FEYNMAN RULES FOR TREE GRAPHS
IN QED, QCD AND THE STANDARD MODEL

2→2 cross section formula
\[ \sigma = \frac{1}{4[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}} |A|^2 \frac{d\text{Lips}}{d\omega} \frac{1}{(2\pi)^{3n-4}} \]

1→2 decay formula
\[ d\text{Lips} = \frac{n \frac{d^3\Phi_n}{2E_n}}{2E_n} \delta(p_c - p_f) \]
\[ d\Gamma = \frac{1}{2m_1} |A|^2 d\text{Lips} (m_1; p_2, p_3). \]

Note that for two identical particles in the final state an extra factor of \( \frac{1}{2} \) must be included in these formulae.

The amplitude \( A \) is the invariant matrix element for the process under consideration, and is given by the Feynman rules of the relevant theory. For particles with non-zero spin, unpolarised cross sections are formed by averaging over initial spin components and summing over final.

\( A \) is a complex number, function of momenta & helicity of the particles.

F.1. qed: rules for tree graphs

F.1.1 External particles

Spin-\( \frac{1}{2} \). For each fermion or antifermion line entering the graph include the spinor (column vector in 4-d space). \( \bar{\psi} \) creation
\[ \bar{\psi} \rightarrow u(p, s), \quad \text{or} \quad \bar{\psi}(p, s) \rightarrow \psi(-p, -s) \]
and for spin-\( \frac{1}{2} \) particles leaving the graph, the spinor (row vector)
\[ \psi \text{ creation} \]

Photons. For each photon line entering the graph include a polarisation vector \( v_\mu(k, \lambda) \) in 4-d Lorentz Space
\[ 4 \text{ d Lorentz Space} \]
And for photons leaving the graph the vector
\[ \xi^\mu(k', \lambda') \text{ for real photons} \]
Useful Formulae

\[ \sigma \leq (2J+1) \frac{8\pi}{5} \]

\[ h = 6.58 \times 10^{-23} \text{ GeV sec} = 1 \]
\[ \hbar = 0.197 \text{ GeV F} = 1 \]

\[ (1 \text{ GeV})^{-2} = 0.389 \text{ mb} \]
\[ \alpha = \frac{\alpha^2}{4\pi} = \frac{1}{137} \]

\[ x^\mu = (x, x) \]
\[ p^\mu = (E, p) = \left( \frac{\partial}{\partial t} - \nabla \right) = i \partial^\mu \]
\[ p \cdot x = E t - p \cdot x, \quad p^2 = p^\mu p_\mu = E^2 - p^2 = m^2 \]
\[ (\sigma^2 + m^2)\phi = 0, \quad (i\gamma^\mu \partial_\mu - m)\psi = 0. \]

In an electromagnetic field, \( i \partial^\mu \rightarrow i \partial^\mu + eA^\mu \) (charge \( -e \))

\[ j^\mu = -ie(\phi^\mu \partial_\mu \psi - \psi \partial_\mu \phi^\mu), \quad j^\mu = -e\bar{\psi}\gamma^\mu \psi \]

**\( \gamma \)-Matrices**

\[ \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\sigma^\mu \gamma^\nu, \quad \gamma^\mu = \gamma^0 \gamma^\mu \gamma^0. \]
\[ \gamma^0 = \gamma^0, \quad \gamma^0 \gamma^0 = 1; \quad \gamma^k = -\gamma^k, \quad \gamma^k \gamma^k = -1, \quad k = 1, 2, 3. \]
\[ \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3, \quad \gamma^5 \gamma^5 + \gamma^5 \gamma^5 = 0, \quad \gamma^{51} = \gamma^5. \]

(Trace theorems on pages 123 and 261)

**Standard representation:**

\[ \gamma^0 = \beta = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \gamma = \beta \alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]
\[ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

**Spinors**

\[ u = u^\gamma \]
\[ \bar{u} = \bar{u} \gamma^0 \]
\[ u'(\gamma^1 + 2E \delta_{\gamma^1}) = 0, \quad \bar{u}'(\gamma^0 + 2m \delta_{\gamma^0}) = 0 \]
\[ u'(\gamma - \gamma^1) u = u_1, \quad (1 + \gamma^1) u = u_R. \]

