Problem 24.2

a) Let \( x \) be any eigenvalue for \( A \) and \( x \) a corresponding eigenvector with \( \|x\|_2 = 1 \), say \( xi = 1 \).

Then \( A \times x \times x \) implies:
\[
\lambda = \sum_{j=1}^{m} a_{ij} x_j x_j = \sum_{j=1}^{m} \left| a_{ij} \right| x_j x_j \leq \sum_{j=1}^{m} |a_{ij}| \leq 1,
\]
\[
|\lambda| \leq 1.
\]

b) This was discussed in class.

c) Gerschgorin's theorem gives:
\[
|\lambda_1 - 8| \leq 1,
\]
\[
|\lambda_2 - 4| \leq 1 + |\lambda_1|,
\]
\[
|\lambda_3 - 1| \leq 1.
\]

Nice for \( E \) small the disks above are disjoint. If \( E \) is real these become intervals on real line.

d) Take \( D = \text{diag}(1,1,2) \) so
\[
DAx = \begin{pmatrix} 8 & 0 \\ 0 & 4 \end{pmatrix} \text{ and then } 0 \leq x \leq 1.
\]

This gives \( |\lambda_3 - 1| \leq 2 \).

Problem 25.3

a) Take a Householder reflector \( \mu \)-null that \( B_2 Q \mu A = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \).

Take \( Q_2 \) null that \( B_2 = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \).

Now take a \( Q_3 \) null that
\[
Q_3 C = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}
\]

So this is \( Q_3 Q_1 A Q_2 \) and the answer is ii) for this.

b) If we multiply \( \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \) by
\[
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

we get \( \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \) and for this alternative ii) is working.

c) Since \( \det \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = 0 \) we can not generically arrive to this matrix from a general matrix since this is in fact impossible for an invertible matrix.

Problem 27.4

If \( p(x) = \frac{x^3 + 2}{x^2 + 2} \) and \( Q_2 = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \) is unitary then \( p(x) \) is the first diagonal entry of \( Q^* A Q \).

Now if \( z = (Q^* A Q)_{ij} = \overline{r}_2(2i - 2j) \) we have \( z = \overline{Q}_{i,j} A_{j,i} = \overline{r}(2i - 2j) \).