THE INTERACTION OF GRAVITY WAVES WITH A NON-UNIFORM SURFACE CURRENT

by

Jerome H. Milgram

Submitted in Partial Fulfillment
of the Requirements for the
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ABSTRACT

The interaction of two dimensional waves with a two dimensional surface current was studied experimentally by J. T. Evans (Ref. 1) and theoretically by G. I. Taylor (Ref. 2). They found that as waves entered an opposing current, their speed was reduced and their height was increased. This effect increased with current strength until the waves became high enough to become unstable and break. Beyond the breaking region there was calm water. This calming effect was the prime motivation for the works of Evans and Taylor as their interest was in the use of hydrodynamic breakwaters.

It was desired to find the effects of the interaction of water waves with a non-uniform surface current and to compare the results with those of Evans and Taylor. The principal difference between the three dimensional results and the two dimensional results is that with the non-uniform surface current the wave power propagation became spatially varient. With flow directed towards the sides of the tank in which the experiment was run, the waves which were two dimensional before entering the current became highest and showed the largest power density near the walls of the tank. The measured wave velocity was compared with the theoretical wave velocity, and the power propagation through various cross-sections was compared. The difference between measured wave velocity and the theoretical value is attributed to the fact that the current was not infinitely thick. Variations in the power through various cross-sections are attributed to uncertainty in calculating the speed of energy propagation due to the variation of the current with depth.

It is recommended that the experiment be repeated with a current variable in the free surface plane but uniform with depth. It is the contention of the author that in such an experiment, the measured wave speed relative to the current will be equal to the theoretical wave speed for waves of the same wavelength moving in undisturbed water and that the power propagation through a cross-section of the tank will be invarient with the cross-section chosen.

Thesis Supervisor: Martin A. Abkowitz, Ph. D
Title: Professor of Naval Architecture

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SYMBOLS

V_1	Theoretical wave speed in still water
λ_1	Theoretical wavelength for waves in still water
λ_2	Theoretical wavelength for waves in the moving current
g	Acceleration due to gravity
V_r	Theoretical wave speed for waves in a current relative to the moving current
V_2	Theoretical wave speed relative to the ground for waves in a current
V_c	Velocity of the current
H_2	Theoretical wave height for waves in a current
H_1	Theoretical wave height for waves in still water
x, y	Coordinates for the surface plane of the tank
y'	$-y$
V_n	The wave velocity in the direction of wave propagation
t	Time for a wave to move one foot longitudinally
λ	Wavelength
ϕ	Angle of current flow with respect to $-x$
θ	Angle of wave propagation direction with respect to $-x$
T	Wave period
P	Power
P_t	Power propagation through a cross-section
P_{t_m}	Approximate power propagation through a cross-section

CHAPTER I

INTRODUCTION

The effects of the interaction of waves with surface currents has been of interest for the past fifty years. These effects were first observed when it was found that waves could be calmed by a curtain of bubbles rising from a submerged perforated pipe. The mechanism by which the bubbles calmed the waves remained unexplained until 1942 when Sir Geoffrey Taylor and C.M. White put forward the suggestion that the bubbles induced a surface current and that this current is what stopped the waves. Taylor (1943) did a mathematical analysis of the effect of a uniform surface current of finite depth, with motionless water beneath it, upon waves. This analysis remained unpublished until 1955 when it as well as Taylor's analysis of the effects of a current whose speed varied linearly with depth were published in Ref. 1. Also in 1955, J.T. Evans performed an experiment to determine the effects on waves of both water jets and rising bubbles. Both the works of Taylor and Evans are for the two dimensional case (no variation across the width of the tank).

In his paper, Evans gives a simple analysis which brings out salient features of the interaction effects for infinitely deep currents. To familiarize the reader with these effects, Evans' analysis is repeated here in slightly modified form. Calling the wave velocity in still water (before the waves reach the current) V_1 ,

$$V_1 = \lambda_1^{1/2} \left(\frac{g}{2\pi} \right)^{1/2} \qquad \text{(I-1)}$$

After the waves have entered the uniform surface current they have a velocity relative to the moving current. This velocity is denoted by Vr

$$V_r = \lambda_2^{1/2} \left[\frac{g}{2\pi} \right]^{1/2} \qquad \text{(I-2)}$$

λ_2 is a new wavelength.

Calling the wave velocity relative to the ground V_2 ,

$$V_2 = V_r + V_c \tag{I-3}$$

where the current velocity V_c is defined such that it is positive when it is directed in the same direction as the direction of wave propagation. The period of the waves must be the same both in and out of the current for the wave crests to retain their identity.

$$T = \frac{\lambda_1}{V_1} = \frac{\lambda_2}{V_2} = \frac{\lambda_2}{V_r + V_c} \tag{I-4}$$

$$\lambda_2 = \lambda_1 \frac{V_r + V_c}{V_1} \tag{I-5}$$

The rate of transmission of wave energy in the still water is $1/2\, V_1$. The rate of wave energy transmission in the moving current is,

$$\frac{1}{2} V_r + V_c$$

The total power propagation through a cross-section of the tank must remain constant unless turbulence or another form of wave energy dissipation occurs. The average energy of a region in which waves exist is proportional to the wave height squared.

Denoting wave height by H,

$$\left[1/2\ V_r + V_c\right] H_2^2 = 1/2\ V_1\ H_1^2 \tag{I-6}$$

Solving for H_2 ,

$$H_2 = H_1 \left[\frac{1/2\ V_1}{1/2\ V_r + V_c}\right]^{1/2} \tag{I-7}$$

combining (I-1) (I-2), (I-3) and (I-4), we obtain

$$\frac{V_r}{V_1} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{V_c}{V_1}} \tag{I-8}$$

If $\frac{V_c}{V_1} = -1/4$, the travel of wave energy relative to the moving water becomes equal and opposite to the current velocity and there is no movement of energy relative to the ground. Energy piles up at one place and the waves break causing the energy to be dissipated in turbulence. For positive V_c, $H_2 < H_1$ and $V_2 > V_1$. For negative V_c, $H_2 > H_1$ and $V_2 < V_1$, until breaking occurs. The interaction of a non uniform surface current with waves has recently come of interest in connection with a number of effects. The purpose of the experiment described on the following pages was to determine what some of these interaction effects are and how they differ from the effects in the two dimensional case.

When a surface current, whose velocity (magnitude and direction) varies with position, interacts with waves which are initially two dimensional, a redistribution of wave energy results. The relation between current speed and wave height derived for the two dimensional case (Eq. I-7) does not hold locally in the three dimensional case, but the general picture of waves getting higher and moving more slowly in opposing currents, lower and faster in similarly directed currents is valid for the most part. The most striking single difference is that the effects on the waves seem to occur further downstream than predicted by two dimensional theory (see Fig. 2).

CHAPTER II

DESCRIPTION OF TERMINOLOGY AND SYMBOLS USED FOR DIRECTION

For the following description refer to Figure 1.

The positive x direction is referred to as "upstream."

The general term referring to the positive or negative x direction is longitudinal direction.

The general term referring to the y or y' direction is "transverse direction."

The general term referring to the directions into or out of the free surface plane is "vertical direction."

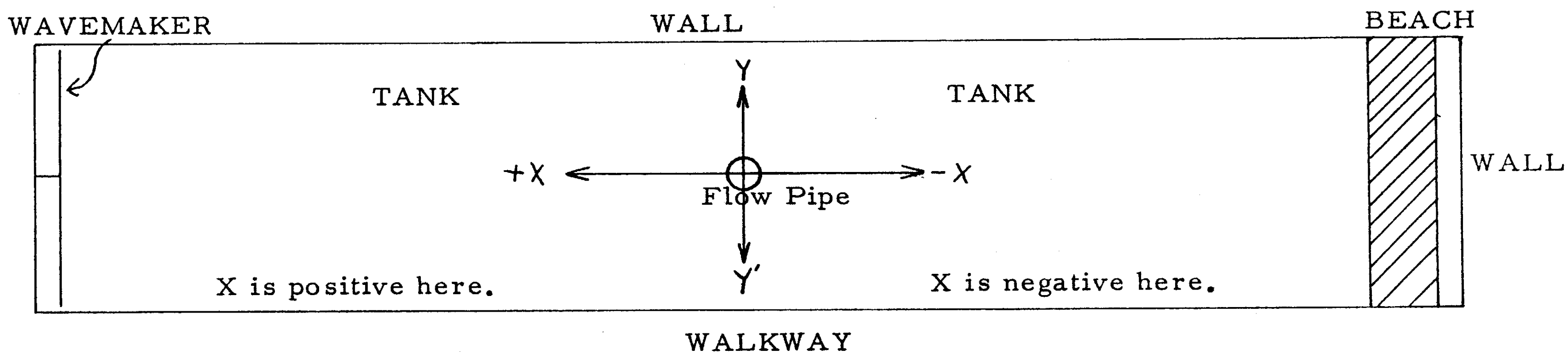

Figure 1; showing the coordinates used and the location of walls and the walkway. There is a vertical window in the side of the tank beside the walkway. The supports for this window altered the flow slightly so wave measurements were taken for positive y. The method of measuring the flow field necessitated taking velocity measurements in the region of negative y (positive y'). Symmetry of the flow field about the plane (y=0) is assumed.

FIGURE 2

THE CHANGE IN HEIGHT OF A WAVE AS IT PASSES THROUGH THE SURFACE CURRENT

Figure 2a	The wave just after it is formed by the wavemaker.
Figure 2b	The wave passing the flow pipe. Notice the increased height.
Figure 2c	The wave as it appears fifteen feet downstream from the flow pipe. This is the region of smallest wave height. By two dimensional theory the region of smallest wave-height is expected to occur much nearer to the flow pipe than is the observed region of smallest wave height.
Figure 2d	The wave far downstream. The region shown is fifty feet from the flow pipe. Notice that the wave height has increased. Since there is no breaking of the wave the effect of height increase downstream is expected, but two dimensional theory predicts that the height increases much nearer the flow pipe than is observed.