If \( m = 0 \) or \( E \gg m \), then \( u_1 \) has helicity \( \lambda = -\frac{1}{2} \), \( u_R \) has \( \lambda = \frac{1}{2} \).
TABLE 6.2
Feynman Rules for $-i\mathcal{M}$

<table>
<thead>
<tr>
<th>Multiplicative Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>External Lines</strong></td>
</tr>
<tr>
<td>Spin 0 boson (or antiboson)</td>
</tr>
<tr>
<td>Spin $\frac{1}{2}$ fermion (in, out)</td>
</tr>
<tr>
<td>antifermion (in, out)</td>
</tr>
<tr>
<td>Spin 1 photon (in, out)</td>
</tr>
<tr>
<td><strong>Internal Lines—Propagators (need $+i\epsilon$ prescription)</strong></td>
</tr>
<tr>
<td>Spin 0 boson</td>
</tr>
<tr>
<td>Spin $\frac{1}{2}$ fermion</td>
</tr>
<tr>
<td>Massive spin 1 boson</td>
</tr>
<tr>
<td>Massless spin 1 photon (Feynman gauge)</td>
</tr>
<tr>
<td><strong>Vertex Factors</strong></td>
</tr>
<tr>
<td>Photon—spin 0 (charge $-e$)</td>
</tr>
<tr>
<td>Photon—spin $\frac{1}{2}$ (charge $-e$)</td>
</tr>
</tbody>
</table>

Loops: $\int d^4k/(2\pi)^4$ over loop momentum; include $-1$ if fermion loop and take the trace of associated $\gamma$-matrices

Identical Fermions: $-1$ between diagrams which differ only in $e^- \leftrightarrow e^-$ or initial $e^- \leftrightarrow$ final $e^+$
F.1.2 Propagators

Spin-0

\[ \frac{1}{p^2} \]

Spin-\(\frac{1}{2} \)

\[ \frac{1}{p^2 - m^2 + i\epsilon} \]

Photon

\[ \frac{i}{k^2} \left( -\frac{\not{q}^2}{k^2} + (1 - \xi) \frac{k^2 k'^2}{k^2} \right) \]

\[ \frac{i}{k^2} \left( \frac{\not{k}^2}{k^2} \right) \]

for a general \( \xi \) gauge. Calculations are usually performed in Lorentz or Feynman gauge with \( \xi = 1 \) and photon propagator
F.1.3 Vertices

Spin-0

\[ -ie(p + p')_\mu \]
(for charge \( +e \))

\[ 2ie^2 g_{\mu\nu} \]

Spin-\( \frac{1}{2} \)

\[ -ie\gamma_\mu \]
(for charge \( +e \))
F.2 QCD: rules for tree graphs

F.2.1 External particles

**Quarks.** The SU(3) colour degree of freedom is not written explicitly: the spinors have $3(\text{colour}) \times 4(\text{Dirac})$ components

\[
\text{ingoing: } u(p, s) \quad \text{or} \quad v(p, s) \\
\text{outgoing: } \bar{u}(p', s') \quad \text{or} \quad \bar{v}(p', s')
\]
as for QED.

**Gluons.** Besides the spin-1 polarisation vector, external gluons also have a 'colour polarisation' vector $a^\mu(\alpha = 1, 2, \ldots, 8)$ specifying the particular colour state involved:

\[
G_\mu^\alpha
\]

\[
\text{ingoing: } e_\mu^\alpha(k, \lambda) a^\alpha \\
\text{outgoing: } e_\mu^\alpha(k', \lambda') a^\alpha
\]

F.2.2 Propagators

**Quark**

\[
\frac{i}{\not{p} - m} = \frac{i}{\not{p}^2 - m^2}
\]

**Gluon**

\[
\frac{i}{q^2} \left( -g^{\mu\nu} + (1 - \xi) \frac{q^\mu q^\nu}{q^2} \right) \delta^{\alpha\beta}
\]

for a general $\xi$ gauge. In Feynman gauge this reduces to

\[
\frac{i}{q^2} (-g^{\mu\nu}) \delta^{\alpha\beta}
\]

which is usually the most convenient form.

\[
\lambda_1 \equiv \begin{pmatrix}
\sigma^1 \\
\sigma^2 \\
\sigma^3
\end{pmatrix}, \begin{pmatrix}
0 \\
0 \\
\sigma^3
\end{pmatrix}, \begin{pmatrix}
\sigma^1 \\
\sigma^2 \\
0
\end{pmatrix}, \begin{pmatrix}
0 \\
\sigma^3 \\
0
\end{pmatrix}
\]

\[
\lambda_3 = \left( \sigma_{+1} + i \sigma_{-2} \right) / \sqrt{3}
\]

F.2.3 Vertices
It is important to remember that the rules given above are only adequate for tree diagram calculations in QCD (see Chapter 14.4).

