MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science,
Department of Mechanical Engineering,
Division of Bioengineering and Environmental Health,
Harvard-MIT Division of Health Sciences and Technology

Quantitative Physiology: Cells and Tissues
2.791J/2.794J/6.021J/6.521J/BE370J/BE470J/HST541J

Homework Assignment #6 Issued: Day 29
Due: Recitation 12

Reading

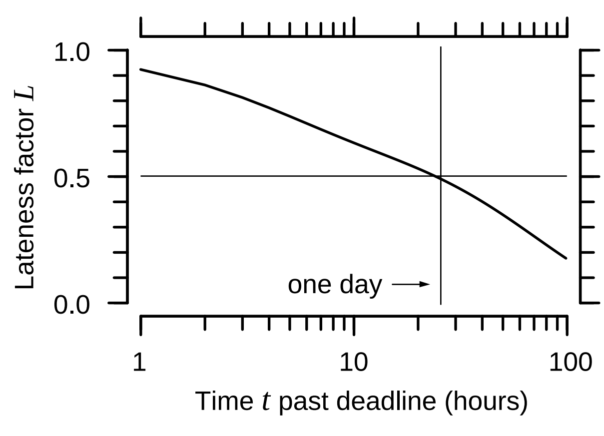
Lecture 19 — Volume 2: Chapter 1
Lecture 20 — Volume 2: 2.1-2.4.2
Lecture 21 — Volume 2: 2.4.3-2.5

Announcements

Laboratory reports are on **Lecture 21 Day.**You should submit one copy of the final draft of the draft of the report. You should include the following items in an appendix: final proposal, copy of your critique of a peer's first draft, peer critique of your first draft, critique of your first draft by writing program, critique of your first draft by technical staff, photocopies of notes taken during original lab session. Note that there is a **SEVERE LATENESS PENALTY.** The grade for a late report will be multiplied by a lateness factor

$$L = 0.3e^{-t/4} + 0.7e^{-t/72}$$

where t is the number of hours late. The lateness factor is plotted below. Notice that the maximum grade for a report that is more than ONE DAY LATE is less than 50%.



This lateness factor is applied regardless of the reason for the lateness, except for health related problems or personal problems certified by the Dean's office. Specifically, lateness due to computer and/or printer problems is not exempt.

Exercise 1. Let ψ represent electrical potential, ϕ_n represent solute flux, c_n represent concentration, z_n represent valence, F represent Faraday's constant, u_n represent molar mechanical mobility, R represent the gas constant, D_n represent the diffusion coefficient, and T represent absolute temperature. Identify the units of the expressions in parts a)-g) as one of: A, moles/s; B, moles/cm·s; C, moles/cm^2·s; D, moles/cm^3·s; E, amps/cm^2; F, amps; G, coul/mol; H, coul/cm^3; I, volts/cm; J, volts; X, none of the above. If the answer to any part is X, provide proper units.

a) $\dfrac{\partial \psi(x,t)}{\partial x}$

b) $z_n F c_n(x,t)$

c) $u_n z_n F c_n(x,t)$

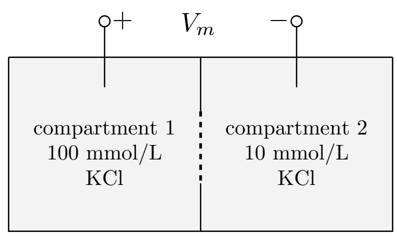
d) $D_n \dfrac{\partial c_n(x,t)}{\partial x}$

e) F

f) $z_n F \phi_n$

g) $\dfrac{RT}{z_n F}$

Exercise 2. As shown in the following figure, compartments 1 and 2 contain well-stirred solutions of potassium chloride and are separated by a membrane that is permeable to potassium only.



The potential between compartment 1 and 2 is V_m. The concentration of KCl in compartments 1 and 2 are 100 mmol/L and 10 mmol/L, respectively.

