OPTIMAL HEDGING UNDER PRICE, QUANTITY AND EXCHANGE RATE UNCERTAINTY

by

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ABSTRACT

After devising expectational measures of production, price and exchange rate uncertainty, this paper presents a model to derive an optimal hedging strategy for a primary producer who is subject to variability on the price and the amount of its output and on the exchange rate risk when the final proceeds of its sales are not denominated in its currency of numeraire. In this model, it is assumed that the producer is a mean-variance maximizer.

The analysis shows that the optimal hedge ratio for the commodity is proportional to the coefficient of the commodity futures prices bias in a linear regression where the producer's revenue uncertainty is the dependent variable. The ratios do not differ from the results that would have been obtained under price and quantity uncertainty only except when revenues are significantly correlated with the exchange rate variability.

An empirical test of the model is carried out wherein cocoa is chosen as a case study. The countries selected in this test are three of the world's largest producers. Since their output is primarily sold on London markets, the dollar and the French franc have been empirically designated to hedge pound revenues. The ratio for cocoa is well below unity and ranges from 78 to 57 percent. For Ghana and the Ivory Coast, there is little correlation between revenue and exchange rate variability. Therefore, the cocoa ratio is not significantly affected by the introduction of this additional source of uncertainty and the currency ratio in the joint hedge is trivial for both the dollar and the franc. Nigeria's revenues, on the other hand, are correlated to the dollar and the cocoa ratio is de facto lowered by the currency effect in a joint cocoa-dollar hedge.

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I dedicate my thesis to my parents, my family and particularly my brother Roger, before all of whom I can do little more than bow in gratitude. My graduate experience in the United States is theirs, as a meager tribute to their constant and inspiring support and as a modest testimony to our shared commitment to initiative and personal achievement through perseverance.

There is no useless undertaking,
What matters is how it is accomplished.
Lao Tseu

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Introduction

Among the prime features that characterize the economies of the Less Developed Countries (LDCs), the dependence of many upon a single major export and the limited, if not insignificant control that each exercises along the value-added chain for this export is probably the most discussed topic in both academic and non-academic forums. The severe debt crisis that some of these countries have been facing during the last five years has highlighted the fragility that such a poor diversification entails and also fueled the debate on sovereign debt management. The purpose of this paper is to explore an aspect of this issue by analyzing how a LDC can optimally manage its commodity-dependent exports in order to secure the repayments on liabilities which are denominated in foreign currencies.

A large number of hedging decisions are made in a context where the only source of uncertainty is the price. Treasurers in commercial banks and corporations essentially use currency futures in order to cover a transaction exposure the amount of which is determined at the moment when the decision is made. Likewise, bullion trading houses, food-packaging firms and shipping companies take certain positions in their respective futures markets (gold, commodities, oil). These examples illustrate a hedging decision in a situation where the settlement of the position is clearly not contingent on the performance of another event - in which case using options would be preferable to entering futures contracts.

In some other cases, the hedger is also uncertain about the quantity to cover. This is typical of the primary producer who would like to secure a given price for each unit of his crop but cannot be sure of the output. The hedging pattern changes dramatically here. Consider the case where the producer sells all his expected output forward at the price prevailing on the futures market. If his production comes below his expectation, his end of period wealth may be greater or lower depending on how spot prices have moved relative to the futures price that he originally agreed upon. If they are higher, he will loose money on the settlement of the contracts since he virtually has to purchase the shortfall on the commodity before selling it back at a lower price as per the contract. In the contrary event, he is better off. This example suggests that hedging under both price and quantity uncertainty will depend on the distributions of prices and output - and on their interaction - and on the producer's attitude toward risk.

Finally, the foreign exchange dimension enters the analysis when the currency of numeraire of the hedger is different from that of the proceeds of the forward sale. The term numeraire refers not only to the domestic currency for a LDC, but also to any other currency in which the hedger would preferably have the proceeds denominated. A Brazilian cattle breeder selling in the U.S where revenues are constant in dollar terms - strong domestic competition - might want to enter a transaction that provides him with German Marks if by so doing, he can decrease the economic exposure brought by his sourcing in the latter country.

In this study, we concentrate on an implicit relationship between debt and commodity revenues. This focus adds a non-trivial dimension to the existing work in this area. As a matter of fact, the earlier conclusions were drawn based on broad macro-economic factors among which the income stabilization goal. McKinnon, for example, constructed a very attractive model to derive optimal hedge ratios for a primary producer and asserted that futures prices were of significant use to the analysis only in so far as their year to year variations did not exceed that of cash prices. However, this view is not consistent with the volume of trade on most commodity futures if we can consider this data as a valid measure of their economic utility from the perspective of primary producers. Also, there might exist an economic rationale for erratic cash and futures prices : unexpected increases in production can depress spot prices relative to futures prices because it is this dynamic which will allow merchants to sustain holding buffer stocks. Finally, and even more important is the fact that the main determinent of the hedge ratio is not the total variability in prices but rather that part of the observed uncertainty that deviates from the producer's anticipations.

In the subsequent papers, this basic contribution of modern porfolio analysis was introduced in the derivations in which either price only as in Peck or both price and quantity as in Rolfo were the sources of uncertainty. The latter proposed and empirically tested two models in a domestic context and obtained ratios that were substantially lower than 1 for the world's four largest cocoa producers. The most insightful aspect of this work is an analytic support to the intuition that when output is uncertain, the producer is likely not to hedge the total amount of his expected production in order to protect himself in the unfavorable state of the world where his crop would fall short of his expectations and spot prices are high - which implies a further drop in income due to the loss incurred to settle the short position on futures contracts .

Though the present work draws substantially on the existing papers on the subject, the focus of the hedging decision from a country's perspective in lieu of a firm and the link between commodity revenues and debt service add some specificity to the analysis. First, the foreign exchange dimension is extremely important because futures prices are quoted in a different currency than that of most LDCs'. The major african producers , for example, sell their cocoa on the London Cocoa Terminal. It follows that a hedging decision which is aimed at securing resources to meet debt service in dollar or in French franc must incorporate the price distribution of those currencies vis a vis of British pound . Also, this allows one to develop a more comprehensive understanding of the hedger's economic exposure. Namely, declining dollar coffee prices are a serious concern for a country depending on this commodity for its revenues. An obvious result would be the reduced flexibility of the Marketing Board in its ability to keep real prices constant in domestic terms. However, the hardship will no doubt be less awesome if the country mostly trades with France and the French franc has depreciated enough against the dollar to more than offset the decline in coffee revenues.

In this paper, we advocate hedging through forward sales as a strategy for a LDC to handle uncertainty on its commodity revenues. In this approach , the hedging horizon is limited to the short-term, where futures prices are critical to the investment decision. They essentially serve two purposes. First, they provide the producer with some valuable information on the future course of commodity prices and form a basis for forecasts (Black; Parsons). Second, they provide an efficient and rather inexpensive mechanism for risk transfer from merchants to speculators. The first section of this paper indicates the methodology and sets up a model for a primary producer who faces output, price and exchange rate uncertainty. In the second and third sections, the model is empirically tested, using data from the cocoa markets and the test results are analyzed in the last section.

