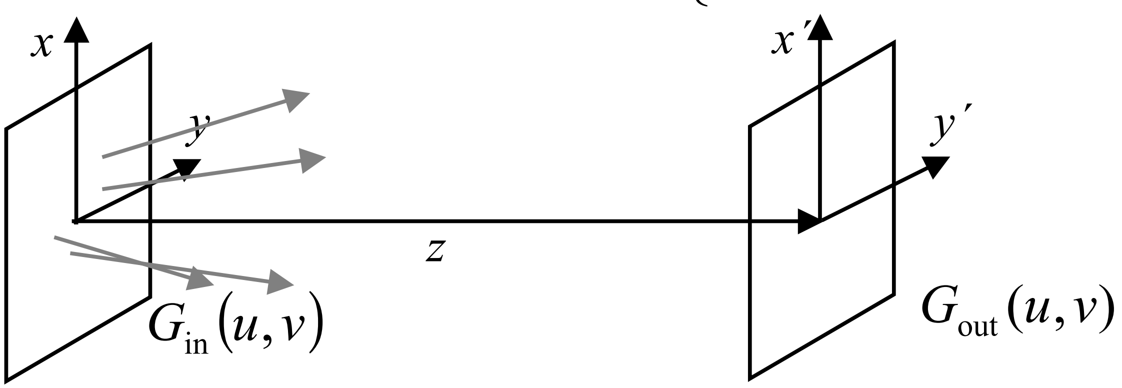
Grating dispersion



Fresnel diffraction formulae



$$g_{\text{out}}(x',y';z)=\frac{1}{i\lambda z}\exp\left\{i2\pi\frac{z}{\lambda}\right\}\int g_{\text{in}}(x,y)\exp\left\{i\pi\frac{(x'-x)^2+(y'-y)^2}{\lambda z}\right\}\mathrm{d}x\mathrm{d}y$$



$$G_{\text{out}}(u,v;z)=\exp\left\{i2\pi\frac{z}{\lambda}\right\}G_{\text{in}}(u,v)\exp\left\{-i\pi\lambda z\left(u^2+v^2\right)\right\}$$

Fresnel diffraction
as a linear, shift-invariant system

Thin transparency $t(x,y)$

$$h(x,y)=\frac{1}{i\lambda z}\exp\left\{i2\pi\frac{z}{\lambda}\right\}\exp\left\{i\pi\frac{x^2+y^2}{\lambda z}\right\}$$

$g_1(x,y)$

$g_2(x,y)=$
$=g_1(x,y)t(x,y)$

impulse response

convolution

output amplitude

$g_3(x',y')=$
$=g_2(x,y)*h(x,y)$

Fourier transform

Fourier transform

(≡plane wave spectrum)