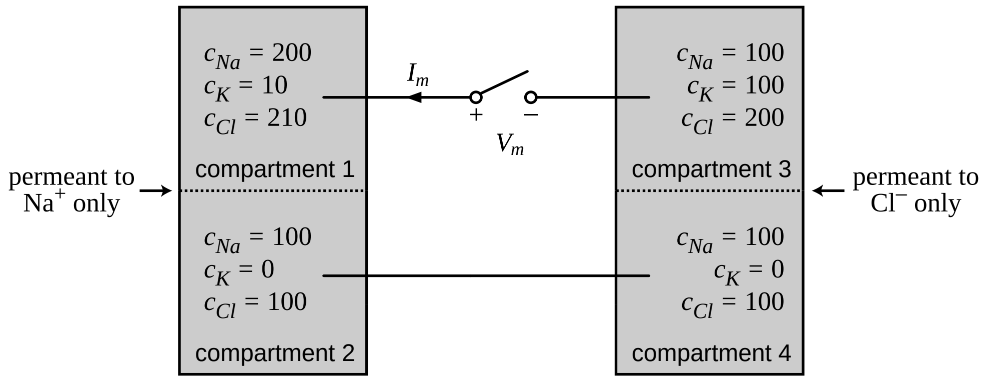
a) Determine the equilibrium value of V_m and give a *physical* explanation of the sign of the potential.

b) A battery is now connected to the solutions so that $V_m = -30$ mV. In which direction will current flow through the membrane? Explain.

Exercise 3. Define the Nernst equilibrium potential and briefly explain its physical basis.

Exercise 4. Active ion transport is said to have a *direct* and an *indirect* effect on the resting potential of a cell. Define both effects and discuss the distinction between the two effects.

Problem 1. Two chambers are each divided into two parts by semi-permeable membranes, as illustrated in the following figure.



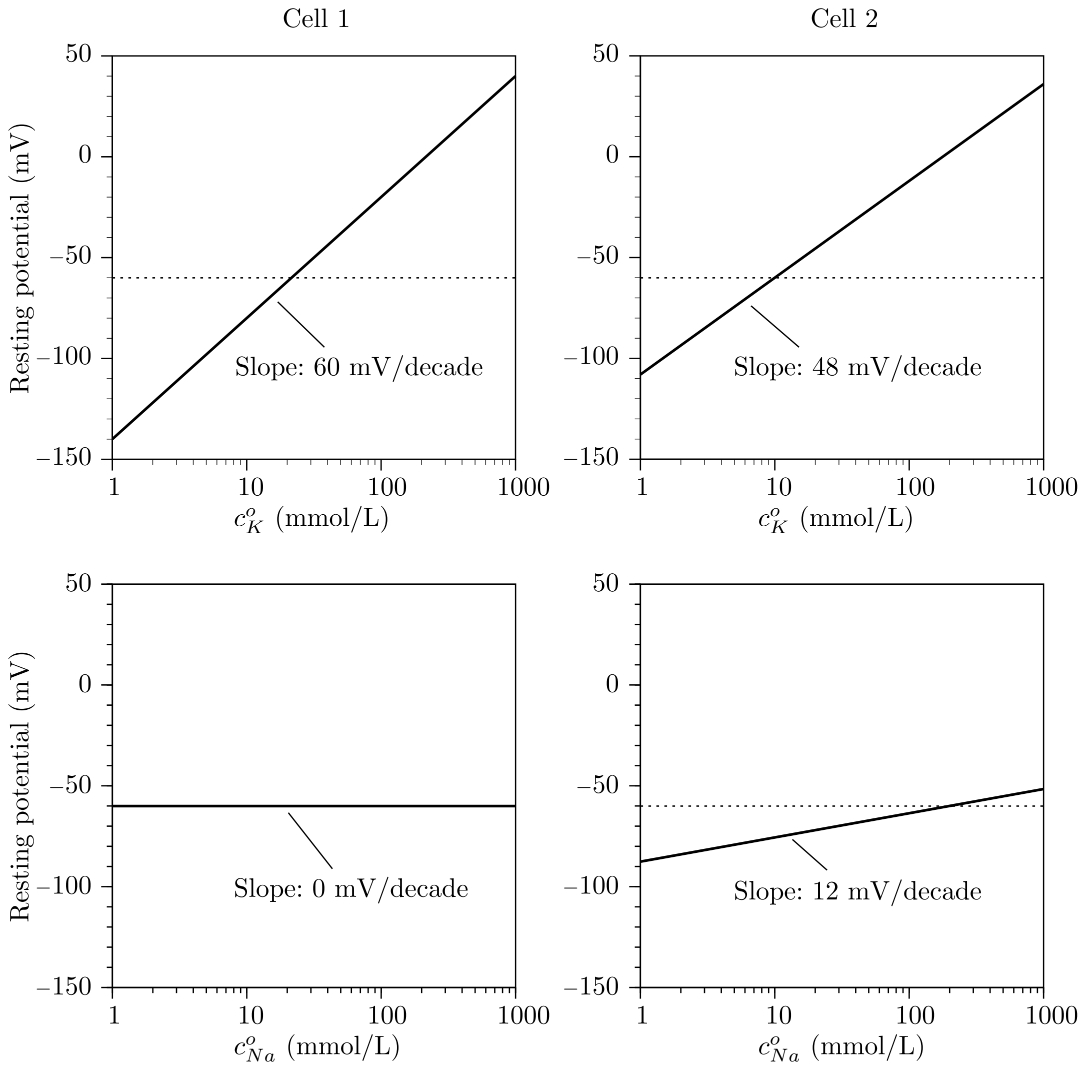
Each compartment contains well-stirred solutions of sodium, potassium, and chloride ions, with concentrations indicated in the figure (in mmol/L). The membrane between compartment 1 and 2 is permeant to sodium ions only, and its specific electrical conductivity G_{Na} is 5 mS/cm^2. The membrane between compartment 3 and 4 is permeant to chloride ions only, and its specific electrical conductivity G_{Cl} is 2 mS/cm^2. Both membranes have areas $A = 10$ cm^2. The temperature T is such that $RT/(F \log e) = 60$ mV. Electrodes in chambers 2 and 4 are connected via a wire. Electrodes in chambers 1 and 3 are connected through a switch with wires.

