

Simplified Model and Numerical Analysis of Multi-layered Piezoelectric Diaphragm

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Abstract—The validity of the dynamic analysis based on simplified plate model was investigated using of FE-codes ANSYS in the present paper. The simplified clamped multi-layered plate model was first verified by comparison with the exact model. The simply supported plate model was confirmed to be not a suitable model due to its large error as comparing with exact model. Influence of dimensions of laminar diaphragm on nature frequencies was studied. Deflection and voltage response driven by mechanical and electric loads were described. The optimized thickness ratio of PZT layer to SiO₂ and Si layers was given in the paper to obtain the best deflection export of actuator in design.

Keywords—piezoelectric diaphragm, laminated plate, nature frequency, FEM

I. INTRODUCTION

Due to their intrinsic electromechanical coupling behaviors, piezoelectric materials have been widely made into actuators and sensors used in smart structures and MEMS [1-3]. The structure of actuators or sensors in general is constructed mainly by multi-layer diaphragm (or film) with piezoelectric layer embedded in it and its support, substrate (Figure 1). The multi-layer PZT diaphragms act as a mechano-electrical converter. They convert values of mechanical quantities (deformation, stress, velocity, acceleration, etc.) into suitable electrical signals as being utilized as sensors, whereas the converse effect, as being designated as actuators, are associated with the mechanical movement generated by the application of electrical field. So the properties of multi-layer PZT diaphragm are the basic parameters in devices design.

For the important roles played by multi-layer PZT diaphragms in micro-device, its laminated structures were investigated for their dynamic behaviors [4-10]. Approximate theories [5-8] and computational models [9, 10] were widely used for the purpose of providing accurate dynamic characteristic of the diaphragms. The interest for exact, closed form and other accurate analytic solutions has much increased during the last few years. Both coupled [7, 11] and uncoupled [6, 12] solutions were proposed. The in-plane strains of piezoelectric actuator were used to induce bending of rectangular plate [13-15], shell [16], multi-layer shell [17], circular plates [18], and disk [19, 20] structures.

Though the primal structures of actuator or sensor were simplified into laminated plates with clamped or simply supported boundary condition in above research since the small ratio of thickness of multi-layer piezoelectric

diaphragm to its other dimensions, the validity of such simplification was not verified. In present work, the validity of simplified clamped and simply supported laminated plate models in the calculation of nature frequency was first verified by undergoing comparison with the exact model by using FE-codes ANSYS. Clamped laminated plate model was confirmed to be a suitable model because of its small error as comparing with exact model and was finally adopted in latter frequency analysis. But then, simply supported plate model is cancelled from modal analysis since its large error comparing with exact model. Influence of dimensions of laminar diaphragm on nature frequencies has been investigated and reported by using the clamped plate model. Mechanical and electric loads induced deflection and voltage response on the diaphragm of exact model were described. For the purpose of obtaining optimum shape or robust control algorithm on design, adjusted SiO₂/PZT (thin film) and Si/PZT (thick film) thickness ratio, with which diaphragm could produce maximum deflection under voltage load, were investigated versus the variation of PZT thickness and diaphragm width.

II. CONFIGURATION OF MICRO-DEVICE

The structure of the micro-piezoelectric thin film diaphragm is shown in Figure 1 and its cross section is schematically illustrated in Figure 2. The PZT film was deposited on Pt/Ti/Si₃N₄/SiO₂/Si wafer. Au layer was evaporated on the surface of PZT film as the top electrode. The backside silicon was wet etched off till the SiO₂ layer. The actuator is actually a square laminar diaphragm. The materials properties of each layer within the diaphragm are listed in Table 1.

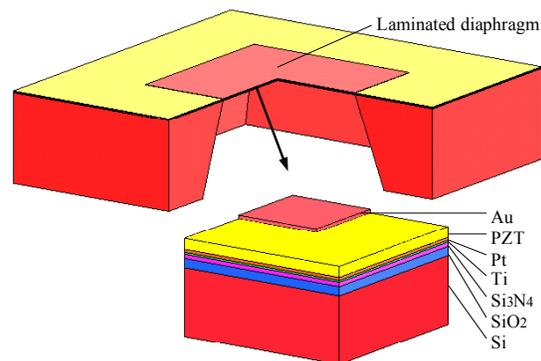


Figure 1. Structure of the micro-actuator and sensor.

