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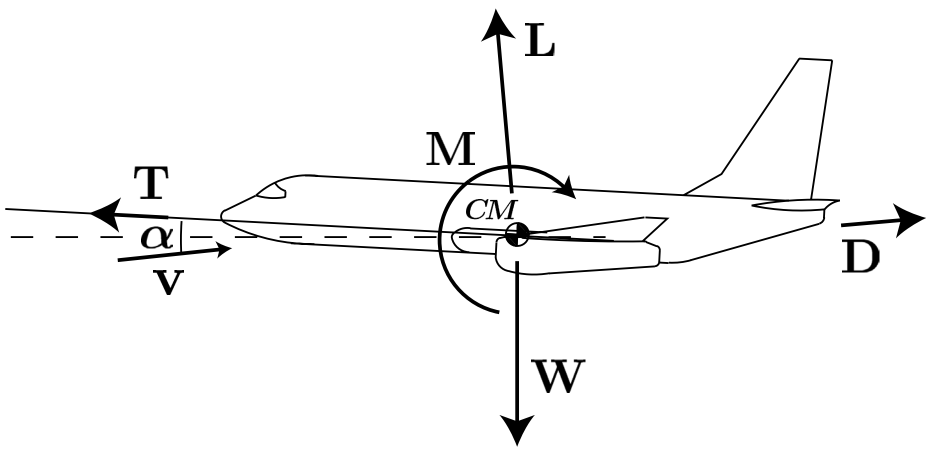
Lecture D6 - Aircraft Stability : Forces on the Aircraft

In the next two lectures, we will introduce the topic of aircraft stability. This topic is rather large and well developed. Here, we will limit ourselves to introduce the basic ideas and illustrate them with the help of a simplified aircraft model. We will consider only, *static* stability and *longitudinal* motion.

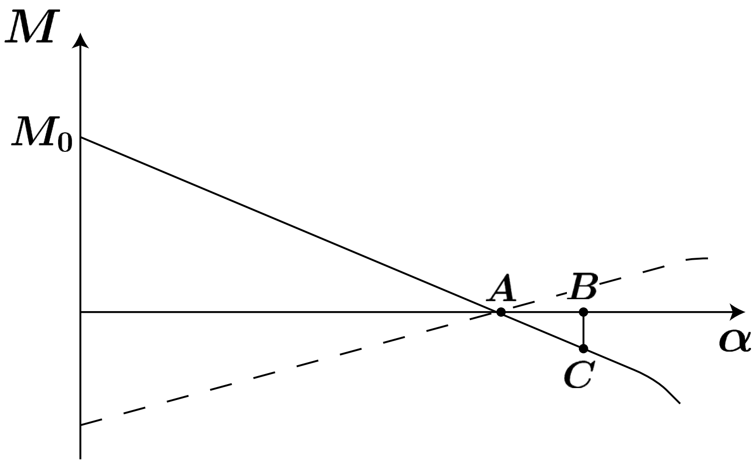
> *Longitudinal* or symmetric motion means that the wings of the aircraft remain level and the center of mass moves in the vertical plane. This excludes lateral motions such as rolling or yawing.
>
> *Static* stability, as opposed to dynamic stability, means that we only look at forces on the aircraft about the equilibrium condition – steady level flight, in our case –, and consider whether these forces tend to destabilize or restore equilibrium, when the aircraft is perturbed from its equilibrium position. We will assume that the aerodynamic characteristics of the aircraft do not change. For instance, we do not consider aeroelastic effects due to the changes in loading conditions of the wings. The study of dynamic stability requires the construction of a dynamical model of the aircraft, which we will only consider, later on in the course, once we have studied rigid body dynamics. It turns out that, static stability is a necessary condition for stable flight. That is, if the aircraft is statically unstable it will also be dynamically unstable.

Equilibrium

Let us briefly discuss the different forces acting on the aircraft. The aerodynamic forces, are resolved into a lift component $\mathbf{L}$, orthogonal to the velocity, and a drag component $\mathbf{D}$, parallel to the velocity. In addition, we will have, the weight of the vehicle, $\mathbf{W}$, and the thrust $\mathbf{T}$, due to the propulsive system. When these forces are referred to the center of mass of the aircraft, we may have an additional pitching moment $\mathbf{M}$. We will use the convention that the pitching moment is positive as drawn, that is, when it tends to force the nose of the aircraft to move upwards.



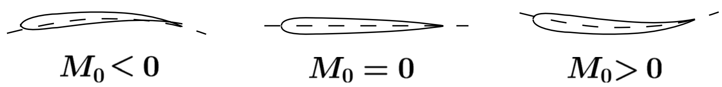
For steady level flight, the sum of the total forces and moments acting on the aircraft must vanish. It is clear that this must be the case, otherwise the aircraft would experience acceleration.
Beyond establishing equilibrium, we will be interested in whether the equilibrium position is stable or unstable. In order to do that, it is useful to look at the variation of $\mathbf{M}$, when the angle of attack is changed. Consider two different variations of $\mathbf{M}$ with α for two different aircraft shown in the graph with continuous and dashed lines.



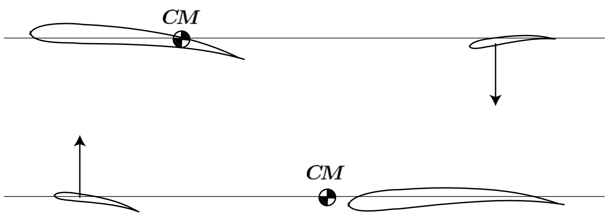
Since the moment must vanish, it is clear that equilibrium can only occur at point A. Now, suppose that the aircraft, flying in equilibrium, is hit by a gust which increases the angle of attack α to that of, say, point B. The aircraft corresponding to the solid line will experience a negative moment (point C), which will push the node down, thus restoring it to the equilibrium position. On the other hand, the aircraft corresponding to the dashed line, will experience a positive moment that will tend to push the node of the aircraft further upwards. Therefore, we conclude that, for the equilibrium point A to be stable, the derivative of $\mathbf{M}$ with respect to α, must be negative. An additional consequence that can be easily inferred from the graph, is that, for stable flight, the pitching moment on the aircraft when $\alpha = 0$ (zero lift), $\mathbf{M_0}$, must be positive.

