A Study of the Behavior of the GRP Hat-Stiffened Panel Bondline Under High Strain Rate Loading

by

Michael Ziv

B.S., Mechanical Engineering
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Submitted to the Department of Ocean Engineering and the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degrees of

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and

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Signature of Author . . . .

Certified by . . . .
Dr. Alan J. Brown, Professor, Department of Ocean Engineering
Thesis Advisor

Certified by . . . .
David M. Parks, Professor, Department of Mechanical Engineering
Thesis Advisor

Accepted by . . . .
A. Douglas Carmichael, Chairman, Departmental Committee on Graduate Studies
Department of Ocean Engineering

Accepted by . . . .
Ain Sonin, Graduate Officer
Department of Mechanical Engineering

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ABSTRACT

The effects of strain rate on the large strain compressive behavior of polymer blends is investigated using Dow Chemical Derakane 8084 rubber toughened vinyl ester and Scott Bader Crestomer 1080 as constituent materials. The goal is to evaluate their suitability for use in the stiffener/panel bondline of GRP minsweeping vessels which need to be able to continue their mission in the event of a near field mine detonation.

The experimental procedure involved the testing of five blends ranging from 0-100 percent Derakane 8084 by weight at 25 percent increments under compressive loading in a hydraulic testing machine at constant loading rates of 0.01, 0.10, 1.00 and 2.50 mm/sec. These test results were then used to generate a predictive model for the lower yield stress as a function of the applied strain rate which was used to predict the yield stress of each of the blends as well as the pure Derakane 8084 material out to strain rates on the order of 10^3 sec^{-1}. The prediction was then compared against results experimentally obtained using a Hopkinson bar. The correlation between the predicted and experimental values was generally good, although universally low by about 10-18% when the Crestomer 1080 concentration was less than 50%. The cause of this deviation is not known, but is likely due to an inertial effect. At concentrations of Crestomer higher than 50%, the rate dependency of the elastic modulus becomes significant due to the viscoelastic nature of Crestomer 1080. Since the model does not account for rate dependency in the elastic modulus, these results are not as accurate.

Thesis Advisor: Dr. Alan J. Brown

Title: Professor of Ocean Engineering

Thesis Advisor: David M. Parks

Title: Professor of Mechanical Engineering

DEDICATION AND ACKNOWLEDGMENTS

I would like to dedicate this thesis to my loving wife Kay and my son Scott without whom this project could not have been accomplished.

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1.0 INTRODUCTION

In naval warship applications such as minehunting vessels, particularly under impact and shock loading, frame to shell connections may be exposed to substantial through-thickness tensile stresses. These stresses are primarily due to initial tensile loading and subsequent peel caused by shock concentration at the frame to shell interface. They are further aggravated by severe frame inertia loading.[1] The current practice is to assume that the resistance of the joint to failure under shock loading may be improved by increasing the pull-off strength under quasi-static loading, in which a top-hat stiffener is pulled from a base panel at a rate of approximately 1 mm/minute. Under these conditions the secondary bond between the stiffener and the base panel is assumed to be isothermal during the pull-off process, and any strain rate effects which may be present due to the shock loading phenomena are ignored.

The assumption of strain rate independence of the bondline material cannot be made without careful consideration of the nature of polymeric materials. Polymers are rate dependent materials whose mechanical properties are often strongly effected by both time and temperature. The strain rate corresponding to "static loads" may be taken to be between 10^{-4} and 10^{-3} $\sec^{-1}$. Under conditions of structural vibration with frequencies in the range of 10-50 Hz, strain rates of approximately 1.0 $\sec^{-1}$ occur while in the case of air blast or hydrodynamic impact, strain rates as high as 10^{+3} $\sec^{-1}$ typically occur.[2]

A comprehensive investigation of improvements in stiffener-base plate secondary bond interfaces has been conducted. The results of which may be found in Appendix A. The failure to consider the effects of a six order of magnitude difference in strain rate when using polymer materials represents the single largest deficiency in the study of these interfaces.

The most recent studies indicate that the use of low modulus/high strength materials in the bondline will provide the best stiffener to shell connection. The effects of

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[1] Michael Trimming, "Monocoque GRP Minehunters," *The Royal Institution of Naval Architects London International Symposium on Mine Warfare Vessels and Systems 1989.*

[2] C. S. Smith, *Design of Marine Structures in Composite Materials* (New York: Elsevier Applied Science, 1990), pp. 95-96.

strain rate on the proposed secondary bondline materials needs to be investigated to ensure that they remain compliant at the higher loading rates.

In the most recent studies, Crestomer 1080, a compliant urethane acrylate produced by Scott Bader, was proposed for this application. Due the large strain to failure of this material, which often exceeds 150% true strain, a dramatic increase in the load to failure during quasi-static pull-off testing was achieved as compared to previous testing which used polyester and acrylic materials in the bondline.

Crestomer 1080 is often blended with a vinyl ester to increase its stiffness to a desired level. The most commonly used vinyl ester for this application is Derakane 8084, which is a rubber-toughed variant produced by Dow Chemical. The purpose of this thesis is to investigate the effects of strain rate on the behavior of several blends of these materials and draw conclusions related to their use in this application.

1.10 Scope of the Project

Chapters 2-6 will develop the theoretical background needed to study high strain rate behavior in polymers. Chapter 2 will give a brief introduction to the mechanical behavior of polymers including a review of the so called "four regions of mechanical equivalence" which define the behavior of a polymers over a wide range of temperatures and loading rates. Chapters 3 through 5 will discuss the theory and implications of the three regions of mechanical equivalence which are important in this study. These include linear elasticity, finite-strain elasticity and viscoelaticity. Chapter 6 will describe a model and methodology for analyzing the yield behavior of these blends when they are subjected to high strain-rate loading. Only compressive loading behavior will be investigated. This will eliminate the complexities inherent in tensile testing which include crazing and fracture, and allow for the large plastic strain behavior of the materials to be studied.

The remaining chapters will discuss the experimental procedure and results. Chapter 7 outlines the specimen fabrication procedure, including resin formulation, mold selection, and the machining processes. Chapter 8 introduces the process of Dynamic Mechanical Analysis (DMA), which was used in this project to determine the glass transition temperature of each of the blends. This parameter is often viewed as one of the most important in the study of polymer materials. Chapter 9 discusses the initial compression tests which were conducted using an hydraulic Instron testing machine. The tests, which spanned more than three orders of magnitude of strain rate, were used to formulate a model which was used to extrapolate the yield stress corresponding to strain rates on the order of 1000 sec^{-1}. In Chapter 10, the results of several tests conducted at strain rates of approximately $3x10^{+03}$ sec^{-1} using a Hopkinson Bar are presented and compared with the results from the slower strain rate Instron tests including an evaluation of the agreement between the predicted and measured values. Chapter 11 will conclude with a discussion of the results and their implications along with suggestions for future study in this area.

2.0 MECHANICAL BEHAVIOR OF POLYMERS

To understand the effects of strain rate on the behavior of either a single polymer or a mixture of several polymers, it is first necessary to review some of the relevant topics in the study of polymer material behavior. There are several ways in which polymers can be grouped. In one typical grouping, they are classified as either thermoplastic or thermosetting. The thermoplastic polymers are those which soften and eventually flow as a viscous liquid when heated. They consist of linear branched-chain molecules with strong intramolecular (within the molecular chain) and weak intermolecular (between chain) bonding. Melting and solidification of these polymers is reversible, and they can be reshaped by the application of heat and pressure. Thermosetting polymers, by contrast, have a highly crosslinked or network structure with strong covalent bonds between chains, which form during the curing process. These crosslinks inhibit flow and, as a consequence, thermosets do not undergo liquid flow but decompose when heated to sufficiently high temperatures. Examples of thermosets include polyester, epoxy, vinyl ester, and phenolics.

As a result of cost and production constraints, virtually all marine composites utilize thermosetting polymers. Thermosets always have an amorphous structure, so only this class will be considered in this study. The next section will discuss the assumptions made in the idealized modeling of amorphous macromolecular chains.

2.10 The Glass Transition Temperature

Polymer properties are known to be very sensitive to both temperature and load rate, particularly near the **glass transition temperature (Tg)**. An understanding of the this material parameter is perhaps the most important piece of information when considering the behavior of amorphous polymer materials.

The glass transition temperature separates the region of glassy behavior (below Tg) from the region of elastomeric behavior (above Tg). In the temperature range very near Tg, the polymer is in a state of transition, and it behaves in a viscoelastic manner. It is in this region that temperature and strain rate effects have the greatest impact on the mechanical behavior of polymer materials.

There is a dramatic change in most of the material properties of a polymer at the glass transition temperature. For example, there is a sharp increase in modulus (about three orders of magnitude) when the temperature is reduced from about 30^0 C above Tg to 30^0 C below Tg. There are also large changes in other physical properties such as the specific volume, the heat capacity and the coefficient of thermal expansion.

The abrupt changes in physical properties have allowed for the development of a wide variety of tests to measure the glass transition temperature. Among these tests are Differential Scanning Calorimetry (DSC) which measures changes in heat capacity during the transition from the glassy to the rubbery state, and Dynamic Mechanical Analysis (DMA) which directly measures the modulus by loading a thin specimen sinusoidally in the elastic range as the temperature is incrementally increased from a temperature below Tg to above Tg.

In the case of thermoset materials, the crosslink density is very high and the change in heat capacity during phase changes is difficult to detect using DSC techniques. For these materials, the use of DMA allows for a clear, accurate and repeatable measurement of Tg. DMA analysis will be discussed in greater detail in a later section.

2.11 Kinetic Versus Thermodynamic Interpretations of the Glass Transition Temperature

As a result of the abrupt changes in the some of the physical properties, attempts have been made to analyze the glass transition temperature as either a first-order or second-order thermodynamic transition process. In a first-order thermodynamic transition, a sharp change occurs in one of the fundamental thermodynamic properties such as enthalpy or volume, while a second-order transition is reflected by a sharp change in the first derivative of one of these properties. Several investigators have shown a connection between the glass transition temperature and second-order thermodynamic properties such as the specific heat and the volume thermal expansion coefficient, but it was later shown that the measured value of the glass transition temperature depends on the rate at which the temperature is changed[1]. It was found that the lower the heating rate used in raising the temperature from temperatures corresponding to the region of glassy behavior, the lower the measured glass transition temperature.

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[1] Kovacs, *J. Polym. Sci.*, **30**, (1958), 131.

Because of the dependence on the cooling rate of the measured value of the glass transition temperature, it cannot be strictly described as a thermodynamic transition event. Thermodynamics considers steady state events which are assumed to be time invariant. The glass transition is therefore generally accepted as a kinetic rather than a thermodynamic event. It should be noted, however, that the change in Tg with decreasing cooling rate is very slow, changing by only about 2-3 degrees Celcius when the cooling rate is varied from 1°C/min to 1°C/day[2], so it is possible to define a value of the glass transition temperature which is nearly independent of the cooling rate. The glass transition temperature will be discussed in a later section dealing with linear viscoelastic theory.

2.12 Effects of Polymer Structure on the Glass Transition Temperature

The glass transition temperature (Tg) varies over a fairly wide range from one polymer to another. These differences are due to the molecular differences between polymers. A brief list is given below[3]:

1. Inflexibility of the repeat unit of the macromolecule increases Tg. This is related to the absence of hinges in the chain backbone.

2. The presence of atoms or groups of atoms in the repeat unit which participate in dipolar or secondary interactions with segments of neighboring macromolecules tend to increase Tg.

3. Tg increases with increasing average macromolecular weight.

4. As the degree of crosslinking increases, the glass transition temperature increases slightly. This is an expected result, since crosslinking suppresses the large elastic deformations typically associated with elastomeric materials.

Examples of the glass transition temperature for a variety of materials are shown in Figure 2-1. Note that the presence of aromatic rings, bulky side groups, and cyanide groups result in much larger values of Tg than in the linear chain molecules.

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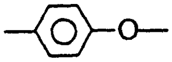
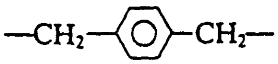
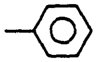
[2] I.M. Ward, *Mechanical Properties of Solid Polymers 2nd ed.* (New York:John Wiley and Sons, 1990), p. 150.

[3] I.V. Yannas, Introduction to Polymer Science and Engineering: A Set of Lecture Notes, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1981, pp. 77-78.

Approximate values of glass transition temperature, T_g, for various polymers

Repeat unit		T_g/K
$-CH_2-CH_2-$		140–270
$-CH_2-CH_2-O-$		206
		357
		353
	Side group (X)	
$-CH_2-CHX-$	$-CH_3$	250
	$-CH_2-CH_3$	249
	$-CH_2-CH_2-CH_3$	233
	$-CH_2-CH(CH_3)_2$	323
		373
	$-Cl$	354
	$-OH$	358
	$-CN$	370

R.J. Young and P.A. Lovell. *Introduction to Polymers 2nd ed.*
(New York:Chapman & Hall, 1992), p.296.

Figure 2-1

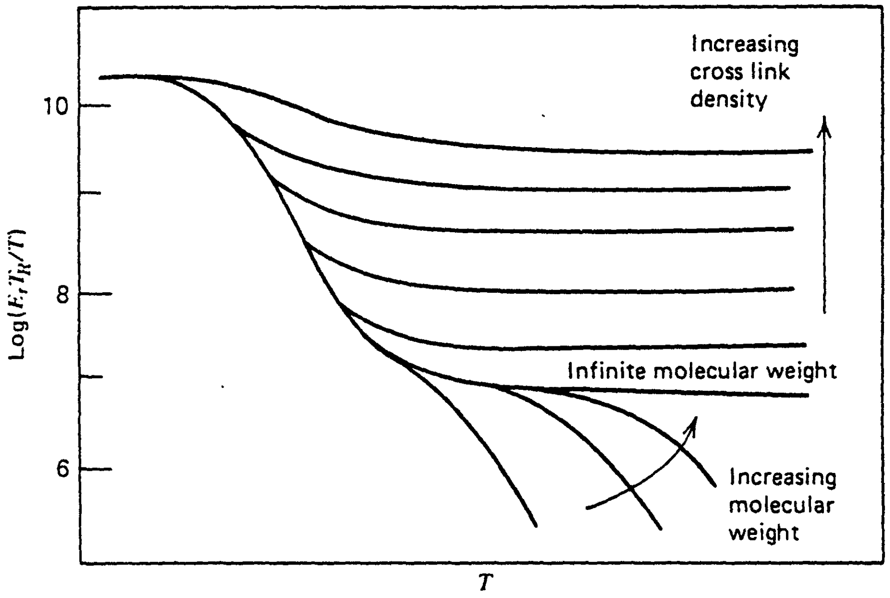
2.20 Representation of Amorphous Polymers Using Modulus-Temperature Curves

Due to the strong temperature dependence on the behavior of polymeric materials, the mechanical behavior of polymers subjected to small strain is often represented by plots of temperature versus the logarithm of modulus (E-T curves). These curves are plotted assuming a fixed strain rate since, in the case of amorphous polymers, an increase in strain can be equated to a decrease in temperature. The equivalence of time and temperature for an amorphous polymer under small strain will be discussed in Chapter 5.

The shape of the modulus-temperature curve is effected to a large extent by the molecular details of the polymer in use and reflects the versatility offered by these materials. As an example, Figure 2-2 shows the effect of increasing the polymer average molecular weight and crosslink density. As the molecular weight of the polymer macromolecule is increased, the elastomeric region is increased, while increasing the density of crosslinks increases the modulus in the elastomeric region and also increases Tg slightly. Recall that themosetting polymers typically have a very high crosslink density.

2.30 The Four Regions of Mechanical Equivalence

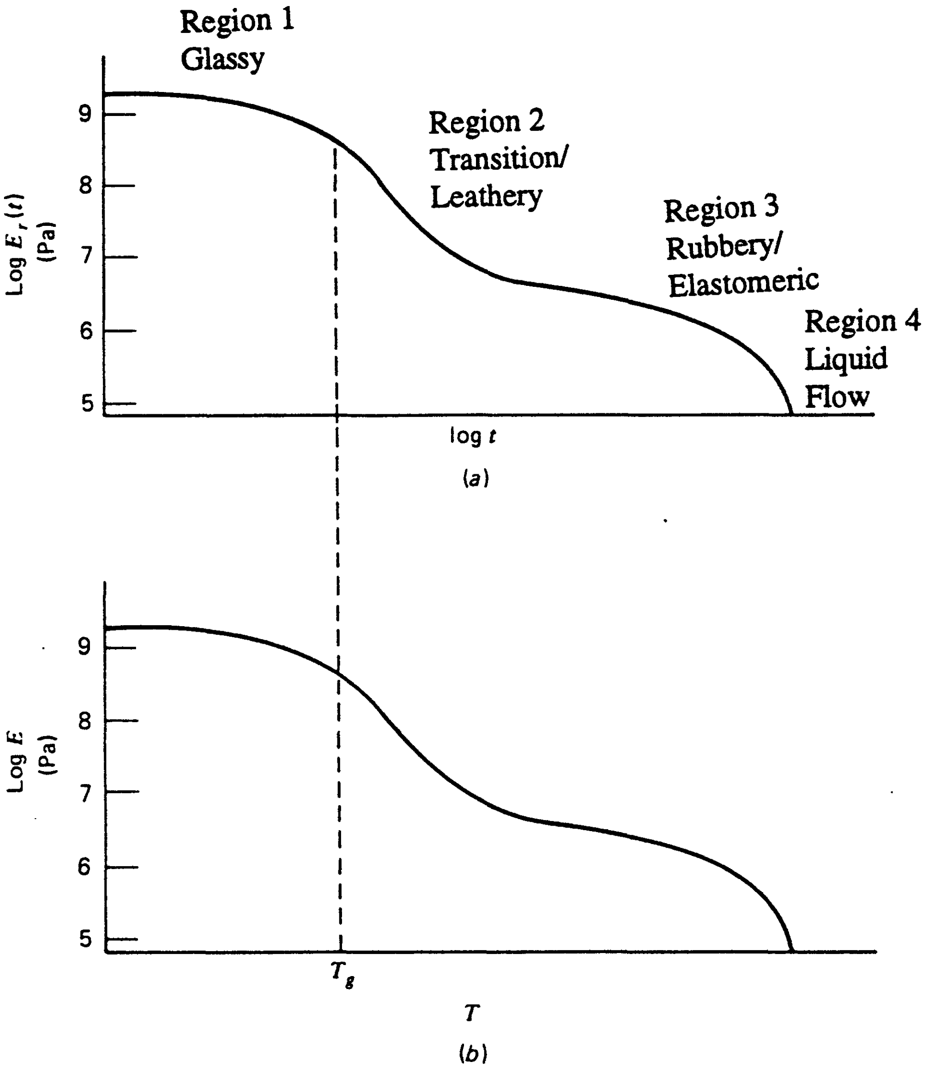
The modulus-temperature (or modulus-time) diagram is typically divided into the four regions of mechanical equivalence in which different constitutive laws apply. These regions are known as glassy (Region I/below Tg), transition/leathery (Region II/near Tg), rubbery/elastomeric (Region III/above Tg), and liquid flow (Region IV/well above Tg) as shown in Figure 2-3. It was mentioned earlier that thermosetting polymers are highly crosslinked and decompose rather than flow when heated, so they do not display region IV behavior. Table 2-1 indicates, qualitatively, the time dependence, mechanical memory, and the simplest applicable branch of mechanics which can apply to a crosslinked amorphous polymer in the three applicable regions of mechanical equivalence. Time dependency reflects how important the loading rate is in determining the mechanical response, and mechanical memory indicates both how rapidly and how completely the material regains its original shape following load removal.



Effect of molecular weight and degree of cross-linking on relaxation modulus.

Richard W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials 2nd ed.* (New York:John Wiley and Sons, 1983), p. 205.

Figure 2-2



Time-temperature dependence of elastic modulus in thermoplastic polymeric solids: (*a*) change in relaxation modulus $E_r(t)$ as function of time; (*b*) change in tensile modulus as function of temperature.

Richard W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials 2nd ed.* (New York:John Wiley and Sons, 1983), p. 203.

Figure 2-3

TABLE 2-1

REGION	TIME DEPENDENCE	MECHANICAL MEMORY	APPL. BRANCH OF MECHANICS
1	WEAK	STRONG	LINEAR ELASTICITY
2	STRONG	MODERATE	LINEAR VISCOELASTICITY
3	WEAK	STRONG	RUBBER (LONG-RANGE) ELASTICITY

The following sections will review the essential elements and limitations of material behavior in the three regions of mechanical equivalence as indicated in table 1. The theory of linear elasticity will be presented first, followed by the theory of rubber elasticity. This is a convenient progression since rubber elasticity can be viewed macroscopically as an extension of linear elasticity which accounts for finite strains. Phenomenologically, however, linear elasticity models materials as "linear energy (Hookian) springs", while rubber materials are modeled as "entropy springs". Finally the elements of the theory of linear viscoelasticity will be presented. This theory is the only one which allows for rate dependency in the modulus by the addition of viscous damping terms to the constitutive relations.

2.40 Yielding, Crazing, and Fracture of Amorphous Polymers

The yielding behavior of metals has been widely studied and is well understood. One of the main assumptions in these theories is that the yield stress is independent of hydrostatic pressure. In contrast, yield in polymers is a strong function of the hydrostatic pressure. It should be noted at this point that yield only occurs in polymers which are in the glassy phase. This will be discussed in greater detail in Chapter 6. Recall that the pressure is defined as:

$$P = -\frac{1}{3}\mathrm{tr}[\sigma] = \frac{1}{3}[\sigma_1 + \sigma_2 + \sigma_3] \qquad (2\text{-}1)$$

where: $\mathrm{tr}[\sigma]$ is the trace of the stress tensor

The effect of hydrostatic pressure on the yield stress is particularly significant in amorphous polymers. This can be explained by viewing the yield stress as the point at which the barriers to chain segment rotation have been overcome and the material is able to flow.

Some of the barriers to chain segment rotation are configurational while others are intermolecular. The configuratonal barriers are those which are due to the resistance of a chain to rotation about its backbone, independent of its neighbors. These effects are not strongly dependent on pressure.

The intermolecular barriers to chain segment rotation arise due to the interactions between chains. It is reasonable to assume that the pressure contribution to the yield stress results in a decreased distance between polymer chains. As a result, there is an increase in the intermolecular barriers to chain segment rotation, and a higher yield stress with increasing pressure.

Since the hydrostatic pressure level is higher in compression than in tension, polymers often have a higher yield stress when tested in compression than when tested in tension. This necessitates a modification of the applicable yield criteria before they can be used to predict the yield stress.

One of the most commonly used criteria to predict yield is the Von Mises yield criteria. This criteria is most easily implemented using principle stress values. Recall that any stress tensor can be transformed into a stress tensor which only has terms on the diagonal (no shear terms) by a transformation of coordinates. In terms of principle stresses, and accounting for the pressure dependence of yield, the Von Mises yield criteria is given as follows:

$$2(\sigma_{yc}-\sigma_{yt})[\sigma_1+\sigma_2+\sigma_3]+\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\right]=2\sigma_{yc}\sigma_{yt}$$

where: σ_{yc} and σ_{yt} are the compressive and tensile yield stress (2-2)

σ_1, σ_2, and σ_3 are the three components of the principle stress tensor

Equation (2-2) reduces to the traditional pressure-independent Von Mises yield criteria when the tensile and compressive yield stresses are equal.

Another effect of pressure on the behavior of a glassy polymer is the formation of crazes. Crazes are a localized form of plastic deformation which occur when the polymer is subjected to an overall hydrostatic tensile stress. This process, which occurs in the elastic regime at approximately 1/2 the yield stress, results in a significant increase in volume as chain segments are separated from each other. In the process of chain segment separation, regions of cavitated polymer are produced. This process does not usually affect the modulus of the material, so the elastic response is not effected. The voids do, however, serve as nucleation sites for cracks which can lead to failure by fracture prior to the yield point.

This study will only consider the compressive behavior of a series of polymer blends as a function of strain rate. This decision was motivated by the general lack of information on the particular blends of interest. To begin to investigate the behavior of these materials under high strain rate loading, it was necessary to minimize the number of design variables. The choice of compressive loading eliminates the possibility of crazing and fracture when the material undergoes homogeneous deformation.

3.0 LINEAR ELASTICITY

The theory of linear elasticity is important in the description of the small strain behavior of glassy polymers. In this regime, the true-stress is linearly related to the true-strain by a constant factor known as the **modulus of elasticity** which represents the rate-independent stiffness of a glassy polymer. This important material parameter is experimentally obtained in a simple compression or tension test as the slope of stress-strain curve.

The theory of linear elasticity is represented by the well known generalized Hooke's Law. This theory is a good approximation to the behavior of many materials when the strains are small, typically on the order of 1%. Its simplicity of use makes it a popular choice for the initial design of a wide variety of structures. There are, however, some important limitations of the theory which limit its use, particularly in the study of polymeric materials.

A primary assumption of the theory is that the material behaves like a linear/Hookean spring. As a consequence, the deformations resulting from an applied load are independent of the history or rate of loading. When a load or displacement is applied to a linear elastic material, the load does not change as a function either of time or temperature. When the load is removed, the material is assumed to return instantaneously and completely to its original state.

The assumption of linearity in this theory indicates that the tensor components of stress may be represented by a linear function of all the tensor components of strain and vice versa. The stress can be computed by multiplying the strain tensor by a stiffness matrix, which is a 6x6 matrix in the most general case of a fully anisotropic material. The number of independent constants may be reduced when material symmetry exists. In the limit, a material which has an infinite number of planes of symmetry such that its behavior is independent of orientation is said to be **isotropic**. In this case, the slope of the tensile or compressive stress strain curve is known as the modulus of elasticity or the Young's Modulus.

This linear behavior is strictly applicable in polymeric materials only for very small values of strain. When strained beyond this limit, known as the proportional limit, many polymers behave in a non-linear fashion. It should be noted that unlike metals, in which the yield point closely corresponds to the proportional limit, polymers are often capable of considerable non-linear deformation without suffering any permanent deformation.

Time independent behavior is a reasonable assumption for glassy polymers when the strains are small (on the order of 1% or less) and molecular motion is limited to only a few segments of any macromolecular chain. The small strain behavior of real polymeric materials is time dependent to a greater or lesser extent depending on the imposed temperature and strain rate. This is particularly important in the vicinity of the glass transition temperature (Tg), which defines the transition region between glassy and elastomeric behavior. Linear elasticity is modified to account for the effects of time in the theory of linear viscoelasticity, at the expense of a significant increase in computational complexity. This theory, which is discussed in Chapter 5, incorporates time dependency into the constitutive relationships by the addition of one or more viscous damping terms.

The assumption of small strain implies that any terms in the strain-displacement equations which are of second order can be neglected. This theory fails when an amorphous polymer is in the region of rubber elasticity where elastic strains on the order of 300-700% are possible. Elastic strains on this level are unique to amorphous polymers, and are modeled by the theory of finite strain elasticity, which will be discussed in Chapter 4.

Despite the limitations of its use, the theory of linear elasticity is widely used in the stress analysis of components constructed of polymeric materials particularly in defining the behavior of glassy polymers prior to yield and is therefore worthy of some discussion.

The generalized Hooke's law relating stresses to strains can be written in contracted notation as follows:

$$\sigma_i = C_{ij} * \varepsilon_j \quad i,j = 1, 2, \dots, 6 \qquad (3\text{-}1)$$

where: C_{ij} is the stiffness matix

or:

$$\varepsilon_i = Sij * \sigma_i \quad i,j = 1,2, \dots, 6 \qquad (3\text{-}2)$$

where: S_{ij} is the compliance matrix

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} \qquad \varepsilon_{ij} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}$$

where: σ_j are normal stresses and τ_j are shear stresses, and

ε_j are normal strains and γ_j are shear strains

note that strain notation in contracted notation is as follows:

$$\varepsilon_1 = \frac{\partial u}{\partial x} \quad \varepsilon_2 = \frac{\partial v}{\partial y} \quad \varepsilon_1 = \frac{\partial w}{\partial z} \quad \gamma_{23} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\gamma_{23} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \gamma_{31} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \quad \gamma_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

where: u, v, and w are displacements in the x, y, and z directions

Note that both the stiffness and compliance matrices are represented by a 6 x 6 matrix and $[C_{ij}] = [S_{ij}]^{-1}$. These matrices are fully populated in the case of a fully anisotropic material, but the number of independent coefficients can be reduced by noting material planes of symmetry.

For the most general case of an element of **anisotropic** material, also known as one with triclinic symmetry, the 6 x 6 compliance matrix can be written as follows with 21 independent compliance coefficients since $S_{ij} = S_{ji}$:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \qquad (3\text{-}3)$$

If one plane of symmetry is present, in this case through thickness midplane symmetry, **monoclinic symmetry** exists and the compliance matrix reduces to the following 13 independent compliance coefficients:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix} \qquad (3\text{-}4)$$

If there are two planes of material property symmetry then symmetry will also exist on a third mutually orthotropic plane. A material displaying this form of symmetry is said to be **orthotropic** with 9 independent constants. Note that there is no interaction between normal stresses and shear strains for orthotropic materials aligned with the principal material directions. These directions are parallel to the intersections of the three orthogonal planes of material symmetry.

The stiffness matrix in coordinates aligned with the principal material directions is as follows:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \tag{3-5}$$

The elements of the compliance matrix are related by simple relationships to the engineering constants Young's modulus (E), Poisson's ratio(v), and shear modulus(G). The compliance matrix (Sij) for an orthotropic material in terms of engineering constants is can be represented as follows:

$$S_{ij} = \begin{bmatrix} \frac{1}{E_{11}} & \frac{-v_{12}}{E_{22}} & \frac{-v_{13}}{E_{33}} & 0 & 0 & 0 \\ \frac{-v_{21}}{E_{11}} & \frac{1}{E_{22}} & \frac{-v_{23}}{E_{33}} & 0 & 0 & 0 \\ \frac{-v_{31}}{E_{11}} & \frac{-v_{32}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{3-6}$$

In a wide variety of materials under small strain, the material properties do not vary significantly with orientation. This is true of amorphous polymers under small strain due to the lack of significant chain alignment. In this case, an infinite number of planes of material symmetry exist. The number of independent constants reduces to 2, and the material is said to be **isotropic**. The compliance matrix for such a material is as follows:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) \end{bmatrix} \tag{3-7}$$

For an isotropic material, the compliance matrix (Sij) can be written in terms of a single value of Young's modulus (E), Poisson's ratio(v), and shear modulus (G) as follows:

$$S_{ij} = \begin{bmatrix} \frac{1}{E} & \frac{-v}{E} & \frac{-v}{E} & 0 & 0 & 0 \\ \frac{-v}{E} & \frac{1}{E} & \frac{-v}{E} & 0 & 0 & 0 \\ \frac{-v}{E} & \frac{-v}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \tag{3-8}$$

From these relations, the well know stress-strain relationships for isotropic linear elasticity can be written as follows:

$$\begin{aligned}
\varepsilon_{xx} &= \frac{1}{E}\sigma_{xx} - \frac{\nu}{E}\left(\sigma_{yy} + \sigma_{zz}\right) \\
\varepsilon_{yy} &= \frac{1}{E}\sigma_{yy} - \frac{\nu}{E}\left(\sigma_{xx} + \sigma_{zz}\right) \\
\varepsilon_{zz} &= \frac{1}{E}\sigma_{zz} - \frac{\nu}{E}\left(\sigma_{xx} + \sigma_{yy}\right) \\
\varepsilon_{xz} &= \frac{\tau_{xz}}{G} \\
\varepsilon_{yz} &= \frac{\tau_{yz}}{G} \\
\varepsilon_{xy} &= \frac{\tau_{xy}}{G}
\end{aligned} \tag{3-9}$$

The isotropic shear modulus is related to the Young's modulus and Poisson's ratio by the following expression:

$$G = \frac{E}{2(1+\nu)} \tag{3-10}$$

Another basic quantity which is of importance in linear elasticity, particularly in the case of polymers, is the bulk modulus (K), which determines the dilation (Δ) produced by a uniform hydrostatic pressure (p):

$$\begin{aligned}
&K = \frac{p}{\Delta} \\
&\textit{where}: \Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}
\end{aligned} \tag{3-11}$$

Using the isotropic linear elastic stress strain relationships it can be shown that the strains produced by a uniform hydrostatic pressure (p) are as follows:

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \left(S_{11} + 2S_{12}\right)p \tag{3-12}$$

Then the bulk modulus is equal can be written as:

$$K = \frac{p}{\Delta} = \frac{1}{3(S_{11} + 2S_{12})} = \frac{E}{3(1 - 2\upsilon)} \tag{3-13}$$

Table 3-1[1] is an order of magnitude comparison of the elastic engineering constants for an isotropic amorphous polymer in each of the three relevant regions of mechanical equivalence, showing a difference in Young's modulus (E) of approximately three orders of magnitude between the glassy and elastomeric regions. It should be noted that the values given for the Young's modulus and the shear modulus (G) are represented as a constant value. This is only strictly true when the stress strain behavior is linear, as was assumed for glassy polymers. Viscoelastic materials are rate dependent so the modulus values given represent a range from the glassy to the elastomeric values. In the elastomeric region, the modulus is known as the rubbery modulus. This value represents the elastic response which occurs when an elastomer is loaded at a high rate. It should be noted that the bulk modulus of an elastomer is very large resulting in Poisson ratio very close to 0.50, indicating that elastomeric materials are essentially incompressible. The assumption of incompressibility is important in the development of constitutive models for the finite strain elasticity of elastomeric materials.

TABLE 3-1

REGION	E (Pa)	G (Pa)	K (Pa)	v
1	10^9 - 10^{10}	10^9	10^9 - 10^{10}	0.33
2	10^6 - 10^9	10^5 - 10^9	10^6 - 10^9	0.40 - 0.45
3	10^5 - 10^6	10^4 - 10^5	$\propto$	0.50

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[1] I.V. Yannas, "Introduction to Polymer Science and Engineering: A Set of Lecture Notes," Deformation and Fracture of Polymers. Massachusetts Institute of Technology, 1994, 101.

4.0 FINITE STRAIN / RUBBER ELASTICITY

Finite strain behavior is important in the large deformation behavior of amorphous polymers. The premise behind this behavior is that large strain results in a decrease in the configurational entropy of the material which gives rise to a stiffening effect. Most of the work in the area of entropic response has been directed towards the development of predictive models for the behavior of elastomeric materials, so this is the primary focus of this section. The theories have also been use to model the entropic resistance to plastic deformation in glassy polymers.

Elastomeric material behavior (also known as rubber elasticity, finite strain elasticity, and long-range elasticity) is unique to crosslinked amorphous polymers which have an average macromolecular weight greater than approximately 10,000 and have been heated to temperatures of $T_g + 30°C$ or greater. Elastomers meeting these requirements are capable of non-linear elastic strains on the order of 300-700%. Polymers with an average macromolecular weight significantly less than 10,000, semicrystalline polymers, and uncrosslinked polymers do not exhibit characteristic rubberlike behavior.

In addition to the large elastic strain limits exhibited by elastomers, several other unusual characteristics have been observed. As an example, it has been observed by many investigators that the modulus of an elastomer increases with increasing temperature. This is in contrast to what is seen in nearly all other engineering materials which soften with increasing temperature. An understanding of this unusual behavior begins with the thermodynamics of elastomeric behavior.

4.10 Thermodynamics of Elastomeric Behavior

The thermodynamic properties of interest in the study of elastomeric material behavior depend on the conditions under which the material parameters in the constitutive equations are experimentally obtained.

Recall that the first law of thermodynamics states:

$$dW = dU - dQ \tag{4-1}$$

where: dW is the work done on the material

dU is the increase in internal energy

dQ is the heat supplied to the material

For an adiabatic experiment conducted at constant volume (V), the work done on the material is equal to the increase in internal energy (U). In this case,

$$dQ = 0 \text{ and } dW = dU \tag{4-2}$$

For an adiabatic experiment conducted under constant pressure (P), the work done on the material is equal to the enthalpy (H), where H=U+PV. In this case,

$$dQ = 0 \text{ and } dW = dU + PdV = d(U+PV)p = dH \tag{4-3}$$

For an isothermal experiment conducted at constant volume, the work done on the material is equal to the Helmholtz free energy (A), where A = U - TS. The variable (T) is the absolute temperature, and (S) is the entropy. In this case,

$$dQ = TdS \text{ and } dW = dU-TdS = d(U-TS)t,v = dA \tag{4-4}$$

Finally, for an experiment conducted at constant temperature (T) and pressure (P), the work done on the material is equal to the Gibbs free energy (G), where G = U + PV - TS = H - TS. In this case,

$$dQ = TdS \text{ and } dW = dU + PdV - TdS = d(U+PV-TS)t,p = dG \tag{4-5}$$

Most of the experimental observations in the study of elastomers have been made under conditions of constant pressure. This indicates that work done on an element of material is best represented by the Gibbs free energy. Several investigators have used the simplified Helmholts free energy equation which differs from the Gibbs free energy by the single differential term PdV. This simplification can often be made in the case of elastomers since they behave in a nearly incompressible manner, so the change in volume (dV) is nearly zero.

The work done on an elastomeric material (dW), originally of length (l), when it is extended under the action of a tensile force (f) is given as:

$$dW = f\,dl \tag{4-6}$$

Using the first law of thermodynamics and setting the work done on the material equal to the Helmholtz free energy gives:

$$f\,dl = dU - TdS \tag{4-7}$$

At constant volume the tensile force can be written as:

$$f = \left(\frac{\partial A}{\partial l}\right)_T = \left(\frac{\partial U}{\partial l}\right)_T - T\left(\frac{\partial S}{\partial l}\right)_T \qquad (4\text{-}8)$$

The entropy term cannot be measured directly by any known experimental method, so a further manipulation is required. The Helmholtz free energy in its most general form is:

$$dA = dU - TdS - SdT \qquad (4\text{-}9)$$

From equation (4.9),

$$\left(\frac{\partial A}{\partial l}\right)_T = f \quad \text{and} \quad \left(\frac{\partial A}{\partial l}\right)_l = -S$$

but,

$$\frac{\partial}{\partial l}\left(\frac{\partial A}{\partial T}\right)_l = \frac{\partial}{\partial T}\left(\frac{\partial A}{\partial l}\right)_T \qquad (4\text{-}10)$$

which gives,

$$\left(\frac{\partial S}{\partial l}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_l$$

Combining equations (4.8) and (4.10) gives the following expression for the tensile force:

$$f = \left(\frac{\partial U}{\partial l}\right)_T + T\left(\frac{\partial f}{\partial T}\right)_l \qquad (4\text{-}11)$$

It was shown by Meyer and Ferri[1] (1935) and later by Gee[2] (1946) that the tensile force of a rubber specimen at constant length is almost linearly proportional to the absolute temperature and the force can be extrapolated nearly to zero tension at absolute zero. This was an early indication that the contribution of the internal energy to the tensile force is small.

Although when modeling elastomeric behavior, the internal energy contribution to the tensile force is typically neglected, its contribution is not always insignificant. As an

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[1] K.H. Meyer and C. Ferri, *Helv. Chim. Acta*, **18**, (1935), 570.

[2] G. Gee, *Trans. Faraday Soc.*, **42**, (1946), 585.

example, it was shown[3] that, for the case of natural rubber at room temperature, the internal energy contribution dominates at true strains below about 40% at which point the energy and entropy contributions were nearly equal. At larger values of strain, the entropy term dominates. For strain values greater than 150%, the force could be explained almost completely on the basis of entropy changes. In the case of natural rubber, the contribution of the internal energy to the force is about 15% of the total. Other studies[4,5] (1961, 1962) confirmed these results, claiming that the internal energy contribution to the total tensile force is approximately 20% of the total force at room temperature. Although these values are not trivial, the internal energy contributions to the deformation of elastomeric materials are neglected in most of the commonly used constitutive modes. The result is that while the constitutive models in linear elasticity consider a material to behave like a linear (Hookian) energy spring, rubber elasticity considers the material to act like an entropy spring. The models which completely neglect the contribution of internal energy are known as **ideal rubber** models.

It has been observed that the temperature of some elastomers increase when stretched to very high values of strain. This behavior can be partly due to crystallization which can occur in some amorphous polymers at very large values of strain. Recall the Gibbs free energy (G):

$$dG = dH - TdS \qquad (4\text{-}12)$$

A system is in a state of thermodynamic equilibrium when the Gibbs free energy is at a minimum. In the case of semi-crystalline polymers (which do not display any elastomeric behavior), the temperature at which phase equilibrium exists between the crystalline form and the amorphous form is known as the **melting point (Tm)**. Above this temperature the polymer is in the amorphous phase and below it is semi-crystalline. The melting point occurs when the Gibbs free energy difference between phases is equal to zero.

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[3] I.V. Yannas, "Introduction to Polymer Science and Engineering: A Set of Lecture Notes," Deformation and Fracture of Polymers. Massachusetts Institute of Technology, 1994, p. 149.

[4] G. Allen, U. Bianchi and C.Price, *Trans. Faraday Soc.*, **59**, (1963), 2492.

[5] A. Ciferri, *Makromolek. Chem.*, **43**, (1961), 152.

From (4-12), the following expression is obtained for the melting point:

$$T_m = \frac{\Delta H}{\Delta S}$$

where: ΔH is the enthalpy of fusion (4-13)

ΔS is the entropy of fusion

Strain crystallization occurs in some amorphous polymers at large values of strain because, as polymer chains are stretched, they become more highly aligned. In this highly aligned state, the entropy associated with the chain is very low. The entropy of a polymer can be calculated using the Boltzmann relationship from statistical thermodynamics as follows:

$$S = k \ln \Omega$$

where: k is Boltzmann's constant (1.38066×10^{-23} J / K) (4-14)

Ω is the number of possible confomations which a polymer chain can adopt

The entropy of a polymer chain in a highly extended state is low by equation (4-14) because the number of possible configurations is small. This is offset somewhat by the large decrease in enthalpy which occurs during crystallization. If the change in enthalpy (latent heat) is greater than the product of the melting temperature and the entropy change, crystallization is thermodynamically favored since this phase has a lower value of the Gibbs free energy. The formation of crystals results in an increase in modulus. An increase in temperature which occurs in strain crystallizing elastomers is due to the change in enthalpy which occurs during the change in phase from the amorphous to crystalline state.

The degree and rate of crystallization depends on the extension of the material and the length of time the extension is maintained. As an example, natural rubber at strains of 300-400% crystallizes very rapidly to a degree of crystallinity of over 30%[6]. By contrast, experiments on butadiene rubbers[7] show that these materials do not crystallize at all when

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[6] R.J. Young and P.A. Lovell, *Introduction to Polymers Second Edition* (London: Chapman and Hall, 1994), p. 303.