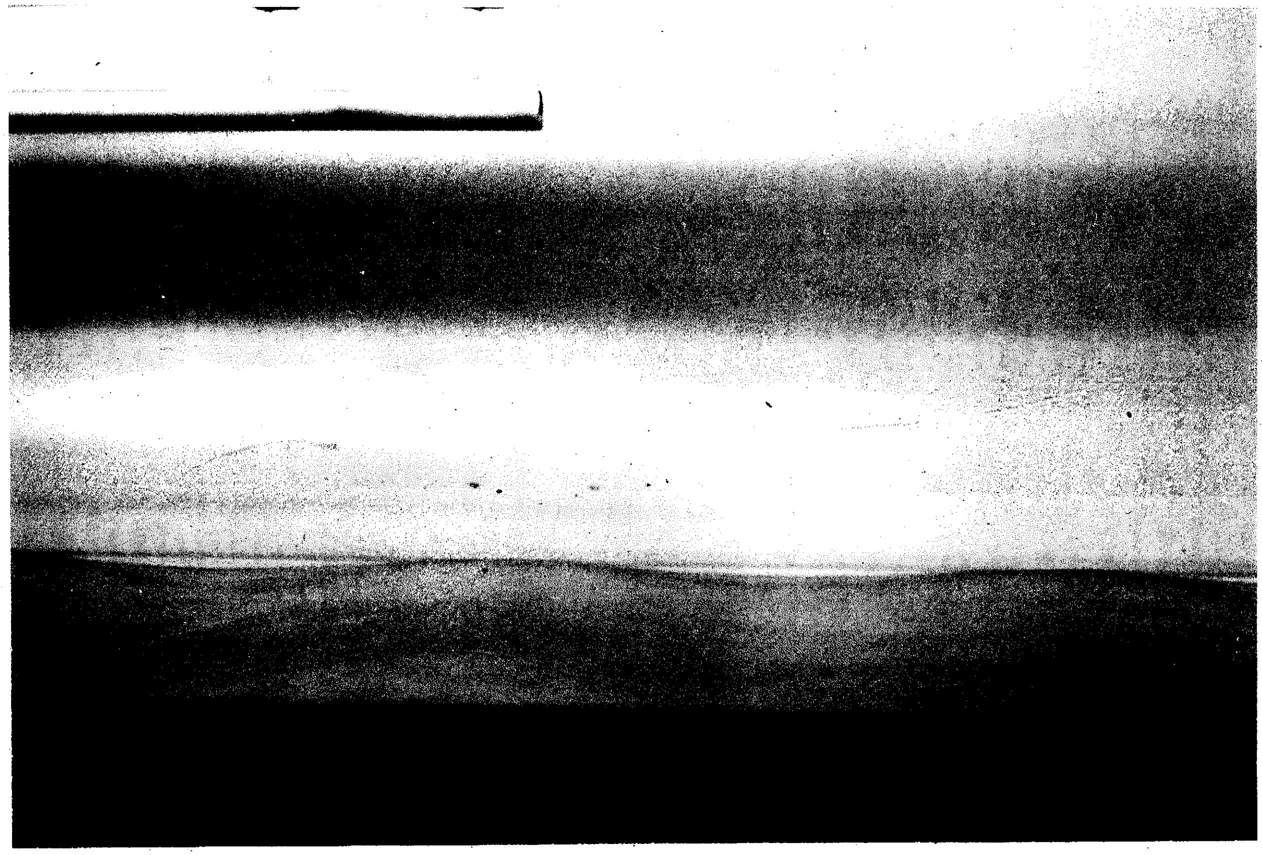

Figure 2(a). The wave just after it is formed by the wavemaker

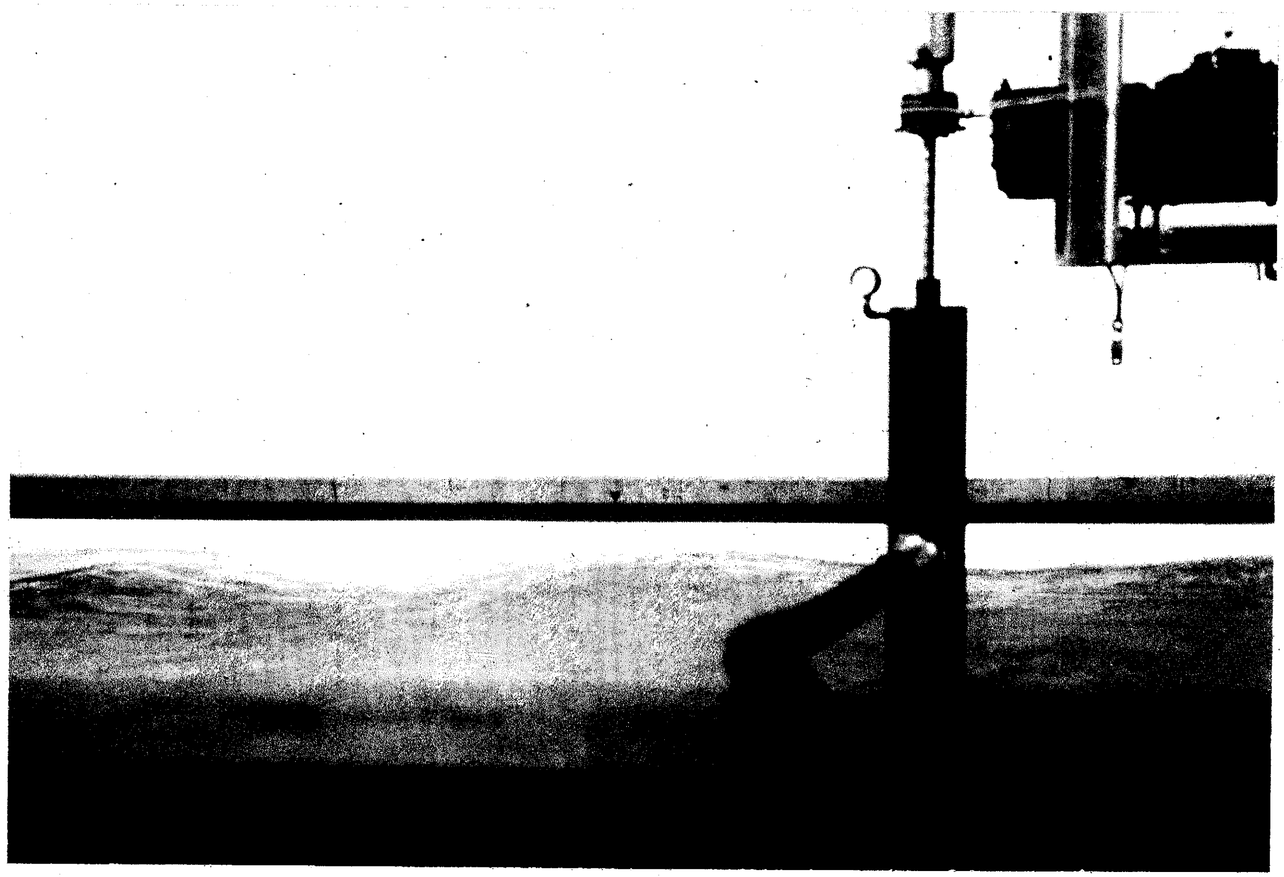

Figure 2(b). The wave passing the flow pipe

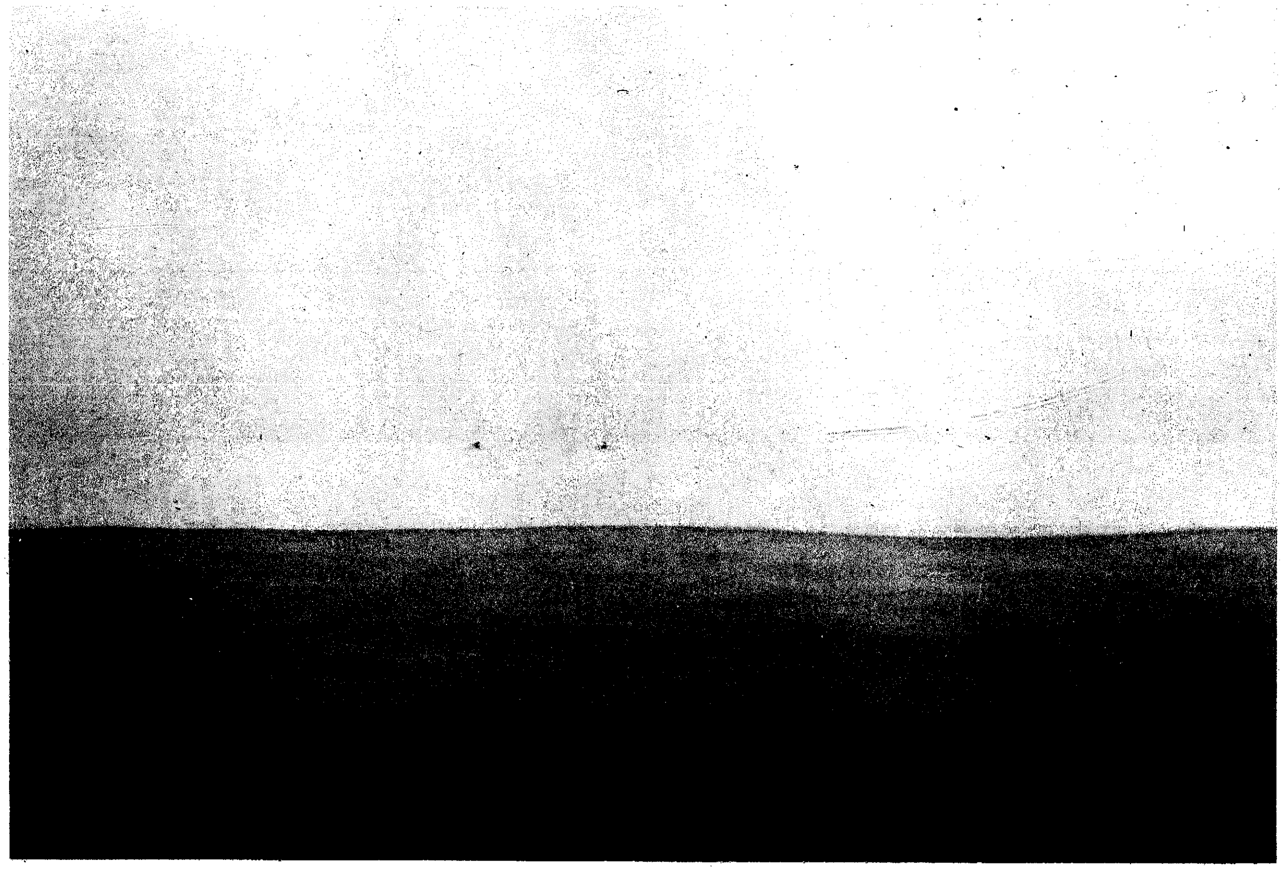

Figure 2(c). The wave as it appears fifteen feet downstream from the flow pipe

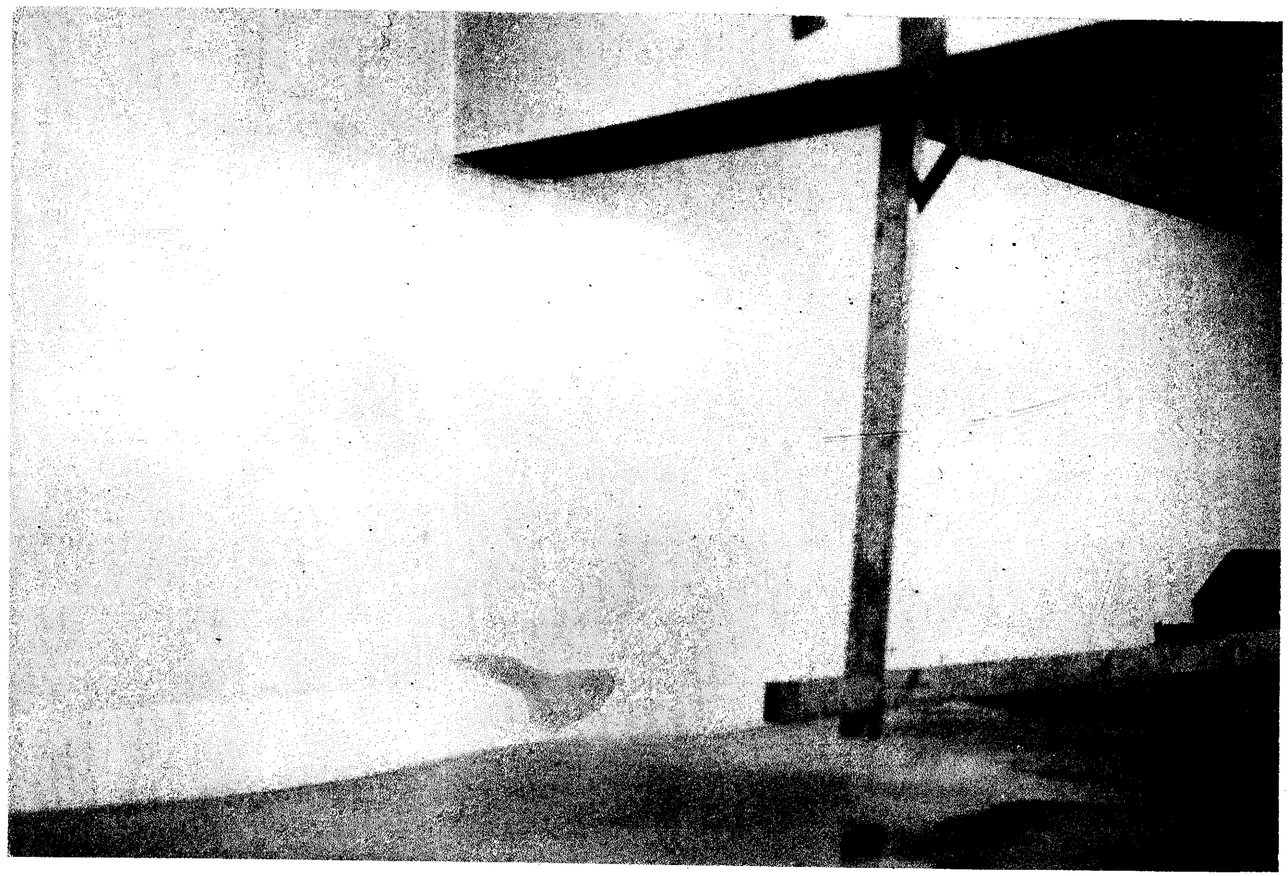

Figure 2(d). The wave as it appears far downstream

CHAPTER III

PRODUCING THE NON-UNIFORM FLOW FIELD

The first meyhod attempted for the production of a non-uniform flow field was to rotate a vertical cylinder located at the center of the tank. This cylinder extended from the bottom of the tank to a point well above the free surface. The cylinder was made out of a six inch outside diameter aluminum pipe with a one-quarter inch wall thickness. A cylindrical plate was fastened to the bottom of the pipe and a one inch diameter, 2 inch long bronze shaft was fastened to the plate. This short shaft fit into a plastic lined bearing at the bottom of the tank. A plate and shaft were affixed at the upper end of the cylinder. The cylinder was driven in rotation by a zero to 100 rpm variable speed drive (Fig. 3). A four to one speed increase as well as a right angle turn between the variable speed drive unit and the shaft was accomplished by the use of a rear axle drive unit from an automobile. In this unit only the gear action from the ring and pinion gears were utilized. The differential gears were welded together.