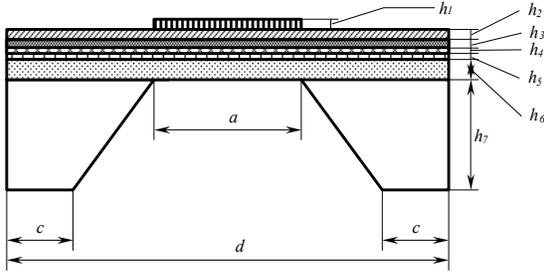


Figure 2. Sketch of cross section of the micro-actuator and sensor.

TABLE 1
THE PROPERTIES OF EACH MATERIAL IN THE STRUCTURE*

Layer	Material	E	μ	ρ	ϵ_{33}	e_{31}	h
1	Au	80	0.42	19.32			0.2
2	PZT	86.2	0.287	7.62	3.98e-6	-7.278	0.6
3	Pt	146.9	0.39	21.45			0.1
4	Ti	102.1	0.3	4.85			0.05
5	Si ₃ N ₄	150	0.24	3.24			0.2
6	SiO ₂	72.4	0.16	2.07			0.5
7	Si	190	0.33	2.33			

* E is Young's modulus (GPa); μ is Poisson ratio; ρ is density ($\times 10^3 \text{kg/m}^3$); ϵ_{33} is dielectric parameter (F/m); e_{31} is piezoelectric parameter (C/m²); and h is thickness (μm).

III. NUMERICAL MODEL VALIDATION

For the analyses, FE-codes of ANSYS [21] were used and three different elements were adopted in decentralization process to characterize different media in device. The first kind of elements is known as Solid5 in ANSYS FE-codes. Solid5 is an 8-nodes element and has the capability of modeling three-dimensional piezoelectric field. Therefore, it was used here to model PZT layer in the device. The second kind element is Solid46. It is a layered version of the 8-node structural solid designed to model layered thick shells or solids. Thus, it was selected to construct elements of the lamina, which was composed by elastic layers of Pt, Ti, Si₃N₄ and SiO₂. The third one is widely used 8-nodes three-dimensional solid element, which is named as Solid45 in ANSYS. It was used to construct FEM model of top Au electrode layer and Si substrate.

Because the thickness of multi-layer diaphragm is much thinner than its other dimensions, a large amount of elements are needed in the laminar plane and the substrate, leading to a huge cost of CPU time, and large requirement on memory and HD space of computer. Hence, simplified models were employed in the analysis for the purpose of saving CPU. According to the structure characteristics of the device, the center square laminar diaphragm with dimensions of $a \times a$ was adapted and the surrounding film and substrate were neglected in the model simplification. The square diaphragm is regarded as a laminated plate in the analysis. Clamped and simply supported boundary conditions for four edges are two commonly adopted boundary conditions, which replace the support function of substrate in device.

To confirm the validity of such simplification and estimate errors as well as other differences caused from model simplification, an exact model shown in Figure 2 was also introduced based on a real structure of the device

shown in Figure 1. In the exact model (Figure 2), a expresses the width of square laminated diaphragm, d is the width of the actuator which is equal to $3a$, and h_7 is thickness of the substrate. Since the substrate is made of etched Si(100) wafer, the etched angle is known of around 54.7° . Therefore, the dimension c can easily be determined by

$$c = \frac{1}{2}(d - a) - h_7 / \tan(54.7^\circ).$$

$h_1, h_2 \dots h_6$ are the thickness of each films and listed in Table 1, respectively. Electrical field is applied between the upper (Au) and bottom (Pt) electrodes of the PZT film.

