

Essays on Informal Banking

by

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B.A., Economics and Mathematics
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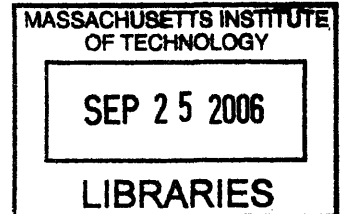
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Abstract

This thesis is a collection of three theoretical essays that examine the role of time-inconsistent preferences in informal banking. The first two chapters focus on specific banking institutions, while the third studies individual welfare more generally.

In Chapter 1, I develop a model of rotating savings and credit associations (roscas) where members are quasi-hyperbolic discounters. I show that, in this setting, roscas function as commitment savings devices, and can survive in equilibrium even in the absence of formal contracting or informal social sanctions.

In Chapter 2, I study the behavior of quasi-hyperbolic discounters who have access to credit and a non-secure savings technology. I show that these agents might simultaneously save and borrow to create optimal investment incentives for future selves.

Chapter 3 evaluates and compares the welfare outcomes for time-inconsistent agents under several banking environments.

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¹ The process of preparing the thesis for the final print is somewhat complicated, especially when one is halfway across the world. This could not have been done without Saugato's help.

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Chapter 1

Hyperbolic Discounting and the Sustainability of Rotational Savings Arrangements

Summary 1 *People across the developing world join rotational savings and credit associations (rosca) to fund repeated purchases of nondivisible goods. When the scope for punishment is weak, there is a natural question about why agents do not defect from these groups. This paper models a rosca as a commitment savings device for hyperbolic discounters. Roscas are attractive for two reasons: the possibility of getting the nondivisible good early (the standard reason), and the externally imposed saving (valued only by time-inconsistent agents). We find explicit conditions under which an agent strictly prefers to remain in a rosca, even in the absence of formal contracting or social punishment. We show why, unlike with standard commitment products, a hyperbolic discounter will never postpone joining a rosca. Finally, this paper makes predictions about the relative survival of random and fixed rosca. Random rosca are more resilient and beneficial than fixed rosca when information is limited and matching for new rosca is fast.¹*

¹I thank Jean-Marie Baland and Sylvain Chassang for additional comments on this chapter.

1.1 Introduction

Roscas (rotating savings and credit associations) are a prominent form of saving across much of the developing world. This paper studies roscas from the perspective of hyperbolic discounting, and models them as equilibrium phenomena in environments with no scope for formal contracting or social punishment. The model allows us to solve some puzzles about the high observed survival of roscas, and to derive implications for existence and sustainability under varying underlying conditions.

1.1.1 Rosca Basics

A rosca consists of a group of individuals who meet at regular intervals. At every meeting, each member contributes a fixed amount to the collective "pot." One member then takes the entire pot home. A "rosca round" consists of exactly as many meetings as there are members. Within a rosca round, each member gets to take the pot home exactly once. The order in which members are given the pot can be determined in several ways. In this paper, we focus on "fixed" and "random" roscas. In a fixed rosca, the order is randomly determined at the first meeting of the rosca, and then repeated indefinitely through future rounds. In a random rosca, the order is randomly determined at the start of each new round. Often, rosca members who leave at the end of a round are free to rejoin a rosca at a later date. However, people who choose to leave *during* a round are punished more heavily—they are not allowed to rejoin any future rosca and, if possible, punished with other forms of social sanctions.

Roscas are widespread across developing countries (and immigrant communities in developed countries), and often survive in environments with poor contracting and limited or nonexistent formal banking. Members of the Tidiane community in Senegal use them as a means to save for annual pilgrimages to Mecca. In Philadelphia, women from the Ivory Coast join roscas to pay for childbirths and funerals. Roscas can be found in several parts of Kenya, where the money is put to various uses, from home repairs to the purchase of food and clothing. Levenson and Besley (1996) point out that, in any given year, one-fifth of all households in Taiwan participate in a rosca. Bouman (1995) cites several studies of African roscas where participation rates are

even higher, and are a significant saving vehicle.²

1.1.2 Theoretical Foundations

Following the model proposed by Besley, Coate, and Loury (1993), we assume that agents would like to save for a nondivisible good. If they were to save alone, each would have to wait a certain number of periods before she could consume the good. If agents instead join a rosca, they pool their savings and allow some members to get the nondivisible good sooner. The likelihood of an early nondivisible gives agents an incentive to join a rosca. This expected benefit, however, does not persist after agents have actually joined. As Anderson, Baland, and Moene (2003) show, when agents are exponential discounters, they will have an ex-post incentive to leave any rosca. Consider an agent in a fixed rosca who has just received the nondivisible good. If she now leaves the rosca, she can replicate the rosca outcome by saving alone (there is no longer a positive probability of getting the good sooner through the rosca). Furthermore, if she prefers a declining pattern of savings, she can do strictly better by saving alone. In a random rosca, some agents have an even greater incentive to leave. Consider the first-ranked agent in a round. If she leaves the rosca after receiving the nondivisible, she can ensure that she continues to be "first-ranked" by walking out and saving alone. If she stays in the rosca, there is no guarantee that she will again be ranked first in the next round.³ The authors conclude that, in the absence of contracting, the threat of social punishment must be severe enough that an agent who would otherwise choose to leave a rosca will now choose to continue participating.

1.1.3 Questions to be Answered

The above papers provide a compelling framework for understanding the ex-ante appeal and subsequent sustainability of roscas, but they also lead to some interesting questions. First, there is some evidence that roscas survive even when there is no credible threat of social punishment (Gugerty, 2005). Why do agents not leave after receiving the nondivisible? Second, for any given outside option, agents in random roscas have greater incentives to leave than agents in

²Roscas and other informal savings groups are the source of half the national savings in Cameroon. Savings through roscas amount to 8-10% of Ethiopia's gross national product.

³By staying on in the rosca, the agent is effectively continuing to save at a negative interest rate.

fixed roscas do (the first-ranked member's expected value of staying on in a random rosca is lower than anyone's expected value of staying on in a fixed rosca). Why do random roscas exist? Third, even if the threat of social sanctions can be used to ensure participation within a round, it is rarely the case that such threats persist between rounds. Then, why does the last-ranked member in a fixed rosca stay on? If she were an exponential discounter, she should leave at the end of a round if there is any probability that she will find a rosca that ranks her higher (since, even if she does not find a rosca, she can save alone and do no worse than in her original rosca).

1.1.4 Outline of Arguments

This paper is built on two key assumptions. First, agents in our model have time-inconsistent preferences and are aware of it. There is empirical evidence that this is indeed the case—Gugerty (2005) shows that members of her dataset most often cite self-control problems as the reason for joining a rosca.⁴ Second, we take seriously the fact that roscas are informal institutions that exist even in environments where contracting and social networks are weak. We assume that participation cannot be contracted upon and that social punishment is infeasible. Roscas, then, can be viewed as commitment savings devices with particular advantages. Not only do they improve savings behavior, but they can survive without social sanctions, and will be adopted without postponement. Unlike with exponential discounters, roscas generate two complementary benefits for hyperbolic discounters—value from high expected rank, and value from commitment. The value from high expected rank ensures that agents will actually enter a rosca, and the value of future commitment gives agents who receive the pot early a reason to repay the "debt." Our arguments are developed in four broad parts.

First, we describe a sophisticated quasi-hyperbolic discounter who values the nondivisible good, and we solve for her autarky equilibrium across the β -parameter space⁵. We show that, as β goes down, the agent saves more slowly.

Second, we show how roscas can be effective commitment devices. Roscas provide com-

⁴Some quotes: "You can't save alone—it is easy to misuse money;" "Saving money at home can make you extravagant in using it."

⁵The " β " refers to the agent's hyperbolic discount factor, by which she discounts her entire future in any given period.

mitment in the following sense: if, when the agent is furthest away from the nondivisible, she prefers to remain in the rosca, then she will always prefer to remain in the rosca. This is because, as she gets closer to the nondivisible, the rosca locks in more of her savings and thus reduces her incentive to leave. Furthermore, roscas have the particularly appealing property of enticing agents to join without delay. With standard commitment devices, costs are incurred in the present while benefits (in the form of matured savings) arrive in the future. This can cause hyperbolic discounters to delay take-up of such devices, even if they value the commitment. However, in the case of roscas, an agent knows that there is a likelihood of her being an instant winner (she might get ranked first). We show that this ensures that the agent will not postpone entry into a rosca.

Third, we study the rosca sustainability problem in a benchmark case. While we have described how roscas can operate as commitment devices, we still need to establish conditions under which the agent will actually choose to stay when she is furthest away from her next nondivisible. In other words, when is the commitment offered by a rosca sufficiently valuable to her? We assume there is a single provider of roscas who cannot make credible threats of punishment. The only enforceable rule, then, is that any agent who leaves a rosca can never rejoin. We find that there is always a parameter region in which an agent will never leave a rosca, regardless of her rank within it. This is the region where the promise of good future behavior (induced by the rosca) outweighs the agent's desire to over-consume in the present. We also find here that the parameter region that supports random roscas is a strict subset of the region that supports fixed roscas.

Fourth, we model roscas as noncooperative equilibria by lifting two assumptions of the benchmark case—we vary the available information about agents' past rosca behavior, and allow the generation of new roscas in any period (there is some exogenous probability with which people get matched into new roscas). Roscas here are the outcomes of repeated games with credible punishment strategies (which can be conditioned on reputation, to the extent that it is available). The objective is to find equilibria that prevent frivolous defection, which involves leaving one rosca in search of a higher rank in another.

This setup gives us two further insights. First, we find that even in completely anonymous environments, roscas can survive if the probability of matching into new roscas is sufficiently low.

Second, we find conditions under which random roscas can be preferred to fixed roscas. To do so, we model what is perhaps the most realistic reputational environment—“partial reputation.” This is the case where only the agents who leave without completing a round develop a bad reputation—these agents can be barred from future roscas. This allows us to restrict our focus to agents’ desires to leave *between* rounds. The last-ranked member in a fixed rosca is permanently last ranked, so she might have an incentive to leave after a round, in anticipation of a better rank in a new rosca. Now, random roscas have a particular advantage over fixed roscas—by re-randomizing at the start of every round, they internalize the attractiveness of the outside option. There is no incentive for a member of a random rosca to leave between rounds. We find that, under certain conditions, random roscas survive longer and increase welfare relative to fixed roscas.

The paper is organized as follows. Section 2 provides a literature review. Section 3 characterizes the autarky equilibrium. Section 4 discusses roscas as commitment devices, and studies the entry problem. Sections 5 and 6 model roscas as equilibria in the benchmark case and the decentralized case, respectively. Section 7 discusses empirical implications of the model. Section 8 concludes.

1.2 Related Literature

The benefit of a fixed or random rosca is not immediately evident, since it provides no interest. Furthermore, since the initial ordering is randomly determined, individuals cannot join with a sole purpose of borrowing or saving. (Roscas that provide informal insurance typically allow for bidding within a round.⁶ We focus on cases where this flexibility in allocation order is not permitted.) Standard explanations of roscas rely on the individual’s desire to purchase an expensive, nondivisible good. As described above, Besley, Coate, and Loury (1993) show that, in expectation, roscas allow individuals to purchase a nondivisible sooner than if each saved alone. Anderson, Baland, and Moene (2003) point out the need for social sanctions in the absence of contracting.

Several papers look at alternative explanations for rosca participation. Anderson and Baland

⁶See, for example, Calomiris and Rajaraman (1998), Klonner (2005), and Klonner and Rai (2005).

(2002) find evidence that roscas are used by women to restrict their spouses' access to their savings. In their model, women have a greater preference for the nondivisible than men, but have limited power over expenditures within the household. If women were to save at home, their husbands would direct too much of their savings towards immediate consumption. On the other hand, if women save in a rosca, husbands have no access to their savings until the pot is received. At this stage, assuming the woman has sufficient bargaining power to purchase the nondivisible good, it is in fact purchased. In this setting, roscas can again be viewed as commitment savings devices. The woman would like to save for the nondivisible, but she knows that if she saves at home, she will not be able to save as fast as she would like to. A rosca, by locking in savings, allows her to prevent over-consumption by her household in future periods.

Gugerty (2005) finds direct evidence that individuals use roscas to overcome their own time-inconsistency.⁷ In her dataset, self-control problems are cited as the most common reason for joining a rosca (36% of the members say it is the primary reason). Based on anecdotal evidence, there appears to be very limited scope for credible social sanctions. Gugerty's study is set in a rural community in Kenya where banks, if available, are very far away. The average rosca in her dataset is 6.5 years old, with the average round lasting a little under a year. She finds that only 6% of members left the rosca in the last round studied. 37% of the roscas are fixed, 58% are random, and the rest use other forms of negotiation/randomization. On the other hand, Anderson, Baland, and Moene study a poor urban neighborhood near Nairobi, where 71% of the roscas are fixed and 29% are random. Funds generated through roscas are more often spent on goods with immediate benefits than on large durables.⁸

There is a wide range of empirical papers on roscas in several parts of the world.⁹ Most rely on an informal notion of social punishment to explain why agents don't defect. A recent theoretical paper by Ambec and Treich (2005) shows how roscas can be efficient institutions, ex-ante, when agents value commitment. However, the paper assumes contracts are binding and that agents can commit to joining a rosca at a future date. The effectiveness of commitment savings devices in other forms has also been studied widely, and the fact that agents with

⁷In a recent paper on Benin, Dagnelie and LeMay (2005) provide further evidence of roscas being used as commitment devices.

⁸Gugerty (2005) finds that the largest proportions of rosca funds are spent on household cooking items, school fees, and food. In Dagnelie and LeMay's (2005) dataset, only 19% of funds are spent on durable goods.

⁹In addition to papers mentioned above, see Bouman (1994), Handa and Kirton (1999), and Kimuyu (1999).

time-inconsistent preferences value commitment is widely understood.¹⁰

In this context, the point of this paper is to show that roscas are effective commitment devices even without "commitment" in the standard sense (in settings where agents cannot pre-commit to join and cannot be forced, through contracts or social punishment, to continue participating).

Quasi-hyperbolic discounting has been used as a substitute for standard exponential discounting in several papers now.¹¹

1.3 Autarky Model

In this section, we assume the individual does not have access to a rosca, and study her behavior in terms of equilibria played by her per-period selves.

1.3.1 Assumptions

- Infinitely lived, sophisticated quasi-hyperbolic discounter.
- Per-period non-stochastic income y ; zero initial endowment.
- No borrowing; no interest.
- Two goods: consumption good (denoted c ; price 1) and nondivisible good (denoted d ; price ky , where k is a positive integer)
- Saving has to be lump-sum, in multiples of y . (This assumption allows us to model autarky equilibria using mixed strategies.)
- Per-period utility: $u(c + bd)$, where u is strictly concave and is defined over the domain $[0, \infty)$, with $u(0) = 0$; b is some constant.
- Intertemporal utility at time t :

¹⁰For examples, see Ashraf, Gons, Karlan and Yin (2003), Thaler and Bernartzi (2004), and Ashraf, Karlan and Yin (2005).

¹¹Originally proposed by Phelps and Pollack (1968), it has been developed in several papers by Laibson, Harris, Rabin, and others. More recently, Krusell and Smith (2003b) characterize the mixed-strategy equilibria in a Ramsey-style consumption-savings problem with lump-sum investment.

$$U_t = u(c_t + bd_t) + \beta \sum_{i=1}^{\infty} \delta^i u(c_{t+i} + bd_{t+i}), \text{ where } \beta \in (0, 1) \text{ and } \delta \in (0, 1)$$

Finally, we assume that the nondivisible good is "desirable":

$$\delta^{k-1} u(b) > \sum_{i=0}^{k-1} \delta^i u(y) \quad (1.1)$$

This ensures that, if the agent were an exponential discounter, she would repeatedly save all her wealth for the nondivisible good (any other saving rule would violate either time consistency or the condition above).

1.3.2 Equilibrium concept

We treat the individual as a time-indexed series of independent selves with utility functions $\{U_t\}$, and assume they play a Markov Perfect Equilibrium. In any period, the agent observes her total wealth, w_t , and makes a decision about how much to save, s_t (s_t is a gross saving decision). Wealth and savings are related in the following way: $w_t = s_{t-1} + y$. Since saving is lump-sum and initial wealth is 0, it must be that $w_t = ay$, where $a \in \{1, 2, 3, \dots\}$. In any state ay the action set is $\{0, y, 2y, \dots, ay\}$, which includes all feasible levels of saving. A strategy associates every state $w = ay$ with a sequence of positive probabilities, $(\rho_0, \rho_1, \rho_2, \dots, \rho_a)$, that sums to 1 and denotes a probability distribution over all feasible actions.

We can immediately restrict our state set to: $\{y, 2y, 3y, \dots, (k-1)y\}$. Since the only incentive to save is for the nondivisible, and since initial wealth is 0, there will never be an equilibrium where the agent encounters wealth higher than ky . The agent with wealth ky will always save 0, regardless of future behavior.

A strategy is an equilibrium if and only if, for any state $w \in \{y, 2y, 3y, \dots, (k-1)y\}$, every action s that is played with positive probability satisfies:

$$s \in \max_{s' \in \{0, 1, 2, \dots, w\}} [u(w - s') + \beta \delta V(s' + y)]$$

Here, $V(\cdot)$ is the value function of the exponential discounter, defined recursively:

$$V(ay) = \sum_{j=0}^a \rho_j [f(ay - jy) + \delta V((j+1)y)]$$

where

$$f(x) = \begin{cases} u(x), & \text{if } x < ky \\ u(b), & \text{if } x = ky \end{cases}$$

1.3.3 Predicting Equilibrium Choice

Multiple-self models with hyperbolic discounting typically lead to a multiplicity of equilibria. In this section, we present some results that (1) allow us to restrict the set of strategies that are candidates for equilibrium, and (2) predict which equilibrium will be chosen in the case of multiplicity. The proofs of the following lemmas and proposition are in Appendix A.

Consider any strategy in which some saving occurs. For this to be an equilibrium, it must be the case that all deviations (in terms of lower saving) at all levels of wealth are dominated. It follows directly from concavity that if an agent with low wealth chooses to save a certain amount, an agent with higher wealth cannot possibly wish to save any less. The stock must always weakly rise until ky is reached.

Lemma 2 *If at wealth w the agent (weakly) prefers to save s , then at any wealth $w' > w$ the agent will never save any $s' < s$ with positive probability.*

This means that in any equilibrium in which the nondivisible is purchased probabilistically, an agent with wealth w will either save w or mix between w and $w - y$.

If there are multiple equilibria in certain regions, we need to predict which among them the agent will actually play. This is relatively straightforward if we establish that the best equilibrium in any one state is also the best equilibrium in all other states. We can separate equilibria into three broad categories—that in which the nondivisible is purchased with certainty every k periods, those in which the nondivisible is purchased probabilistically, and those in which the nondivisible is never purchased. Clearly, the full-saving equilibrium, when it exists, dominates all other equilibria in all states (all future players save optimally, and the current player voluntarily chooses to save). For a similar reason, any equilibrium with saving dominates any equilibrium in which the nondivisible is never purchased. Finally, we need a rule to choose between two equilibria when each involves probabilistic saving.

Lemma 3 *Consider any two equilibria, A and B , in which the nondivisible is purchased with some probability. (1) If the agent with wealth w saves w with higher probability in A than in*

B , then $V_A(w + y) > V_B(w + y)$. (2) $V_A(w + y) > V_B(w + y)$, then the agent with wealth w prefers equilibrium A .

Lemma 4 Consider any two equilibria, A and B , in which the nondivisible is purchased with some probability. If $V_A(w + y) > V_B(w + y)$, then $V_A(w + 2y) > V_B(w + 2y)$.

From these lemmas, it follows that, between any two equilibria with probabilistic saving, the one with higher saving at wealth y is preferred in all states. The agent always wants her future selves to save more; and the more they save, the more she is willing to save in the current period.

Proposition 5 There is always an equilibrium that is "optimal" in the sense that, at any level of wealth that is reached in equilibrium, the agent does not strictly prefer to play any other equilibrium.

