MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Fall '01**

Problem Set #9 Posted Nov. 21, 2001 — Due Wednesday Nov. 28, 2001

1. **Parseval's theorem.** Let $f(x)$ denote a square-integrable and sufficiently smooth function and $F(u)$ its Fourier transform.

   1.a) Show that

   $$\int_{-\infty}^{+\infty} \left|f(x)\right|^2 \mathrm{d}x = \int_{-\infty}^{+\infty} \left|F(u)\right|^2 \mathrm{d}u.$$

   1.b) Argue that this result expresses energy conservation in the context of an optical system.

2. **Telescopes and magnification.** A 4–F system (*i.e.*, a telescope) is constructed with two lenses L1, L2 of focal lengths f_1 and f_2, respectively, so that $f_1 > f_2$. Light enters L1 first, and then L2.

   2.a) Is the system operating as a magnifier or demagnifier in the lateral coordinate?

   2.b) Is the system operating as a magnifier or demagnifier in the angular coordinate?

   2.c) Are the two answers above consistent and why? (Discuss as thoroughly as you can.)

3. The Fourier transform may be regarded as a mapping of functions into their transforms and therefore satisfies the definition of a system.

   3.a) Is this system linear?

   3.b) Can you specify a transfer function for this system? If yes, what is it? If no, why not?

4. **Spatial filtering.** Figure A (next page) shows a 4–F optical imaging system with 1:1 magnification. Lenses L1, L2 are identical with focal length $f = 10$cm. Figure B shows the amplitude transmittivity $t(x)$ of the thin transparency which is located at the input plane. The modulation of the transparency is periodic with period $L = 20\mu$m. The transparency is illuminated by a plane wave, which is generated by a laser at wavelength $\lambda = 0.5\mu$m, and is incident in a direction parallel to the optical axis. At the Fourier plane of L1 there is another transparency which transmits the optical field within distances $d_\mathrm{i} = 1.5$mm and $d_\mathrm{o} = 8.5$mm from the optical axis, and is opaque everywhere else (see also Fig. A.) Compute

the simplest possible expression for the amplitude distribution $a(x')$ at the image plane of the system. Treat the system as 1D and ignore the effect of the finite lens apertures.

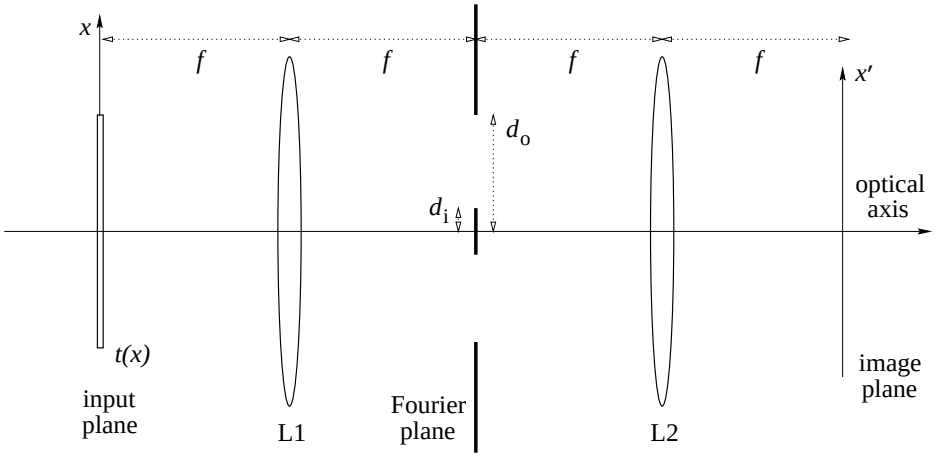

Figure A: 4F imaging system with Fourier-plane filter.

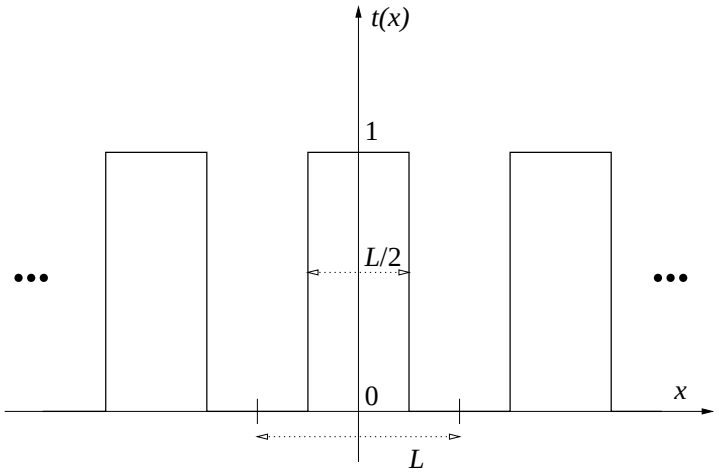

Figure B: Amplitude transmittance of the input transparency.