For easy and accurate comparison of the results, the same discrete schemes were adopted on the two plate models and the diaphragm at the center of the exact model during FEM modeling process (Figure 3). The same mode extraction method, reduced method, and the same master degrees of freedom were chosen in dynamic solution.

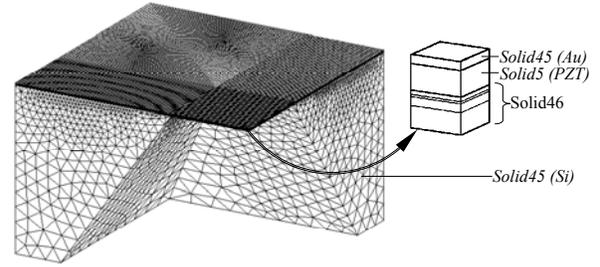


Figure 3. Mesh sketch of a quarter of exact model.

3.1 Validation of numerical analysis

The finite element method (FEM) is a kind of approximation of numerical method. Since the approach of accurate solution strongly depends on adopted elements, model discretization, and type of solution method etc., the selected element types, mesh generation and solution procedures supplied by FEM software, need to be evaluated before concrete model analysis. The numerical (FE-codes, ABAQUS) and analytical results of frequencies of two-layered clamped PZT plate [15] were employed to verify our numerical analysis process by using ANSYS FE-codes. As the same as the model described in [15], the clamped square plate was laminated by an elastic layer of SiO₂ and a PZT layer of PbZr_{0.54}Ti_{0.46}O₃ with 1- μm in thickness, respectively. Table 2 compares the first five frequencies obtained by ANSYS FEM software and by theoretic calculation at different width of the square plate. The errors, based on ANSYS data, are within 2% as comparing with the results in [15], and indicate the acceptance of the analysis method using the ANSYS FEM software.

3.2 Effect of supported condition

In the present modeling, it was assumed that the width of square laminar diaphragm, a , was 200 μm and the thickness of substrate, h_7 , was equal to a in exact model. The first six nature frequencies of exact model, clamped laminated plate model and simply supported laminated

plate models are listed in Table 3. It is noted that the values of the frequency obtained by the exact model are smaller than that of the clamped plate, but much greater than that of the simply supported plate. The errors in the frequencies between the exact model and the clamped plate are almost constant and around 1%, whereas, the errors of simply supported plate are not only very large but decrease rapidly with the increase of the order of the resonance frequency (Table 3).

TABLE 2.
COMPARISON OF THE FIRST FIVE NATURE FREQUENCIES FOR A CLAMPED SQUARE $\text{PbZr}_{0.54}\text{Ti}_{0.46}\text{O}_3/\text{SiO}_2$ LAMINATED PLATE WITH EACH LAYER OF THICKNESS AT $1\ \mu\text{m}^*$

Width	Method	Mode 1	Mode 2; 3	Mode 4	Mode 5
100 (μm)	ANSYS	1378	2821	4157	5013
	Theory	1389 (0.79%)	2832 (0.38%)	4175 (0.45%)	5073 (1.2%)
	ABAQUS	1364 (0.99%)	2774 (1.69%)	4074 (1.99%)	4954 (1.19%)
200 (μm)	ANSYS	343.6	701.3	1034	1260
	Theory	347.2 (1.06%)	707.9 (0.95%)	1044 (1.00%)	1268 (0.69%)
	ABAQUS	342.4 (0.35%)	697.9 (0.48%)	1028 (0.56%)	1251 (0.65%)
300 (μm)	ANSYS	152.6	311.3	458.8	558.5
	Theory	154.3 (1.14%)	314.6 (1.08%)	463.9 (1.11%)	563.7 (0.93%)
	ABAQUS	152.3 (0.20%)	310.6 (0.23%)	457.5 (0.29%)	557.2 (0.23%)
400 (μm)	ANSYS	85.78	175.0	258.0	313.9
	Theory	86.80 (1.18%)	177.0 (1.14%)	261.0 (1.15%)	317.1 (1.01%)
	ABAQUS	85.67 (0.13%)	174.8 (0.13%)	257.5 (0.19%)	313.6 (0.08%)

* The percent ratios in brackets are the frequency errors ANSYS with respect to Theory and ABAQUS.