It was hoped that this device would cause a two dimensional rotating type of flow field with the flow pattern near the pipe approximating that of a vortex. But, such a flow was not obtained. In fact, the only flow which was observed at the surface moved very slowly and was directed radially away from the pipe. One possible explanation for this is that it was related to viscous forces, varying along the length of the pipe and strongly influenced by the proximity of the bottom of the tank. Because of the dynamical restrictions of no fluid motion at the bottom of the tank no fluid motion with respect to the rotating pipe at the pipe surface and a spatially continuous velocity distribution; the fluid velocity near the pipe increased with distance from the bottom of the tank. Since the only possible deviation in pressure from hydrostatic pressure was due to the fluid motion itself, the tendency of fluid particles to move radially away from the pipe increased with distance from the bottom of the tank. This resulted in a reduction in pressure from the hydrostatic value, which increased with distance from the bottom of the tank. Hence, fluid particles were accelerated upwards along the pipe by pressure forces. To keep continuity of flow, the vertically moving fluid assumed a radial motion upon reaching the surface.

In an attempt to overcome this effect, longitudinal vanes were attached to the lower fourteen inches of the pipe. The only effect of these vanes was to increase the production of turbulence.

To be assured that the failure of obtaining a circulating flow field was not caused by insufficient time allowed for the flow field to form, the pipe was rotated continuously for 15 hours without satisfactory results. It is unfortunate that this experiment could not be run with stationary circulating flow because the results of the interaction of surface waves with such a flow may have shed some light on the problem of the interaction of surface waves with turbulence.

Then it was decided to run the experiment with a radially directed flow. Since Evans (Ref. 1) used a thin surface current in his experiment and the use of a thin surface current facilitated drawing water in the bottom of the tank and thus preserving symmetry, the use of a surface current was decided upon.

The same aluminum tube was used as before. It was partitioned at its center and a row of holes was drilled around the pipe just below the water surface from which the outward flow would originate. Water was pumped into the upper portion of the pipe and drawn from the lower end through a second row of holes drilled near the bottom of the pipe. The two sections were separated by the aformentioned partition. With this arrangement too large a pressure drop occurred before the suction side of the pump for efficient operation, so water was drawn from a four inch pipe located at the center of the tank about 18 feet from the discharge pipe on the side toward which the waves would progress (Fig. 4).

Three pumps were tested each of larger capacity than the previous one; the third being the only one which produced a satisfactory flow field. (Fig. 5b).

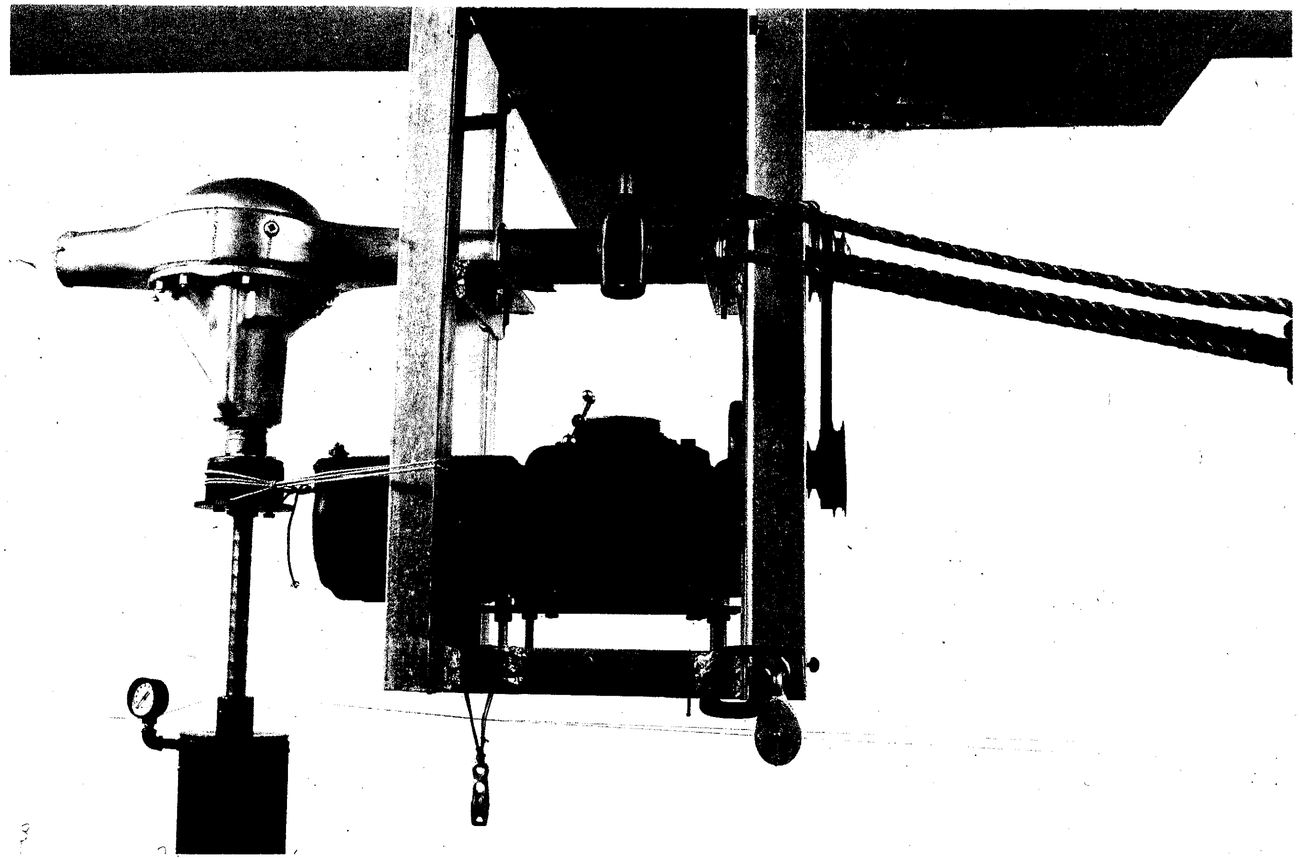

Figure 3. The device for driving the cylinder in rotation
Since this device did not produce the desired circulating flow it was used only as a support for the flow pipe.