[7] W.O.S. Doherty, K.L. Lee and L.R.G. Treloar, *Brit. Polymer J.*, **12**, (1980), 19.

extended. These and other elastomers which do not crystallize at large values of strain still demonstrate a temperature increase when stretched at high strain rates.

The increase in temperature due to high strain rate loading can be explained by looking at the adiabatic extension of an elastomer[8]. When the load is applied very rapidly, there is not enough time for the material to reach thermal equilibrium with the environment. This behavior can be modeled as an adiabatic process (dQ = 0), which is also isentropic (dS = 0). The rise in temperature as the length is increased in this process can be written as:

$$\left(\frac{\partial T}{\partial \lambda}\right)_S = -\frac{T}{C_l}\left(\frac{\partial S}{\partial \lambda}\right)_T \tag{4-15}$$

where: C_l is the heat capacity of the elastomer held at constant length
λ is the extension ratio (l / l_0)

Using equation (4-10), this relationship can be rewritten as:

$$\left(\frac{\partial T}{\partial \lambda}\right)_S = \frac{T}{C_l}\left(\frac{\partial f}{\partial T}\right)_l \tag{4-16}$$

where: C_l is the heat capacity of the elastomer held at constant length

It can be concluded from equation (4-16) that the temperature of an elastomer will rise when it is stretched adiabatically as long as the derivative of force with respect to temperature for a specimen held at a fixed length is positive. The earlier discussion shows that this is usually the case. The temperature rise resulting from the adiabatic extension of natural rubber is shown in Figure 4-1. It should be noted that this material crystallizes at large values of strain, which likely accounts for the increase in slope at strains greater than about 300%.

The key observation of this section is that the deformation is assumed to be controlled almost entirely by changes in entropy for large strain as a randomly oriented macromolecular chain undergoes deformation. The next section will look at the entropy of an individual chain, and how this type of model can be used to model the deformation of an elastomeric material.

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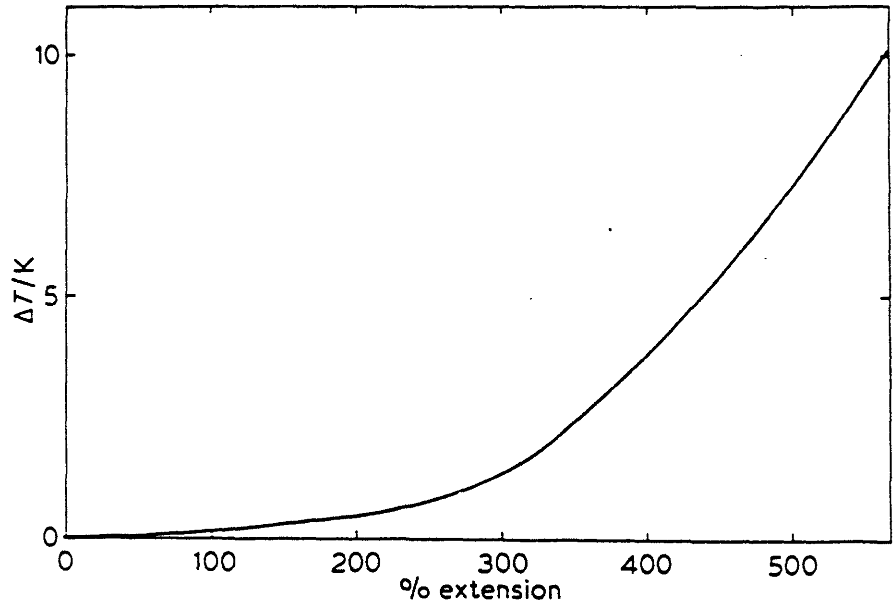
[8] R.J. Young and P.A. Lovell, *Introduction to Polymers Second Editon* (London: Chapman and Hall, 1994), p. 349.



Increase in temperature, ΔT, upon the adiabatic extension of an elastomer (vulcanized natural rubber) (data of Dart, Anthony and Guth reported by Treloar).

R.J. Young and P.A. Lovell. *Introduction to Polymers 2nd ed.* (New York:Chapman & Hall, 1992), p.349.

Figure 4-1

4.20 Entropy of a Single Polymer Chain

The entropy of a polymer chain was derived based on a mathematical model known as the "random walk" by Kuhn[9] (1939) and Guth and Mark[10] (1934). The model used in this analysis was a freely joined chain composed of (N) links, each of length (l). In a freely joined chain, each link is allowed to freely rotate and pivot without restriction relative to its neighbor. The chain end-to-end distance (r) is the distance from point (P), located at the origin to point (Q) located at a point in space as shown in Figure 4-2. If the end-to-end distance is much less than the fully extended chain length (Nl), the probability that (Q) lies within an elemental volume dx dy dz at a distance (r) from the origin is given represented by a Gaussian function as follows:

$$p(\bar{r})\,dr = \frac{b^3}{\pi^{3/2}} \exp\left[-b^2 r^2\right] dr \qquad \text{where: } b^2 = \frac{3}{2Nl^2} \qquad (4\text{-}17)$$

This vector probability can be expressed as a scalar probability, P(r) dr, by noting that the set of all possible values of the vector probability described above would trace out a spherical shell of radius (r) and thickness (dr). In other words, the scalar probability, (P(r)dr) that the scalar value of (r) lies between position r and r+dr is:

$$P(r)dr = 4\pi r^2 * p(\bar{r})dr$$

so,

$$P(r)dr = \left(\frac{4b^3}{\sqrt{\pi}}\right) r^2 \exp\left[-b^2 r^2\right] dr \qquad (4\text{-}18)$$

The number of possible conformations is proportional to the probability, P(r), defined above. The entropy of a single macromolecular chain can be expressed as:

$$s = c - kb^2 r^2 = c - kb^2\left(x^2 + y^2 + z^2\right) \qquad (4\text{-}19)$$

where: c is an arbitrary constant

This entropy equation is based on the assumption that the chain end-to-end distance (r) can be represented by a Gaussian function. A chain which conforms to these

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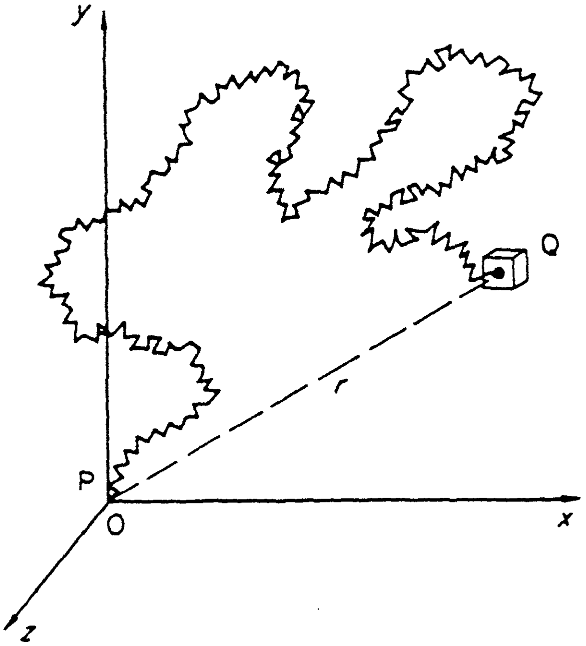
[9] W. Kuhn, *killoidzeitschrift*, **76**, (1936), 258; **87**, (1939), 3 .

[10] E. Guth and H. Mark, *Lit. Chem.*, **65**, (1934),93.



Freely jointed chain made of (N) identical links, each of length (l).

I.M. Ward, *Mechanical Properties of Solid Polymers 2nd ed.* (New York:John Wiley and Sons, 1990), p. 66.

Figure 4-2

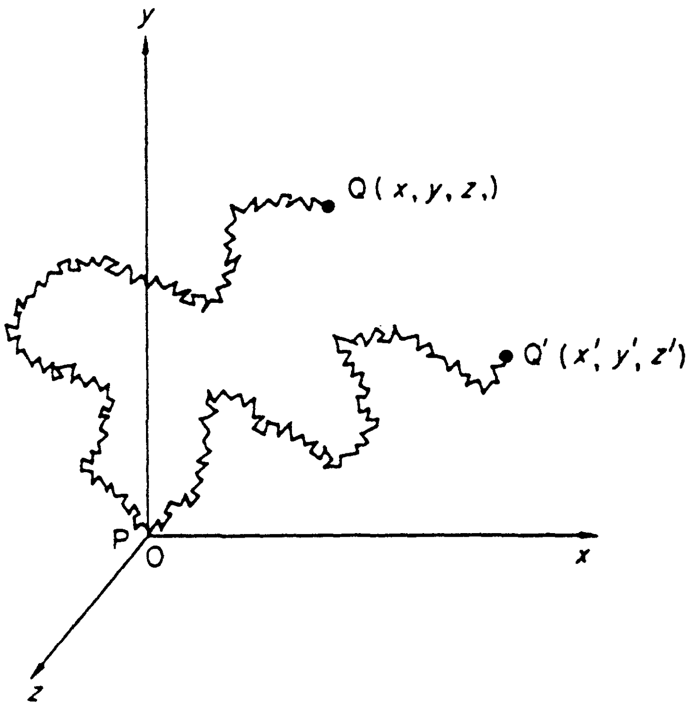
assumptions is known as a **Gaussian chain,** and the bulk materials are known as **neo-Hookian.**

The elasticity of a molecular network can be derived directly from the equation for the entropy of a single macromolecular chain. This analysis will consider the deformation of a chain from point (Q) to (Q') as shown in Figure 4-3.

In finite strain elasticity, the deformations are typically defined in terms of extension ratios. These extension ratios, designated λ, are the ratios of the deformed to undeformed lengths in the three principal directions and are given as:

$$\begin{aligned} \lambda_1{}^2 &= 1+2e_{xx} \\ \lambda_2{}^2 &= 1+2e_{yy} \\ \lambda_3{}^2 &= 1+2e_{zz} \\ \text{and } e_{yz} &= e_{zx} = e_{xy} = 0 \end{aligned} \tag{4-20}$$

At large values of strain, the second order effects in the strain-displacement equations which are typically ignored in the case of small strain elasticity (linear elasticity) become significant and need to be reintroduced.



The end of the chain Q (x, y, z) is displaced to Q (x′, y′, z′).

I.M. Ward, *Mechanical Properties of Solid Polymers 2nd ed.* (New York:John Wiley and Sons, 1990), p. 68.

Figure 4-3

The six components of finite strain are given by the following equations:

$$
\begin{aligned}
e_{xx} &= \frac{\partial u}{\partial x}+\frac{1}{2}\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right\} \\
e_{yy} &= \frac{\partial v}{\partial y}+\frac{1}{2}\left\{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right\} \\
e_{zz} &= \frac{\partial w}{\partial z}+\frac{1}{2}\left\{\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\} \\
e_{yz} &= \frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}+\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}+\frac{\partial v}{\partial y}\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\frac{\partial w}{\partial z} \\
e_{zx} &= \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}\frac{\partial v}{\partial x}+\frac{\partial w}{\partial z}\frac{\partial w}{\partial x} \\
e_{xy} &= \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}
\end{aligned}
\tag{4-21}
$$

where: e_{xx} , e_{yy} , and e_{zz} , are the normal finite strain components in the x, y, and z directions respectively

e_{yz} , e_{zx} , and e_{xy} , are the shear finite strain components

The system of equations has been simplified by considering an isotropic element of material subjected to **homogeneous pure strain**. In this state, the three mutually orthogonal axis known as the **principal strain directions** are not rotated during deformation. As a consequence, the displacements can be represented by three normal strains known as the **principal strain components** oriented in a set of three mutually orthogonal directions known as the **principal strain directions.** In this case, all of the shear strain components are identically zero. Any mutually orthogonal material orientation system can be transformed into the principal strain orientation by the well known eigenvalue/eigenvector transformation. As a consequence, the remainder of this discussion will only consider a state of homogeneous pure strain with no loss in generality.

When a chain end is deformed from point (Q) to (Q') it moves from point (x,y,z) to point (x',y',z'), where:

$$\begin{aligned} x' &= \lambda_1 x \\ y' &= \lambda_2 y \\ x' &= \lambda_3 z \end{aligned} \tag{4-22}$$

The change in entropy when the chain deforms from (Q) to (Q') can be expressed as follows:

$$\Delta s = -kb^2\left[\left(\lambda_1^2 - 1)x^2\right)+\left(\lambda_2^2 - 1)y^2\right)+\left(\lambda_3^2 - 1)z^2\right)\right] \tag{4-23}$$

After summing the contribution of all chains in a network, the following expression is obtained for the total change in entropy as the entire chain network is deformed:

$$\Delta s = -\frac{1}{2}nk\left(\lambda_1^2 + \lambda_1^2 + \lambda_1^2 - 3\right) \tag{4-24}$$

where: n is the number of chains per unit volume in a network

Recalling the change in Helmholtz free energy:

$$\Delta A = \Delta U - T\Delta S \tag{4-9}$$

Assuming the change in internal energy is negligible gives:

$$\Delta A = -T\Delta S \quad \text{so,}$$

$$\Delta A = \frac{1}{2}nkT\left(\lambda_1^2 + \lambda_1^2 + \lambda_1^2 - 3\right) \tag{4-25}$$

The strain tensor (a second-rank tensor) possesses three quantities known as strain invariants which are independent of the choice of coordinate system. These strain invarients are, in general, functions of both normal and shear components of strain. In the case of homogeneous pure strain, the three strain invarients I_1, I_2, and I_3 can be reduced to principal extension values as follows:

$$\begin{aligned} I_1 &= \lambda_1^2+\lambda_2^2+\lambda_3^2 \\ I_2 &= \lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2 \\ I_3 &= \lambda_1^2\lambda_2^2\lambda_3^2 \end{aligned} \tag{4-26}$$

As a simplifying assumption, elastomeric materials are often assumed to be incompressible so the strain invariant I_3 is approximately equal to 1.0.

The change in Helmholtz free energy can be expressed in terms of strain invariants as follows:

$$\Delta A = \frac{1}{2} nkT(I_1 - 3) \tag{4-27}$$

The stress-strain relationships in rubber elasticity are most often defined in terms of a strain-energy functions which are given the symbol (U). This strain energy function is actually one of several thermodynamic quantities with experimentally determined constants, the choice of which depends on the conditions under which the experimental constants were determined. Four strain-energy functions are typically defined which are numerically equal to either the internal energy, the enthalpy, the Helmholtz free energy, or the Gibbs free energy.

In the case of the Gaussian chain network, the strain-energy function is equal to the change in Helmholtz free energy if the strain-energy function is assigned a value of zero in the undeformed state as follows:

$$U = \Delta A = \frac{1}{2} nkT(I_1 - 3) \tag{4-28}$$

The constant nkT is equal to the shear modulus (G) of the material. It can also be represented in terms of the density of the material as follows:

$$G = nkT = \left(\frac{\rho RT}{M_c} \right) \tag{4-29}$$

where: ρ is the density of the material
R is the universal gas constant
M_c is mean molecular weight of the chains

It can be shown that the true stresses which result from the deformation of an incompressible material in the pricipal directions can be given in indicial notation as:

$$\sigma_{ii} = 2\left[\lambda_i^2 \frac{\partial U}{\partial I_1} - \frac{1}{\lambda_i^2} \frac{\partial U}{\partial I_2} \right] + p; \quad \sigma_{ij} = 0 \tag{4-30}$$

where: p is an arbitrary hydrostatic pressure and,
i, j = 1, 2, and 3

The principal stresses are only determinant to within an arbitrary hydrostatic pressure. As a consequence, the stresses are sometimes represented as the differences between principal stresses.

As an example, consider the case of uniaxial tension or compression of a specimen made of a Gaussian elastomer loaded in the 1-direction,

$$\frac{\partial U}{\partial I_1} = \frac{G}{2}\,;\ \frac{\partial U}{\partial I_2} = 0$$

$$\sigma_{11} = 2\left[\lambda_1^2\frac{\partial U}{\partial I_1}\right] + p\,;\ \sigma_{22} = \sigma_{33} = 2\left[\lambda_2^2\frac{\partial U}{\partial I_1}\right] + p = 0$$

so:

$$p = -2\left[\lambda_2^2\frac{\partial U}{\partial I_1}\right]$$

and:

$$\sigma_{11} = 2\left[\lambda_1^2\frac{\partial U}{\partial I_1}\right] - 2\left[\lambda_2^2\frac{\partial U}{\partial I_1}\right] = 2\frac{\partial U}{\partial I_1}\left[\lambda_1^2-\lambda_2^2\right] = G\left[\lambda_1^2-\lambda_2^2\right]$$

incompressibility gives, $\lambda_1\lambda_2\lambda_3 = 1$ and since $\lambda_2 = \lambda_3$ for a specimen loaded uniaxially:

$$\lambda_1 = \frac{1}{\lambda_2^2} = \frac{1}{\lambda_3^2}$$

which gives:

$$\sigma_{11} = G\left[\lambda_1^2 - \frac{1}{\lambda_1}\right] \tag{4-31}$$

The neo-Hookian form of the strain energy function is the simplest function which can be used to describe the deformation of a polymer chain and is limited by the assumption that the chain end-to-end distance (r) is much less than the fully extended length (Nl). A more general description based on Langevin statistics which allows for the modeling of large stretch behavior will be discussed in the next section.

4.30 Non-Gaussian Finite Strain Models

The Gaussian model mentioned in the previous section is a simple representation of an amorphous polymer based on the assumption that the chain end-to-end distance (r) is small relative to the fully extended or locking stretch of the chain (Nl). A modification to the Gaussian theory to allow for accurate modeling of the large stretch behavior of elastomers was initially developed by Kuhn and Grun[11] (1942). The modification began with the introduction of the Langevin probability distribution denoted by the symbol ($\mathcal{L}$,) which led to the following probability distribution for a freely joined chain:

$$\ln p(r) = c - n\left[\frac{r}{Nl}\beta + \ln\frac{\beta}{\sinh(\beta)}\right]$$

where: $\mathcal{L}(\beta) = \frac{r}{Nl} = \coth\beta - \frac{1}{\beta}$

so the inverse Langevin funtion (β) is given as: (4-32)

$$\beta = \mathcal{L}^{-1}\left(\frac{r}{Nl}\right)$$

and the entropy can be expressed as:

$$s = k\left[\ln p(r)\right] = kN\left[\frac{r}{Nl}\beta + \ln\frac{\beta}{\sinh(\beta)}\right]$$

When expanded into an infinite series, the probability distribution ln p(r) and the entropy equation can be expressed as follows:

$$\ln p(r) = c - N\left[\frac{3}{2}\left(\frac{r}{Nl}\right)^2 + \frac{9}{20}\left(\frac{r}{Nl}\right)^4 + \frac{99}{350}\left(\frac{r}{Nl}\right)^6 + \dots\right]$$

so the entropy of a single chain can be expressed as: (4-33)

$$s = c - kN\left[\frac{3}{2}\left(\frac{r}{Nl}\right)^2 + \frac{9}{20}\left(\frac{r}{Nl}\right)^4 + \frac{99}{350}\left(\frac{r}{Nl}\right)^6 + \dots\right]$$

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[11] Kuhn, W. and Grun, F., *Killoid Z.* **101**, (1942), 248.

The first term of the series representing the entropy for the Langevin distribution recovers the Gaussian distribution. This can be shown as follows:

$$s = c - kN\left[\frac{3}{2}\left(\frac{r}{Nl}\right)^2\right]$$

recall, from equation (17):

$$b^2 = \frac{3}{2Nl^2} \qquad (4\text{-}34)$$

so:

$$s = c - kb^2r^2$$

which is the entropy of a single Gaussian chain given in equation (4-19)

The Langevin probability distribution has been used by several investigators in multiple chain models to predict the large stretch behavior of elastomers. These models include a three chain model developed by Wang and Guth[12] (1952), and a tetrahedal model developed by Flory and Rehner[13] (1943). In a recent study, an eight chain model was developed by Boyce and Arruda[14] (1993) as shown in Figure 4-4. In this model, it is assumed that the material behaves incompressibly and the sides of the element shown in Figure 4-4 remain aligned with the principal stretch directions during deformation. The unstretched network contains eight chains, each of length $r_o = \sqrt{N}\,l$ in a cube of dimension $a_o = \frac{2}{\sqrt{3}} r_o$. Following a development similar to that for the Gaussian chain, the strain energy potential for this eight chain model can be expressed as a function of the first strain invariant as follows:

$$U = nkT\left[\begin{array}{l}\frac{1}{2}(I_1 - 3) + \frac{1}{20N}(I_1{}^2 - 9) + \frac{11}{1050N^2}(I_1{}^3 - 27) + \ldots \\ \ldots + \frac{19}{7000N^3}(I_1{}^4 - 81) + \frac{519}{673750N^4}(I_1{}^5 - 243) + \ldots\end{array}\right] \qquad (4\text{-}35)$$

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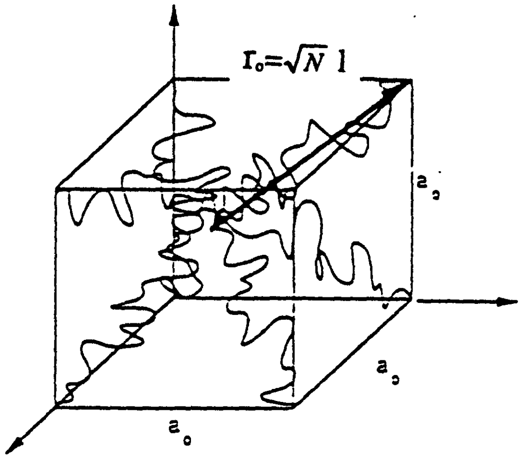
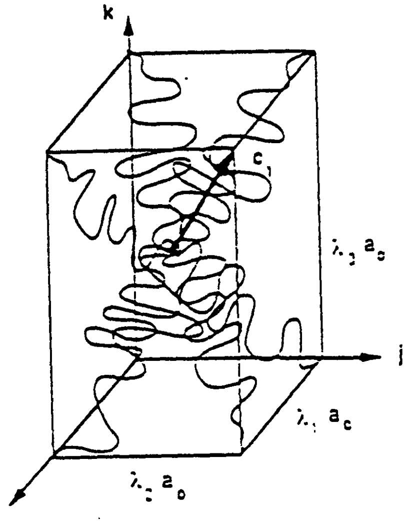
[12] Wang, M.C. and Guth, E., *J. Chem. Phys.* **20**, (1952), 1144.

[13] Flory, P.J. and Rehner, J., *J. Chem. Phys.* **11**, (1943) 512.

[14] Boyce, M.C. and Arruda, E.M., *J. Mech. Phys. Solids* **41**, (1993) 389.



The unstretched network for the proposed eight chain model.



The eight chain network in a stretched configuration

Ellen M.Arruda and Mary C. Boyce, "A Three-Dimensional Constitutive Model for the Large Strech Behavior of Rubber Elastic Materials," *J. Mech. Phys. Solid*, Vol.41, No.2,(1993), 395-396.

Figure 4-4

4.40 Phenomenological Invariant-Based Models

Phenomenological invariant based models attempt to model the strain energy potential of an elastomer as a function of the first two strain invariants (I_1 and I_2), assuming the materials are incompressible and initially isotropic. Several of these theories including those of Mooney[15] (1940), Rivlen[16] (1948), Valanis and Landel[17] (1967) and Ogden[18] (1972) are discussed in two articles by Treloar[19] [20] (1975, 1976). These models are essentially curve fits to experimentally obtained data.

The most commonly used representations are the Ogden and the so called Polynomial representations. The polynomial representation for the strain-energy function is expressed as follows:

$$U = \sum_{i=0,\, j=0}^{\infty} C_{ij}(I_1 - 3)^i (I_2 - 3)^j$$

where: C_{ij} are experimentaly determined and describe (4-36)
the shear behavior of the material

The neo-hookian form presented earlier is the first term in the polynomial series, and can be obtained by setting i=1 and j=0. This model may be viewed as a full generalization of the relationship derived by Mooney in 1940.

Mooney derived his strain energy potential relationship under the assumption of a linear stress-strain relationship in shear. The Mooney equation can be expressed as follows:

$$U = C_1(I_1 - 3) + C_2(I_2 - 3) \tag{4-37}$$

The Mooney equation is therefore the simplest version of the polynomial representation which contains both the first and second strain invariant, and is obtained by setting both i=1 and j=1.

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[15] Mooney, M., *J. app. Phys*, **11**, (1940), 582.

[16] Rivlin, R.S., *Phil. Trans. R. Soc. Lond. A.* **241**, (1948), 565.

[17] Valanis, K.C. and Landel, R.F., *J. appl. Phys.* **38**, (1967), 2997.

[18] Ogdon, R.W., *Proc. R. Soc. Lond. A* **326**, (1972), 565.

[19] Treloar, L.R.G., *The Physics of Rubber Elasticity.* Oxford University Press. Oxford. 1975.

[20] Treloar, L.R.G., *Proc. R. Soc. Locnd. A.* **351**, (1976), 301.

The Ogden strain-energy function of order, N, is of the form:

$$U = \sum_{n=1}^{N} \frac{\mu_n}{\alpha_n}\left(\lambda_1^{\alpha_n}+\lambda_2^{\alpha_n}+\lambda_3^{\alpha_n}-3\right) \tag{4-38}$$

where: μ_n and α_n are expermentally determined constants

The stresses in the principal coordinate system, resulting from the use of the Ogden strain-energy function is represented in indicial notation as follows:

$$\sigma_{ii} = \sum_{n=1}^{N} \mu_n \lambda_i^{\alpha_n} - p \tag{4-39}$$

4.5 Further Uses of Finite Strain Elasticity

The finite strain theories presented in this chapter were developed to model the behavior of elastomeric materials, but they can be used to model the entropic component of any amorphous polymer finite strain event.

Finite strain elasticity is often used in the modeling of finite strain plastic deformation in glassy polymers. After the yield stress is reached, a polymer begins to flow as plastic deformation occurs. During this process, the polymer chains become increasingly more aligned, thereby decreasing their configurational entropy. The decrease in configurational entropy results in a significant increase in modulus as the plastic strain becomes large. This change in entropy of the polymer can be viewed as a component of the total plastic work which is stored and therefore not dissipated during plastic deformation. Eventually the alignment reaches a point where nearly all of the plastic work goes to stored energy and plastic deformation cannot occur. This point is reached near the locking stretch of the material. The use of finite strain elasticity in the modeling of the yield process will be discussed in greater detail in Chapter 6.

5.0 LINEAR VISCOELASTIC BEHAVIOR

This section will review the basic underpinnings of linear visoelastic theory, which introduces rate dependence to small strain theory. Visoelastic effects are important in polymer materials because the measured value of the modulus of elasticity is often rate-dependent. This rate dependency can only be quantified by the inclusion of a viscous term in the constitutive equations.

Linear viscoelastic behavior is exhibited by polymer materials subjected to small strain at temperatures within approximately 30° C of the glass transition temperature (Tg). This theory can be viewed as the counterpart of the small strain elastic model (Hooke's Law), which is modified to account for time-dependent behavior. The effects of time are not included either in linear or finite strain elasticity. Viscoelastic theory strictly applies only to the case of very small strain. Reasonable limits of linear viscoelastic theory for amorphous polymers vary by region of mechanical equivalence as follows[1]:

TABLE 5-1

REGION	STRAIN (%)
1-Glassy	1
2-Visocelastic	5
3-Rubbery	50

When these strain limits are exceeded, the stress/strain relationships become nonlinear, and the theoretical analysis becomes significantly more complex.. As a consequence, the analysis of real polymers in the non-linear visoelastic region is typically handled by means of numerical, empirical or semi-empirical methods.

The most important practical use of linear viscoelastic theory lies in the experimental determination of phase transformations, such as the glass transition temperature (Tg), by means of Dynamic Mechanical Analysis (DMA). In DMA, a

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[1] I.V. Yannas, "Introduction to Polymer Science and Engineering: A set of Lecture Notes," Deformation and Fracture of Polymers, Massachusetts Institute of Technology, 1994, p. 127.

specimen is subjected to a small amplitude sinusoidal load, so the assumptions made in linear visoelastic theory are valid. DMA will be discussed in greater detail in Chapter 8.

The following sections outline the basic mechanical models used to describe viscoelastic behavior including creep and stress relaxation, the Boltzmann integral, and the time-temperature superposition principle.

5.10 Mechanical Analogies Representing Linear Visoelastic Behavior

The three most common models used to describe linear viscoelastic behavior are the Kelvin-Voight, Maxwell, and Standard linear solid models (see Figure 5-1). These models contain various combinations of Hookian springs and Newtonian dampers, and form the fundamental basis for more complex visoelastic models used to describe the time dependent behavior of materials subjected to small strain.

The two most common experiments used in the description of viscoelastic material behavior are **stress relaxation** and **creep**. In a stress relaxation experiment, a material is subjected to a constant strain, and the stress decreases over time with a characteristic time constant. In a creep experiment, a constant stress is applied and the strain increases exponentially with time at the same characteristic time constant.

The **Maxwell model** consists of one Hookian spring and one Newtonian damper in series and is the simplest model which can be used to describe stress relaxation. When loaded under a uniform axial stress, σ, the total stress and strain in the system is given by:

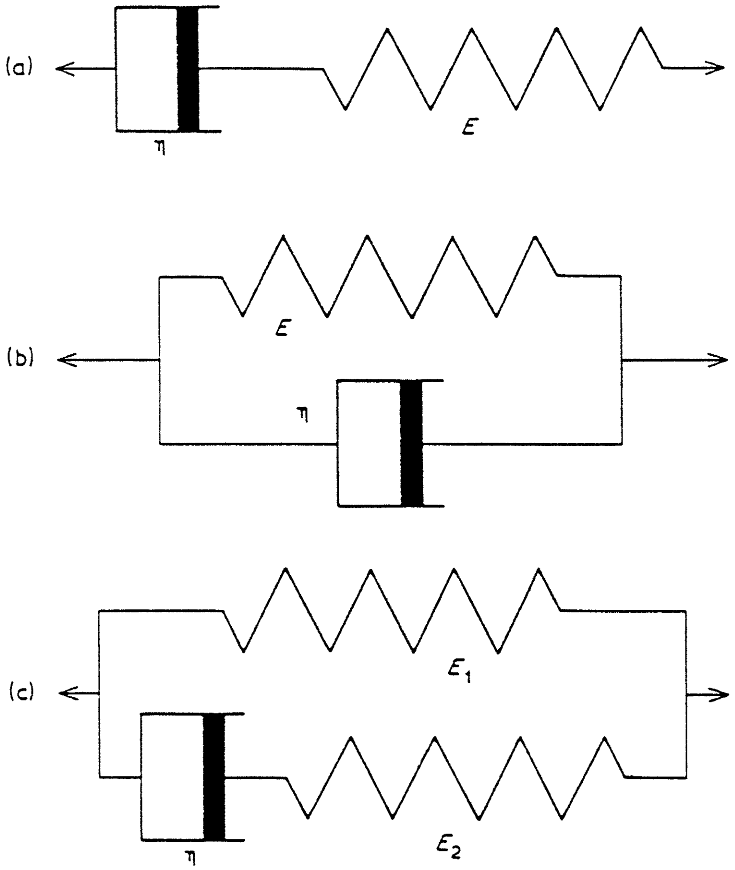
$$\sigma = \sigma_S = \sigma_D$$
$$\varepsilon = \varepsilon_S + \varepsilon_D \qquad (5\text{-}1)$$

where: σ_S and ε_S are the stress and strain in the spring
σ_D and ε_D are the stress and strain in the spring

The stress in the spring and the damper are given by the following:

$$\sigma_S = E\varepsilon_S$$
$$\sigma_D = \eta \frac{d\varepsilon_D}{dt} \qquad (5\text{-}2)$$

where: E is the modulus of elasticity
η_D is the viscosity



Mechanical models used to represent the viscoelastic behaviour of polymers. (a) *Maxwell model,* (b) *Voigt model,* (c) *Standard linear solid.*

R.J. Young and P.A. Lovell. *Introduction to Polymers 2nd ed.*
(New York:Chapman & Hall, 1992), p.325.

Figure 5-1

Combining equations (5-1) and (5-2) gives the following linear ordinary differential equation representing the response of a Maxwell model under an applied stress:

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \tag{5-3}$$

In the case of stress relaxation, the stress is given as:

$$\sigma(t) = \sigma_0 \exp(-t/\tau_0)$$

where: σ_0 is the inital stress at $t = 0$ (5-4)

$\tau_0 = \eta/E$ is the relaxation time

The negative exponential term in the stress relaxation equation gives an indication as to how quickly the stress will relax to a steady state value. The most common definition of the relaxation time, known as the **characteristic relaxation time (CRT)**, is the time required for the stress to reduce to 63% of its original value. Small values of CRT indicate a material which relaxes quickly, while large values result in behavior which approaches linear elastic material behavior in the limit of an infinitely large relaxation time.

The **Kevin-Voight model** consists of a Hookian spring and a Newtonian damper in parallel and represents the simplest model which can adequately describe creep behavior. Under the action of a uniform strain (ε), the total stress and strain in the system is given as follows:

$$\varepsilon = \varepsilon_S = \varepsilon_D$$

$$\sigma = \sigma_S + \sigma_D$$

where: σ_S and ε_S are the stress and strain in the spring (5-5)

σ_D and ε_D are the stress and strain in the spring

The behavior of the Kelvin-Voight model is represented by the following ordinary differential equation:

$$\frac{d\varepsilon}{dt} + \frac{E\varepsilon}{\eta} = \frac{\sigma_0}{\eta} \tag{5-6}$$

In the case of creep, the strain decays with time as follows:

$$\varepsilon(t) = \frac{\sigma_0}{E}\left[1 - \exp(-t/\tau_0)\right] \tag{5-7}$$

The Maxwell and Kelvin-Voight models represent first order approximations of stress relaxation and creep behavior respectively. Neither is capable of adequately describing both of these important phenomenon. Figure 5-2 shows the stress relaxation and creep behavior of both the Kelvin-Voight and Maxwell models. Note that the Kelvin-Voight model displays no stress relaxation, and the Maxwell model shows a linear increase in strain with time. Both of these are clearly non-physical.

To capture the exponential response both in stress relaxation and creep, a model known as a **standard linear solid** was developed[2]. It consists of a Maxwell element in parallel with a Hookian spring. It can be shown that the differential equation for this model can be expressed as:

$$\sigma + \left(\frac{\eta_m}{E_m}\right)\frac{d\sigma}{dt} = E_a\varepsilon + (E_m + E_a)\left(\frac{\eta_m}{E_m}\right)\frac{d\varepsilon}{dt}$$

where: η_m and E_m are the elements of the Maxwell model (5-8)

E_a is the parallel spring element

This model adequately represents the qualitative behavior of a polymer under both stress relaxation and creep. The behavior in the case of creep is shown below to demonstrate the increase in complexity as compared with either the Maxwell or Kelvin-Voight models. The solution of (5-8) in the case of creep is given as follows:

$$\varepsilon(t) = \sigma_0\left[\frac{1}{E_{a1}} + \left[\frac{1}{E_a + E_m} - \frac{1}{E_a}\right]\exp(-t/\xi)\right] \tag{5-9}$$

$$\text{where: } \xi = \frac{(E_a + E_m)\eta_m}{E_a E_m}$$

The standard linear solid can be used to qualitatively represent the behavior of a polymer, with a significant increase in complexity as compared to either the Maxwell or Kelvin-Voight models. In order to quantify the viscoelastic response of a real polymer, a large number of series and parallel spring/damper combinations are required, a process which can quickly become computationally intractable. A more usable approach involves the use of combinations of empirically obtained data, and curve fitting techniques, at the expense of physical significance. These approaches require the introduction of the Boltzmann integral and the Boltzmann superposition principle.

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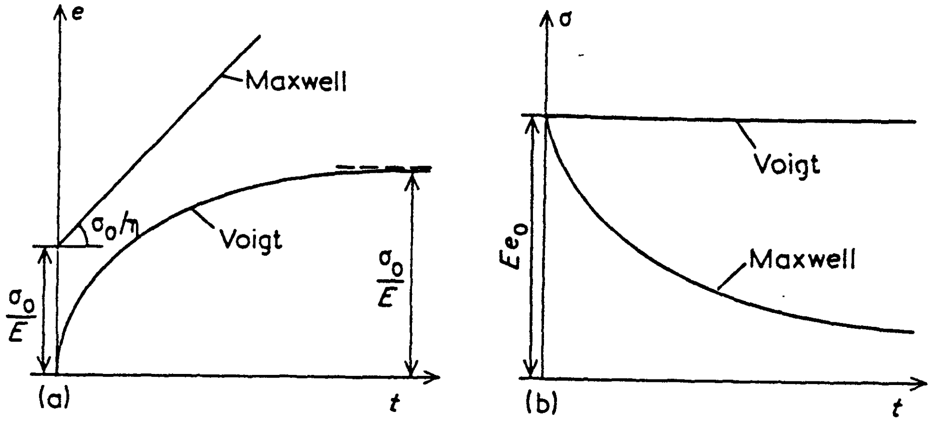
[2] C. Zener, *Elasticity and Anelasticity of Metals,* Chicago University Press, Chicago, 1948.



The behaviour of the Maxwell and Voigt models during different types of loading. (a) *Creep (constant stress* σ_0*),* (b) *Relaxation (constant strain* e_0*) (after Williams).*

R.J. Young and P.A. Lovell. *Introduction to Polymers 2nd ed.*
(New York:Chapman & Hall, 1992), p.326.

Figure 5-2

5.20 The Boltzmann Integral and the Boltzmann Superposition Principle

The assumption that the relationship between stress and strain is linear allows for the definition of two important quantities in the study of linear viscoelastic behavior: the **creep compliance J(t)** and the **stress relaxation modulus E(t).** These quantities are defined as follows:

$$J(t) = \frac{\varepsilon(t)}{\sigma_0} \qquad E(t) = \frac{\sigma(t)}{\varepsilon_0} \tag{5-10}$$

The **Boltzmann superposition principle** states that the strain (ε_i) at any time (t) due to an increment in stress ($\Delta\sigma$) applied at time (τ_i) is given as follows:

$$\varepsilon_i(t) = (\Delta\sigma_i)\, J(t-\tau_i) \tag{5-11}$$

An important premise in the theory is that all previous loadings effect the state of stress. In practice, the effects of all previous loading histories can be eliminated from a polymer by raising the temperature to approximately 30-40 degrees Celcius above the glass transition temperature in a process which is similar to annealing. The sum of all the loadings can be expressed as:

$$\varepsilon(t) = \sum_{n=0}^{n} (\Delta\sigma_n)\, J(t-\tau_n) \tag{5-12}$$

In integral form, the Boltzmann integral takes the form of a Duhamel integral as follows:

$$\varepsilon(t) = \int_{-\infty}^{t} J(t-\tau) \frac{d\sigma(\tau)}{d\tau} d\tau \tag{5-13}$$

The Boltzmann integral can also be expressed in terms of the stress relaxation modulus as:

$$\sigma(t) = \int_{-\infty}^{t} E(t-\tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau \tag{5-14}$$

An exact relationship can be derived between creep compliance and stress relaxation using Boltzmann integrals and convolution as follows:

$$\int_0^t E(\tau)J(t-\tau)d\tau = \int_0^t J(\tau)E(t-\tau)d\tau = t \tag{5-15}$$

The exact relationship is often implemented in numerical codes. An approximate relationship between the stress relaxation and creep compliance is given as[3]:

$$J(t) = \frac{\sin(m\pi)}{m\pi}\frac{1}{E(t)} \tag{5-16}$$

where: m is the negative slope of the stress relaxation curve at t

In the case of linear and finite strain elasticity, the compliance is the inverse of the modulus. Equation (5-16) can be used to qualitatively estimate the effect of neglecting viscous effects from the slope of the relaxation modulus master curve. Since the sine(θ) is approximately equal to θ when θ is small, a relatively small error is incurred when neglecting viscous effects near the beginning of the sloping regions of the stress relaxation curve. As an example, the error when the slope is 20 degrees from horizontal is only 2%. The production of the stress relaxation master curve will be discussed in a later section.

The Boltzmann integral can be used to obtain a qualitative description of the viscoelastic response of a real polymer based on relaxation and retardation time spectra. The interested reader is referred to any standard text on the mechanics of solid polymers for more information on these topics. The next section will focus on numerical approximations used by several finite element codes such as ABAQUS[4] to describe the viscoelastic response of real polymers.

Most finite element codes use experimentally obtained data and curve fitting methods to approximate visoelastic behavior. Often, the codes split the problem into deviatoric and volumetric components which are solved in separate operations. The

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[3] I.V. Yannas, "Introduction to Polymer Science and Engineering: A set of Lecture Notes," Deformation and Fracture of Polymers, Massachusetts Institute of Technology, 1994, p. 127.

[4] Hibbitt, Karlsson, and Sorenson, eds., *Analysis of Viscoelastic Problems with ABAQUS,* (1994), pp.3-1 - 3-23.

deviatoric component of the stress tensor is obtained by subtracting the hydrostatic pressure from the stress tensor. The pressure is defined as one third of the trace of the stress tensor. For simplicity, an explanation of semi-empirical methods will be presented assuming incompressible material behavior. In this case, the bulk modulus is assumed to be infinitely large and the volumetric effects can be neglected.

5.30 Semi-Empirical Methods Used in Modeling Viscoelasticity

In most of the numerical approximations of linear viscoelatic behavior, the Boltzmann integral for stress relaxation is rewritten entirely in terms of stress. This is done to allow the same form to be used for finite strain viscoelasticity, which will be discussed shortly. Using integration by parts, equation (5-14) can be written as follows:

$$\sigma(t) = \sigma_0(t) + \frac{1}{G_0}\int_0^t \frac{dG(\tau)}{d\tau}\sigma_0(t-\tau)d\tau \qquad (5\text{-}17)$$

where: G_0 is the instantanous elastic shear modulus

The relaxation modulus can be determined using a **Prony series**, which is given as follows:

$$G(\tau) = G_0\left(1-\sum_{i=1}^{N}\bar{g}_i^P\left(1-e^{-\tau/\tau_i^G}\right)\right)$$

where: $\bar{g}_i^P$ is obtained by applying several stress levels,
measuring the relaxation time, and curve fitting the results (5-18)
τ is the time, and τ_i^G is the characteristic relaxation time
N is the number of terms in the Prony series

This procedure uses a set of experimentally obtained relaxation times and curve fits the results to within a default (or user-defined) average RMS error in the non-linear least squares estimate. Once obtained, the Prony series is used with concepts from the most elementary models of viscoelasticity to quantify the visoelastic response of real polymers.

The same numerical procedure can be used to add rate dependency to isotropic finite strain viscoelasticity. Although viscoelasticity strictly applies only to small strain time-dependent behavior, finite strain visoelasticity of elastomers can be analyzed utilizing a Prony series as described in the small strain viscoelastic case.

Equations (4-36 to 4-39) described the Polynomial and Ogden functions, two of the phenomenological models used to describe the strain energy potential for the finite strain behavior of elastomers. Rate-dependency is added to these models by means of a Prony series modification of the elastic constants.