1.3.4 Best Autarky Equilibria Across Parameter Regions

We are now in a position to find the most preferred autarky equilibrium for any value of β . While it might not be the case that the agent actually plays the best possible equilibrium, we use it a reasonable benchmark. Also, this stacks the odds against us (the better the autarky option, the lower the agent's incentive to stay on in a rosca). We know that if $\beta = 1$, the agent behaves exactly like an exponential discounter, and if $\beta = 0$, she never wishes to save any amount. To map out the equilibria within these boundaries, we focus on the optimal equilibrium, which involves the highest possible level of saving sustainable at wealth y , for every value of β . We find that there is always a region in which the agent behaves like an exponential discounter. Once β gets sufficiently small, she no longer wishes to save today if she knows she would start saving tomorrow anyway. Then, to re-induce saving, she plays a mixed strategy equilibrium (which worsens the consequences of not saving, thereby giving her an incentive to save today). Finally, there is always a region that cannot support any saving equilibrium.

A full-saving strategy is defined as the following: at any $w \in \{0, 1, 2, \dots, (k - 1)y\}$, the agent saves w .

Lemma 6 Consider a strategy with full saving at all $w > w'$. If the agent at wealth w' (weakly)

prefers to save w' than to save any lower amount, agents at all higher levels of wealth strictly prefer to save fully.

Proof. The strategy determines some continuation value $V(\cdot)$. If the agent at w' weakly prefers to save fully, this means:

$$\beta\delta V(w' + y) \geq u(y) + \beta\delta V(w')$$

To prove the lemma, we need to show that $V(x)$ is strictly convex for $x \geq w'$. Note that $V(x) = \delta^{k-x}u(b) + \delta^{k-x+1}V(y)$. Since $V(y) > 0$, V is strictly convex. ■

Proposition 7 Consider $\bar{\beta}$ as defined in Equation 1.2 below. When $\beta \in [\bar{\beta}, 1)$, the full-saving strategy is an equilibrium.

Proof. A set of necessary and sufficient no-deviation conditions must be satisfied for the full-saving strategy to be an equilibrium. Specifically, at every wealth level $w \in \{0, 1, 2, \dots, (k-1)y\}$, the agent must prefer to save her entire wealth relative to any lower level of saving. If we can show that, at each w , the agent prefers to save w over $w - y$, Lemma 1 ensures that all other conditions will be satisfied. Thus, a full-saving equilibrium exists if and only if, for each w :

$$\beta\delta V(w + y) \geq u(y) + \beta\delta V(w)$$

By Lemma 4, a necessary and sufficient condition for all the above conditions is:

$$\begin{aligned} \beta\delta V(2y) &\geq u(y) + \beta\delta V(y) \\ \Leftrightarrow \beta &\geq \frac{u(y)}{\delta[V(2y) - V(y)]} = \frac{1 - \delta^k}{\delta^{k-1}[u(b)]} \cdot \frac{u(y)}{1 - \delta} \\ \bar{\beta} &= \frac{1 - \delta^k}{\delta^{k-1}[u(b)]} \cdot \frac{u(y)}{1 - \delta} \end{aligned} \tag{1.2}$$

Since $u(y) > 0$ and $V(2y) > V(y)$, $\bar{\beta}$ is above 0. Also, expanding the term, we see that it must be less than 1 if the nondivisible is good. ■

The above proposition also tells us that at $\bar{\beta}$, agents at higher levels of wealth still strictly prefer to save fully if all future selves save. Therefore, when the full-saving equilibrium can

no longer be supported, we expect to find a region in which the agent with y plays a mixed strategy, but all others save fully. Below $\bar{\beta}$, to create incentives to save, the agent with wealth y must play a strategy where she saves with some probability $\pi_1 < 1$. Then, she knows that if she does not save today, there is a likelihood that she will not even save tomorrow. By worsening the consequences of not saving today, she again becomes willing to save.

Proposition 8 *When $\beta < \bar{\beta}$ (defined in equation 1.2), there is always a range of β -values where (1) no full-saving equilibrium exists and (2) there is a mixed-strategy equilibrium that involves mixing at wealth y and full-saving at all higher levels of wealth.*

Proof. Consider β values below $\bar{\beta}$. Full-saving equilibria cannot exist in this region (at the cutoff $\bar{\beta}$, the agent with wealth y is indifferent between saving and not saving if all future selves save fully). This proposition can then be proved in two steps. First, we show that if there is full-saving at all levels of wealth above y , then the agent at y is willing to play a mixed strategy in a region below $\bar{\beta}$. Second, we show that in a subset of this region, all players at wealth above y will in fact be willing to save fully (when a mixed strategy is played at y).

(a) Consider a mixed strategy in which the agent with wealth y saves y with probability π_1 , and 0 with probability $1 - \pi_1$. π_1 is determined by the following indifference condition:

$$\beta\delta V(2y; \pi_1) = u(y) + \beta\delta V(y; \pi_1) \quad (1.3)$$

where,

$$V(y; \pi_1) = \frac{\pi_1 [\delta V(2y)] + (1 - \pi_1) u(y)}{1 - (1 - \pi_1) \delta} = \frac{\pi_1 \delta^{k-1} u(b) + (1 - \pi_1) u(y)}{1 - (1 - \pi_1) \delta - \pi_1 \delta^k}$$

and,

$$V(2y; \pi_1) = \delta^{k-2} u(b) + \delta^{k-1} V(y)$$

$V(y; \pi_1)$ and $V(2y; \pi_1)$ are both continuous and increasing in π_1 , but $\beta\delta \frac{\partial V(y; \pi_1)}{\partial \pi_1} > \frac{\partial V(2y; \pi_1)}{\partial \pi_1}$. Therefore, as π_1 drops, the RHS of Equation 1.3 drops faster than the LHS. For every $\pi_1 \in [0, 1]$, there is some $\beta \leq \bar{\beta}$ such that the indifference condition is again satisfied. For $\pi_1 = 0$, the

corresponding β -value, $\underline{\underline{\beta}}$, is given by:

$$\begin{aligned} \underline{\underline{\beta}} \left(\delta^{k-1}u(b) + \frac{\delta^k u(y)}{1-\delta} \right) &= u(y) + \underline{\underline{\beta}} \delta \frac{u(y)}{1-\delta} \\ \Rightarrow \underline{\underline{\beta}} &= \frac{u(y)}{\delta^{k-1}u(b) - \frac{(\delta - \delta^k)u(y)}{1-\delta}} \end{aligned} \quad (1.4)$$

For $\beta < \underline{\underline{\beta}}$, there is no value of π_1 such that the agent is willing to save at wealth y . It must be the case that $0 < \underline{\underline{\beta}} < \bar{\beta}$.

(b) We now need to establish that there is a region below $\bar{\beta}$ where players at higher levels of wealth still have the incentive to save fully. As shown in Lemma 4, by concavity of u , all agents at higher levels of wealth strictly prefer to save fully at the cutoff $\bar{\beta}$:

$$\bar{\beta} \delta V(3y; 1) > u(y) + \bar{\beta} \delta V(2y; 1)$$

Since $V(3y; \pi_1)$ and $V(2y; \pi_1)$ drop continuously as π_1 drops, there must be a β -region before the inequality becomes an equality for the agent at wealth $2y$. ■

The above proposition establishes that there is always a region in which a mixed strategy is played—in particular, a strategy in which there is full saving above y , and mixing at y . To see how far this region extends, we need to know if full saving at wealth levels above y is sustainable down to $\underline{\underline{\beta}}$. If this is the case, then $\underline{\underline{\beta}}$ is the cutoff below which there will be no more saving in equilibrium. However, if δ is reasonably high, and $u(b)$ sufficiently close to $u(y)$, this will not be the case.¹² Then, there will be a region in which the best possible equilibrium will involve mixing between y and 0 at wealth y , mixing between $2y$ and y at wealth $2y$ (with the two mixing ratios simultaneously determined), and full saving at all higher levels of wealth.

While explicit solutions for mixed strategy equilibria depend on actual parameter values (see Appendix D for an example with $k = 2$), they can always be constructed using the following rule: Moving down from $\bar{\beta}$, solve for π_1 at each β so that the agent at y is indifferent between saving and consuming. Continue until some β_1 where either (a) the agent at $2y$ is indifferent,

¹²As δ gets high, the incentives to defect at $2y$ start to get almost as strong as the incentive to defect at y , because the fact that we are one period closer to the durable good becomes less significant. See Appendix B for an example.

or (b) $\pi_1 = 0$. If (b), then define $\underline{\beta} = \beta_1$. If (a), continue below β_1 , now solving for π_1 and π_2 for indifference at y and $2y$, respectively. Continue until some β_2 where either (a) the agent at $3y$ is indifferent, or (b) $\pi_1 = 0$. If (b), $\underline{\beta} = \beta_2$. By repeating these steps until (b) is satisfied, there will be some $i \in \{1, 2, \dots, k - 1\}$ such that $\beta_i = \underline{\beta}$.

Ultimately, $\underline{\beta}$ is determined by the point at which the agent at y no longer wishes to save ($\pi_1 = 0$). It is clear that π_1 must go to 0 before the mixing ratios for the other players do. If this were not the case, it would mean that the agent at y would be saving at some $\pi_1 > 0$ while an agent at a higher level of wealth, ay , saved with $\pi_a = 0$. Then, clearly it would not be rational for agent y to save, since the nondivisible would never be bought.

Proposition 9 *There is some $\underline{\underline{\beta}} \in [\underline{\underline{\beta}}, \bar{\beta})$ such that, for $\beta > \underline{\underline{\beta}}$, there is always an equilibrium in which the nondivisible is purchased, and for $\beta' \leq \underline{\underline{\beta}}$, the nondivisible can never be purchased in equilibrium. ($\underline{\underline{\beta}}$ is defined in Equation 1.4, and $\bar{\beta}$ is defined in Equation 1.2.)*

Proof. (1) By part (a) of Proposition 3, we know that if full saving is sustainable at wealth higher than y , then $\underline{\beta} = \underline{\underline{\beta}}$. Therefore, $\underline{\underline{\beta}}$ must be a lower bound on $\underline{\beta}$ because, at $\beta < \underline{\underline{\beta}}$, the agent at y will never save.

(2) Take any $\beta' \in [\underline{\underline{\beta}}, \bar{\beta})$ such that there is a saving equilibrium at β' . Then, there must be a saving equilibrium at all $\beta'' > \beta'$, since, given strategies at other states, each agent has a strictly greater incentive to save at β'' than at β' (proof in Appendix C). ■

In Appendix C, we also show that for $\beta \in [\underline{\underline{\beta}}, \bar{\beta}]$, the lifetime autarky utility for the agent at wealth y is weakly concave in β (see Figure 2).

1.4 Roscas as Commitment Devices

When agents are hyperbolic discounters, it is natural to think of roscas as commitment savings devices. We describe a rosca as a group of k people (k as defined in Section 3), where one rosca round lasts k periods. The per-period contribution is y . This rosca can be either fixed or random.

Consider an agent in a fixed rosca, in the period after which she has received the nondivisible. Suppose she values the commitment provided by the rosca and chooses to stay. Then, she knows

that in all future periods she will continue to stay. This is because, for every additional period that she participates in the rosca, more of her savings get locked-in (they are consumed by someone else, so there is no way for her to access them). This argument can be similarly applied to the first-ranked agent in a random rosca. If she chooses to stay in the rosca in the second period, the lock-in property ensures that she will always choose to stay.

This value of commitment that a rosca provides also allows for an effective punishment mechanism to ensure continued participation even without the threat of formal or social punishment. If an agent knows that once she leaves a rosca she can never return, she might strictly prefer to stay even when she is furthest away from her next nondivisible. Conditions under which this happens are discussed in Section 5.

1.4.1 Entry and Welfare

We measure welfare from the point of view of the agent in a hypothetical "period 0" before she has access to a rosca. This is consistent with a social planner's measure of welfare, where we assume the same δ factor as above, but $\beta = 1$. Clearly, the social optimal requires that the agent join a rosca in period 1.

Suppose there is no defection from a rosca once it forms. In this section, we study a single agent's entry decision. An agent faces identical expected values from joining a random rosca or a fixed rosca. In each case, she expects to get the nondivisible once every k periods, with some uncertainty about her exact rank. The expected value is $\frac{u(b)}{k} \left[1 + \frac{\beta\delta}{1-\delta} \right]$.

A potential problem with commitment savings devices, especially when start dates cannot be contracted upon, is that the agent might have an incentive to postpone entry. As O'Donoghue and Rabin (1999) show, when tasks are costly in the present and have delayed benefits, a hyperbolic discounter will procrastinate even though the welfare-maximizing outcome involves completing the task immediately. One way to improve welfare is to place restrictions on dates when a task can be completed. If an agent knows that she has only one chance to join a commitment device, she will join. By a similar argument, sporadic access to commitment devices might improve welfare (by inducing higher take-up), but in a growing population it is still sub-optimal since newborns will occasionally have to wait until the next access date.

This problem disappears with roscas because of the randomization in ranking (in both fixed

and random roscas). We show below that the possibility of getting the nondivisible in the current period ensures that the hyperbolic discounter will not want to postpone entry.¹³

Proposition 10 *If an agent knows that she will always stay in a rosca once she joins, she will never procrastinate while joining.*

Proof. Suppose an agent can join a rosca in any period, and knows she will remain in it forever once she enters. She will join in the first period of her life only if she would rather not postpone by one period:

$$\begin{aligned} \frac{u(b)}{k} \left[1 + \frac{\beta\delta}{1-\delta} \right] &> u(y) + \frac{u(b)}{k} \left[\frac{\beta\delta}{1-\delta} \right] \\ \Rightarrow \frac{u(b)}{u(y)} &> k \end{aligned}$$

It is important to note that this condition does not depend on β . We can see that, as long as the nondivisible is desirable, the condition is always satisfied. The desirability condition (from Equation 1.1) is:

$$\begin{aligned} \delta^{k-1}u(b) &> \sum_{i=0}^{k-1} \delta^i u(y) \\ \Rightarrow \frac{u(b)}{u(y)} &> \frac{\sum_{i=0}^{k-1} \delta^i}{\delta^{k-1}} \end{aligned}$$

For $\delta \in (0, 1)$, it is always true that $\frac{\sum_{i=0}^{k-1} \delta^i}{\delta^{k-1}} > k$. Therefore, if the nondivisible is desirable to exponential discounters, then hyperbolic discounters will never choose to postpone entry into a rosca. ■

1.5 Sustainability–Benchmark Case

In this section, we assume that roscas have the following rule: any agent who leaves can never rejoin a rosca. We also assume that roscas have no access to social sanctions or contracts. This

¹³This result relies on the assumption that goods yield immediate benefits (as is the case in many empirical studies). However, even when goods are more durable, agents will be less likely to postpone entry into roscas than other commitment devices.

can be interpreted as a case where roscas can only form at some central location, which allows agents' past behavior to be monitored.

1.5.1 Fixed Roscas

Again, consider a rosca with k members, with one round lasting k periods, and with a per-period contribution of y . In the first period of a rosca, an ordering is randomly determined (and is maintained for all future rounds). Consider the last-ranked agent in the rosca. How strong is her temptation to defect in the first period? If she has a strict incentive to stay on in the rosca, it follows that she will always have a strict incentive to stay on.

If the agent were an exponential discounter, she would not have a strict incentive to stay on, since she could replicate the rosca outcomes on her own. However, a hyperbolic discounter can have a strict preference for a rosca even in the absence of social sanctions. This happens when she values the commitment provided by the rosca highly enough that she is willing to forego current consumption.

Proposition 11 *Consider β^* as defined in Equation 1.5 below, and $\bar{\beta}$ as defined in Equation 1.2. Suppose a member of a rosca knows that the other members of the rosca will never defect. Then, when the alternative is autarky, she strictly prefers to never defect if $\beta \in (\beta^*, \bar{\beta})$. If $\beta < \beta^*$, there will be periods when she strictly prefers to leave. If $\beta \geq \bar{\beta}$, there will be periods when she weakly prefers to leave.*

Proof. Consider an agent's decision when she is k periods away from the next nondivisible. For simplicity, denote the continuation value from autarky equilibrium at any level of wealth, $V_A(\cdot)$. Denote the continuation value from a strategy in which all future selves save fully, $V_F(\cdot)$. First consider the region, $\beta \in [\underline{\beta}, \bar{\beta})$. If the individual defects from the rosca, she will have to play autarky forever (and forego any contributions she has made to the rosca so far). Since in autarky she is indifferent between saving and not saving at wealth y , her autarky utility will be $\beta\delta V_A(2y)$, where $V_A(2y)$ involves probabilistic saving in at least some future periods. If the agent remains in the rosca, her utility is $\beta\delta V_F(2y)$. Since $V_F(2y)$ involves optimal saving by all future selves, it must be greater than $V_A(2y)$. Therefore, if the agent chooses to save probabilistically in autarky, she will strictly prefer to remain in the rosca.

Second, we show that there is a region below $\underline{\beta}$ in which the agent still strictly prefers to remain in the rosca. Consider the indifference condition at $\underline{\beta}$. The agent at y is made indifferent with $\pi = 0$:

$$\underline{\beta}\delta V_A(2y) = u(y) + \frac{\beta\delta u(y)}{1-\delta}$$

The utility from a rosca, $\underline{\beta}\delta V_F(2y)$, still strictly dominates the utility from autarky. The agent continues to strictly prefer a rosca down to β^* that satisfies:

$$\begin{aligned} \beta^*\delta V_F(2y) &= u(y) + \frac{\beta^*\delta u(y)}{1-\delta} \\ \Leftrightarrow \beta^* &= \frac{u(y)}{\delta \left[V_F(2y) - \frac{u(y)}{1-\delta} \right]} \\ \Leftrightarrow \beta^* &= \frac{u(y)}{\delta \left[\frac{\delta^{k-2}\{u(b)\}}{1-\delta^k} - \frac{u(y)}{1-\delta} \right]} \end{aligned} \quad (1.5)$$

Since $u(y) > 0$, we know that $\beta^* > 0$. ■

It is useful to note here that $\beta^* < \underline{\beta}$.

Source of Commitment

A natural question here is: what exactly about the rosca provides commitment to the hyperbolic discounter? A rosca comes with (1) a different kind of contracting and (2) illiquidity of savings. We can consider two alternative commitment savings devices that separately perform these functions of a rosca: a friend who promises to monitor your saving and credibly threatens to stop helping if you under-save, and a fixed-deposit that locks up your savings until you reach a target amount (in this case, ky).

In the $(\underline{\beta}, \bar{\beta})$ range, illiquidity plays no role, since even in autarky the agent never dips into her savings. As long as we are in a region where some saving occurs in equilibrium, access to a fixed deposit cannot improve savings behavior. Here, a fixed rosca is very similar to a friend who offers to "help". The fact that the rosca can offer a credible threat to deny access to defectors creates a large enough wedge between the rosca and autarky equilibria to ensure participation. Then, if the agent were able to play history dependent (instead of Markov) equilibria with herself, she should be able to replicate rosca behavior. However, an individual's ability to play

such equilibria on her own is severely limited by the fact that the punishment strategy would be dominated by the strategy along the equilibrium path. If she deviates under a history-dependent equilibrium, she can easily renegotiate with herself to not play the punishment strategy, thus making it an unlikely threat.

At lower levels of β , the illiquidity provided by the rosca might become important. Consider the region in which an agent stays in a rosca but would not save in autarky ($\beta < \underline{\beta}$):

$$\frac{\beta\delta^{k-1}u(b)}{1-\delta^k} \geq u(y) + \frac{\beta\delta u(y)}{1-\delta}$$

Now, suppose the agent had the helpful friend instead of the rosca. The above condition might no longer be sufficient to ensure cooperation. She would also need to ensure, for instance, that at wealth $2y$ she did not have an incentive to consume everything:

$$\frac{\beta\delta^{k-2}u(b)}{1-\delta^k} \geq u(2y) + \frac{\beta\delta u(y)}{1-\delta}$$

If $u(\cdot)$ is not very concave, and δ is sufficiently high, then the second condition can fail even if the first is satisfied. With δ high, the agent does not benefit as much from being one period closer to the nondivisible. On the other hand, if u is almost linear, the benefit of consuming $2y$ can be enough to outweigh the fact that she will no longer save in the future.¹⁴ However, even when the illiquidity plays a role, it is useful only when combined with the contracting aspect of a rosca. With a fixed-deposit instead of a rosca, the agent would simply not deposit any money in the first place.

This section does not address the question of entry—clearly, the problem of postponement would exist with the devices mentioned above. The focus here is on commitment, conditional on having entered the savings device.

¹⁴One can expand the region in which illiquidity plays a role if there is either an intermediate "temptation" good or if income is stochastic. In each of these cases, the agent will have a greater incentive to dip into her savings in autarky, and she might therefore further value the fact that a rosca will prevent her from doing so.