TABLE 3.
EFFECT OF SUPPORTED CONDITION ON NATURE FREQUENCIES (kHz)*

Mode	Exact model	Clamped plate model	Simply supported plate model
1	229.73	231.82 (0.92%)	127.13 (80.70%)
2; 3	469.13	473.51 (0.93%)	318.14 (47.46%)
4	692.15	698.41 (0.90%)	509.17 (35.94%)
5	843.17	851.25 (0.96%)	637.60 (32.24%)
6	847.43	855.29 (0.93%)	637.61 (32.91%)

* The percent ratios in brackets are the frequency errors between exact model and simplified model.

To further evaluate errors caused by the simplification, the frequencies obtained by the three models were calculated by varying the width of diaphragm, a . The errors of the first five order nature frequencies are shown in Figures 4 and 5. The errors of clamped plate model are observed to reduce with the increase in the dimension of the square diaphragm, and the errors caused by the simply

supported plate remain unchanged. However, the errors of the simply supported plate model are very large so that the model was no longer adopted in the further analysis.

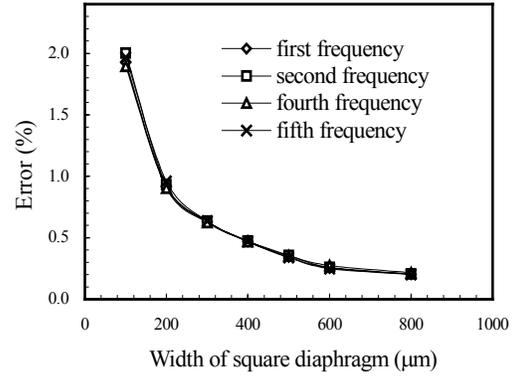


Figure 4. Error caused by the clamped laminated plate model.

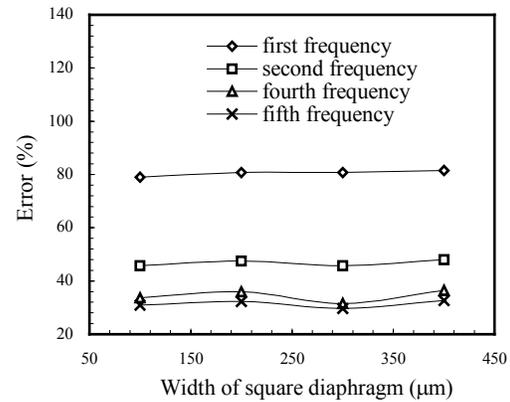


Figure 5. Error caused by the simply supported laminated plate model.

3.3 Effect of dimension of substrate

The thickness of the substrate is another parameter causing errors. To evaluate the significance of the thickness of the substrate, calculation was performed at a constant the diaphragm width, a , at $400\ \mu\text{m}$ by varying the substrate thickness, h_7 . Figure 6 reveals that the errors are almost invariable versus the thickness variation of silicon layer. Thus, an upper limit (average value) of the error can be obtained from Figure 6 and the value of the frequency of the exact model could be approximately obtained by the consideration the error. It is important to realize that the lower limit value of frequency of the exact model can be obtained from the frequency of the clamped plate.

Because the substrate and surround film materials of the diaphragm are elastic in the exact model and the clamped condition is a very rigid condition for diaphragm boundaries, the values of frequency of the exact model and the clamped plate model still possess small errors when the thickness of the substrate is very thin even completely neglected. By introducing a small amount of relaxation in the clamped boundary condition on the clamped plate model, it may help to reduce the errors in further dynamic or static simulations.