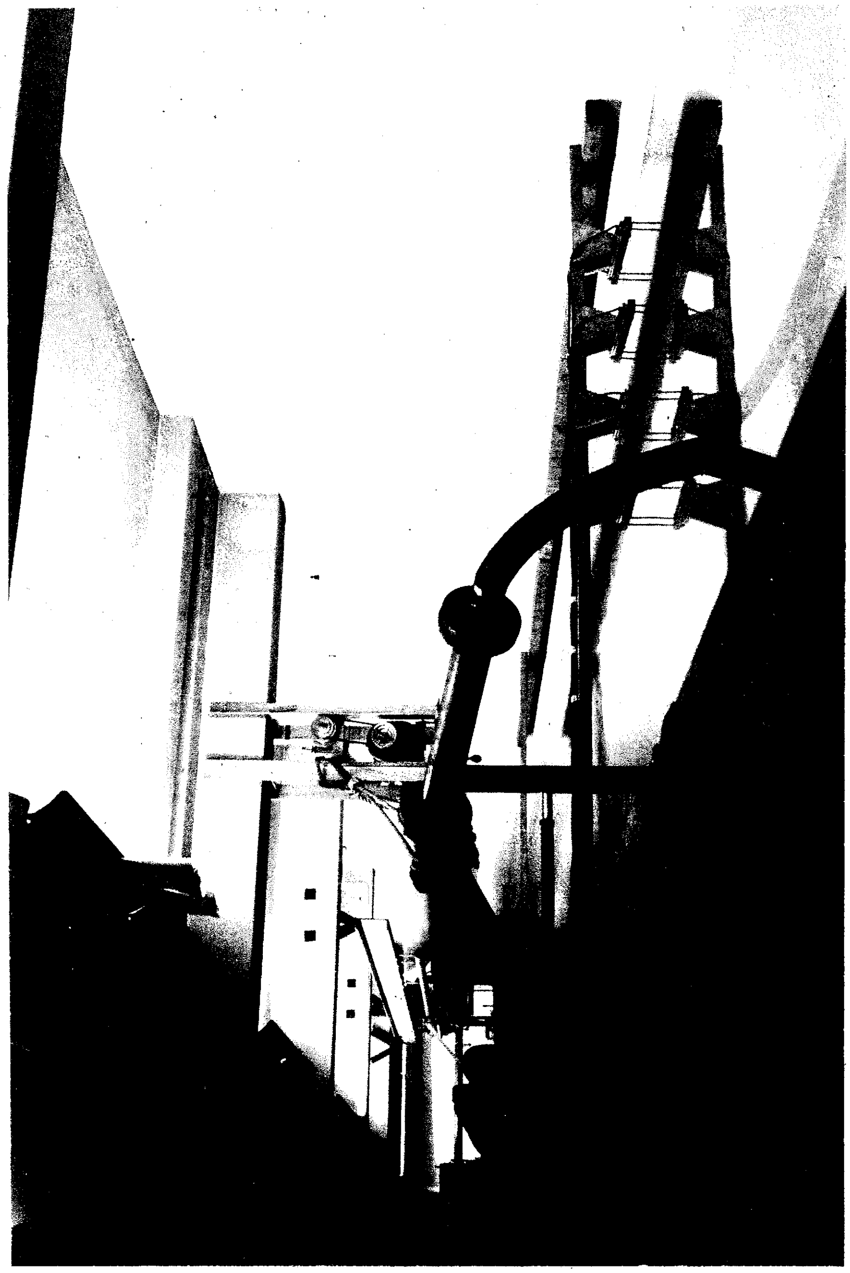

Figure 4. Details of the suction pipe and wave measuring probes

Figure 5. Some of the equipment used in this experiment

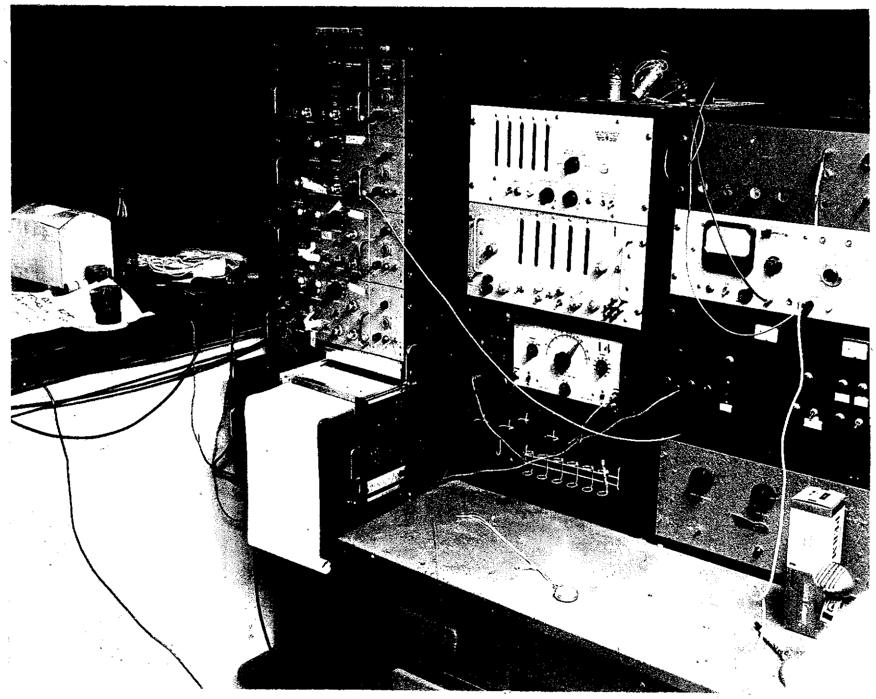

Figure 5(a). Electronic data recording equipment

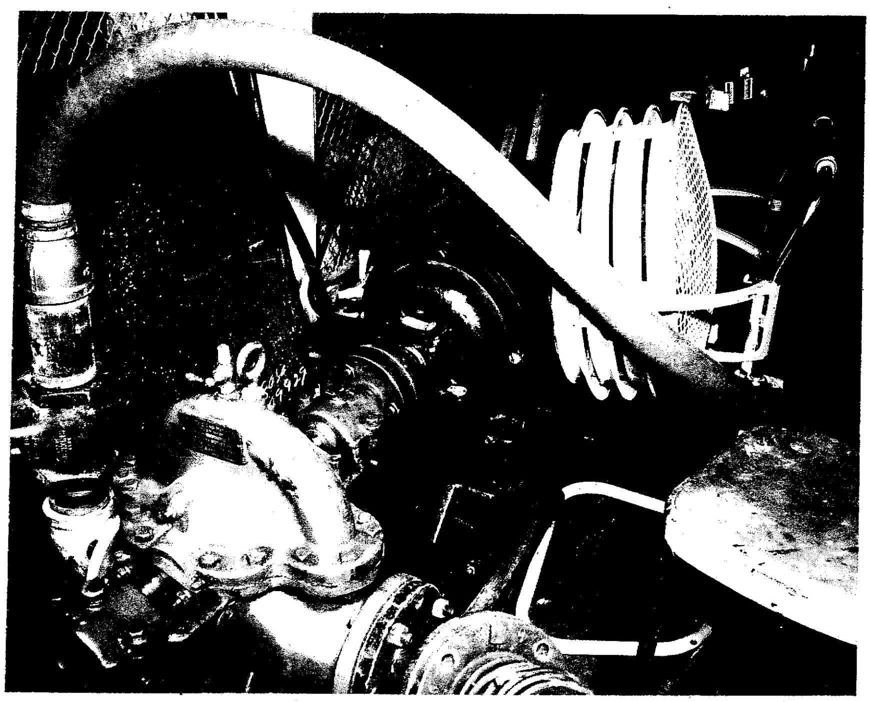

Figure 5(b). The Pump The purpose of the fan was to cool the pump motor.

CHAPTER IV

MEASUREMENTS

The parameters measured were wave height and wave speed. A continuous record of the wave height taken approximately 20 feet from the wavemaker was made by use of a sonic type wave height transducer and one Sanborn Recorder Channel. This transducer and its associated circuitry transmitted a voltage which was proportional to the distance from the transducer to the water surface. The signal which the device sensed was the sound caused by a spark in the transducer and then was reflected from the water surface to a microphone. No part of the device penetrated the surface of the water.

The wave height and wave speed in the region of the surface current were measured at one point at a time. To facilitate taking measurements at many points in a reasonable length of time, four sensing units were constructed. Each unit consisted of one piece of phenolic plastic 1" x 1 1/2" x 14" and four brass rods each 1/4" in diameter and 14" long (Fig. 6). The rods were threaded at one end over two inches of length and were screwed into tapped holes in the plastic. The direction of the longest dimension on the bar of plastic will be referred to as the longitudinal direction. Near each end of the plastic bar were two of the holes spaced one inch apart on a line perpendicular to the longitudinal direction. The longitudinal spacing of the two sets of holes was twelve inches. The two bars in each set of holes formed a capacitor. There was also some conductance between the bars due to impurities in the water. When the bars were partially immersed, the capacitance was dependent upon the depth of immersion, the relative permitivity of water being about 80. When the depth of immersion was somewhat greater than one inch, it could be expected that the capacitance would vary linearly with the depth of immersion. The four measuring units were mounted on a beam such that when the beam was placed across the tank the longitudinal direction of the units was parallel to the longest axis of the tank (Fig. 6). The distance between units was 1 foot. Units were wired so that the two capacitors on any one unit were connected respectively to two cables running from the sensing device to the operating station. The unit to which the cables were connected was determined by the position of a switch mounted on the beam. Each of the two cables was connected to a separate bridge circuit which was excited by alternating current. The bridge was balanced in the calm water condition. Then, when waves existed, a voltage output was obtained from the bridge which was proportional to the water wave disturbance. The output voltage as a function of time was recorded on a Sanborn recorder. By simultaneously recording the imbalances of the two bridges, information sufficient to determine the wave shape, wave height and wave velocity along the longitudinal direction of the tank was obtained. The measuring units were numbered one, two, three and four respectively, number one being the unit centered in the tank and number four being the unit nearest the wall. The sensitivity of the recording instrument was set so that the recording stylii deflected one centimeter per inch change of bar immersion on unit number one. Then, testing the other units showed that they all gave the same sensitivity within one part in one hundred. In making an experimental run, the signal from each capacitor was recorded for fifteen wave periods, the two on each unit being recorded simultaneously. Then the beam position was altered and the process was repeated in numerous beam postions. In this way the character of the waves over the region of the surface current was observed at a number of points. Observations over only one half of the tank were required since the effects on the opposite half must have been identical due to the symmetry of the flow field.

Various methods were attempted for measuring the velocity of the surface current in the absence of waves. The method which was eventually used to obtain information was to measure the time for a small floating cork to travel a given distance at various positions on the surface (Fig. 7). The direction was obtained by setting a small rod parallel to the direction of motion of the cork. This rod was connected to a shaft on which a pointer was mounted. The direction of the pointer was shown on a protractor. Although this method of determining the flow field seems to somewhat primitive, fairly repeatable data was obtained. A chart showing lines of constant velocity was made (Fig. 9).

The pump speed was measured with a "Strobotac," this being a commercial instrument made for measuring rotational speed. The speed was adjusted by means of a valve in the piping system and was kept constant at 1575 rpm.