The case of cocoa has been chosen because it has one of the highest price and quantity volatilities among primary commodities. Also, the largest producers are developing countries and they are largely dependent on cocoa exports. Finally, cocoa prices are largely determined by market forces , allowing to capture this uncertainty in the analysis. This explicitly differentiates cocoa from coffee for that matter since coffee export quotas are determined by multilateral agreements between the International Coffee Organization members. Furthermore, the latter - who account for the bulk of the world trade - agree on formal mechanisms in order to limit the range of fluctuations in price indicators. Since cocoa is sold in pound , the currencies selected to test the model are the French franc and the U.S. dollar because the latter has allegedly been pointed at as the major source of the LDCs' problem while the currency of one of the countries involved in this analysis (Ivory Coast) is directly pegged to the former.

Section 1: Methodology

This paper derives an optimal hedge ratio for a primary producer who is subject to price, quantity and exchange rate uncertainty. In order to set up the model, a few assumptions must be made, regarding how the investor perceives risk in his decision and how this attitude toward risk can be captured. In the following section, we review some of the most commonly used models of uncertain decisions with a brief survey of some of their fundamental characteristics and derive the most relevant model for hedging with the three above mentioned dimensions of uncertainty.

1.1 Alternative models of uncertain decisions

An acceptable model for decision making under uncertainty should optimally be simple to handle, consistent in the empirical results that it provides and most of all based on reasonable assumptions with respect to primary producers' observed behavior.The most important question relative to the choice of a model is how the producer behaves toward risk and how is his risk behavior changes with his level of wealth. Among the different models available , we will concentrate on three of them which are more or less widely recognized and fit the requirement that we stated above.

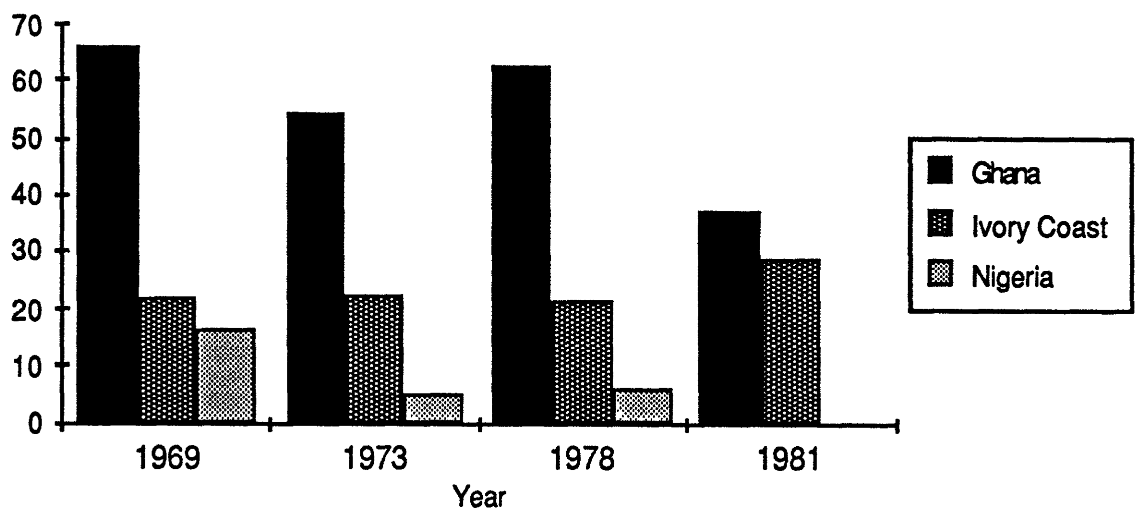
The first model is the expected utility. It relies on the Bernouillan postulate that individuals can order the likelihood of events in a cardinal fashion according to their degree of belief. Whatever the source of uncertainty and regardless to the decision process that is implemented to cope with uncertainty, an individual's belief about the consequences of a particular act are assumed to be summarized by his personal probabilities. As such, the expected utility model is a fully optimal and maximizing method.

The second method is the exponential utility function. Like the former, it is a utility-based method. This representation is directly premised on a negative exponential utility function which implies that the explored dimension of utility - in most cases, wealth - is normally distributed. The risk parameter, which is the increase in expected income required by the producer to compensate for an increase in variance of income is proportional to the producer's risk aversion and the trade-off between expected income and variance of income is independent of the level of expected income. The assumption of normality is attractive from a computational point of view because it allows the third and higher order moments to be easily specified with the mean, the variance and the correlation parameter and leads to closed-form solutions (Kendall).

The third method is the mean-variance rule. Through its use in portfolio analysis and quadratic programming, it has the longest history of both theoritical argument and empirical application. This method is consistent with expected utility theory when the decision maker acts in accordance with a quadratic utility function of income. It follows that the decision maker becomes more averse as his wealth increases.

The selection of a decision rule must account for two important aspects of the farmers' actual behavior. First, Roumasset has shown that yield and profit distribution deviate substantially from normality and, in particular , fertilizer is often a factor in explaining skewness. This suggests that any model assuming a normal distribution of income will be at best unrealistic. Second, Pratt-Arrow (Roumasset) have hypothesized that absolute risk aversion may decline rapidly in the neighborhood of subsistence income. This hypothesis has aroused some controversy . It is indeed empirically difficult to determine what this critical level of income is - it probably depends on the type of commodity that is produced - and also it would require a large decrease in one-period income for the producer to consider that his whole wealth is at risk. Nevertheless, the rationale for this hypothesis is intuitively sound: when subsistence is at stake, primary producers will not take a substantial gamble on variance of income that is they are most unlikely to risk to fall below that critical level of income. This is an illustration of a general controversy regarding investors' attitude to risk at extremely low levels of income. The issue can however be ruled out if we assume thay the countries surveyed are above this subsistence level. One convenient way to suggest this statement is to consider how critical the commodity export is to the country's total export portfolio.

Figure 1
Ratio of Cocoa to Total Exports



Percentage Figures.
Source: International Financial Statistics. Various years.

Figure 1 above attempts to capture this notion. For four selected years, the ratio of cocoa exports to total exports is provided. This ratio does not appear on the chart for Nigeria in 1981 because it is almost equal to zero. The most cocoa dependent country is Ghana and the least is Nigeria, primarily because of the surge in oil production for the latter. It follows that the assumption we made is clearly undebatable for Nigeria, quite acceptable for the Ivory Coast and perhaps more controversial in the case of Ghana. In light of the different assumptions we made, it appears reasonable to adopt the mean-variance rule in which risk aversion increases with wealth. In this paper, we will model a country's preference like that of a rational investor.

1.2 The mean variance framework

What determines the fraction of the output that the producer is willing to hedge is the extent to which his predictions - or more formally the information that he uses as predictors - are accurate. This uncertainty is captured by the standard error of estimates based on expectational data from the past.

We assume that the producer has to face total output uncertainty from such unexpected factors as tree diseases or bad climatic conditions arising between the time the production decision is made and the beginning of the marketing season. For this uncertainty to take full effect, we assume that there is no buffer stock - or that the yearly variations in stocks over the covered period are not significant, which is not too strict an assumption since cocoa is a perishable commodity.

