Power Method

x_0 given, $||x_0||_2 = 1;$

$i = 0;$

while (convergence criterion)

$$y_{i+1} = Ax_i;$$

$$x_{i+1} = \frac{y_{i+1}}{||y_{i+1}||_2};$$

$$\mu_{i+1} = x_{i+1}^T A x_{i+1};$$

$$i \quad = i + 1;$$

end

$O(n^2)$ / step

Inverse Iteration
Power Method for $(A - \mu I)^{-1}$

$$
\begin{array}{rl}
x_0 \text{ given,} & ||x_0||_2 = 1; \quad i = 0; \\
[L_\mu,\ U_\mu] & = \ \text{lu}(\ A - \mu I\ ); \\
\text{while} & \text{(convergence criterion)} \\
 & \begin{array}{lcl}
 y_{i+1} & = & U_\mu \setminus (L_\mu \setminus x_i); \\
 x_{i+1} & = & \dfrac{y_{i+1}}{||y_{i+1}||_2}; \\
 \mu_{i+1} & = & x_{i+1}^T A x_{i+1}; \\
 i & = & i + 1;
 \end{array} \\
\text{end} &
\end{array}
$$

$$O(n^3) \quad + \quad O(n^2) \ / \ \text{step}$$

Rayleigh Quotient Iteration

$$
\begin{array}{rl}
x_0 & \text{given}, \quad ||x_0||_2 = 1; \\
\lambda_0 = & x_0^T A x_0; \\
i = & 0; \\
\text{while} & (\text{convergence criterion}) \\
& \begin{array}{lcl}
y_{i+1} & = & (A - \mu I)^{-1} x_i; \\
x_{i+1} & = & \dfrac{y_{i+1}}{||y_{i+1}||_2}; \\
\mu_{i+1} & = & x_{i+1}^T A x_{i+1}; \\
i & = & i + 1;
\end{array} \\
\text{end} &
\end{array}
$$

$O(n^3)$ / step

Orthogonal (Simultaneous) Iteration

$Z_0 \in \mathbb{R}^{n+p}$ orthogonal matrix

$i = 0$

while (convergence criterion)

$$
\begin{aligned}
Y_{i+1} &= AZ_i; \\
Y_{i+1} &= Z_{i+1}R_{i+1}; \quad \text{(QR decomposition)} \\
i &= i+1;
\end{aligned}
$$

end

$O(n^3)$ / step

QR iteration

given A, $i = 0$; $A_0 = A$;

while (convergence criterion)

$$
\begin{aligned}
A_i &= Q_iR_i; \quad \text{(QR decomposition)} \\
A_{i+1} &= R_iQ_i; \\
i &= i+1;
\end{aligned}
$$

end