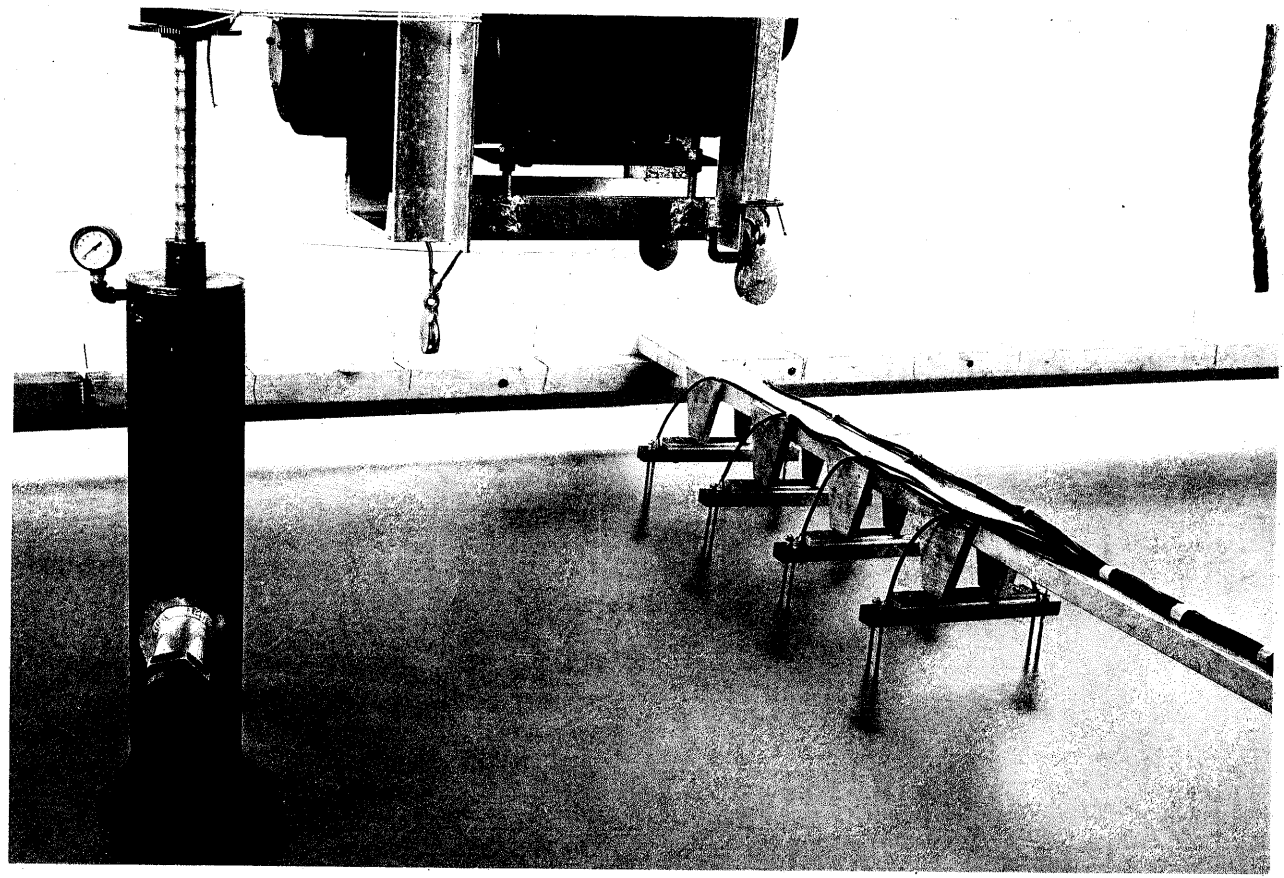

Figure 6. The flow pipe and the wave measuring probes

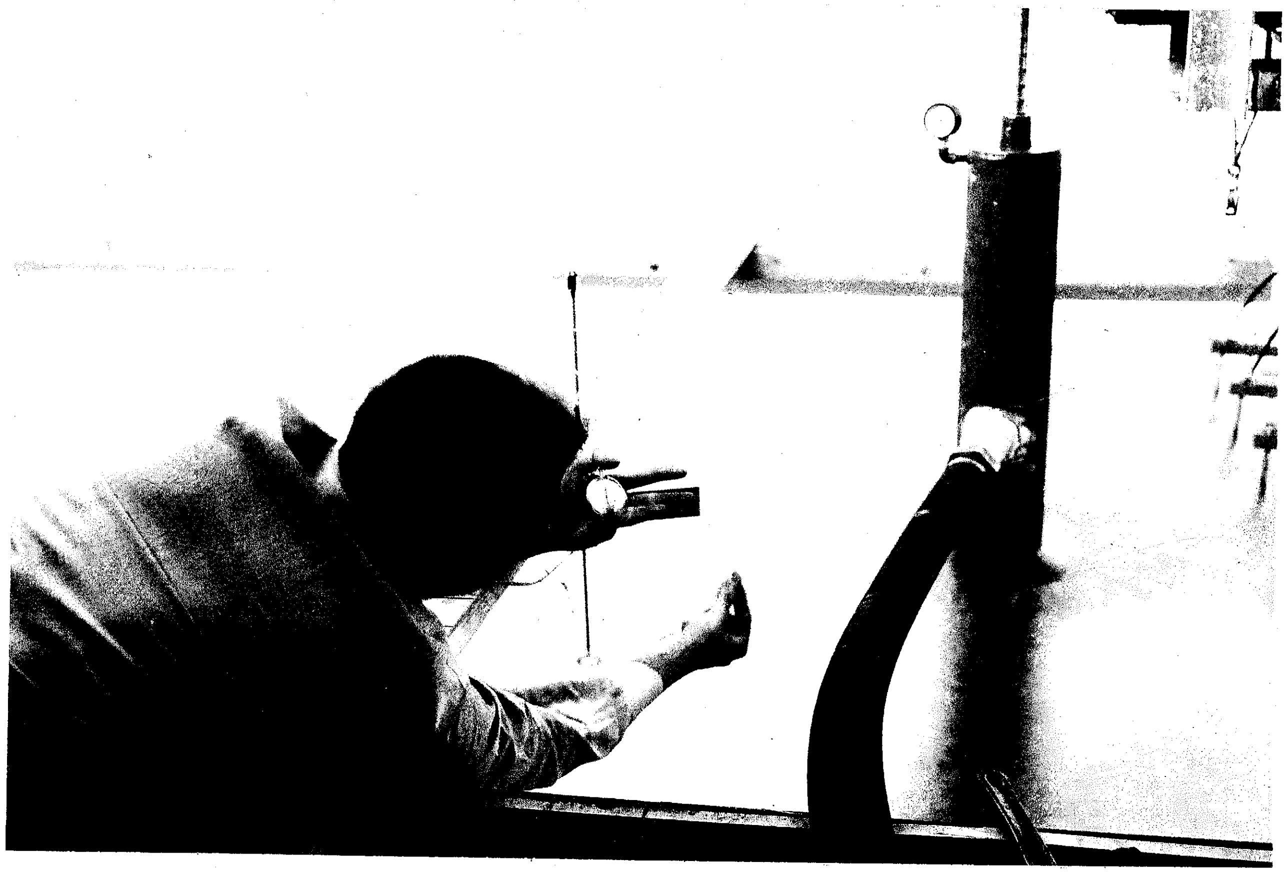

Figure 7. Measuring the flow field. The small object in the person's right hand is a piece of cork about to be dropped into the tank. The rod directly behind the person's left hand is a ruler. The vertical rod is part of the flow angle measuring device.

CHAPTER V

PLOTTING THE FLOW FIELD

For this experiment, the pump speed was held constant at 1575 revolutions per minute. Measurements of the surface current were taken twice on successive days and are described under "measurements." In the first set of measurements, data at each point was taken once. In the second set of measurements, data at each point was taken many times, until a repeatable result was obtained. Therefore, data recorded in the second measurement is considered to be more accurate than data recorded in the first measurement.

A chart was made which shows coordinates of the tank (Fig. 8). At each point of this chart corresponding to a point in the tank where current measurements were taken, the current data was recorded. Then lines of constant velocity were drawn. These lines were based upon the measured data, the boundary conditions at the wall of the tank; and the physical conditions of straight line flow down the center of the tank due to symmetry, and the fact that the flow velocity must be a continuous function of space. The streamlines were based upon the measured current direction and the physical fact that the streamlines should make angles of approximately 90° to the constant velocity lines. For irrotational flow the angle between streamlines and constant velocity lines is exactly 90°. For the flow field in question, the vorticity normal to the surface was assumed small because the velocity associated with this vorticity was small compared to the velocity associated with the source flow. Therefore, the angle between streamlines and constant velocity lines was approximately 90°. It should be noted that the component of vorticity parallel to the surface was very large compared to the component normal to the surface. To facilitate using data concerning the flow field, a chart showing only streamlines and constant velocity lines was drawn (Fig. 9).

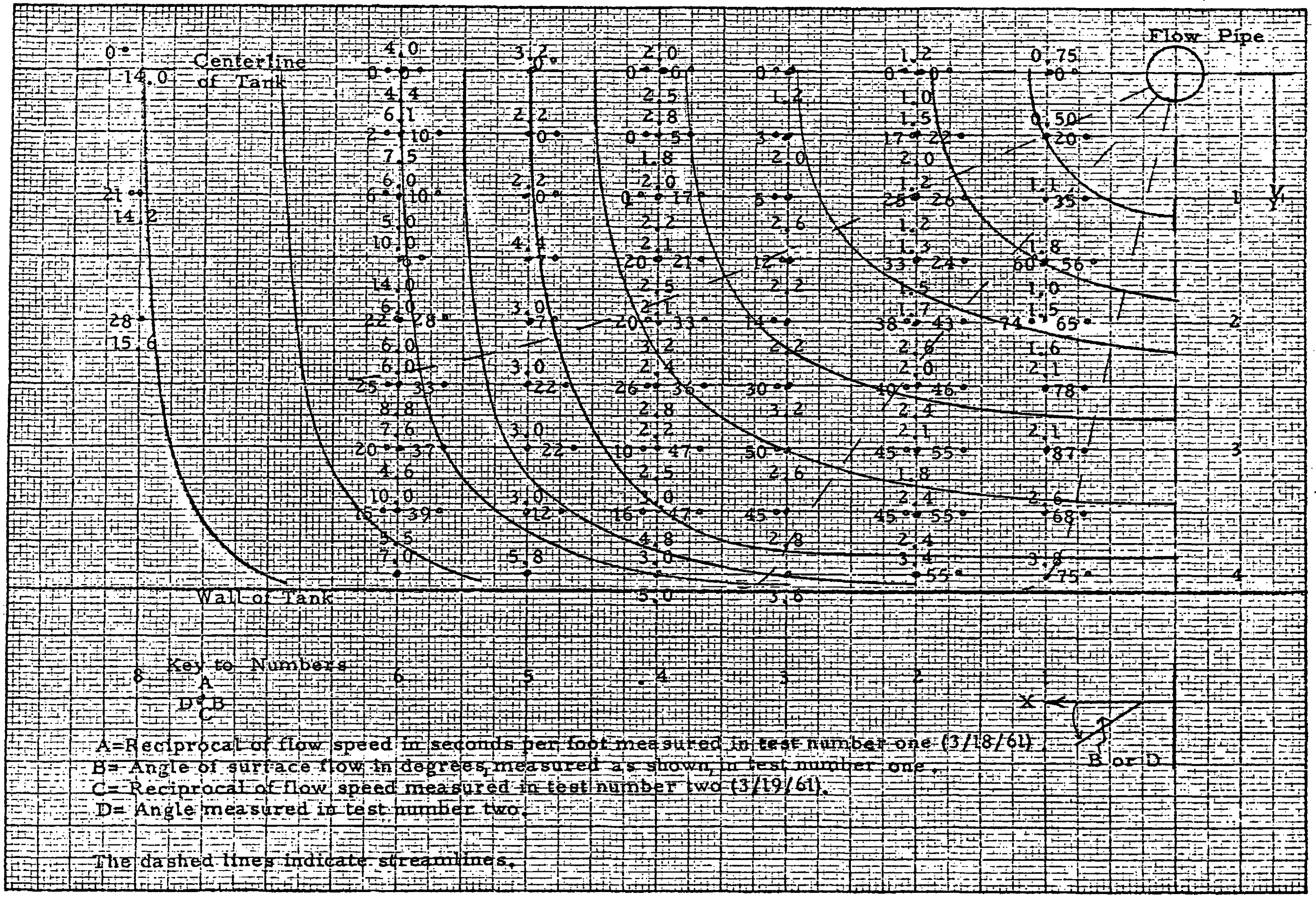

Figure 8. The current velocity at various points in the tank. Lines of constant velocity and streamlines are also shown.

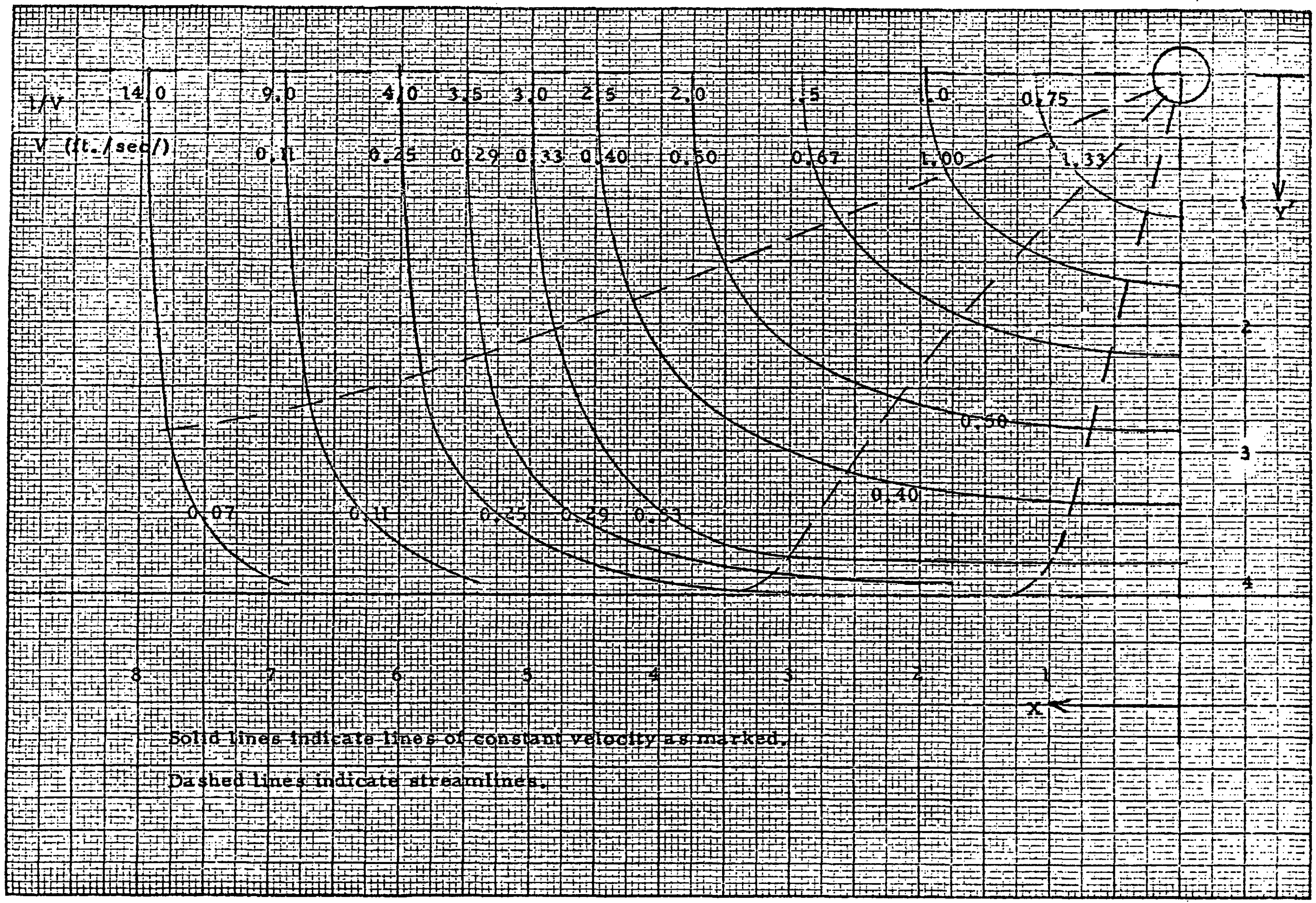

Figure 9 Lines of constant velocity and streamlines.

CHAPTER VI

DATA REDUCTION AND ACCURACY

Data was recorded for a number of different wave periods. The data for three different wave periods was reduced to tabular form and comprises the "data" section of this report.