As a simplifying paradigm, the production cycle is divided in two periods. Before the harvest, all three variables are uncertain. The uncertainty is resolved at the harvest. Distributions of price, quantity and exchange rates are empirically determined, as explained in a later part of the paper. A distinction is made between the price distribution on the physical market, $\tilde{\mathbf{P}}$, and the price distribution on the futures market $\tilde{\mathbf{p}}_1$. Hedging n_1 commodity futures contracts contracts and n_2 futures contracts on a given currency X will enable the producer to change his revenue distribution from $\tilde{P}\tilde{Q}$ to:

$$\tilde{W} = \tilde{P}\tilde{Q} + n_1(f_1 - \tilde{p}_1) + n_2(f_2 - \tilde{p}_2) \qquad (1)$$

where $\tilde{Q}$ is the output distribution, f_1 the futures cocoa price prevailing before harvest, f_2 the futures exchange rate and $\tilde{p}_2$ the price distribution on the currency futures market. The rationale is that the producer sells short a fraction n_1 of its output at a futures price f_1, virtually becoming long the currency - pound - in which the forward sale is denominated and therefore simultaneously buys n_2 units of currency X at the futures rate f_2. In the above definition of the random end-of-period income, n_1 has the dimension of expected production $\tilde{Q}$ while n_2 has the dimension of expected revenues $\tilde{P}\tilde{Q}$. Note that the difference between f_i and $\tilde{p}_i$ has a positive sign because the producer benefits if the futures price that he agreed upon turns out to exceed the price distribution at maturity.

With this model, the expected utility as a function of expected income and variance of income can be defined as:

$$E [U (\tilde{W})] = E (\tilde{W}) - m \ [var (\tilde{W})] \qquad (2)$$

where E (.) denotes the expected value operator. If we combine equations (1) and (2) and then follow the two usual first-order conditions $dE [U (\tilde{W})] / dn_1 = 0$ and $dE [U (\tilde{W})] / dn_2 = 0$, we have (see Appendix D for the complete derivation of the optimal values):

$$n_1^* = \frac{var (\tilde{p}_2)[f_1 - E (\tilde{p}_1)] + 2m \ cov (\tilde{P}\tilde{Q} , \tilde{p}_1)] - cov (\tilde{p}_1, \tilde{p}_2)[f_2 - E (\tilde{p}_2) + 2m cov (\tilde{P}\tilde{Q} , \tilde{p}_2)]}{2m [var (\tilde{p}_1) var (\tilde{p}_2) - cov (\tilde{p}_1, \tilde{p}_2)^2]} \qquad (3)$$

and

$$n_2^* = \frac{var(\tilde{p}_1)[f_2 - E(\tilde{p}_2)] + 2m\, cov(\tilde{P}\tilde{Q}, \tilde{p}_2)] - cov(\tilde{p}_1, \tilde{p}_2)[f_1 - E(\tilde{p}_1) + 2m cov(\tilde{P}\tilde{Q}, \tilde{p}_1)]}{2m[var(\tilde{p}_1)\, var(\tilde{p}_2) - cov(\tilde{p}_1, \tilde{p}_2)^2]} \quad (4)$$

We should emphasize that in order to derive these ratios , no assumption was made regarding the joint distribution of output and prices. As we mentioned in section 1.1, the yield distribution is skewed and we cannot assume normality in revenues distribution. However, by assuming that the producer is mean-variance maximizer, we only say that its utility is quadratic in revenues .

If we define $ß_i$ as the coefficient of $\tilde{p}_i$ in a linear regression where the revenue $\tilde{P}\tilde{Q}$ is the dependent variable and $\tilde{p}_i$ the independent variable and in a similar form $ß'_1$ ($ß'_2$) as the coefficient of $\tilde{p}_1$ ($\tilde{p}_2$) in a regression where $\tilde{p}_2$ ($\tilde{p}_1$) is the dependent variable, equations (3) and (4) can be rewritten as:

$$n_1^* = \frac{[\,[f_1 - E(\tilde{p}_1)]/var(\tilde{p}_1)] + 2mß_1 - ß'_1[[f_2 - E(\tilde{p}_2)]/var(\tilde{p}_2) + 2mß_2]}{2m(1 - ß'_1ß'_2)} \quad (5)$$

and

$$n_2^* = \frac{[\,[f_2 - E(\tilde{p}_2)]/var(\tilde{p}_2)] + 2mß_2 - ß'_2[[f_1 - E(\tilde{p}_1)]/var(\tilde{p}_1) + 2mß_1]}{2m(1 - ß'_1ß'_2)} \quad (6)$$

It can be observed from equations (5) and (6) that each ratio is proportional to the coefficient parameter in a linear regression where the revenue is the dependent variable and the price (cocoa or currency) the explaining variable. In other words, the larger the increase in revenues related to an increase in cocoa futures prices, the larger the cocoa ratio i.e. the better off is the primary producer by selling forward a more substantial portion of the expected output. Likewise, it is economically sound to sell more pounds forward for dollars or for francs if pound revenues are highly positively proportional to the currency.

Both ratios are jointly determined in a symmetric way. However, a peculiarity seems to emerge from the first observation of the results. The cocoa ratio can be split into two terms:

$$\frac{[\, [f_1 - E(\tilde{p}_1)] / var(\tilde{p}_1)] + 2m\beta_1}{2m(1 - \beta'_1\beta'_2)} \quad \text{and} \quad \frac{-\beta'_1[\,[f_2 - E(\tilde{p}_2)] / var(\tilde{p}_2) + 2m\beta_2]}{2m(1 - \beta'_1\beta'_2)}$$

The first term is essentially the ratio which would have prevailed if there were no currency uncertainty. As such, it would be equal to the ratio calculated by Rolfo except for the cross-product of price and currency prices in the denominator; however, we should expect trivial values for β'_1 and β'_2. This term shows that the ratio is inversely proportional to a negative bias in the cocoa price forecast. The second term in n_1^* is the adjustment factor. It shows that when the currency fitures prices exhibit a positive bias or when revenues increase with higher values of the currency relative to the pound, then the ratio is reduced. However, this reduction in the ratio is proportional to the correlation coefficient β'_1. In other words, the adjustment factor ultimately depends on the magnitude of co-movements of prices and currency.

Rolfo suggested that in the case of an unbiased forecast, the optimal hedge would be independent of the producer's attitude toward risk. When exchange rate is an additional source of uncertainty, then both the latter and price must be unbiased for the ratios to be independent from the parameter m. When this is the case, then we have:

$$n_1^* = \beta_1 / (1 - \beta'_1 \beta'_2) \quad \text{and} \quad n_2^* = \beta_2 / (1 - \beta'_1 \beta'_2) \qquad (7)$$

An intuitive understanding of the above equation emerges from the following example. Consider the situation where $\beta_1 = .6$, $\beta_2 = .2$ and we further assume for simplicity that prices and currencies are not correlated. If the producer is absolutely confident in the accuracy of the forecasts, he has to hedge 60 percent of his expected production. This is so because if he hedges more, he might not be able to deliver the amount of output required to cover his commitment. Alternatively, there is no incentive to forego some potential revenus by hedging less. By a similar argument, he has to hedge 20 percent of his expected pound revenues. So, when the forecast is consistently accurate for both the commodity and the currency, futures prices can directly guide the hedge decision: the higher the impact of futures prices increases on revenues, the higher the ratio. In equation (7), the denominator can be interpreted as a noise between exchange rate and cocoa prices.

