16.100 Homework Assignment # 8
Due: Wednesday, November 5, 9am

Reading Assignment
Notes on Karman's Integral Momentum Equation and Correlation Methods

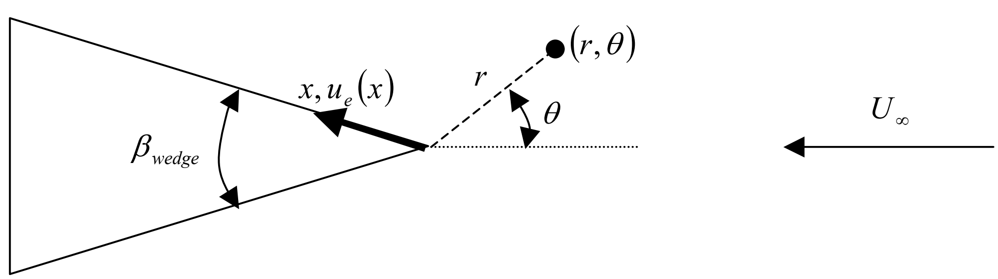
Problem 1

In this problem, we will apply the approximate method to solve the momentum integral boundary layer equation developed by Thwaites to laminar flat plate flow. NOTE: Thwaites method only works well for laminar boundary layers. For turbulent boundary layers, a more sophisticated method is required.

Using the Thwaites method, calculate $\theta(x)$, $\delta^*(x)$, and $c_f(x)$ and compare the results to the Blasius flat plate solution (which is the exact solution for this problem). Is the scaling with respect to Reynolds number (based on distance from the leading-edge of the flat plate) the same for Thwaites as it is for the Blasius solution? What is the percent error in the Thwaites estimates for these three quantities relative to the Blasius solution?

Problem 2

In this problem, we will continue the application of Thwaites method. Now, we will consider how a boundary layer behaves on a symmetric wedge at zero angle of attack as shown below:



a) In order to estimate the behavior of the boundary layer on the wedge, the edge velocity is needed. Assuming the boundary layers are thin, we can estimate the edge velocity as the potential flow velocity on the body surface. The potential for wedge flow has the following form:

$$\phi = -\frac{K}{m+1} r^{m+1} \cos[(m+1)\theta]$$

where $K, m > 0$ are positive constants. By applying the flow tangency condition on the wedge, prove that the wedge angle is related to m by:

$$\beta_{wedge} = 2\pi \frac{m}{m+1}$$

b) Prove that the edge velocity is $u_e(x) = Kx^m$

c) Using Thwaites' method, prove that the boundary layer displacement thickness on the wedge grows as $x^{(1-m)/2}$. Does the boundary layer grow faster or slower (with x) for increasing wedge angles?

d) One particularly interesting case for the wedge flow is for flow normal to a wall (in that case, $\beta_{wedge} = \pi$. How does the boundary layer grow in that case?

Problem 3

In this problem, we will apply Thwaites method to predict the separation location on a cylinder. Note: the separation location is the point on the cylinder at which the flow reverses directions just above the cylinder surface. When the flow is attached, the velocity just off the surface will be positive (though near zero since the no-slip condition requires zero velocity on the wall). The derivative of the velocity normal to the wall in attached flows is thus positive, $\partial u/\partial y > 0$. When the flow reverses, the velocity gradient is now negative, $\partial u/\partial y > 0$. Thus, the point at which the flow reverses directions will also be the location at which $\partial u/\partial y = 0$. Since the skin friction is proportional to this gradient, the skin friction also goes to zero at the separation location, $\tau_{wall} = \mu\,\partial u/\partial y = 0$.

a) Using the potential flow solution about a non-lifting cylinder, show that the edge velocity is:

$$\frac{u_e(x)}{U_\infty} = 2\sin\frac{x}{R}$$

where x is the distance along the surface.

b) Using Thwaites method, estimate the angle along the cylinder at which separation would occur.

c) The experimental observation for laminar flows over cylinders is that separation occurs around 80.5 degrees. Assuming you found the correct value in part b), you should observe that the actual flow separates much earlier than predicted. Explain why this occurred. Hint: it is NOT because the Thwaites method itself is inaccurate. Hint #2: see Figure 3.49 on page 262 of Anderson (3rd edition) and the discussion of it. In the 2nd edition, see Figure 3.44 on page 234.