The Polynomial form is given as:

$$U = \sum_{i=0, j=0}^{\infty} C_{ij} (I_1 - 3)^i (I_2 - 3)^j$$

where: C_{ij} are experimentaly determined and describe the shear behavior of the material (4-36)

The Ogden form is given as:

$$U = \sum_{n=1}^{N} \frac{\mu_n}{\alpha_n} \left(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3 \right) \qquad (4\text{-}38)$$

where: μ_n and α_n are expermentally determined constants

Time-dependency is introduced by modifying the coefficients C_{ij} in the Polynomial form and μ_n in the Ogden form using a Prony series as follows:

$$C_{ij}(\tau) = C_{ij}^{\,0} \left(1 - \sum_{i=1}^{N} \bar{g}_i^{\,P} \left(1 - e^{-\tau / t_i^G} \right) \right)$$

$$\mu_n(\tau) = \mu_n^{\,0} \left(1 - \sum_{i=1}^{N} \bar{g}_i^{\,P} \left(1 - e^{-\tau / t_i^G} \right) \right) \qquad (5\text{-}19)$$

where: $C_{ij}^{\,0}$ and $\mu_n^{\,0}$ are the instantanuous values (at $t = 0$)

5.40 The Time-Temperature Superposition Principle

The assumption of linear viscoelasticity allows for the production of a master stress relaxation (or creep compliance curve) for amorphous polymers based on an extrapolation procedure developed by Williams, Landel, and Ferry[5]. These curves are expressed as log(time) versus log(creep compliance or stress relaxation). An example stress relaxation curve for polyisobutylene is shown in Figure 5-3. The theory states that a master curve can be obtained which spans many decades in time from data obtained over a much narrower range of time. The extrapolation is performed using a shift factor which equates the effect of time and temperature on the behavior of an amorphous polymer. This is known as the **time-temperature superposition principle**. The creep test is easier

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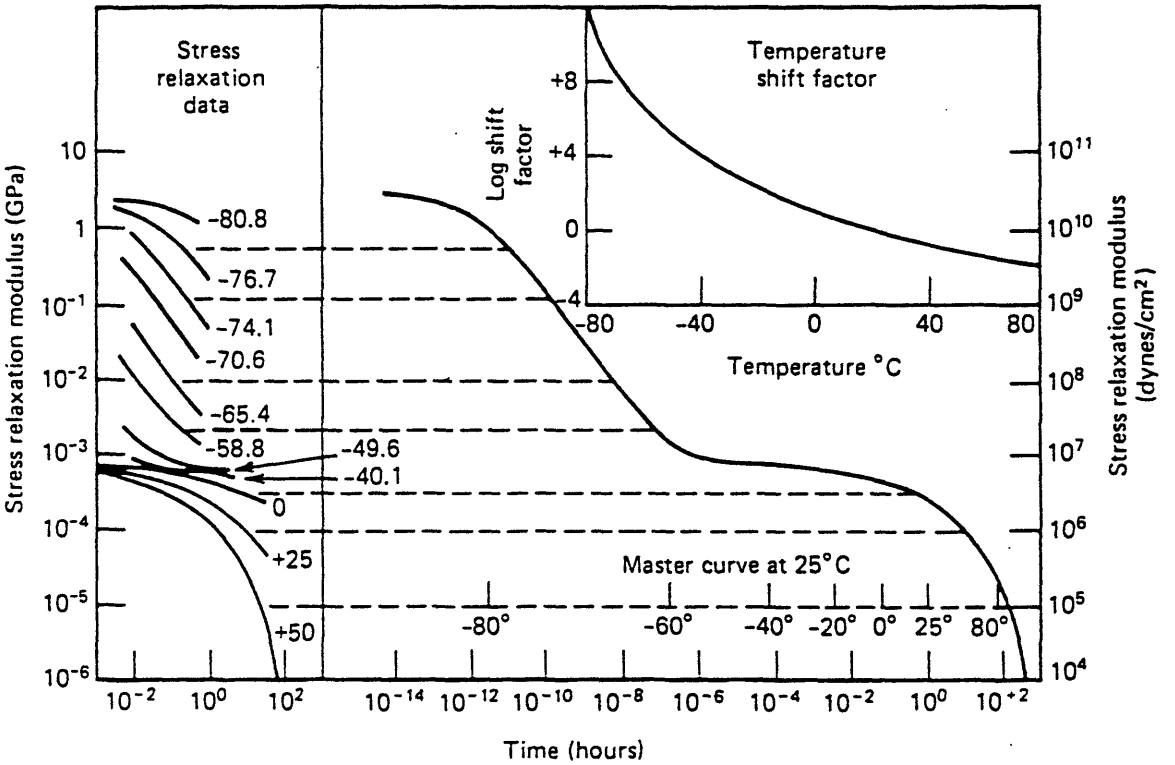
[5] Ferry, J.D., *Viscoelastic Properties of Polymers*, Wiley, New York, 1961, Chapter 11.



Modulus-time master plot for polyisobutylene based on time-temperature superposition of data to a reference temperature of 25°C.

Richard W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials 2nd ed.* (New York:John Wiley and Sons, 1983), p. 206.

Figure 5-3

to perform than stress relaxation, and the curve can be transformed into a master stress relaxation curve using equation (5-15).

The first step in the production of a master curve is to perform a series of creep or stress relaxation tests over a relatively short but fixed time interval (usually between one hour and one day) at a variety of temperatures. The range of test temperatures should be wide enough to capture the glassy behavior at the lowest temperatures as well as the elastomeric behavior at the highest temperatures. The master curve will represent the behavior of an amorphous polymer over many decades of time at a desired temperature.

The temperature corresponding to a master curve is the experimentally determined temperature which is not shifted. The curve can be obtained by shifting the curves taken above the desired temperature to the right (forward in time), and those taken below the test temperature to the left (backward in time) along the horizontal (time) axis. Once completed, the entire master curve can be shifted to represent any temperature. The horizontal shift has been shown to be nearly identical for all amorphous polymers and can be defined by a quantity known as **shift factor**.

The shift factor, denoted by the symbol a_T, which was determined experimentally by Williams, Landel, and Ferry can be expressed as follows:

$$\log(a_T) = \frac{17.44\left(T - T_g\right)}{51.6 + \left(T - T_g\right)} \qquad (5\text{-}20)$$

The shift factor is used to equate the effect of time and temperature in ^{0}C. The time variable can take on any of the following forms, representing effects of time directly, as well as changes in frequency, or changes in strain rate. The effect of an increase in one of the time variables can be represented as a decrease in temperature as follows:

$$\log(a_T) = \log\left[\frac{t_1}{t_2}\right] = \log\left[\frac{\omega_1}{\omega_2}\right] = \log\left[\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}\right] \qquad (5\text{-}21)$$

Using equations (5-20) and (5-21), it can be shown that a increase of one decade in one of the time variables, results in an upward shift of 3.14 degrees in Tg.

Although this expression was obtained experimentally, it is often given physical significance based on arguments related to fractional free volume[6]. Figure 5-4 shows the specific volume versus temperature for a typical amorphous polymer. Below Tg, the free volume (volume occupied by voids) is constant and begins to increase at temperatures above Tg. The occupied volume (the volume which is occupied by the polymer macromolecules themselves), increases linearly with increasing temperature.

The fractional free volume which can be expressed as follows:

$$f = f_g + \alpha_f (T - T_g)$$

where: $f = v_f / v$ is the fractional free volume

f_g is the fractional free volume at Tg

v_f is the free volume (5-22)

v is the total volume

$\alpha_f = \alpha_{solid} - \alpha_{liquid}$ is the coefficient of expansion of the free volume

The Doolittle viscosity equation is used to relate viscosity to free volume as follows:

$$\eta = a \exp\left(b\left(v - v_f\right)/ v_f\right) \text{ or}$$

$$\ln(\eta) = \ln(a) + b(1/f - 1) \tag{5-23}$$

where: a and b are experimentally obtained constants

A shift factor (a_T) can be defined by assuming that the polymer behaves as a viscoelastic material with a characteristic relaxation time (τ). Recall from the Maxwell model, that the relaxation time is given as the ratio of the viscosity to the material modulus of elasticity.

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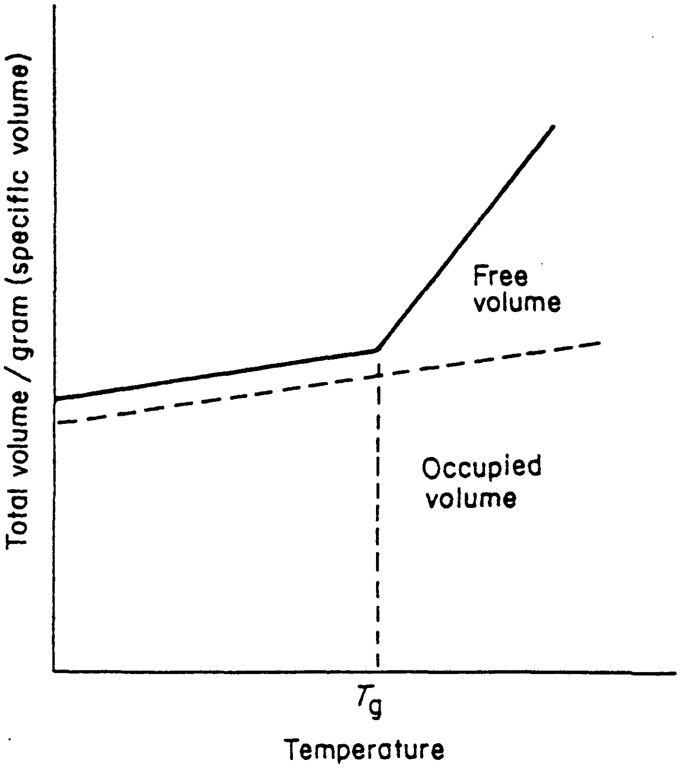
[6] Williams, M.L., R.F. Landel and J.D. Ferry, *J. Amer. Chem. Soc.*, **77**, 3701 (1955).



The volume–temperature relationship for a typical amorphous polymer.

I.M. Ward, *Mechanical Properties of Solid Polymers 2nd ed.* (New York:John Wiley and Sons, 1990), p. 150.

Figure 5-4

Assuming the temperature sensitivity of the viscosity is much greater than the change in modulus, the shift factor can be expressed as:

$$a_T = \frac{\tau_T}{\tau_{TG}} = \frac{\eta_T}{\eta_{Tg}}$$

where: η_T is the viscosity at the temperature T (5-24)

η_{Tg} is the viscosity at the temperature Tg

Using equations (5-17) and (5-18), the shift factor can be expressed as:

$$\log a_T = -\frac{\left(b/2.303f_g\right)\left(T-T_g\right)}{f_g/\alpha_f+\left(T-T_g\right)} \qquad (5\text{-}25)$$

From equations (5-15) and (5-19), it can be seen that the $f_g = 0.025$ so the glass transition temperature is reached when the fractional free volume reaches 2.5% and $\alpha_f = 4.85 \times 10^{-4}$ degC^{-1}. It has been shown experimentally that the fractional free volume for most amorphous polymers at Tg is 0.025 ± 0.003 and the coefficient of thermal expansion has a "universal average" value of 4.8 x 10-4 C^{-1}.[7]

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[7] I.M. Ward, *Mechanical Properties of Solid Polymers*,(New York: John Wiley and Sons, 1983), p.152.

6.0 YIELD AND POST-YIELD BEHAVIOR OF GLASSY POLYMERS UNDER COMPRESSIVE LOADING

The previous chapters have discussed the small strain behavior of amorphous polymers in the elastic and viscoelastic regime, as well as the finite strain behavior of elastomeric materials. This chapter will utilize and expand on the concepts developed in chapters 2-5 in the presentation of a 3-D coupled thermal-displacement viscoplastic constitutive model which has been used successfully to describe the temperature and rate dependent plastic deformation of glassy polymers. The theory will be reduced to the case of uniaxial compression, therefore crazing and fracture will not be discussed. The interested reader is referred to one of the many texts on polymer material mechanics for information on these topics. The uniaxial compression model will be further reduced to model the yield stress as function of strain rate.

6.10 Description of the Stress-Strain Curve

The stress-strain curve for an amorphous polymer loaded isothermally in compression has a characteristic shape which is shown in Figure 6-1. In the glassy regime, the small strain behavior is modeled using the theory of linear elasticity, which is used to determine the modulus of elasticity of a glassy polymer. When strained beyond the limits of linear elasticity, the slope of the stress/strain curve decreases until it reaches a maxima. The stress at that point is known as the **yield stress**, which typically occurs between 5 and 10 percent true strain.

Following yield, glassy polymers undergo **strain softening**. This is seen in the stress/strain curve as a significant drop in load as further strain is applied beyond yield point. This behavior is a global response to microscale inhomogeneous deformation mechanisms such as shear banding. As the polymer continues to be strained, the stress reaches a steady state value where it remains until alignment of the macromolecular chains results in **strain hardening**.

Strain hardening occurs as the chains within the plastically flowing polymer becomes increasingly more aligned. This increase in alignment results in a decrease in configurational entropy, which can be equated to an increase in the modulus of the material. As a consequence, strain hardening results in an increasing barrier to plastic

Plot of True Stress vs. True Strain

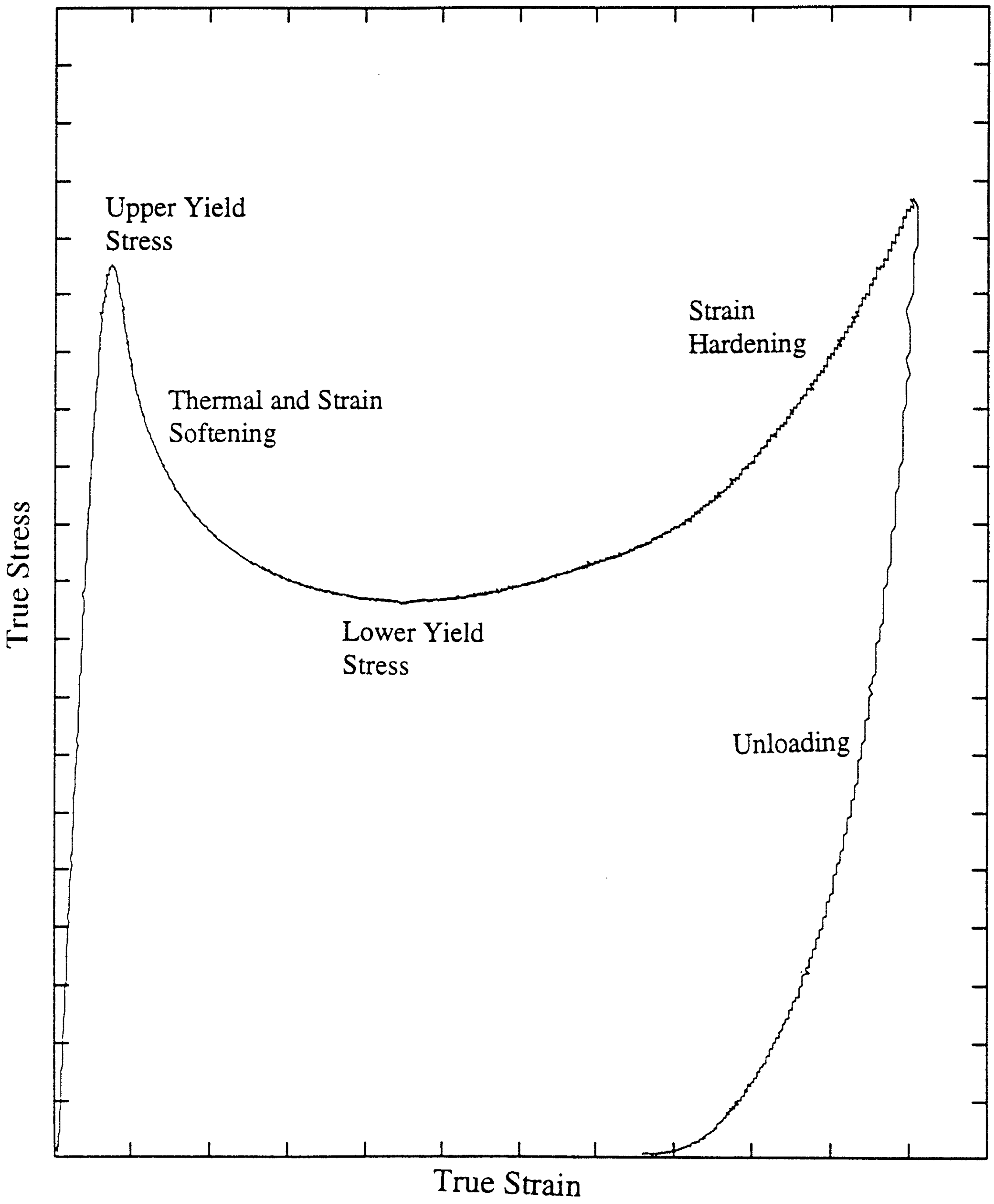

Figure 6-1

deformation with increasing strain. This effect is primarily entropic and can be modeled using Langevin statistical finite strain theory.

The yield stress is a strong function of both temperature and the strain rate. It generally decreases with increasing temperature and decreasing strain rate, eventually decreasing to zero as the polymer approaches the glass transition temperature. As a result, the process of yielding only occurs in polymers which are in the glassy region. The remainder of this chapter will discuss the constitutive modeling of the yield and post-yield behavior of glassy amorphous polymers as a function of temperature and strain rate.

6.20 Constitutive Modeling

The constitutive modeling of the yield and post-yield behavior of glassy polymers is a relatively new endeavor. To capture the effects of temperature and load rate, the model must consider the temperature and displacement components in a fully coupled form. A successful approach was developed by Boyce[1] et.al. (1988) based on a model originally proposed by Argon[2] (1973). A physically-based three dimensional approach is used, which includes a fully coupled temperature-displacement viscoplastic model. Following the presentation of the complete theory, the model will be reduced to the case of uniaxial compression in a somewhat simplified form for use in this study.

The model is based on the assumption that the total resistance to plastic deformation in a glassy polymer is due to two distinct physical barriers as described by Boyce and Arruda (1995)[3]. The first of these is an isotropic barrier to chain segment rotation, and the second is an anisotropic resistance to chain alignment. This implies that some of the total plastic work is dissipated during plastic deformation, and the remainder is stored, providing additional resistance to further plastic deformation.

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[1] Boyce, M.C., D.M. Parks and A.S. Argon (1988), Large Inelastic Deformation in Glassy Polymers, Part 1: Rate Dependent Constitutive Model, *Mech. Mater* **7**, pp. 15-33.

[2] Argon, A.S. (1973), A Theory for the Low Temperature Plastic Deformation of Glassy Polymers, *Philos. Mag.* **28**, pp 839-865.

[3] Arruda, E.M., M.C. Boyce, and R. Jayachandran (1995), Effects of strain rate, temperature and thermomechanical coupling on the finite strain deformation of glassy polymers, *Mech. Mater.* **19**, pp.193-212.

The force which drives the plastic deformation in a glassy polymer is the effective equivalent shear stress (the shear flow stress), which is given as follows:

$$\tau = (s+\alpha p)\left[1+\frac{k_B \Theta}{A(s+\alpha p)} \ln\left(\frac{\dot{\gamma}^p}{\dot{\gamma}_o}\right)\right]^{6/5}$$

where: s is the athermal shear strength
α is the pressure coefficient
p is the pressure
k_B is the Boltzmann constant
Θ is the Absolute temperatue (6-1)
$A(s+\alpha p)$ is the zero stress level activation energy modified to include pressure effects
$\dot{\gamma}_o$ is the pre - exponential factor proportional to the attempt frequency
$\dot{\gamma}^p$ is the applied shear strain rate

The athermal shear strength is assumed to be constant prior to the onset of plastic deformation, and evolves following yield as follows:

$$s = \frac{0.077\mu}{(1-\nu)}$$

where: μ is the elastic shear modulus
ν is the Poisson ratio

$$\dot{s} = h\left(1-\frac{s}{s_{ss}}\right)\dot{\gamma}^p \tag{6-2}$$

where: h is the softening slope
s_{ss} is the preferred state of the material associated with a completely de - aged condition

The shear modulus has a strong dependence on temperature. This dependency is quantified by generating a series of isothermal stress/strain curves at a loading rate which is slow enough to ensure that no temperature rise occurs during the deformation process. The results are curve fitted using a logarithmic function to obtain the temperature dependent shear modulus.

The stored portion of the plastic work provides an increasing resistance to plastic deformation with increasing strain, an effect is captured using a second-order tensor known as the **back stress**. The back stress can be modeled using equations (4-30) and (4-35) which were developed to model the finite strain behavior of rubber elastic materials assuming that the back stress evolves with increasing chain alignment. Combining equations (4-30) and (4-35), the back stress can be written as follows:

$$B_i = \frac{nk_B\Theta}{3}\sqrt{N}\,\mathcal{L}^{-1}\left\{\frac{\Lambda^P{}_{chain}}{\sqrt{N}}\right\}\frac{\left(\Lambda^P{}_i\right)^2 - \frac{1}{3}I_1}{\Lambda^P{}_{chain}}$$

where: $nk_B\Theta$ is the rubbery shear modulus

$\sqrt{N}$ represents the limiting chain extensibility

N is the number of statistically rigid links

$\mathcal{L}^{-1}\left\{\frac{\Lambda^P{}_{chain}}{\sqrt{N}}\right\}$ is the inverse Langevin funtion (6-3)

$\Lambda^P{}_i$ is the applied plastic principal stretch of component i

$\Lambda^P{}_{chain}$ is the strech on any chain in the 8 - chain network

I_1 is the first stetch invariant

The first stretch invariant is given as:

$$I_1 = \left(\Lambda^P{}_1\right)^2 + \left(\Lambda^P{}_2\right)^2 + \left(\Lambda^P{}_3\right)^2 \tag{6-4}$$

Using the first stretch invariant, $\Lambda^P{}_{chain}$ can be expressed in terms of the plastic stretch components as follows:

$$\Lambda^P{}_{chain} = \frac{1}{\sqrt{3}}\sqrt{\left(\Lambda^P{}_1\right)^2 + \left(\Lambda^P{}_2\right)^2 + \left(\Lambda^P{}_3\right)^2} = \frac{1}{\sqrt{3}}\sqrt{I_1} \tag{6-5}$$

The strain chain density (n) in equation (6-3) is actually a function of the absolute temperature due to chain dissociation[4]:

$$n(\Theta) = B - De^{\left(\frac{-E_a}{R\Theta}\right)}$$

where: B and D represents the portion of the network which
non - dissociating, and dissociating respectively (6-6)
E_a is the thermal dissocitating energy
R is the universal gas constant (0.001695 kcal / mol K)

It has been shown that the thermally evolving chain density results in an increase in the number of statistically rigid links (N) due to conservation of mass[5]. The chain density and the number of statistically rigid links are related as follows:

$$n(\Theta)N(\Theta) = \text{constant} \qquad (6\text{-}7)$$

The kinetics of deformation of a glassy polymer begins with the polar decomposition of the deformation gradient into elastic, thermal and plastic components[6] which are used to define the components of the plastic velocity gradient as follows:

$$F = F^e F^{th} F^p$$

where: F^e, F^{th}, F^p are the elastic, thermal, and plastic components of the total deformation gradient

$$L = \dot{F}F^{-1} = L^e + F^e\left[L^{th} + F^{th}L^p(F^{th})^{-1}\right](F^e)^{-1} \qquad (6\text{-}8)$$

where: L is the plastic deformation velocity gradient

$$L^e = \dot{F}^e(F^e)^{-1},\ L^{th} = \dot{F}^{th}(F^{th})^{-1},\ L^p = \dot{F}^p(F^p)^{-1}$$

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[4] Raha, S. and P.B. Bowden (1972), Birefringence of plastically deformed polymethylmethacrylate, *Polymer 13*, 174-183.

[5] Boyce, M.C. (1986) Large inelastic deformation of glassy polymers, Ph.D. thesis, The Massachusetts Institute of Technology.

[6] Lee, E.H. (1969), "Elasto-plastic deformation at finite strains". ASME *J. App. Mech.* 36, 1.

The thermal and plastic components of the deformation and velocity gradient are typically grouped together as follows:

$$F^{thp} = F^{th}F^{p}$$

and,

$$L^{thp} = D^{thp} + W^{thp} = \dot{F}^{thp}(F^{thp})^{-1}$$

where: W^{thp} is known as the spin and is often set equal to zero

D^{thp} is the defomation rate (6-9)

$$D^{thp} = \overline{D}^{p} + \alpha(\Theta)I\dot{\Theta}$$

where: $\overline{D}^{p}$ is the plastic stetch

$\alpha(\Theta)$ is the temperature dependent coefficient of thermal expansion

The plastic stretch tensor is given in terms of the applied shear strain rate as follows:

$$\overline{D}^{p} = \frac{1}{\sqrt{2}}\dot{\gamma}^{p}N$$

where: N is the normalized tensorial direction of the shape change

$$N = \frac{1}{\sqrt{2\tau^{2}}}\overline{T}^{*\prime}$$

and, (6-10)

$$\overline{T}^{*\prime} = \left[R^{eT}TR^{e}\right]^{\prime} - B$$

where: $\overline{T}^{*\prime}$ is the deviatoric component of the driving stress state

R^{e} is the rotation matix to priciple strain space

The quantity (T) in equation (6-10) is the Cauchy (true) stress and T* represents the portion of the stress which continues to activate plastic flow and is the tensorial difference between the Cauchy stress deviator and the convected back stress.

The shear stress from equation (6-1) can be expressed as follows:

$$\tau = \sqrt{\frac{1}{2}\overline{T}^{*\prime}\cdot\overline{T}^{*\prime}} \qquad (6\text{-}11)$$

The kinematic relationship from equation (6-11) is set equal to the rate and temperature dependent flow stress equation (6-1) to quantify the plastic deformation process.

The elastic constitutive relationship is given by:

$$T = \frac{1}{J} L^{e}\left[\ln V^{el}\right]$$

where: J is the volume change (6-12)

V^{el} is the Hencky strain

6.30 Heat Generation Due to Plastic Deformation

To adequately capture the thermomechanical interaction at high strain rates, the effect of internal heat generation due to plastic deformation must be included. The general energy balance equation can be written as:

$$\rho\, c\dot{\Theta} - \mathrm{div}(k\ \mathrm{grad}\ \Theta) = \dot{q}$$

where: ρ is the density

c is the specific heat (6-13)

k is the thermal conductivity

$\dot{q}$ is the rate of heat generation

The rate of heat generation due to plastic flow can be determined using the following equation:

$$\dot{q} = \left[\mathrm{tr}\left(\overline{T}^{*\prime}\,\overline{D}^{P}\right)\right] \qquad (6\text{-}14)$$

The heat generation can result in a temperature rise in the material if the heat is not allowed sufficient time to be conducted away. By comparing the time scale of the test to the thermal diffusion time, the deformation process may be classified as adiabatic, isothermal, or fully coupled.

The thermal diffusion time is given as:

$$t_d = \frac{L^2}{2\alpha}$$

where: L is the distance from the center of the specimen to the nearest heat sink (6-15)
α is the thermal diffusivity

A dimensionless thermal time parameter can be defined as:

$$\tau = \frac{t_{test}}{t_d}$$

where: $\tau >> 1$ indicates an isothermal process (6-16)
$\tau \approx 1$ indicates fully coupled thermal - displacement
$\tau << 1$ indicates an adiabatic process

The temperature rise during plastic deformation contributes an additional thermal component to the strain softening of the material. When the test is conducted at a very slow rate, the deformation process is nearly isothermal (no thermal softening occurs). As the strain rate is increased, the softening effect becomes increasingly more pronounced.

6.40 Experimental Methods for Determining Material Properties

This section will discuss some of the possible methods which could be used to determine the material properties required to conduct the analysis procedure. The first step is to conduct a series of stress-strain experiments at several strain rates and a constant ambient temperature. The value of the compressive yield stress resulting from an applied strain rate as well as the athermal shear stress can be determined directly from the experimental results. When attempting to predict the yield stress, the temperature can be taken as a constant equal to the ambient temperature for the entire range of strain-rates. This is a valid assumption since the plastic deformation is negligibly small and equation (6-14) indicates that the heat generation is also small.

The value of the pressure coefficient can be determined directly by conducting tests in both tension and compression, since these tests contain different pressure levels. The pressure coefficient is the slope of the curve of the peak yield stress versus the applied pressure.

Once the pressure coefficient is known, the value of the constant A and $\dot{\gamma}_0$ can be determined. Rewriting equation (6-1) in terms of the applied shear strain rate gives:

$$\ln\left[\frac{\dot{\gamma}_p}{\dot{\gamma}_0}\right]=\left[-\frac{A(s+\alpha p)}{k_B\Theta}\left\{1-\left(\frac{\tau_{peak}}{(s+\alpha p)}\right)^{5/6}\right\}\right]$$

$$\ln(\dot{\gamma}_p)=\left[\ln(\dot{\gamma}_0)-\frac{A(s+\alpha p)}{k_B\Theta}\right]+\left[\frac{A(s+\alpha p)}{k_B\Theta}\right]\left[\left(\frac{\tau_{peak}}{(s+\alpha p)}\right)^{5/6}\right] \qquad (6\text{-}17)$$

Setting $B=\left[\ln(\dot{\gamma}_0)-\frac{A(s+\alpha p)}{k_B\Theta}\right]$ and $C=\left[\frac{A(s+\alpha p)}{k_B\Theta}\right]$ gives

$$\ln\left[\dot{\gamma}_p\right]=B+C\left(\frac{\tau_{peak}}{(s+\alpha p)}\right)^{5/6}$$

The reduced form of equation (6-17) is the equation of a straight line. By plotting the values of $\ln\left[\dot{\gamma}_p\right]$ as a function of $\left(\frac{\tau_{peak}}{(s+\alpha p)}\right)^{5/6}$, the value of A/k_B can be determined directly from the slope (C). With A/k_B known, the value of $\dot{\gamma}_0$ can be determined from the intercept (B). Note that equation (6-17) could also be used to determine the lower yield stress. If tensile test data is not available, a value of the pressure coefficent can be estimated between the values of 0.10 and 0.30. The constant A/k_B is generally in the range of 100-175 MPa/K.

To quantify the strain softening behavior, the softening slope (h) and the value of s/s_{ss} needs to be determined. It has been shown[7] that the an amorphous polymer in the glassy phase has a relatively constant value of s/s_{ss}. This value can be estimated as the ratio of the maximum peak yield stress to the minimum value of the stress obtained after softening has occurred.

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[7] Boyce, M.C., D.M. Parks and A.S. Argon (1988), Large Inelastic Deformation in Glassy Polymers, Part 1: Rate Dependent Constitutive Model, *Mech. Mater* 7, pp. 15-33.

The softening slope can be estimated from the plot of true stress versus true strain using the following equation:

$$h \approx \frac{\Delta s}{\Delta\gamma^{p}}\left[\frac{1}{1-s/s_{ss}}\right]$$

$$\text{where: } \Delta s = s - s_{ss} \qquad (6\text{-}18)$$

$\Delta\gamma^{p}$ is the increment in plastic strain over which Δs occurs

The strain hardening properties require the experimental determination of the rubbery shear modulus as a function of temperature, $G_R(\Theta) = nk_B\Theta$. The rubbery modulus can be estimated by heating the polymer to a few degrees above the glass transition temperature, and conducting a compression or tension test at a high loading rate. The initial slope of the true stress-true strain curve is the rubbery shear modulus at the test temperature. The value of the rubbery shear modulus at a desired temperature can be obtained using the following expression:

$$G_R(\Theta) = G_R(\Theta_{test})\left[\frac{\Theta}{\Theta_{test}}\right] \qquad (6\text{-}19)$$

The final piece of information required is the number of rigid links between entanglements (N). Using Langevin statistics, N is defined as the square of the locking stretch. The locking stretch can be determined from a plot of true stress versus true strain as the asymptotically approached value of true strain reached during the strain hardening process. The locking stretch is the exponential of the limiting value of the true strain.

6.50 Uniaxial Compression

This section will reduce the general tensorial equations given in the previous section to the simplified case of uniaxial compression. The Cauchy stress tensor for uniaxial compression is given as:

$$T = \begin{bmatrix} -\sigma_c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where: σ_c is the applied compressive true stress, $\sigma_c > 0$ (6-20)

The deviatoric component of the stress tensor is given as:

$$T' = -\sigma_c \begin{bmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix} \tag{6-21}$$

It can be shown that the backstretch tensor is given as:

$$B = -B_{11} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \tag{6-22}$$

Using equation (6-10), the deviatoric component of the driving stress can be expressed as:

$$\overline{T}^{*\prime} = T' - B$$

so, (6-23)

$$\overline{T}^{*\prime} = \begin{bmatrix} -\frac{2}{3}\sigma_c - B_{11} & 0 & 0 \\ 0 & \frac{1}{3}\sigma_c + \frac{B_{11}}{2} & 0 \\ 0 & 0 & \frac{1}{3}\sigma_c + \frac{B_{11}}{2} \end{bmatrix}$$

The shear stress can be found using equation (6-11) as follows:

$$\tau = \sqrt{\frac{1}{2}\overline{T}^{*\prime}\cdot\overline{T}^{*\prime}} = \tau = \frac{1}{\sqrt{3}}\left[\sigma_c + \frac{3}{2}B_{11}\right] \tag{6-24}$$

The shear stress from equation (6-24) can be compared to the result from the Von Mises equation which predicts the compressive stress, neglecting the backstretch tensor, is $\sqrt{3}\tau$.

In uniaxial compression, the (1,1) matrix element is of primary concern. The (1,1) components of the normalized tensorial direction of the shape change is given as follows:

$$\begin{aligned}
&N_{11} = \frac{1}{\sqrt{2\tau^2}}\overline{T}^{*\prime}{}_{11} \\
&\text{from } (6-23), \\
&\overline{T}^{*\prime}{}_{11} = -\frac{2}{3}\sigma_c - B_{11} \\
&\text{so,} \\
&N_{11} = \frac{-\sqrt{\frac{3}{2}}\left[\frac{2}{3}\sigma_c + B_{11}\right]}{\left[\sigma_c + \frac{3}{2}B_{11}\right]}
\end{aligned} \tag{6-25}$$

The (1,1) component of the stretch tensor can be expressed as the sum of a plastic and elastic components:

$$\begin{aligned}
&D_{11} = \overline{D}^{p}{}_{11} + D_{11}{}^{e} \\
&D_{11}{}^{e} \text{ can be appoximated by } \frac{-\dot{\sigma}_c}{E} \text{ so,} \\
&\overline{D}^{p}{}_{11} \approx \dot{\varepsilon}
\end{aligned} \tag{6-26}$$

Using equations(6-10) and (6-25), the plastic component of the stretch tensor can be given as:

$$\overline{D}^{p}_{11} = -\dot{\gamma}^{p}\frac{\sqrt{3}\left[\frac{2}{3}\sigma_{c} + B_{11}\right]}{2\left[\sigma_{c} + \frac{3}{2}B_{11}\right]} \tag{6-27}$$

Combining equations (6-26) and (6-27) and simplifying gives the a relationship for the shear strain rate in terms of the back stretch, strain rate, and compressive stress.

$$\dot{\gamma}^{p} = 2\sqrt{\frac{1}{3}}\dot{\varepsilon}\left[\frac{\sigma_{c} + (3/2)B_{11}}{(2/3)\sigma_{c} + B_{11}}\right] \tag{6-28}$$

where: $\dot{\varepsilon}$ is the compressive strain rate

Equation (6-28) can be combined with equation (6-1) to quantify the response of an amorphous polymer subjected to a homogeneous compressive stress. These equations will be used to predict the yield stress as a function of strain rate for a specific set of polymer blends in Chapter 9.

7.0 SPECIMEN FABRICATION

The production of high quality polymer test specimens is required to produce repeatable test results. To this end, a great deal of time and effort must be directed towards the fabrication process. The key concerns will be addressed in this chapter. These include the resin catalyst and promoter selection and concentration, precautions needed to produce void free specimens, the selection of appropriate test specimen dimensions and the machining operations required to produce them.

7.10 Resin Formulation

Both the Derakane 8084 and Crestomer 1080 were obtained from the manufacturer in an uncatalyzed and unpromoted form. The **catalyst** is the material which initiates the crosslinking reaction, and the **promoter** is an additive which is used to speed up and enhance both the degree and quality of the resin cure.

The amount of promoter and catalyst to use for a given formulation is, in general, a weak function of relative humidity and a strong function of ambient temperature. As the temperature drops, the required amount of the catalyst and promoter tends to increase. It is often a difficult time consuming trial and error process to determine the correct resin formulation for a particular thermoset polymer. The temperature and humidity were measured using a digital hydrometer with the following specifications:

TABLE 7-1

Measuring Range:	10.0%-95.0% Rel. Hum. 32.0-140.0 Degrees F
Maximum Response time:	3.5 Minutes
Resolution:	0.10 % Rel. Humidity 0.10 % Farenheit
Accuracy:	1.5% Rel. Humidity 0.40 Farenheit

When considering blends of two or more polymers, the determination of the correct formulation becomes even more complex. The formulations are usually proprietary and are therefore difficult to obtain.

Dow Chemical has produced a fabrication guide which outlines some catalyst and promoter concentrations. The recommended formulation for Derakane 8084 was given as a function of gel time and ambient temperature. The selected formulation will be discussed.

It was also stated that the ratio of catalyst to promoter should be in the range of 3:1 to 10:1. These values served as a valuable guide in the resin formulation process. An additional complication was introduced by a lack of information regarding "typical" formulations for Crestomer 1080 due to its proprietary nature.

The first catalyst used in this study was Lupersol DHD-9 MEKP (Methyl Ethyl Ketone Peroxide) with 9% active oxygen produced by Elf Atochem of North America. This catalyst was recommended by Dow Chemical as a good general choice for vinyl ester resins, however the use of this catalyst resulted in a slow exotherm development and a poor cure in some of the test specimens containing more than 25% Crestomer. These problems were attributed to the low ambient temperatures in the fabrication facility (58-63^{0}F). This problem was solved by changing the catalyst to Lupersol DELTA-X-9, which is specifically formulated for use in ambient temperatures in range of 55-65^{0}F. The promoter used throughout this study was Cobalt Naphthenate with 6% cobalt content by weight produced by Mooney Chemical.

The combination of Lupersol Delta-X-9 MEKP and Cobalt Naphthenate (CONAP) was used as the catalyst/promoter combination throughout this study. The concentration of MEKP was 3.0% by weight, and the concentration of CONAP was 0.60% by weight. Although recommended in the Dow fabrication guide, DMA (N,N-Dimethylaniline) was not used due to the health risks involved. DMA is a resin cure accelerator which is often used in very small quantities (approximately 0.10 % by weight) to speed up the curing reaction. The specific gravity is important in the fabrication process since the constituants are measured on a volumetric basis. The specific gravity along with the viscosity may be found in Table 7-2.

TABLE 7-2

MATERIAL	SPECIFIC GRAVITY	VISCOSITY (@ 77^0F)
DERAKANE 8084	1.02	350 cps
CRESTOMER 1080	1.04	900 cps
LUPERSOL DELTA-X-9 MEKP	1.15	15.8 cps
MOONE CHEMICALS CONAP	0.93	N/A

7.20 Preparation of Cylindrical Test Specimens

The preparation of test specimens requires a considerable amount of planning and forethought in order to obtain repeatable experimental results. One of the key concerns in the fabrication process was the production of a suitable mold. The key consideration in the selection of the mold material is its ability to resist warping and deformation during the curing exotherm, which typically exceeds 350^0F for approximately 10 minutes. In addition, the free ions of metals such as copper and zinc have an adverse effect on the curing process. Molds constructed of stainless steel and aluminum, as well as glass are acceptable options.

Excellent results were obtained by pouring the resin into a test tube composed of the "hard glass" variant (S-34) of KIMEX glassware, which was coated with several layers of PVA (Poly-Vinyl Alcohol) mold release agent. When cheaper tubes of borosilicate glass were used, the resin did not separate from the tube wall during the curing process. This resulted in severe cavitation of the polymer as shrinkage occurred. This shrinkage, which is on the order of 10%, begins as the crosslinking reaction and the corresponding exotherm occurs, and is complete by the time the resin returns to room temperature.

A final issue in the fabrication process is the elimination of bubbles in the cured resin. The bubbles were caused by entrained air which was trapped in the resin as it was being blended, as well as by the foaming which occurs in the initial stages of the crosslinking reaction. The elimination of these bubbles is a function viscosity which strongly related to temperature as well as the aspect ratio of the container. If the aspect ratio is too large, the rise of the bubbles will be slowed due to an interaction with the walls of the container. As a result, the resin was mixed in a large beaker and allowed to sit for approximately 10 minutes. The resin was then drawn from the bottom of the beaker and injected into the test tubes. The injection process was conducted very slowly, with the test tubes tilted at a 45 degree angle. By allowing the resin to run down the tube in a thin film, the remainder of the bubbles were liberated. The only blend which could not be deairated in this manner was the 100% 1080, due to the high viscosity which was approximately the consistency of molasses. Air removal could have been aided by the use of a vacuum mixing apparatus, which was not available for this study.

The test tubes used had an outer diameter of 18 mm and a length of 150 mm. This allowed for the production of approximately seven cylindrical test specimens with a length to diameter ratio of 1.0. This ratio was chosen for the Instron testing to satisfy two conflicting goals. The first is the need for sufficient cross-sectional area to minimize the errors in the load cell force measurement. The second is to keep the cross sectional area small enough to minimize the effects of friction which can result in inhomogeneous deformation in the specimen.

The specimens were marked and cut using a band saw and left slightly oversized in length (approximately 15%). They were then carefully centered in a lathe, and both sides were faced until parallel. The specimens were each individually inspected to ensure that the faces were parallel to within approximately 0.005 inches, to prevent shearing during the loading process. Despite all these precautions, as well as liberal lubrication and the use of Teflon sheets in the interface between the specimen and the compression plates, approximately 5% of the specimens exhibited some degree of shearing during the tests. It was later discovered that this was due to a misalignment between the compression plates in the Instron machine.

Several other precautions were taken to ensure the repeatability of the results. Prior to testing, the sharp edges of each specimen were removed with fine grit sandpaper to minimize the possibility of tearing the Teflon sheets during testing. In addition, the specimens were heated to 180^0F for 10 hours to ensure that all of the resin was fully reacted prior to testing.

It was discovered during the testing process that the measured value of the peak yield stress is a strong function of the imposed thermal history. These effects may be removed by raising the temperature of the material several degrees above the glass transition temperature for several hours in a process similar to annealing. At the time the tests were conducted, the glass transition temperatures were not known. Several attempts were made to determine an upper-bound limiting temperature, beyond which the material would be damaged. It became clear that a complete analysis of this type could quickly become intractable and the 180^0F value was adopted.