The hedge ratio is inversely proportional to the risk aversion parameter. In the extreme case where the producer is infinitely risk averse, the cocoa hedge ratio depends only on both the correlation of revenue with exchange rate and the cross product of the price-revenue and price-exchange rate correlations as shown in equation (7). The above example allows us to relate the hedging position of an investor facing unbiased forecasts with that of an infinitely risk-averse producer . For the latter, the bias in the forecasts does not affect his decision. Any ratio different from 60 percent exposes him to bias risk. If he hedges more that 60 percent of his expected cocoa production, then he gambles on settlement prices being lower than the contract price so that he can increase his income. Likewise, by hedging less than 60 percent, he actually hopes that the settlement prices will be higher than the contract price. In both cases, this contradicts his risk averse behavior.

When it is assumed that the futures prices are unbiased, we derive some interesting forms of n^*_1 and n^*_2. However, we should clarify the conditions under which futures prices have strong forecast power and inquire into the sources of the bias in futures prices. There are two complementary theoretical approaches to this problem.

First, the theory of storage provides us with some statistically powerful explanations in this respect. According to this theory, the basis - that is the difference between contemporaneous spot and futures prices - is equal to the sum of the opportunity cost of capital, marginal storage costs and marginal convenience yield. While the two first items are pretty staightforward, the notion of marginal convenience yield requires explanation. It actually is an elusive concept which captures the shadow price of holding inventories of some commodities and therefore is generally explained with examples. There may be a convenience yield for wheat because it is an input to the production of other commodities the demand of which is extremely volatile (for example, flour). Another intuition of the concept of convenience yield would be the return that is required to meet unexpected demand. A direct implication of this definition of convenience yeld is that when the marginal storage costs are high - this is actually the case for a perishable commodity like cocoa , changes in the convenience yield mostly explain the variance in the basis. Before harvest, when inventories are low, marginal convenience yields outweigh storage costs to produce inverse carrying charges - alternatively, the expected change in spot prices is negative across periods when a harvest will substantially increase output . Likewise, the basis is positive between harvests because at this point in time, convenience yields decline with an increased availability of the commodity - alternatively, spot prices are expected to increase in order to induce storage between harvest .

Another view of the basis defines it as the sum of the expected change in spot prices until maturity and an expected premium around the expected future spot price - Breeden defines it as the compensation for the basis risk . Since both the theory of storage and the forecast power approach are alternative views of the same economical issue, the former can be empirically tested in order to assess whether the non-zero value of the expected premium is an acceptable hypothesis. The evidence provided by Fama are highly conclusive in this respect for some commodities, although statistically unreliable for a few other commodities (Fama). In essence, the combination of both theories explains why unbiased cocoa forecasts are most unlikely on an expected basis.

Note also that the theory of storage explains why futures and spot prices do not converge at maturity - the futures prices are actually lower . Typically, a hedger does not deliver the commodity but simply reverses the position at maturity by going long a futures contract. The latter generally stipulates the location at which the commodity is due for delivery. The holder of a futures contract does not receive the full cash price and the difference between the last day's futures price and the prevailing spot price is at least equal to transportation costs (Peck).

As far as the currency is concerned, substantial evidence based on empirical testing of the Unbiased Expectations Hypothesis (UEH) is available to show that currency futures prices are biased and the premium generally included in the basis is equal to the expected real return differential across different currencies (Korajczyk).

Nevertheless, the above equations suggest that, if the covariance of the producer's revenue with the exchange rate is large - which may well be if the country the currency of which is hedged imports a sizeable portion of the LDC output and at the same time, accounts for a substantial share of the world consumption - but the exchange rate is very volatile relative to the pound, then the ratio of cocoa hedged would be substantially reduced, as compared to the results obtained by Rolfo.

Section 2: Empirical data

An appropriate measure of price, quantity and exchange rate uncertainties is only that portion of the variance that is left unexplained. Indeed, the systematic component allows to evaluate the change in expected returns from one period to another but does not affect the risk associated with the non-attainment of that expected value.The variance of the disturbance term, on the other hand, does not contribute anything to changes in expected returns since, by definition, it is a random variable that cannot be specified except by a probability distribution with constant mean. Subsequently, the price series that are used are expectational and measure the difference between realized and forecasted values. These measures are considered to be randomly drawn from four different sets of population - no assumption is made on the dependence of the four drawings.

2.1 Price Series

There are two cocoa seasons yearly but the major one starts on the first of October. In accordance with the definition ot the production cycle that we made above, preharvest refers to late September and harvest refers to March, when the uncertainty is resolved. Three price series are used. The cocoa and currency futures prices as reported on the last day of September (or on the first business day thereafter in the case cocoa markets were closed) are the price and exchange rates forecasts. The futures prices reported on the first day of March are the settlement prices for both the commodity and the currency whereas the shipment prices are used to measure the real cocoa transaction spot rates. Note that for most traded futures contracts, there is no specific maturity date and short traders can deliver at any time during a three to four week period stretching from the beginning of the month. In assuming that positions are settled on the first trading day of the month, we essentially state that the commodity and currency contracts have a six-month maturity.

The data have been collected for the last sixteen years because currency volatility became a real issue only when the industrialized nations entered an area of managed float at the begining of the seventies.The forecast errors as specified below are plainly printed to differentiate from the bold print of the price series:

$$p_t = (\mathbf{P_t} - f_{1t}) / f_{1t} \qquad t = 1,16 \tag{8}$$

and

$$p_{1t} = (\mathbf{p_{1t}} - f_{1t}) / f_{1t} \qquad t = 1,16 \tag{9}$$

where f_{1t}, the cocoa futures price, is the price forecast; $\mathbf{P_t}$ is the shipment price and $\mathbf{p_{1t}}$ the settlement cocoa futures price for season t. The error terms have been divided by the forecast price to allow for different historical rates of inflation. Likewise, the forecast error on the currency X is:

$$p_{2t} = (\mathbf{p_{2t}} - f_{2t}) / f_{2t} \qquad t = 1,16 \tag{10}$$

These price series are collected from the London Financial Times for the last 16 seasons. The construction of these series is rather straightforward, not only because they are readily available, but also because they reflect a market consensus. The production forecast data, on the other hand, raise a the question of reliability. We use the data provided by Gill & Duffus for the three largest cocoa producers (Ghana, Ivory Coast and Nigeria) because it is generally believed that Gill & Duffus have the best information on the prevailing conditions of cocoa production ; we will assume that the production data incorporate the best available information. The figure used for the forecast are recorded from Rolfo from 1966 to 1975 and from the most recent preharvest forecasts published by Gill & Duffus from 1976 onwards.