$G_2(u,v)$

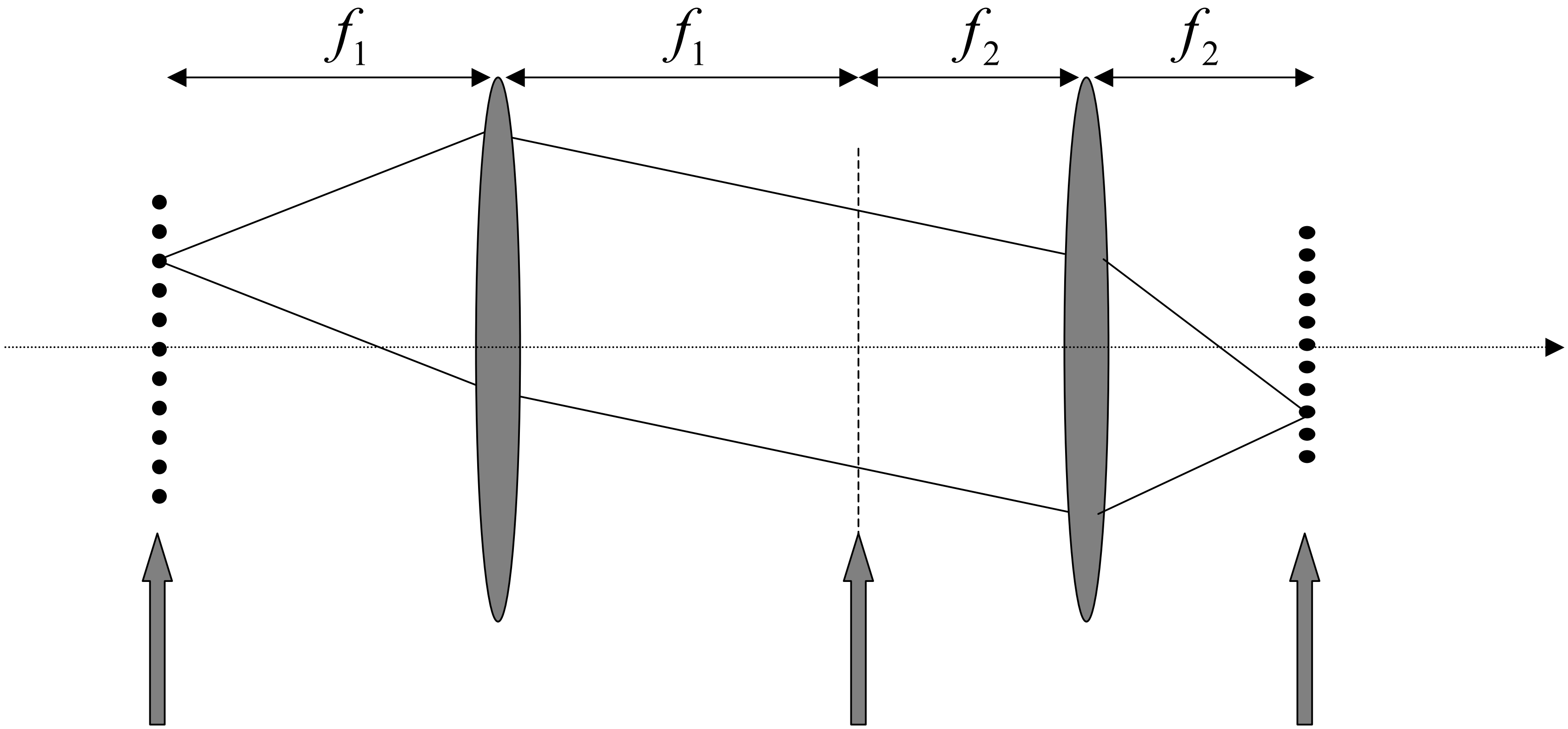
transfer function

multiplication

$G_3(u,v)=$
$=G_2(u,v)H(u,v)$

$$H(u,v)=\exp\left\{i2\pi\frac{z}{\lambda}\right\}\exp\left\{-i\pi\lambda\left(u^2+v^2\right)z\right\}$$