a) Sketch an electrical circuit that represents the steady-state relation between current and voltage for the four compartments. Label the nodes that correspond to compartments 1, 2, 3, and 4. Include the switch in your sketch. Label I_m, V_m, and the conductances.

b) Let V_1 and V_2 represent the steady-state potentials in compartments 1 and 2 with reference to compartment 3 when the switch is open. Calculate numerical values for V_1 and V_2.

c) Compute the steady-state value of the current I_m when the switch is closed.

Problem 2. The resting membrane potential, V_m^o, of two uniform, isolated cells is measured as a function of the external concentration of potassium, c_K^o, with the sodium concentration held fixed at its normal value, c_{Na}^{on}, and then as a function of the external sodium concentration, c_{Na}^o, with the potassium concentration held fixed at its normal value, c_K^{on}. The results for these two cells are shown in the following figure.



You may assume that for each cell: (1) external solutions are isotonic; (2) the membranes are impermeable to ions other than potassium and sodium; (3) the internal concentrations of potassium and sodium are maintained constant by a non-electrogenic active transport mechanism; (4) the total membrane conductance is 10 nS; (5) the normal resting potential is −60 mV; (6) the internal concentration of sodium is 20 mmol/L.

a) For each cell, determine the total conductance of the membrane to potassium and to sodium, $\mathcal{G}_K$ and $\mathcal{G}_{Na}$, respectively. If either value is indeterminate from the information given, describe what additional information would be needed.

b) For each cell, determine the internal and external concentrations of potassium and the external concentration of sodium at the normal resting potential (−60 mV). If the value is indeterminate from the information given, describe what additional information would be needed.

c) For each cell, determine the simplest equivalent electric network model that relates the dependence of the resting potential of the cell on the ion concentrations. Indicate the values of all elements in the network.