Using the same expectational measure as before, the forecast error on production is:

$$Q_t = (Q_t - Q^e{}_t) / Q^e{}_t \qquad t = 1, 16 \tag{11}$$

where $Q^e{}_t$ is the forecast production for year t. Note here that we assume that production is primarily exported, which is consistent with the actual policy of the major producers.

2.2 Optimal Hedge Ratios

Expressing $\tilde{\mathbf{P}}$, $\tilde{\mathbf{Q}}$, $\tilde{\mathbf{p}}_1$ and $\tilde{\mathbf{p}}_2$ as a function of the errors around the forecasts as defined in equations (8) to (11), the optimal hedge ratios can be expressed as a function of the error terms, the notations being the same as in equations (5) and (6) but with the statistics on the error terms in plain print :

$$n_1 / Q^e = \frac{\beta_1 - \beta_2\beta'_1}{1 - \beta'_1\beta'_2} + \frac{\beta'_1 E(\tilde{p}_2) / var(\tilde{p}_2) - E(\tilde{p}_1) / var(\tilde{p}_1)}{2mf_1(1 - \beta'_1\beta'_2)} \qquad (12)$$

and

$$n_2 / f_1Q^e = \frac{\beta_2 - \beta_1\beta'_2}{f_2(1 - \beta'_1\beta'_2)} + \frac{\beta'_2 E(\tilde{p}_1) / var(\tilde{p}_1) - E(\tilde{p}_2) / var(\tilde{p}_2)}{2mf_1f_2(1 - \beta'_1\beta'_2)} \qquad (13)$$

Equations (12) and (13) confirm some of the findings from equations (5) and (6). However, we can see that in the case of unbiased forecasts, the ratios become respectively:

$$n_1 / Q^e = \frac{\beta_1 - \beta_2\beta'_1}{1 - \beta'_1\beta'_2} \quad \text{and} \quad n_2 / f_1Q^e = \frac{\beta_2 - \beta_1\beta'_2}{f_2(1 - \beta'_1\beta'_2)}$$

There is a level of the parameter m at which reverse hedging - namely, a long position taken by the primary producer on the futures market - on both cocoa and the currency might become optimal. Though this might be somewhat strange, the following example illustrates the value of reverse hedging. In the scenario where the cocoa futures price is 1000 pounds per ton and the producer's expected output is 100 tons, let us suppose that the producer hedges 80 tons - 80 percent of the expected output - and accordingly expects to receive 80,000 pounds at maturity of the contract. If spot prices double to 2000 pounds per ton at maturity and the producer's output is finally only half what he expected, he will have to buy 30 tons of the commodity therefore loosing 60,000 pounds or 75 percent of his the expected income. Had the producer hedged only 50 percent of expected production, his final income would not be affected. However, had the ratio been 25 percent, he would have been better off 50,000 pounds which represent the proceeds of the spot sale of production in excess of the forward commitment. In general terms, suppose that in years of unusually small crops prices do not rise enough to compensate for the low output, therefore leaving the farmer with an unusually low income. He clearly would have benefited from buying cocoa forward instead of selling: the profit made on settling the futures position would increase his income. It should be emphasized that reverse hedging presents some interest only insofar as the same economic factors behind it also determine why hedge ratios could be substantially lower than unity when there are multiple sources of uncertainty.

The critical value of m at which reverse hedging becomes optimal is obtained for the first ratio by simply setting $n_1^* = 0$ which yields:

$$m^* = \frac{\beta'_1 E(\tilde{p}_2) / var(\tilde{p}_2) - E(\tilde{p}_1) / var(\tilde{p}_1)}{2f_1 \left(\beta_2 \beta'_1 - \beta_1 \right)} \qquad (13)$$

Section 3: Test results

The following results are obtained from the data presented in Appendix (A to D). The most important point to notice is that overall, cocoa futures prices tend to underestimate both the contracts' settlement prices - by 2.90% in average - and the shipment prices - by 10.11% in average. On the currency side, the forecast are slightly biaised downward for both the dollar and the franc -1.8% vs .08%. However, the variance of errors is still significant for both statistics - 9.87% for the dollar and 8.24% for the franc .

3.1 Production

Table 2 below presents the variance-covariance and correlation matrix of production uncertainty for the producing countries. The data show little correlation among these uncertainties. Only between Ghana and the Ivory Coast and also between the latter and Nigeria is the correlation not insignificant. This may come as some surprise since a direct source of production variability can be traced to climatic conditions. Since the three countries are situated

Table 2

	Ghana	Ivory Coast	Nigeria
Ghana	.00538* (1.0000)**	-.00113 (-.15839)	-.00079 (-.08477)
Ivory Coast	-.00113 (-.15839)	.01349 (1.0000)	-.00321 (-.21652)
Nigeria	-.00079 (-.08477)	-.00321 (-.21652)	.01628 (1.0000)

* The first figure is Variance-Covariance
** The second figure is Correlation

in the same geographical area, one could have expected their crops to be affected by the same hazards. However, this view clearly neglects two other factors, namely the yield pattern of the cocoa tree over time and the amount of attention paid by the cocoa farmer before harvest. Roughly, an acre of newly planted cocoa tree yields little or no cocoa before the fifth year. It then generates a sharp increase in yield until approximatively the fifteenth year at which point the continued increase in yield tends to be offset by the incidence of diseases in some of the less vigourous trees. After about the twentieth year, the yield levels off and starts to decline. This gross representation of the cocoa tree lifecycle just illustrates how uncertain the output is before harvest, the successive phases varying significantly in their duration. The second point is that the most commom disease on West-African crops come from a virus that the farmer can avoid by weeding or spraying more thoroughly (Weymar). The point, however, is that there is no reason why these factors could not be country specific and therefore explain the lack of correlation between the production uncertainties.

Table 3 below displays the correlations between production errors and price uncertainty. It is implicitly assumed that supply is the only source of uncertainty and the driving force of prices. Whether such an assumption is realistic or not depends on the velocity of changes in the patterns of consumption . Some of the major determinents of demand are consumers' tastes, competition from substitutes, new uses and government policies. The latter are infrequent, as illustrated by the British confectionery after Second World War (Vitton). The former are likely to take time before being effective, bearing therefore a limited impact on the period of uncertainty that we defined earlier.

Table 3

	Ghana	Ivory Coast	Nigeria
Covariance	.001964	-.000143	-.008822
Correlation	.10429	-.00480	-.26920

We should have expected large and negative correlations in accordance with the leading positions of these countries in the trade of cocoa. Unfortunately, with the exception of Nigeria (-.27), this is not the case. This result, however, is supportive of the conclusions drawn from the first table in the sense that the main factors of uncertainty are not systematic and the likelihood of a simultaneous decrease in output due to the same factors is reduced.

In more general terms, one would expect a negative correlation between production and prices because any farmer expects his output to be positively correlated with the aggregate output of all producers and hence negatively with prices. From this perspective, a situation likely to guarantee a zero correlation for most of the producers is that when the sources of supply of the commodity are largely dispersed geographically.

3.2 Revenues

The results of the correlation between forecast errors on revenues and prices are show in Table 4 below. The revenue error is computed as the sum of the quantity, the physical price errors and their cross-product.