The 4F system



$$g_1(x, y)$$

object plane

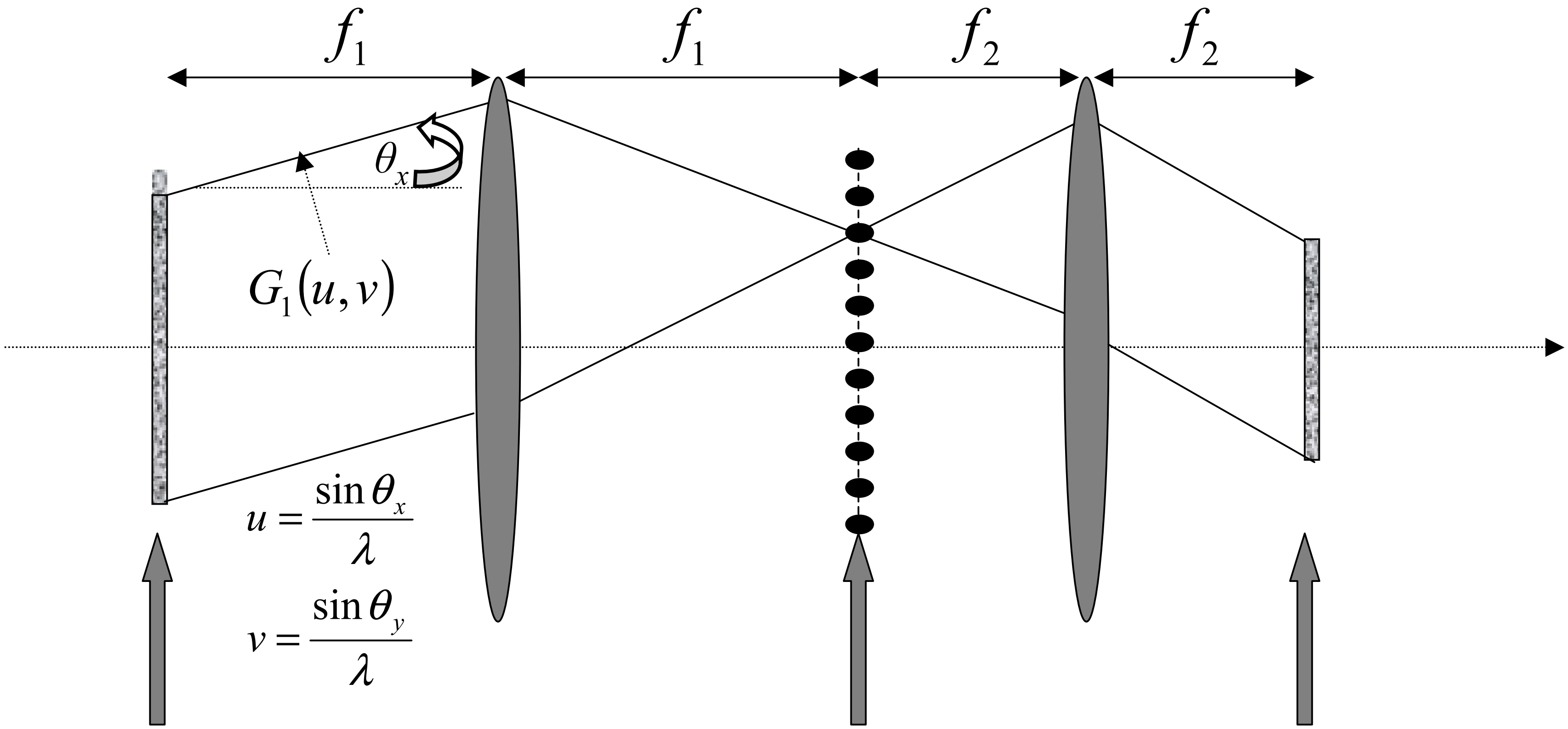
$$G_1\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right)$$

Fourier plane

$$g_1\left(-\frac{f_1}{f_2}x', -\frac{f_1}{f_2}y'\right)$$

Image plane

The 4F system



$$g_1(x,y)$$

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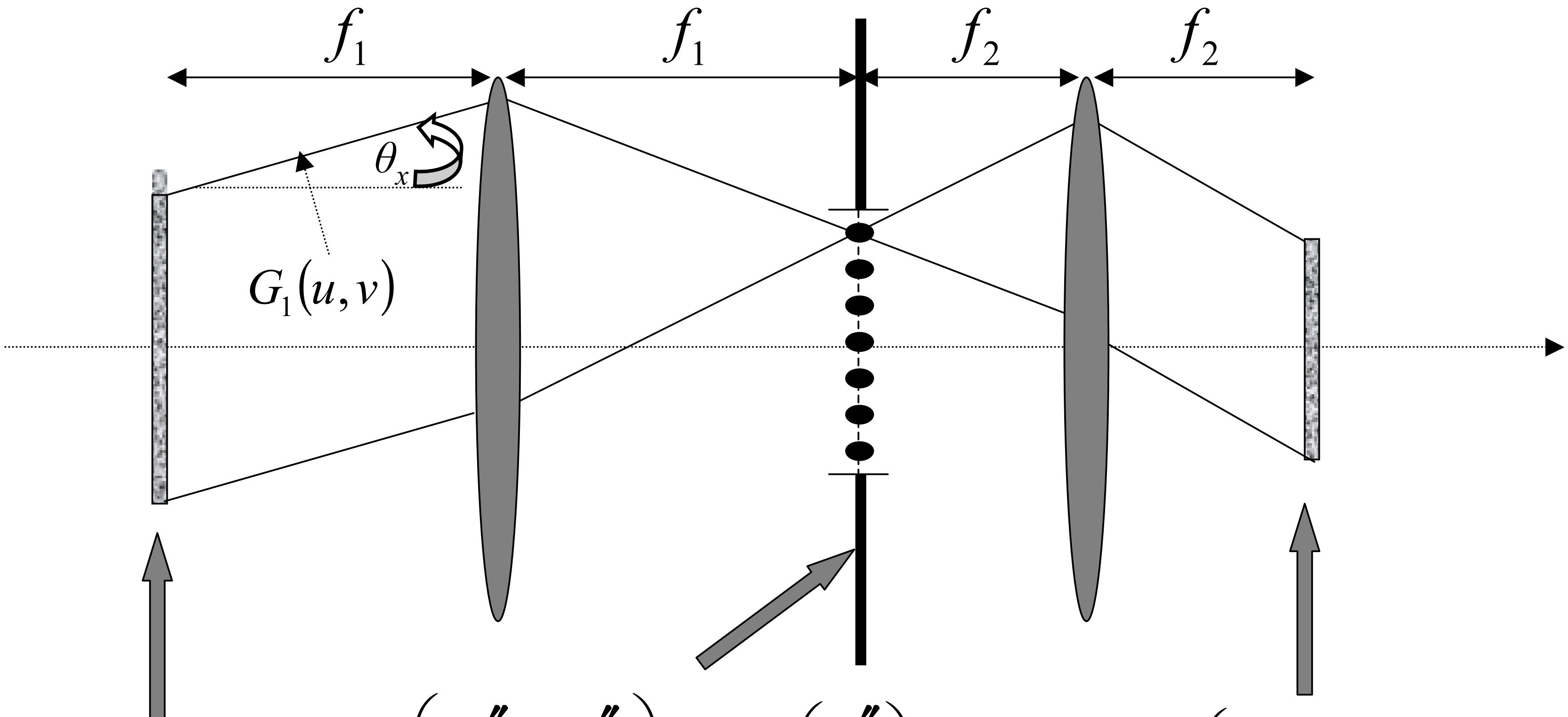
$$G_1\left(\frac{x''}{\lambda f_1}, \frac{y''}{\lambda f_1}\right)$$