Problem 3. Figure 1 shows measurements of the resting potential of a glial cell for different values of extracellular potassium concentration (left panel). These measurements are to be interpreted in terms of the network model shown in the right panel of the same figure. Assume that $c^o_{Na} = 150$

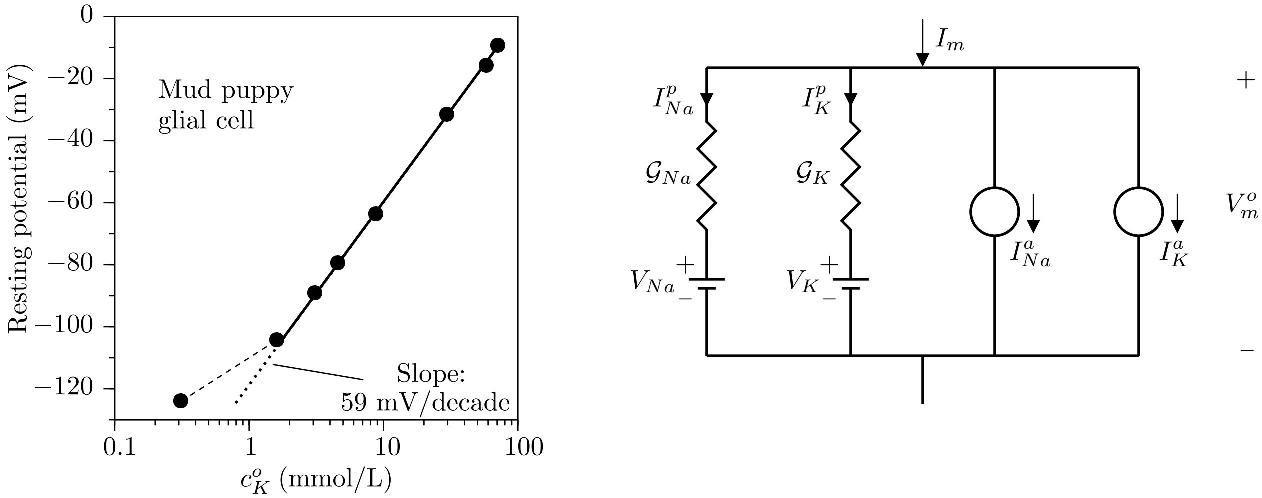


Figure 1: Measurements and model of electrical responses of a glial cell.

mmol/L, $c^i_{Na} = 15$ mmol/L and that the external solution is maintained isotonic with the cytoplasm by controlling impermeant solutes. Assume that sodium and potassium concentrations are constant, except for c^o_K, and that the pump system, which consists of I^a_{Na} and I^a_K, is nonelectrogenic.

a) Consider only the region for which the data are well fit by the straight line of slope 59 mV/decade. Indicate whether the following statements are true or false and give a brief reason for each answer.

   i $I_m = 0$.

   ii) $V^o_m \approx V_K$.

   iii) $\mathcal{G}_{Na} \gg \mathcal{G}_K$.

   iv) $V_{Na} > V_K$.

   v) $c^i_K = 100$ mmol/L.

   vi) $I^a_K = -I^a_{Na}$.

   vii) $I^p_K = -I^p_{Na}$.

   viii) $I^a_{Na} = -\mathcal{G}_{Na}(V^o_m - V_{Na})$.

b) It is proposed that deviation of the data from the straight line for the lowest c^o_K is a result of a change in $\mathcal{G}_K$ that occurs when $V^o_m < -110$ mV. For the data shown, is this a reasonable hypothesis? Does it require that $\mathcal{G}_K$ for $V^o_m = -125$ mV is larger or smaller than $\mathcal{G}_K$ for $V^o_m > -100$ mV? Explain.

Problem 4. The membrane of a cell contains an active transport mechanism that pumps three sodium ions out of the cell for every two potassium ions that it pumps into the cell. The membrane also supports the passive transport of sodium and potassium ions, but is impermeant to all other ions and is impermeant to water. The sodium conductivity is 10^{-5} S/cm^2 and the potassium conductivity is 10^{-4} S/cm^2. The cell is allowed to come to steady state and its membrane potential is -52.5 mV. The Nernst equilibrium potential for sodium is 60 mV and the Nernst equilibrium potential for potassium is -60 mV. The net outward current density due to active transport is $\frac{3}{8}$ μA/cm^2.

a) Draw an electrical circuit to represent ionic transport across the membrane of this cell. Include labels for each of the 6 numbered parameters provided in the problem statement.

b) Is the cell at rest? If yes, prove that it is at rest. If no, explain why not.

c) Is the cell in quasi-equilibrium? If yes, prove that it is at quasi-equilibrium. If no, explain why not.

d) Is the active transport mechanism electrogenic? Explain.