Table 4

Revenue

	Ghana	Ivory Coast	Nigeria	Price
Ghana	1.00000	.85879	.78819	.63337
Ivory Coast	.85879	1.00000	.70212	.53737
Nigeria	.78819	.70212	1.00000	.61586

The table shows a strong correlation for all countries . Higher prices tend to be associated with higher country revenues, regardless of the difference between realized and forecast output. This is consistent with the lack of systematic production error correlation among the three countries that we outlined above. Had it not been the case, the comovements of production would have had an impact on prices because the three countries account for most of the world production. This effect in turn would have signified a lower correlation between revenues and price uncertainties. The regional revenues are also strongly correlated, the highest correlation existing between the two countries sharing a common border (.86 for the Ivory Coast and Ghana).

3.3 Exchange rates

As earlier mentioned in the discussion of equations (12) and (13), the exchange rates act as a premium or a discount over the ratio that would have prevailed under pure quantity and price uncertainties, depending essentially on how they are correlated to cocoa futures prices and how they are correlated with pound revenues. An interesting pattern seems to emerge from Table 5 below.

Table 5

	Revenue			
	Ghana	Ivory Coast	Nigeria	Price
Dollar	.04695	-.01381	.41530	.03120
Franc	.06463	.15131	.19690	.08356

There is little correlation between the pound prices errors and the currency errors. Also, the revenues of the three countries seem to present distinct patterns. Nigeria's revenue uncertainty is relatively more tied to that of the dollar, the Ivory Coast's to the error on the franc forecast and Ghana's pretty much indifferent. However, only in the case of Nigeria does this result appear significant (Appendix E). Note also that cocoa prices do not seem to be strongly correlated with either the franc or the dollar - which is somehow to be expected for a commodity .

Section 4: Hedge ratios Analysis

4.1 General Observations

The hedge ratios on both futures markets are presented in Table 6 and Table 7. The results obtained are striking on many counts. There are some common features across countries and also some specific aspects for each country in both hedges.

First, the range over which both the commodity and the currency ratio is by far narrower than the one exhibited in Rolfo's conclusions (Table 8). For both the cocoa-dollar and cocoa-franc hedge, the commodity ratio starts to decrease when the risk parameter reaches 100 thousandths . At this level of m, the ratio determined under only price and quantity uncertainty would be below zero meaning that reverse hedging is optimal. However, the ratios' rate of decrease is small and for the different levels of m that are illustrated, it appears that the commodity ratios are in the 78 to 57% range. At the same time the currency ratios are insignificant except for Nigeria .

Table 6

Ratios for Cocoa -Dollar Hedge

m	Ghana	Ivory Coast	Nigeria
.01	.7704 (.0446)	.7275 (-.0557)	.6650 (.5881)
.0001	.7704 (.0446)	.7275 (-.0557)	.6650 (.5881)
.0000001	.7591 (.0223)	.7218 (-.0669)	.6495 (.5587)
.00000005	.7477 (.0)	.7161 (-.0781)	.6346 (.5293)
.00000002	.7136 (-.0671)	.6991 (-.1116)	.5898 (.4412)
.00000001	.6567 (-.1789)	.6707 (-.1675)	.5151 (.2943)

First figure is Cocoa Ratio
Second figure is Dollar Ratio

The stickiness of the cocoa ratio around the coefficient β_1 is largely explained by the small value of the cocoa price-currency rates beta and the limited explanatory power of this statistic. As illustrated in Appendix E, these coefficients are in most cases small, and when they are not, their estimates exhibit a large variance i.e the significance of the slope in the linear regression is trivial. In the analysis below, this patterm will be shown for each country.

Table 7

Ratios for Cocoa -Franc Hedge

m	Ghana	Ivory Coast	Nigeria
.01	.7703 (.0032)	.7141 (.0323)	.6646 (.0360)
.0001	.7703 (.0032)	.7141 (.0323)	.6646 (.0360)
.0000001	.7584 (.0032)	.7081 (.0323)	.6490 (.0360)
.00000005	.7465 (.0032)	.7022 (.0323)	.6334 (.0360)
.00000002	.7109 (.0032)	.6844 (.0323)	.5865 (.0360)
.00000001	.6514 (.0032)	.6546 (.0323)	.5083 (.0360)

First figure is Cocoa Ratio
Second figure is Franc Ratio

Another interesting result is that for each country, the choice of the currency in which the final proceeds are denominated hardly affects the cocoa ratio. However, there are important differences in the currency ratios prescribed for a dollar vs a franc hedge. In the case of Nigeria , the difference is large: the dual cocoa-dollar ratio for a value of m equal to .0001 is 66.4%-58.8% whereas the cocoa-franc optimal ratio is 66.4%-3.6% at the same level of the risk parameter m (the first figure is the cocoa ratio and the second one is the currency ratio in the hedge). The increase of the currency ratio from one hedge to the other is due to the stronger correlation of the country's revenue uncertainty with forecast error on the dollar .

Table 8

Ratios for Cocoa Hedge

m	Ghana	Ivory Coast	Nigeria
.01	.599	.737	.633
.001	.502	.370	.446
.0001	-.461	-3.295	-1.431
.00001	-10.09	-39.949	-20.201

Source: Rolfo

The boundaries for reverse hedging are specified in Table 9 and the values do not depend on the currency used in the hedge (except in the case of Nigeria), which confirms the stickiness of the cocoa ratio around the coefficient β_1.

Table 9

Reverse Hedging Levels

	Ghana	Ivory Coast	Nigeria
Dollar	1.5	.8	2.2
Franc	1.5	.8	2.4

Figures in billionths

4.2 Country Analysis

With a level of the risk parameter of one thousandth (m = .0001), the cocoa ratio is about the same for Ghana in both a dollar and a franc hedge at 77%. The optimal hedge ratio for a short sale of dollars is 4% whereas the ratio for franc is insignificant at about .032%. Clearly, the difference in the currency included in the hedge adds little to the results. This conclusion relates to the fact that Ghanean revenues are roughly uncorrelated to both the Franc and the Dollar. Also, β_1 is large (.77) and also significant (Appendix E). Obviously, this tends to validate the assumption we earlier made to the effect that short-term price fluctuations were probably caused by supply shifts. On the other hand, β_2 is not only small (.13 for the dollar and .21 for the franc) but is not significantly different from zero.

In the case of the Ivory Coast, the cocoa ratio slightly changes from one currency to another (.73 in the dollar case vs .71 for the franc hedge). The coefficient of regression of revenues on pound prices is .73. One of the earlier test results (Table 5) indicated that Ivory Coast's revenues tended to be correlated with the franc and exhibited an insignificant correlation with the dollar (-.014). Based on that, one could have thought that the franc ratio would be non trivial. As a matter of fact, it does not show out that way primarily because β_2 is small for the dollar (-.04) and large for the franc (.56) but at the same time very volatile.

Nigeria on the contrary is a case in point for a strong significant correlation between revenues and currency. The dollar ratio in the joint hedge is .58 at the same level of the risk parameter as in the case of Ghana and the Ivory Coast. The cocoa ratio is .66 in both cases and the franc ratio is small (.03) because β_2 though large (.60) has a large variance (.80).