1.5.2 Random Roscas

The only difference between a random and a fixed rosca is that, in a random rosca, the ordering is re-randomized at the start of each round. Consider an agent who has received the nondivisible in period 1. In period 2, her expected value from staying in the rosca is $\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k}$. If, in this period, she stays in the rosca, she will always stay in the rosca, since this is the furthest away from the nondivisible she can ever be.

Proposition 12 *If δ is sufficiently large, there is a β -region, bounded by $(\beta_{ran}^*, \bar{\beta}_{ran})$, within which an agent will always strictly prefer to remain in a random rosca (when the alternative is autarky). The region will be a strict subset of the region in which an agent stays in a fixed rosca: $\beta_{ran}^* > \beta^*$, $\bar{\beta}_{ran} < \bar{\beta}$.*

Proof. First, we show that the β -region that supports full participation in a random rosca must be smaller than the equivalent region for a fixed rosca. The lowest expected value from staying in a random rosca is smaller than the lowest expected value from staying in a fixed rosca:

$$\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k} < \frac{\beta\delta^{k-1}u(b)}{1-\delta^k}$$

(This is always true for $\delta < 1$ and $k > 1$). In each case, the outside option (autarky) is identical. Therefore, if an agent weakly prefers to stay in a random rosca, she will strictly prefer to stay in a fixed rosca.

Second, consider the first-ranked agent's decision in the second period of a rosca round. She will remain in the rosca if $\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k}$ is higher than her autarky equilibrium. If there is a region in which this is true, the lower bound, β_{ran}^* , must lie in $(0, \underline{\beta})$ and the upper bound, $\bar{\beta}_{ran}$, must lie in $(\underline{\beta}, \bar{\beta})$. This is because, at $\beta = 0$ and $\beta = \bar{\beta}$, autarky is strictly preferred, and for $\beta \in (0, \underline{\beta})$, autarky utility is linear and increasing in β , and for $\beta \in [\underline{\beta}, \bar{\beta}]$, autarky utility is concave and increasing in β . β_{ran}^* is given by:

$$\begin{aligned} \frac{\beta_{ran}^* \delta^{k-1} u(b)}{(1-\delta)k} &= u(y) + \frac{\beta_{ran}^* \delta u(y)}{1-\delta} \\ \beta_{ran}^* &= \frac{(1-\delta)u(y)}{\delta^{k-1}u(b) - k\delta u(y)} \end{aligned} \tag{1.6}$$

In Appendix E, we show that, for $\delta \rightarrow 1$, $\underline{\beta} \rightarrow \frac{u(y)}{u(b)-(k-1)u(y)} > 0$. Thus, for any δ , however large, there will be no autarky saving for $\beta < \frac{u(y)}{u(b)-(k-1)u(y)}$. As $\delta \rightarrow 1$, $\beta_{ran}^* \rightarrow 0$. This ensures that, for sufficiently high δ , there will be a region in which an agent always prefers to remain in a random rosca. ■

The intuition for the above result is the following: as δ gets large, the agent does not mind the fact that her rank in the next round is uncertain (her expected value gets closer to the expected value of staying on in a fixed rosca). The graphs below provide a summary of the results from the sections above.

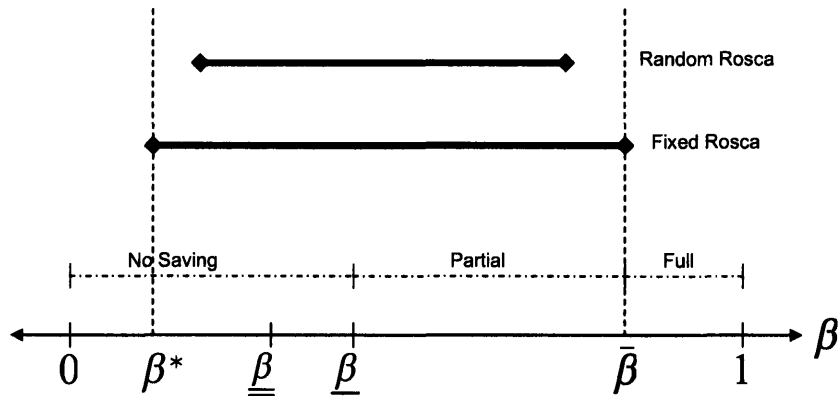


Figure 1-1: Comparison of β -regions that support random and fixed rosca. The lowest line indicates the type of equilibrium played in autarky.

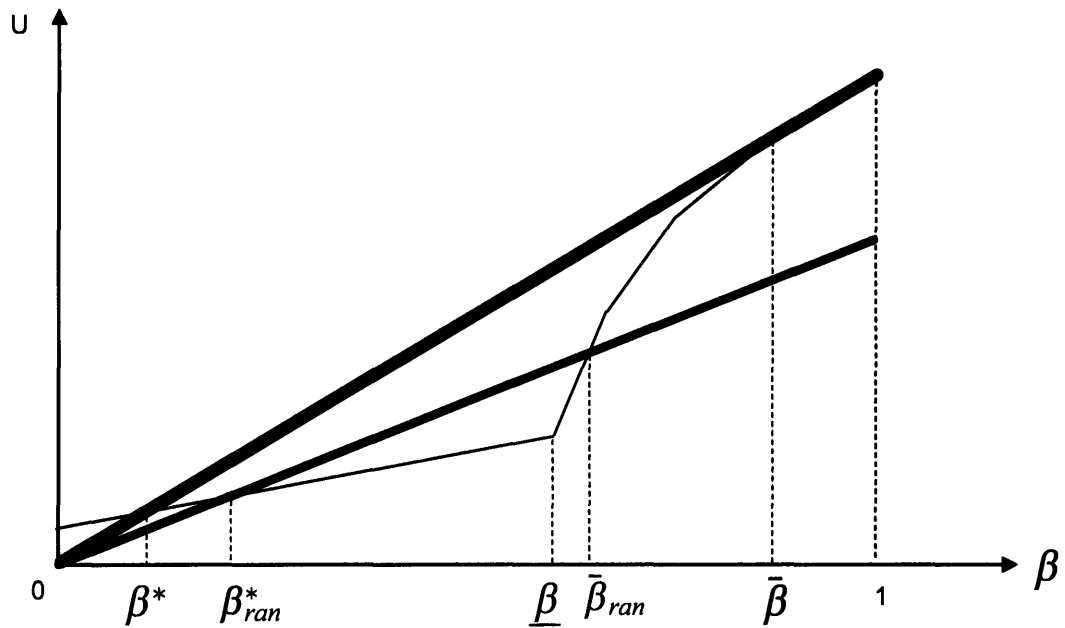


Figure 1-2: Comparison of lifetime utilities across β . The thickest line is the utility of an agent in a fixed rosca who is k periods away from her next nondivisible. The lower line of medium thickness is the utility of an agent in period 2 of a random rosca, conditional on having received the nondivisible in period 1. The thinnest (crooked) line is the autarky utility at wealth y . In $(\beta^*, \bar{\beta})$, fixed roscas are preferred to autarky. In $(\beta_{ran}^*, \bar{\beta}_{ran})$, random roscas are preferred to autarky.

1.6 Roscas as Decentralized Equilibria

In the previous section, we have shown that it is possible for a hyperbolic discounter to strictly prefer to remain in a rosca at all times if departure results in being banned from future roscas. If all agents in a rosca are within the necessary parameter region, then no individual has an ex-post incentive to leave if others choose to stay. In this section, we loosen two assumptions. We limit the available information about an agent's past rosca behavior. This places a restriction on what strategies can be conditioned on. We also allow the formation of new roscas in any period. The rate at which this happens is pinned down by an exogenous probability with which rosca aspirants get matched into groups of size k .

This gives us two key results. First, while we have shown that neither contracts nor social sanctions are needed to prevent defection from a rosca, we would like to know how roscas might survive if there is limited information about agents' past behavior. We find that, even under complete anonymity, roscas can survive if the exogenous probability of finding new roscas is sufficiently low. This follows directly from the commitment value to hyperbolic discounters. If all agents were exponential discounters, even an infinitesimal probability of finding a higher rank in a new rosca would create a strict incentive to leave.

Second, the previous section does not provide a reason for random roscas to exist (the greatest incentive to defect from random roscas is stronger than for fixed roscas). In this section we show that under limited reputation, when matching is fast, random roscas are more resilient than fixed roscas. Suppose we have "partial reputation"—this is a case where agents who have left roscas in the middle of past rounds are remembered as defectors, but agents who leave after completing a round are indistinguishable from all others (this is reasonable—we are most likely to remember the people who owe us money). This limits the problem of defection during rounds, but leaves open the possibility that an agent might wish to leave after completing a round. Now, a particular advantage of random roscas becomes salient. Since the ordering is re-randomized at the end of each round, no agent has an incentive to leave a random rosca between rounds. On the other hand, the last-ranked member of a fixed rosca knows that she will be last-ranked forever. Therefore, she might still wish to leave at the end of a round, which would keep her reputation intact and allow her to rejoin a new rosca with a higher expected rank. This gives us conditions under which random roscas are both more sustainable and more

welfare-generating than fixed roscas.

1.6.1 Assumptions

We study three reputational environments. Under *anonymity*, agents' past behavior is completely invisible. Under *partial reputation*, agent's histories become public to the extent that others know if they have ever defected from a rosca *during* a round. This can be thought of as a black mark that defectors acquire (this is a natural outcome if memory is costly). Finally, under *full reputation*, an agent's entire past rosca behavior is publicly known (strategies can be conditioned on whether the agent has ever left a rosca, and if so, at which stage). The better the reputational environment is, the easier it is for roscas to condition strategies on agents' past behavior, thus increasing the sustainability of and total benefits from roscas.

We assume a large, growing population of identical hyperbolic discounters. For analytical convenience, we restrict the agents to $\beta \in (\beta^*, \underline{\beta})$. In this region, agents value a rosca and would not save in autarky (so their actions are limited to rosca-related decisions). Finally, we assume that in any period, an infinitesimal proportion of the population becomes completely myopic ($\beta = 0$) for life.

Timing and Strategies

We assume there are two rosca "pools"—the pool of agents looking for a new rosca (pool *New*), and the pool of agents who wish to fill an open slot in an existing rosca (pool *Old*). The timing of the game is as follows. Each period is divided into 5 sub-periods:

- a) Agents in existing roscas choose whether to stay (*Y*) or leave (*N*). Agents who are not in a rosca choose whether to move to *Old* (by default, they are in *New*).
- b) Each existing rosca (defined as a rosca with at least 1 remaining member) with vacated slots makes rank-specific offers to agents in pool *Old*. In pool *New*, some proportion, p , of agents are randomly matched into groups of size k . (p is exogenously determined. We assume this is an index of how easily people are able to find each other and form groups).
- c) Agents accept (*Y*) or reject (*N*) offers of membership in roscas (in either pool).

- d) New roscas randomly determine the ordering. Existing random roscas randomly determine the ordering if a new round is starting.
- e) Agents in roscas decide whether to stay (Y) or leave (N).

If any agent leaves a rosca or rejects an offer, she can only re-enter the pool in the following period. Any rosca that is unable to fill its slots breaks up and agents re-enter the pool in the following period.

In any period, the following actions are available to agents: those in an existing rosca observe their state (profile of other members and distance to the next nondivisible) and must choose Y or N in sub-periods (a), (c), and (e). Agents who are not in a rosca choose whether to move to *Old* in sub-period (b). If they receive an offer, they choose Y or N in sub-period (c). Finally, in sub-period (e), agents can again choose Y or N after learning their rank.

A rosca round starts in sub-period (d) of period 1 and continues for k periods. Then, for example, under partial reputation, if an agent leaves in subperiod (c) of round 1 (before the ordering is determined), she does not acquire a reputation as a defector.

A rosca strategy is a decision about how to choose members from pool *Old* if there is an opening in the rosca (all members are aware of their rosca strategy). An equilibrium is an action associated with each information set (for individuals) and a rosca strategy for each rosca configuration, such that no agent has an incentive to deviate from her strategy at any information set.

1.6.2 Outside Option

The problem of sustainability is directly affected by an agent's outside option. This is defined as the expected lifetime utility for an agent who leaves a rosca in any period. Especially when available information is low, an agent might know that she has a realistic chance of leaving a rosca in which she has a low rank, and re-entering one with a higher expected rank.

Assume all roscas survive forever. Consider an agent in a fixed rosca who is k periods away from the next nondivisible. Suppose she is free to leave the rosca and re-enter any other rosca starting in the next period. If she is certain to get a new rosca ($p = 1$), clearly she prefers to leave (if not, it would violate the assumption that $\beta < \beta^*$). We would like to find conditions

under which she will not leave her rosca. The agent will have a strict incentive to stay if $p < p^*$, where p^* is defined by:

$$\begin{aligned} \frac{\beta\delta^{k-1}u(b)}{1-\delta^k} &= u(y) + \beta\delta \left[\frac{p^* \left(\frac{u(b)}{k(1-\delta)} \right) + (1-p^*)u(y)}{1-(1-p^*)\delta} \right] \\ \Rightarrow p^* &= \frac{(1-\delta) \left(\frac{\beta\delta^{k-1}u(b)}{1-\delta^k} \right) - (1-\delta + \beta\delta)u(y)}{\frac{\beta\delta u(b)}{k(1-\delta)} - \frac{\beta\delta^k u(b)}{1-\delta^k} + (\delta - \beta\delta)u(y)} \end{aligned} \quad (1.7)$$

Similarly, consider the first-ranked agent in a random rosca that survives forever. If, by leaving, she can find a new rosca with some probability p , she will only stay on in her current rosca for $p \leq p_{ran}^*$, where p_{ran}^* is given by:

$$\begin{aligned} \beta\delta^{k-1} \left(\frac{u(b)}{k(1-\delta)} \right) &= u(y) + \beta\delta \left[\frac{p_{ran}^* \left(\frac{u(b)}{k(1-\delta)} \right) + (1-p_{ran}^*)u(y)}{1-(1-p_{ran}^*)\delta} \right] \\ \Rightarrow p_{ran}^* &= \frac{(1-\delta) \left(\frac{\beta\delta^{k-1}u(b)}{1-\delta^k} \right) - (1-\delta + \beta\delta)u(y)}{\frac{\beta\delta u(b)}{k(1-\delta)} - \frac{\beta\delta^k u(b)}{1-\delta^k} + (\delta - \beta\delta)u(y)} \end{aligned} \quad (1.8a)$$

It follows that $p_{ran}^* < p^*$.

1.6.3 Fixed Roscas

Anonymity

Here, we assume that strategies cannot be conditioned on any aspect of an agent's past behavior. If there is an opening in a rosca, the rosca simply decides whether to make an arbitrary offer to a person in the pool. Similarly, an agent has no information about the other members' past rosca experience.

First we consider fixed roscas. We can show that if p is low enough, we can have equilibria with roscas surviving over time. Without such limits on p , every agent will have an incentive to leave her rosca immediately after receiving the nondivisible—she can consume her income today and join a new rosca in the next period.

Proposition 13 *Assume complete anonymity. Consider p^* as defined in equation 1.7. Then,*

strategies in which agents always stay in fixed roscas constitute an equilibrium only when $p \leq p^$.*

Proof. Suppose $p \leq p^*$. Consider the following strategy: Agents in roscas, or with rosca offers, always play Y ; agents outside roscas always enter New ; roscas with openings randomly make offers from Old . Then, by definition of p^* , the outside option from defection is sufficiently small that the agent who is k periods away from the next nondivisible prefers to remain in the rosca. Therefore, every group of k agents who are lucky enough to form a rosca will never separate.

(2) Suppose $p > p^*$. By the definition of p^* , a rosca cannot survive forever in equilibrium: if all agents have a strategy of never leaving a rosca, then any individual who has just received a nondivisible has a strict incentive to deviate. ■

Partial Reputation

Under partial reputation, an agent develops a bad reputation if she has left during any round, which is defined as starting in the sub-period in which the ordering is randomized in period 1, and ending after round k (and continuing every k periods after that).

Now, the problems associated with anonymity are alleviated to some extent. It is possible for agents to have strategies where they refuse to join roscas with people who have left during a past round (believing that anyone who has done so is now a $\beta = 0$ type). However, such strategies cannot stop the last-ranked member of a fixed rosca from leaving at the end of a round. Since she cannot be distinguished from those who have never been in a rosca, she will have an incentive to join a new rosca if p is high enough.

Proposition 14 *Assume partial reputation. Consider p^* as defined in Equation 1.7. Then, strategies in which agents always stay in fixed roscas constitute an equilibrium only when $p \leq p^*$.*

Proof. (1) Assume $p \leq p^*$. Then, agents can play the same equilibrium as under anonymity.

(2) Assume $p > p^*$. Roscas cannot last forever with the same membership. Since there is no strategy that can be conditioned on whether an agent left at the end of a round, the last ranked player has a strict incentive to leave after a round if she knows that, in the future, all agents will stay in a rosca forever. ■

Full Reputation

In this section, we assume that strategies can be conditioned on whether an agent has left a rosca in the past, and if so, at what stage of a round she left. Here, fixed roscas can be sustained even with $p = 1$. Consider the following beliefs: Any agent who has ever left a past rosca is now a $\beta = 0$ type. Consider the following strategies: a rosca with open slots accepts any people who have never left a rosca before; agents outside roscas remain in *New*; agents in roscas or with rosca offers always play *Y* unless there is a $\beta = 0$ type in the group. Since any agent who leaves a rosca must play autarky, we know that no agent will leave. The strategies described are an equilibrium, and the beliefs are justified.

1.6.4 Random Roscas

Under full reputation, random roscas cannot last longer than fixed roscas, since fixed roscas survive forever at any p . Also, under anonymity, random roscas can never last longer than fixed roscas. To see why, first note that, if $p \leq p_{ran}^*$ and $\beta \geq \beta_{ran}^*$, then random roscas survive forever and yield the same expected benefit as fixed roscas. Now, consider $p > p_{ran}^*$. There can be no equilibrium in which random roscas last forever. If they exist at all, they must disintegrate with positive probability. Suppose there is such an equilibrium. It must be the case that the first-ranked member plays a mixed strategy in the second period of a round, while all others continue to play with certainty (if the first-ranked member is indifferent in the second period, all other agents will strictly prefer to stay in the rosca). However, with any such strategy, the first-ranked member will strictly prefer to leave—since she knows that her current rosca might break up in the next round anyway, the outside option becomes relatively more attractive. To see this, note that for $p > p_{ran}^*$:

$$\beta\delta^{k-1} \left(\frac{u(b)}{k(1-\delta)} \right) < u(y) + \beta\delta \left[\frac{p \left(\frac{u(b)}{k(1-\delta)} \right) + (1-p)u(y)}{1 - (1-p)\delta} \right]$$

This follows from Equation 1.8a. Any equilibrium with some survival must satisfy:

$$\beta\delta^{k-1} (V) = u(y) + \beta\delta \left[\frac{p_{ran}^* (V) + (1-p_{ran}^*)u(y)}{1 - (1-p_{ran}^*)\delta} \right]$$

where V is the expected value of being in a random rosca at the start of a round. Since V will be smaller than $\frac{u(b)}{k(1-\delta)}$, this condition cannot be satisfied. Therefore, under anonymity, random rosca cannot exist at all when $p > p_{ran}^*$.

However, with partial reputation, when p is high, we find that random rosca can be more resilient than fixed rosca, and also give greater utility to the participants. The intuition for this result is the following. When $p > p^*$, there cannot be a fixed rosca in which all agents stay forever, because the last ranked member would have an incentive to leave. However, there is no such incentive to leave a random rosca between rounds. Therefore, if β is such that agents *within* the rosca prefer to stay rather than play autarky, then random rosca will survive forever, and any agent's expected value from a rosca will be higher than it would be under fixed rosca (see Figure 3 in Section 7).

Proposition 15 *Assume partial reputation. If $\beta > \beta_{ran}^*$ (as defined in Equation 1.6), then random rosca can survive forever in equilibrium. If, in addition, $p > p^*$, then random rosca are strictly welfare improving relative to fixed rosca.*

Proof. Consider the following beliefs: any agent who has left a rosca during a past round has $\beta = 0$. Consider the following strategies: a rosca with open slots accepts any people who have never left a past rosca; individuals outside rosca enter *New*; agents in rosca or with rosca offers play *Y* only if the other members have never left a past rosca.

Suppose there is an equilibrium with random rosca. It must be the case that the first-ranked agent always prefers to remain in the rosca:

$$\begin{aligned} \frac{\beta\delta^{k-1}u(b)}{(1-\delta)k} &> u(y) + \frac{\beta\delta u(y)}{1-\delta} \\ \Rightarrow \beta &> \beta_{ran}^* \end{aligned}$$

If this condition is satisfied, then random rosca will exist forever for any value of p .