In addition to the compression specimens required for the Instron testing, several specimens were required for the high strain-rate testing which was conducted using a Hopkinson bar. The compression surfaces in the testing apparatus were only 0.75 inches in diameter. To ensure that the specimens remained within these limits during the test, the maximum allowable diameter of the specimens is only on the order of 8 mm.

The Hopkinson bar testing introduced several other issues which required the specimens to have an aspect ratio of 0.5 to 0.75 (length/diameter ratio). Some of these issues will be addressed in Chapter 10. The size made the fabrication of these specimens particularly difficult. The resin rods were centered and machined down in a lathe to the required diameter. The specimens which were produced measured approximately 8 mm in diameter and, therefore needed to be approximately 4-6 mm in length. Specimens of this length are difficult to face-off in a lathe. As a result, most of the specimens were close to the imposed upper limiting aspect ration of 0.75.

Several other specimens were cut using a low speed diamond saw. This apparatus produced high quality specimens of any desired length but required approximately 1 hour per cut. The specimens containing more than 50% Crestomer 1080 could not be cut at room temperature by this method. In order to cut these specimens, they were cooled in liquid nitrogen for 10 minutes prior to cutting.

Chapters 9 and 10 describe the low and high strain rate testing which was conducted using these specimens. These sections begin by presenting the relevant theory and background, followed by the results and experimental observations. Chapter 8 will present the theory and results of the DMA testing. The specimens used in these tests were provided by the research staff of the NASA Langley Research Center.

8.0 FREQUENCY DEPENDENT VISCOELASTIC BEHAVIOR AND DYNAMIC MECHANICAL ANALYSIS (DMA)

The previous discussion of viscoelastic behavior considered the important properties of stress relaxation and creep compliance. It was discussed earlier that the determination of the glass transition temperature (Tg) is accomplished most effectively by means of **Dynamic Mechanical Analysis (DMA).** This process involves loading a specimen of a specific size with a sinusoidal loading program. The amplitude of the load is small enough to ensure that the specimen remains in the elastic range. The test can be conducted either at a fixed temperature over a range of frequencies or at a fixed frequency with a varying temperature.

In the case of linear elastic or elastomeric materials, the stress applied on the specimen is in phase with the strain. When viscoelastic effects are present, the strain lags the stress by a phase angle δ. The stress and strain in a visoelastic material under sinusoidal loading can be can be expressed most conveniently using complex notation as follows:

$$\begin{aligned} \varepsilon(t) &= \varepsilon_0 e^{i\omega t} \\ \sigma(t) &= \sigma_0 e^{i(\omega t+\delta)} \end{aligned} \tag{8-1}$$

The stress can be viewed as a phasor rotating counter clockwise at angular frequency ω composed of two orthogonal components. The first of these component is in phase with the strain and the other component is 90 degrees out of phase. Dividing the stress by the strain gives the following:

$$G^* = \frac{\sigma_0}{\varepsilon_0} e^{i\delta} = \frac{\sigma_0}{\varepsilon_0}(\cos(\delta) + i\sin(\delta)) = G_1 + i\,G_2$$

where: G^* is the complex modulus (8-2)
G_1 is the storage modulus
G_2 is the loss modulus

Using the definition of the of the complex modulus, the phase angle δ can be found as follows:

$$\delta = \tan^{-1}\left(\frac{G_2}{G_1}\right) \tag{8-3}$$

Viscoelasticity is associated with a coupled elastic and damping response. When a polymer undergoes a phase transition, such as the transformation from glassy to rubbery behavior, a change in damping occurs due to changes in molecular mobility. The damping response of an amorphous polymer reaches a maximum at the glass transition temperature which can be thought of as the natural frequency of the main chain rotation at the test frequency. The damping component of the response is represented by the loss modulus and reflects the dissipation of energy during a cycle. The energy loss per cycle (ΔE) can be expressed as follows:

$$\Delta E = \pi G_2 \varepsilon_0^2 \qquad (8\text{-}4)$$

The energy dissipation appears in the polymer in the form of internal heat generation. If the loss modulus is large or the frequency is high, a significant temperature rise can occur.

The next section will use the ideas presented in this section to predict the value of the glass transition temperature resulting from the blending of two dissimilar materials.

8.10 Experimental Determination of the Glass Transition Temperature for Various Blends of Derakane 8084 Vinyl Ester and Crestomer 1080 Urethane Acrylate

The glass transition temperature for five blends of Dow Chemical Derakane 8084 rubber toughened vinyl ester and Scott Bader Crestomer 1080 Urethane Acrylate was determined using DMA techniques. The testing was conducted at the Composites and Polymers Branch of the NASA Langley Research Center in Hampton, Virginia on a Polymer Labs MKII DMTA (Dynamic Mechanical Testing Apparatus). The test was conducted using the parameters shown on Table 8-1.

TABLE 8-1

DMA test frequency:	10 Hz
Thermal Loading Rate	10^0C / minute
Testing Mode:	Single Cantilever Beam

Two runs were conducted for each of five compositions which ranged from 100% 8084 by weight to 100% 1080 by weight in 25% increments. The convention used in defining the glass transition temperature from a DMA test varies from one investigator to the next. In this case, the value was taken as the intercept of approximated lines drawn through the glassy and viscous region regions. The results may be found in Figures 8-1 through 8-10 and are summarized in Table 8-2.

TABLE 8-2

% 8084	% 1080	Tg - Trial 1 (C)	Tg - Trial 2 (C)	Mean Tg (C)	Stdev Tg C)
100	0	113	114	113.5	0.5
75	25	101	102	101.5	0.5
50	50	89	90	89.5	0.5
25	75	66	65	65.5	0.5
0	100	27	27	27.0	0.0

Using the information in Table 8-2, a predictive model for the glass transition temperature as a function of the weight percentage of one of the constituents can be developed. To this end, a curve of Tg versus the weight percentage of Crestomer 1080 was produced.

The form of the fitting equation is based on the premise that the value of the glass transition temperature of a copolymer composed of two polymers can be calculated by the following empirical relationship[1]:

$$\frac{1}{T_g} = \frac{1}{w_1 + B * w_2} \left[\frac{w_1}{T_{g1}} + \frac{B * w_2}{T_{g2}} \right]$$

where: w_1, w_2 are the weight fractions of the individual polymers
as given in Table 8 - 2 (8-5)
T_{g1}, T_{g2} are the glass transition temperatures of the individual polymers
B is a constant which is close to unity

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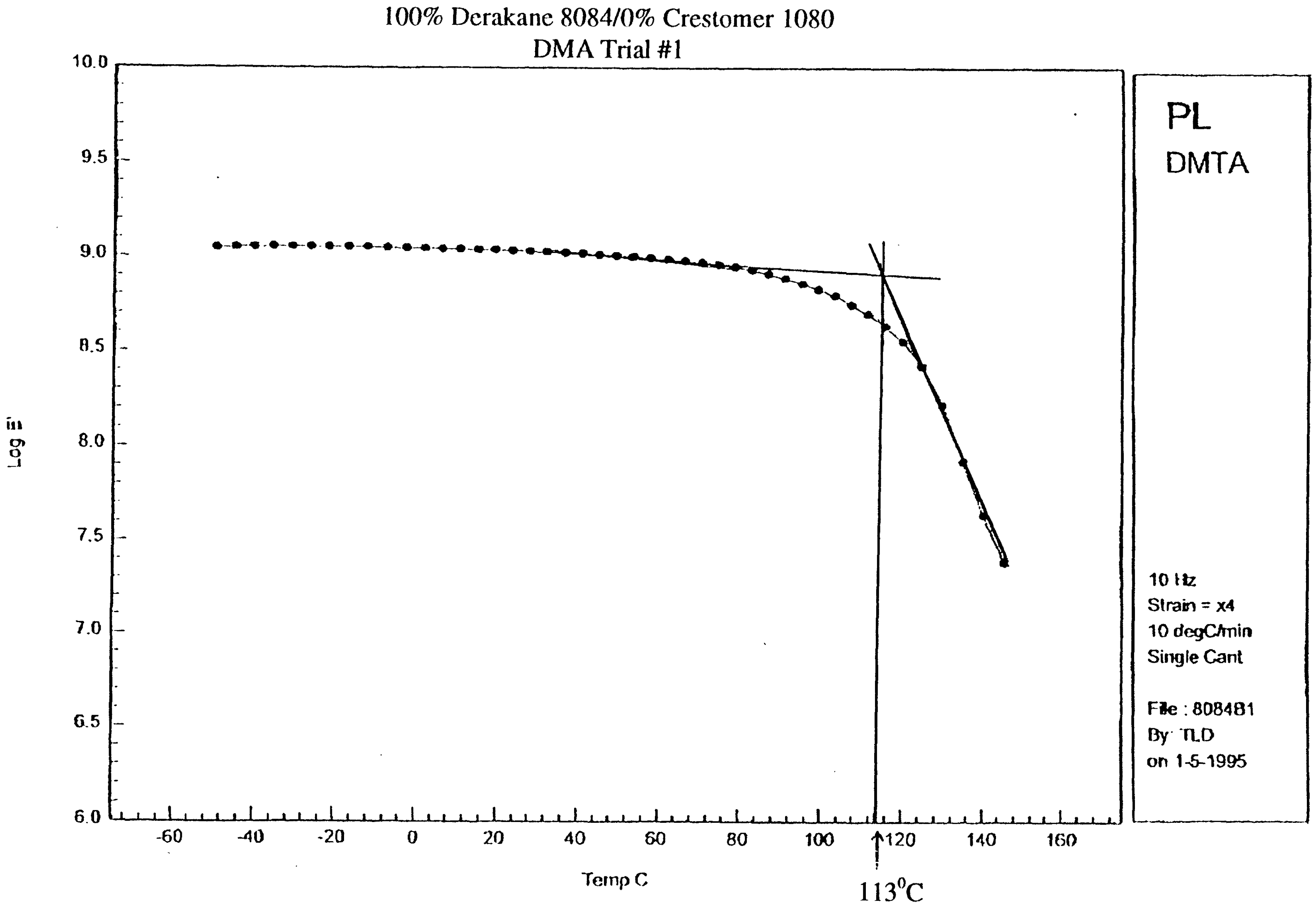
[1] L. Mandelkern, G.M. Martin and F. A. Quinn, *J. Res. Natl Bur. Stand.*, **58**, (1959) 137.

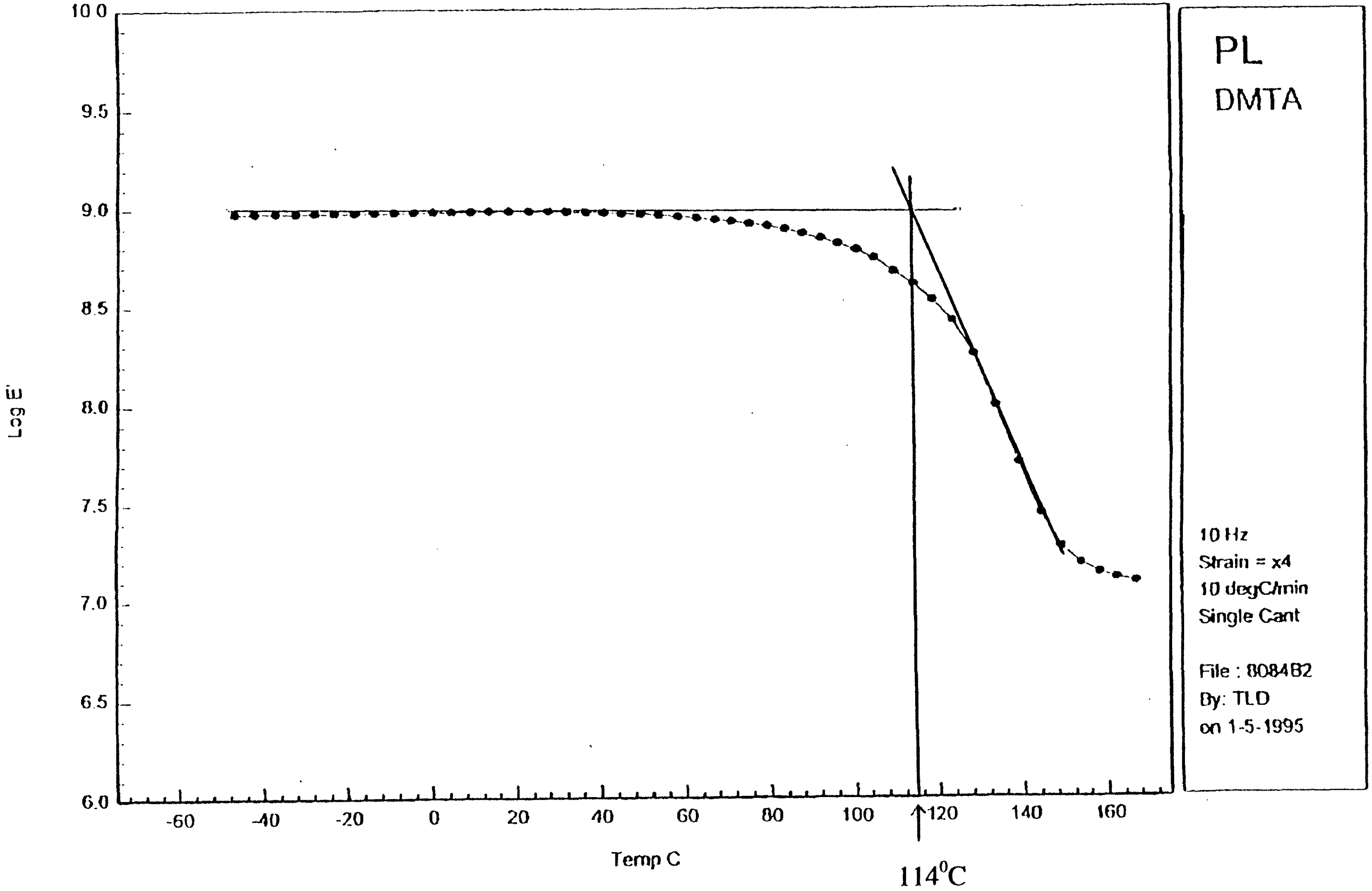


100% Derakane 8084/0% Crestomer 1080
DMA Trial #1

Juska, “DMA Data,” NASA Langley Research Center, 1995.

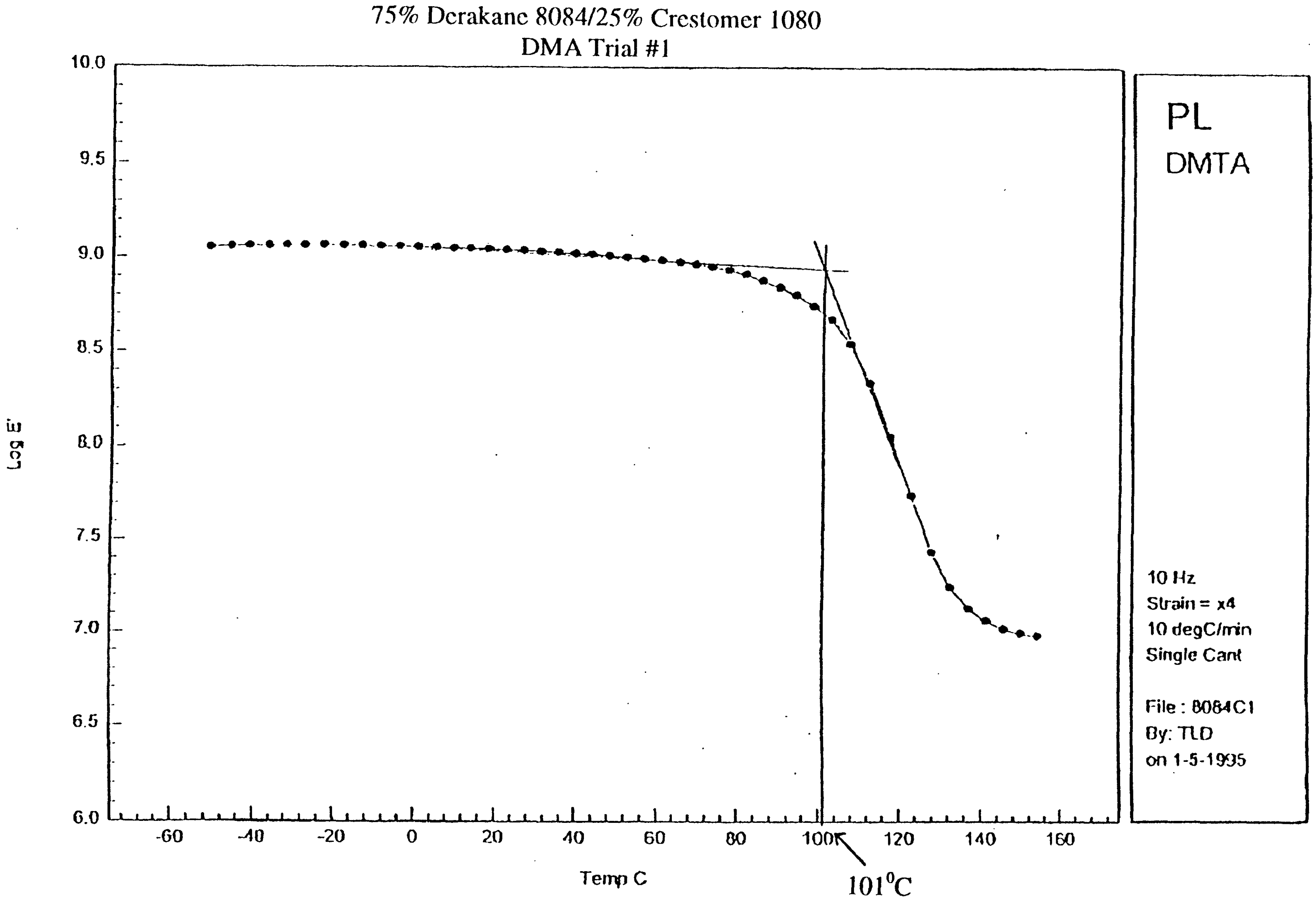
Figure 8-1

100% Derakane 8084/0% Crestomer 1080
DMA Trial #2



Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-2



75% Derakane 8084/25% Crestomer 1080
DMA Trial #1

Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-3

75% Derakane 8084/25% Crestomer 1080
DMA Trial #2

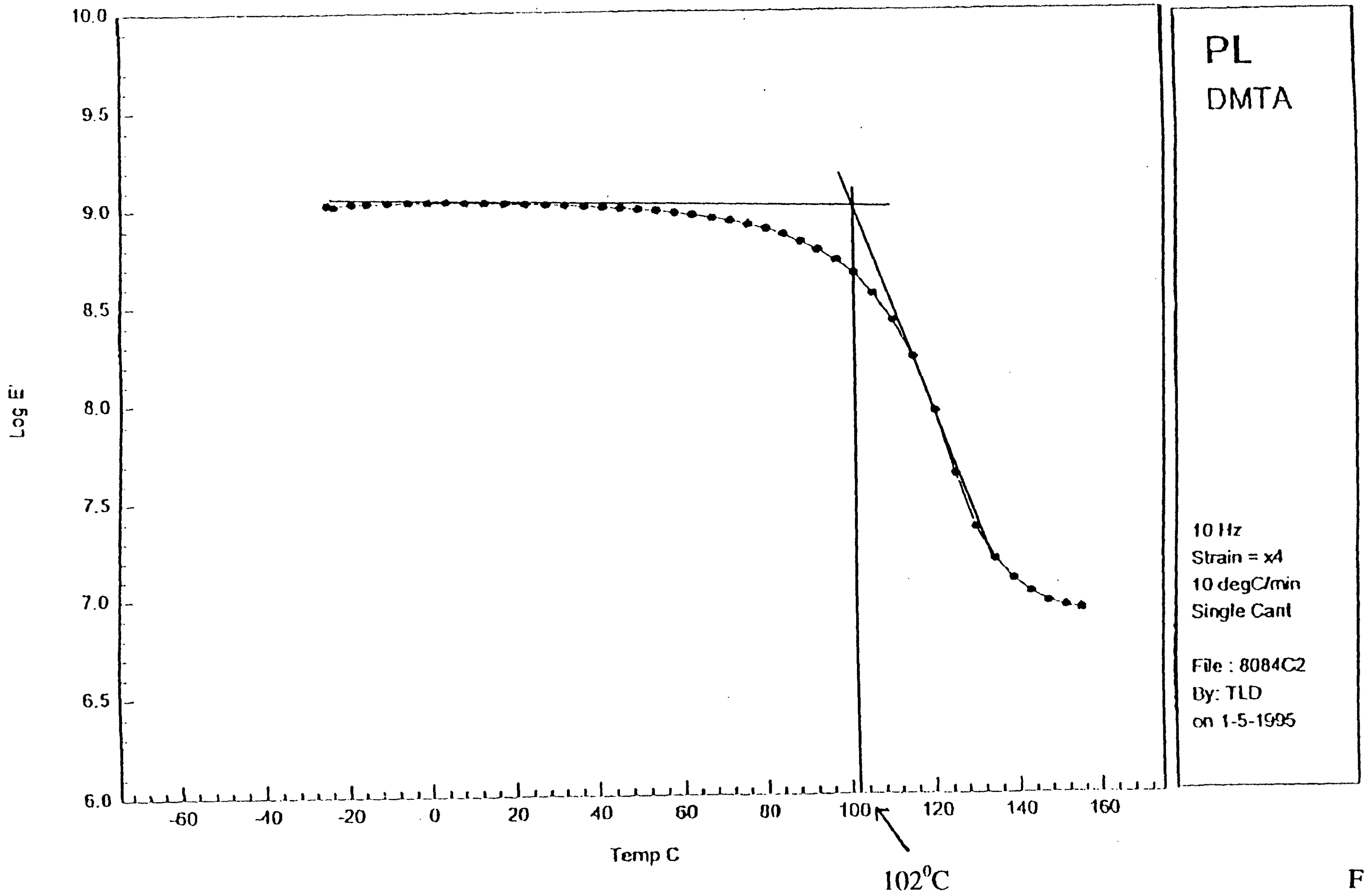
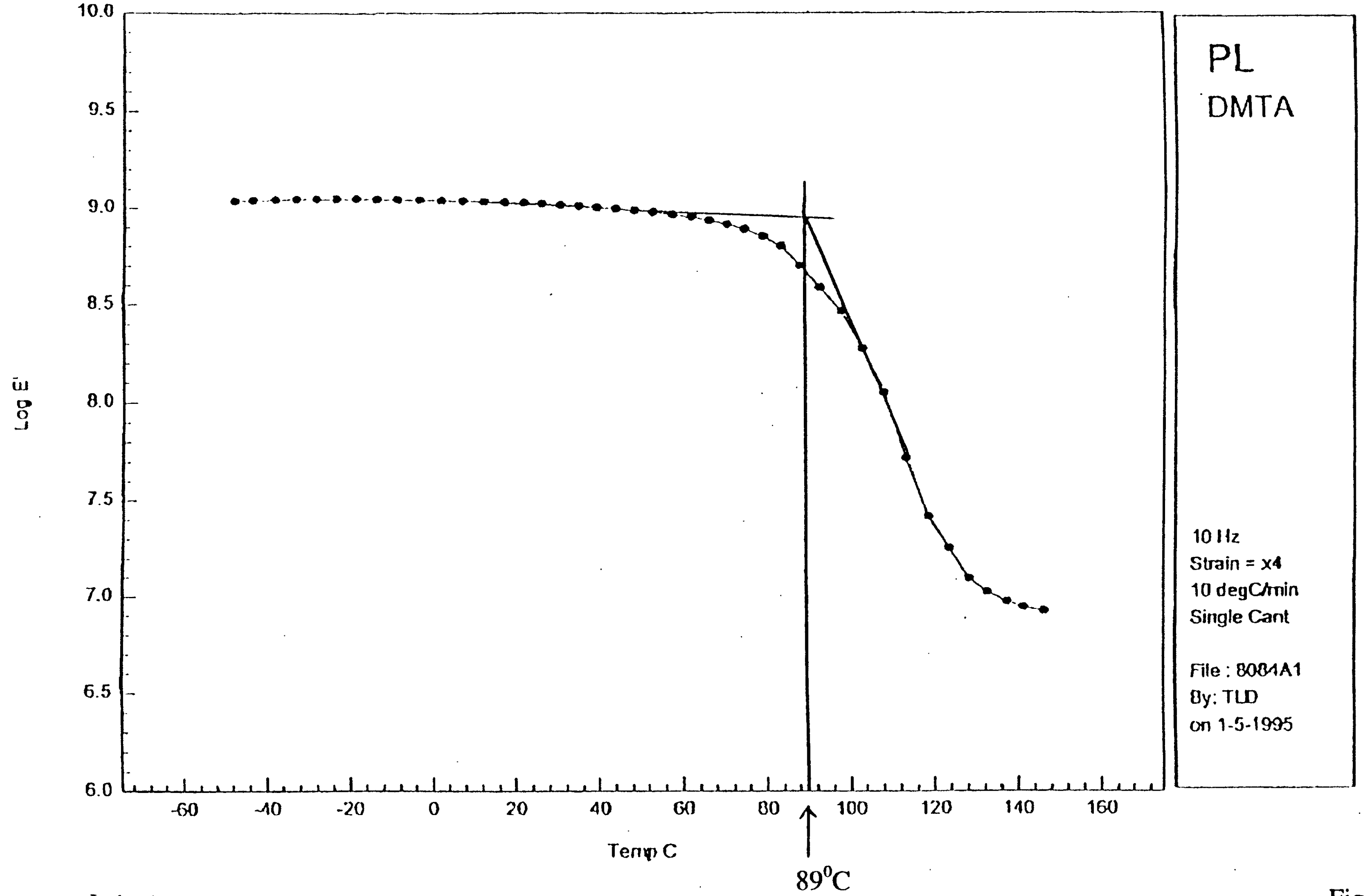


Figure 8-4

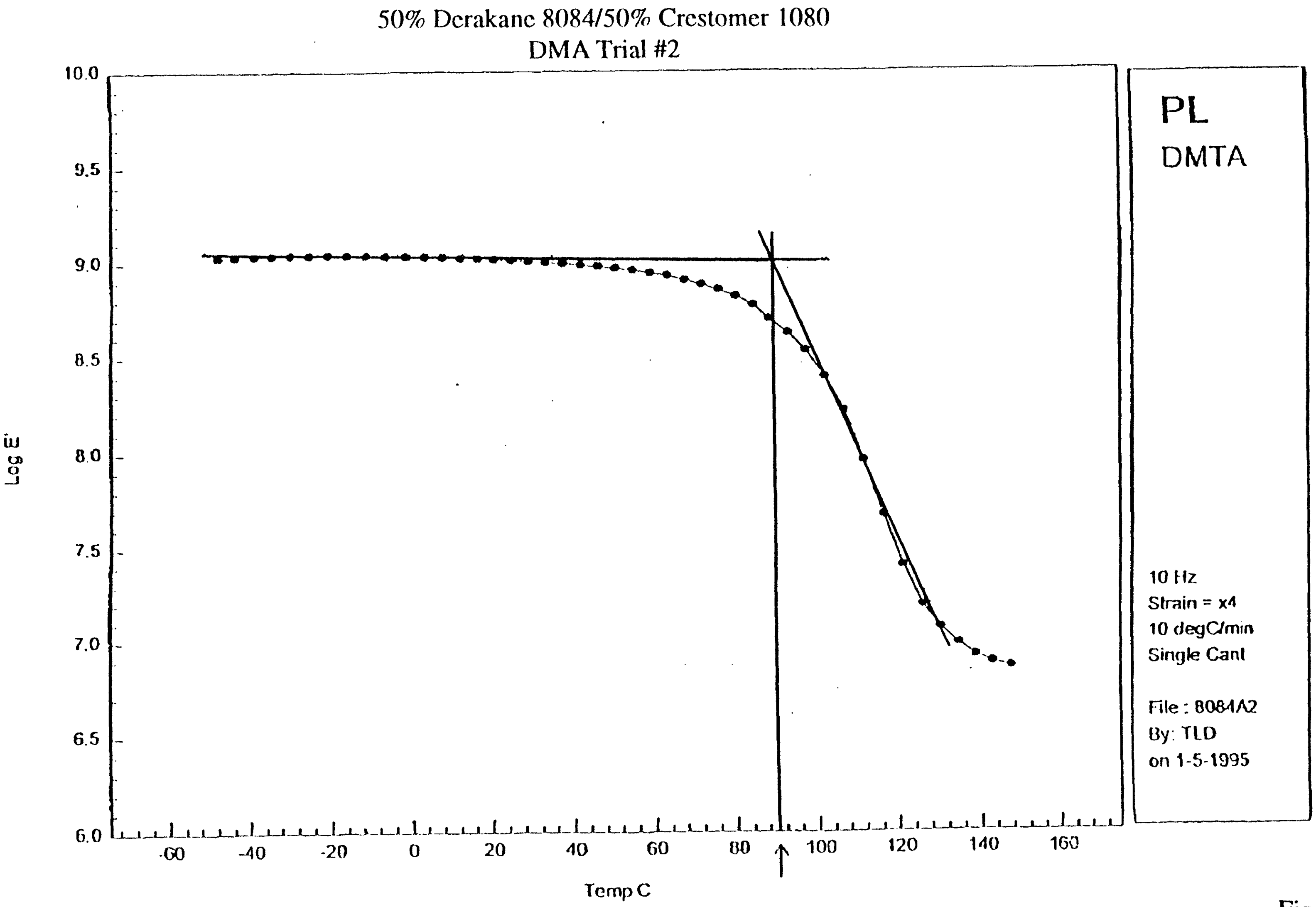
Juska, "DMA Data," NASA Langley Research Center, 1995.

50% Derakane 8084/50% Crestomer 1080
DMA Trial #1



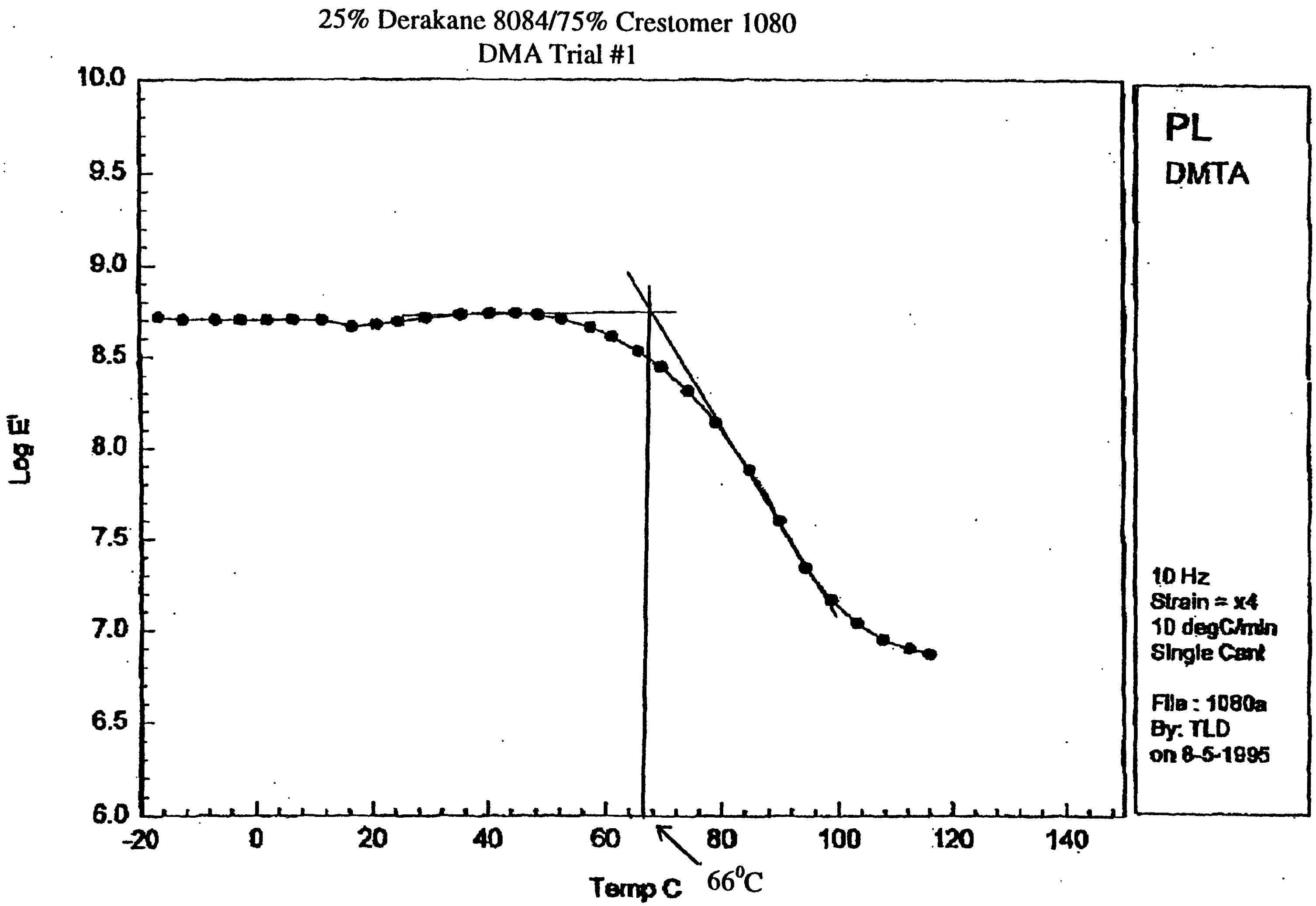
Juska, “DMA Data,” NASA Langley Research Center, 1995.

Figure 8-5


50% Derakane 8084/50% Crestomer 1080
DMA Trial #2

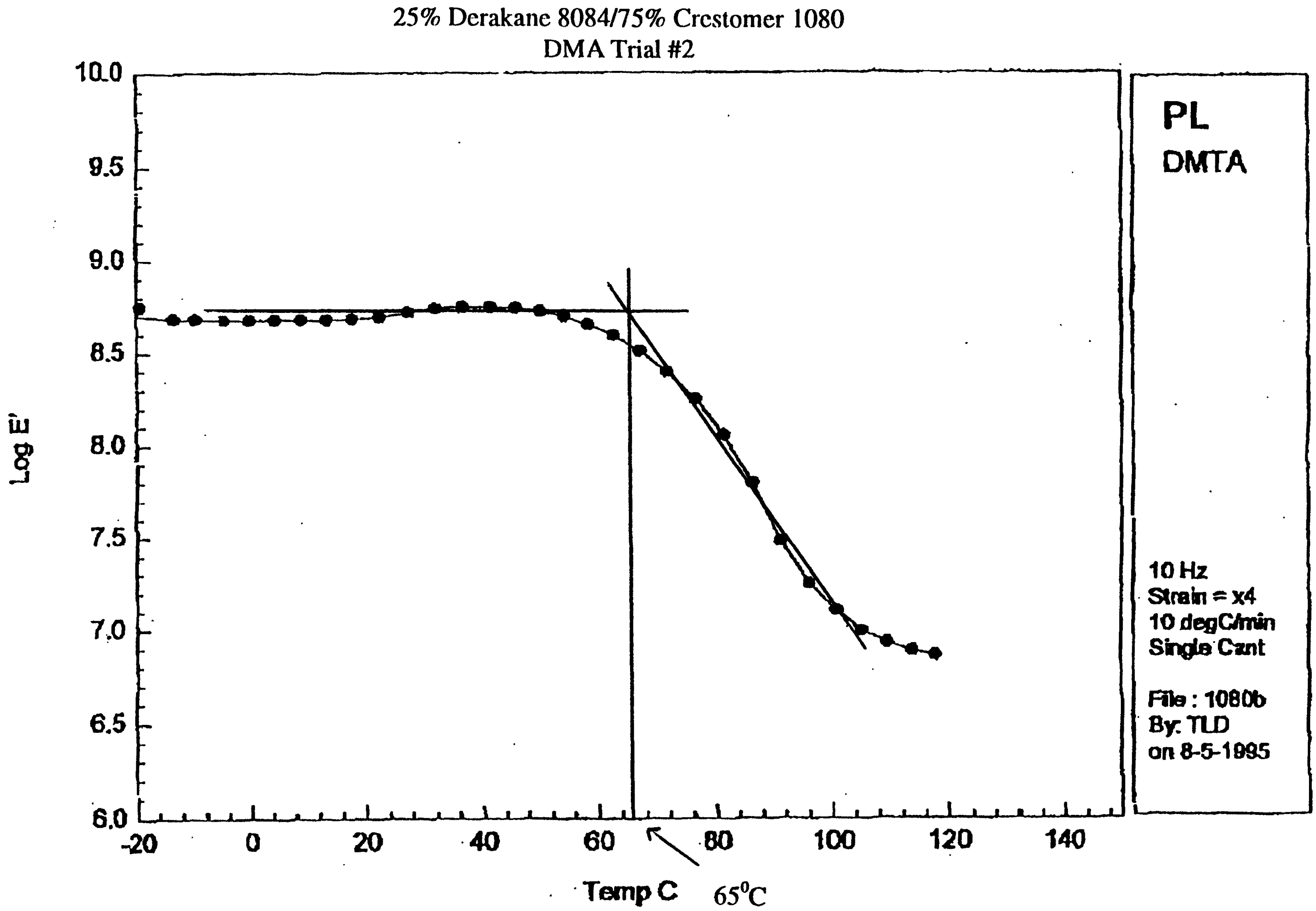
Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-6



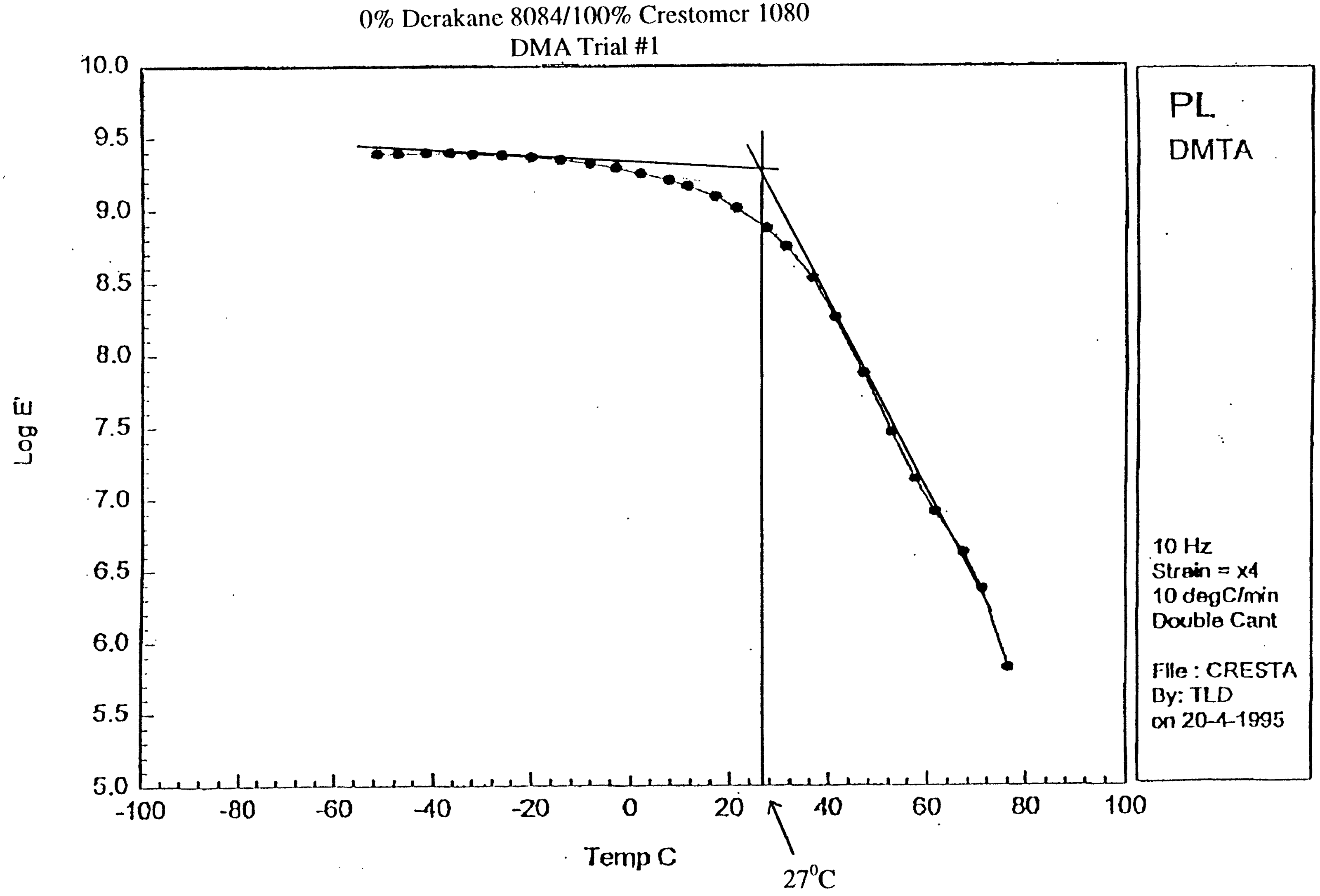
Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-7



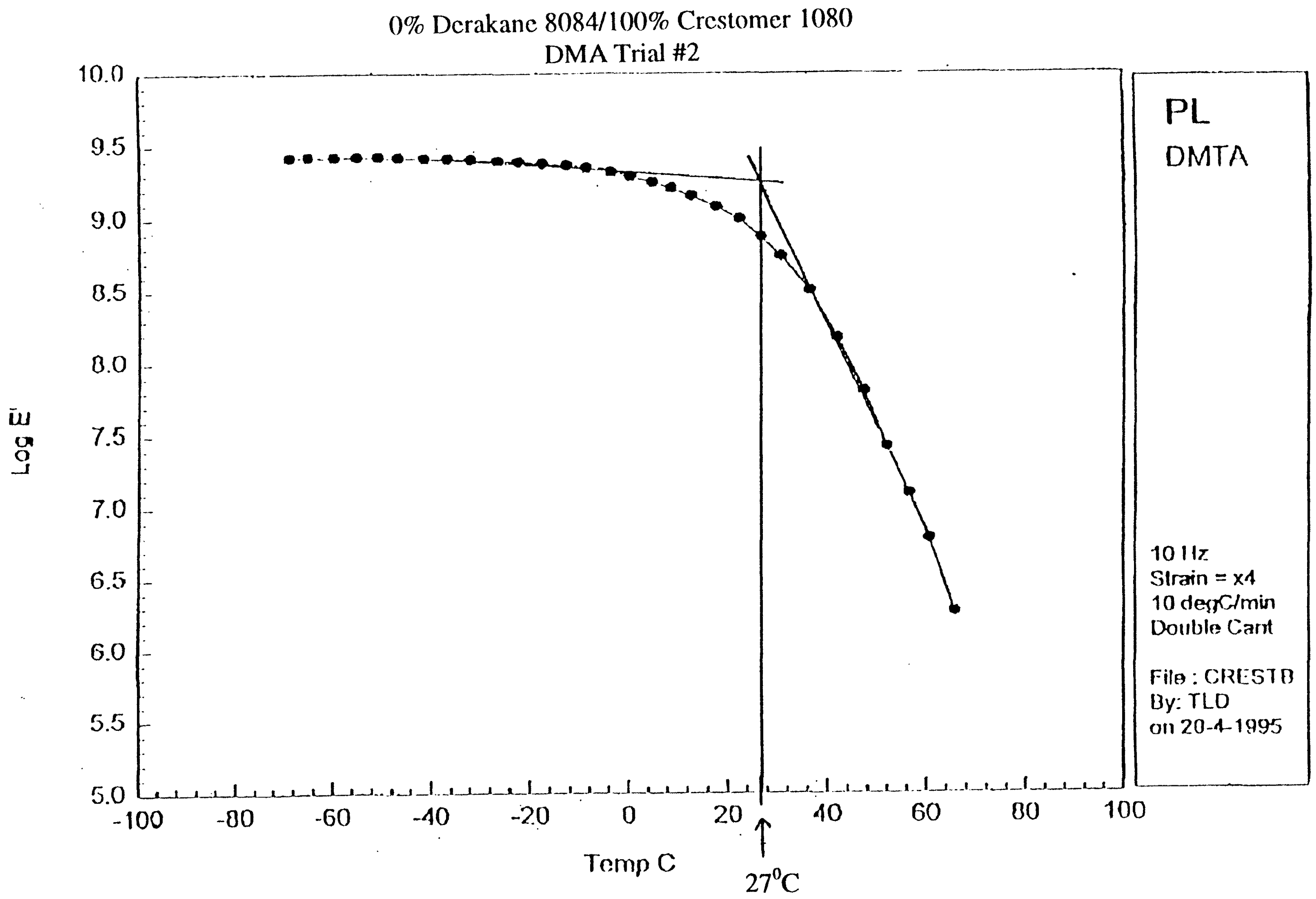
Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-8



Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-9



0% Derakane 8084/100% Crestomer 1080
DMA Trial #2

Juska, "DMA Data," NASA Langley Research Center, 1995.