Configurations

It turns out that, for virtually any combination of lifting surfaces, $\partial M/\partial\alpha$, can be made to be negative by simply moving the center of mass forward. We will make this statement more precise below. The condition which is actually most restrictive, is that of having a positive $\mathbf{M_0}$. If we look at an isolated airfoil section we have that, for positive camber airfoils, the zero lift pitching moment, is usually negative, whereas for negative camber, the pitching moment is usually positive.



This implies that, in principle, it would be possible to have a straight wing tailless aircraft, provided the wing section had a negative camber. It turns out that negative camber airfoils do not have a good aerodynamic behavior and hence, are not generally used. The solution is to use a positive camber wing in combination with additional lifting surfaces, such as a wing-tail, or wing-canard configurations, to obtain a positive M_0 about the aircraft's CM.



For the wing-tail configuration we can see that, by setting the tail at a lower incidence than the wing, and even creating a negative lift if necessary, we can balance a negative wing pitching moment. For the wing-canard configuration, the same effect can be obtained by setting up the canard at a higher incidence than the wing. In this case, the lift generated by the canard will be upwards.

Forces on the Aircraft

For a standard wing-tail configuration aircraft we have the following:

Lift

Most of the aircraft's lift is generated by the wing and horizontal stabilizer. For a given angle of attack α, the lift is proportional to the relative wind's dynamic pressure, $q = (1/2)\rho v^2$, and to the wing's area S,

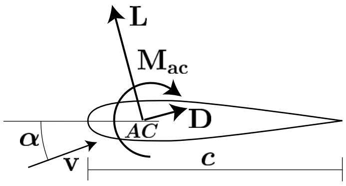
$$L = \frac{1}{2}\rho v^2 S C_L.$$

Here, C_L, is the non-dimensional lift coefficient, which is a function of the shape, and the angle of attack α. The variation of C_L with the angle of attack is linear, up to a maximum value $C_{L_{max}}$, of the order of

1.5 − 2.0. Thus, we have

$$C_L = a\alpha \ ,$$

where $a = \partial C_L/\partial\alpha$. For an idealized flat plate wing, $a = 2\pi$ rad^{-1}, and more generally for finite wings $a = 5-6$ rad^{-1}. Note that for $\alpha = 0$, $C_L = 0$, which implies that we are assuming that $\alpha = 0$ corresponds to the zero-lift line.



Pitching Moment

The lift force, **L**, is the result of integrating a distributed force over the surface of the wing. This distributed force is equipollent to **L**, acting through a particular point called the center of pressure. For symmetric sections, changing the angle of attack does not change the position of the center of pressure. This is because all the distributed forces on the wing surface, increase in the same proportion, i.e. they are self-similar. The theoretical location of the center of pressure for an idealized flat plate wing is at 25% of the chord, starting from the wing's leading edge. For a cambered wing, on the other hand, it is possible that the moment of the distributed surface forces is non zero even when the total lift is equal to zero ($\alpha = 0$). Changing α, generates a net lift force and the resulting center of pressure is no longer fixed. If we divide the surface forces into those present at $\alpha = 0$, $\mathbf{L} = 0$, and those at $\alpha \neq 0$, it turns out that the part due to $\alpha \neq 0$, is self-similar and the resultant force always acts through the same point. This point is called the *aerodynamic center* (AC) of the wing and is usually around the 25% chord (note that for a flat plate the center of pressure and the aerodynamic center, coincide). Therefore, the moment of the aerodynamic forces, with respect to the AC, is only due to the zero lift force distribution, and remains constant when α changes. Thus $M = M_0$ and is independent of α. This property of moment invariance with α, is often used to define the aerodynamic center of a wing.

The moment M, is proportional to the dynamic pressure, q, the surface of the wing, S, and the airfoil chord c. Thus,

$$M_{ac} = \frac{1}{2}\rho v^2 \, S \, c \, C_M \ .$$

Here, C_M, is the non-dimensional pitching moment coefficient which is, to first order, a function of the airfoil shape only. For positive camber airfoils C_M is usually negative, with typical values between −0.02 and −0.3.

Drag

The drag force can also be expressed as a function of a non-dimensional drag coefficient, C_D, as

$$D = \frac{1}{2}\rho v^2 \, S \, C_D.$$

The drag coefficient in general has a complicated dependence on α and C_L. But commonly it is assumed to have two components, one which is independent of the lift, and one which increases quadratically with the lift,

$$C_D = C_{D0} + KC_L^2 \ .$$

Typical values for C_{D0} for the whole aircraft, are of the order of 0.003 to 0.02. An important ratio used to determine the aircraft's performance is $L/D = C_L/C_D$, which is typically of the order of $10 - 20$ for most subsonic aircraft, and up to 60 for some sailplanes.

Although the drag plays an essential role in determining the aircraft's performance, its role in aircraft dynamics is minimal as we shall see in the next lecture.

References

[1] M. Martinez-Sanchez, *Unified Engineering Notes*, Course 95-96.

[2] B. Etkin and L.D. Reid, *Dynamics of Flight, Stability and Control*, Third Edition, Wiley, 1996.