Fourier plane

$$g_1\left(-\frac{f_1}{f_2}x', -\frac{f_1}{f_2}y'\right)$$

Image plane

The 4F system with FP aperture



$$g_1(x,y)$$

object plane

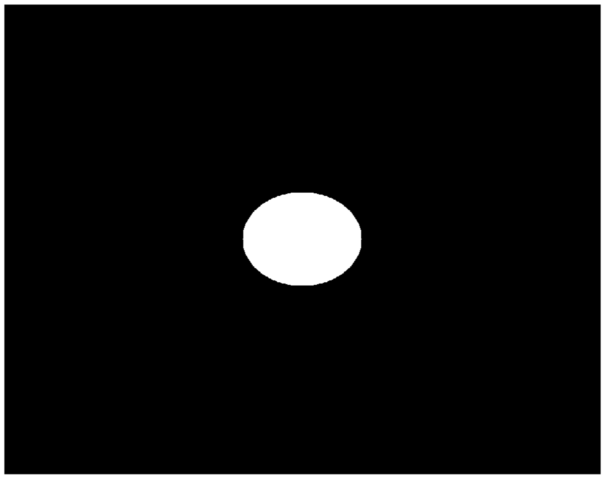
$$G_1\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times \mathrm{circ}\left(\frac{r''}{R}\right)$$

Fourier plane: aperture-limited

$$(g_1 * h)\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

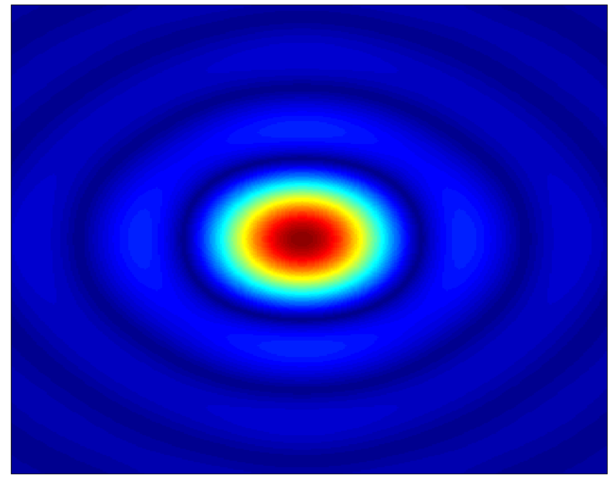
Image plane: blurred
(i.e. low-pass filtered)