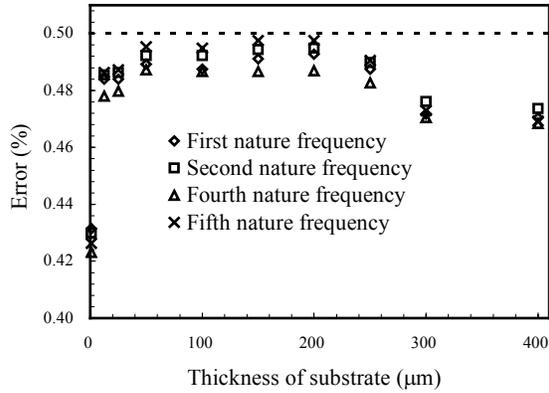


Figure 6. Influence of substrate thickness effect on errors between clamped laminated plate model and exact model.

IV. NUMERICAL RESULTS

4.1 Effect of dimension of diaphragm on nature frequency

According to the evaluation of the error, it has been known that the range of the error resulted from the simplification in the clamped laminar plate model is acceptable and hence it can be used to replace the exact model.

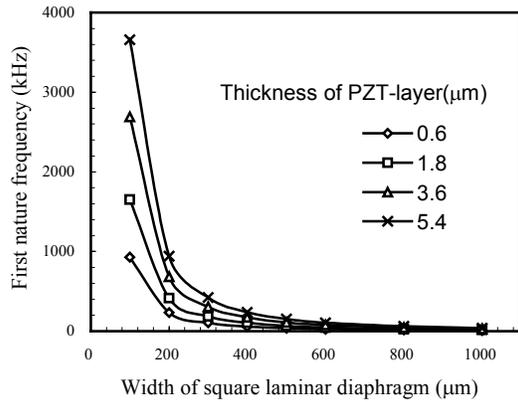


Figure 7. Change of the first nature frequency of structure via the variation of width of square laminated diaphragm

Figures 7 and 8 show the influence of the width of the square laminar diaphragm on the first nature frequency. The frequency value reveals a rapid decrease with the increase in the width, especially when the width is small. For the PZT films with thickness of 0.6 μm , the resonance frequencies of the diaphragm with width of 1000 to 300 μm are at the range of 10 to 100 kHz (Figure 7). This frequency range is particularly suitable for the ultrasonic audio beam application. A simple and easy used linear relation on frequencies could be obtained by changing x-coordinate in to the reciprocal area of diaphragm, $1/a^2$, as shown in Figure 8.

Figure 9 shows the change of the first nature frequency via the variation of the thickness of PZT-layer. The values of frequency almost increase linearly with the increase in the thickness.

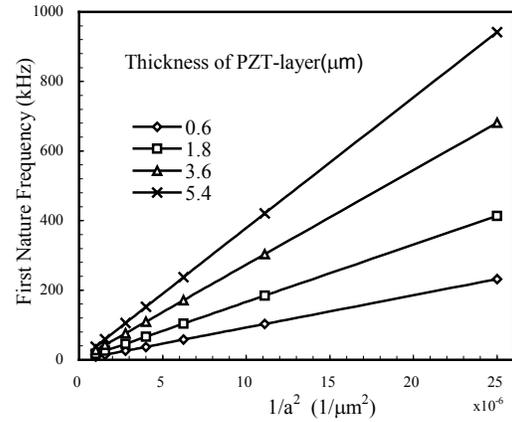


Figure 8. Change of the first nature frequency of structure via the inverse of square of width, $1/a^2$, of square laminated diaphragm.

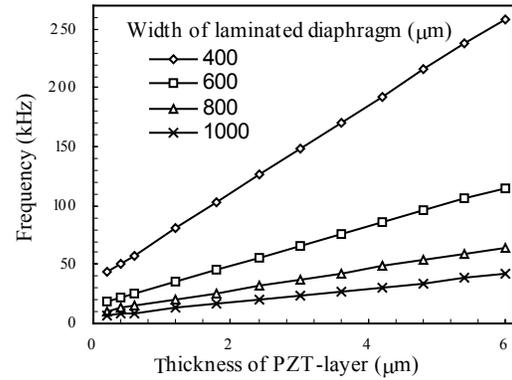


Figure 9. Effect of thickness of PZT-layer on the structure nature frequency.