Now suppose $p > p^*$. Then, the expected value for any agent entering a rosca is: $\frac{u(b)}{k} \left(1 + \frac{\beta\delta}{1-\delta}\right)$. If fixed rosca survived forever, this would be identical to the expected value from fixed rosca. Since there is no such equilibrium, the expected value from fixed rosca must be lower. ■

1.7 Empirical Implications

In this section, we look at how the results in previous sections can be related to our empirical understanding of roscas. Our focus is on implications that are directly related to hyperbolic discounting, including (1) comparative statics generated by the model, (2) predictions about the survival of random and fixed roscas, and (3) comparisons between durable and nondurable goods.

If hyperbolic discounting is indeed a primary explanation of rosca participation, then we predict that members of long-lasting roscas will exhibit intermediate levels of time-inconsistency in hypothetical preference reversal games. The actual size of the β -region within which roscas survive depends on several parameter values. All else equal, roscas are more likely to survive as the nondivisible gets more expensive relative to income. Also, roscas are more likely to survive as the utility from small units of consumption increases relative to the utility from the nondivisible (until the point where the nondivisible is no longer valued even by exponential discounters).

In the previous section, we have seen how it is possible for roscas to survive as non-cooperative equilibria. The defection incentive in these cases is created by the option value of a higher rank in a future rosca. We see that in environments that are completely anonymous, fixed roscas can survive forever only if matching for new roscas is sufficiently low. When there is some reputation, fixed roscas again survive when matching is sufficiently slow, but random roscas can be welfare-improving when matching probabilities are high. When roscas can access more information about an agent's past rosca behavior, equilibria with repeating fixed roscas can exist even under perfect matching (see Figure 3).

This gives us some testable predictions. We conjecture that the likelihood of matching is positively correlated with population density, which suggests that p rises as communities get urbanized. When reputation is informal, the availability of information is likely to be inversely correlated with urbanization (full reputation is a feature of very small rural communities, while urban areas are more anonymous). Then, our model predicts that fixed roscas are more likely than random roscas at fully rural and fully urban extremes. In semi-rural communities, we are more likely to encounter conditions suited to the survival of random roscas.

The Anderson, Baland (2002) study, set in an urban neighborhood, finds that a majority of

roscas are fixed. The Gugerty (2005) study, set in rural Kenya, finds that a majority of roscas are random. These patterns appear consistent with our predictions, but we would require more information for a more accurate analysis.

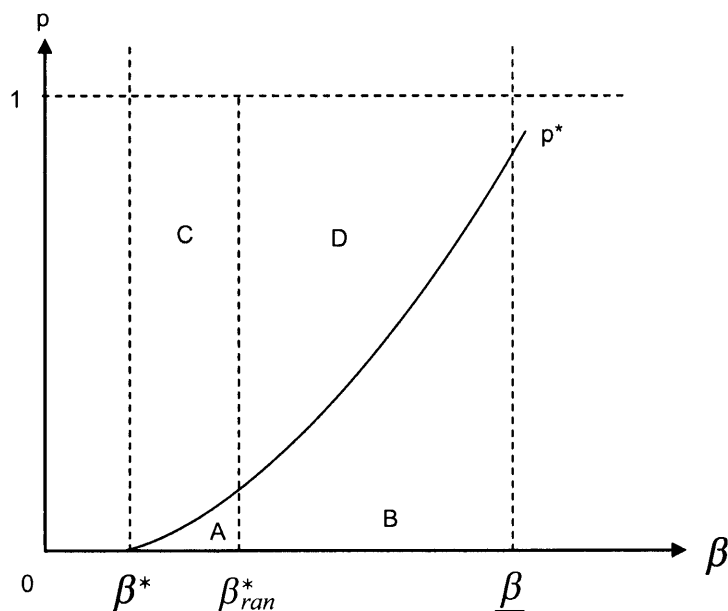


Figure 1-3: Partial Reputation (p is the exogenous probability of matching into new roscas groups; assume p_{ran}^* is close to p^*): In region A, only fixed roscas can survive forever. In region B, fixed and random roscas can survive forever. In region C, no roscas survive forever (if any roscas exist, they must be fixed). In region D, random roscas are more likely to survive, and provide higher welfare than any fixed roscas equilibrium.

In this model, as in some previous models, we describe the nondivisible good as yielding a one-period benefit. This is not an unreasonable assumption, since there is a range of empirical evidence suggesting that roscas members often do not spend the money on durable goods. However, it is useful to identify implications for roscas survival if agents save for durable goods instead. When goods are durable, the benefits are spread across multiple periods, and agents place less value on immediate consumption. This has two implications. First, as the good becomes more durable, agents are more likely to postpone entry into roscas. Second, once an agent is actually in a roscas, the conditions under which roscas survive will expand. Since the benefit from a potentially higher rank in a new roscas is dampened, agents are more willing to

remain in an existing rosca than they would if they were purchasing nondurables. This implies that, as goods get more durable, individuals may choose to delay entry into roscas but existing roscas are more likely to survive over time.

1.8 Conclusion

We have shown that roscas can be effective commitment savings devices even without formal contracting or social sanctions. Agents with self-control problems derive benefits from staying in a rosca and improving their savings behavior. A particularly useful feature of roscas is that hyperbolic discounters have no incentive to postpone take-up. Unlike other commitment savings devices that yield delayed benefits, roscas always come with the possibility that an agent might be an instant winner. The randomization of rank and subsequent commitment play complementary roles—the first draws an agent into a rosca, and the second gives her a reason to stay.

In this paper, we also highlight the relative advantages of fixed and random roscas. Within a round, agents in random roscas have a weaker incentive to stay than they would in fixed roscas. However, between rounds, agents in random roscas never have an incentive to leave, while late ranked agents in fixed roscas might prefer to leave if they can join a new rosca. Empirically, both random and fixed roscas exist in large numbers. We make predictions about the survival of each, based on the depth of reputation and the speed of matching in a community.

This paper gives us several directions for further research. Models that allow for heterogeneous populations (for instance, exponential discounters, or agents who want different kinds of goods) are likely to alter conditions under which roscas survive in the absence of contracting. Also, there is scope for testing some of the predictions of our model, especially if convincing measures of time-inconsistency, reputation, and matching speed can be constructed. However, the overall idea of roscas as commitment savings devices seems to be realistic, and helps explain their resilience across the developing world.

1.9 Appendix

1.9.1 Appendix A: Proofs for Section 3.3

Proof of Lemma 1. Suppose not. If at wealth w the agent saves s with positive probability, it means:

$$u(w - s) + \beta\delta V(s + y) \geq u(w - s') + \beta\delta V(s' + y)$$

Now, if at wealth $w' > w$ the agent saves s' with positive probability, it means:

$$u(w' - s) + \beta\delta V(s + y) \leq u(w' - s') + \beta\delta V(s' + y)$$

This implies that:

$$u(w - s') - u(w - s) \leq u(w' - s') - u(w' - s)$$

This violates strict concavity of u . ■

Proof of Lemma 2. Suppose the agent with wealth w saves with higher probability in A than in B , but both play a mixed strategy. Then, in each equilibrium, the following condition must hold (for $i \in \{A, B\}$):

$$\beta\delta V_i(w + y) = u(y) + \beta\delta V_i(w)$$

Assuming the agent saves w with probability π_i , we can solve for $V_i(w)$. Note that this means she saves $w - y$ with probability $1 - \pi_i$.

$$\begin{aligned} V_i(w) &= \pi_i [\delta V_i(w + y)] + (1 - \pi_i) [u(y) + \delta V_i(w)] \\ \Rightarrow V_i(w) &= \frac{\pi_i [\delta V_i(w + y)] + (1 - \pi_i) u(y)}{1 - (1 - \pi_i) \delta} \end{aligned}$$

Reinserting this into the indifference condition and rearranging terms, we get:

$$\beta\delta(1 - \delta)V_i(w + y) = [u(y)] [1 - \delta + \pi_i\delta(1 - \beta)] + \beta\delta u(y)$$

Comparing across equilibria, $\pi_A > \pi_B \Leftrightarrow V_A(w + y) > V_B(w + y)$. Since in each case the

agents are willing to save, this also means that the agent with wealth w prefers equilibrium A relative to B . ■

Proof of Lemma 3. Assume $V_A(w + y) > V_B(w + y)$. Now suppose the proposition is false, and assume $V_A(w + 2y) > V_B(w + 2y)$. The following chain of inequalities must hold:

$$\begin{aligned} \beta\delta V_B(w + 2y) &> \beta\delta V_A(w + 2y) \\ &\geq u(y) + \beta\delta V_A(w + y) > u(y) + \beta\delta V_B(w + y) \\ &\Rightarrow \beta\delta V_B(w + 2y) > u(y) + \beta\delta V_B(w + y) \end{aligned}$$

Therefore, the agent with $w + y$ saves with certainty, which means $V_B(w + y) = \beta\delta^2 V_B(w + 2y)$. Furthermore:

$$V_A(w + y) \leq \beta\delta^2 V_A(w + 2y) < V_B(w + y)$$

This contradicts our assumption. ■

Proof of Proposition 1. 1. At least one Markov Perfect Equilibrium exists (Fudenberg and Tirole, 1991).

2. Consider any two equilibria, A and B , each of which involves saving. If the agent at wealth y saves with higher probability under A than B , then A is strictly preferred by the agent at y (Lemma 1). Then, A is strictly preferred at all higher levels of wealth (Lemma 2).

3. Consider any two equilibria, A and B . Suppose the agent at wealth y saves with identical probability, π , in A and B . This can happen in two ways: (a) $0 < \pi \leq 1$ or (b) $\pi = 0$. Consider (a). Find the lowest level of wealth, \hat{w} , at which saving probabilities differ (if this never happens, A and B are identical). At \hat{w} , suppose $\pi_A > \pi_B$ (where the probabilities are of saving \hat{w}). By Lemma 1, $V_A(\hat{w} + y) > V_B(\hat{w} + y)$. It follows from Lemma 3 that $V_A(\hat{w}) > V_B(\hat{w})$. Therefore, the agent at $\hat{w} - y$ prefers A . Similarly, A is preferred to B at all levels of wealth. Now consider (b). Under any two such equilibria, a higher level of wealth is never reached, so the agent is always indifferent.

4. Consider any two equilibria, A and B . Suppose A involves saving but B does not. Then $V_B(y) = \frac{u(y)}{1-\delta}$. Since $\frac{\partial}{\partial \pi_1} V_A(y) > 0$ and, at $\pi_1 = 0$, $V_A(y) = \frac{u(y)}{1-\delta} = V_B(y)$, we know that

$V_A(y) > V_B(y)$. Since there is saving in A but no saving in B , we know:

$$\beta\delta V_A(2y) \geq u(y) + \beta\delta V_A(y) > u(y) + \beta\delta V_B(y) > \beta\delta V_B(2y)$$

Therefore, the agent at y prefers equilibrium A to B . Also, $V_A(2y) > V_B(2y)$. At wealth $2y$, utility from equilibrium A is $\beta\delta V_B(3y)$, which is weakly greater than $u(2y) + \beta\delta V_A(y)$ and $u(y) + \beta\delta V_A(2y)$. Utility from equilibrium B is:

$$\max\{u(2y) + \beta\delta V_B(y), u(y) + \beta\delta V_B(2y), \beta\delta V_B(3y)\}$$

The first two possibilities are strictly dominated by the utility from A . Furthermore, it must be the case that $V_B(3y) < V_B(3y)$ (if not, the agent at wealth y in equilibrium B would have an incentive to save). By iteration, at any higher level of wealth, iy , any possible utility from equilibrium B must be strictly dominated by $\beta\delta V_A(iy + y)$.

5. This gives us a complete and transitive ordering over any set of equilibria. Any equilibrium in which the nondivisible is ever purchased is always preferred to any equilibrium in which the nondivisible is not purchased. Among equilibria in which the nondivisible is purchased, the best involves the highest saving at y (or the lowest level of wealth at which a unique equilibrium has higher saving than all others). ■

1.9.2 Appendix B: Example for Section 3.4

Suppose the nondivisible costs $3y$. Then a full saving equilibrium exists as long as the agent at y strictly prefers to save fully:

$$\frac{\beta\delta^2 u(b)}{1 - \delta^3} > u(y) + \frac{\beta\delta^3 u(b)}{1 - \delta^3}$$

Once β falls low enough so that the above condition fails, the agent at y can start playing a mixed strategy, which must satisfy:

$$\beta\delta^2 u(b) + \beta\delta^3 V(y) = u(y) + \beta\delta V(y)$$

The RHS responds faster to π than the LHS does. If the agent at $2y$ continues to save fully, the agent at y can be kept indifferent down to some $(\hat{\beta}, 0)$, which in this case corresponds to:

$$\begin{aligned}\hat{\beta}\delta^2u(b) + \frac{\hat{\beta}\delta^3u(y)}{1-\delta} &= u(y) + \frac{\hat{\beta}\delta u(y)}{1-\delta} \\ \Rightarrow \hat{\beta} &= \frac{u(y)}{\delta[\delta u(b) - (1+\delta)u(y)]}\end{aligned}$$

If the agent at $2y$ is still willing to save fully at the $(\hat{\beta}, 0)$ combination, then the following condition must hold:

$$\begin{aligned}\hat{\beta}\delta u(b) + \frac{\hat{\beta}\delta^2u(y)}{1-\delta} &\geq u(y) + \hat{\beta}\delta^2u(b) + \frac{\hat{\beta}\delta^3u(y)}{1-\delta} \\ \Rightarrow \hat{\beta} &\geq \frac{u(y)}{\delta[(1-\delta)u(b) + \delta u(y)]}\end{aligned}$$

For both of these conditions to be simultaneously satisfied, we must satisfy:

$$\begin{aligned}(1-\delta)u(b) + \delta u(y) &\geq \delta u(b) - (1+\delta)u(y) \\ \Rightarrow (1+2\delta)u(y) &\geq (2\delta-1)u(b)\end{aligned}$$

which is only true if δ is sufficiently small, or if $u(y)$ is sufficiently large relative to $u(b)$.

1.9.3 Appendix C: Mixed Strategy Equilibria

First (for Proposition 4), we show that $\pi'_i(\pi_j) > 0$, for all $i, j \in \{1, 2, \dots, k-1\}$, $j \neq i$. The utility from saving is:

$$\beta\delta V(w+y)$$

The utility from not saving is:

$$u(y) + \beta\delta[\pi_i\delta V(w+y) + (1-\pi_i)(u(y) + \delta V(w))]$$

If some π_j rises, then the utility from saving rises faster than the utility from not saving does. To re-establish indifference, π_i must rise.

Second, we show that the utility from autarky (from the point of view of the agent at wealth

y) is weakly concave over $[\underline{\beta}, \bar{\beta}]$. Take $\beta \in (\beta_i, \beta_{i-1})$. Recall that this is a region in which agents at wealth iy and below play a mixed strategy. Since they are all indifferent, we know that for $j \leq i$:

$$u(y) + \beta\delta V(jy) = \beta\delta V(jy + y)$$

and for $j > i$:

$$V(jy) = \delta^{k-j}u(b) + \delta^{k-j+1}V(y)$$

Combining these, we get:

$$V(y) = \frac{\delta^{k-i}u(b)}{1 - \delta^{k-i+1}}$$

which means that the lifetime utility at wealth y is: $\beta \frac{\delta^{k-i}u(b)}{1 - \delta^{k-i}} - \frac{iu(y)}{1 - \delta^{k-i}}$. Since i is weakly decreasing in β and $\frac{\delta^{k-i}u(b)}{1 - \delta^{k-i}}$ is strictly increasing in i , lifetime utility is less responsive to β as β rises; i.e. it is weakly concave.

1.9.4 Appendix D: The $k = 2$ Case

We solve an example in which $k = 2$. First, we provide explicit solutions for all key terms. Then, we make specific assumptions about δ , $u(y)$, and $u(b)$, and graph the resulting values.

Here, $\bar{\beta} = \left(\frac{1+\delta}{\delta}\right) \frac{u(y)}{u(b)}$ and $\underline{\beta} = \underline{\underline{\beta}} = \frac{1}{\delta} \frac{u(y)}{u(b) - u(y)}$. For $\beta \in (\underline{\beta}, \bar{\beta})$, the agent plays a mixed strategy, where she saves with probability π at wealth y . The mixed strategy satisfies the indifference condition:

$$\begin{aligned} \beta\delta u(b) + \beta\delta^2 V &= u(y) + \beta\delta V \\ \Rightarrow V &= \frac{\beta\delta u(b) - u(y)}{\beta\delta - \beta\delta^2} \end{aligned}$$

V is also given by,

$$V = \frac{\pi\delta u(b) + (1 - \pi)u(y)}{(1 - \delta)(1 + \pi\delta)}$$

where

$$\pi = \frac{\beta}{1 + \beta} \frac{u(b)}{u(y)} - \frac{1 + \beta\delta}{\delta + \beta\delta}$$

Then, in autarky, the agent's utilities from equilibrium (at wealth y) are given by:

$$\begin{aligned}
& \frac{\beta\delta u(b)}{1-\delta^2}, \text{ for } \beta \geq \bar{\beta} \\
u(y) + & \left[\frac{\beta\delta u(b) - u(y)}{1-\delta} \right], \text{ for } \beta \in (\underline{\beta}, \bar{\beta}) \\
& u(y) + \frac{\beta\delta u(y)}{1-\delta}, \text{ for } \beta \leq \underline{\beta}
\end{aligned}$$

Now we look at roscas. The lower bound of the region in which the agent always prefers a fixed rosca is:

$$\beta^* = \frac{(1-\delta^2)u(y)}{\delta u(b) - (1+\delta)\delta u(y)}$$

In the region with no autarky saving, the agent always strictly prefers a random rosca if:

$$\frac{\beta\delta u(b)}{2(1-\delta)} > u(y) + \frac{\beta\delta u(y)}{1-\delta}$$

This determines β_{ran}^* (assuming δ is high enough):

$$\beta_{ran}^* = \frac{2(1-\delta)u(y)}{\delta(u(b) - 2u(y))}$$

In the region with partial saving in autarky, the agent always strictly prefers a random rosca if:

$$\frac{\beta\delta u(b)}{2(1-\delta)} > u(y) + \left[\frac{\beta\delta u(b) - u(y)}{1-\delta} \right]$$

This determines $\bar{\beta}_{ran}$:

$$\bar{\beta}_{ran} = 2 \left(\frac{u(y)}{u(b)} \right)$$

Finally, under anonymity and partial reputation, fixed roscas survive forever if $p < p^*$, where:

$$p^* = \frac{(1-\delta) \left(\frac{\beta\delta u(b)}{1-\delta^2} \right) - (1-\delta + \beta\delta)u(y)}{\frac{\beta\delta u(b)}{2(1-\delta)} - \frac{\beta\delta^2 u(b)}{1-\delta^2} + (\delta - \beta\delta)u(y)}$$

Let us assume that $\delta = .95$, $u(y) = 1$, $u(b) = 2.5$. This ensures that the nondivisible is desirable to an exponential discounter. Then, $\bar{\beta} = 0.821$, $\underline{\beta} = 0.702$, $\beta^* = 0.187$. The lifetime

utilities from equilibrium (at wealth y) are:

$$\begin{aligned} & 24.359\beta, \text{ for } \beta \geq \bar{\beta} \\ & 47.5\beta - 19, \text{ for } \beta \in (\underline{\beta}, \bar{\beta}) \\ & 19\beta + 1, \text{ for } \beta \leq \underline{\beta} \end{aligned}$$

The agent will always strictly prefer to stay in a random rosca if $\beta \in (\beta_{ran}^*, \bar{\beta}_{ran}) = (0.211, 0.8)$.

(See Figure 4 for a graphical summary of these results.)

Under the assumptions above, $p^* = \frac{.05 - .268\beta}{.341\beta - .95}$. In Figure 5, we plot p^* as a function of β .