At each point of measurement, the surface level as a function of time was recorded for six or more wave periods with only one of the two channels of the measuring unit in operation. Then the surface level was recorded for an additional six wave periods with both channels in operation. The reason for this procedure is that there was mutual electromagnetic coupling between the two measuring elements on each unit and when both channels were operating there was a beat frequency signal caused since the carrier frequencies of the two preamplifiers were unequal. To obtain an accurate record of the wave shape some data was recorded at each point with only one channel operating. The only need for having two channels operating was to determine the wave speed. From the number of preamplifiers available the two chosen for use were those which gave the highest beat frequency. Since this frequency was considerably higher than the fundamental wave frequencies, it was possible to ascertain the time at which a wave crest passed each measuring probe. From the one channel record at each point the time interval for six wave crests to pass the point was determined and dividing by six gave the wave period. For one particular unit data was recorded at fifty-six points and the calculation of the wave period at each point gave the same result accurate to one part in one hundred. Similar accuracy in determining the wave period was experienced in the other runs. The wave period for each run is printed at the head of the corresponding table in the data section. Also printed there are the dial setting and the crank setting for each run. These refer to the settings on the wavemaker which determine wave period and wave amplitude respectively. These settings are included to facilitate repeating part of the experiment if this is ever necessary. From the recordings with both channels operating the time for a wave crest to traverse the distance between the two measuring probes on each unit was ascertained. The distance between probes was one foot and the time was determined in seconds. The reciprocal of this number gave the apparent longitudinal wave velocity in feet per second. The velocity for each of the six crests recorded with both channels operating was determined and the numerical average of these velocities is listed in the data tables. The maximum deviation of the recorded wave velocities from the numerical averages given in the tables was 10 percent, and in most cases was not more than 5 percent. Any value differing by more than 10 percent from the average was not included in computing the average. This accuracy does not apply to regions within one foot of a place where waves are stopped: a point where the waves are stopped is denoted by the words "Waves Stopped," in the Data Tables.

The accuracy of the wave height data is limited to one part in ten by the readability of the recorded data. In many instances the wave heights at a point varied slowly with time. At such points at least fifteen wave periods were recorded. The wave height recorded in the tables is the largest repeatable wave height observed. Occasionally a solitary wave considerably larger than most was observed but the heights of such waves are not considered repeatable.

CHAPTER VII

DATA ANALYSIS

One of the parameters measured in this experiment was the interval between the time when a wave crest was under a measuring probe and the time when the same crest was under the second probe on the same unit; this probe being displaced from the first one foot in the longitudinal direction. This time interval is called t. $\frac{1}{t}$ is called the longitudinal wave speed.

The angle between the negative x direction and the direction of flow is called ϕ. (Fig. 10) and the angle between the negative x direction and the direction of wave propagation is called θ. (Fig. 10) The wave velocity in the direction of wave propagation is called V_n. The wave-length is called λ and the wave period is called T.

$$V_n = \frac{1}{t} \cos \theta \qquad \text{(VII-1)}$$

$$\lambda = V_n \, T \qquad \text{(VII-2)}$$

Substitution of Eq. (VII-1) in Eq. (VII-2)

$$\lambda = \frac{T}{t} \cos \theta \qquad \text{(VII-3)}$$

It is desired to find the relationship between wavelength and wave speed. One difficulty encountered in this respect is the determination of what frame of reference should be used in measuring the wave speed. If the current were infinitely thin, it would be advantageous to measure wave speed relative to the fixed tank. If the current were more than one half of a wavelength thick and uniform, it would be advantageous to measure wave speed relative to the current.

For this experiment, the surface current distribution as a function of depth was not determined quantitatively. Only the velocity on the surface was measured. To obtain a qualitative knowledge of the variation of the surface current with depth, small air bubbles were released from a deeply submerged tube and the path of these bubbles was observed through a window in the side of the tank. The current was very thin near the flow pipe and was less than 3 inches thick three feet from the flow pipe. Also the velocity distribution in the current was very non-linear with depth. For this experiment, the wave speed was referred to the moving current at the surface. The current velocity at the surface is called V_c and the wave velocity relative to the moving current is called V_r.

$$V_r = V_n - V_c \tag{VII-4}$$

The V's are taken in the vector sense.

The angle θ was not measured in this experiment, but qualitative observation showed it was always smaller than 20°. Cos 20° = 0.94. The approximation

$$V_r \cong \frac{1}{t} - V_c \cos \phi \tag{VII-5}$$

introduces an error in V_r of less than 10 percent if $V_c < \frac{1}{2t}$ which is a relation always true for the data which was anlayzed. The approximation

$$\lambda \cong \frac{T}{t} \tag{VII-6}$$

introduces an error in λ of less than 6 percent.

All the data for T = 0.76 seconds was analyzed as well as some of the data for T = 0.42 seconds. The numerical values calculated in this analysis are shown in Table I. It was desired to determine whether or not the three dimensional waves on a non-uniform current satisfied the familiar relationship,

$$V = \sqrt{g\lambda/2\pi} \tag{VII-7}$$

A glance at Table I shows the difference quotient

$$D = \frac{\left(\frac{1}{t} - V_c \cos\right) - \sqrt{g\lambda/2\pi}}{\frac{1}{t} - V_c \cos \phi} \tag{VII-8}$$

is in most instances less than 0.08 in magnitude and often negative near the side of the tank. Proceding from the side of the tank toward the center of the tank, the value of the difference quotient increases to as much as 0.18. The difference quotient is the fractional increase of the measured wave speed ($\frac{1}{t} - V_c \cos\theta$) over the theoretical value ($\sqrt{g\lambda/2\pi}$), this being based on the measured wavelength. It seems reasonable to conclude that the theoretical value ($\sqrt{g\lambda/2\pi}$ is either the actual wave speed or a very good approximation to the wave speed for waves on a current with variable velocity in the surface plane, but uniform with respect to depth at each point (x, y).

In this experiment and method of data analysis there are two effects which cause the quantity $\left[\left(\frac{1}{t} - V_c \cos\theta\right) - \sqrt{g\lambda/2\pi}\right]$ to be non-zero. One is due to the approximation used in computing values of the difference quotient listed in Table I. This approximated difference quotient is designated by D. The difference quotient computed without approximation is designated by D'

$$D = \frac{\left[\frac{1}{t^2}\cos^2\theta - V_c^2 - 2V_c\frac{1}{t}\cos\theta\cos(\phi-\theta)\right]^{\frac{1}{2}} - \left[\left(\frac{gT\cos\theta}{2\pi t}\right)\right]^{\frac{1}{2}}}{\left[\frac{1}{t^2}\cos^2\theta + V_c^2 - \frac{2}{t}V_c\cos\theta\cos(\phi-\theta)\right]^{\frac{1}{2}}} \qquad \text{(VII-9)}$$

(See appendix for derivation)

The second effect which causes D to non-zero is that V_r is referred to the current motion at the free surface. The equation $V_r = \sqrt{g\lambda/2\pi}$ (VII-7) applies to waves on a fluid whose velocity is spatially constant. If the relation were to be valid at a point on the surface then there would have to be no variation of current velocity with depth at this point. A detailed calculation of waves on a current whose velocity is constant in x and y but linearly decreasing to a depth h and is zero for depths greater than h can be found in Ref. 2. This solution, though complicated, is conceptually straightforward because in the moving current the vorticity is constant. For the flow distributions in this experiment an exact prediction of the effects of current depth upon the wave speed would be extremely difficult, if not impossible. However, one can make the qualitative prediction that the thicker the current is, the smaller will be the deviation of V_r from $\sqrt{g\lambda/2\pi}$ due to non-infinite depth of current. This is readily observed in Table I where it is seen that the numerical values of the difference quotient decrease with distance from the flow pipe. The thickness of the surface current increases with distance from the flow pipe.

The next quantity to be considered is the energy propagation. For waves in deep water (depth greater than $1/2\,\lambda$) the speed at which the energy propagates relative to the water is one half of the wave speed. If the surface current were deep, the speed of energy propagation in the longitudinal direction relative to the current would be $\frac{V_r}{2}\cos\theta$. Denoting the rate of energy propagation in the longitudinal direction relative to the ground by $P(x_1\ y)$ (for power) we know:

$$P(x, y)\ (\frac{V_r}{2}\cos\theta + V_c\cos\phi)\ h^2 \qquad \text{(VII-10)}$$

H is the wave height.

The units of power here are not those conventionally used due to the absence of a constant, but the units do not concern as so long as consistancy of units is maintained.

Again we make the approximation $\theta = 0$ and say:

$$P \cong (\frac{V_r}{2} + V_c\cos\phi)h^2 \qquad \text{(VII-11)}$$

Substituting Eq.(VII-5)in Eq.(VII-11) there results:

$$P(x, y) \cong h^2\left[-\frac{1}{2}(\frac{1}{t} - V_c\ \cos\phi) + V_c\cos\phi\right] \qquad \text{(VII-12)}$$

Values of $P(x_1\ y)$ at various points for T = 0.76 are shown in Table I. We see at once that the energy propagation is not spatially constant as it is in two dimensional waves.

Unless wave energy is lost through viscous sheer forces at the tank walls or through turbulence in the fluid, the total wave power through each cross section of the tank must be the same. Denote the total power through a crossection by P_t and the half width of the tank by a

$$P_t = \int_0^a P(x, y)\, dy \qquad \text{(VII-13)}$$

We take the sum of the values of $P(x_1\ y)$ at the four measurement stations for each value of x as a measure of the integral in Eq. (VII-13) and call this sum $P_{t_m}(x)$. Values of $P_t(x)$ as well as $P_{t_m}(x)$ are shown in Table I. The variation of $P_{t_m}(x)$ with (x) is much greater than can be accounted for by turbulence. The cause of this variation is that the formulation for P was based upon surface current deeper than one half a wavelength. As was previously stated, this was not the case for this experiment. The values of $P_{t_m}(x)$ increase with (x). The reason for a reduction in $P_{t_m}(x)$ near the pipe is that the current is thinner there than elsewhere and the approximation to the speed of energy propagation used is more inaccurate for thin currents than thick currents. Since the current velocity in the region under consideration opposes the wave motion the term in the energy propagation speed due to the moving current subtracts from the term due to the wave speed. Since thin currents have less effect on the waves than thick currents, the term subtracted near the flow pipe is larger than it should be for an exact answer. This gives the result that $P_{t_m}(x)$ decreases as we approach the flow pipe.

Observations of the waves in the vicinity of the flow pipe show that wave breaking occurs on the upstream side of the pipe as expected. The relation that the stopping velocity is one fourth of the wave velocity which is valid for the two-dimentional case with a deep current does not hold for the three dimensional case in a thin current. However, this relationship does yield the order of magnitude of the current speed needed to cause the waves to break.

CHAPTER VIII

CONCLUSIONS

For waves on a non-uniform surface current we find that:

$$V_r = f(c)\sqrt{\frac{g\lambda}{2\pi}} \qquad \text{(VIII-1)}$$

where the term f(c) takes into account the variation of the current with depth. For a current varying on the surface plane but constant with depth there is considerable evidence that

$$V_r = \sqrt{\frac{g\lambda}{2\pi}} \qquad \text{(VIII-2)}$$

Also, the wave power density does not remain constant in space when the waves pass over a non-uniform surface current so that the two-dimensional results of Evans (Ref. 1) and Taylor (Ref. 2) cannot be applied to obtain quantitative answers. The general form of the two dimensional results does give a qualitative picture of the effects as shown in Fig. (2). It is noted here that the correlation between wave height and surface current speed for the results of this experiment does not agree with the results of Evans (Ref. 1) and Taylor (Ref. 2) which are valid for the two-dimensional case. As the waves approach the opposing surface current the wave height increases to a maximum which occurs after the waves pass through the strongest opposing current (Fig. 2b). Then if the current is sufficiently strong, considerable attenuation occurs (Fig. 2c). The return of waves far downstream (Fig. 2d) is not explained by the results of Evans(Ref.1) and Taylor (Ref. 2).