F.3 The standard model of electroweak interactions: rules for tree graphs.

F.3.1 External particles

Leptons and quarks

Ingoing: \( u(p, s) \) or \( v(p, s) \)

Outgoing: \( \bar{u}(p', s') \) or \( \bar{v}(p', s') \).

Vector bosons

Ingoing: \( \varepsilon_\mu^\lambda(k, \lambda) \)

Outgoing: \( \varepsilon_\nu^\hat{\lambda}(k', \hat{\lambda}) \).

Take \( k \parallel \Sigma \), \( \lambda = \pm 1 \), \( \varepsilon_\pm = \pm \sqrt{\frac{1}{2}}(0, 1, \pm i, 0) \)

F.3.2 Propagators

Leptons and quarks

\[
\frac{i}{p - m} = i \frac{p + m}{p^2 - m^2}.
\]
Vector mesons (U gauge)

\[ W^\pm, Z^0 \quad = \quad \frac{i}{k^2 - M_V^2} \left( -g^\mu + k^\mu k^\nu / M_V^2 \right) \quad -M_V^2 \Rightarrow -M_V^2 + i M_V \]

where the mass \( M_W \) of the charged W bosons is given by

\[ \frac{G_F}{2^{1/2}} = \frac{g^2}{8 M_W^2} \]

with \( g \sin \theta_W = e \) (where, in our convention, \( e > 0 \)) so that

\[ M_W = \frac{\sqrt{(1 + \Delta r)} e (m_e)}{2^{5/4} G_F^{1/2} \sin \theta_W} \approx \left( \frac{37.3}{\sin \theta_W} \right) \text{GeV}/c^2 \sqrt{(1 + \Delta r)} \]

The mass of the neutral Z boson is related to that of the charged W bosons by

\[ M_Z = M_W / \cos \theta_W . \]

**Higgs scalar**

\[ \quad = \quad \frac{i}{p^2 - \mu^2} \]

**F.3.3 Vertices**

**Charged current weak interactions**

\[ \quad -i \frac{g}{2^{1/2} \gamma_5} \frac{1 - \gamma_3}{2} \]

\[ \quad -i \frac{g}{2^{1/2} \cos \theta_W \gamma_5} \frac{1 - \gamma_3}{2} \]
Neutral current weak interactions

Massless neutrinos

Massive fermions

where 
\[ c_L = -\frac{1}{2} + \sin^2 \theta_W, \quad c_L = +\frac{1}{2} - \frac{3}{2} \sin^2 \theta_W, \quad c_L = -\frac{1}{2} + \frac{1}{2} \sin^2 \theta_W, \]
\[ c_R = \frac{1}{2} \left( 1 - \gamma_5 \right), \quad c_R = -\frac{1}{2} \sin^2 \theta_W, \quad c_R = \frac{1}{2} \sin^2 \theta_W, \]
\[ c_R = \frac{1}{2} + \frac{1}{2} \sin^2 \theta_W, \]

(massless neutrinos have \( c_L = \frac{1}{2}; c_R = 0 \)).

Vector boson couplings. (a) Trilinear couplings

\[ YW^+W^- \text{ vertex} \]

\[ i e \left[ g_{\mu\nu}(k_1 - k_2) + g_{\mu\nu}(k_2 - k_2) + \frac{1}{2} (k_1 - k_1) \right] \]
Fermion Yukawa couplings (massive fermions, mass $m_i$)

\[ \frac{ie}{\sin 2\theta_w} M_Z g_{\alpha i} \]

Trilinear self-coupling

\[ \frac{ie}{2 \sin \theta_w M_W} m_i \]

(b) Quadrilinear couplings

$\sigma \sigma W^+ W^-$ vertex

\[ -i \frac{3\mu^2 e}{2 M_W \cos \theta_W} \]

$\sigma \sigma ZZ$ vertex

\[ \frac{ie^2}{4 \sin^3 \theta_W} g_{\mu \nu} \]

\[ \frac{ie^2}{2 \sin^2 2\theta_W} g_{\mu \nu} \]