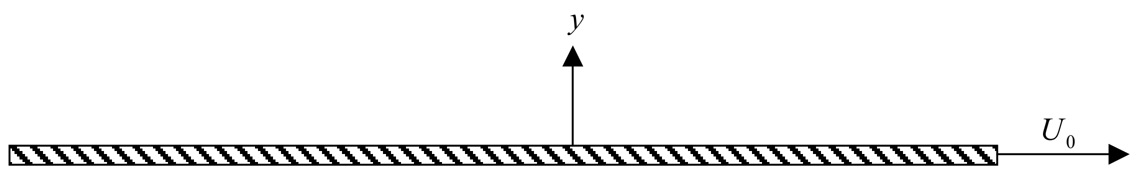
16.100 Homework Assignment # 7
Due: Monday, October 27, 9am

Reading Assignment
Anderson, 3rd edition: Chapter 17, pages 787 – 801
Chapter 18, pages 803 – 810
Anderson, 2nd edition: Chapter 17, pages 711 – 729

Problem 1



Consider an initially stationary, long flat plate. At time $t = 0$, the plate is set in motion at velocity U_0. As time evolves, the air above the plate will begin to move as the viscous effects diffuse the momentum away from the plate. We will define the height of the boundary layer of non-negligible momentum, $\delta(t)$, as the y-location at which the velocity is only 1% of the wall velocity. Assume the flow is incompressible.

a) Apply the conservation of mass in differential form (i.e. not integral form) to show that the vertical velocity, v, is zero everywhere in the flow for all time.

b) Next, show that the conservation of x-momentum can be reduced to the following form:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

c) Applying an order of magnitude analysis to Equation (1), show that:

$$\delta = O\left(\sqrt{\nu t}\right).$$

d) Show that the following x-velocity is a solution to Equation (1) and that it satisfies the initial and boundary conditions:

$$\frac{u}{U_0} = 1 - erf(\eta)$$

$$\eta \equiv y/\sqrt{4\nu t}$$

$$erf(\eta) \equiv \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi$$

Note that $erf(\eta)$ is known as the error function and its values are available in Matlab and Excel using the erf function.

e) Plot the velocity distribution as a function of η. Determine the boundary layer thickness as a function of time. For air at standard day conditions, at what time does the boundary layer thickness reach 1 inch above the plate? What time for 1 foot above the plate?

f) Calculate the vorticity. Plot the vorticity (divided by the wall velocity) at the two times calculated in part e). Describe the behavior of the vorticity as time evolves.

g) Determine the skin friction coefficient c_f as a function of time where

$$c_f = \frac{\tau_{wall}}{\frac{1}{2}\rho U_0^2} \qquad \tau_{wall} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

Plot c_f as a function of $U_0^2 t / \nu$. As time proceeds, is the plate easier or harder to keep moving at the same velocity? Notice that at $t = 0$, the skin friction has some rather large values. Obviously, in a real situation, the friction does not reach these values. What basic part in the specification of the problem is the cause for this? Hint: it is not the infinite length of the plate.

Problem 2

In this problem, we will use Xfoil to simulate the flow of over an airfoil at relatively low Reynolds number, and compare the results to some simple flat plate results. To run Xfoil, you need to use Athena. Directions are available on the website, and have been sent in an email.

a) Using Xfoil, calculate the drag coefficient versus Reynolds number for the NACA 0004 airfoil from $\text{Re} = 1 \times 10^3$ to $\text{Re} = 1 \times 10^6$ at zero degrees angle of attack. Plot the results on a log-log scale and include the drag coefficient from the laminar flat plate solution by Blasius. How does the Xfoil result compare to the flat plate theory?

b) A form factor approach is often used to estimate the drag on an airfoil (or other body) by multiplying the skin friction drag estimated from flat plate results. Specifically, assume the drag can be estimated as follows:

$$c_d = K c_d^{flat}$$

where K is the form factor and c_d^{flat} is the drag coefficient from the Blasius flat plate theory for a plate of equal chord length. From the Xfoil results in a), estimate the form factor K. Does a constant value of K (i.e. independent of Reynolds number) exist which provides a good estimate of the Xfoil drag coefficient?

c) The displacement thickness of the boundary layer predicted using Xfoil can also be compared to the flat plate results. Plot the displacement thickness at the trailing edge as predicted by Xfoil versus the displacement thickness at the trailing edge estimated from Blasius flat plate theory for the range of Reynolds numbers given above. How do the displacement thicknesses for the airfoil and the flat plate results compare?