It seems reasonable to conclude that until breaking and the resulting turbulence occurs that the total wave energy propagated through a cross-section of the tank per unit time is constant. In fact, the difference between P_{t_m} (x) far from the current and P_{t_m} (x) in the current is a measure of the reduced effect of the current due to its limited depth.

CHAPTER IX

RECOMMENDATIONS FOR FURTHER RESEARCH

It is highly recommended that this experiment be repeated for waves with a period of 0.76 seconds with a deep surface current. To obtain such a current the flow pipe must have holes over its full depth. The suction pipe should be identical with the flow pipe so that the fluid is removed in a two-dimensional manner. The pump needed for such an experiment will have to rated at 20 H.P. or more.

The flow field for such an experiment can be calculated from a mathematical model. Let the outlet pipe and suction pipe be set vertically along the center line of the tank with the distance between the two pipes equal to four feet. (Fig. 10) The width of the tank is 8.6 ft. The outlet pipe and suction pipe can be approximated by a two-dimensional source and a two-dimensional sink respectively. Using the directions and dimensions shown in Fig. 10, we obtain;

$$w = m\left[\log \sinh \frac{\pi(z+2)}{8.6} - \log \sinh \frac{\pi(z-2)}{8.6}\right] \qquad \text{(IX-1)}$$

where w is the complex potential $\phi + i\psi$ and $z = x + iy$.
Denoting the x directed velocity by u and the y directed velocity by v, we obtain:

$$u - iv = \frac{dw}{dz} = m\left[\frac{\frac{\pi}{8.6}\cosh\frac{\pi(z+2)}{8.6}}{\sinh\frac{\pi(z+2)}{8.6}} - \frac{\frac{\pi}{8.6}\cosh\frac{\pi(z-2)}{8.6}}{\sinh\frac{\pi(z-2)}{8.6}}\right]$$

$$u - iv = \frac{m\pi}{8.6}\left[\text{ctan}\frac{\pi(z+2)}{8.6} - \text{ctan}\frac{\pi(z-2)}{8.6}\right] \qquad \text{(IX-2)}$$

$$\phi = R_e\, w = m\left[\log \sinh \frac{\pi(z+2)}{8.6} - \log \sinh \frac{\pi(z-2)}{8.6}\right] \qquad \text{(IX-3)}$$

Lines of ϕ = constant are lines on which the speed is constant. The value of the speed on each line is $\left|\frac{dw}{dz}\right|^2$ and can be calculated from Eq. (14).

For the flow proposed there is sufficient theoretical information to plot the flow field very accurately except in the immediate vicinity of the flow pipe. In this experiment the angle (θ) which the wave normal makes with the negative x direction should be measured. This can be accomplished by taking a series of photographs from different positions above the tank.

It is the opinion of the author that the results of the proposed experiment will show that the wave velocity relative to the current will be $\sqrt{\frac{g\lambda}{2\pi}}$ and that taking the speed of energy propagation as one half the wave velocity plus the current velocity, will show that the power propagation through any cross-section is constant.

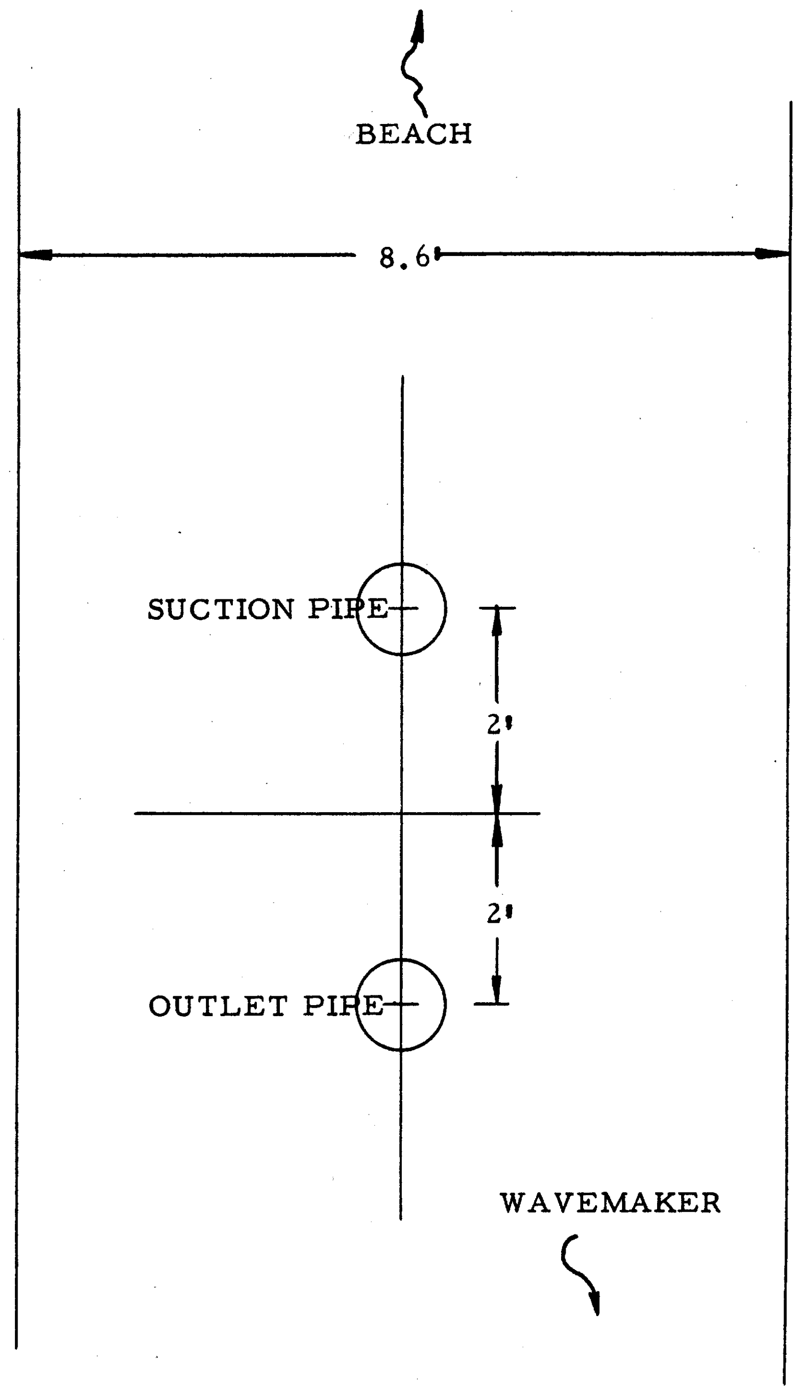

Figure 10. Top view of pipes for producing a two dimensional flow which is variable in the surface plane.

APPENDIX A

<u>Calculation of the exact difference quotient.</u>

Refer to Fig. (11)

To find V_r , we apply the law of cosines.

$$V_r^2 = V_a^2 + V_c^2 - 2 V_c V_n \cos (\phi - \theta) \qquad \text{(A-1)}$$

$$V_n = \frac{1}{t} \cos \theta \qquad \text{(VII-1)}$$

Substituting (1) in (A-1) and then taking the square root yields;

$$V_r = \left[\frac{1}{t^2} \cos^2 \theta + V_c^2 - 2 V_c \frac{1}{t} \cos \theta \cos (\phi - \theta) \right]^{\frac{1}{2}} \qquad \text{(A-2)}$$

$$\lambda = \frac{T}{t} \cos \theta \qquad \text{(VII-3)}$$

$$D = \frac{V_r - \sqrt{\frac{g\lambda}{2\pi}}}{V_r} = \frac{\left[\frac{1}{t^2} \cos^2 \theta + V_c^2 - 2V_c \frac{1}{t} \cos \theta \cos (\phi - \theta) \right]^{\frac{1}{2}} - \left[\frac{gT\cos\theta}{2\pi t} \right]^{\frac{1}{2}}}{\left[\frac{1}{t^2} \cos^2 \theta + V_c^2 - 2V_c \frac{1}{t} \cos \theta \cos (\phi - \theta) \right]^{\frac{1}{2}}}$$

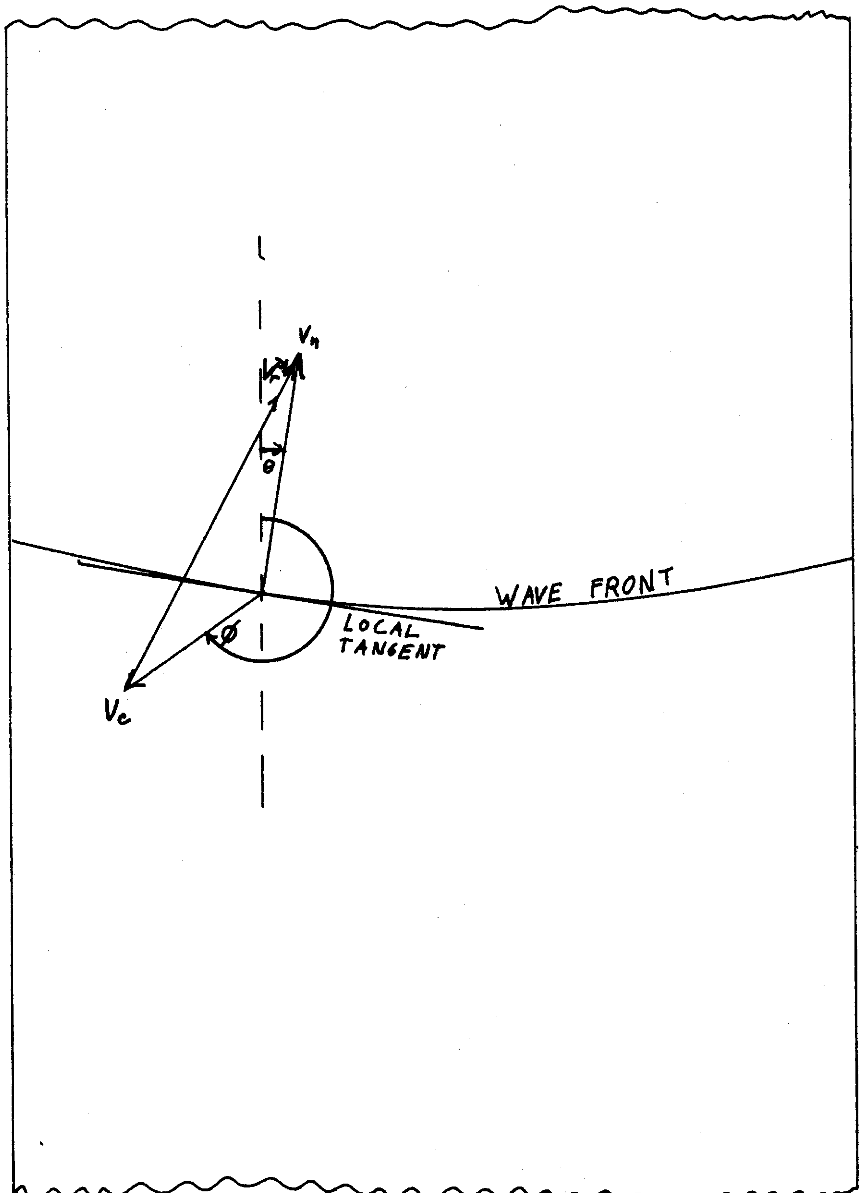

Figure 11. Definitions of Velocities and Angles

APPENDIX B

DISCUSSION OF A MATHEMATICAL SOLUTION

To find even a linearized mathematical solution for the experiment performed would be quite impossible. The simplest model which would bring forth the salient points of this experiment is described below.

Consider a body of water infinite in its transverse dimensions and infinite in depth. We set-up a coordinate system at a point on the surface with the x and y axis lying in the surface and the z axis oriented downwards. A two dimensional source is situated on the line $x = 0$, $y = 0$, z is positive. Two dimemensional waves originate at $x = \infty$ and propagate in the negative x direction. As these waves approach the source there will be an interaction between them and the source current.