4.2 Harmonic response of the micro-actuator

Since the supported silicon and surrounding laminar film of diaphragm are not rigid solids and the boundary condition is equivalent to a clamped boundary, the exact model was adopted in investigation of the harmonic responses of the diaphragm. The dimensions of the exact model were $a=200\mu\text{m}$, $b=600\mu\text{m}$, $h_1=(b-a)/2=200\mu\text{m}$ and $h_2=2.4\mu\text{m}$.

A concentrate force was applied at the center point to induce the symmetric first and fifth harmonic responses. The responses of deflection at center point are shown in Figure 10, and the voltage response of PZT layer is shown in Figure 11. Damping ratio, $\gamma=0.01$, was considered in calculation.

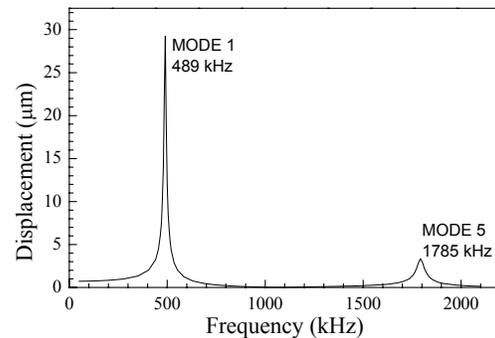


Figure 10. Deflection responses at centre point.

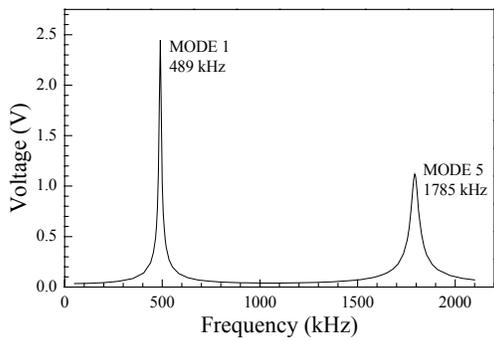


Figure 11. voltage response in PZT-layer at centre point.

4.3 Effect of thickness ratio on dynamical deformation

For a certain thickness of elastic layers (Pt, Ti, Si_3N_4 and SiO_2 layers), especially the thickness SiO_2 of layer, as the thickness of piezoelectric layer equals zero or tends to infinity, the electric load applied on top and bottom electrodes will not induce deformation in normal direction of the diaphragm. There exists an optimum thickness ratio of a PZT to an elastic layers under which the deflection of the diaphragm will reach a maximum value. In the calculation, the thickness of SiO_2 layer was varied, and the thickness of the PZT layer was remained at $h_2 = 0.6\mu\text{m}$. Figure 12 shows the relation between the thickness ratio and the deflection of the diaphragm with given widths when driven by a voltage of 1V. The optimized values of thickness ratio for given widths versus the variations of PZT thickness are shown in Figure 13, and the first nature frequencies of the diaphragm with the corresponding optimum thickness ratio are shown in Figure 14.

Sometime since the electrode layers are too thin to be considered, the laminated films can be simplified into two layers, PZT and Si. Figure 15 shows the optimized thickness ratio when only Si and PZT layers were considered. The corresponding first nature frequencies are shown in Figure 16. It is interested to note that the frequencies obtained from the optimum thickness ratio are approximately increase linearly with the increasing thickness of PZT layers at the given diaphragm width in logarithm coordinate systems.

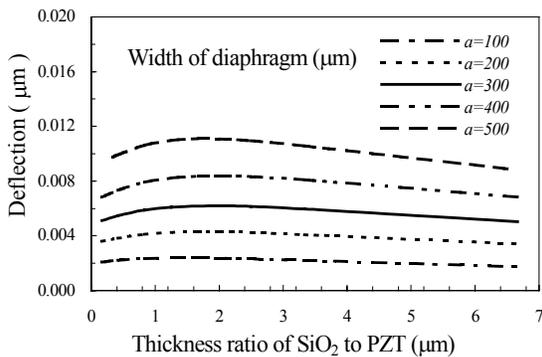


Figure 12. Dependence of the diaphragm deflection via the thickness ratio of SiO_2/PZT layers.