Figure 8-10

Equation (8-5) was coded into a MATHCAD file which may be found in Appendix B. The value of the parameter B was iterated until a reasonable fit was obtained. The results of the final iteration are shown in Figure 8-11. Superimposed on the same graph is a curve for a typical polymer in which B is 1.0. The value of B obtained in this study was 0.11. Most polymer blends obey what is essentially a series model in which a small quantity of the softer material results in a rapid drop in stiffness of the blend. In this material blend (B=0.11) the stiffness drops very slowly with increasing weight percentage of the softer material (Crestomer).

The glass transition curve, which was obtained at a frequency of 10 Hz, was extrapolated out to 100 Hz., 1,000 Hz., and 10,000 Hz using shift factorss and the WLF equation, equations (5-20 and 5-21). These are shown in Figure 8-12.

The glass transition temperature can be used to make some general statements about the response of the material under load. To first order, if the glass transition temperature of a blend of two polymers is very close to one of the component homopolymers, it is expected that the modulus of elasticity will also be similar. Since the first three compositions are only separated by 20^0C, it is expected that the mechanical loading behavior of these compositions will not differ to any significant degree and it is expected that the behavior will be dominated by the Derakane 8084 material. This indicates that the rate dependency of the modulus is likely to be small. By contrast, the blend consisting of 25% Derakane 8084 and 75% Crestomer 1080 shows a significantly lower value of Tg. In this blend the Crestomer 1080 is exertion a considerable influence on the blend. As a consequence, the loading behavior of this blend will likely be significantly different than the previous three including a significant increase in rate dependency of the modulus since the pure Crestomer 1080 will show a very strong dependence on load rate at room temperature.

A key assumption in this analysis is that the polymer blends behave, to first order, in a manner similar to a single homogeneous polymer with the estimated value of Tg. This assumption requires further investigation. The assumption that an equivalent value of Tg could be used to characterize the behavior of a polymer blend might begin to break down when plastic deformation occurs. In particular, the initial stages of plastic deformation

FIGURE 8-11

PLOT OF PREDICTED GLASS TRANSITION TEMPERATURE VERSUS WEIGHT% CRESTOMER 1080:

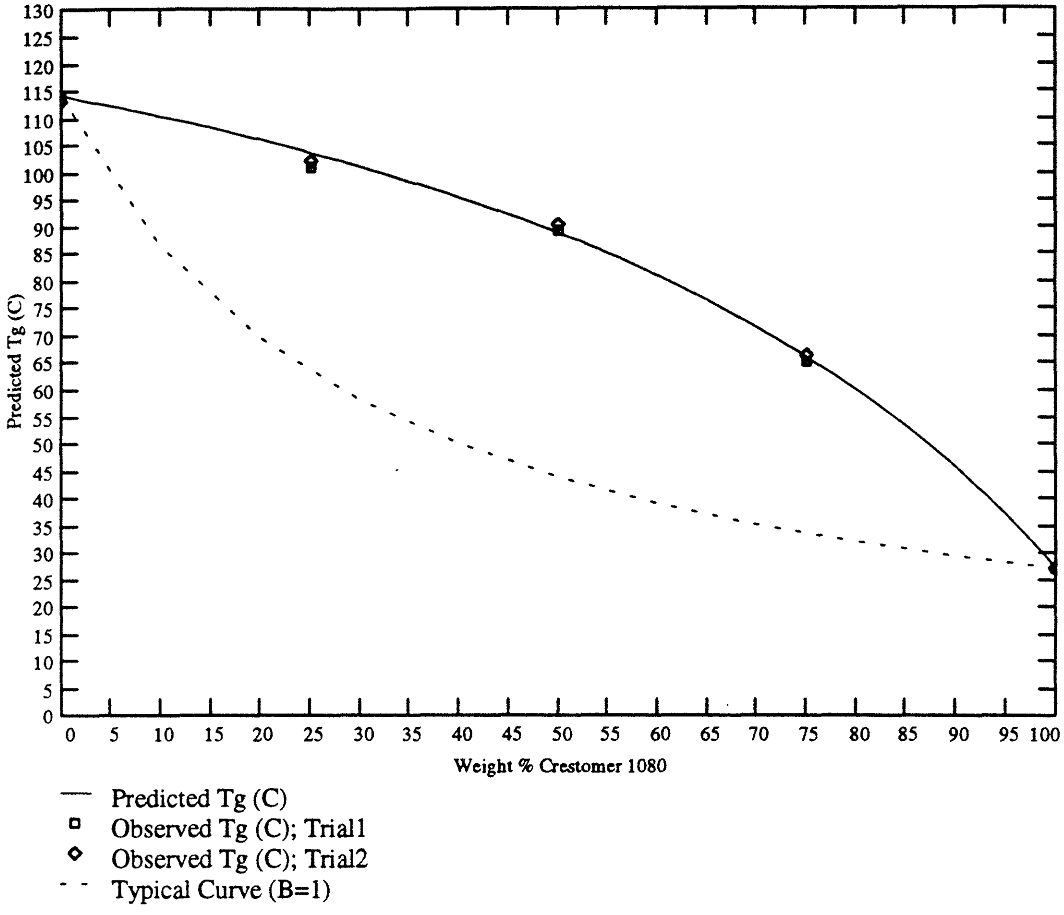
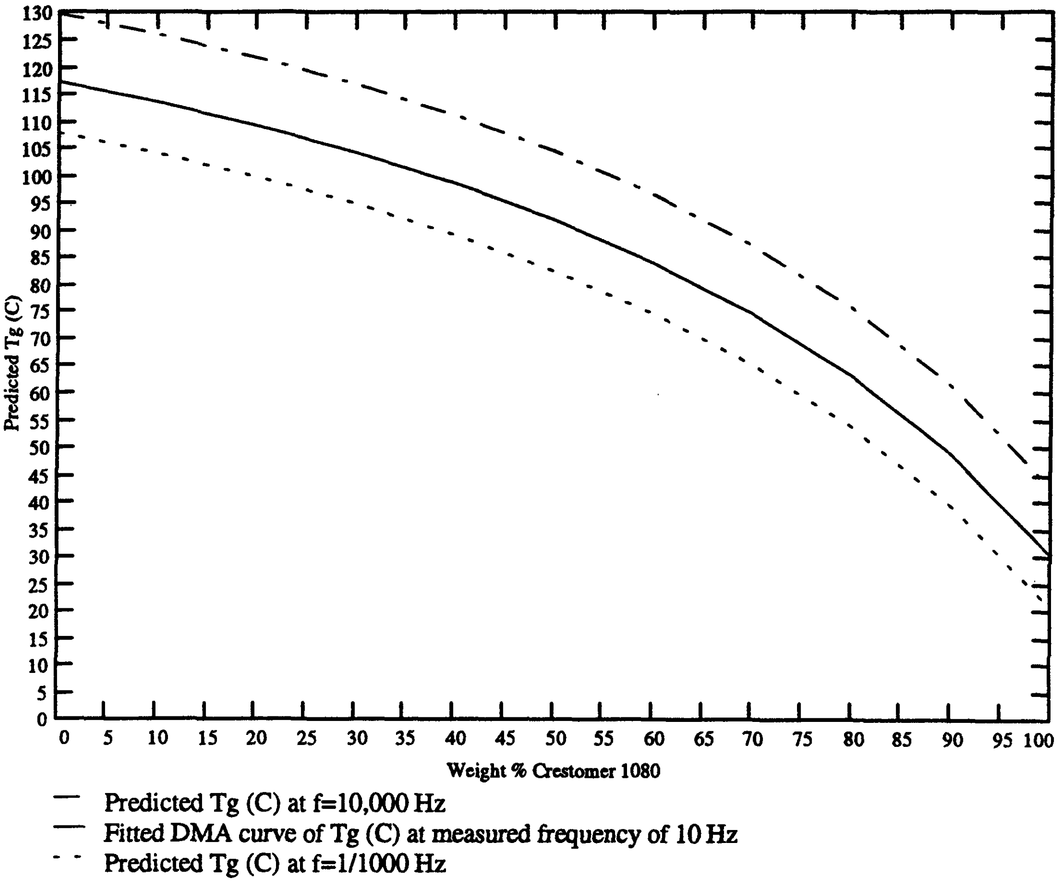


FIGURE 8-12

PLOT OF THE GLASS TRANSITION TEMPERATURE VERSUS WEIGHT %CRESTOMER 1080 EXTRAPOLATED TO VARIOUS FREQUENCIES



following yield is represented by a softening effect. This effect is due to small-scale inhomogeneous deformation mechanisms such as shear banding. At this scale, the assumption that the polymer behaves in a homogeneous manner is in question, and a composite material constitutive model should be used to describe the behavior.

9.0 INSTRON TEST RESULTS

The purpose of this series of tests is to evaluate the behavior of the proposed blends of Derakane 8084 and Crestomer 1080 under a variety of loading rates. The resets of these tests were used to formulate a predictive model of the yield behavior as a function of axial strain rate. The goal was to be able to extrapolate the yield stress out to strain rates on the order of 1000 sec-1. The next chapter contains a comparison of the projected and experimentally obtained values of the yield stress at these high strain rates.

The compressive stress-strain behavior of several blends of Derakane 8084 and Crestomer 1080 were determined using an Instron hydraulic testing machine. The tests were conducted at the Material Science and Testing Lab at MIT on an Instron Model #8501 hydraulic testing machine. The data was obtained using and IBM compatible PC with a Kiethley model 500 data acquisition board which had a maximum data acquisition frequency of 500 Hz. The analysis was performed using LABTEC, a Windows based data collection program.

The testing was conducted on each of the five blends of Derakane 8084 and Crestomer 1080 which ranged from 100 wt.% 8084 to 100 wt.% 1080 in 25% increments. The loading for the first four materials was applied at a constant crosshead velocity of 0.01, 0.10, 1.00, and 2.50 mm/sec, at data collection frequencies of 5 hz, 50 hz, 400 hz, and 500 hz respectively. To minimize frictional effects during the loading process (such as barreling), lubricated Teflon sheets were inserted between the specimen surfaces and the compression plates. The 100 wt% 1080 specimen tests were only included for the first three loading rates due to a lack of reliable (repeatable) specimens. The results, which may be found in the Appendix C, are summarized in Table 9-1.

Appendix C contains a testing log which reflects all of the Instron testing which was conducted in this study. Many of the tests produced poor results due to manufacturing and resin formulation errors which were later resolved. The manufacturing and testing lessons learned were discussed in Chapter 7.

The loading was conducted at a constant loading velocity, instead of a constant strain rate. The strain rates shown below represent the initial (nominal) strain rate which was determined as follows:

$$\dot{\varepsilon}_{nom} = \frac{V_0}{L_0}$$

where: V_0 is the crosshead velocity (mm / sec) (9-1)

L_0 is the inital length of the specimen (mm)

The true (instantaneous) strain rate is defined as follows:

$$\dot{\varepsilon}_{true} = \frac{V_0}{L_i}$$

where: V_0 is the crosshead velocity (mm / sec) (9-2)

L_i is the instantaneous length of the specimen (mm)

The true strain rate can be determined from the initial strain rate as follows:

$$\dot{\varepsilon}_{true} = \frac{V_0}{L_i} = \frac{V_0}{L_0}\frac{L_0}{L_i} = \frac{\dot{\varepsilon}_{nom}}{\lambda} \qquad (9\text{-}3)$$

where: λ is the extension ratio

Figure 9-1 shows the difference between the true strain rate and the nominal strain rate for a specimen with an initial length of 15.5 mm subjected to a loading rate of 0.01 mm/sec to a true compressive strain of 150%. Note that this curve maintains the same shape for all the applied load rates.

The deviation in strain rate shows approximately a four fold increase in strain rate over the nominal value at 150% true compressive strain. These effects are minimal when determining the yield stress which occurs at strains below 10% true strain. Another observation is that the stress is generally assumed to be a function of log($\dot{\varepsilon}$), and the difference in the logarithmic values is only on the order of 1.50 at 150% true compressive strain. Because of these factors, substitution of the nominal strain rate for the true strain rate is a valid assumption when attempting to predict the yield stress as a function of strain rate, and an acceptable one up to the testing limits in this study.

FIGURE 9-1

COMPARISON BETWEEN TRUE STRAIN RATE AND NOMINAL STRAIN RATE AS A FUNCTION OF TRUE STRAIN

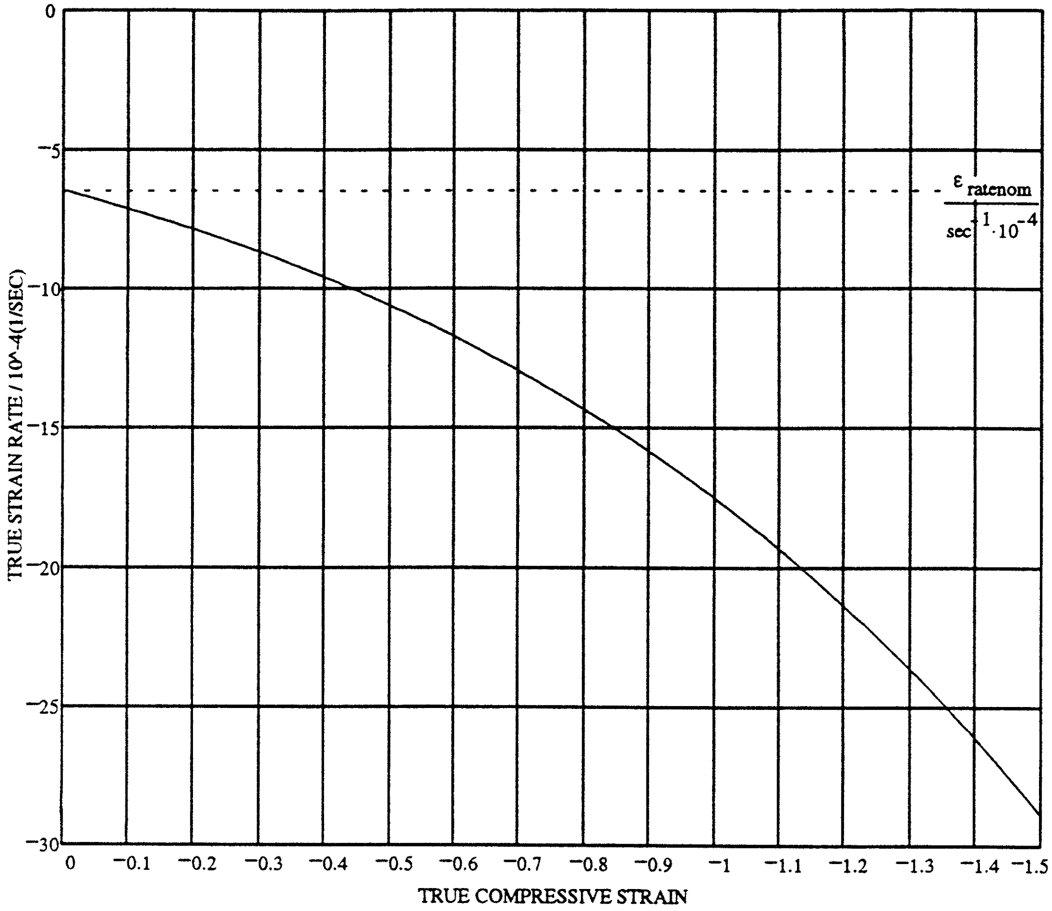


TABLE 9-1

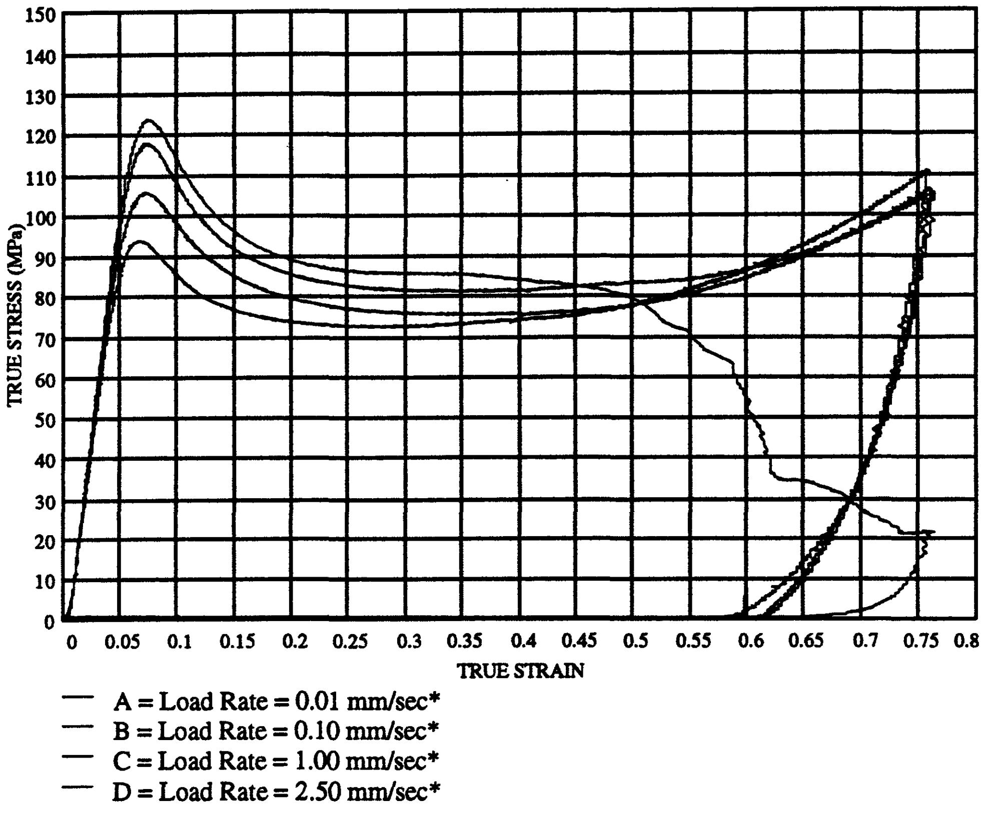
Wt%8084/ Wt%1080	Initial Strain Rate (1/sec)	Number of Runs	Mean True Peak Yield Stress (MPa)	Stdev. True Peak Yield Stress (MPa)	Mean True Peak Yield Strain	Stdev. True Peak Yield Strain
100/0	6.33x10^{-4}	3	93.409	0.040	0.065	0.002
100/0	6.41x10^{-3}	2	105.201	0.221	0.069	0.002
100/0	6.44x10^{-2}	3	118.436	0.740	0.071	0.003
100/0	1.64x10^{-1}	3	123.647	0.517	0.073	0.002
75/25	6.66x10^{-4}	3	67.997	0.154	0.068	0.002
75/25	6.68x10^{-3}	3	81.961	1.719	0.065	0.002
75/25	6.68x10^{-2}	3	95.180	0.599	0.070	0.003
75/25	1.70x10^{-1}	3	100.843	0.519	0.070	0.000
50/50	6.73x10^{-4}	3	61.213	0.664	0.061	0.002
50/50	6.60x10^{-3}	4	*74.165	*0.096	*0.062	*0.001
50/50	6.82x10^{-2}	3	86.508	0.615	0.068	0.003
50/50	1.64x10^{-1}	3	92.670	0.390	0.067	0.001
25/75	6.50x10^{-4}	3	*22.445	*0.015	*0.0585	*0.005
25/75	6.38x10^{-3}	3	30.087	1.279	0.067	0.001
25/75	6.42x10^{-2}	4	49.841	0.698	0.065	0.001
25/75	1.59x10^{-1}	4	57.367	2.419	0.066	0.002

* Each of these runs had an outlier which was removed prior to the calculation of the mean and standard deviation. The individual values of the peak yield stress and the corresponding yield strain may be found on the stress-strain curves in Appendix C.

Figures 9-2 through 9-6 show a comparison of the true stress versus true strain curves for each of the compositions showing the effect of strain rate on the behavior of the blend. Figures 9-7 through 9-10 compare the behavior of each of the materials at a given load rate. These traces shown in these plots correspond to a "typical" run, and not an average response. This simplification can be made due the high degree of repeatability in the experimentally obtained stress strain curves.

The 100% Derakane and 75% Derakane / 25% Crestomer blends failed by a fracture mechanism when tested at relatively high loading rates. In the former, this occurred at the two highest loading rates, and in the later fracture only occurred at the highest loading rate of 2.5 mm/sec. None of the other specimens failed by this mechanism.

Figure 9-2
100% 8084 / 0% 1080 at various load rates

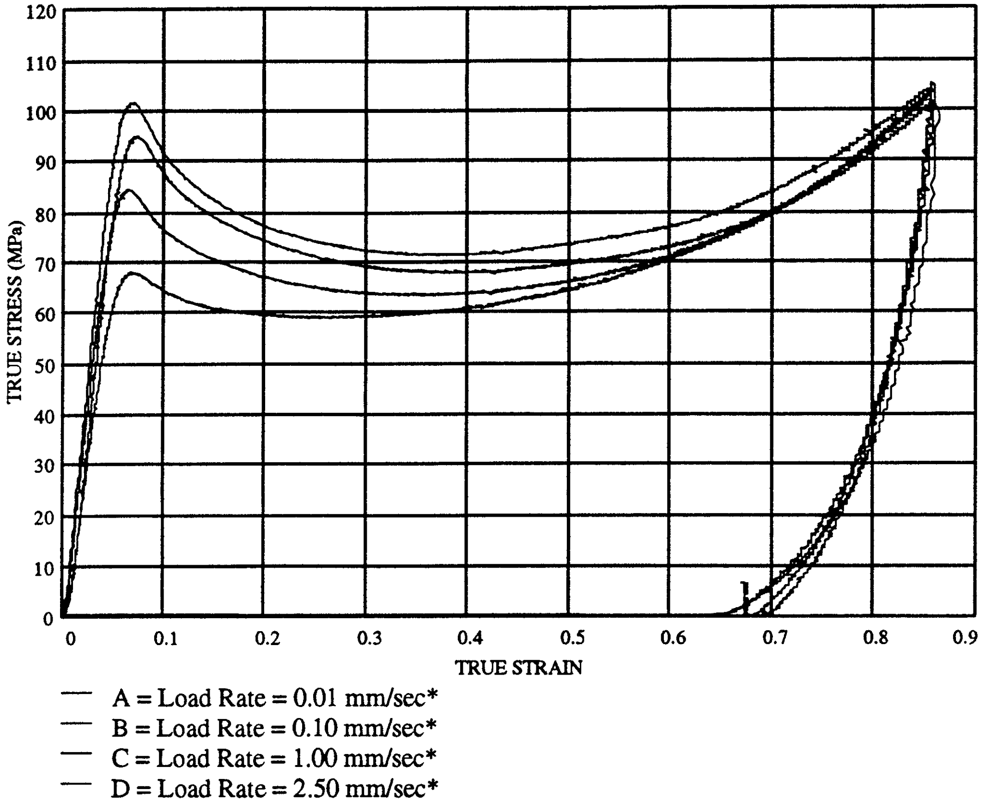


1. **A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec**
2. **A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec**
3. **A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec**
4. **A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec**

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 0.759$	$\max\left(\sigma_{\text{trueA}}\right) = 110.58\cdot\text{MPa}$	$\sigma_{yA} = 93.767\cdot\text{MPa}$	$\varepsilon_{yA} = 0.065$
$\max\left(\varepsilon_{\text{trueB}}\right) = 0.761$	$\max\left(\sigma_{\text{trueB}}\right) = 106.084\cdot\text{MPa}$	$\sigma_{yB} = 105.442\cdot\text{MPa}$	$\varepsilon_{yB} = 0.071$
$\max\left(\varepsilon_{\text{trueC}}\right) = 0.763$	$\max\left(\sigma_{\text{trueC}}\right) = 117.63\cdot\text{MPa}$	$\sigma_{yC} = 117.63\cdot\text{MPa}$	$\varepsilon_{yC} = 0.073$
$\max\left(\varepsilon_{\text{trueD}}\right) = 0.763$	$\max\left(\sigma_{\text{trueD}}\right) = 123.683\cdot\text{MPa}$	$\sigma_{yD} = 123.683\cdot\text{MPa}$	$\varepsilon_{yD} = 0.073$

Figure 9-3
75% 8084 / 25% 1080 at various load rates



1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

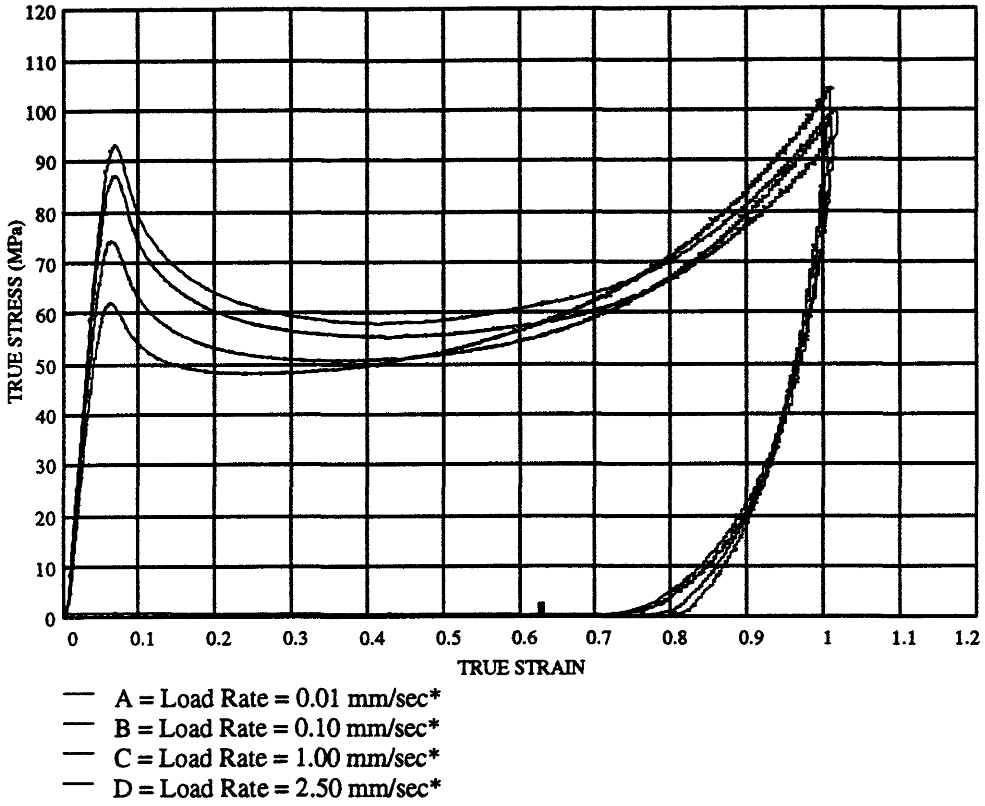
$\max(\varepsilon_{\mathrm{trueA}}) = 0.855 \quad \max(\sigma_{\mathrm{trueA}}) = 103.201 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yA}} = 67.896 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yA}} = 0.066$

$\max(\varepsilon_{\mathrm{trueB}}) = 0.86 \quad \max(\sigma_{\mathrm{trueB}}) = 104.253 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yB}} = 84.366 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yB}} = 0.064$

$\max(\varepsilon_{\mathrm{trueC}}) = 0.862 \quad \max(\sigma_{\mathrm{trueC}}) = 101.544 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yC}} = 94.798 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yC}} = 0.071$

$\max(\varepsilon_{\mathrm{trueD}}) = 0.864 \quad \max(\sigma_{\mathrm{trueD}}) = 104.907 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yD}} = 101.388 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yD}} = 0.07$

Figure 9-4
50% 8084 / 50% 1080 at various load rates



1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

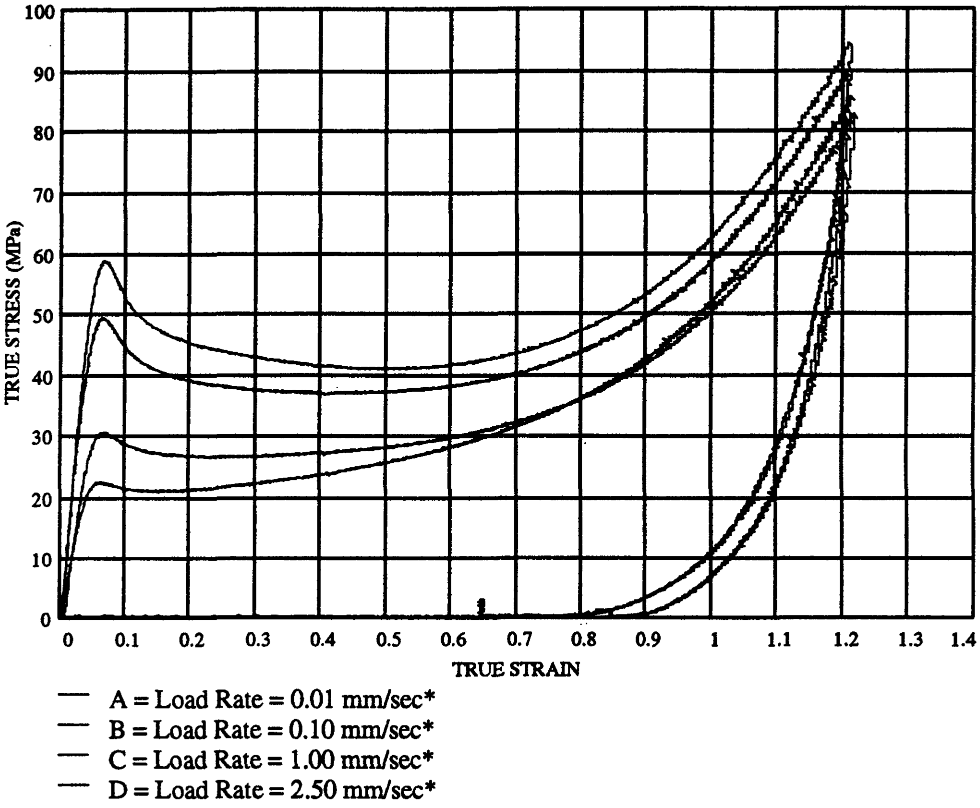
$\max(\varepsilon_{\text{trueA}}) = 1.012$ $\max(\sigma_{\text{trueA}}) = 104.044 \cdot \text{MPa}$ $\sigma_{yA} = 62.089 \cdot \text{MPa}$ $\varepsilon_{yA} = 0.059$

$\max(\varepsilon_{\text{trueB}}) = 1.01$ $\max(\sigma_{\text{trueB}}) = 98.384 \cdot \text{MPa}$ $\sigma_{yB} = 74.187 \cdot \text{MPa}$ $\varepsilon_{yB} = 0.063$

$\max(\varepsilon_{\text{trueC}}) = 1.012$ $\max(\sigma_{\text{trueC}}) = 94.066 \cdot \text{MPa}$ $\sigma_{yC} = 87.054 \cdot \text{MPa}$ $\varepsilon_{yC} = 0.067$

$\max(\varepsilon_{\text{trueD}}) = 1.017$ $\max(\sigma_{\text{trueD}}) = 100 \cdot \text{MPa}$ $\sigma_{yD} = 93.068 \cdot \text{MPa}$ $\varepsilon_{yD} = 0.067$

Figure 9-5
25% 8084 / 75% 1080 at various load rates



1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

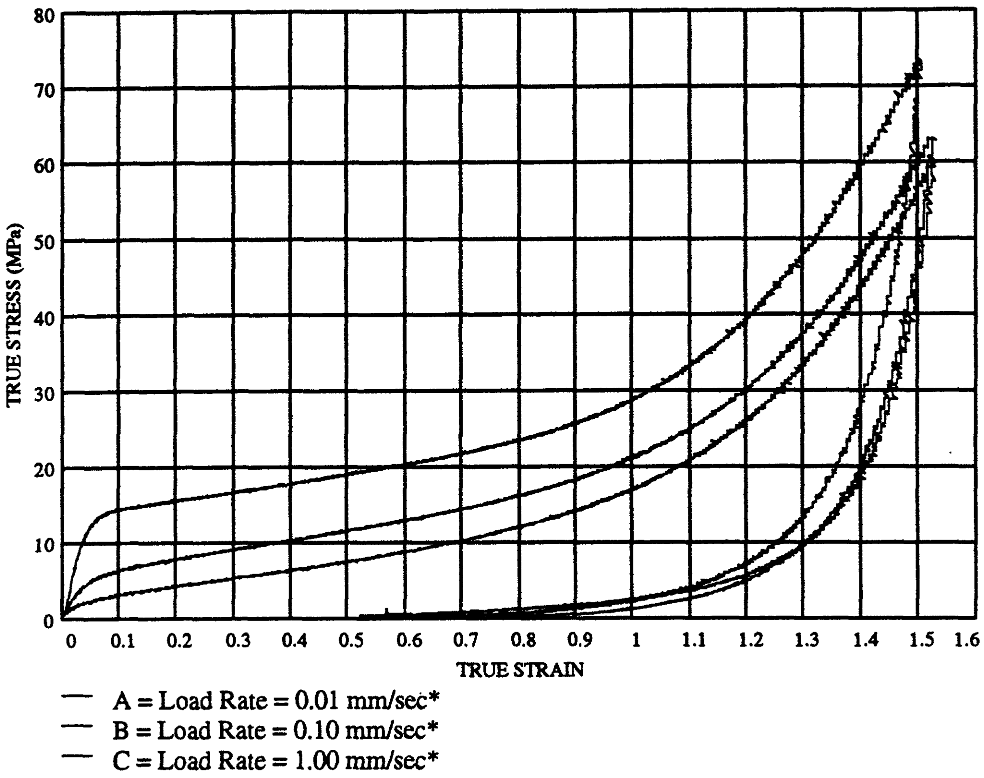
$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 1.216$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 85.539 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 22.46 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.059$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 1.222$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 82.846 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 30.551 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.068$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.211$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 89.4 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 49.394 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.066$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.212$ $\max\left(\sigma_{\mathrm{trueD}}\right) = 94.333 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yD}} = 58.77 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yD}} = 0.069$

Figure 9-6
0% 8084 / 100% 1080 at various load rates



1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

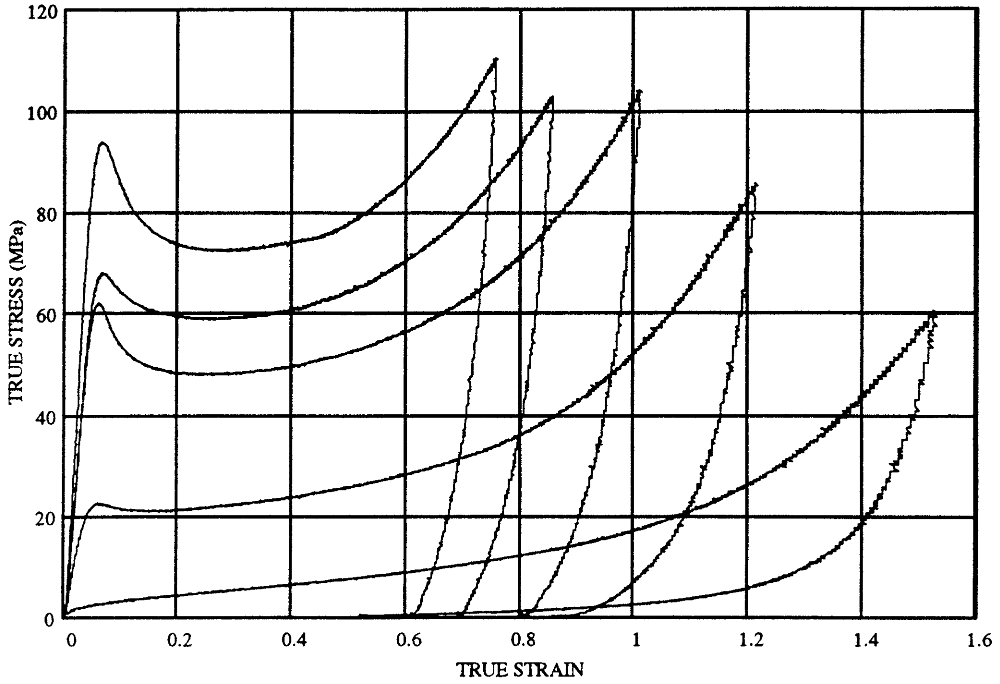
$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 1.529 \quad \max\left(\sigma_{\mathrm{trueA}}\right) = 60.495 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 1.53 \quad \max\left(\sigma_{\mathrm{trueB}}\right) = 63.194 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.506 \quad \max\left(\sigma_{\mathrm{trueC}}\right) = 73.66 \cdot \mathrm{MPa}$

FIGURE 9-7

Plot of true stress (MPa) versus true strain for a loading rate of 0.01 mm/sec for various compositions



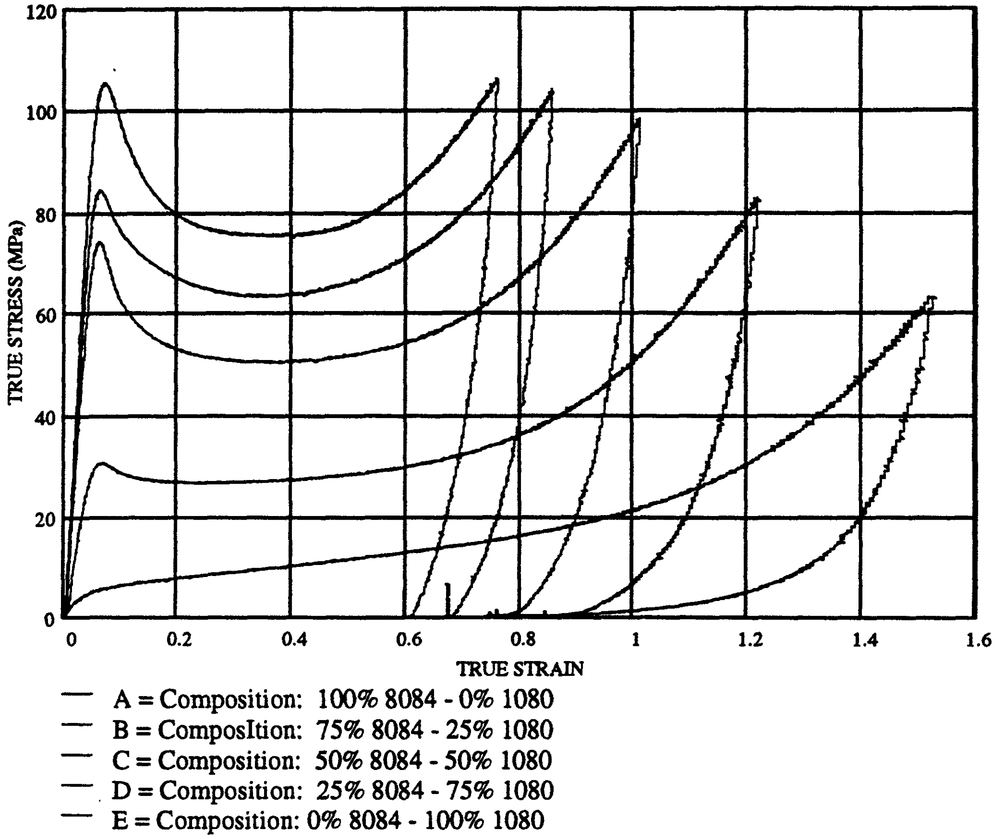
— A = Composition: 100% 8084 - 0% 1080
— B = Composition: 75% 8084 - 25% 1080
— C = Composition: 50% 8084 - 50% 1080
— D = Composition: 25% 8084 - 75% 1080
— E = Composition: 0% 8084 - 100% 1080

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\text{trueA}}) = 0.759$ $\max(\sigma_{\text{trueA}}) = 110.58 \cdot \text{MPa}$ $\sigma_{\text{yA}} = 93.767 \cdot \text{MPa}$ $\varepsilon_{\text{yA}} = 0.065$
$\max(\varepsilon_{\text{trueB}}) = 0.855$ $\max(\sigma_{\text{trueB}}) = 103.201 \cdot \text{MPa}$ $\sigma_{\text{yB}} = 67.896 \cdot \text{MPa}$ $\varepsilon_{\text{yB}} = 0.066$
$\max(\varepsilon_{\text{trueC}}) = 1.012$ $\max(\sigma_{\text{trueC}}) = 104.044 \cdot \text{MPa}$ $\sigma_{\text{yC}} = 62.089 \cdot \text{MPa}$ $\varepsilon_{\text{yC}} = 0.059$
$\max(\varepsilon_{\text{trueD}}) = 1.216$ $\max(\sigma_{\text{trueD}}) = 85.539 \cdot \text{MPa}$ $\sigma_{\text{yD}} = 22.46 \cdot \text{MPa}$ $\varepsilon_{\text{yD}} = 0.059$
$\max(\varepsilon_{\text{trueE}}) = 1.529$ $\max(\sigma_{\text{trueE}}) = 60.495 \cdot \text{MPa}$