To find a solution for this problem we make the assumptions that the flow is irrotational an inviscid. For these assumptions we can define a velocity potential, the Laplacian of which is zero, i.e.

$$\nabla^2 \phi = 0 \qquad \text{(A-3)}$$

$$\text{velocity} = \vec{q} = \nabla \phi \qquad \text{(A-4)}$$

The boundary conditions are:

1. dynamic and kinematic conditions at the free surface
2. at $z = \infty$ the only motion is that due to the source
3. at $y = \pm \infty$ the waves are not affected by the source
4. the boundary condition at $x = -\infty$ is not determined.

It is possible that there is no affect from the source at $x = -\infty$, but I know of no reason why the source cannot affect the wave phase at $x = -\infty$.

Assuming the boundary condition at $x = -\infty$ can be found, the task of finding the solution for ϕ is difficult indeed because the solution is one in which the variables do not separate. For separation of variables to exist we find two types of spatial dependence for ϕ.

They are:

1. exponential dependence
2. sinusoidal dependence

For the problem at hand, we obtain a variation in wave height along the same direction where the spatial dependence is oscillatary in nature. Hence, the form of the solution is one in which the variables do not separate. Techniques for solving Laplace's equation where the variables do not separate exist for some cases. The possibility that one of these techniques can be used for the problem at hand is worthy of investigation.

REFERENCES

1. Evans, J. T., "Pneumatic and Similar Breakwaters," 1955, Proc. Roy. Soc. A, 231, 457.

2. Taylor, G. I., "The Action of a Surface Current Used as a Breakwater," 1955, Proc. Roy. Soc. A, 231, 466.

DATA

TABLE I

STATION	$\frac{L}{t}$	V_c	ϕ	$-V_c \cos\phi$	$\frac{L}{t} - V_c \cos\phi$	$\lambda \cong \frac{T}{t}$	$\sqrt{\frac{g\lambda}{2\pi}}$	$\frac{L}{t} - V_c\cos\phi - \sqrt{\frac{g\lambda}{2\pi}}$
		T = 0.76						
X =7								
1	3.79	0.10	180	0.10	3.89	2.88	3.86	0.03
2	3.68	0.10	180	0.10	3.78	2.80	3.79	-0.01
3	3.24	0.09	175	0.09	3.33	2.46	3.56	-0.23
4	3.62	0.07	120	0.04	3.66	2.75	3.75	-0.11
X=6								
1	3.57	0.25	180	0.25	3.82	2.71	3.74	0.08
2	3.52	0.24	175	0.24	3.96	2.68	3.72	0.24
3	3.57	0.20	160	0.17	3.74	2.71	3.74	0.00
4	3.52	-.10	115	0.04	3.56	2.70	3.73	-0.17
X=5								
1	3.33	0.33	180	0.33	3.66	2.53	3.52	0.14
2	3.52	0.32	173	0.32	3.84	2.68	3.72	0.12
3	3.68	0.31	160	0.29	3.97	2.80	3.79	0.18
4	3.91	0.17	130	0.11	4.02	2.97	3.89	0.13
X=4								
1	3.47	0.47	180	0.47	3.94	2.64	3.68	0.26
2	3.91	0.44	162	0.42	4.33	2.97	3.90	0.43
3	3.57	0.37	150	0.32	3.89	2.71	3.74	0.15
4	3.38	0.25	115	0.11	3.49	2.57	3.53	-0.04
X=3								
1	3.85	0.65	180	0.65	4.50	2.92	3.86	0.64
2	3.21	0.59	143	0.47	3.68	2.44	3.54	0.14
3	3.42	0.44	120	0.22	3.44	2.60	3.65	-0.21
4	3.47	0.28	96	0.03	3.50	2.64	3.68	-0.18
X=2								
1	3.21	1.00	180	1.00	4.21	2.44	3.54	0.75
2	3.79	0.72	140	0.55	4.34	2.88	3.95	0.39
3	3.52	0.50	115	0.22	3.74	2.68	3.72	0.02
4	3.57	0.30	94	0.02	3.59	2.71	3.73	-0.14
X=1								
Waves very confused for station one. Waves stopped for station 2.								
3	3.57	0.51	95	0.05	3.62	2.71	3.73	-0.11
4	3.75	Flow velocity not determined.						
X=-1								
Waves stopped for stations one and two.								
3	5.00	0.51	85	-0.05	4.95	3.80	4.40	0.55
4 Flow velocity not determined.								
X =-2								
4	5.00	0.30	86	-0.02	4.98	3.80	4.40	0.58
		T = 0.42						
X=7								
1	1.82	0.10	180	0.10	1.92	0.765	1.98	-0.08
2	1.97	0.10	180	0.10	2.07	0.826	1.93	0.14
3	1.61	0.09	175	0.09	1.70	0.676	1.46	-0.14
X=5								
2	1.33	0.32	173	0.32	1.65	0.559	1.69	-0.04
3	1.70	0.31	160	0.24	1.94	0.713	1.91	0.03

TABLE I (Continued)

STATION	$\sqrt{\lambda}$	$\frac{\frac{1}{t} - V_e \cos\phi - \sqrt{\frac{g\lambda}{2\pi}}}{\frac{1}{t} - V_e \cos\phi}$	h^2	$\frac{\frac{1}{t} - V_e \cos\phi}{2}$	V_f	$P(X,Y)$	$P_{T_m}(X)$
X=7		T =0.76					
1	1.70	0.0077	1.69	1.95	1.85	3.12	
2	1.68	-0.0026	1.69	1.89	1.79	3.02	13.80
3	1.57	-0.069	1.69	1.67	1.68	2.84	
4	1.66	-0.030	2.89	1.83	1.79	4.82	
X=6							
1	1.65	0.021	1.69	1.91	1.76	2.98	
2	1.64	0.063	1.69	1.98	1.74	2.94	12.70
3	1.65	0.000	1.95	1.87	1.70	2.33	
4	1.65	-0.048	2.56	1.78	1.74	4.45	
X=5							
1	1.59	0.038	1.69	1.83	1.50	2.54	
2	1.64	0.031	1.69	1.92	1.60	2.71	11.42
3	1.67	0.045	1.44	1.99	1.70	2.44	
4	1.73	0.032	1.96	2.01	1.90	3.73	
X=4							
1	1.62	0.066	1.44	1.97	1.50	2.16	
2	1.72	0.099	1.44	2.17	1.75	2.52	10.52
3	1.65	0.039	1.00	2.95	2.65	2.63	
4	1.60	-0.011	1.96	1.75	1.64	3.21	
X=3							
1	1.71	0.140	1.44	2.25	1.40	2.02	
2	1.56	0.038	1.44	1.84	1.36	1.96	14.85
3	1.61	-0.061	1.69	1.72	1.52	2.57	
4	1.63	-0.051	4.84	1.75	1.72	8.30	
X=2							
1	1.56	0.180	1.44	2.11	1.11	1.60	
2	1.70	0.090	1.21	2.17	1.62	1.96	10.46
3	1.64	0.0053	1.44	1.87	1.65	2.37	
4	1.65	-0.039	2.56	1.79	1.77	4.53	
X=1							
Waves very confused for station one. Waves stopped for station 2.							
3	1.65	-0.030					
4	Flow velocity not determined.						
X=-1							
Waves stopped for stations one and two.							
3	1.95	0.140					
4	Flow velocity not determined.						
X =-2							
4	1.95	0.150					
		T=0.42					
X=7							
1	0.875	-0.031					
2	0.851	0.067					
3	0.821	-0.082					
X=5							
2	0.747	0.024					
3	0.844	0.015					

TABLE II

Wave Period = 0.76 Seconds Dial Setting= 1805 Crank Setting = 25

Y in feet	Wave Height in inches	Wave Velocity in ft./sec.	Y in feet	Wave Height in inches	Wave Velocity in ft./sec.
	X = 10 Ft.			X = 2 Ft.	
1	1.2	4.24	1	1.2	3.21
2	1.4	4.03	2	1.1	3.79
3	1.3	4.10	3	1.2	3.52
4	1.3	3.73	4	1.6	3.57
	X = 9 Ft.			X = 1 Ft.	
1	1.3	3.68	1	0,9	3.12
2	1.2	3.21	2	Waves Stopped.	0.7 in. height variation.
3	1.2	3.57	3	1.5	3.57
4	1.6	3.47	4	1.6	3.57
	X = 8 Ft.			X = -1 Ft.	
1	1.2	3.73	1	Almost all Waves Stopped	
2	1.4	4.81	2	Waves Stopped	
3	1.2	4.71	3	1.5	5.00
4	1.3	3.91	4	1.7	3.12
	X = 7 Ft.			X = -2 Ft.	
1	1.3	3.79	1	Waves Stopped	
2	1.3	3.68	2	Waves Stopped	
3	1.3	3.24	3	1.0	6.24
4	1.7	3.62	4	1.3	5.00
	X = 6 Ft.			X = -3 Ft.	
1	1.3	3.57	1	Waves Stopped	
2	1.3	3.52	2	Waves Stopped	
3	1.4	3.57	3	1.1	3.33
4	1.6	3.52	4	2.2	3.68
	X = 5 Ft.			X = -8 Ft.	
1	1.3	3.33	1	1.1	3.62
2	1.3	3.52	2	0.8	4.16
3	1.2	3.68	3	0.8	4.16
4	1.4	3.91	4	1.2	4.81
	X = 4 Ft.				
1	1.2	3.47			
2	1.2	3.91			
3	1.0	3.57			
4	1.4	3.38			
	X = 3 Ft.				
1	1.2	3.85			
2	1.2	3.21			
3	1.3	3.42			
4	2.2	3.47			

TABLE III

Wave Period = 0.52 Seconds. Dial Setting = 1700 Crank Setting = 10

	Y in feet	Wave Height in inches	Longitudinal Wave Speed in feet/Second
		X = 5 Ft.	
	1	0.9	1.92
	2	0.9	2.27
	3	1.0	1.92
	4	0.7	2.50
		X = 4 Ft.	
	1	0.7	2.08
	2	0.8	2.08
	3	0.9	1.92
	4	1.0	3.12
		X = 3 Ft.	
	1	0.7	1.66
	2	1.2	1.66
	3	1.5	*
	4	1.7	2.27
		X = 2 Ft.	
	1	1.0	*
	2	Waves Stopped	Random Looking Signal Remains.
	3	1.8	2.50
	4	1.4	3.57
		X = 1 Ft.	
	1	Waves Stopped	
	2	1.1	*
	3	1.1	*
	4	1.1	*
		X = -8 Ft.	
	1	Waves Stopped	
	2	1.2	3.38
	3	0.8	3.33
	4	0.6	2.78
		X = -2 Ft.	
	1	Waves Stopped	
	2	Waves Stopped	
	3	1.0	3.97
	4	1.2	3.56

* indicates that this data was unobtainable.

TABLE IV

Wave Period = 0.42 Seconds. Dial Setting = 1633 Crank Setting = 10

	Y in feet	Wave Height in inches	Longitudinal Wave Speed in feet/Second
		X = 11 Ft.	(This data taken with Pump off.)
	1	0.9	2.17
	2	0.9	2.17
	3	0.9	2.17
	4	0.8	2.17
		X = 9 Ft.	
	1	o.8	2.00
	2	0.8	*
	3	1.1	2.00
	4	1.2	*
		X = 7 Ft.	
	1	2.1	1.82
	2	2.0	1.97
	3	2.0	1.61
	4	2.1	*
		X = 5 Ft.	
	1	1.4	*
	2	1.6	1.33
	3	1.2	1.70
	4	0.9	1.64
		X = -8Ft.	
	1	Waves Stopped	
	2	0.7	2.50
	3	0.6	*
	4	0.6	3.12
		X = -5 Ft.	
	1	0.5	*
	2	Waves Stopped with occasional variations of as much as 0.5 inches in the remaining random like signal.	
	3		
	4		

Height as recorded on Sonic Wave Transducer = 0.85 inches.

At X = 1 and X = 2, the waves were stopped, but the amplitude of the remaining random like signal was somewhat more than in the "no wave" case. Variations of as much as o.5 inches were observed.

* indicates that this data was unobtainable.