The 4F system with FP aperture



Transfer function:
circular aperture

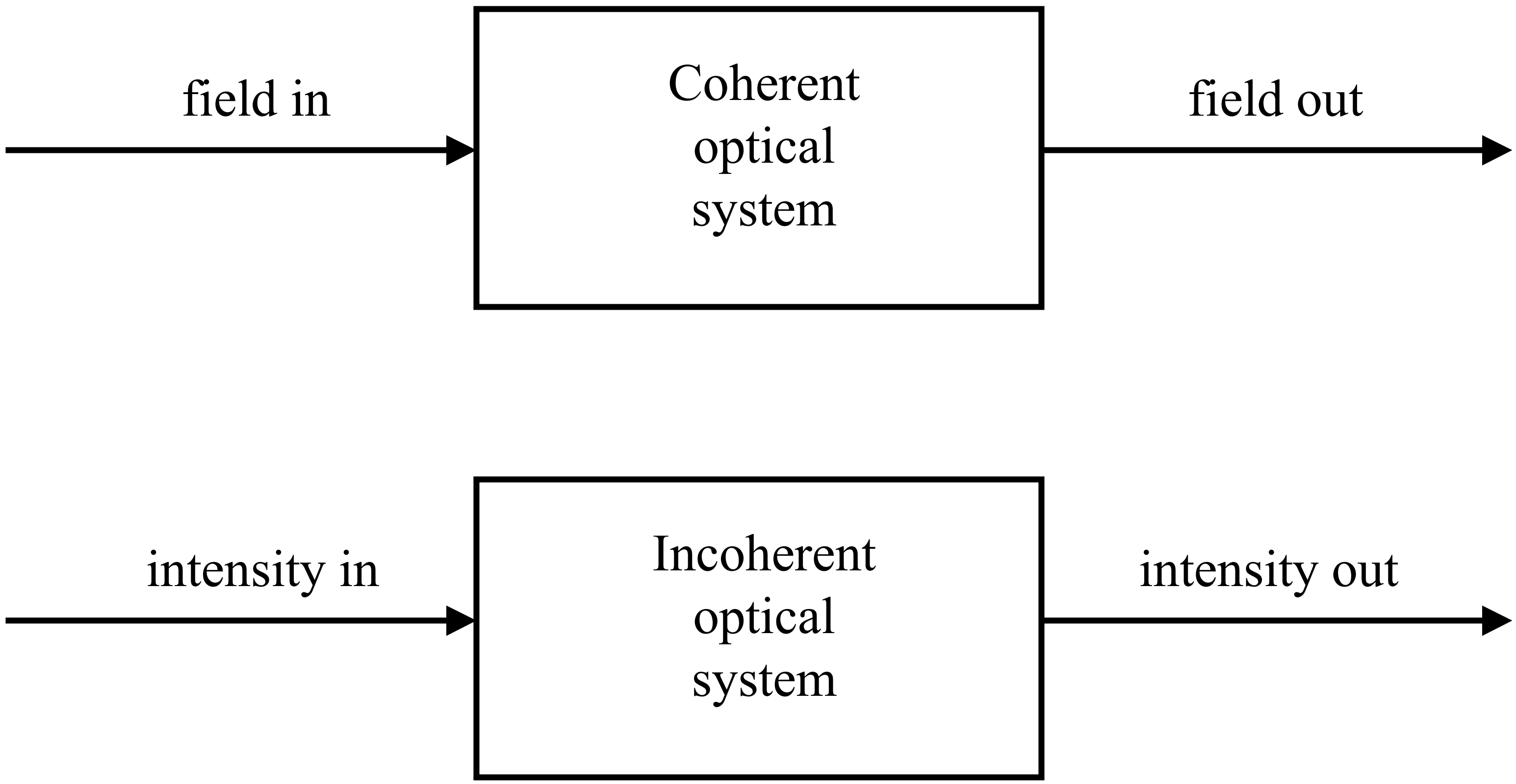
$$\mathrm{circ}\left(\frac{r''}{R}\right)$$