The yield stress could not be defined for the 100% Crestomer 1080 sample

FIGURE 9-8
Plot of true stress (MPa) versus true strain for
a loading rate of 0.10 mm/sec for various compositions



List maximum true strain obtained and corresponding stress (MPa),
yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 0.761$ $\max\left(\sigma_{\text{trueA}}\right) = 106.084 \cdot \text{MPa}$ $\sigma_{\text{yA}} = 105.442 \cdot \text{MPa}$ $\varepsilon_{\text{yA}} = 0.071$

$\max\left(\varepsilon_{\text{trueB}}\right) = 0.86$ $\max\left(\sigma_{\text{trueB}}\right) = 104.253 \cdot \text{MPa}$ $\sigma_{\text{yB}} = 84.366 \cdot \text{MPa}$ $\varepsilon_{\text{yB}} = 0.064$

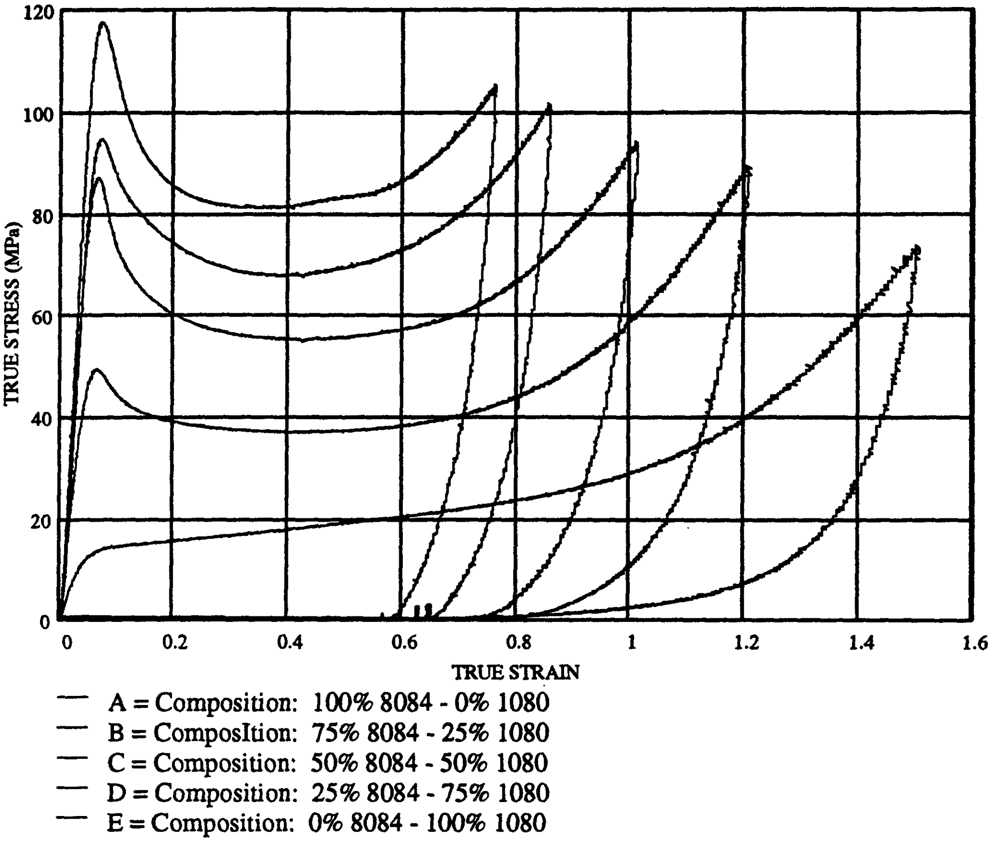
$\max\left(\varepsilon_{\text{trueC}}\right) = 1.01$ $\max\left(\sigma_{\text{trueC}}\right) = 98.384 \cdot \text{MPa}$ $\sigma_{\text{yC}} = 74.187 \cdot \text{MPa}$ $\varepsilon_{\text{yC}} = 0.063$

$\max\left(\varepsilon_{\text{trueD}}\right) = 1.222$ $\max\left(\sigma_{\text{trueD}}\right) = 82.846 \cdot \text{MPa}$ $\sigma_{\text{yD}} = 30.551 \cdot \text{MPa}$ $\varepsilon_{\text{yD}} = 0.068$

$\max\left(\varepsilon_{\text{trueE}}\right) = 1.53$ $\max\left(\sigma_{\text{trueE}}\right) = 63.194 \cdot \text{MPa}$

The yield stress could not be defined for the 100% Crestomer 1080 sample

FIGURE 9-9
Plot of true stress (MPa) versus true strain for a loading rate of 1.00 mm/sec for various compositions

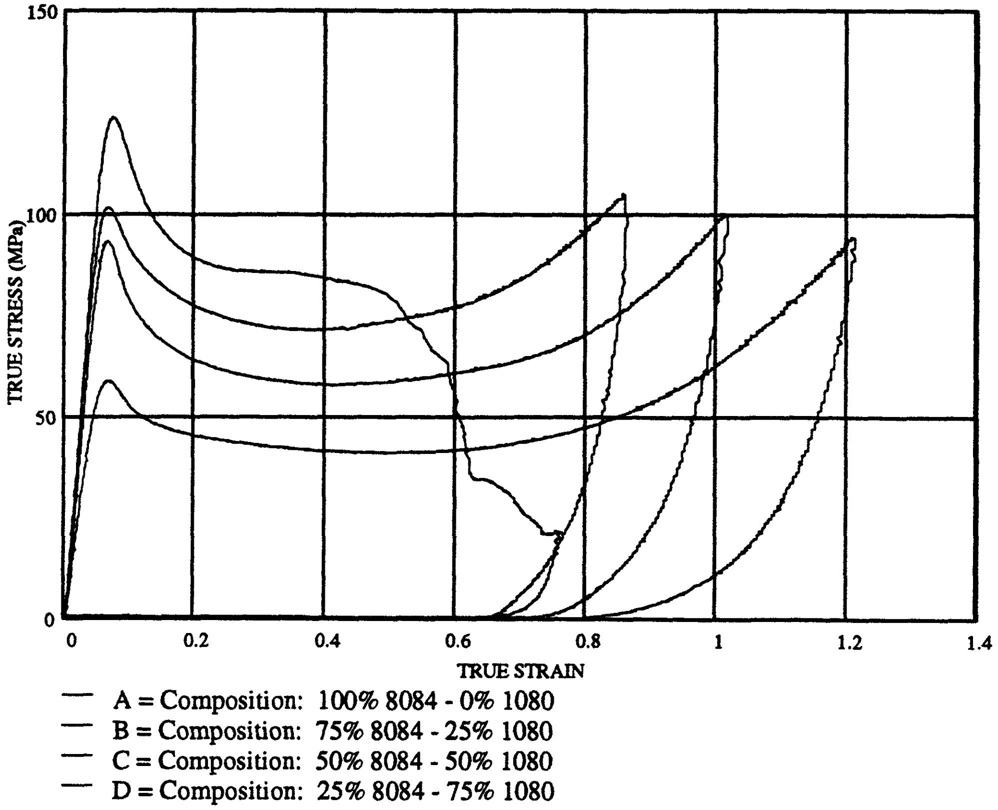


List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 0.763$	$\max\left(\sigma_{\text{trueA}}\right) = 117.63 \cdot \text{MPa}$	$\sigma_{\text{yA}} = 117.63 \cdot \text{MPa}$	$\varepsilon_{\text{yA}} = 0.073$
$\max\left(\varepsilon_{\text{trueB}}\right) = 0.862$	$\max\left(\sigma_{\text{trueB}}\right) = 101.544 \cdot \text{MPa}$	$\sigma_{\text{yB}} = 94.798 \cdot \text{MPa}$	$\varepsilon_{\text{yB}} = 0.071$
$\max\left(\varepsilon_{\text{trueC}}\right) = 1.012$	$\max\left(\sigma_{\text{trueC}}\right) = 94.066 \cdot \text{MPa}$	$\sigma_{\text{yC}} = 87.054 \cdot \text{MPa}$	$\varepsilon_{\text{yC}} = 0.067$
$\max\left(\varepsilon_{\text{trueD}}\right) = 1.211$	$\max\left(\sigma_{\text{trueD}}\right) = 89.4 \cdot \text{MPa}$	$\sigma_{\text{yD}} = 49.394 \cdot \text{MPa}$	$\varepsilon_{\text{yD}} = 0.066$
$\max\left(\varepsilon_{\text{trueE}}\right) = 1.506$	$\max\left(\sigma_{\text{trueE}}\right) = 73.66 \cdot \text{MPa}$		

The yield stress could not be defined for the 100% Crestomer 1080 sample

FIGURE 9-10
Plot of true stress (MPa) versus true strain for
a loading rate of 2.50 mm/sec for various compositions



List maximum true strain obtained and corresponding stress (MPa),
yield stress (MPa) and true strain at yield:

$$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.763 \quad \max\left(\sigma_{\mathrm{trueA}}\right) = 123.683 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yA}} = 123.683 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yA}} = 0.073$$

$$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.864 \quad \max\left(\sigma_{\mathrm{trueB}}\right) = 104.907 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yB}} = 101.388 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yB}} = 0.07$$

$$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.017 \quad \max\left(\sigma_{\mathrm{trueC}}\right) = 100 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yC}} = 93.068 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yC}} = 0.067$$

$$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.212 \quad \max\left(\sigma_{\mathrm{trueD}}\right) = 94.333 \cdot \mathrm{MPa} \quad \sigma_{\mathrm{yD}} = 58.77 \cdot \mathrm{MPa} \quad \varepsilon_{\mathrm{yD}} = 0.069$$

$$\max\left(\varepsilon_{\mathrm{trueE}}\right) = 1.643 \quad \max\left(\sigma_{\mathrm{trueE}}\right) = 60.466 \cdot \mathrm{MPa}$$

The yield stress could not be defined for the 100% Crestomer 1080 sample

The fracture always occurred in the hoop direction, usually along four radial lines which were separated by 90 degrees. At the center, the material was usually crushed.

The cause of this behavior was attributed to frictional effects. At the highest loading rates, the upper surface of the specimen would remain fixed resulting in barreling of the specimen. The barreling resulted in the formation of a tensile hoop stress which, under the applied loading, favored fracture to yield. Several attempts were made to lubricate the interface, to avoid this problem. In all cases in which this fracture occurred, the failure initiated at the upper compression plate of the testing machine which was affixed to the anchored section. It is possible that the failure occurred due to some small initial misalignment of the compression plates.

Another interesting observation was made concerning thermal softening effects in each of the glassy materials (up to 50% Crestomer by weight). Following some degree of plastic deformation, the true stress true strain curves are seen to cross one another. As an example, the 50% Derakane / 50% Crestomer sample at the lowest load rate crossed the curves for load rate 2, 3, and 4 at -0.45, -0.65, and -0.75 percent true strain respectively as shown in Figure 9-4. This occurs due to thermal softening effects caused by a temperature rise which occurred as the plastic work is dissipated during large strain deformation. No temperature measurements were taken during the Instron testing phase of this study, however, studies have been conducted[1] on PMMA at strain rates as high as 0.10 sec^{-1}, which showed a temperature rise of approximately 18^0C above ambient.

Some in-situ temperature measurements were conducted during the high strain-rate testing phase which will be discussed in the following chapter. The next section will discuss the use of the plastic deformation model presented in Chapter 6 in the prediction of the compressive yield stress of each of the blends as well as the pure Derakane 8084 material.

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[1] Arruda, E.M., M.C. Boyce and R. Jayachandran (1995), "Effects of strain rate, temperature, and thermomechanical coupling on the finite strain deformation of glassy polymers", *Mech. Mater.* 19,193-212.

9.10 Modification of the plastic deformation model

Section 6.50 presented a simplification of the visoplastic model proposed by Boyce et al., which was reduced to represent the behavior of material under homogeneous compression. This model will be used to predict the compressive yield stress for each of the material blends in this study.

Recall from Chapter 6, the (1,1) component of the stretch tensor can be expressed as the sum of a plastic and elastic components:

$$D_{11} = \overline{D}^{p}_{11} + D_{11}^{e}$$

If the loading is not applied at a constant strain rate,
but at a constant crosshead velocity(V) then,

$$D_{11} = \frac{V}{L(t)}$$

where: L(t) is the instantaneous length (9-4)

D_{11}^{e} can be appoximated by $\frac{-\dot{\sigma}_c}{E}$ so,

$$\overline{D}^{p}_{11} = \frac{V}{L_o\left[1 - \frac{\sigma_c}{E}\right]}$$

where: L_0 is the original length of the specimen

Using equations(6-10) and (6-25), the plastic component of the stretch tensor can be given as:

$$\overline{D}^{p}_{11} = -\dot{\gamma}^{p} \frac{\sqrt{3}\left[\frac{2}{3}\sigma_c + B_{11}\right]}{2\left[\sigma_c + \frac{3}{2}B_{11}\right]} \tag{9-5}$$

Combining equations (9-4) and (9-5) and simplifying gives the a relationship for the shear strain rate in terms of the back stretch, loading velocity, and compressive stress.

$$\dot{\gamma}^{p} = 2\sqrt{\frac{1}{3}}\left[\frac{V}{L_0(1-\sigma_c/E)}\right]\left[\frac{\sigma_c+(3/2)B_{11}}{(2/3)\sigma_c+B_{11}}\right]$$

$$\dot{\gamma}^{p} = 2\sqrt{\frac{1}{3}}\dot{\varepsilon}\left[\frac{\sigma_c+(3/2)B_{11}}{(2/3)\sigma_c+B_{11}}\right] \qquad (9\text{-}6)$$

where: $\dot{\varepsilon}$ is the compressive strain rate

Equation (9-6) can be used to compute the yield stress as a function of strain rate. At the yield point, the back stretch tensor is equal to zero. This is because no significant chain rotation can occur until after the yield point, when the intermolecular barriers to chain segment rotation are overcome. Setting B_{11}=0 gives the following equation which is consistent with the Von Mises yield equation:

$$\dot{\gamma}^{p} = 2\sqrt{\frac{1}{3}}\dot{\varepsilon}\left[\frac{\sigma_c}{(2/3)\sigma_c}\right] = 2\sqrt{\frac{1}{3}}\dot{\varepsilon}\left[\frac{3}{2}\right] = \sqrt{3}\dot{\varepsilon} \qquad (9\text{-}6)$$

Recall the equation for the shear stress is given as follows:

$$\tau = (s+\alpha p)\left[1+\frac{k_B\Theta}{A(s+\alpha p)}\ln\left(\frac{\dot{\gamma}^{p}}{\dot{\gamma}_o}\right)\right]^{6/5} \qquad (6\text{-}1)$$

Rewriting equation (6-1) in terms of the compressive stress and compressive strain rate, and combining the Boatsmann constant (k_B) and A into a single constant denoted by X gives:

$$\frac{\sigma_y}{\sqrt{3}} = (s+\alpha\frac{\sigma_y}{3})\left[1+\frac{\Theta}{X(s+\alpha\frac{\sigma_y}{3})}\ln\left(\frac{\sqrt{3}\dot{\varepsilon}}{\dot{\gamma}_o}\right)\right]^{6/5} \qquad (9\text{-}8)$$

Equation (9-8) will be used to predict the yield stress using the methods outlined in Section 6.40. The results will include plots of compressive yield stress versus axial strain rate for each of the first four compositions. The 100% Crestomer 1080 tests will not be analyzed, as no clear yield behavior was observed during the Instron testing.

The analysis began with the prediction of the peak yield stress, which was performed using MATHCAD. The complete files may be found in Appendix D. The required inputs were the mean and standard deviation of the peak compressive yield stress found in Table 9-1. The average modulus of elasticity for each of the blends is listed in Table 9-2.

TABLE 9-2

Composition	Modulus of Elasticity
100% 8084 / 0% 1080	2020 MPa
75% 8084 / 25% 1080	1880 MPa
50% 8084 / 50% 1080	1680 MPa
25% 8084 / 75% 1080	1125 MPa

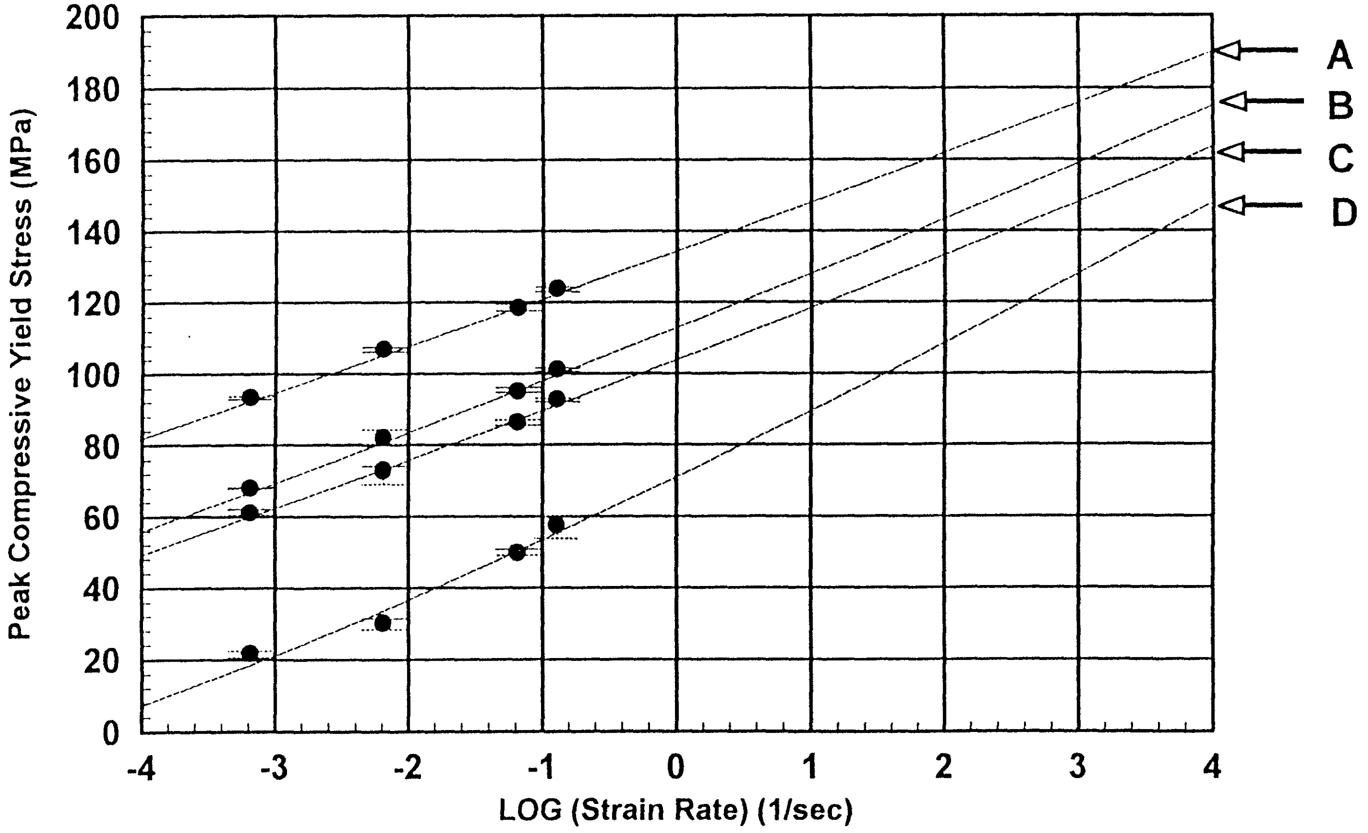
These values are used to calculate the athermal shear strength which represents the behavior at 0 Kelvin. As a result, the modulus values used in this study were obtained from the elastic response of each of the blends at the highest Instron loading rate of 2.5 mm/sec. An average specimen length was also provided.

The program assumes a constant value for the pressure coefficient of 0.20 for all calculations in this study. The determination of the pressure coefficient requires a series of tensile tests, which were not available. With an assumed pressure coefficient, the value of $\dot{\gamma}_0$ and X can then be determined using equations (6-17). Note that there is one unique solution to the model for a given pressure coefficient, but the resulting fitted curve is the same for any reasonably selected value of the pressure coefficient. Since the values of $\dot{\gamma}_0$ and X do not effect the final shape of the fitted curve and therefore do not effect the analysis, no attempt will be made to adjust the pressure coefficient to bring the other values near the "typical" values given in Chapter 6.

The individual results for each of the material blends may be found in Appendix D. Figure 9-11 shows a plot of the peak yield stress versus log(strain rate) for each of the various compositions, and Figure 9-12 shows the effect of the weight percentage of Crestomer 1080 on the peak yield stress for a series of strain rates.

Figure 9-12 indicates that there is a "jog" in the model, particularly at the lowest strain rates. This indicates that the yield stress of the 50% Derakane / 50% Crestomer is nearly the same at these rates as the 75% Derakane / 25% Crestomer material. This

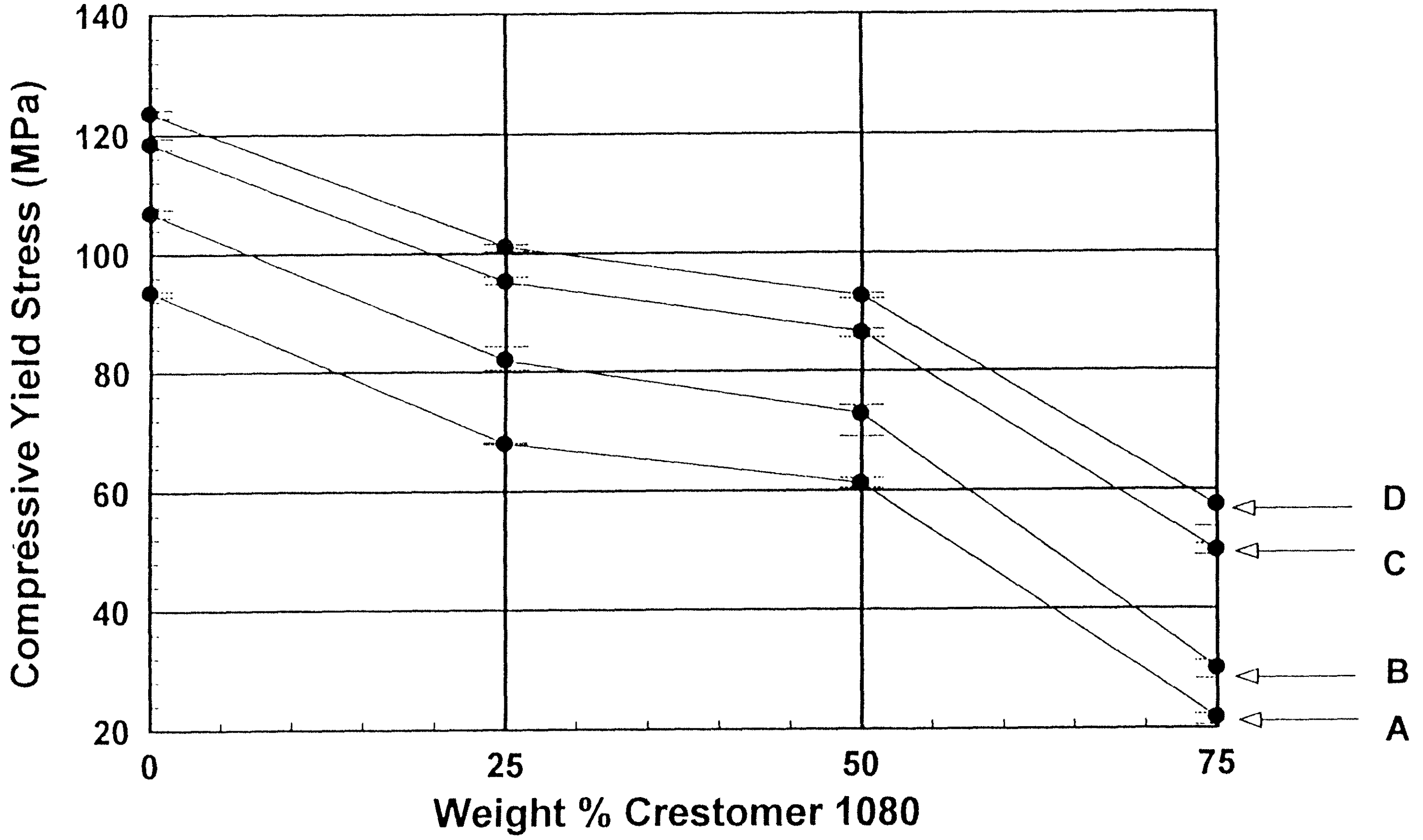
PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR VARIOUS BLENDS



A = 100% 8084 / 0% 1080 B = 75% 8084 / 25% 1080
C = 50% 8084 / 50% 1080 D = 25% 8084 / 75% 1080

FIGURE 9-11

PLOT OF TRUE COMPRESSIVE UPPER YIELD STRESS (MPa) VERSUS WEIGHT % CRESTOMER 1080



A = LOAD RATE 1 (0.01 mm./sec)
B = LOAD RATE 2 (0.10 mm/sec)
C = LOAD RATE 3 (1.00 mm/sec)
D = LOAD RATE 4 (2.50 mm/sec)

FIGURE 9-12

discrepancy could have been predicted from the composite Instron test results shown in Figures 9-7 through 9-10, where the peak yield behavior of the 75% Derakane / 25% Crestomer is clearly different from its neighbors.

This series of observations led a shift from the prediction of the peak yield stress to that of the lower yield stress. It was observed[2] in the testing of Polycarbonate, that quenched specimens often displayed very little softening. When the same material was annealed for several hours above its glass transition temperature prior to testing, a much higher peak yield stress was observed with a corresponding increase in the degree of softening. The surprising result of these tests was that the measured lower yield stress was nearly independent of any previous thermal treatment. The lower yield point is a strength level which is thought to be associated with a preferred structure which is reached during plastic deformation, possibly due to a local increase in free volume associated with small scale shear banding[3].

These observations indicated that the lower yield stress was likely a better indicator on which to build a predictive model. As a result, the model was rerun by substituting the lower yield stress for the upper yield stress used in the previous iteration. The lower stress values are listed in Table 9-3 and summarized in Table 9-4.

The results of the predictive model applied to the lower yield stress are given in Figures 9-13 through 9-14. As in the previous case, the individual runs may be found in Appendix D. A plot was included which compares the lower yield stress with the applied strain rate as well as one which shows the effect of the weight percentage of Crestomer 1080 on the lower yield stress.

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[2] Hayward, R.N. (1980), "The effect of chain structure on the annealing and deformation behavior of polymers," *Coll. Poly. Sci.* 258, 42.

[3] Boyce, M.C., Parks, D.M. and A.S. Argon (1988), " Large inelastic deformation of glassy polymers. Part 1: Rate dependent constitutive model," *Mechanics of Materials* 7, 22.

Table 9-3

100% Derakane / 0% Crestomer

	Load Rate # 1		Load Rate # 2		Load Rate # 3		Load Rate # 4	
	Stress	Strain	Stress	Strain	Stress	Strain	Stress	Strain
	71.420	0.279			81.124	0.353	85.293	0.347
	72.340	0.277	75.290	0.360	80.911	0.343	83.896	0.328
	72.250	0.275	75.380	0.347	81.073	0.336	84.149	0.332
Average	**72.003**	**0.277**	**75.335**	**0.354**	**81.073**	**0.344**	**84.446**	**0.336**
STDEV	**0.507**	**0.002**	**0.064**	**0.009**	**0.111**	**0.009**	**0.744**	**0.010**

75% Derakane / 25% Crestomer

	Load Rate # 1		Load Rate # 2		Load Rate # 3		Load Rate # 4	
	Stress	Strain	Stress	Strain	Stress	Strain	Stress	Strain
	58.942	0.257	62.493	0.338	67.741	0.391	70.725	0.395
	59.076	0.256	63.463	0.337	68.938	0.380	71.552	0.404
	59.239	0.263	62.561	0.324	68.342	0.390	71.260	0.385
Average	**59.086**	**0.259**	**62.839**	**0.333**	**68.340**	**0.387**	**71.179**	**0.395**
STDEV	**0.149**	**0.004**	**0.541**	**0.008**	**0.599**	**0.006**	**0.419**	**0.010**

50% Derakane / 50% Crestomer

	Load Rate # 1		Load Rate # 2		Load Rate # 3		Load Rate # 4	
	Stress	Strain	Stress	Strain	Stress	Strain	Stress	Strain
			50.641	0.361	XXX	XXX	XXX	XXX
	47.137	0.249	50.536	0.366	54.608	0.457	57.399	0.401
	48.015	0.240	50.391	0.332	55.161	0.437	57.851	0.432
	47.045	0.247	49.646	0.347	54.555	0.440	56.423	0.411
Average	**47.399**	**0.245**	**50.304**	**0.352**	**54.775**	**0.445**	**57.224**	**0.415**
STDEV	**0.535**	**0.005**	**0.450**	**0.015**	**0.336**	**0.011**	**0.730**	**0.016**

25% Derakane / 75% Crestomer

	Load Rate # 1		Load Rate # 2		Load Rate # 3		Load Rate # 4	
	Stress	Strain	Stress	Strain	Stress	Strain	Stress	Strain
	XXX	XXX	XXX	XXX	36.941	0.365	41.886	0.488
	20.760	0.157	25.208	0.210	37.041	0.388	40.950	0.501
	21.001	0.158	26.432	0.213	36.436	0.384	40.465	0.508
	19.858	0.147	27.295	0.212	37.524	0.415	39.417	0.488
Average	**20.540**	**0.154**	**26.312**	**0.212**	**36.986**	**0.388**	**40.680**	**0.496**
STDEV	**0.603**	**0.006**	**1.049**	**0.002**	**0.446**	**0.021**	**1.028**	**0.010**

PLOT OF LOWER COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR EACH OF THE BLENDS

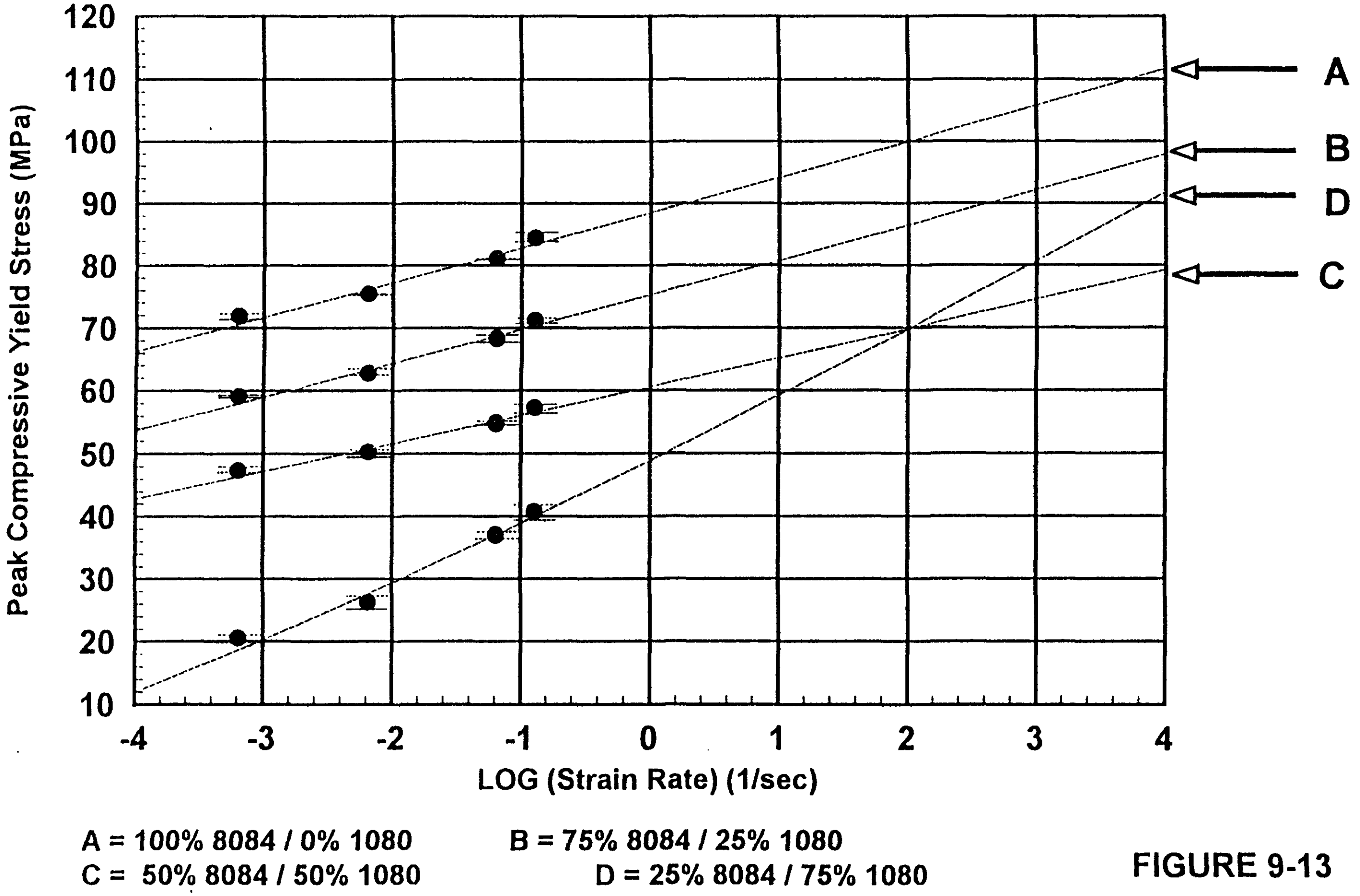
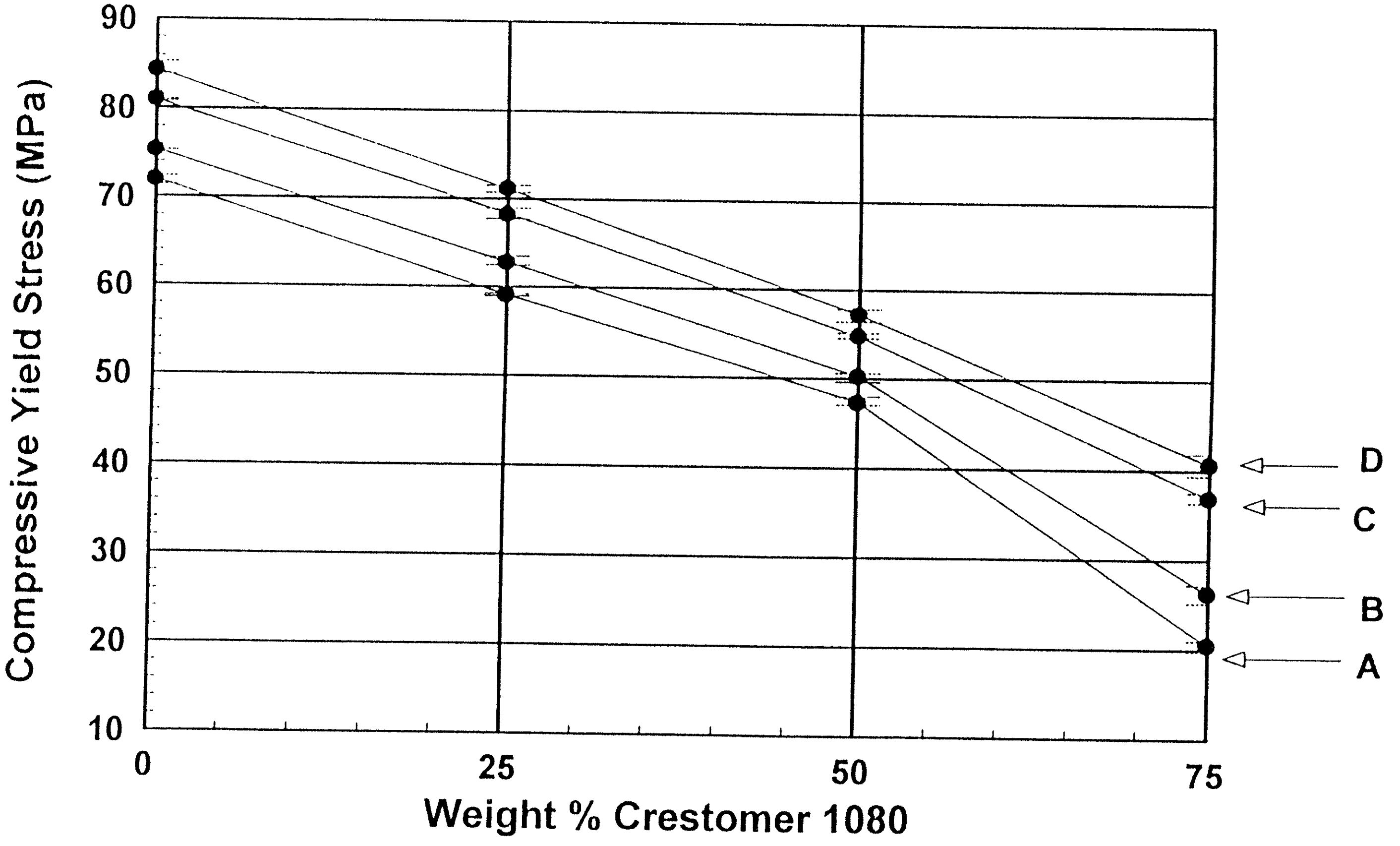


FIGURE 9-13

PLOT OF TRUE COMPRESSIVE LOWER YIELD STRESS (MPa) VERSUS WEIGHT % CRESTOMER 1080



A = LOAD RATE 1 (0.01 mm/sec)
B = LOAD RATE 2 (0.10 mm/sec)
C = LOAD RATE 3 (1.00 mm/sec)
D = LOAD RATE 4 (2.50 mm/sec)

FIGURE 9-14

TABLE 9-4

Wt%8084/ Wt%1080	Initial Strain Rate (1/sec)	Number of Runs	Mean True Lower Yield Stress (MPa)	Stdev. True Lower Yield Stress (MPa)	Mean True Lower Yield Strain	Stdev. True Lower Yield Strain
100/0	$6.33\text{x}10^{-4}$	3	72.003	0.507	0.277	0.002
100/0	$6.41\text{x}10^{-3}$	2	75.335	0.064	0.354	0.009
100/0	$6.44\text{x}10^{-2}$	3	81.073	0.111	0.344	0.009
100/0	$1.64\text{x}10^{-1}$	3	84.446	0.744	0.336	0.010
75/25	$6.66\text{x}10^{-4}$	3	59.086	0.149	0.259	0.004
75/25	$6.68\text{x}10^{-3}$	3	62.839	0.541	0.333	0.008
75/25	$6.68\text{x}10^{-2}$	3	68.340	0.599	0.387	0.006
75/25	$1.70\text{x}10^{-1}$	3	71.179	0.419	0.395	0.010
50/50	$6.73\text{x}10^{-4}$	3	47.399	0.535	0.245	0.005
50/50	$6.60\text{x}10^{-3}$	4	50.304	0.450	0.352	0.015
50/50	$6.82\text{x}10^{-2}$	3	54.775	0.336	0.445	0.011
50/50	$1.64\text{x}10^{-1}$	3	57.224	0.730	0.415	0.016
25/75	$6.50\text{x}10^{-4}$	3	20.540	0.603	0.154	0.006
25/75	$6.38\text{x}10^{-3}$	3	26.312	1.049	0.212	0.002
25/75	$6.42\text{x}10^{-2}$	4	36.986	0.446	0.388	0.021
25/75	$1.59\text{x}10^{-1}$	4	40.680	1.028	0.496	0.010

Figure 9-14 indicates that the behavior of the blends up to a Crestomer concentration of 50% are a linear function of the weight percentage of Derakane 8084. The curves for these blends are nearly parallel, which further supports the assumption that they are dominated by the behavior of the Derakane. This behavior was supported by the DMA results presented in Chapter 8, which showed the glass transition temperature of these compositions to be nearly the same. By contrast, the 75% Crestomer 1080 composition shows a significant degree of rate dependency, indicating that the Crestomer was exerting a significant influence on the blend. Recall that Crestomer 1080 has a glass transition temperature of 27^0C, and is therefore viscoelastic (and highly rate dependent at room temperature). This behavior also could have been foreseen from the measured glass transition temperature of 67^0C, which is nearly midway between that of the two constituent materials.

10.0 THE COMPRESSION SPLIT HOPKINSON (KOLSKI) BAR

This section will describe the results of a series of high strain rate compression tests of several blends of Derakane 8084 and Crestomer 1080. This chapter will begin with the theoretical development of the relevant equations needed to obtain the true stress and true strain results from a Hopkinson bar experiment. Subsequent sections will include the experimental procedure as well as a MATHCAD data reduction program with the corresponding results. The results of these experiments will be compared to the projected values obtained from the Instron tests described in Chapter 9.

10.10 Theoretical Development

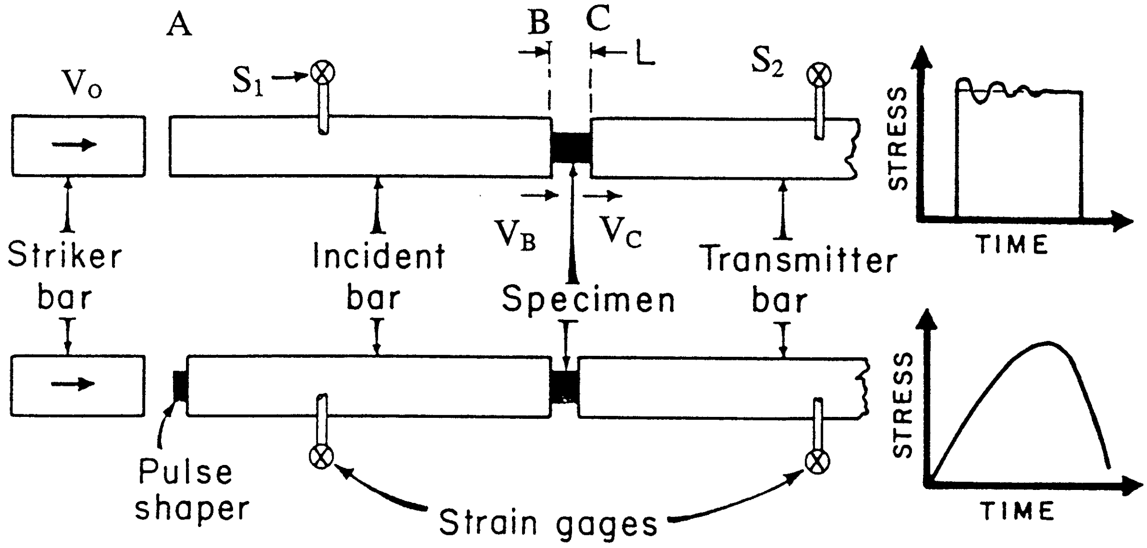
The Compression Split-Hopkinson Bar is shown schematically in Figure 10-1. The technique involves impacting the incident bar with a striker bar at a known velocity (V_0) at point A. The impact produces a longitudinal compressive stress wave in the incident bar with a strain amplitude of $\varepsilon_i(t)$. The pulse width of this wave can be estimated as twice the time required for an elastic wave to travel the length of the striker bar. This assumes that the striker bar is made of the same material and has the same diameter as the incident bar. The compressive wave travels the length of the incident bar until it reaches the specimen/bar interface at point B. At this point, part of the wave is reflected back down the incident bar as a tensile wave with a strain amplitude of $\varepsilon_r(t)$, and part is transmitted through the specimen to the transmitter bar with a strain amplitude of $\varepsilon_t(t)$. The strain is measured using one strain gage located on the incident bar and one located on the transmitter bar at locations S_1 and S_2 as shown on Figure 10-1.