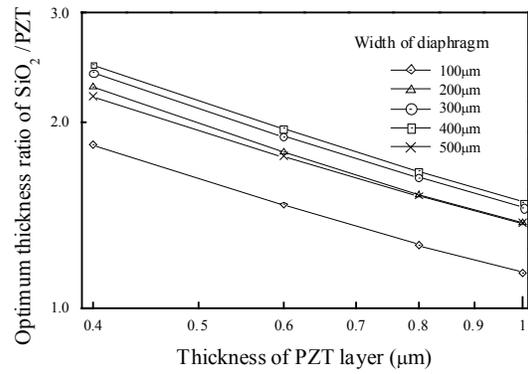


Figure 13. Change of optimum thickness ratio of SiO_2/PZT plate versus variation of PZT thickness and diaphragm width.

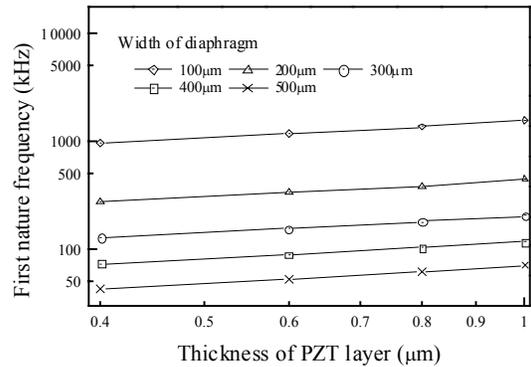


Figure 14. Change of first nature frequency of SiO_2/PZT plate versus variation of PZT thickness and diaphragm width.

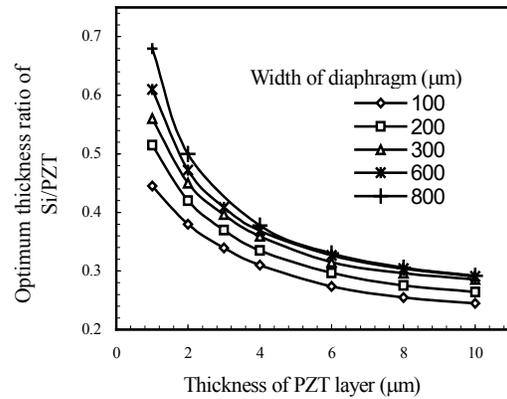


Figure 15. Change of optimum thickness ratio of Si/PZT plate versus variation of PZT thickness and diaphragm width.

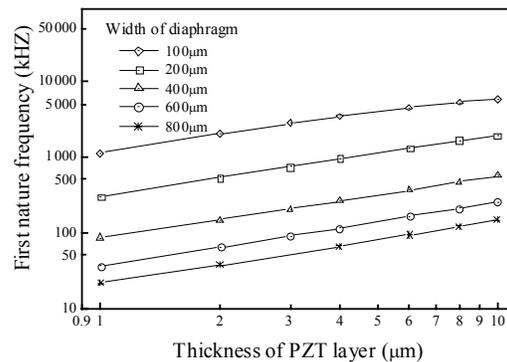


Figure 16. Change of first nature frequency of Si/PZT plate versus variation of PZT thickness and diaphragm width.

V. CONCLUSIONS

Dynamic characteristics of a multilayer piezoelectric/elastic diaphragm were investigated using FE-codes ANSYS. The comparison between the dynamic results obtained from the laminated plate models and from the exact model shows that the clamped plate model was a valid simplified model in analysis, especially in the case where the diaphragm has a small ratio of thickness to width. The simply supported model was confirmed to be not suitable due to its large error as comparing with exact model.

The influence of dimensions of laminar diaphragm on nature frequency was studied. The frequency values were found to rapidly decrease rapidly with the increase in the diaphragm width, especially in the small width range, and to increase linearly with the increase in the thickness of the PZT-layer. The deflection and the first nature frequency of diaphragm as a function of the thickness ratio of PZT layer to SiO₂ (thin film) and Si (thick film) layer were presented for the design of actuator or sensors in MEMS applications.

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