Impulse response:
Airy function

$$\mathrm{jinc}\left(\frac{r'R}{\lambda f_2}\right)$$

Coherent vs incoherent imaging



Coherent vs incoherent imaging

Coherent impulse response
(field in $\Rightarrow$ field out)

$$h(x,y)$$

Coherent transfer function
(FT of field in $\Rightarrow$ FT of field out)

$$H(u,v) = \mathrm{FT}\{h(x,y)\}$$

Incoherent impulse response
(intensity in $\Rightarrow$ intensity out)

$$\tilde{h}(x,y) = |h(x,y)|^2$$

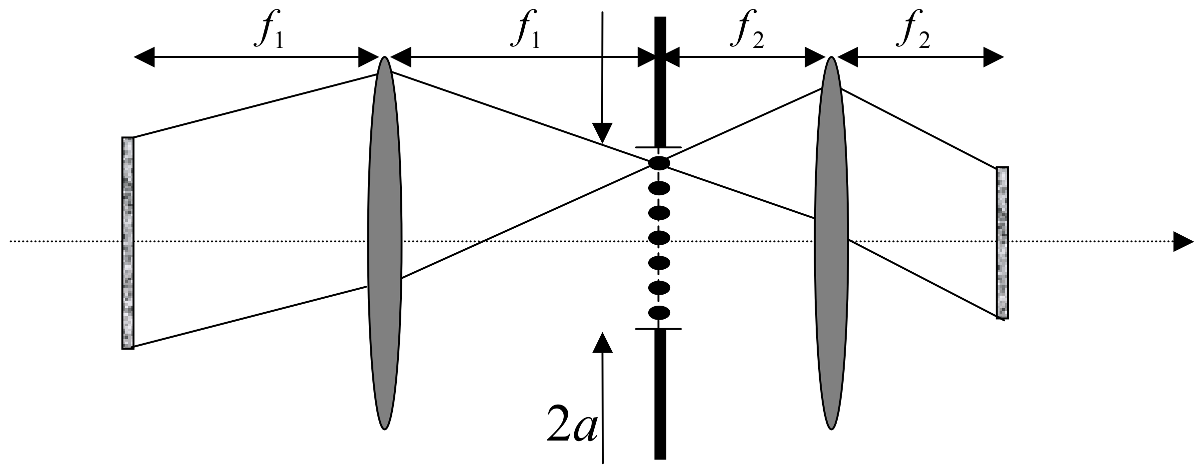
Incoherent transfer function
(FT of intensity in $\Rightarrow$ FT of intensity out)

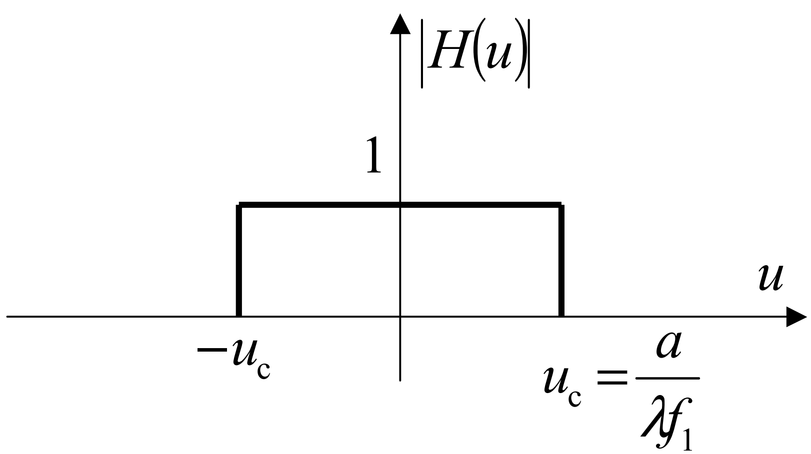
$$\tilde{H}(u,v) = \mathrm{FT}\{\tilde{h}(x,y)\} = H(u,v) \otimes H(u,v)$$

$\left|\tilde{H}(u,v)\right|$: Modulation Transfer Function (MTF)

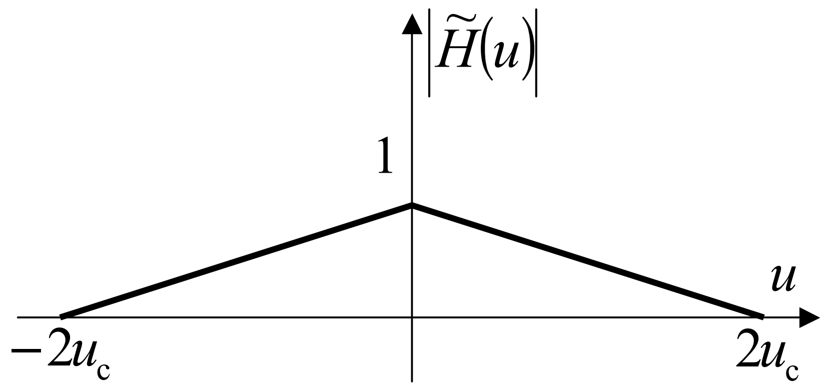
$\tilde{H}(u,v)$: Optical Transfer Function (OTF)