The separation of the wave into a transmitted and reflected component is due to differences in acoustic impedance at the bar/specimen interface. The acoustic impedance (Z_c) is defined as follows:

$$Z_c = \rho A c_L \tag{10-1}$$

where: ρ is the density (mass / volume)
A is the cross sectional area
c_L is the velocity of propagation of a quasi - longitudinal (stress) wave

Hopkinson Bar



Marc A. Meyers, *Dynamic Behavior of Materials* (New York:John Wiley and Sons, 1994), p. 306.

Figure 10-1

The quasi-longitudinal (stress) wave speed can be determined using the following equation:

$$c_L = \sqrt{\frac{E}{\rho}}$$

where: E is the modulus of elasticity of the incident and transmitted bars (10-2)

When an incident stress wave traveling in material A of cross sectional area, A_A, meets a boundary separating it from another material B of cross sectional area A_B, the transmitted and reflected stress components may be found as follows:

$$\sigma_t = \sigma_i \left[\frac{2A_A\rho_B c_{LB}}{A_B\rho_B c_{LB} + A_A\rho_A c_{LA}} \right]$$

$$\sigma_r = \sigma_i \left[\frac{A_B\rho_B c_{LB} - A_A\rho_A c_{LA}}{A_B\rho_B c_{LB} + A_A\rho_A c_{LA}} \right] \qquad (10\text{-}3)$$

The transmitted and reflected strain amplitudes are given by the same relationship, since the stress and strain values are related by the elastic modulus. From equation (10-3), it can be seen that when the acoustic impedance of material B is greater than that of material A, a pulse of the same sign as the incident pulse is reflected at the interface. When the acoustic impedance of material B is less than that of material A, a reflected pulse which is opposite in sign is created. When polymers are tested using a Hopkinson bar, the acoustic impedance of the sample is much less than that of the steel used in the incident and transmitter bars, so the incident wave which was initially compressive is reflected from interface B as a tensile wave.

The stress and strain in the specimen can be determined completely from the transmitted and reflected strain traces. The derivation of the stress/strain relationships begins with the definition of the axial strain, ε, and the particle velocity, v, as follows:

$$\varepsilon = \frac{\partial u}{\partial x} \qquad v = \frac{\partial u}{\partial t} \tag{10-4}$$

Taking the partial derivative of the strain with respect to time (t), and the velocity with respect to x gives:

$$\frac{\partial \varepsilon(t)}{\partial t} = \frac{\partial v(t)}{\partial x} \tag{10-5}$$

The average strain rate in the deforming specimen can then be expressed as:

$$\dot{\varepsilon}(t) = \frac{d\varepsilon(t)}{dt} = \frac{v_B(t) - v_C(t)}{L} \tag{10-6}$$

where: $v_B(t)$ is the particle velocity at point B
$v_C(t)$ is the particle velocity at point C
L is the length of the test specimen

The strain at the interfaces B and C can be written as:

$$\begin{aligned} \varepsilon_B(t) &= \varepsilon_i(t) - \varepsilon_r(t) \\ \varepsilon_C(t) &= \varepsilon_t(t) \end{aligned} \tag{10-7}$$

The particle velocity is related to the strain by the relation, $v=c_L\varepsilon$, so the particle velocity at interfaces B and C can be written as:

$$\begin{aligned} v_B(t) &= c_L(\varepsilon_i(t)-\varepsilon_r(t)) \\ v_C(t) &= c_L\varepsilon_t(t) \end{aligned} \tag{10-8}$$

Combining equations (10-6) and (10-8) gives:

$$\dot{\varepsilon}(t) = \frac{c_L}{L}\left[\varepsilon_i(t)-\varepsilon_r(t)-\varepsilon_t(t)\right] \tag{10-9}$$

The average stress in the specimen can be expressed using Hook's Law as:

$$\sigma(t) = \frac{F_B(t)+F_C(t)}{2A}$$

where: $F_B(t) = E\,(\varepsilon_i(t)+\varepsilon_r(t))\,A_0$

$$F_C(t) = E\,(\varepsilon_t(t))\,A_0 \tag{10-10}$$

A is the cross sectional area of the specimen

A_0 is the cross sectional area of the incident and transmitter bai

If the specimen undergoes homogeneous deformation, the stress at point B is equal to the stress at point C. The assumption of homogeneous deformation in the specimen results in force equilibrium across the specimen so $F_B(t)=F_C(t)$ and $\varepsilon_i(t) + \varepsilon_r(t) = \varepsilon_t(t)$. The stress, strain rate, and strain in the specimen can be expressed as follows:

$$\begin{aligned} &\text{(a)}\ \sigma(t) = E\frac{A_0}{A}\varepsilon_t(t) \\ &\text{(b)}\ \dot{\varepsilon}(t) = \frac{-2C_L}{L}\varepsilon_r(t) \\ &\text{(c)}\ \varepsilon(t) = \int_0^t \dot{\varepsilon}(\tau)d\tau \end{aligned} \tag{10-11}$$

The stress-strain equations listed in equations (10-11) represent engineering stress and engineering strain. In the study of polymer materials, the stresses and strains are more appropriately given in terms of true stress and true (logarithmic) strain.

The true strain in the specimen is defined as:

$$\varepsilon_{true} = \ln\left[\frac{L_i}{L_o}\right] = \ln[\lambda_i]$$

where: L_i is the instantaneous length (10-12)

L_o is the initial length

λ_i is the extension ratio

The engineering strain can be expressed in terms of the extension ratio as:

$$\varepsilon_{eng} = \left[\frac{L_i - L_o}{L_o}\right] = \frac{L_i}{L_o} - 1 = \lambda_i - 1$$

so; (10-13)

$$\lambda_i = \varepsilon_{eng} + 1$$

Combining equations (10-12) and (10-13) gives the relationship between true stress and engineering stress as follows:

$$\varepsilon_{true} = \ln(\varepsilon_{eng} + 1) \qquad (10\text{-}14)$$

Since most of the deformation occurs in the plastic region, the material is considered to behave in an incompressible manner. Assuming incompressibility,

$$AL_o = A_i L_i$$

so;

$$\frac{A}{A_i} = \frac{L_i}{L_o} = \lambda_i \qquad (10\text{-}15)$$

and;

$$\frac{\sigma_{true}}{\sigma_{eng}} = \frac{A}{A_i}$$

From equations (10-15), the true stress is related to the engineering stress as follows:

$$\sigma_{\text{true}} = \lambda_i \left[\sigma_{\text{eng}}\right] \tag{10-16}$$

There are three key assumptions made in the derivation of the stress-strain relationships for the Hopkinson Bar. These are worthy of discussion and will be described briefly.

(1) The incident, transmitter and striker bars remain elastic.

This is accomplished by ensuring that the material used for the transmitter and incident bars has a much higher yield strength than the material to be tested. The most common material used in the construction of these bars is maraging steel, which has a yield strength of approximately 350 ksi (2500 Mpa). This is much higher than the yield strength of most metals and all polymers.

(2) The wave propagation is one dimensional.

In the derivation of the Hopkinson bar equations, it was assumed that the impactor creates a rectangular stress pulse which travels down the incident bar at the longitudinal wave speed of the incident bar material (c_L). In reality, the measured stress pulses show significant fluctuations due to wave dispersion effects, which have a significant effect on the interpretation of the stress/strain results obtained using this method.

When the striker bar impacts the incident bar, a very complex stress field is established due to end effects. Within a distance of about 10 bar diameters, the end effects essentially disappear, and the resulting pressure can be evaluated using the frequency or dispersion equation. This problem was studied extensively by Pochhammer[1] and Chree[2] who showed that the pressure pulse was actually composed of an infinite number of modes

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[1] Pochhammer,L., "On the Propagation Velocities of Small Oscillations in an Unlimited Isotropic Circular Cylinder," *Journal fur die Reine und Angewandte Mathematik*, Vol. 81, 1876, pp. 324-326.

[2] Chree, C., "The Equations of an Isotropic Elastic Solid in Polar and Cylindrical Coordinates, Their Solutions and Applications," Cambridge Philosophical Society, *Transactions*, Vol. 14, 1889, pp. 250-369.

which correspond to solutions of the dispersion equation. The fundamental mode is associated with the longest wavelengths which carry most of the wave energy and travel at the highest speed which is equal to the bar longitudinal wave speed (c_L). Higher order modes are associated with higher frequency multiples of the fundamental frequency (shorter wavelengths) which travel at slower speeds as compared to the lower modes. These higher modes give rise to dispersion of the initially sharp pulse, causing oscillations known as Pochhammer-Chree oscillations. For sufficiently long incident bars with l/d ratios of 20 or greater, the fundamental frequency dominates and the deformation process is essentially one dimensional.

The above discussion related to the effects of dispersion in the incident pulse. Some dispersion effects also occur in the reflected and transmitted pulse. The strain in the specimen is assumed to be measured at the interfaces between the specimen and the bars. The actual strain measurements are made some distance from these interfaces, resulting in additional dispersion of the pulse as it travels from the specimen to the strain gage location.

The effects of dispersion on the interpretation of split Hopkinson Bar data can be minimized by applying a dispersion correction. Follansbee and Franz[3] have demonstrated that correcting for dispersion in the fundamental mode can remove most of the fluctuations in the stress/strain curve. The process begins by performing a fourier decomposition of the fundamental mode of the strain pulses measured at the strain gage locations. After decomposition, the phase angle of each of the fourier components is adjusted to account for the dispersion which occurs as the component travels from the specimen/bar interface to the strain gage location. Once a sufficient number of components are adjusted, the wave can be reconstructed at the specimen bar interface.

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[3] Follansbee, P.S. and Franz, C., "Wave Propagation in the Split Hopkinson Bar," *Journal of Engineering Materials and Technology*, Vol. 105, 1983, pp. 61-66.

(3) The specimen undergoes homogeneous deformation.

When the stress pulse enters the sample, it undergoes deformation both axially and radially. The resultant stress field is inhomogeneous in this region, but the stress equilibrates and the deformation becomes essentially homogeneous after three transit times in the sample. The equilibration time (τ) which is actually equal to π transit times, can be calculated using the following expression:

$$\tau = \sqrt{\frac{\pi^2 \rho L^2}{E_s}} \tag{10-18}$$

where: E_s is the modulus of elasticity of the sample

In the acoustic analysis of polymers, the modulus of elasticity is represented by a complex number, due to the presence of viscous damping. It was demonstrated in Chapter 8 that when viscoelastic materials are loaded at high frequencies, they tend to behave in a glassy manner. As a result, the equilibrium time in the specimens will be determined using the glassy modulus of the material.

Equation (10-12) indicates that the results obtained in the initial part of the elastic deformation phase may be in error when using the split-Hopkinson bar technique. The effects of initial inhomogeneity in this study were minimized by using specimens with a length/diameter ratio between 0.5 and 0.75. The effects of initial inhomogeneity were minimized by reducing the slope of the rise time in the incident pulse in a process known as **pulse shaping**. This was achieved by placing an interface material such as a few sheets of paper or a soft metal such as copper between the striker bar and the incident bar at interface A. The interfaces between the bar and the specimen were lubricated with Molybdenum Disulfide grease to minimize the radial constraints and to prevent barreling at large strains.

10.20 Experimental Procedure

The high strain-rate testing was conducted at the California Institute of Technology in Pasadena, California on a split compression Hopkinson bar which was built by the staff. The incident, transmitter, and striker bars were composed of high strength maraging steel with an outer diameter of 0.75 in. The striker bar was propelled using air pressure which was provided by a low pressure air compressor and could be regulated between 0-80 psi by a pressure control valve. The strain gage data was acquired using a Nicolet 400 digital oscilloscope. The environmental conditions at the time of the test are listed in Table 10-1.

TABLE 10-1

Start Date:	4/20/95
Finish Date:	4/21/95
Temperature:	71.6 F
Humidity:	32.3 %

The temperature and humidity were measured using a VWR Scientific digital humidity and temperature meter. The specifications of the meter are listed in Table 7-1.

The desired strain rate and the desired strain can be estimated from the following equations:

$$\dot{\varepsilon} \leq \frac{V_0}{L}$$

$$\varepsilon = 2\dot{\varepsilon}\frac{l_0}{c_0} = \dot{\varepsilon}(PD) \tag{10-19}$$

where: V_0 is the inital velocity of the striker bar

L is the inital length of the specimen

l_0 is the length of the stiker bar

c_0 is the longitudinal compressive wave speed in the striker bar

PD is the duration of the compressive stress pulse

The goal of the high strain rate test in this study was to obtain the greatest possible true strain while maintaining a strain rate on the order of 1000 $\sec^{-1}$. To achieve this goal, the longest available striker bar (12 inch) was selected. The 12 inch bar was used to provide the longest possible compressive pulse duration, resulting in the largest total plastic strain in the specimen. The striker bar was propelled using the highest possible pressure of 80 psi. Since the initial velocity is directly related to the propelling pressure, higher pressure will result in higher strain rates in the specimen.

Some trial and error testing was required prior to this selection. Since the velocity of the shorter striker bars will be greater than that of the longer ones for a given pressure, the strain rate could have increased enough to compensate for the shorter pulse duration resulting in a higher overall strain. The strain rate obtained using the shortest bar (4 inches) was approximately 4500 $\sec^{-1}$, and the resulting engineering strain was approximately 20%. Tests conducted with the longest bar (12 inch) resulted in a strain rate of approximately 3000 $\sec^{-1}$ and the total strain was approximately 30-40%. As a result, the 12 inch striker bar and a propelling pressure of 80 psi was selected.

Three to four runs were obtained using each of the five material compositions. The data acquisition system generated separate files for each of the transmitted and reflected strain histories. Each of these files contained 6000 data points, each taken at an equal time interval of 0.1 microseconds.

10.21 Description of data reduction program

The first page of the reduction file translates the voltage signals to values of reflected and transmitted strain using the following equation:

$$\varepsilon = \frac{4(\varepsilon_{\text{volts}})}{(GF)(V_0)} \qquad (10\text{-}20)$$

where: $\varepsilon_{\text{volts}}$ is the strain gage voltage measured
by the occiliscope
$GF = 2.1$ is the gage factor
$V_0 = 30$ Volts is the reference voltage

The next step in the analysis was to convert the reflected strain history into a strain rate using equation (10-11(b)). This requires the user to input values for the initial specimen length and diameter. The longitudinal wave speed in the steel bars is also required. A value of 5000 m/sec will used throughout this study.

The portion of the trace which is needed in the analysis is the inverted section of the transmitted pulse. This pulse was isolated and integrated over time using a midpoint-rectangle rule using approximately 200-250 equal time increments. The integral of the strain rate with respect to time over the duration of the inverted pulse gives the total compressive engineering strain as shown in equation 10-11(c). This numerical integration procedure was used to obtain values for the engineering strain over the entire range of time defined by the pulse duration.

The engineering stress was obtained from the transmitted strain history using equation 10-11(a). A value of 210 Gpa was assumed for the modulus of elasticity of the steel bars. With the engineering stress and strain known over the range of time defined by the reflected pulse, a plot of engineering stress versus engineering strain was drawn. The true stress and true strain were computed using equations (10-14) and (10-16) and plotted. Beneath each of these plots is the maximum value of stress which occurs at the yield point. This value is obtained by locating the minimum value of the stress (most negative value) since the compressive stresses are reflected as negative numbers.

10.30 Results

The reduction files are organized by material composition. The compositions range from 100% Derakane 8084 to 100% Crestomer 1080 at 25% intervals in composition. The complete reduction files are included in Appendix E, and the results are summarized in Table 10-2 below:

TABLE 10-2

%8084	%1080	Trial Number	Strain Rate(sec^{-1})	Maximum True Strain	Maximum True Stress (MPa)
100	0	1 of 4	-3000	-0.488	-189.806
100	0	2 of 4	-2500	-0.382	-200.044
100	0	3 of 4	-3200	-0.506	-208.491
100	0	4 of 4	-3200	-0.506	-203.560
75	25	1 of 3	-3000	-0.470	-179.287
75	25	2 of 3	-3600	-0.588	-187.317
75	25	3 of 3	-3000	-0.440	-182.353
50	50	1 of 3	-2500	-0.372	-171.791
50	50	2 of 3	-2500	-0.385	-174.609
50	50	3 of 3	-2100	-0.312	-176.762
25	75	1 of 4	-2800	-0.425	-145.159
25	75	2 of 4	-2600	-0.383	-140.000 (est)
25	75	3 of 4	-2900	-0.454	-137.791
25	75	4 of 4	-3000	-0.457	-147.004
0	100	1 of 3	-2000	-0.339	-88.720
0	100	2 of 3	-2200	-0.352	-85.937
0	100	3 of 3	-2200	-0.359	-87.000 (est)

The yield stress for two of the runs were visually approximated from the curves because of voltage spikes in the signal which occurred near the yield point.

The data in Table 10-2 will be reduced by assuming that all runs for a given material occur at the same strain rate. The strain rate will be taken as the mean value for all runs on given material. This is a valid assumption since the stress is related to the logarithm of the strain rate which varies very little over the range of concern. With this assumption a mean value of the maximum true stress (yield stress) and a corresponding standard deviation can be calculated. The results are given in Table 10-3.

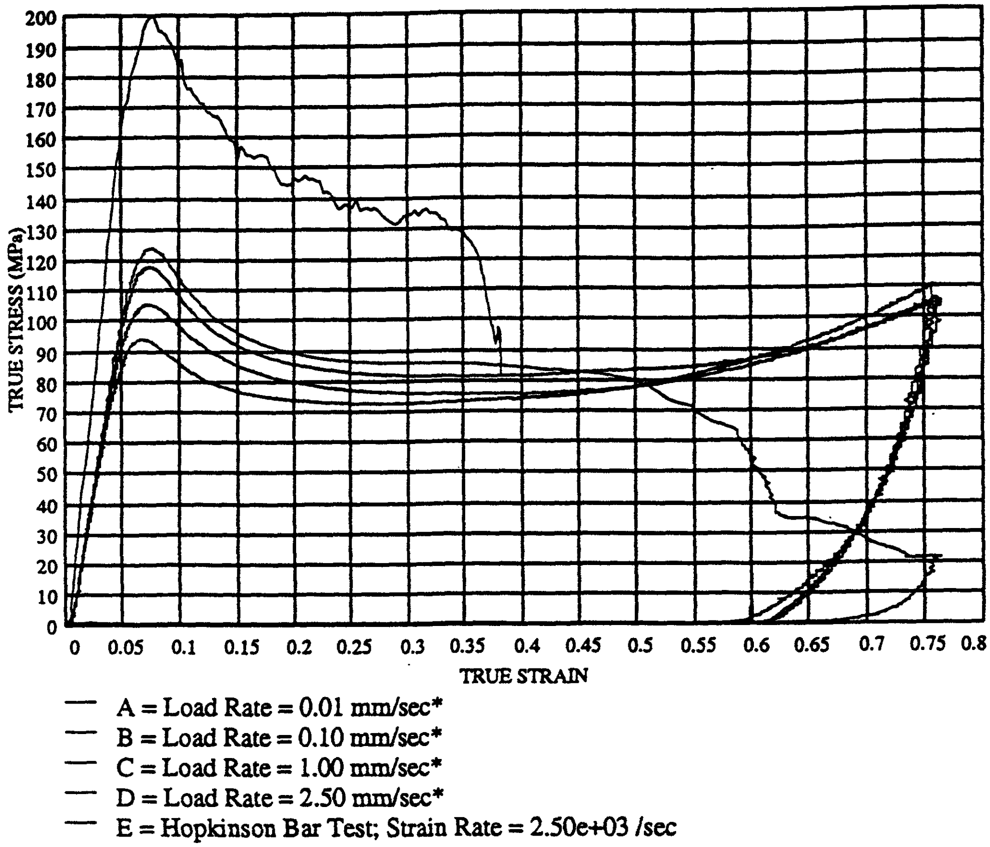
TABLE 10-3

%8084	%1080	Strain Rate(sec^{-1})	Mean True Yield Stress (MPa)	Stdev. True Yield Stress (MPa)
100	0	-2.97x10^{+3}	-200.48	-6.85
75	25	-3.20x10^{+3}	-182.99	-3.31
50	50	-2.37x10^{+3}	-174.39	-2.04
25	75	-2.83x10^{+3}	-142.49	-3.73
0	100	-2.13x10^{+3}	-87.22	-1.15

The Hopkinson bar results in Table 10-3 can be compared to the peak yield stress in Table 9-1 which were obtained using an Instron testing machine. Figures 10-2 through 10-6 superimposed a representative Hopkinson bar true stress/true strain on the Instron machine testing results for each of five material blends. These curves show the dramatic increase in the yield stress as a function of strain rate.

Of particular interest is the behavior of the pure Crestomer 1080. In all of the Instron tests, which were conducted to true strains of 150%, the material never displayed a clearly defined yield stress. This was not surprising since the glass transition temperature of the material was determined to be 27°C at a DMA testing frequency of 10 Hertz. To first order, this is approximately one decade higher than that of the highest loading rate Instron test. This places the glass transition temperature at approximately 23^{0}C which was within a few degrees of the ambient temperature during the test. This indicates that the material was in the viscoelastic regime. Since only glassy polymers display true yield behavior, yield could not occur in these tests.

FIGURE 10-2
100% 8084 / 0% 1080 at various loads



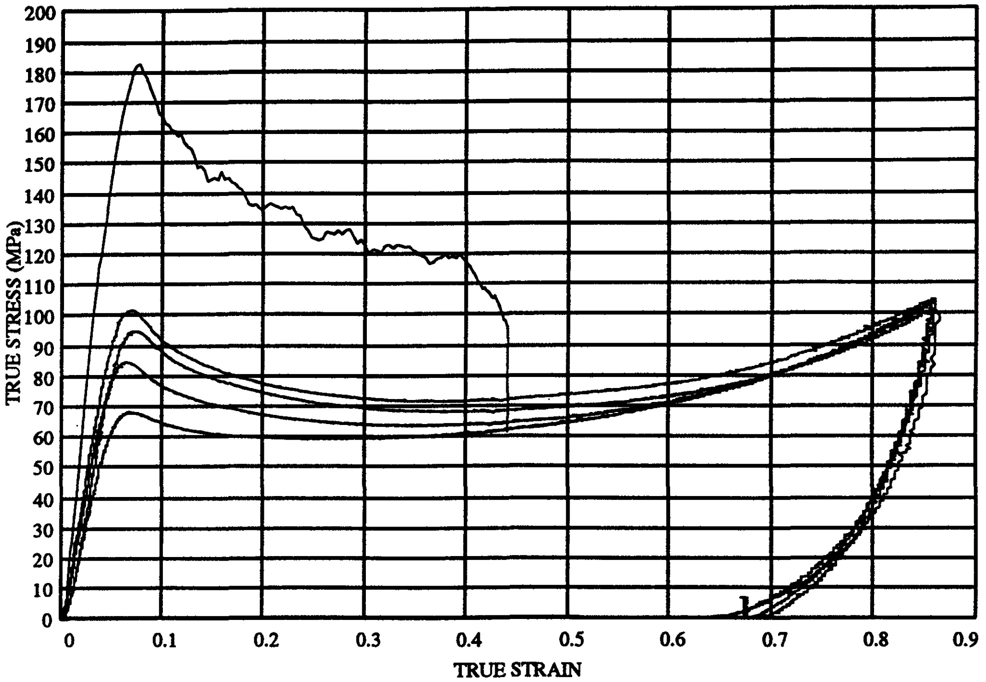
* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate/initial specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.759$	$\max\left(\sigma_{\mathrm{trueA}}\right) = 110.58 \cdot \mathrm{MPa}$	$\sigma_{yA} = 93.767 \cdot \mathrm{MPa}$	$\varepsilon_{yA} = 0.065$
$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.761$	$\max\left(\sigma_{\mathrm{trueB}}\right) = 106.084 \cdot \mathrm{MPa}$	$\sigma_{yB} = 105.442 \cdot \mathrm{MPa}$	$\varepsilon_{yB} = 0.071$
$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 0.763$	$\max\left(\sigma_{\mathrm{trueC}}\right) = 117.63 \cdot \mathrm{MPa}$	$\sigma_{yC} = 117.63 \cdot \mathrm{MPa}$	$\varepsilon_{yC} = 0.073$
$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 0.763$	$\max\left(\sigma_{\mathrm{trueD}}\right) = 123.683 \cdot \mathrm{MPa}$	$\sigma_{yD} = 123.683 \cdot \mathrm{MPa}$	$\varepsilon_{yD} = 0.073$
$\max\left(\varepsilon_{\mathrm{trueh}}\right) = 0.382$	$\max\left(\sigma_{\mathrm{trueh}}\right) = 200 \cdot \mathrm{MPa}$	$\sigma_{yh} = 200 \cdot \mathrm{MPa}$	$\varepsilon_{yh} = 0.077$

FIGURE 10-3
75% 8084 / 25% 1080 at various load rates



— A = Load Rate = 0.01 mm/sec*
— B = Load Rate = 0.10 mm/sec*
— C = Load Rate = 1.00 mm/sec*
— D = Load Rate = 2.50 mm/sec*
— E = Hopkinson Bar Test; Strain Rate = 3.00e+03 /sec

* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.855$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 103.201 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 67.896 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.066$

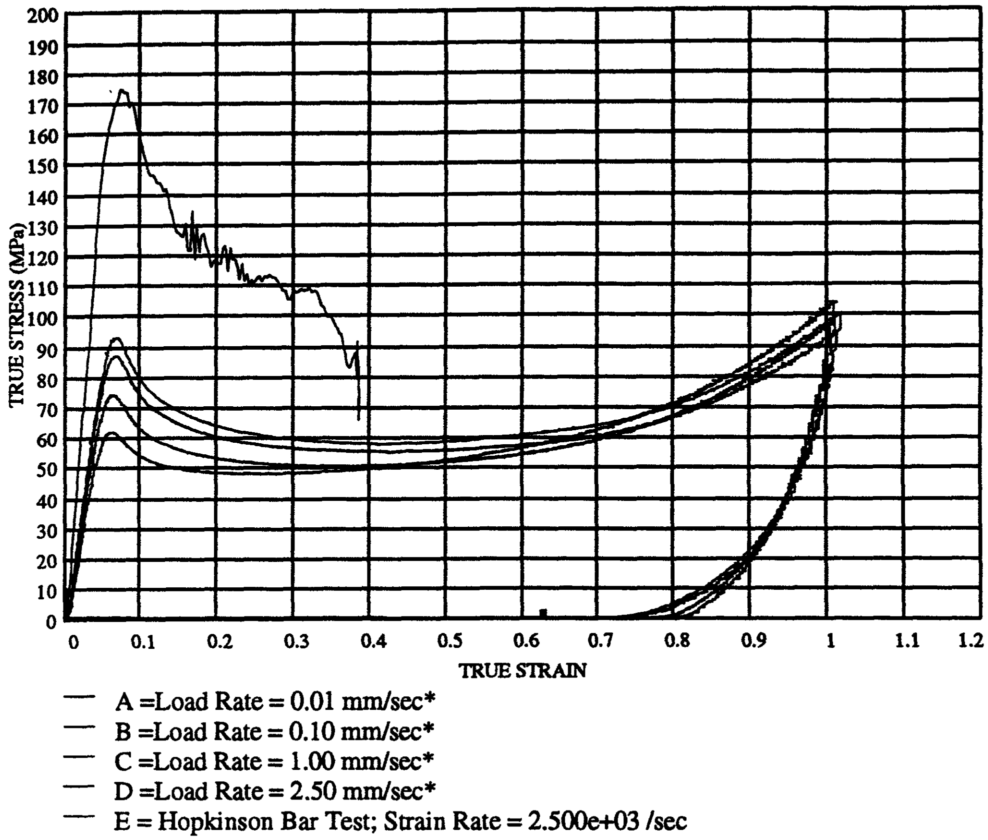
$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.86$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 104.253 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 84.366 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.064$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 0.862$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 101.544 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 94.798 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.071$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 0.864$ $\max\left(\sigma_{\mathrm{trueD}}\right) = 104.907 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yD}} = 101.388 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yD}} = 0.07$

$\max\left(\varepsilon_{\mathrm{trueh}}\right) = 0.44$ $\max\left(\sigma_{\mathrm{trueh}}\right) = 182.4 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yh}} = 182.4 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yh}} = 0.077$

FIGURE 10-4
50% 8084 / 50% 1080 at various load rates:



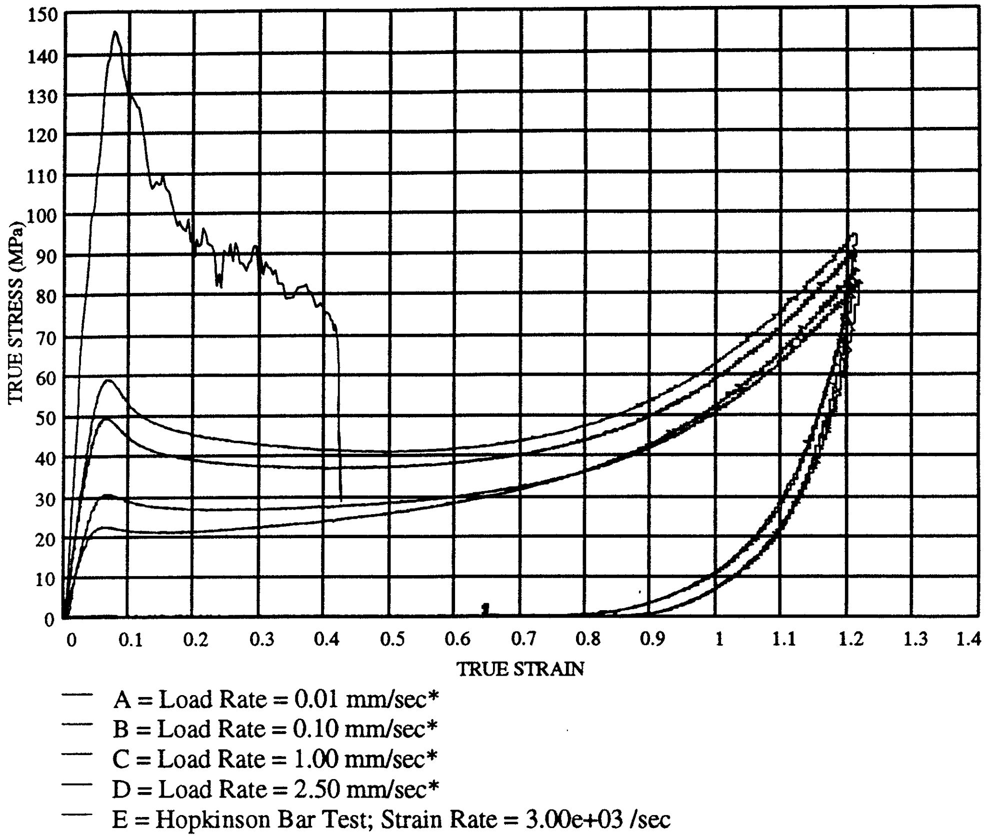
* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 1.012$	$\max\left(\sigma_{\mathrm{trueA}}\right) = 104.044 \cdot \mathrm{MPa}$	$\sigma_{yA} = 62.089 \cdot \mathrm{MPa}$	$\varepsilon_{yA} = 0.059$
$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 1.01$	$\max\left(\sigma_{\mathrm{trueB}}\right) = 98.384 \cdot \mathrm{MPa}$	$\sigma_{yB} = 74.187 \cdot \mathrm{MPa}$	$\varepsilon_{yB} = 0.063$
$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.012$	$\max\left(\sigma_{\mathrm{trueC}}\right) = 94.066 \cdot \mathrm{MPa}$	$\sigma_{yC} = 87.054 \cdot \mathrm{MPa}$	$\varepsilon_{yC} = 0.067$
$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.017$	$\max\left(\sigma_{\mathrm{trueD}}\right) = 100 \cdot \mathrm{MPa}$	$\sigma_{yD} = 93.068 \cdot \mathrm{MPa}$	$\varepsilon_{yD} = 0.067$
$\max\left(\varepsilon_{\mathrm{trueh}}\right) = 0.385$	$\max\left(\sigma_{\mathrm{trueh}}\right) = 174.6 \cdot \mathrm{MPa}$	$\sigma_{yh} = 174.6 \cdot \mathrm{MPa}$	$\varepsilon_{yh} = 0.074$

FIGURE 10-5
25% 8084 / 75% 1080 at various load rates



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\mathrm{trueA}}) = 1.216$ $\max(\sigma_{\mathrm{trueA}}) = 85.539 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 22.46 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.059$

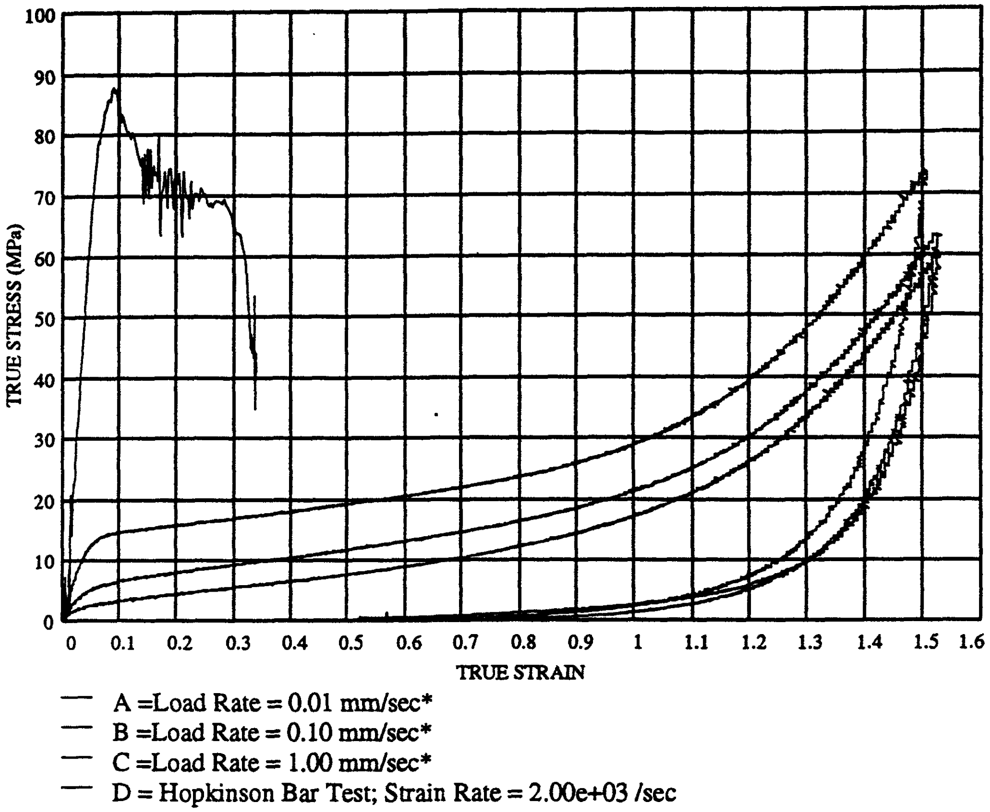
$\max(\varepsilon_{\mathrm{trueB}}) = 1.222$ $\max(\sigma_{\mathrm{trueB}}) = 82.846 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 30.551 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.068$

$\max(\varepsilon_{\mathrm{trueC}}) = 1.211$ $\max(\sigma_{\mathrm{trueC}}) = 89.4 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 49.394 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.066$

$\max(\varepsilon_{\mathrm{trueD}}) = 1.212$ $\max(\sigma_{\mathrm{trueD}}) = 94.333 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yD}} = 58.77 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yD}} = 0.069$

$\max(\varepsilon_{\mathrm{trueh}}) = 0.425$ $\max(\sigma_{\mathrm{trueh}}) = 145.2 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yh}} = 145.2 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yh}} = 0.076$

FIGURE 10-6
0% 8084 / 100% 1080 at various load rates:



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - C:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 1.529 \qquad \max\left(\sigma_{\text{trueA}}\right) = 60.495 \cdot \text{MPa}$

$\max\left(\varepsilon_{\text{trueB}}\right) = 1.53 \qquad \max\left(\sigma_{\text{trueB}}\right) = 63.194 \cdot \text{MPa}$

$\max\left(\varepsilon_{\text{trueC}}\right) = 1.506 \qquad \max\left(\sigma_{\text{trueC}}\right) = 73.66 \cdot \text{MPa}$

$\max\left(\varepsilon_{\text{trueh}}\right) = 0.339 \qquad \max\left(\sigma_{\text{trueh}}\right) = 87.72 \cdot \text{MPa} \qquad \sigma_{yh} = 87.72 \cdot \text{MPa} \qquad \varepsilon_{yh} = 0.089$

Since the frequency in Hertz is the inverse of the period, an average compressive stress wave in a Hopkinson bar with an pulse duration of 100 microseconds can be equated to a frequency of approximately 5000 Hz. Using the time-temperature superposition principle the glass transition temperature has effectively increased by 10^0C over the DMA result to approximately 37^0C. This can be achieved either by shifting the stress relaxation curve (DMA Figure 8-9) to the right or assuming that the ambient temperature has decreased by the same amount. After performing this operation, it can be seen that the material has not shifted completely into the glassy regime, however equation (5-16) indicates that the material will behave essentially in a glassy manner. This was borne out in the Hopkinson bar test.

Figures 10-7 through 10-10 show the plots of the predictive model of peak compressive yield stress as a function of LOG(strain rate) which were previously presented in Figures 9-11 and 9-12. To aid in the clarity of the presentation, each material blend was plotted individually. The results of the comparison is summarized in Table 10-4.

TABLE 10-4

COMPOSITION	MEAN PEAK EXPERIMENTAL COMPRESSIVE YIELD STRESS (MPa)	PREDICTED VALUE OF THE PEAK COMPRESSIVE YIELD STRESS (MPa)	DIFFERENCE BETWEEN MEAN AND PREDICTED VALUES (MPa)
100% 8084 / 0% 1080	-200.48	-179.10	21.38
75% 8084 / 25% 1080	-182.99	-163.90	19.09
50% 8084 / 50% 1080	-174.39	-150.13	24.26
25% 8084 / 75% 1080	-142.49	-132.14	10.35

Table 10-4 indicates that the predictive equation underestimated the peak yield stress in all cases. The first three values showed no obvious trend as a function of composition, but instead maintained a steady value of approximately 20 Mpa.

