28.2a) Looking at Gram-Schmidt algorithm we see that each column \( j \) of \( Q \) is a linear combination of the columns \( 1 \) to \( j \) of \( A \), so \( Q \) is upper Hessenberg. On the other hand since \( A \) is tridiagonal we have \( r_{ij} = 2 \) for all \( j > i + 2 \). So \( R = \) 

\[
\begin{bmatrix}
2 & 1 \\
0 & 2
\end{bmatrix}
\]

and \( Q = \) 

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

b) The matrix \( RQ \) must be upper Hessenberg and also \( RQ = QRT \) is symmetric since \( A \) is symmetric so \( RQ \) is upper Hessenberg and symmetric which means tridiagonal.

c) At step \( i \) only the element \( a_{ii} \) needs to be made 0. This can be done by using a 2x2 Householder reflection based on \( a_{ii} \) and \( a_{ii} \). This will affect only 6 elements of the matrix, namely \( a_{ii} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), \( a_{ii+1} \), so we need 6 flops for multiplications and 24 flops at each step. A total of \( m-1 \) steps have to be done, ending with 24 \( m \) operations, compared with \( O(m^3) \) for a full matrix.

30.3 Let \( A \) be the matrix and \( A' \) the updated matrix. Let \( |a_{jj}'| = \max |a_{ij}|. \)

If \( a_{jj} \) is zeroed out then:

\[
a_{jj}' = a_{jj} = a_{jj} + a_{jj}' + 2a_{jj}'^2 = (a_{jj} + a_{jj}') + 2a_{jj}'^2 = (a_{jj})^2 + 2a_{jj}'^2.
\]

Now \( \| A' \| = \sum_{i,j} |a_{ij}'|^2 \)

\[
\| A \| = \sum_{i,j} |a_{ij}|^2
\]

since \( \| A' \|^2 = \sum_{i,j} |a_{ij}'|^2 \leq m(m-1) \sum_{i,j} |a_{ij}|^2 \)

we get \( \| A' \| \leq \| A \| (1 - \frac{2}{m(m-1)}) \)

or \( \| A' \| \leq (1 - \frac{2}{m(m-1)}) \| A \| \).

30.6 Simple computations show that:

\[
\begin{align*}
\text{If } & p(z) \neq 0, \\
p^{(1)}(2) &= 0, & p^{(0)}(2) &= 1, & p^{(2)}(2) &= -1, \quad \text{if } z = 2 \\
p^{(1)}(2) &= 0, & p^{(0)}(2) &= 1, & p^{(2)}(2) &= 1 \\
p^{(1)}(2) &= 0, & p^{(0)}(2) &= 0, & p^{(2)}(2) &= 0
\end{align*}
\]

The sign sequence is +1, +1, -1, +1, which means that no eigenvalues of \( A \) are smaller than 2.

Now \( p(x) = x^4 + 1 \), \( p^{(4)}(x) = 4x^3 + 1 \), \( p^{(3)}(x) = 12x^2 + 0 \), \( p^{(2)}(x) = 24x + 0 \), \( p^{(1)}(x) = 24x^2 + 0 \).

The sign sequence is +1, -1, -1, +1 and there is one sign change, which means that 1 eigenvalue of \( A \) is smaller than 1.

So there is only one eigenvalue of \( A \) in \([2, 2]\).