1.9.5 Appendix E: Approximations of Critical Values

To see how the critical β values relate to each other, we take first-order approximations for δ close to 1. Let $\delta = 1 - \varepsilon$, where ε is very close to 0.

$$\begin{aligned} \bar{\beta} &= \frac{1 - \delta^k}{\delta^{k-1} - \delta^k} \left(\frac{u(y)}{u(b)} \right) = \frac{1 - (1 - \varepsilon)^k}{(1 - \varepsilon)^{k-1} - (1 - \varepsilon)^k} \left(\frac{u(y)}{u(b)} \right) \\ &\rightarrow k \left(\frac{u(y)}{u(b)} \right) \end{aligned}$$

$$\begin{aligned} \underline{\beta} &= \frac{u(y)(1 - \delta)}{(1 - \delta)\delta^{k-1}u(b) - \delta u(y)(1 - \delta^{k-1})} \\ &= \frac{u(y)(1 - (1 - \varepsilon))}{(1 - (1 - \varepsilon))(1 - \varepsilon)^{k-1}u(b) - (1 - \varepsilon)u(y)(1 - (1 - \varepsilon)^{k-1})} \\ &\rightarrow \frac{u(y)}{u(b) - (k - 1)u(y)} > 0 \end{aligned}$$

$$\begin{aligned} \beta^* &= \frac{(1 - \delta^k)(1 - \delta)u(y)}{(\delta^{k-1} - \delta^k)u(b) - (\delta - \delta^{k+1})u(y)} \\ &= \frac{(1 - (1 - \varepsilon)^k)(1 - (1 - \varepsilon))u(y)}{\left((1 - \varepsilon)^{k-1} - (1 - \varepsilon)^k \right)u(b) - \left((1 - \varepsilon) - (1 - \varepsilon)^{k+1} \right)u(y)} \\ &\rightarrow 0 \end{aligned}$$

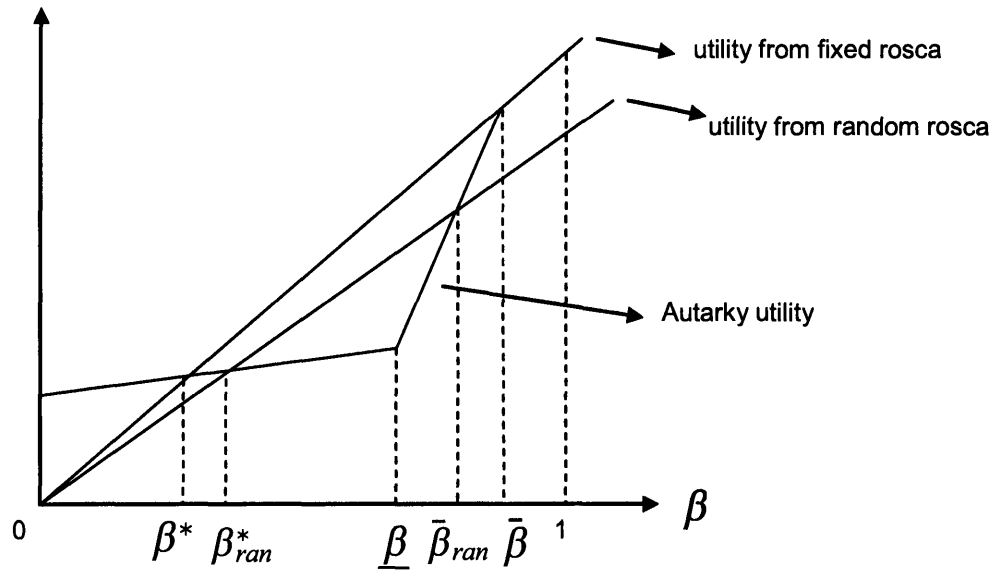


Figure 1-4: The lowest utility from a random rosca is lower than the lowest utility from a fixed rosca. The region in which a random rosca survives is smaller than the region in which a fixed rosca survives.

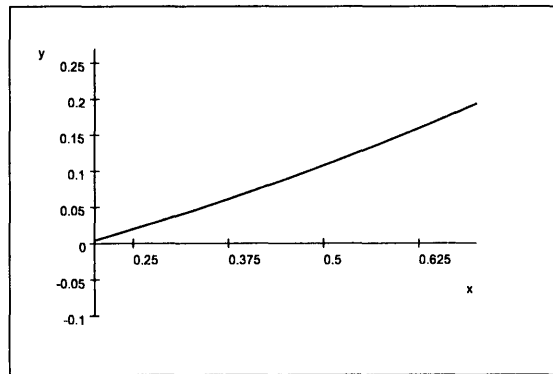


Figure 1-5: The x axis spans the relevant β -range. The y axis shows the corresponding value of p^* .

The comparative statics suggested by these approximations are discussed in Section 7.

1.10 References

1. AMBEC, S., and TREICH, N. (2005), "Roscas as Financial Agreements to Cope with Self-Control Problems", *Journal of Development Economics* (forthcoming).
2. ANDERSON, S., and BALAND, J. (2002), "The Economics of Roscas and Intrahousehold Resource Allocation", *Quarterly Journal of Economics*, 117 (3), 963-995.
3. ANDERSON, S., BALAND, J., and MOENE, K. (2003), "Enforcement and Organizational Design in Informal Saving Groups" (mimeo).
4. ASHRAF, N., GONS, N., KARLAN, D., and YIN, W. (2003), "A Review of Commitment Savings Products in Developing Countries", *Asian Development Bank Economics and Research Department Working Paper #45*.
5. ASHRAF, N., KARLAN, D., and YIN, W. (2005), "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines", *Quarterly Journal of Economics* (forthcoming).
6. BERTRAND, M., KARLAN, D., MULLAINATHAN, S., SHAFIR, E., and ZINMAN, J. (2005), "What's Psychology Worth? A Field Experiment in the Consumer Credit Market", (mimeo).
7. BERTRAND, M., MULLAINATHAN, S., and SHAFIR, E. (2004), "A Behavioral Economics View of Poverty", *American Economic Review Papers and Proceedings*, 419-423.
8. BESLEY, T. and COATE, S. (1995), "Group Lending, Repayment Incentives, and Social Collateral", *Journal of Development Economics*, 46, 1-18.
9. BESLEY, T., COATE, S., and LOURY, G. (1993), "The Economics of Rotating Savings and Credit Associations", *American Economic Review*, 83, 792-810.
10. BESLEY, T., COATE, S., and LOURY, G. (1994), "Rotating Savings and Credit Associations, Credit Markets and Efficiency", *Review of Economic Studies*, 61, 701-719.

11. BOUMAN, F. (1994), "ROSCA and ASCRA: Beyond the Financial Landscape", in: F.J.A. Bouman and O. Hospes (eds.), Financial Landscapes Reconstructed: The Fine Art of Mapping Development, Westview Press, Boulder, 375-394.
12. BOUMAN, F. (1995), "Rotating and Accumulating Savings and Credit Associations: A Development Perspective", *World Development*, 23, 371-384.
13. CALOMIRIS, C. and RAJARAMAN, I. (1998), "The Role of ROSCAS: Lumpy Durables or Event Insurance?", *Journal of Development Economics*, 56, 207-216.
14. DAGNELIE, O., and LeMAY, P. (2005), "Rosca Participation in Benin: a Commitment Issue", (mimeo).
15. FUDENBERG, D., and TIROLE, J. (1991), Game Theory, The MIT Press, Cambridge, MA.
16. GUGERTY, M. (2005), "You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya", (mimeo).
17. HANDA, S. and KIRTON, C. (1999), "The Economics of Rotating Savings and Credit Associations: Evidence from the Jamaican 'Partner'", *Journal of Development Economics*, 60, 173-194.
18. HARRIS, C. and LAIBSON, D. (2000), "Dynamic Choices of Hyperbolic Consumers", *Econometrica* (forthcoming).
19. KARLAN, D. (2005), "Social Connections and Group Banking", (mimeo: Yale University).
20. KIMUYU, P. (1999), "Rotating Saving and Credit Associations in Rural East Africa", *World Development*, 27, 1299-1308.
21. KLONNER, S. (2005), "Rotating Savings and Credit Associations when Participants are Risk Averse", *International Economic Review* (forthcoming).
22. KLONNER, S. and RAI, A. (2005), "Adverse Selection in Credit Markets: Evidence from South Indian Bidding Roscas", (mimeo).

23. KRUSELL, P., and SMITH, A. (2003a), "Consumption-Savings Decisions with Quasi-Geometric Discounting", *Econometrica*, 71, 365-375.
24. KRUSELL, P., and SMITH, A. (2003b), "Consumption-Savings Decisions with Quasi-Geometric Discounting: The Case with a Discrete Domain", (mimeo).
25. LAIBSON, D. (1997), "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics*, 62, 443-479.
26. LEVENSON, A., and BESLEY, T. (1996), "The Anatomy of an Informal Financial Market: Rosca Participation in Taiwan", *Journal of Development Economics*, 51, 45-68.
27. O'DONOGHUE, T., and RABIN, M. (1999), "Doing it Now or Later", *American Economic Review*, 89, 103-124
28. PHELPS, E., and POLLACK, R. (1968), "On Second-Best National Saving and Game-Equilibrium Growth", *Review of Economic Studies*, 35, 185-199.
29. THALER, R. and BERNARTZI, S. (2004), "Save More Tomorrow: Using Behavioral Economics to Increase Employee Savings." *Journal of Political Economy*, 112, 164-187.

Chapter 2

A Behavioral Model of Simultaneous Borrowing and Saving

Summary 16 *We present a model in which agents choose to simultaneously save and borrow money in equilibrium. The model describes the behavior of sophisticated hyperbolic discounters who have access to a non-secure savings technology. The combination of savings and a loan generates incentives for future selves to invest optimally, by punishing over-consumption relatively more severely. We show that these results cannot hold if agents are time-consistent, or if savings are fully secure.¹*

¹The puzzle tackled in this chapter was proposed by Dean Karlan.

2.1 Introduction

Why would an individual simultaneously save and borrow when the interest rate on saving is no higher than on borrowing? Given the prevalence of such behavior, several economic explanations, both traditional and behavioral, have been proposed. Traditional explanations usually rely on the option value of savings—under risky conditions, an agent might maintain savings for use in case of an emergency (if, for example, there are transaction costs with taking a loan on short notice, or if bankruptcy laws don't require the agent to repay a loan even if there are assets in the bank). Behavioral explanations, most notably Laibson, Repetto and Tobacman (2001), focus on illiquid savings as a self-control device. Agents lock assets for future consumption while smoothing short-term consumption with high-interest credit card debt.

In this paper, we suggest an alternative model. The model is motivated by a similar phenomenon observed among participants in FINCA, a microcredit organization in Peru. We argue that, in this specific context, existing explanations of simultaneous borrowing and saving are insufficient. As in Laibson et al, agents have time-inconsistent preferences, but in this model savings serve a different purpose. We exploit the fact that savings with FINCA are not entirely secure—this allows agents to generate uncertainty that can improve the behavior of future selves. We show that the combination of a non-secure savings technology and a future investment opportunity can induce an agent to borrow and save simultaneously, and that this behavior is not optimal if either of these elements is absent. We also show that access to non-secure savings can make an agent better off than if savings are always secure.

2.2 Motivating Background

FINCA provides banking services to the very poor in the cities of Lima and Ayacucho. The majority of its clients are women who own and operate small informal businesses. Individuals are allowed to take out loans which must be repaid over a 4-month loan cycle. The average loan is \$203, and is typically used for business investment purposes (often inventory). All borrowers are required to also maintain a savings account. The saving is intra-group—i.e. an agent who saves is giving a loan to some other FINCA member. Savings and borrowing take place at the same fixed interest rate. When agents save internally, they effectively get a lower return than

they pay on their loans because there is some risk of their savings being defaulted upon. For any agent who does not repay a loan, the punishment (apart from the seizure of assets from the savings account) includes expulsion from future access to the bank.

A significant proportion of borrowers maintain savings that are above the required minimum. At any time, 15% save *more* than they borrow (30% have done this at some time). This behavior is most common in Ayacucho, where incomes are relatively low and access to credit is mostly limited to moneylenders who charge high interest rates. In his work on FINCA, Karlan finds that those who save and borrow simultaneously tend to express greater than average risk aversion in hypothetical questions.

2.3 Description of the Model

In this section, we argue against alternative explanations and broadly describe the intuition of our model. The goal of our model is to explain an individual's decision to make a savings and borrowing decision at the same time and in the same bank.

The possibility that savings are maintained for their option value is unlikely for two reasons. First, since the lender organization is the same as the borrower, it is not possible for an agent to default on a loan while still having access to her savings in the case of a negative shock. Second, savings are relatively illiquid during a loan cycle. Then, if an agent has a sudden need for liquidity, it is not clear that it is easier for her to access her savings than it is to simply take out a fresh loan from the bank. Since alternative banking services are very limited, it is unlikely that a savings account with FINCA would be useful as collateral for other banks.

It also seems that, unlike in Laibson, Repetto and Tobacman, agents are not using savings for their illiquidity. In their model, agents save to ensure consumption in the distant future, and take out high-interest loans if they suffer income shocks. However, in our context, while savings are not liquid within a loan cycle, they can nevertheless be used to pay back loans at the end of a cycle. Also, agents make the decision to maintain debt and savings *simultaneously*, so the loans are not a response to an unanticipated shock after savings have been locked up.

In our model, the agent is a quasi-hyperbolic discounter who has the opportunity to make an investment in the middle of her life. While her early selves would like her to invest, she is

The agent has an endowment w in period 0. The price of investment is p , and the monetary benefit of investment is b . The agent has a per-period utility function u , which is strictly concave and differentiable, with $u(0) = 0$ and $u'(0)$ finite. The agent is a quasi-hyperbolic discounter with $\delta = 1$ (the exponential discount factor) and $0 < \beta \leq 1$ (the hyperbolic discount factor).

If banking services are used, savings and borrowing takes place at an exogenous interest rate r ($R \equiv 1 + r$), such that x in period 0 yields Rx in period 2. If an agent saves within the bank, they are not repaid to her with some probability ε . This is also taken as exogenous at this stage. If an agent does not repay her loan in full, she faces a punishment of F . This can be interpreted as sanctions (financial or monetary), restricted access to future banking services, or seizure of durable assets. This is in addition to the bank collecting any amount that the agent has saved (at home or with the bank). We assume that F is large enough that it is always worth reducing consumption to avoid default.

2.4.2 Exponential Case

As a benchmark case, we assume $\beta = 1$. Here, an agent has time-consistent preferences and behaves like an exponential discounter. If there is no banking, the agent will invest in period 1 (we assume from here on that it is worthwhile to invest):

$$\max_{s \geq 0} u(w - p - s) + u(b + s) > 2u\left(\frac{w}{2}\right)$$

Now suppose the agent has access to banking as described above. If there was no investment to be made, she would save s_1 (in the bank) and s_2 (in period 1) to satisfy:

$$\max_{s_1, s_2} [u(w - s_1 - s_2) + (1 - \varepsilon)u(s_1 + Rs_2) + \varepsilon u(s_1)]$$

She will not borrow any money since this will be seized in case of default anyway (we are assuming punishment is sufficiently large that she will not default).

Now we bring back the possibility of investment. The agent will borrow to fund consumption in period 1:

$$\max_{l \geq 0} [u(w - p + l) + u(b - Rl)]$$

Assume R is low enough here that there is no incentive to save.

2.4.3 No Investment Case

Now we turn to the agent of interest—the quasi-hyperbolic discounter ($\beta < 1$). First, we analyze behavior in the absence of the investment opportunity. Since we are considering the case without banking, there is no action the period 0 agent can take. The agent in 1 will save s_1 for consumption in period 2 according to:

$$u'(w - s_1^*) = \beta u'(s_1^*) \quad (2.1)$$

This will involve lower saving than the period 0 agent would like.

Now suppose banking is available. The agent in 0 would like to improve period 2 consumption. First, why would she not borrow money? Suppose she borrowed money in 0. She would do this only if it would induce agent 1 to save more. The agent in 1 will save more than in the no-banking case only if there is inducement in the form of a period 2 threat. In this case, we need to show that any outcome that would be achieved by a combination of borrowing and saving can be achieved (or improved on) by simply saving less and borrowing 0.

Proposition 17 *When there is no investment to be made, the agent will never borrow in period 0.*

Proof. Suppose the agent in 0 saves some amount s_0 and borrows $l > 0$. The period 1 wealth, W_1 , is $w - s_0 + l$. The agent in 1 will consume some $0 \leq c_1^* \leq W_1$ and save $s_1^* = W_1 - c_1^* \geq Rl$ (to avoid default in the bad state). From the period 0 perspective, lifetime utility is:

$$u(c_1^*) + (1 - \varepsilon) u(Rs_0 + s_1^* - Rl) + \varepsilon u(s_1^* - Rl) \quad (2.2)$$

Now, consider the same s_0 as above, but change the loan to $\hat{l} = 0$. The period 1 wealth is $\hat{W}_1 = w - s_0$. Consider the following consumption and savings in period 1: $\hat{c}_1 = c_1^* + (Rl - l)$; $\hat{s}_1 = s_1^* - Rl$. This is an outcome where the agent in 1 consumes what was previously interest on the loan, while leaving period 2 consumption unchanged. This gives us a lower

bound on welfare from the period 0 perspective (if the agent in 1 deviates from this plan, it will be to transfer more consumption to period 2).

Utility from the period 0 perspective is bounded below by:

$$u(c_1^* + (Rl - l)) + (1 - \varepsilon)u(Rs_0 + s_1^* - Rl) + \varepsilon u(s_1^* - Rl) \quad (2.3)$$

The utility in 2.3 is strictly higher than the utility in 2.2. ■

Then, the agent in 0 will save s_0 to satisfy:

$$\begin{aligned} & \max_{s_0} \beta [u(w - s_0 - s_1) + (1 - \varepsilon)u(s_1 + Rs_0) + \varepsilon u(s_1)] \\ \text{s.t. } s_1 & \in \arg \max_{s_1 \geq 0} [u(w - s_0 - s_1) + \beta(1 - \varepsilon)u(s_1 + Rs_0) + \beta\varepsilon u(s_1)] \end{aligned}$$

2.4.4 Investment Case without Banking

We assume here that, in the absence of banking, the agent in 1 will not invest:

$$u(w - s^*) + \beta u(s^*) > u(w - p) + \beta u(b)$$

where the LHS is the agent's optimal saving behavior as described in Equation 2.1.

2.4.5 Behavior with Banking

This is the case of interest. Since an investment needs to be made in period 1, good behavior requires that the agent have enough available money for it, and that there be sufficient incentive for her to do so. First, we know that agent 0 can simply use saving to at least ensure the no-investment banking outcome described above. Assume again that F is high enough that the agent will repay the loan if at all possible. We use backward induction to analyze this problem.

The agent in period 2 has wealth $W_2 = s_1 + f(i) + Rs_0$ if her savings are repaid, and $W_2 = s_1 + f(i)$ if her savings are defaulted upon. Here, $f(i)$ is the output from investment, which is either 0 or b . Suppose also the agent owes a loan repayment of size Rl . If $W_2 \geq Rl$, she will repay the loan, and get utility $u(W_2 - Rl)$. If she defaults, her utility is $u(W_2 - F)$.

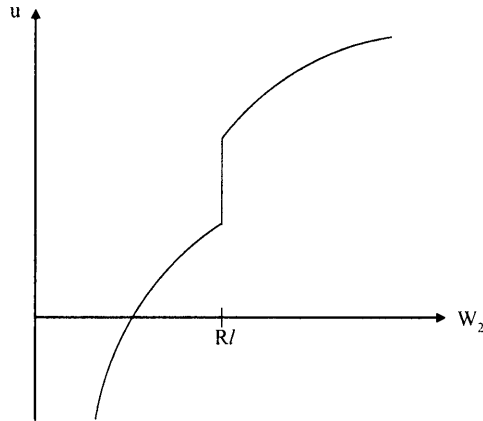


Figure 2-2: Period 2 utility

Now consider the decision in period 1. Her current wealth is $W_1 = w + l - s_0$. She chooses to consume some amount c_0 to maximize her discounted sum of period 1 and period 2 utilities. If c_0 is small enough, the investment is made.

If the agent in period 0 could control future behavior, she would choose some loan \bar{l} to compensate agent 1 for the cost of investment, while ensuring that investment actually does happen. However, given that agent 1 is free to make her investment decision, if she is sufficiently hyperbolic she would rather consume more (and not invest) than consume the amount necessary to invest. If this is the case, agent 0 must choose from the following options:

1. Give up on investing, and simply save some amount for agent 2.
2. Borrow more (some $\hat{l} > \bar{l}$) so that the agent in 1 is willing to invest.
3. If ε is low enough, borrow more than \bar{l} but also save some amount s_0 . This creates a threat for the period 1 agent—if she does not invest, there is a possibility that she will be unable to pay her loan in period 2. If F is large, this threat can create incentives to invest.