Coherent vs incoherent imaging



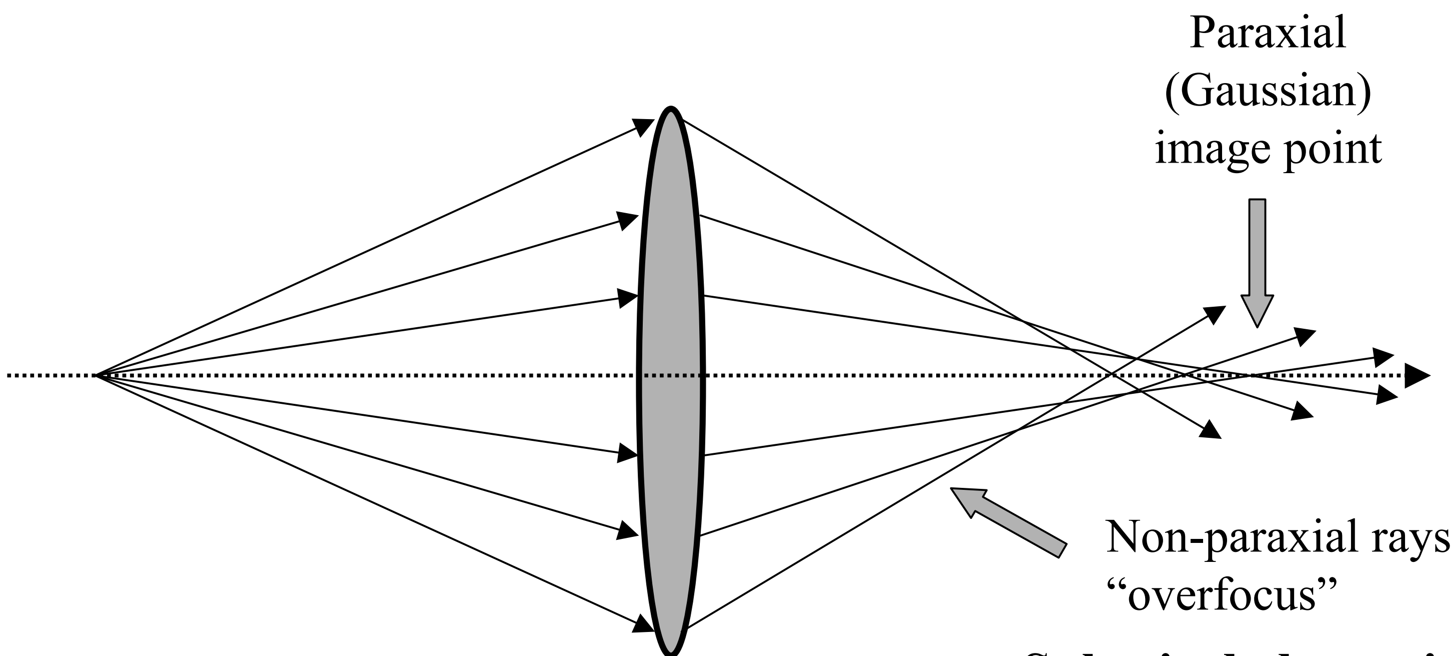


Coherent illumination



Incoherent illumination

Aberrations: geometrical



Spherical aberration

• Origin of aberrations: nonlinearity of Snell’s law ($n \sin\theta$=const., whereas linear relationship would have been $n\theta$=const.)
• Aberrations cause practical systems to perform *worse* than diffraction-limited
• Aberrations are best dealt with using optical design software (Code V, Oslo, Zemax); optimized systems usually resolve ~3-5λ (~1.5-2.5μm in the visible)

Aberrations: wave

Aberration-free impulse response $h_{\substack{\text{diffraction}\\\text{limited}}}(x,y)$

Aberrations introduce additional phase delay to the impulse response

$$h_{\text{aberrated}}(x,y)=h_{\substack{\text{diffraction}\\\text{limited}}}(x,y)\,\mathrm{e}^{i\varphi_{\text{aberration}}(x,y)}$$

Effect of aberrations on the MTF