The most interesting outcome of this analysis is the peculiar role that the currency plays when it is added as an additional source of uncertainty. We expected the currency ratios to be rather large in order to offset unanticipated changes in pound revenues. As a matter of fact, our prior intuition was construed on a definition of uncertainty that did not differentiate the expected part of the variance from the random variation. It followed that we expected the dollar ratio to be large and positive given the depreciation of the pound against the former for the last few years. However, in addition to this bias, the fact that both ratios are jointly determined has a surprising consequence: the cocoa ratio is changed - upward or downward - in order to accomodate for this additional source of uncertainty but the currency ratio is almost equal to zero. In essence, it seems that the cocoa ratio alone captures the impact of the currency of the hedging strategy (except in the case of Nigeria) .

Conclusion

In summary, the optimal cocoa hedge ratios prescribed for the three countries chosen for the empirical test of the model show little difference with the values that they would have taken under price and quantity uncertainty only. Furthermore, this cocoa ratio does not change with the currency assigned in the hedge - U.S dollar or French franc . This characteristic can be explained by the relative importance of the correlation between revenues and futures prices over the correlation of revenues with exchange rates. As a matter of fact, with the exception of Nigeria, the latter correlation has been weak or insignificant.This lack of correlation between revenues and exchange rates has led to trivial values of the currency ratio in the hedge in both cases. Only Nigeria exhibits a strong currency ratio in a cocoa-dollar hedge.

The ratios are extremely dependent on the quality of the data used to derive them. Market sources were relied upon as future prices predictors when they were available. Production forecasts were obtained from the most largely recognized specialist of the cocoa market. But the difference in the cocoa ratios as compared to prior derivations (Rolfo) highlights another important issue, namely the sensitivity of the results to the sample from which the empirical data is constructed. The lower sensitivity of the cocoa ratio to changes in the level of the risk parameter are explained by the lower value of the forecast bias in our sample the effect of which is to reduce the impact of the adjustment factor on the final value of the hedge as defined in equation (12) above (Section 2.2).

Overall, the results of the analysis are limited by the constraints of the model itself. With a one period model, futures prices are assumed to guide the hedging decision. It follows that if they are to be used in order for commodity revenues to back debt service, then the debt issuance decision should be endogeneous to the the hedging scheme. However, a country's borrowing needs cannot depend only on their one-period expected income. With the exception of revolving facilities and other types of short-term financing the existence of which are contingent on some form of immediate performance, borrowing generally implies a scheduled stream of future payments, stretching far after the maturity of the longest futures contracts traded today. Since there are no long horizon commodity futures contracts currently traded in an organized and centralized fashion, which could be used as price predictors (Parsons), it appears that hedging with today's contracts only will be of limited value when it comes to long-term debt management and that alternative approaches and analytical tools must be developed for this problem te be fully addressed.

Appendix A

Cocoa Production (Realized and Forecast)

Season	Ghana		Ivory Coast		Nigeria	
1966	369.4	400	139.3	135	263	210
1967	389.7	400	130.2	115	235	320
1968	297.9	360	129.4	125	189	200
1969	375.9	355	157.7	140	219	200
1970	359.0	370	159.5	170	303	250
1971	440.0	400	205.4	200	251	265
1972	405.0	400	172.7	205	237	250
1973	337.5	325	205.6	170	212	240
1974	361.4	387	241.4	210	211	225
1975	382.5	400	231.1	232	213	240
1976	320.0	330	230.0	235	165	160
1977	268.0	320	304.0	255	205	220
1978	250.0	250	312.0	255	137	145
1979	275.0	305	379.0	310	169	160
1980	250*	270	400*	390	155*	149
1981	230*	230	430*	460	160*	175

The output data are provided by Gill & Duffus in their bimonthly reports. The actual and realized production are obtained by from the most recent statistics for the 16 years covered in the study. When Gill & Duffus statistics were not available, FAO statistics were used for the actual production. In this case, the data is followed with an asterix sign.

Figures are in thousand tons and represent the main crop only, except for Nigeria where the total crop is provided.

Appendix B

Cocoa Prices

Season	Futures Price	Settlement Price	Shipment Price
1966	190.5	216.0	228.5
1967	228.0	267.5	262.0
1968	346.0	401.0	384.5
1969	394.5	292.0	384.5
1970	303.0	243.5	225.0
1971	213.0	210.0	226.5
1972	316.5	322.0	408.0
1973	572.5	659.0	975.0
1974	697.0	734.5	681.0
1975	600.5	771.0	826.0
1976	1619	644.0	2320.5
1977	2192	1715	2220
1978	1972	1762	1660
1979	1519.5	1379	1440
1980	1013	866	1012
1981	1320	1176	994

The futures prices are the closing quotes on the last day of September for the March futures contract. The settlement prices are the price quoted on the first trading day of March for the contract maturing the same month. The shipment prices are the prices ex-warehouse on the last day of March. The three price series are reported in the London Financial Times.

Prices quoted in pounds per metric ton.

Appendix C

Currency Prices (Futures and Spot Prices)

Season	Dollar		French Franc	
1966*	2.1664	2.7955	13.6807	13.8050
1967*	2.7792	2.4006	13.6100	11.8145
1968*	2.7795	2.3937	11.8750	11.8463
1969*	2.3582	2.4078	13.2610	11.1655
1970*	2.3800	2.4625	13.1650	13.3275
1971*	2.4988	2.6140	13.5700	13.1500
1972*	2.4088	2.4780	12.0700	11.2800
1973*	2.3863	2.3940	10.1600	11.4000
1974	2.3935	2.4035	11.1400	10.1600
1975	2.0105	1.9161	9.1405	8.9650
1976	1.5750	1.7200	8.1025	8.5400
1977	1.7505	1.8630	8.6000	8.5000
1978	1.9440	2.0660	8.4950	8.8800
1979	2.1907	2.1640	8.9920	9.7230
1980	2.3778	2.2450	10.0000	11.1450
1981	1.8063	1.7820	10.3400	11.1550

The first figure is the six month futures price on the currency as reported in the London Financial Times on the last day of September. The second figure is the spot price on the first day of March.Seasons when there is no six-month quote available are designated with an asterix. The three-month quote is used instead.

The spot rate is the average closing bid and asked rates as reported in the London Financial Times. The forward rate is equal to the spot plus (minus) the average bid and asked premium (discount).

Appendix D

Production Forecast Errors

Season	Ghana	Ivory Coast	Nigeria
1966	-.0765	.0318	.2524
1967	-.0257	.1322	-.2656
1968	-.1725	.0352	-.0550
1969	.0589	.1250	.0950
1970	-.0297	-.0629	.2120
1971	.1000	.0270	-.0528
1972	.0125	-.1576	-.0520
1973	.0378	.2094	-.1167
1974	-.0661	.1500	-.0622
1975	-.0437	-.0039	-.1125
1976	-.0303	-.0213	.0312
1977	-.1625	.1922	-.0682
1978	.0000	.2235	-.0552
1979	-.0984	.2226	.0565
1980	-.0741	.0256	.0403
1981	.0000	-.0652	-.0857

Forecast Error is the difference between actual and forecast ouput per unit of expected output.

