16.100 Homework Assignment \# 4

Due: Monday, September $29^{\text {th }}$, 9 am

## Reading Assignment

Anderson, $3^{\text {rd }}$ edition: Chapter 3, pages 247-256
Anderson, $2^{\text {nd }}$ edition: Chapter 3, pages 218-228

## Problem 1 (70\%)



- nodes where vortices are located

X control points where
$\vec{u} \cdot \vec{n}=0$ is enforced

Consider the isosceles triangle airfoil with $10 \%$ thickness as shown above. A simple potential flow model for the flow over the triangle is to place point vortices at each of the three nodes and then determine the strengths (i.e. the circulations $\Gamma_{i}$ ) which satisfy flow tangency at three control points. For this model, let the control points be located at the midpoints of the three edges of the triangle.
a) Set-up the $3 \times 3$ system of equations that expresses $\vec{u} \cdot \vec{n}=0$ at the control points.
b) Assuming one solution exists for this system of equations, prove that in fact there are an infinite number of solutions to this system of equations.
c) The solution that is most often of interest in aerodynamics is the one satisfying the Kutta condition. The Kutta condition requires the flow to leave the trailing edge smoothly. Any non-zero value of the circulation at node 3 would imply the flow wraps around the trailing edge (note: the velocity becomes infinite at a point vortex and therefore cannot be counteracted by the other point vortices). Thus, in this simple flow model, the Kutta condition would require that $\Gamma_{3}=0$. Replace the flow tangency condition at the control point between nodes 2 and 3 with the Kutta condition. What is the solution for $\Gamma_{1}$ and $\Gamma_{2}$ that satisfies the remaining flow tangency conditions and the Kutta condition as a function of the freestream angle of attack?
d) Calculate and plot the lift coefficient as a function of angle of attack. What is the lift slope and what is the angle of zero-lift? Note: use the Kutta-Joukowski theorem to find the lift.

Problem \#2 (30\%)


Consider the semi-cylinder aircraft hangar shown above. Assume:

- Far upstream of the hangar, the wind has uniform speed $U$ and is perpendicular to the hangar length. The upstream pressure is $p_{\infty}$.
- The end effects are small.
- The flow is inviscid to good approximation.

Answer the following questions:
a) Assume the pressure in the interior of the hangar is $p_{H}$. If the circular roof can withstand a maximum net vertical force of $F_{\max }$, what is the maximum velocity the hangar can withstand?
b) To reduce the pressure differential between the inside and outside of the hangar, vents are to be placed at positions $\theta_{v}$ as shown in the figure below. What should $\theta_{v}$ be to make the net vertical force on the roof zero?