To see when c might be the best option, consider the tradeoff that agent 1 has to make. Suppose s_1 is the amount she chooses *not* to consume (if $s_1 > p$, then the investment is made). Her utility from current consumption is:

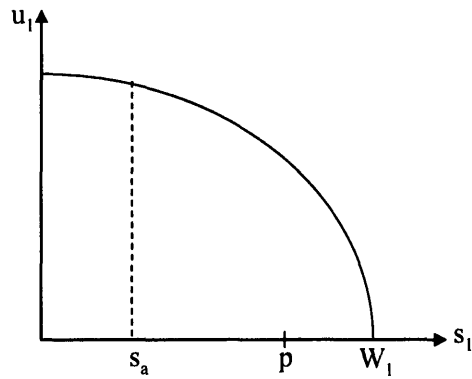


Figure 2-3: Period 1 utility

Suppose the agent in 0 only took a loan. Then, the period 2 utility is given by the thick curve below. The lowest segment is the region in which the agent in 2 might default. The middle segment is the case where there is no chance of her defaulting, but investment has not taken place. The uppermost segment is one in which the agent invested in period 1. Suppose agent 1's optimal choice is s_a , so that there is no punishment for default but the investment is made.

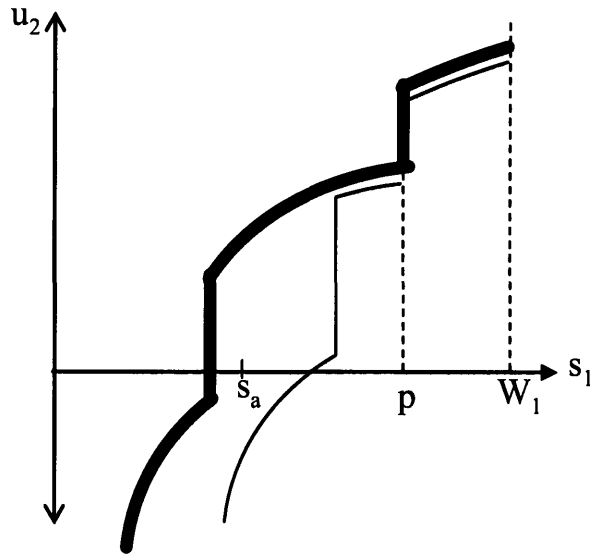


Figure 2-4: Period 2 utility as a function of period 1 saving

Suppose the agent in 0 saves more and borrows more. Now, period 2 utility is represented by the thin curve. If agent 1 continues to save only s_a , she might face punishment in period 2. To prevent this, she must raise the amount she saves. Here, it can become optimal for the agent to invest. Note that if ε is low, the final outcome is close to the optimal outcome (from the perspective of period 0).

In this case, saving creates the incentive to invest, while borrowing is used to actually fund the investment.

Proposition 18 *There is a parameter region in which the agent will save $s_0 > 0$ and borrow $l > 0$ in period 0.*

Proof. First, consider the optimal outcome under investment:

$$U_{inv-\max} = \max_{\bar{l} \geq 0} [u(w + \bar{l} - p) + u(b - R\bar{l})] \quad (2.4)$$

Now, consider the optimal outcome under no investment:

$$U_{noinv-max} = \max_{s_0, s_1 \geq 0} [u(w - s_0 - s_1) + (1 - \varepsilon)u(Rs_0 + s_1) + \varepsilon u(s_1)] \quad (2.5)$$

By the assumption that investment is worthwhile, $U_{inv-max} > U_{noinv-max}$.

Consider the outcome if the agent does not plan to invest. She will save some s_0 that satisfies:

$$\begin{aligned} U_{noinv} &= \max_{s_0} [u(w - s_0 - s_1(s_0)) + (1 - \varepsilon)u(Rs_0 + s_1(s_0)) + \varepsilon u(s_1(s_0))] \quad (2.6a) \\ \text{s.t. } s_1(s_0) &= \arg \max_{s_1} [u(w - s_0 - s_1(s_0)) + \beta(1 - \varepsilon)u(Rs_0 + s_1(s_0)) + \beta\varepsilon u(s_1(s_0))] \end{aligned}$$

Since the agent in 1 is maximizing a different utility function than in 0, $U_{noinv} < U_{noinv-max} < U_{inv-max}$.

Consider the case where β is low enough that $U_{inv-max}$ is not achievable. To induce saving, the agent can take a loan $\hat{l} > \bar{l}$ that satisfies (assume F is large enough that default is not desirable):

$$\max_{s_1 \geq 0} [u(w + \hat{l} - s_1) + \beta u(s_1 - R\hat{l})] = [u(w + \hat{l} - p) + \beta u(b - R\hat{l})] \quad (2.7)$$

This gives the following utility:

$$U_{inv-loan} = u(w + \hat{l} - p) + u(b - R\hat{l})$$

$$U_{inv-loan} < U_{inv-max}$$

The optimal pure-saving or pure-borrowing strategy yields a utility that is lower than the true optimal by some amount D , which is given by:

$$D = U_{inv-max} - \max\{U_{inv-loan}, U_{noinv}\}$$

There is a lower bound on D that does not depend on ε (set $\varepsilon = 0$ in Condition 2.6a). We now show there is a special case where simultaneous borrowing and saving can raise utility. Consider borrowing $l = \bar{l} + w$ and saving $s_0 = w$. Then, for any non-investment outcome

($s_1 < p$), agent 1's utility is:

$$u(w + \bar{l} - s_1) + \beta [(1 - \varepsilon) u(s_1 - R\bar{l}) + \varepsilon u(-F)] \quad (2.8)$$

If the agent invests (assuming $\bar{l} + w \leq b$), her utility is:

$$u(w + \bar{l} - p) + \beta [(1 - \varepsilon) u(b - R\bar{l}) + \varepsilon u(b - R(\bar{l} + w))] \quad (2.9)$$

If F is large enough, the agent will choose to invest. If ε is small enough, the agent in 0 prefers this outcome to any pure borrowing or pure saving outcome.

We now have sufficient conditions under which simultaneous borrowing and saving dominates:

- $\bar{l} + w \leq b$
- β is sufficiently low that D is large.
- ε is sufficiently low.
- For β and ε that satisfy above conditions, F is sufficiently high.

■

The actual optimal point will not involve pinning the agent in 1 to the original optimal consumption. Consider a deviation that leads to a change in c_1 . If the agent lowers l , she must raise s_0 to maintain incentives for agent 1 to invest (and vice versa). First, we consider a drop in l and a corresponding rise in s_0 . This lowers W_1 , which results in a lowering of c_1 and raising of c_2 . While this brings the marginal utilities of consumption between the two periods closer to each other, it also lowers the total amount of wealth to be shared. Alternatively, consider a rise in l and a drop in s_0 . While this raises c_1 and lowers c_2 (thus pushing marginal utilities further apart), the change in c_1 is greater than the change in c_2 .

To determine the actual savings-loan combination used in equilibrium, let $s_o(l)$ be the level of savings required to maintain investment incentives for any l . The marginal utility from

raising l (from a period 0 perspective) is:

$$u'(l - p) + (1 - \varepsilon) u'(b + Rs_0) Rs'_0(l) - R$$

The agent will choose l and s_0 so that the above term is 0.

It is useful to note here that if, instead, the individual only had access to secure savings (at a lower interest rate), then she would never choose to borrow and save simultaneously. In this case, it is impossible for agent 0 to create a discontinuity in period 2 utility that gets exacerbated if agent 1 over-consumes. To induce her future self to save, she will have to create incentives by lowering the relative marginal cost of saving in period 1. Rather than use the costly device of saving *and* borrowing, she will simply borrow to the point where agent 1 is willing to save. This is because, in either case, the period 0 agent must appeal to the period 1 agent's incentive to invest without a new threat being created. To see this, consider any loan-savings combination that induces investment. As shown in the proposition below, the agent in 0 can reduce both loan and savings in such a way that total wealth rises (less money is burned), and the benefits accrue to the period 1 agent. If the agent in period 1 has money at her disposal, her incentive to invest remains intact. Thus, investment continues to happen with less money wasted due to simultaneous saving and borrowing.

Proposition 19 *Suppose the agent can borrow at interest rate r ($R \equiv 1 + r$) and can save at interest rate t ($T = 1 + t$), where $r > t$. Then she will never save and borrow simultaneously.*

The proof of this proposition is in the appendix.

2.5 Comparative Statics

2.5.1 Example

We use a simplified example to make cross-sectional predictions about the survival of simultaneous saving and borrowing in equilibrium. Consider individuals who have a linear utility function: $u(c) = c$. Let the initial endowment w be normalized to 1.

If the agent in 0 does not plan to invest, she will either save her entire endowment or nothing at all. If $(1 - \varepsilon)R \geq 1$, she will save 1. Otherwise she will save 0. This yields the following utility from the period 0 perspective: $U_N = \max\{1, (1 - \varepsilon)R\}$.

Now we consider the investment opportunity. Since the per-period utility functions are linear, the agent would prefer not to borrow. Since $b > p$, the investment is always preferred. This gives us the optimal utility level:

$$U_O = \max\{b + 1 - p, b + R(1 - \varepsilon)(1 - p)\}$$

Now we look at the intra-personal equilibrium in the absence of banking. The agent in period 1 must decide whether to consume 1 or invest p and consume $1 - p$. She will choose to invest if: $p > \beta b$. Suppose this condition fails. Now consider a pure loan to induce investment (we assume that F is large enough that the agent will never default). Then, the agent in 1 will invest if investment dominates just repaying the loan. Agent 1's utility from just repaying is:

$$U_1^N = \begin{cases} l + 1 - Rl, & \text{if } l \leq \frac{1}{r} \\ l + 1 - \beta F, & \text{otherwise} \end{cases}$$

Agent 1's utility from investment is:

$$U_1^I \begin{cases} (l + 1 - p) + \beta(b - Rl), & \text{if } l < \frac{b}{1+r} \\ (l + 1 - p - (Rl - b)), & \text{if } \frac{b}{1+r} \leq l \leq \frac{1+b-p}{r} \\ l + 1 - p - \beta F, & \text{otherwise} \end{cases}$$

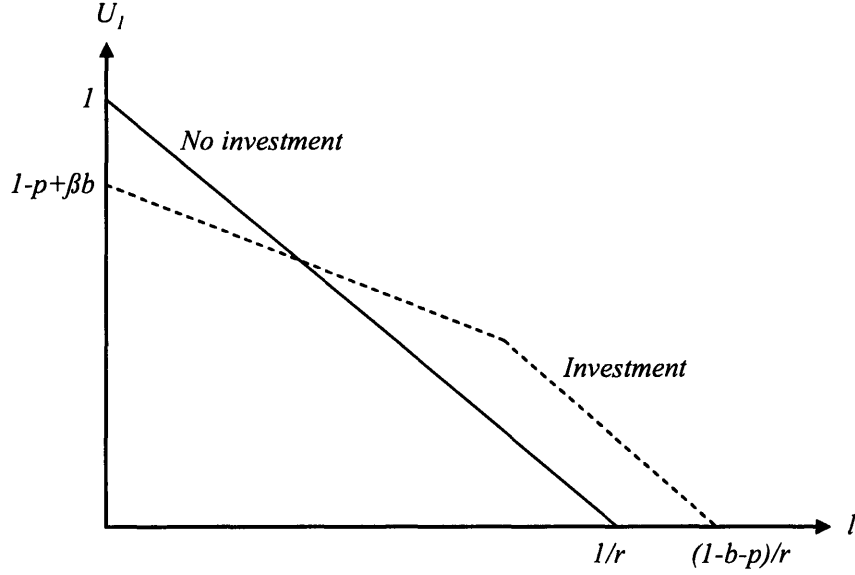


Figure 2-5: Agent 1's utility as a function of loan size.

Since $b > p$, there will always be a loan that can induce investment. The smallest possible loan size, l_A , is given by:

$$\begin{aligned}
 (l + 1 - p) + \beta(b - Rl) &\geq l + 1 - Rl \\
 \Rightarrow l_A &= \frac{p - \beta b}{R(1 - \beta)}
 \end{aligned}$$

This yields the following utility for agent 0: $U_L = b + \left(\frac{p - \beta b}{R(1 - \beta)} + 1 - p\right) - R\left(\frac{p - \beta b}{R(1 - \beta)}\right) < U_O$.

Now consider a loan l and savings of s_0 . We are interested in conditions under which this will raise agent 0's utility. Agent 0 will either choose to leave agent 1 with no consumption, or allow agent 1 to consume all leftover funds. Suppose she chooses to leave agent 1 with no extra consumption. Then $W_1 = 1 - s_0 + l = p$. Assume the agent in 0 will never take a loan so large that it cannot be repaid even under investment. Consider agent 1's utility from just repaying

(without investing):

$$U_1^{NS} = \begin{cases} p - Rl + \beta(1 - \varepsilon)Rs_0, & \text{if } l < \frac{p}{1+r} \\ p + \beta(1 - \varepsilon)R(s_0 - l) - \beta\varepsilon F, & \text{otherwise} \end{cases}$$

Her utility from repaying through investment is:

$$U_1^{IS} = \beta[b - Rl + (1 - \varepsilon)Rs_0]$$

We know that $\frac{p}{1+r} > \frac{p - \beta b}{R(1 - \beta)}$. This determines agent 0's optimal savings-loan combination:

$$l_B = \frac{p - \beta b}{R(1 - \beta)}; \quad s_B = 1 - p + l_B$$

If agent 0 prefers to allow the agent in 1 to consume the uninvested amount, she will not save any amount. Then, if the agent simultaneously saves and borrows, her utility in period 0 is: $b - Rl_B + (1 - \varepsilon)R(1 - p + l_B)$.

2.5.2 Implications

We are now in a position to predict the agent's optimal behavior based on exogenous parameter values: $\beta, b, p, R, \varepsilon$. The utility of the agent in period 0 from (a) no investment, (b) pure loan, and (c) loan and savings, is:

$$\begin{aligned} \text{(a)} & \max \{1, (1 - \varepsilon)R\} \\ \text{(b)} & b - R \left(\frac{p - \beta b}{R(1 - \beta)} \right) + \left(\frac{p - \beta b}{R(1 - \beta)} + 1 - p \right) \\ \text{(c)} & b - R \left(\frac{p - \beta b}{R(1 - \beta)} \right) + (1 - \varepsilon)R \left(\frac{p - \beta b}{R(1 - \beta)} + 1 - p \right) \end{aligned}$$

This gives us the following implications. First, the relative values of (b) and (c) depend entirely on $(1 - \varepsilon)R$. If ε goes down or R goes up, the agent is more likely to use simultaneous saving and borrowing (if she chooses to invest). If $(1 - \varepsilon)R < 1$, then the agent will either engage in no banking, or will invest using a pure loan. If $(1 - \varepsilon)R > 1$, then the agent will either save her entire wealth, or will use a combination of saving and borrowing to invest.

To find conditions under which the agent will invest at all, we compare (b) and (c) to (a). Interestingly, we find that, in all cases, if R rises, the agent is more likely to invest. This is because it is now easier for the agent to create incentives for future selves to save. Also, as we would expect, the agent has a greater incentive to invest as the investment gets cheaper relative to the benefits.

Finally, as long as the punishment for default is sufficiently large, the agent is more likely to invest as ε drops. We see, then, that the hyperbolic agent is able arrive arbitrarily close to the optimal outcome if there is a very small possibility that her savings will not mature. The actual value of ε would depend on the number of agents in this setting who were not susceptible to punishment for default.

2.6 Conclusion

We have attempted to solve a puzzle of simultaneous borrowing and saving by providing a new rationale for this phenomenon. When agents are sophisticated hyperbolic discounters, access to a non-secure source of saving can be useful—by creating the threat of a large punishment in the event of default, the agent can induce her future selves to invest. Actual utility loss in equilibrium is limited if the probability of default is low.

We have shown that, in this setting, simultaneous borrowing and saving cannot be optimal if agents have time-consistent preferences. We have also shown that, if savings are secure, an interest rate differential cannot explain this behavior. The agent is always better off when she simply borrows to fund investment. When there is a small chance that savings will disappear, the agent can find herself better off than if savings mature with certainty.

2.7 Appendix

2.7.1 Appendix A: Differential Interest Rates

Statement of Proposition 3: Suppose the agent can borrow at interest rate r ($R \equiv 1 + r$) and can save at interest rate t ($T = 1 + t$), where $r > t$. Then she will never save and borrow simultaneously.

Proof. The agent in 0 will either plan for the investment to be made, or not. If the investment is not made, clearly the optimal strategy is to save some amount s_0 such that:

$$u'(w - s_0) = u'(Ts_0)$$

In this case, no loan will be taken.

If the agent in 0 takes a loan, it must be to induce investment in period 1. Suppose the agent borrows $l > 0$ and saves $s_0 > 0$. It must be the case that the investment is made in period 1. The utility from period 0 perspective is:

$$u(w - s_0 + l - s_1) + u(b + s_1 - p + Ts_0 - Rl)$$

Now suppose we lower l and s_0 such that $\Delta s_0 = \frac{R}{T}\Delta l$. The period 1 incentive to save rises. Utility from the period 0 perspective must go up. ■

2.8 References

1. ASHRAF, N., GONS, N., KARLAN, D., and YIN, W. (2003), "A Review of Commitment Savings Products in Developing Countries", *Asian Development Bank Economics and Research Department Working Paper #45*.
2. ASHRAF, N., KARLAN, D., and YIN, W. (2005), "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines", *Quarterly Journal of Economics* (forthcoming).
3. BERTRAND, M., KARLAN, D., MULLAINATHAN, S., SHAFIR, E., and ZINMAN, J. (2005), "What's Psychology Worth? A Field Experiment in the Consumer Credit Market", (mimeo).
4. BERTRAND, M., MULLAINATHAN, S., and SHAFIR, E. (2004), "A Behavioral Economics View of Poverty", *American Economic Review Papers and Proceedings*, 419-423.
5. HARRIS, C. and LAIBSON, D. (2000), "Dynamic Choices of Hyperbolic Consumers", *Econometrica* (forthcoming).

6. KARLAN, D. (2005), "Social Connections and Group Banking", (mimeo: Yale University).
7. LAIBSON, D. (1997), "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics*, 62, 443-479.
8. LAIBSON, D., REPETTO, A., and TOBACMAN, J. (2001), "A Debt Puzzle", (mimeo).

Chapter 3

Time-Inconsistency in Informal Credit Markets: A Welfare Analysis

Summary 20 *We study the relationship between informal banking markets and the welfare of hyperbolic discounters. We consider three settings that are relevant to developing economies—monopolistic banking, commitment saving, and competitive banking. Welfare is defined as the discounted sum of utilities from a hypothetical "period 0" perspective. First, with a monopolist banker, agents fare better when lending contracts cannot be enforced. Second, a single moneylender can cause zero takeup of any sustainable commitment saving product. Nevertheless, the availability of commitment saving has the desirable effect of lowering the moneylender's interest rate. Third, in a competitive banking environment, agents borrow less than they would from a monopolist. The equilibrium loan size drops further if contracts are vulnerable to renegotiation. In either case, competition raises the hyperbolic agent's welfare relative to monopoly.*

3.1 Introduction

This paper is a study of the welfare implications of time-inconsistency in informal credit markets. While time-inconsistency (in various guises) is now widely invoked in the literature, the study of its role in developing economies has been a more recent phenomenon. Several empirical and experimental papers find evidence for behavioral biases and time-inconsistent preferences in the banking sectors of developing countries. However, the fact that agents are "behavioral" and

that markets respond to this does not in itself help inform us about welfare. In this paper, we focus on simple banking environments to try and understand when, in the presence of time-inconsistency, a banking service provider might exploit an agent's temptation at the expense of the agent's lifetime welfare. Our ultimate objective is to arrive at implications for policy and experiments.

While a major purpose of credit and saving is to smooth consumption and facilitate investment, it is also recognized as a source of commitment for time-inconsistent agents. Here, we abstract away from uncertainty and ask how credit markets can provide commitment in settings where contracting possibilities and banking facilities are limited. We consider three settings—monopolistic moneylending, paternalistic commitment saving, and competitive credit markets. We study how the level of competition as well as the level of contracting ability in a market can affect welfare. We find that when there is a monopolist banker, an expansion in the set of available contracts can lower welfare. We also try to answer the question of why, in practice, there is a gap between the desire for commitment saving and the provision of it. We show that the presence of a single moneylender can result in zero takeup of well-meaning commitment savings. However, this should not be viewed as a failure of the commitment savings product—the availability of it can result in lower interest rates charged by the moneylender, thus improving welfare. Finally, we also consider competitive credit markets. We show that while competition creates pressure to return surplus to the client, it also exposes contracts to renegotiation. While the list of settings studied here is not exhaustive, we hope they capture some aspects of informal banking in developing countries. In this paper, we avoid what are perhaps the most commonly studied forms of informal banking—microcredit and microfinance. While there have been several stories of success and failure in both areas (see, for example, Morduch, 1999 and 2000), our main focus here is on those environments where loan repayment is harder to enforce.