Appendix D

Cocoa prices and Currency Forecast Errors

Season	Futures Prices	Shipment Prices	Dollar Rates	Franc Rates
1966	.1338	.1995	.2904	.0124
1967	.1732	.1491	-.1362	-.1319
1968	.1590	.1561	-.1388	-.0018
1969	-.2598	-.0253	.0210	-.1580
1970	-.1964	-.2574	.0347	.0123
1971	-.0141	.0634	.0461	-.0309
1972	.0174	.2891	.0287	-.0654
1973	.1511	.7030	.0032	.1220
1974	.0538	-.0229	.0480	-.0880
1975	.2939	.3755	-.0469	-.0192
1976	.6331	.4333	.0921	.0540
1977	-.2176	.0128	.0643	-.0116
1978	-.1065	-.1582	.0627	.0453
1979	-.0925	-.0523	-.0122	.0813
1980	-.1451	-.0009	-.0559	.1145
1981	-.1091	-.2470	-.0134	.0788
Mean	.0290	.1011	.0180	.0008
Variance	.0516	.6599	.0097	.0068

Forecast errors are the difference between the realized and the forecast values per unit of forecast.

Appendix E

Derivation of Hedge Ratios

The random end-of period income can be expressed in probabilistic terms according to equation (1):

$$\tilde{W} = \tilde{P}\tilde{Q} + n_1 (f_1 - \tilde{p}_1) + n_2 (f_2 - \tilde{p}_2)$$

leading to the following expression of expected income:

$$E (\tilde{W}) = E (\tilde{P}\tilde{Q}) + n_1 [f_1 - E (\tilde{p}_1)] + n_2 [f_2 - E (\tilde{p}_2)]$$

Combining the two equations above, we have:

$$\tilde{W} - E (\tilde{W}) = \tilde{P}\tilde{Q} - E (\tilde{P}\tilde{Q}) - n_1 [\tilde{p}_1 - E (\tilde{p}_1)] - n_2 [\tilde{p}_2 - E (\tilde{p}_2)] \quad \text{and}$$

therefore,

$$[\tilde{W} - E (\tilde{W})]^2 = [\tilde{P}\tilde{Q} - E (\tilde{P}\tilde{Q})]^2 + n_1^2 [\tilde{p}_1 - E (\tilde{p}_1)]^2 + n_2^2 [\tilde{p}_2 - E (\tilde{p}_2)]$$

$$- 2n_1 [\tilde{P}\tilde{Q} - E (\tilde{P}\tilde{Q})][\tilde{p}_1 - E (\tilde{p}_1)] - 2n_2 [\tilde{P}\tilde{Q} - E (\tilde{P}\tilde{Q})][\tilde{p}_2 - E (\tilde{p}_2)]$$

$$+ 2n_1n_2 [\tilde{p}_1 - E (\tilde{p}_1)] [\tilde{p}_2 - E (\tilde{p}_2)]$$

The variance of income, as the expected value of $[\tilde{W} - E (\tilde{W})]^2$ is then:

$$\text{var} (\tilde{W}) = \text{var} (\tilde{P}\tilde{Q}) + n_1^2 \, \text{var} (\tilde{p}_1) + n_2^2 \, \text{var} (\tilde{p}_2) - 2n_1 \, \text{cov} (\tilde{P}\tilde{Q}, \tilde{p}_1)$$

$$- 2n_2 \, \text{cov} (\tilde{P}\tilde{Q}, \tilde{p}_2) + 2n_1n_2 \, \text{cov} (\tilde{p}_1, \tilde{p}_2)$$

In order to fully specify the total variance of end-of-period income, we would need to know more about the joint distribution of $\tilde{P}\tilde{Q}$. In the case, for example,where price and quantity are joint-normally ditributed, closed forms of the revenue variance can be explicited given the individual variances of price and quantity and their correlation. However, since the variance of income does not depend on any of the ratios, there is no need to make such a severe assumption.

Following the two first-order conditions $dE[\,U(W)\,]/dn_1 = 0 = dE[\,U(W)\,]/dn_2$, we derive the system of two equations with two unknowns n_1 and n_2 :

$$\begin{cases} f_1 - E(\tilde{p}_1) - m\,[\,2n_1\, var(\tilde{p}_1) - 2cov(\tilde{P}\tilde{Q}, \tilde{p}_1)\,] + 2n_2\, cov(\tilde{p}_1, \tilde{p}_2) = 0 \\ f_2 - E(\tilde{p}_2) - m\,[\,2n_1\, var(\tilde{p}_2) - 2cov(\tilde{P}\tilde{Q}, \tilde{p}_2)\,] + 2n_1\, cov(\tilde{p}_1, \tilde{p}_2) = 0 \end{cases}$$

Finally, solving for this system of equations in n_1 and n_2 yields the hedge ratios which are presented in equations (3) and (4) :

$$n_1^* = \frac{var(\tilde{p}_2)[\,f_1 - E(\tilde{p}_1)\,] + 2m\, cov(\tilde{P}\tilde{Q}, \tilde{p}_1)] - cov(\tilde{p}_1, \tilde{p}_2)[\,f_2 - E(\tilde{p}_2) + 2m cov(\tilde{P}\tilde{Q}, \tilde{p}_2)]}{2m\,[\,var(\tilde{p}_1)\, var(\tilde{p}_2) - cov(\tilde{p}_1, \tilde{p}_2)^2\,]}$$

and

$$n_2^* = \frac{var(\tilde{p}_1)[\,f_2 - E(\tilde{p}_2)\,] + 2m\, cov(\tilde{P}\tilde{Q}, \tilde{p}_2)] - cov(\tilde{p}_1, \tilde{p}_2)[\,f_1 - E(\tilde{p}_1) + 2m cov(\tilde{P}\tilde{Q}, \tilde{p}_1)]}{2m\,[\,var(\tilde{p}_1)\, var(\tilde{p}_2) - cov(\tilde{p}_1, \tilde{p}_2)^2\,]}$$

Appendix F

Regression Results

Revenues- Futures Prices

	Beta	St. Deviation	t-statistic
Ghana	.77	.25	3.06
Ivory Coast	.73	.30	2.38
Nigeria	.68	.23	2.92

Revenues- Currency

	Beta	St. Deviation	t-statistic
Ghana	.13 (.22)	.17 (.24)	.75 (.90)
Ivory Coast	-.04 (.56)	.83 (.98)	-.05 (.57)
Nigeria	1.05 (.60)	.62 (.79)	1.71 (.75)

The first figure refers to a cocoa-dollar hedge.
The figure between parantheses refers to a cocoa-franc hedge.

Futures Prices-Currency

	Beta	St. Deviation	t-statistic
Dollar	.01 (.07)	.12 (.61)	.12 (.12)
Franc	.03 (.02)	.09 (.73)	.31 (.31)

The first figure is the coefficient in a linear regression where the currency error is the dependent variable.The second figure is the coefficient in a linear regression where the futures prices error is the dependent variable.

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