The question of welfare under hyperbolic discounting is not well resolved. Given that an agent's intertemporal preferences vary during his lifetime, useful welfare criteria tend to privilege an agent of a particular age. One approach would be to simply take the preferences of the agent at the start of his life as his true preferences, and compare lifetime outcomes from his perspective. However, this is unsatisfactory since it (somewhat arbitrarily) legitimizes early

myopia while rejecting the same preferences later in life. In this paper, we follow O'Donoghue and Rabin (1999) in assuming that an agent's true lifetime welfare is as evaluated just before he actually starts making decisions, in some hypothetical "period 0" of his life. This can be interpreted as, say, the preferences parents have over their children's lives. This approach is also reasonable if we are interested in thinking about how people might vote on future changes in policy such as new banking structures and new forms of contract enforcement.

The paper is split into the following sections: literature review, outline of arguments, modeling assumptions, benchmark analysis, monopolist banker, commitment savings, competitive banking, and conclusions.

3.2 Related Literature

This paper relates to two broad areas in the literature on time-inconsistent preferences.

First, several models of saving and credit under time-inconsistency highlight an agent's need for commitment and tendency to overborrow. For example, Laibson (1997) and Harris and Laibson (2000) have extensively studied lifetime saving under hyperbolic discounting. Krusell and Smith (2003) solve a Ramsey-style model when agents are time-inconsistent. O'Donoghue and Rabin (1999) focus on an individual's tendency to procrastinate when faced with costly (but desirable) tasks. Kocherlakota (1996) develops a notion of renegotiation-proofness under hyperbolic discounting.

Second, the development literature on microcredit is increasingly interested in commitment. Ashraf, Gons, Karlan, and Yin (2003) provide a survey of commitment savings products and related research. Ashraf, Karlan, and Yin (2005) find, in a field experiment, that agents most interested in commitment savings devices are those who face relatively greater time inconsistency in their preferences and are aware of it. Among other papers, Dagnelie and LeMay (2005), Gugerty (2005), and Thaler and Bernartzi (2004) provide empirical evidence of the value of commitment in a range of informal financial settings.

3.3 Outline of Arguments

We assume an agent lives for three periods, and must decide whether to consume a nondivisible good. This good is expensive and cannot be purchased unless the agent saves (or borrows). The agent is a sophisticated quasi-hyperbolic discounter—he has time-inconsistent preferences and knows that his future selves do too. The cases of interest are ones where he values the nondivisible good but is tempted to defer the savings burden to his future selves. We evaluate the agent’s welfare from the perspective of a hypothetical “period 0;” i.e. assuming he was an exponential rather than a quasi-hyperbolic discounter. We assume that the banker does not suffer from a time-inconsistency problem—his objective is to maximize his wealth as measured at the end of period 3.

There are two key points that drive the results in this paper. First, when an agent saves, less surplus can be extracted from him than when he borrows. This is because, when an agent borrows, he is much more willing to pay in terms of future utility for increased consumption today. Second, while competitive markets give more of the surplus to the consumer, the total available surplus might be lower than in monopolistic markets. This derives from the fact that, unlike a competitive firm, a monopolist can credibly promise not to renegotiate.

We first study a benchmark case with an exponential discounter in the absence of banking. This allows us to establish conditions under which the nondivisible good is valuable (as opposed to merely being a temptation good), and to describe the agent’s optimal behavior from the period 1 perspective. Then we turn our attention to the quasi-hyperbolic discounter. This agent would like his future selves to save for the good, but is reluctant to sacrifice current consumption. This gives us a parameter region in which the good is valuable but is never purchased in subgame perfect equilibrium.

We then turn to banking. First, we consider the case of a monopolist banker. There are two sub-cases here—one where no lending contracts can be enforced, and ones where both saving and lending contracts are available. When the monopolist cannot lend, he must choose whether to offer saving (or no service at all). With a pure saving contract, the change in the agent’s welfare is ambiguous. We then show that the agent’s welfare drops further when the ability to contract on lending increases. The intuition for this result survives even when there is no nondivisible good—if the banker can lend to the agent, then he is able to lend at a rate that is

particularly disadvantageous to the agent's future selves, thus lowering the borrower's lifetime welfare. However, we see that the presence of the nondivisible good can exacerbate the welfare loss. The good effectively operates as a temptation good by making the agent willing to sacrifice even more for the utility benefit today.

In the succeeding section, we continue to assume the presence of a single banker (who can offer both saving and debt), but allow for the entry of an NGO that provides commitment saving (here we make the assumption that NGOs often lack the information necessary to provide loans). We show that, even if the NGO provides saving at the highest sustainable interest rate, there might be no takeup of the product. Since, all else equal, the hyperbolic discounter prefers credit to saving, the monopolist banker can simply lower the interest rate enough to still induce the agent to purchase his product rather than the NGO's. However, the fact that the banker lowers his interest rate means that the presence of an NGO can have an indirect, positive effect on welfare even if no agent actually takes advantage of the commitment saving offer.

Finally we look at competitive credit markets, and take more seriously the possibility of renegotiation through secondary markets.

3.4 Assumptions

An agent lives for 3 periods, $i \in \{1, 2, 3\}$. He has a non-stochastic income in each period, which we normalize to 1. His per-period utility function, $u(c)$, is strictly concave and differentiable, with $u'(0) = \infty$. The agent can either consume his income directly or purchase a nondivisible good (only one such good can be purchased over a lifetime). This good has a price of p and yields a monetary benefit of b , where $3 \geq p \geq 2$ and $b > p$.

The agent is a sophisticated quasi-hyperbolic discounter. In any period i , he evaluates his lifetime utility as:

$$V_i = u_i + \beta \sum_{j=i+1}^3 u_j$$

where $0 < \beta \leq 1$. We implicitly assume there is no other discounting. Then, his utility from his period 0 perspective, which we describe as his welfare, is given by:

$$W = u_1 + u_2 + u_3$$

Any banker or NGO in this setting is constrained by the extent of contract enforceability. In our model of a monopolistic banker, he is assumed to have a large amount of liquid wealth that he is willing to loan. His objective is to maximize his wealth at the end of period 3. We assume that NGOs and competitive banks have access to external credit markets where they can save and from which he can borrow at some interest rate $r \geq 0$. While an NGO's objective is to maximize the agent's welfare, a competitive bank (like the monopolist) wishes to maximize its wealth at the end of period 3.

3.5 Benchmark Case: No Banking

In this section, we first find the agent's optimal behavior assuming $\beta = 1$, and then characterize the intra-personal equilibrium for $\beta < 1$.

3.5.1 Non-Hyperbolic Agent

When $\beta = 1$, the agent's knows that his optimal plan in period 1 will be carried out by his future selves. If he did not save, he would simply consume his income in each period:

$$W = V_1 = 3u(1)$$

If he saves for the nondivisible, he will evenly share the cost in periods 1 and 2 (since he cannot reduce his period 3 consumption below b):

$$W = V_1 = 2u\left(1 - \frac{p-1}{2}\right) + u(b)$$

The agent will choose to start saving in period 1 if:

$$u(b) \geq 3u(1) - 2u\left(1 - \frac{p-1}{2}\right) \tag{3.1}$$

We assume from now on that Condition 3.1 holds.

3.5.2 Hyperbolic Agent

Hyperbolic Optimum

First, we consider the agent's optimal behavior from two perspectives. From the period 0 perspective, welfare is maximized when behavior matches that of the non-hyperbolic agent described above. Consider now the optimal from the period 1 perspective. If he does not save for the nondivisible, his utility is:

$$V_1 = u(1) + 2\beta u(1)$$

If he saves, he would like to save according to the following maximization problem:

$$\hat{s}_1 = \arg \max_{s_1 \geq 0} [u(1 - s_1) + \beta u(2 - p + s_1)]$$

We assume from here on that even the hyperbolic agent prefers to save:

$$u(1 - \hat{s}_1) + \beta u(2 - p + \hat{s}_1) + \beta u(b) \geq u(1) + 2\beta u(1) \quad (3.2)$$

Condition 3.2 is stronger than Condition 3.1.

To predict actual behavior, we treat the individual as three independent time-indexed agents and use backward induction to solve for the Subgame Perfect equilibrium. In periods 1 and 2, the agent makes a decision of how much money, s_i , to send on to the next period. This decision is a function of the current wealth, $s_{i-1} + 1$.

Period 3

Since, by assumption, $b > p$, the period 3 decision is straightforward. If $s_2 \geq p - 1$, then he will purchase the good, yielding $V_3 = u(1 + s_2 - p + b)$. If $s_2 < p - 1$, then the good is unaffordable. Here, $V_3 = u(1 + s_2)$.

Period 2

We take s_1 as given and evaluate the decisions made by the agent in period 2. First, suppose there was no option of a nondivisible good. Then he would choose s_2 to maximize V_2 :

$$V_2 = \max_{\bar{s}_2 \geq 0} [u(1 + s_1 - \bar{s}_2) + \beta u(1 + \bar{s}_2)]$$

Now we reintroduce the possibility of the nondivisible. If saves for the nondivisible, his utility is given by:

$$V_2 = \max_{\hat{s}_2 \geq p} [u(2 + s_1 - \hat{s}_2) + \beta u(b + \hat{s}_2 + 1 - p)]$$

He will save for the nondivisible only if the following condition holds:

$$u(2 + s_1 - \hat{s}_2) + \beta u(b + \hat{s}_2 + 1 - p) \geq u(1 + s_1 - \bar{s}_2) + \beta u(1 + \bar{s}_2) \quad (3.3)$$

Proposition 21 *There is some $0 < s_1^{\min} < 1$ such that, for all $s_1 \geq s_1^{\min}$, the nondivisible will be purchased in equilibrium.*

Proof. Since $b - p > 1$, we know that at any s_1 , the optimal \bar{s}_2 must be lower than the optimal \hat{s}_2 . By the envelope theorem, when s_1 goes up, the RHS increases at the rate $u'(1 + s_1 - \bar{s}_2)$ and the LHS increases at a higher rate, $u'(2 + s_1 - \hat{s}_2)$. Therefore, if at any s_1 the LHS > RHS, this inequality will continue to hold for all higher values of s_1 . ■

Period 1

A necessary condition for the agent to save at all is that the nondivisible be purchased. Given our assumptions, we also know that the agent will never save an amount that creates a period 2 incentive to save $s_2 > p$. Then, there must be some s_1^{\max} that creates an upper bound on his willingness to save in period 1, which is given by:

$$u(1 - s_1^{\max}) + \beta u(2 + s_1^{\max} - p) + \beta u(b) = u(1) + 2\beta u(1) \quad (3.4)$$

Proposition 22 *There is a parameter region in which the nondivisible is desirable to the hyperbolic discounter (Condition 3.2 is satisfied), but $s_1^{\max} < s_1^{\min}$, where s_1^{\max} is determined by*

Condition 3.4 and s_1^{\min} is determined by Condition 3.3.

The proof of this proposition can be given by example. The special case studied in the appendix satisfies the conditions described in the statement of the proposition.

This is the parameter region we are most interested in. It suggests the agent's willingness to save for the nondivisible if he has access to the appropriate commitment technologies. Saving could improve his welfare both from her period 1 and period 0 perspectives. We now consider the role of banking services.

3.6 Monopolist Banker

We assume there is only one banker in a particular region. His objective, then, is to offer a contract that maximizes the period 1 agent's surplus, and then extract all of it.

3.6.1 Limited Contract Enforceability

This is the case where the banker does not have the capacity to enforce loan repayments. He can, however, specify savings contracts of the form $\{t_1(s_a), t_2(s_a - t_1, s_b)\}$, where $t_1, t_2 \geq 0$ are payments made to the agent if he saves s_a and s_b in periods 1 and 2 respectively. The banker will provide pure commitment saving—by offering the agent in period 1 the option of making his savings illiquid, he provides him with an opportunity to purchase the nondivisible.

Banker's Optimal Contract

There are three autarky cases that we need to consider: (a) the nondivisible is not purchased, (b) the nondivisible is purchased but not optimally from the period 1 perspective, and (c) the nondivisible is purchased and the period 1 optimal plan is implemented. In case (c), the agent is not willing to pay for any saving service. In cases (a) and (b), however, the banker will offer commitment saving. The access to illiquidity can improve the agent's saving pattern, thus raising surplus for the period 1 agent, which can then be extracted by the banker. Since the only incentive to save is for the nondivisible, the banker will offer a contract which involves saving s_a and s_b in periods 1 and 2 respectively, with a yield of $p - 1$, just enough for the

nondivisible to be purchased in period 3. The banker will solve:

$$\begin{aligned} & \max_{s_a, s_b} [s_a + s_b - (p - 1)] \\ \text{s.t.} \quad & u(1 - s_a) + \beta u(1 - s_b) + \beta u(b) \geq V_1^{\text{autarky}} \end{aligned} \quad (3.5a)$$

$$u(1 - s_B) + \beta u(b) \geq u(1) + \beta u(1) \quad (3.5b)$$

We interpret Constraint 3.5a as an individual rationality constraint—the agent in period 1 must be at least as well off as he would be in autarky. Constraint 3.5b is the incentive compatibility constraint—the agent in period 2 must actually be willing to save the remaining amount towards the nondivisible.

The solution to the above maximization problem will result in the following offer to the agent: if he saves s_a and s_b in periods 1 and 2 respectively, he will get $p - 1$ in period 3. For any other amounts saved, he will get 0 in period 3. In terms of our earlier notation, $s_a = s_1$ and $s_b + s_a = s_2$. The reason such a contract can improve performance over autarky is that, by making the saving illiquid, the temptation for the agent in period 2 to overconsume rather than save is dampened (s_1^{\min} is goes down). Note that the agent is never actually being forced to make a payment. The incentive to do so comes directly from the modified tradeoffs that emerge under illiquidity.

What will the actual optimum look like? Suppose IR is the binding constraint. Then s_a and s_b will be chosen to equalize $u'(1 - s_a)$ and $\beta u'(1 - s_b)$. But at this point, IC might fail—the agent in period 2 might be willing to sacrifice locked savings in order to consume her income today. Then, the maximal s_b is determined by IC, and then the highest possible s_a is selected based on IR.

Agent's Welfare

Regardless of which constraint ultimately binds, we can place an upper bound on the utility from period 1 perspective: the agent will never have a higher utility than his optimal savings path gives him. Also, by definition, he will be no worse off than in autarky.

True welfare, however, might be lower than in autarky. Consider case (b) in the previous subsection. If the agent in period 1 was unsatisfied with the saving pattern in autarky, it might

have been the result of an excessively high (from his perspective) s_1^{\min} . Access to a bank allows him to save less in period 1 and more in period 2. This might lower true welfare if the autarky savings path was more equally balanced between periods 1 and 2.

However, in case (a), when there is no saving in autarky, monopolistic commitment saving can only increase welfare. Since IC is satisfied:

$$\begin{aligned} u(1 - s_B) + \beta u(b) &\geq u(1) + \beta u(1) \\ \Rightarrow u(1 - s_B) + u(b) &> u(1) + u(1) \end{aligned}$$

because $u(b) > u(1)$. Then, since IR is satisfied:

$$\begin{aligned} u(1 - s_a) + \beta[u(1 - s_b) + u(b)] &\geq u(1) + \beta[u(1) + u(1)] \\ \Rightarrow u(1 - s_a) + [u(1 - s_b) + u(b)] &\geq u(1) + [u(1) + u(1)] \end{aligned}$$

We see in the next section that, even if commitment saving lowers welfare relative to autarky, the outcome under monopolistic credit is always worse for the agent. Under commitment saving, the agent making the banking decision is giving up current consumption to fund future consumption. Since he has present-biased preferences, he is reluctant to make this trade-off, which constrains the amount of surplus than can be extracted by the banker.

3.6.2 Full Contract Enforceability

Now the banker can also offer loans. First, we show that the banker can always do better with a loan than with a savings contract. Consider the optimal saving contract in the previous section. Now suppose the banker offers a loan of $p - 1$ in period 1 with repayments s_a and s_b in periods 2 and 3. The agent strictly prefers the loan contract and is willing to pay more for it.

Banker's Optimal Contract

A loan contract is one where the agent is offered a loan l , with repayment s_b and s_c in periods 2 and 3 respectively. Assume that, except in the case of bankruptcy, the client's repayment is enforceable by the bank. We assume that, when a contract is signed, the bank can credibly

commit not to renegotiate.

First suppose there was no nondivisible. The banker's objective is:

$$\begin{aligned} & \max_{l, s_b, s_c} [s_b + s_c - l] \\ \text{s.t. } & u(1+l) + \beta u(1-s_b) + \beta u(1-s_c) \geq V_1^{\text{autarky}} \end{aligned} \quad (3.6)$$

The first-order conditions give us the following: First, $s_b = s_c \equiv s$. Since the agent would like to equalize marginal utilities across all future periods, an equal saving plan will maximize the amount the banker can charge for the loan. Second, $\frac{u'(1+l)}{u'(1+s)} = \beta$. Since the agent is a hyperbolic discounter, he will choose a lower marginal utility in period 1 than in future periods. Third, the agent must be made indifferent between the contract and autarky (Constraint 3.6).

The constraints are illustrated in the diagram below. The maximum repayment that the banker can extract for any loan l is given by the indifference constraint. The banker's profits are maximized at the greatest difference between the constraint and the $s = \frac{l}{2}$ line. This gives some optimal loan l^* and repayment s^* .

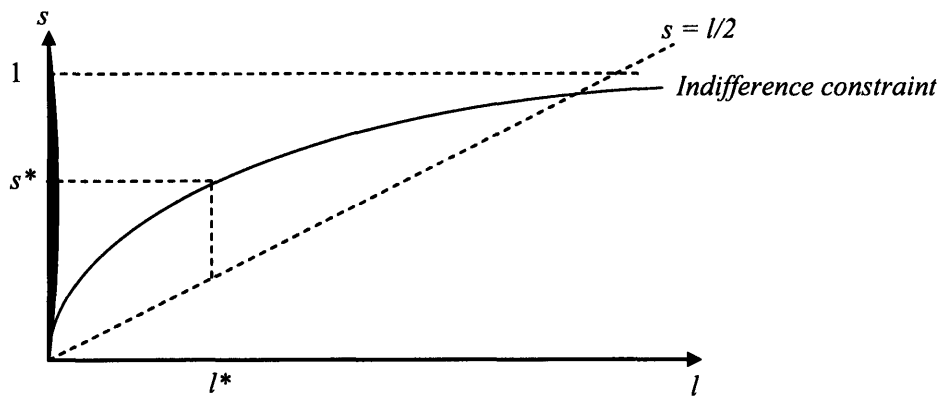


Figure 3-1: For any loan size l , the indifference constraint shows the maximum repayment, s , than can be charged.

Now we reintroduce the nondivisible. The banker's maximization problem is:

$$\begin{aligned} & \max_{l,s} [2s - l] \\ & \text{s.t. } V(l, s) \geq V_1^{\text{autarky}} \\ \text{where } V(l, s) = & \begin{cases} u(1+l) + 2\beta u(1-s), & \text{if } l < p-1 \\ u(b + (l+1-p)) + 2\beta u(1-s), & \text{if } l \geq p-1 \end{cases} \end{aligned}$$

This creates a discontinuity in the indifference constraint, as shown in the diagram below (the thick, discontinuous line is the new constraint). Now, the loan size might be smaller than before, but can be accompanied by a higher repayment.

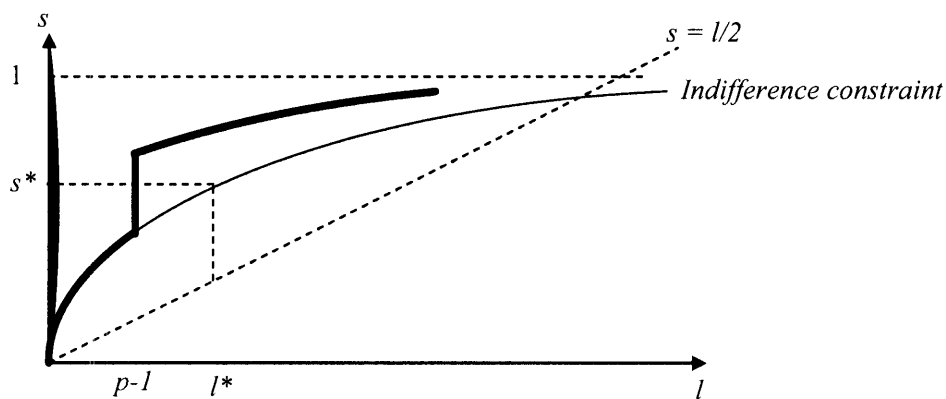


Figure 3-2: The thick line is the new indifference constraint under the presence of the nondivisible good.

Agent's Welfare

Proposition 23 *When a bank is able to enforce loan contracts, the agent's welfare is strictly lower than when the bank can only offer commitment saving.*

Proof. We have shown in the previous section that, under commitment saving, the change in welfare is ambiguous. Consider the two cases: (a) welfare under saving is higher than in autarky, and (b) welfare under saving is lower than in autarky.

(a) It is sufficient to show that welfare under credit is always lower than in autarky. Let the consumptions under autarky and credit be $\{c_1, c_2, c_3\}$ and $\{\bar{c}_1, \bar{c}_2, \bar{c}_3\}$ respectively. By the indifference constraint: $u(c_1) + \beta[u(c_2) + u(c_3)] = u(\bar{c}_1) + \beta[u(\bar{c}_2) + u(\bar{c}_3)]$. Since $c_1 > \bar{c}_1$, $u(c_1) > u(\bar{c}_1)$, and $\beta < 1$, it must be that $u(c_1) + [u(c_2) + u(c_3)] > u(\bar{c}_1) + [u(\bar{c}_2) + u(\bar{c}_3)]$.

(b) Let the consumption under commitment saving be $\{\hat{c}_1, \hat{c}_2, \hat{c}_3\}$. In this case, $u(c_1) + [u(c_2) + u(c_3)] > u(\hat{c}_1) + [u(\hat{c}_2) + u(\hat{c}_3)]$ and $u(c_1) + \beta[u(c_2) + u(c_3)] \leq u(\hat{c}_1) + \beta[u(\hat{c}_2) + u(\hat{c}_3)]$. Since $\bar{c}_1 > \hat{c}_1$, and given the indifference constraint, it must be the case that $u(c_1) + [u(c_2) + u(c_3)] > u(\bar{c}_1) + [u(\bar{c}_2) + u(\bar{c}_3)]$. ■

It is notable here that access to the nondivisible does not directly figure into the above proof. This is because the banker is always able to write a contract that leaves the period 1 agent no better off than in autarky. The nondivisible good, instead of improving the agent's utility, now functions more as a temptation good—by allowing the agent to consume it in period 1, the banker is able to take more of his future income. The graph below depicts the relationship between period 1 consumption and welfare under a monopolistic credit regime.

If the presence of the nondivisible leads to higher total consumption in period 1, then the agent's welfare has been lowered by the presence of the nondivisible.

3.7 Paternalistic Commitment Saving

We now consider commitment saving. Suppose there is an NGO that has access to external banking at some interest rate r . We assume the NGO lacks the resources necessary to enforce loan repayment, so it can only offer commitment saving. Its goal is to maximize the agent's welfare while balancing its own budget. For convenience, in this section we assume $r = 0$. The NGO solves:

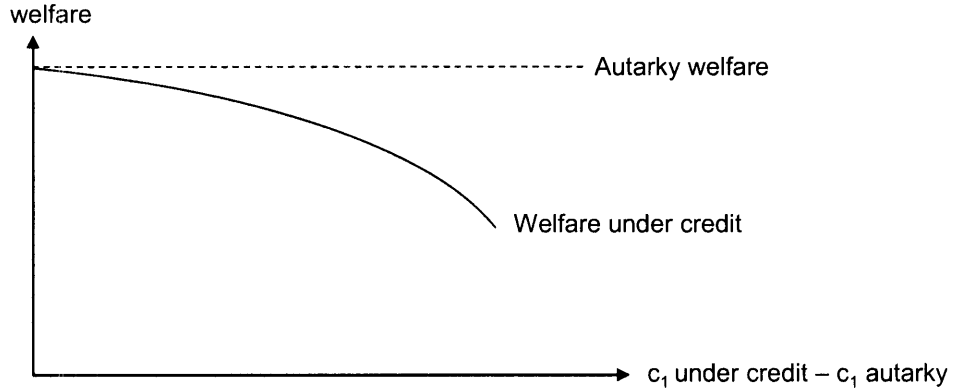


Figure 3-3: As period 1 consumption rises under banking, the agent's welfare drops relative to autarky.

$$\begin{aligned} & \max_{s_b} [u(2 - p + s_b) + u(1 - s_b) + u(b)] \\ \text{s.t. } & u(2 - p + s_b) + \beta u(1 - s_b) + \beta u(b) \geq V_1^{\text{autarky}} \\ & u(1 - s_b) + \beta u(b) \geq u(1) + \beta u(1) \end{aligned}$$

This yields some period 1 saving $s_a = s_b + 1 - p$ and s_b . If they are both between 0 and 1, the NGO can offer a contract that implements exactly this saving path. By offering illiquidity, the agent in period 2 has a greater incentive to save. This raises both welfare and utility from the period 1 perspective.

3.7.1 NGO and Monopolist Moneylender

On the face of it, sustainable commitment saving seems like a good idea. Given that it is even attractive to the agent who must make the decision to join (and given the evidence that there is actual demand for it), the dearth of commitment savings programs around the world is somewhat surprising. In this section, we provide a possible reason for the failure of commitment saving. The presence of a monopolistic moneylender can, for two reasons, drive agents away from commitment saving.

Period 2 Renegotiation

The first problem is that of renegotiation. Consider an agent who has taken up the commitment saving product. The only advantage (over autarky) that this product offers is illiquidity, which leads the period 2 agent to behave differently than he would otherwise. However, consider the decision of the agent in period 2. He is willing to pay the monopolist banker to make his savings from the previous period liquid. The banker can, in effect, make all illiquid savings liquid, rendering commitment savings worthless. The problem of renegotiation is discussed more formally in the section on competition and secondary markets.

It might be possible for the NGO to overcome this problem, either by making final payments in terms of a good that cannot be resold, or by making it difficult for the agent to verify that he has illiquid assets. However, we see in the following section that a fundamental problem in period 1 remains.

Period 1 Problem

Consider any contract offered by the NGO. It involves saving $s_a = s_b + 1 - p$ in period 1 and s_b in period 2, with a return of $p - 1$ in period 3. The banker can always provide a contract that is strictly preferred by the agent in period 1 (for example, a contract in which the agent gets $p - 1$ in period 1 and pays s_a and s_b as above in periods 2 and 3). Therefore, if there is a moneylender, the agent will never take up commitment saving.

However, the presence of commitment saving, by changing the outside option, can nevertheless raise welfare by limiting the banker's ability to extract surplus. Consider Constraint 3.6. Commitment saving raises the right hand side of the constraint:

$$u(1 + l) + \beta u(1 - s_b) + \beta u(1 - s_c) \geq V_1^{commitment} \geq V_1^{autarky}$$

Therefore, the banker must reduce s to induce the agent to take up a credit contract. Then, even though commitment saving fails in terms of take-up, it is successful in lowering the interest rate charged by the moneylender and consequently raising the agent's welfare.

The above analysis has taken place entirely under the assumption of a low external interest rate. If r is sufficiently high (and returns are directly given to the agent), then commitment

saving might be able to defeat the moneylender in both periods. However, we have seen above that this is impossible if r is low.

3.8 Competition and Secondary Markets

In this section we assume a competitive moneylending environment. There are several moneylenders who have access to funds at $r = 0$.

3.8.1 Equilibrium Contracts

In competitive equilibrium every contract, $\{l, s_b, s_c\}$, offered by a bank must satisfy $l = s_b + s_c$. Suppose the banks could offer contracts in period 1 with no risk of renegotiation in period 2. The contract offered would maximize the agent's period 1 utility:

$$\begin{aligned} & \max_{l, s_b, s_c} [u(1 + f(l)) + \beta u(1 - s_b) + \beta u(1 - s_c)] \\ \text{s.t. } & l = s_b + s_c \\ \text{where } f(l) &= \begin{cases} l, & \text{if } l < p - 1 \\ b + (l + 1 - p), & \text{if } l \geq p - 1 \end{cases} \end{aligned}$$

The first order conditions of this problem pin down $s_b = s_c = \frac{l}{2}$, with the optimal loan size being $l = p$, or an interior solution that satisfies $\frac{u'(1+f(l))}{u'(1-\frac{l}{2})} = \beta$. This means that the equilibrium loan size under competition is (weakly) smaller than under a monopolist moneylender.

However, in a competitive market the possibility of renegotiation must be taken more seriously. Any contract that is a solution to the problem above is vulnerable to renegotiation—a banker has an incentive to offer the period 2 agent a new debt repayment plan that transfers more of the burden to period 3. If this is not preventable, a renegotiation-proof period 1 contract must satisfy an additional constraint:

$$\begin{aligned} & \max_{l, s_b, s_c} [u(1 + f(l)) + \beta u(1 - s_b) + \beta u(1 - s_c)] \\ \text{s.t. } & l = s_b + s_c \\ & u'(1 - s_b) = \beta u'(1 - s_c) \end{aligned}$$

The first-order conditions for this problem are $\frac{u'(1-s_b)}{u'(1-s_c)} = \beta$ and a loan size $l = p$ or an interior

solution that satisfies $\frac{u'(1+f(l))}{u'(1-s_b)} = \frac{u'(1+f(l))}{\beta u'(1-s_c)} = \frac{\beta+1}{2} > \beta$. The equilibrium loan size will now be smaller than above, since the marginal repayment cost of every additional loan unit is higher.

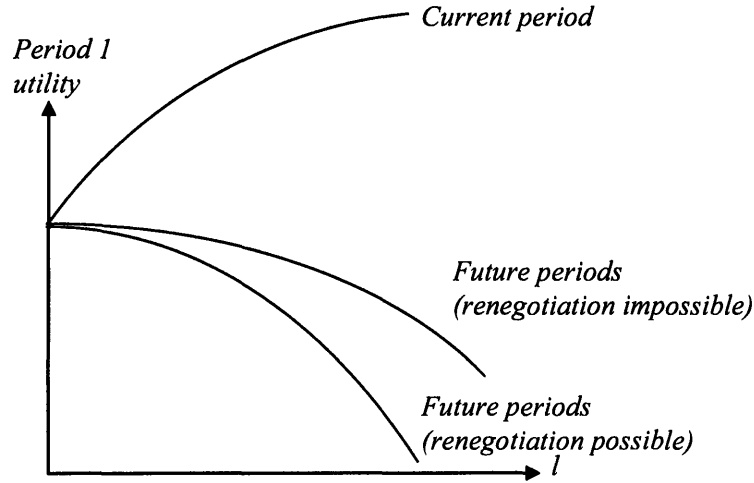


Figure 3-4: Comparison of utility in current and future periods as a function of the loan size.

3.8.2 Welfare

Proposition 24 *Under competition, the agent's welfare is always higher than under a monopolist moneylender.*

Proof. Let the consumption paths under monopoly, competition with renegotiation problem, and competition without renegotiation problem be $\{c_1, c_2, c_3\}$, $\{\bar{c}_1, \bar{c}_2, \bar{c}_3\}$ and $\{\hat{c}_1, \hat{c}_2, \hat{c}_3\}$ respectively. We have shown above that $c_1 > \hat{c}_1 > \bar{c}_1$. By the indifference constraint, $u(c_1) + \beta[u(c_2) + u(c_3)] = V_1^{autarky}$. Since $u(\bar{c}_1) < u(c_1)$ and $u(\bar{c}_1) + \beta[u(\bar{c}_2) + u(\bar{c}_3)] \geq V_1^{autarky}$ and $\beta < 1$, it must be the case that $u(c_1) + [u(c_2) + u(c_3)] < u(\bar{c}_1) + [u(\bar{c}_2) + u(\bar{c}_3)]$. The same argument can be made for $\{\hat{c}_1, \hat{c}_2, \hat{c}_3\}$ in place of $\{\bar{c}_1, \bar{c}_2, \bar{c}_3\}$. ■

3.9 Conclusion

In this paper, we have attempted to characterize banking equilibria and assess their welfare impacts under three different kinds of banking regimes. We focus solely on hyperbolic discounters. We show that when the only banking services are provided by a monopolist (and there is no uncertainty), agents are better off if the banker is not able to enforce loan contracts. The presence of a monopolist moneylender can also drive out sustainable commitment saving programs. However, we argue that zero takeup of commitment saving should not be reason enough for such programs to be discontinued—by forcing the moneylender to lower his interest rates, they can help raise the welfare of hyperbolic agents. Finally, we characterize the problem of renegotiation in secondary markets that might affect competitive banking environments. We show that agents borrow less under competition than under a monopoly, and further reduce their loans if contracts have to be safeguarded against renegotiation.

This suggests several areas for continued research. It would be useful to study settings with heterogenous agents (in terms of both preferences and risk) and consider the ability of monopolist bankers to price discriminate. In addition, banking contracts might be affected if the banker can also contract with the agent in other economic spheres. There is also room for analyzing in greater detail the role of external interest rates on equilibria in informal banking markets. Finally, there is probably much to be learned from a longer-horizon analysis, which includes the accumulation of wealth across generations as well as the banking decisions of individuals who face more distant deadlines than we have studied above. In particular, it seems that the problem of renegotiation in competitive markets might be particularly severe in an infinite horizon setting. If several firms compete to provide commitment saving, for example, there might be an over-provision of this service. Agents who, if compelled, would have taken up commitment saving very early, might now procrastinate indefinitely.

3.10 Appendix

3.10.1 Appendix A: Example with Log Utility

Suppose $u(c) = \ln c$. Additionally, we assume $p = 2$ and $\beta = \frac{1}{2}$. For convenience, denote $u(b) = B$.

First, we look at what an exponential discounter would do. If he does not consume the nondivisible, he consumes his income each period: $V_1 = W = 3u(1) = 0$. If he consumes the nondivisible, he shares the cost evenly. Welfare is $2u\left(1 - \frac{p-1}{2}\right) + u(b) = 2\ln\left(\frac{3-p}{2}\right) + B$. Assume it is worth consuming the nondivisible: $B \geq -2\ln\left(\frac{3-p}{2}\right) = 1.39$ (which means $b \geq 4$).

Now, consider the hyperbolic discounter. If he does not consume the nondivisible, he consumes his income in each period: $V_1 = W = u(1) + 2\beta u(1) = 0$. If he were to consume the nondivisible, his optimal saving path would be the following: save s_1 and s_2 (cumulative) such that: $s_2 = p - 1$. First-order conditions give us $s_1 = \max\left\{\frac{p+\beta-2}{1+\beta}, 0\right\}$ ($s_1 = \frac{1}{3}$). Assuming an interior solution, this would give: $V_1 = \ln\left(\frac{1-p+2}{1+\beta}\right) + \beta \ln\left(\frac{2\beta-p\beta}{1+\beta}\right) + \beta B$ ($V_1 = -.41 - .55 + \frac{B}{2} = \frac{B-1.92}{2}$), and $W = \ln\left(\frac{1-p+2}{1+\beta}\right) + \ln\left(\frac{2\beta-p\beta}{1+\beta}\right) + B < 2\ln\left(\frac{3-p}{2}\right) + B$. Still, let us assume that the agent prefers to save for the nondivisible than not: $\ln\left(\frac{1-p+2}{1+\beta}\right) + \beta \ln\left(\frac{2\beta-p\beta}{1+\beta}\right) + \beta B \geq 0$ (i.e. $b \geq 6.8$).

To find the equilibrium, we first look at period 2 behavior in the absence of a nondivisible. He solves: $\max_{s_2} [u(1 + s_1 - s_2) + \beta u(1 + s_2)]$. This gives FOC: $\frac{1}{1+s_1-s_2} = \frac{\beta}{1+s_2} \Rightarrow s_2 = \max\left\{\frac{\beta-1+\beta s_1}{1+\beta}, 0\right\}$. His utility is: If $s_1 \geq \frac{1-\beta}{\beta}$, then $V_2 = \ln\left(\frac{2+s_1}{1+\beta}\right) + \beta \ln\left(\frac{2\beta+\beta s_1}{1+\beta}\right)$; If $s_1 < \frac{1-\beta}{\beta}$, then $V_2 = \ln(1 + s_1)$. If instead he saves for the nondivisible, he gets: $V_2 = u(2 + s_1 - p) + \beta B$ ($= \ln s_1 + \frac{B}{2}$) compare this to autarky ($\ln(1 + s_1)$). This determines a value of s_1^* above which the agent is willing to save for the nondivisible (to get s_1^* : $\ln\left(\frac{1}{s_1} + 1\right) = \frac{B}{2}$; $s_1^* = \frac{1}{e^{\frac{B}{2}} - 1}$). In period 1, the agent's only incentive to save is for the nondivisible. Let's assume that, if he saves, the agent in period 2 will in fact save for the good. Agent in 1 is willing to save if, at s_1^* , he is better off than not saving: $u(1 - s_1^*) + \beta u(2 + s_1^* - p) + \beta B \geq 0$. If this condition holds, then nondivisible will be bought. If s_1^* is less than the agent's optimal saving, then final outcome will be optimal from period 1 perspective. But if condition fails, then the nondivisible will not be bought, even though it is optimal (in terms of total welfare, and in terms of agent 1's optimization) to do so. This condition fails if $B < 2.13 \Rightarrow b < 8.4$

3.11 References

1. ARMENDARIZ, B., and MORDUCH, J. (2004), "Microfinance: Where do we Stand?", in Goodheart, C. (ed.), *Financial Development and Economic Growth: Explaining the Links*, Palgrave Macmillan, Basingstoke, Hampshire, UK.

2. ASHRAF, N., GONS, N., KARLAN, D., and YIN, W. (2003), "A Review of Commitment Savings Products in Developing Countries", *Asian Development Bank Economics and Research Department Working Paper #45*.
3. ASHRAF, N., KARLAN, D., and YIN, W. (2005), "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines", *Quarterly Journal of Economics* (forthcoming).
4. BERTRAND, M., KARLAN, D., MULLAINATHAN, S., SHAFIR, E., and ZINMAN, J. (2005), "What's Psychology Worth? A Field Experiment in the Consumer Credit Market", (mimeo).
5. BERTRAND, M., MULLAINATHAN, S., and SHAFIR, E. (2004), "A Behavioral Economics View of Poverty", *American Economic Review Papers and Proceedings*, 419-423.
6. BESLEY, T. and COATE, S. (1995), "Group Lending, Repayment Incentives, and Social Collateral", *Journal of Development Economics*, 46, 1-18.
7. BRAVERMAN, A. and STIGLITZ, J. (1982), "Sharecropping and Interlinking of Agrarian Markets", *American Economic Review*, 72, 695-715.
8. DAGNELIE, O., and LeMAY, P. (2005), "Rosca Participation in Benin: a Commitment Issue", (mimeo).
9. GUGERTY, M. (2005), "You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya", (mimeo).
10. HARRIS, C. and LAIBSON, D. (2000), "Dynamic Choices of Hyperbolic Consumers", *Econometrica* (forthcoming).
11. KARLAN, D. (2005), "Social Connections and Group Banking", (mimeo: Yale University).
12. KOCHERLAKOTA, N. (1996) "Reconsideration-Proofness: A Refinement for Infinite Horizon Time Inconsistency", *Games and Economic Behavior*, 15, 33-54.

13. KRUSELL, P., and SMITH, A. (2003), "Consumption-Savings Decisions with Quasi-Geometric Discounting", *Econometrica*, 71, 365-375.
14. LAIBSON, D. (1997), "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics*, 62, 443-479.
15. MORDUCH, J. (1999), "The Microfinance Promise", *Journal of Economic Literature*, 37, 1569-1614.
16. MORDUCH, J. (2000), "The Microfinance Schism", *World Development*, 28, 617-629.
17. O'DONOGHUE, T., and RABIN, M. (1999), "Doing it Now or Later", *American Economic Review*, 89, 103-124
18. PHELPS, E., and POLLACK, R. (1968), "On Second-Best National Saving and Game-Equilibrium Growth", *Review of Economic Studies*, 35, 185-199.
19. THALER, R. and BERNARTZI, S. (2004), "Save More Tomorrow: Using Behavioral Economics in Increase Employee Savings." *Journal of Political Economy*, 112, 164-187.