The 25% 8084 / 75% 1080 blend showed much closer agreement with the predicted value differing only by 10.35 Mpa. The estimated curve in this case was

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS
LOG (STRAIN RATE) FOR THE 100% 8084 MATERIAL

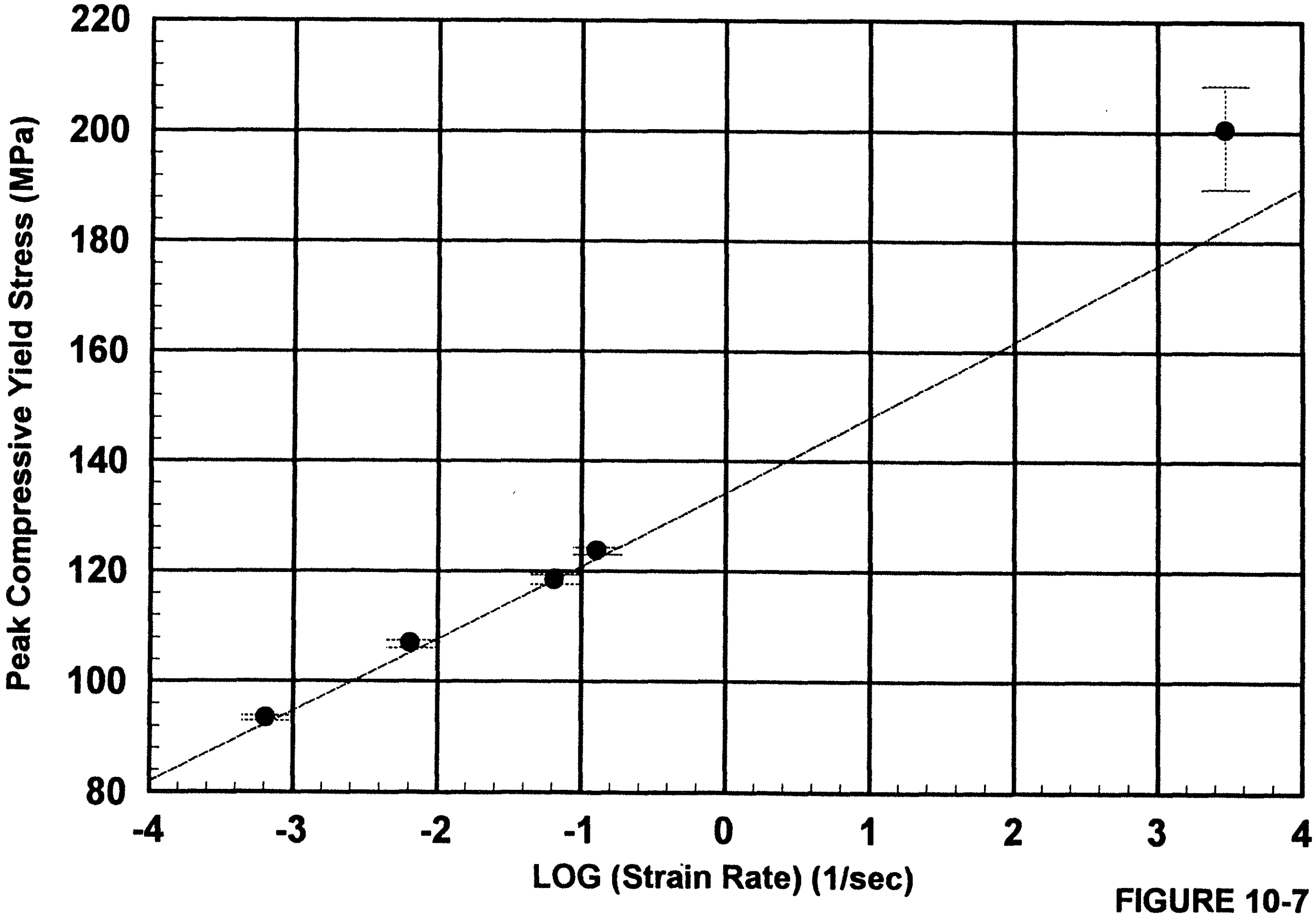


FIGURE 10-7

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS
LOG (STRAIN RATE) FOR THE 75% 8084 / 25% 1080 BLEND

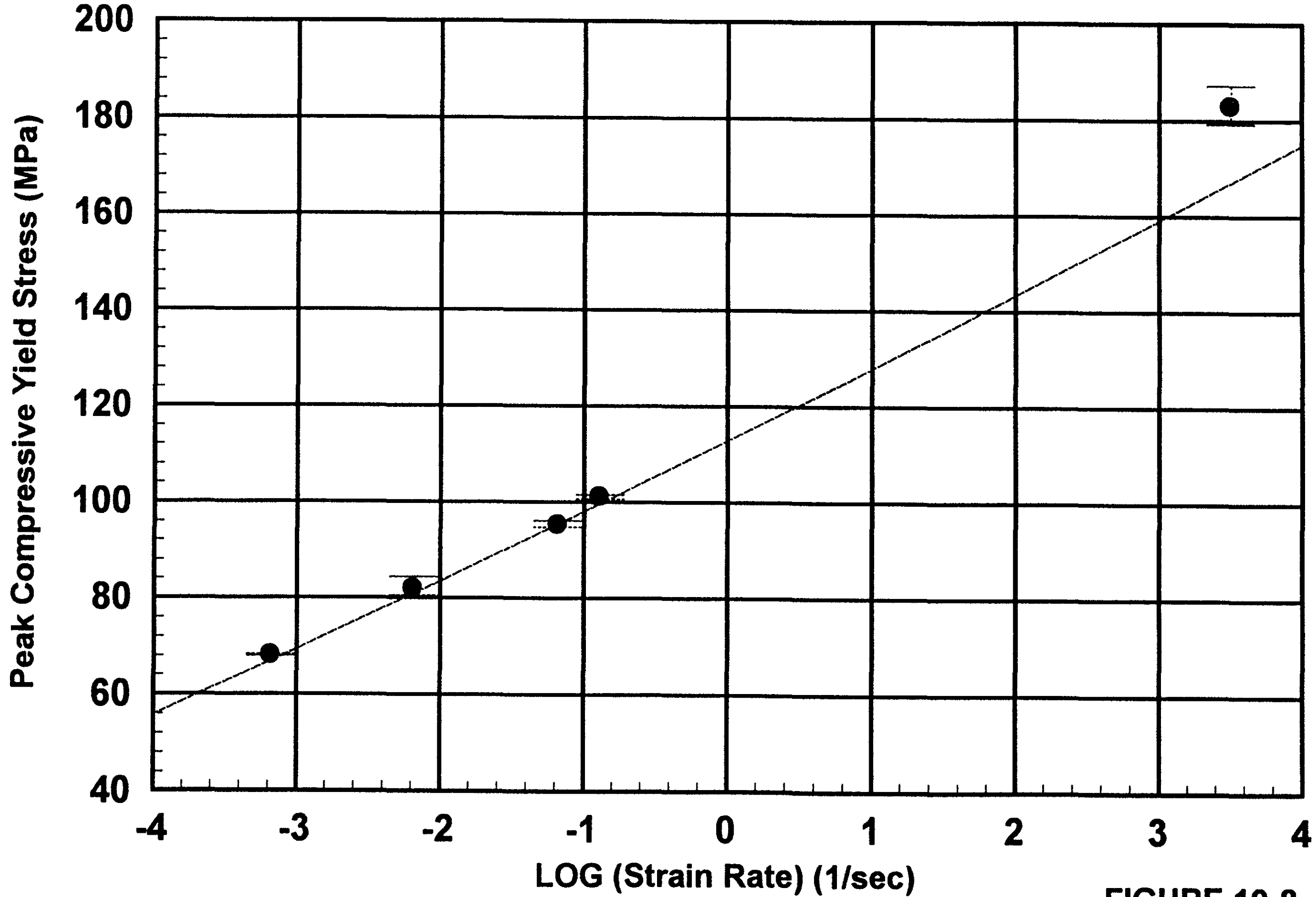


FIGURE 10-8

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS LOG(STRAIN RATE) FOR THE 50% 8084 / 50% 1080 BLEND

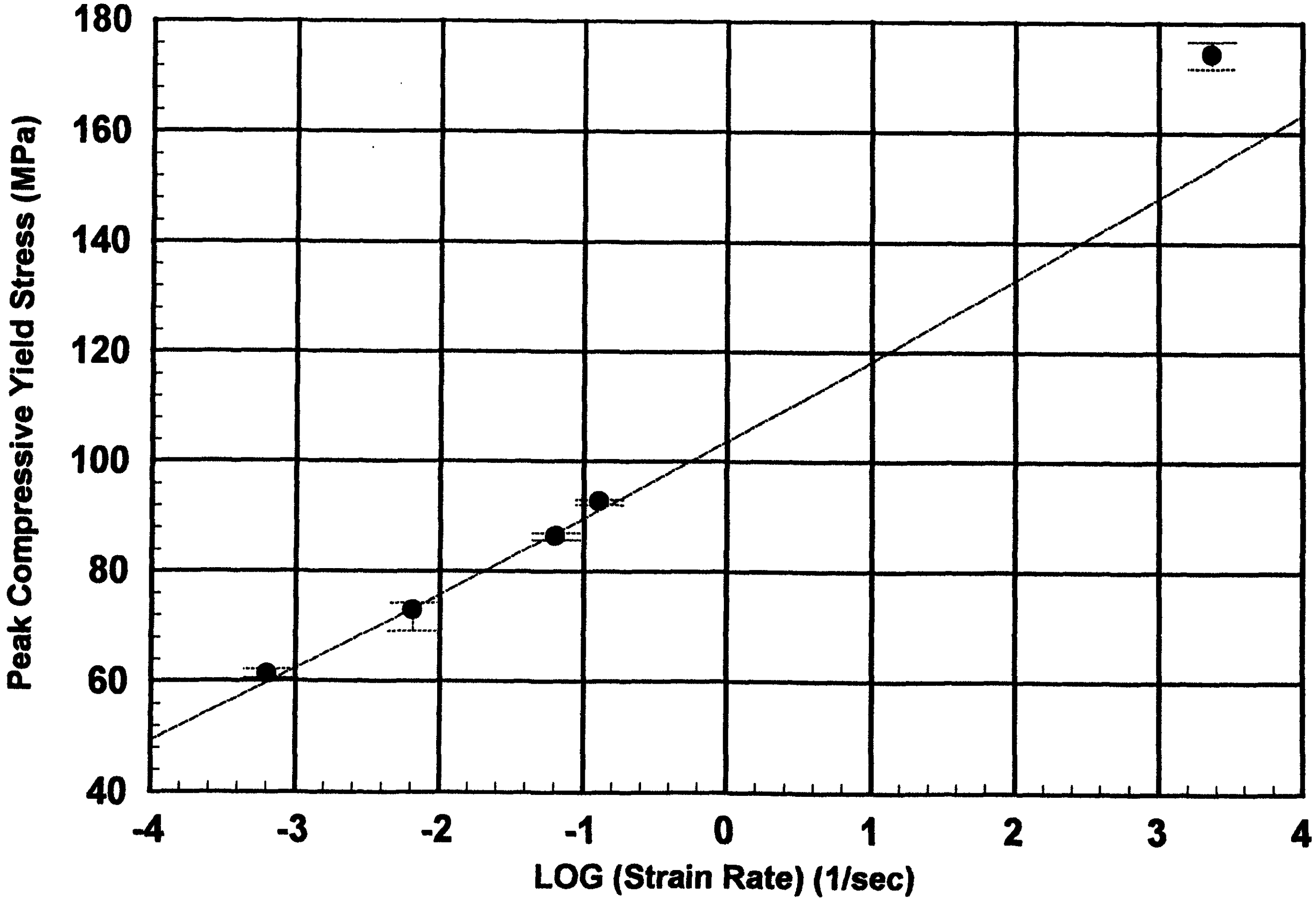


FIGURE 10-9

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR THE 25% 8084 / 75% 1080 BLEND

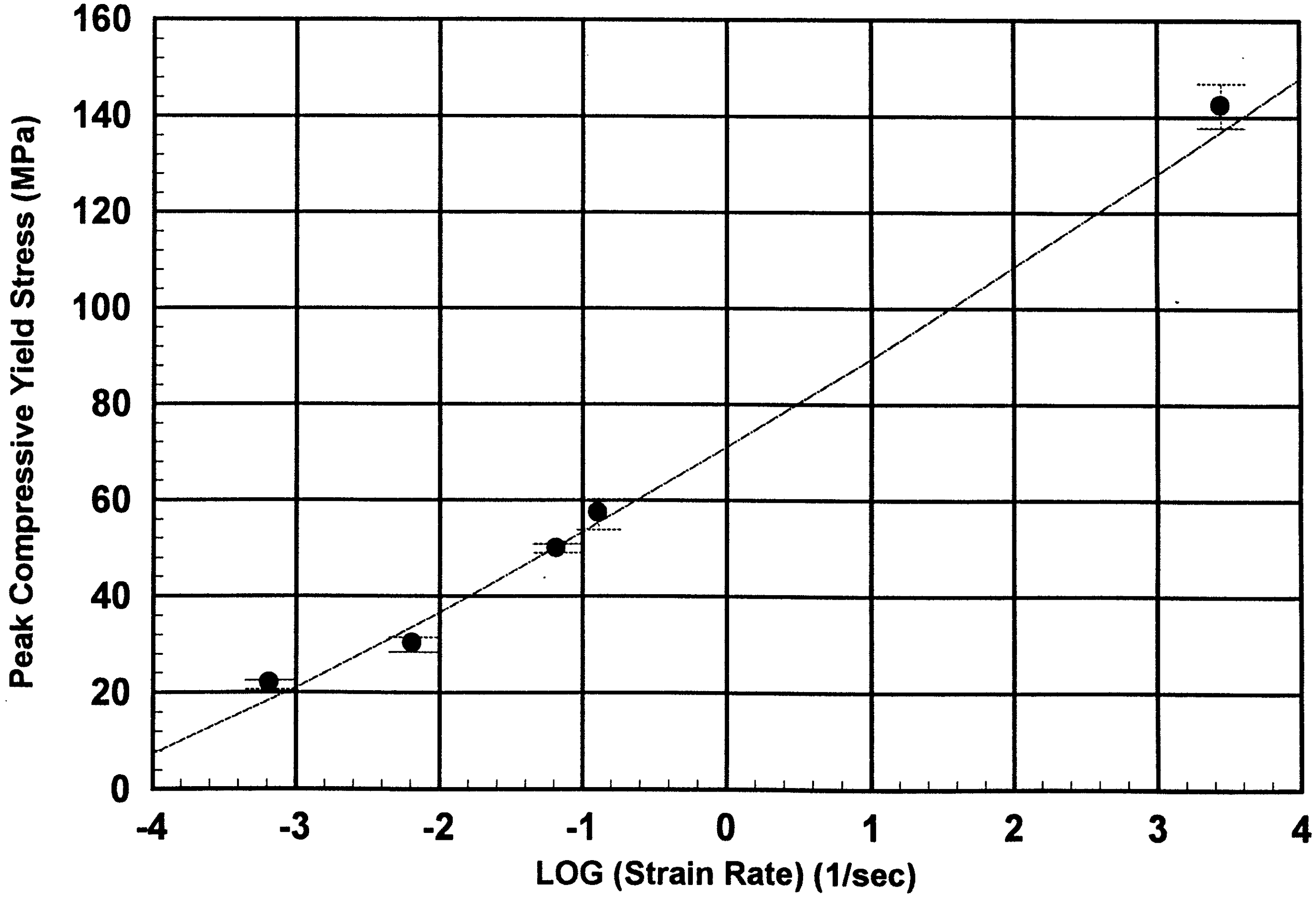


FIGURE 10-10

somewhat different than the others. All of the curves showed a slight upturn due to the 6/5 coefficient in the fitting equation. The curvature of the 25% 8084 / 75% 1080 blend was much more pronounced. It was this extra curvature in the predictive model which brought the experimental and predicted values into such close agreement. The curvature is a strong function of the athermal shear strength. This value is calculated using the elastic modulus of the material which corresponds to absolute zero (or a very high strain rate). The assumption in the model is that the modulus is not a strong function of strain rate. Clearly this blend shows a significant degree of rate dependent behavior, and therefore cannot be adequately modeled using this procedure.

10.40 Temperature Rise During Plastic Deformation

As part of the high strain rate testing, an in-situ measurement of the temperature rise was conducted for several of the specimens. The temperature rise was measured by illuminating the test specimen with a Helium-Neon laser produced by Spectra-Physics. As the temperature rises in the specimen, the emission of photons increases. This increase is measured by a collector dish which results in a change in resistance (and a subsequent voltage drop). The voltage time history was recorded by a Nicolet 400 digital oscilloscope.

In order to translate these voltage signals to a corresponding temperature rise, a calibration process was required. Due to time constraints, only the 100% Derakane specimens were calibrated. The process involved imbedding a thermocouple into one of the specimens, and heating it up to the highest temperature expected during the test. To this end, the specimen was placed in boiling water for several minutes. When temperature equilibrium was achieved (not very critical), the specimen was placed in the same location as it was during the testing procedure and illuminated with the laser. The temperature was then measured as a function of the voltage drop detected by the oscilloscope. The specimen was then removed from the apparatus, reheated and the process was repeated to obtain enough data points for a good fit.

In order to measure the voltage drop, a simulated AC signal was required. This was accomplished by "chopping" the signal. In this process, the laser was alternately blocked, by projecting it though a hole in a rotating disk. This produced a strobe effect, adequately simulating an AC signal. The disk was painted black to ensure a uniform emmisivity and a stable zero reference for the voltage signal.

The calibration data was curve fitted using a quadratic function on a statistical program called SPSS. The resulting calibration equation was:

$$\text{Temp} = 83.0073(\text{Voltage})^2 + 36.7420(\text{Voltage}) + 22.1931$$

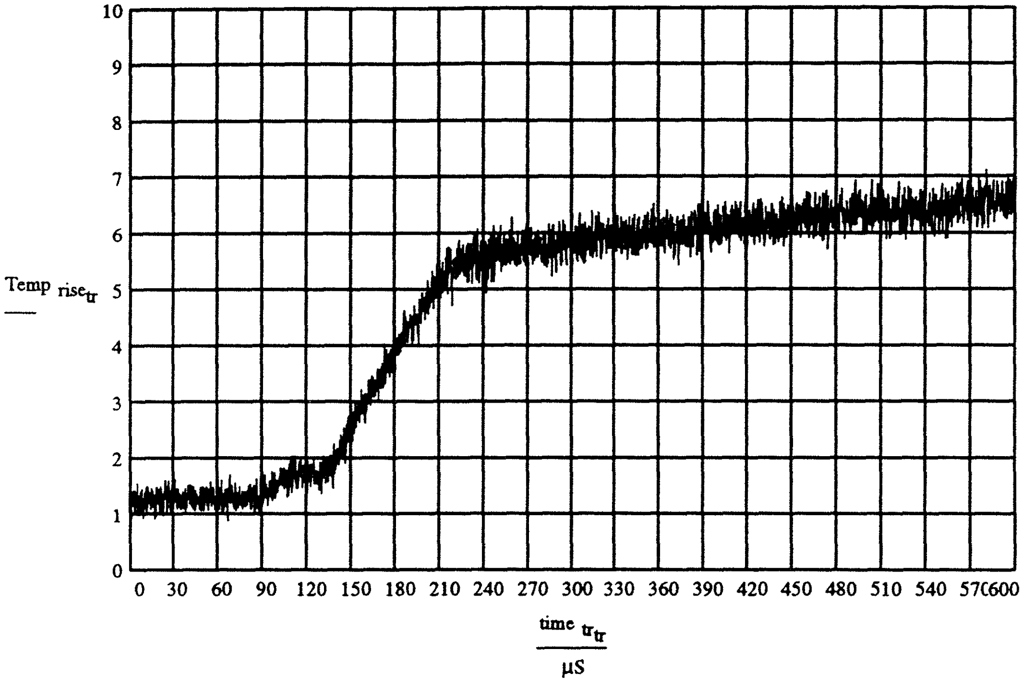
where: Temp is the temperature in Celsius

Voltage is the measured voltage at the oscilloscope

Using this calibration curve, a typical temperature trace as a function of test time can be generated. The temperature traces showed a very high degree of repeatability, and a typical curve is shown in Figure 10-11. The average true strain at the end of the test was approximately -0.40 to -0.50, at an average compressive yield stress of -200.48 Mpa.

The temperature rise was approximately 6-7 ^{0}C above ambient temperature. This value can be used to estimate the maximum possible temperature rise in the specimens (assuming adiabatic conditions) when tested at slower strain rates, since the temperature rise is directly proportional to the plastic work which is the area under the true stress- true strain curve. As a result, even if adiabatic conditions are assumed, it is expected that the temperature rise in the slower strain rate specimens would be significantly less than in the high strain rate testing case since the peak yield stress and the area under the stress strain curve is much reduced.

Figure 10-11
Temperature Rise to to High Strain RatePlastic Deformation



11.0 DISCUSSION AND CONCLUSIONS

The goal of this study was to investigate the compressive behavior of a series of blends of Dow Chemical Derakane 8084 rubber toughened vinyl-ester and a urethane acrylate called Crestomer 1080 produced by Scott Bader and to draw some conclusions about the effects of strain rate on the expected performance of a typical GRP stiffener panel bondline composed of blends of these materials.

A series of compression tests were conducted at strain rates ranging from 6.5×10^{-3} sec^{-1} to 1.3×10^{-1} sec^{-1}. Using these test results, a predictive trend for the lower yield stress as a function of strain rate was established using a procedure which was developed for the large strain viscoplastic analysis of glassy homopolyers. The lower yield stress is a more consistent predictor of performance than the upper one, since the former is nearly insensitive to any previous thermal treatment while the latter requires an annealing procedure to produce consistent results. Annealing in these polymers is achieved by raising the temperature 10-20^{0}C above the glass transition temperature for several hours.

The modeling procedure worked well for the blends containing less than 50% Crestomer by weight. The stress value obtained from the model is a strong function of the elastic modulus via the athermal shear strength. It works best when the elastic modulus does not vary to any large degree as a function of strain rate. In the 75% Crestomer blend and the 100% Crestomer material, a significant increase in the elastic modulus with strain rate was seen. Further review of this model is required to account for the strongly rate-dependent elastic response of these materials.

An attempt was made to extrapolate the Instron test results obtained for the lower yield stress as a function of strain rate out to strain rates as high as 3500 sec^{-1} and compare this prediction to actual test results obtained using a Hopkinson bar. The prediction underestimated the experimental results by approximately 20 Mpa for all the blends up to 50% Crestomer by weight.

The cause of this discrepancy may be attributed to viscous or inertial effects. Both of these would result in an increase in yield stress with increasing strain rate. The exact mechanism of such an inertial effect in these materials is unknown. It was noted by

Meyers[1] that a dividing line typically exists at a strain rate of approximately 5.0 sec^{-1}. Below this value, inertial forces can be neglected and equilibrium can always be obtained. At strain rates higher than 5.0 sec^{-1}, inertial forces have an increasing effect due to wave propagation effects. Further research is required to determine inertial mechanisms of this type in polymer materials.

11.10 Modeling of Themoplastic Versus Thermoset Materials

The model presented for the prediction of the yield stress as a function of strain rate was developed and tested using a variety of amorphous glassy polymers such as PMMA (polymethylmethacrylate). This material is a thermoplastic which has strong intramolecular and weak intermolecular bonds between chains. The intermolecular interactions in thermoplastics occur primarily as a result of chain entanglements. It has been observed that the thermally evolving chain density in PMMA exceeds the molecular density by roughly an order of magnitude, indicating that the chain entanglement density in PMMA is very high[2]. An amorphous material such as PMMA can then be viewed as an entangled network of long macromolecular chains. The effect of these entanglements will likely become increasingly more important as the strain rate is increased, due to friction between chains assuming that the temperature rise is not high enough to cause significant thermally induced chain disassociation.

This concept forms a link between thermoplastics and the thermoset materials studied here. The behavior observed for all of the glassy blends in this study was similar to what was observed in the study of true amorphous materials. In particular, the material exhibited a peak yield stress, followed by combined thermal and strain softening, and finally entropic strain hardening. It is not immediately obvious that a highly crosslinked network structure can produce an entropic hardening response, which is typically associated with the straightening out of a chain or series of chains from their fully coiled configuration.

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[1] Marc A. Meyers, *Dynamic Behavior of Materials* (New York: John Wiley and Sons, 1994), pp. 298-299.

[2] E.M. Arruda, M.C. Boyce, and R. Jayachandran "Effects of strain rate, temperature and thermomechanical coupling on the finite strain deformation of glassy polymers," *Mech. Mater.* 19, (1995) p. 196.

In a thermoset, the network structure is formed during the curing process. This crosslinked network can be viewed as a highly entangled network of chains. Since the entanglement density is high, the observed locking stretch for these materials is often smaller than in the case of true amorphous materials.

To address this issue, an experiment was conducted in which a sample of Derakane 8084, 15.5 mm long (length/diameter ratio of 1.0), was compressed at 0.01 mm/sec (nearly isothermal) until failure. The material underwent plastic flow until a true strain of approximately 0.80 was reached. By a true strain of approximately 0.90, the specimen abruptly shattered. This corresponded to a locking stretch of approximately 2.0-2.5 at the test temperature of approximately 298 K, which corresponds closely to the reported value of 2.1 for PMMA[3]. This lends further evidence that the large stretch deformation behavior of a thermoset could be modeled as a highly entangled amorphous material.

11.20 Modeling of Polymer Blends

Another important issue addressed in this study is the effect of the concentration of constituent materials on the large strain plastic behavior of the blend. Most of the experimental work to date on polymers under large strain has been concerned with a single homopolymer. The material studied here is a blend of two distinct polymer constituents. Not much is known about the structure of these materials, especially Crestomer. The testing conducted in this study gives a fair amount of information about their macroscopic behavior.

Crestomer 1080 is viscoelastic at room temperature with a glass transition temperature of approximately 27^0C at 10 Hz. It will therefore have a highly rate-dependent elastic modulus. In addition, it is capable of large deformation without undergoing yielding which indicates a low crosslink density and a large locking stretch.

The material behaves in many ways like a rate-dependent elastomer. No detectable change in the specimen dimensions was observed as a result of any of the Instron testing up to the maximum limit of 150% true compressive strain. This assumes that the loading

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[3] E.M. Arruda, M.C. Boyce, and R. Jayachandran "Effects of strain rate, temperature and thermomechanical coupling on the finite strain deformation of glassy polymers," *Mech. Mater.* **19**, (1995) p. 205.

high strain rate testing, no change in specimen dimensions was detected down to the resolution of the digital micrometer (0.001 inch). It appears that while the specimens clearly underwent plastic deformation based on the measured stress history, the effects were removed within a very short period of time. This may have been due in part to a temperature rise of approximately 4-6°C, which was measured during the high strain rate testing. It should be noted that this material was not calibrated. If the emmisivity of all of the materials is the approximately the same, then the calibration curve for the Derakane 8084 specimen can be used to interpret the temperature trace.

Derakane, in contrast to Crestomer, is a highly crosslinked vinyl-ester thermoset with a glass transition temperature of 115°C. Since the glass transition temperature is well above room temperature, the material behaves in a glassy manner. As a consequence, the elastic modulus is not highly rate dependent and the material undergoes shear yielding when loaded in compression beyond a true strain of approximately 7%. Along with yielding comes thermal and strain softening effects which become more pronounced as the strain rate is increased.

11.30 Material Blend Morphology

It is not clear what morphology occurs when these materials are blended. Since both constituents are formed from an unpromoted resin in a liquid phase, blending can be very complete. When the blended liquid resin containing a known weight percentage of each of the constituents is promoted and catalyzed, the network of each of the materials will be intimately interwoven.

One issue which is unresolved relates to how the materials combine. The crosslinking reaction may occur by the formation of hybrid regions of material or it may be possible that the each of the constituents reacts only with other material of the same type. If hybrid chains do not form and the precatalyzed resin is thoroughly mixed, the structure may be viewed as a parallel combination of the two materials. Derakane can be modeled to first order as a large spring in series with a small viscous damper, while Crestomer is better modeled as a large viscous damper in series with a small spring. The springs will likely be nonlinear, especially if large stretch behavior is modeled.

This model would lead to small a decrease in stiffness as Crestomer is added to the higher modulus Derakane resin, assuming that the loading rate is low. This behavior was observed both in the plot of glass transition temperature in Chapter 8, where the measured value of Tg varied by only about 24°C from 100% Derakane up to a concentration of 50% 8084 / 50% 1080, and in the Instron testing.

Some valuable information can be obtained from the predicted glass transition temperature as a function of Crestomer concentration. As mentioned, this curve is not typical of what has been observed in the study of polymers to date. Typical polymer blends follow a series type predictive model in which a small weight percentage of the lower stiffness causes a rapid decrease in the stiffness of the blend. This can be viewed as two springs in series, one of which is approximately three orders of magnitude greater than the other.

It is thought that the blending of long chain thermoplastic materials cannot be very thorough unless the materials are polymerized in-situ. This likely results in a morphology in which regions of one material exist near regions of another, resulting is a blend only in the macroscopic sense. Blending on a small-scale would appear to be very difficult. This concept that thermoplastic polymer blends exist on small-scale as localized regions of the constituents, lends itself well to a model which is primarily series in nature.

In the case of the thermoset materials used in this study, the morphology of the structure is likely to be much closer to an idealized parallel model since the chains are formed in-situ during the curing process. In a parallel model, a large quantity of the softer material is required to result in any change in the stiffness of the blend.

The glass transition temperatures cited in this study were all obtained at a testing frequency of 10 Hz. Some conclusions were drawn about the behavior of these blends both at higher and lower load rates based on time-temperature equivalency. This approach, which applies only when the theory of linear viscoelasticy is valid, was justified by noting that the maximum strain imposed on the test specimens during DMA testing are typically less than 1.0%. Nevertheless, actual testing at higher frequencies should be conducted to determine if these extrapolated approximations are valid.

If hybrid chain formation is possible, the next issue which needs to be addressed is the preferred order in which these chains are formed. For example, the behavior of a chain composed of long strings of Crestomer 1080 followed by long strings of Derakane 8084 will differ markedly from one composed of alternating shorter sections of the same constituent materials. The morphological structure in this case would be a strong function of the reaction kinetics of each of the constituents.

The issues relating to the precise morphology of the blends would provide valuable information which could be used to minimize manufacturing variance and build better models to predict the mechanical behavior of these materials under a wide variety of loading conditions. The morphology could be determined by irradiating one of the constituents with Deuterium. The blended material morphology could then be determined using TEM (Transmission Election Microscope) techniques. The morphological characterization of these blends should be a high priority if these materials are to be used in critical structural applications.

11.40 Additional Required Testing

To complete the modeling of these materials under compressive loading using the predictive model presented in this work, several additional material tests are required. First, a series of tensile tests are needed to eliminate the ambiguity in the pressure coefficient, which was assumed to be 0.20 throughout this study. In addition, if the behavior beyond the lower yield stress is desired, the elements of the backstretch tensor (Bij) are required. Recall that the backstretch tensor is assumed to be an entropic force which is modeled using Langevin statistics and finite strain elasticity. To determine this behavior, the rubbery (initial) modulus of the individual blends is required. This information can be obtained by raising the temperature of the material to a few degrees above the glass transition temperature, and loading the specimen at a high rate. The slope of the initial portion of the true stress / true strain response is the rubbery modulus of the material.

The final pieces of information required are the temperature dependence of both the modulus and yield stress. This information is readily obtained from plots of true stress versus true strain which are taken from compression or tension tests conducted at a series of test temperatures at a rate which is slow enough to ensure that isothermal conditions are maintained.

11.50 Conclusions and Implications

The model predicted the behavior of the glassy materials to a high degree of accuracy. In the 75% Crestomer 1080 / 25% Derakane 8084 blend, the Crestomer 1080 began to influence the behavior of the material. The stress levels are a strong function of the athermal shear strength which is dependent on the assumed modulus of elasticity. Some work is needed to implement rate dependency into the model to reflect the rate dependent elastic response of these materials.

This project was motivated by the general lack of information concerning the high strain rate behavior of amorphous polymers and thermosets which are used in critical structural applications requiring a significant degree of high strain rate survivability

The example cited in this study was the secondary bondline between a stiffening member and the hull shell of glass-reinforced plastic (GRP) minesweeping vessel. The current measure of the effectiveness of a bondline improvement in this application is the pull-off strength. These pull-off tests are conducted at very low rates (1.0 mm/sec is typical). Recent increases in pull of strength have been accomplished by increasing the strain to failure of the resins in the most critically stressed areas.

This study has concluded that pure Crestomer 1080 is capable of large deformations without yielding for the observed strain rates which were as high as 0.13 sec^{-1}. When tested at strain rates on the order of 1000 sec^{-1}, which are "typical" of hydrodynamic and air blast, this material behaves in a glassy manner, showing a clearly defined upper yield point as well as thermal and strain softening behavior.

Although most of the stresses in a stiffener pull-off test are tensile, the compressive material behavior obtained in this study can be used to gain some insight into the high strain rate tensile behavior of these blends. In general, fracture toughness decreases as yield stress increases. The large increase in yield stress seen in these Derakane/Crestomer blends will therefore likely lead to significantly lower pull off strength under high pull-off rates as compared to the slow pull-off results. This effect will be especially significant when the Crestomer 1080 concentration is greater than approximately 50% by weight.

Further study of the blends between 50% and 75% Crestomer are required to determine exactly when the rate dependency of the elastic modulus becomes most important.

Large Stiffener pull-off loads rely on the high compliance (low modulus) of the materials in the bondline. If the modulus increases rapidly from the testing load rate to the actual service load rate, all of the advantages of the compliant material will be lost and failure will likely initiate much sooner than predicted.

This study has investigated the compressive behavior of a blend of two distinct polymers as a function of strain rate. Further work is required to fully understand the behavior of these materials. The next step must include some morphological studies to determine both the mechanisms of the formation and final structure of these bends. The other issues which have not been addressed relate to tensile loading and include crazing verses yielding response. This will give some information relating to the determination of the transition from brittle to ductile behavior as a function of strain rate. Crazing occurs in the elastic response regime and the voids which are formed can serve as nucleation sites for fracture to occur. Once these issues are understood, more accurate models can be developed and eventually implemented into a numerical finite element code to model the response of more complex "real world" structures.

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Appendix A
Historical Overview

HISTORICAL OVERVIEW OF STIFFENER TO PANEL SECONDARY BONDLINE IMPROVEMENTS

Introduction:

This section will present a chronological account of methods which have been proposed to improve the stiffener to shell secondary bond. The earliest method involved bolting the stiffener to the main hull laminate, a complicated process which added a great deal of weight and cost to the ship's structure. The need to utilize bolts has been virtually eliminated in the past few years by the use of highly compliant resins initially based on acrylics, and most recently on urethane acrylates.

In addition to the effects of mechanical fastening and modification of bondline materials on secondary bond strength, the effects of the loading mode on the stresses in the bondline will be investigated. During an explosion, a stiffened panel is subjected to a very complicated loading pattern. To simplify the experimental evaluation of a proposed bond line improvement, a variety of clamping modes have been investigated. It was determined that both the required stiffener pull-off load and the mode of failure are strongly dependent on the loading mode used in the experiment.

The ultimate goal of the bondline material improvements is to prevent the separation of top-hat stiffeners from the hull shell under explosive loading. The results which will be presented in this section span approximately 25 years of research, mostly in support of various minesweeper construction projects in the United Kingdom. The results reflect static pull-off tests which are available in the open literature, but details relating to the verification of these results in explosion trials is not available.

Mechanical fastening techniques:

Early frame to shell connection designs, which generally used an all polyester matrix with E-glass woven reinforcement, were prone to failure at very low intensity explosive loads. Initially, this problem was solved by bolting the stiffener to the base panel. An increase in stiffener pull-off strength was obtained solely because the bolt head needed to be pulled through the stiffener flange or base plate prior to final failure. As a consequence, the presence of bolts served only to add damage tolerance to the structure by acting as crack arresters but did not prevent initial failure. In addition, the use of bolts added considerable weight and cost. The bolts initially used were made of aluminum-silicon-bronze and later titanium. These materials were chosen due to the non-magnetic hull requirements of mine sweeping vessels, the corrosive environment, and fatigue and vibration service requirements. Some of the early top-hat stiffener attachment methods were presented, by C. S. Smith and are reproduced in Figure 1.[1]

An article written by Green, A.K. and Bowyer, W.H. in 1981 described alternative mechanical fasteners.[2] The requirement to maintain water tight integrity led to an insertion scheme involving boring and counterboring of the hull laminate, the use of sealant, as well as the need to manually torque tighten the bolts from both inside and outside the hull following demoulding. The high cost associated with the fasteners and the insertion procedure led to the study of other fabrication methods that could resist the effects of shock loading. Green and Bowyer investigated the possibility of replacing the titanium bolts with a relatively inexpensive commercially available mechanical fastener and insertion technique using stainless steel screws. The screws were pneumatically driven into holes filled with liquid polyester resin. It was noted once again that mechanical fasteners were only a partial solution to the problem, since the bond failure initiates at the stiffener web/flange corner, remote from the fastener, and the fastener acts only as a crack arrester. To achieve a fundamental improvement in performance, it was necessary to inhibit the crack initiation process. As a result, they investigated the use of internal flanges produced using tough acrylic resins.

Internal flanging:

Green and Bowyer investigated the use of an internal flange, which is shown in the Figures 1 and 2. This fabrication method is an attempt to utilize the redundant stiffener base area and reduce the stress concentration in the stiffener web/flange corner by bridging the angle between the stiffener web and the base panel with Kevlar fiber stitching. The use of stitching was investigated using both polyester and tough acrylic matrix materials in the

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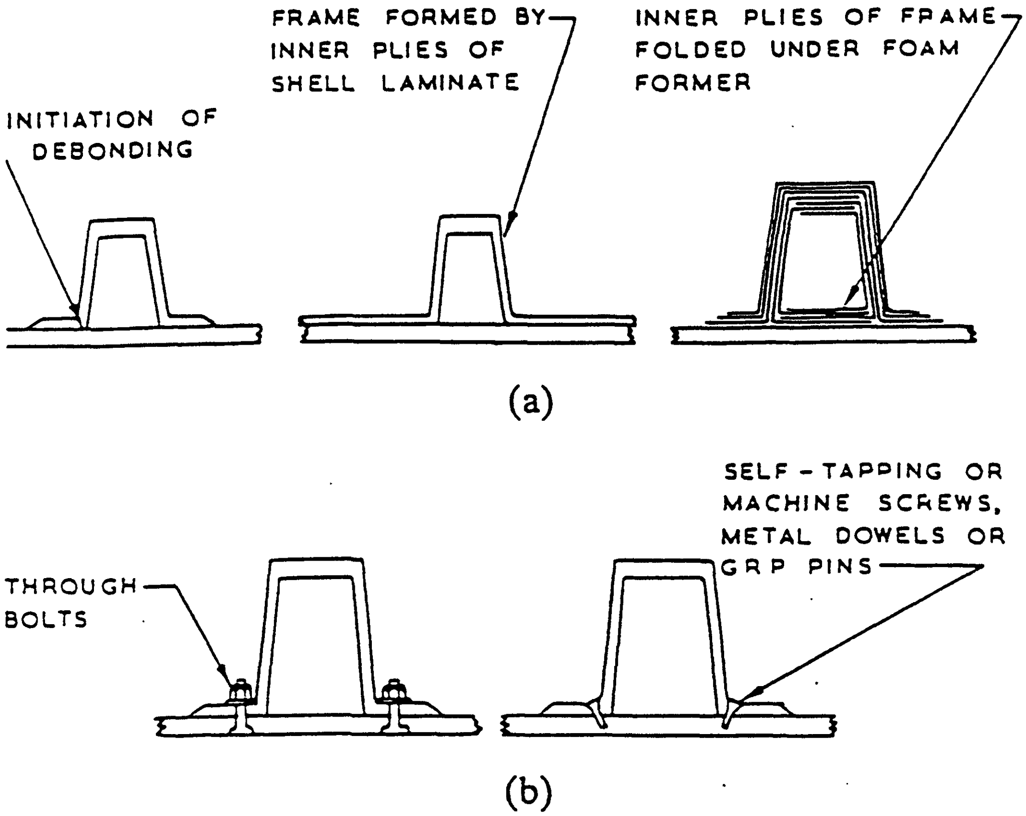
[1] C. S. Smith, Design of Marine Structures in Composite Materials (New York: Elsevier Applied Science, 1990), p. 289.

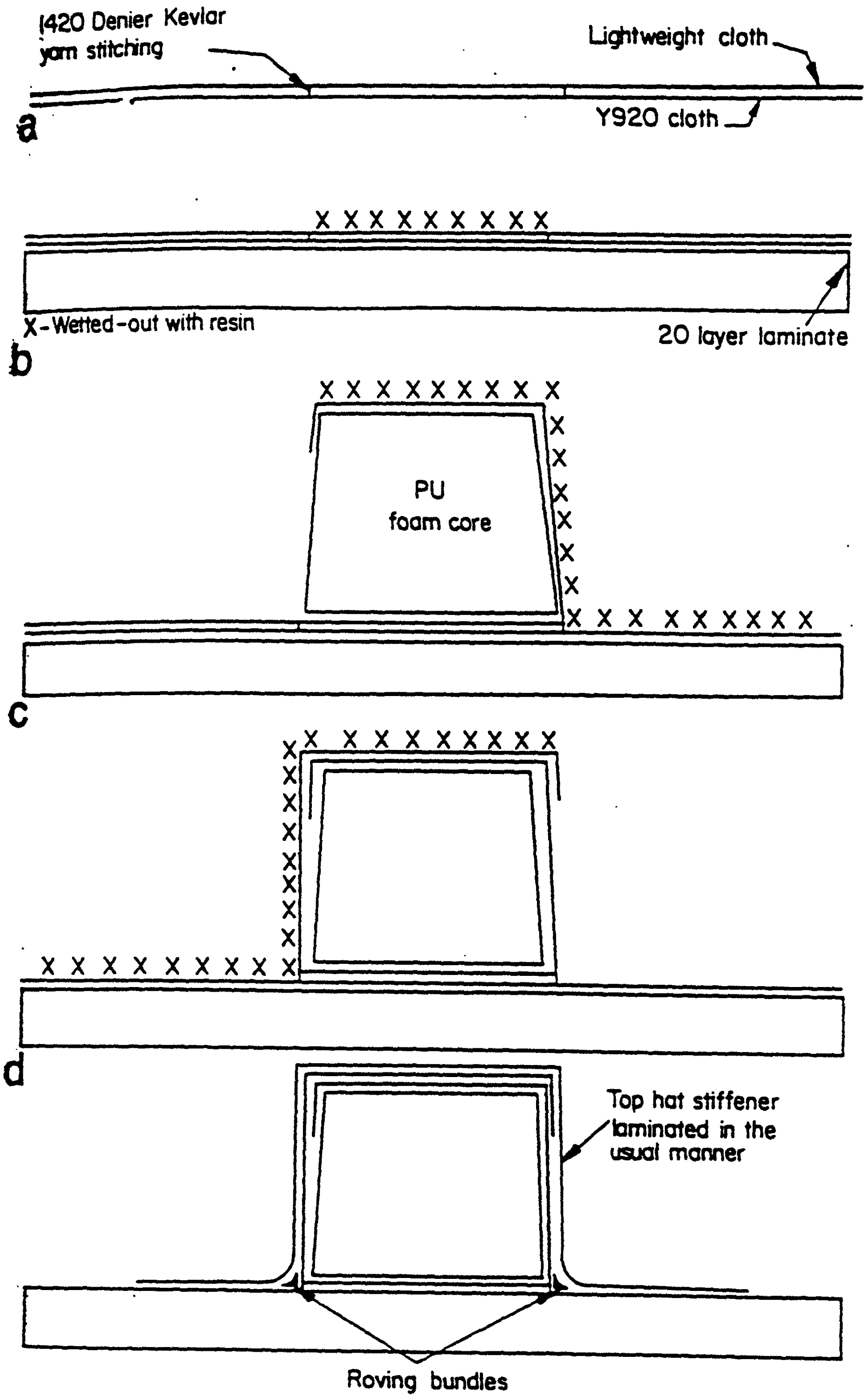
[2] A.K. Green and W. H. Bowyer, "The Development of Improved Attachment Methods for Stiffening Frames on Large GRP Panels," Composites, Jan. 1981, pp. 49-55.



Frame/shell joints: (a) types of frame/shell attachment; (b) reinforcement of frame/shell joint.

C.S. Smith, *Design of Marine Composite Materials* (New York:Elsevier Applied Science, 1990), p.289.

Figure 1



Laminating sequence using stitched cloths

A.K. Green and W.H. Bowyer, "The Development of Improved Attachement Methods for Stiffening Frames on Large GRP Panels," *Composites*, Jan. 1981, p.51.

Figure 2

bondline. Crack initiation in the secondary bond line at the stiffener flange occurred at higher load levels in specimens incorporating an internal flange with stitched cloth construction and acrylic matrix material in the bondline. In the stitched cloth construction specimens fabricated with polyester resin, peak loads were only slightly higher than in the unreinforced specimens. This indicates that the properties of the specimens are limited by the fundamental interlaminar toughness of the glass/polyester laminate, known to be about two orders of magnitude less than the translaminar (cross fiber) toughness in these materials[3]. In the acrylic stitched cloth specimens, the load bearing capability was increased on the order of 30 percent due to the large strain to failure of the acrylic material and subsequent improvement in load distribution in the secondary bond line. This was an early indication that by increasing the strain to failure and the compliance of the material in the bondline, the load distribution in this region could be improved significantly.

Clamping mode:

Green and Bowyer[4] expanded their work to include the effects of the clamping mode on the stress distribution and the final failure of a top-hat stiffener subjected to quasi-static pull-off testing. The two clamping modes were termed centre clamp and two clamp as shown in figure 3. This terminology has been used to describe the quasi-static testing of top-hat stiffeners ever since. The nomenclature used in this and all future studies is shown in figure 4. It was shown in earlier studies that the distribution of the tensile and compressive stresses in the secondary bondline are very sensitive to the form of the applied load.[5] In particular, these stresses are sensitive to the magnitude of the bending stresses present in the stiffener table and more importantly in the side web.

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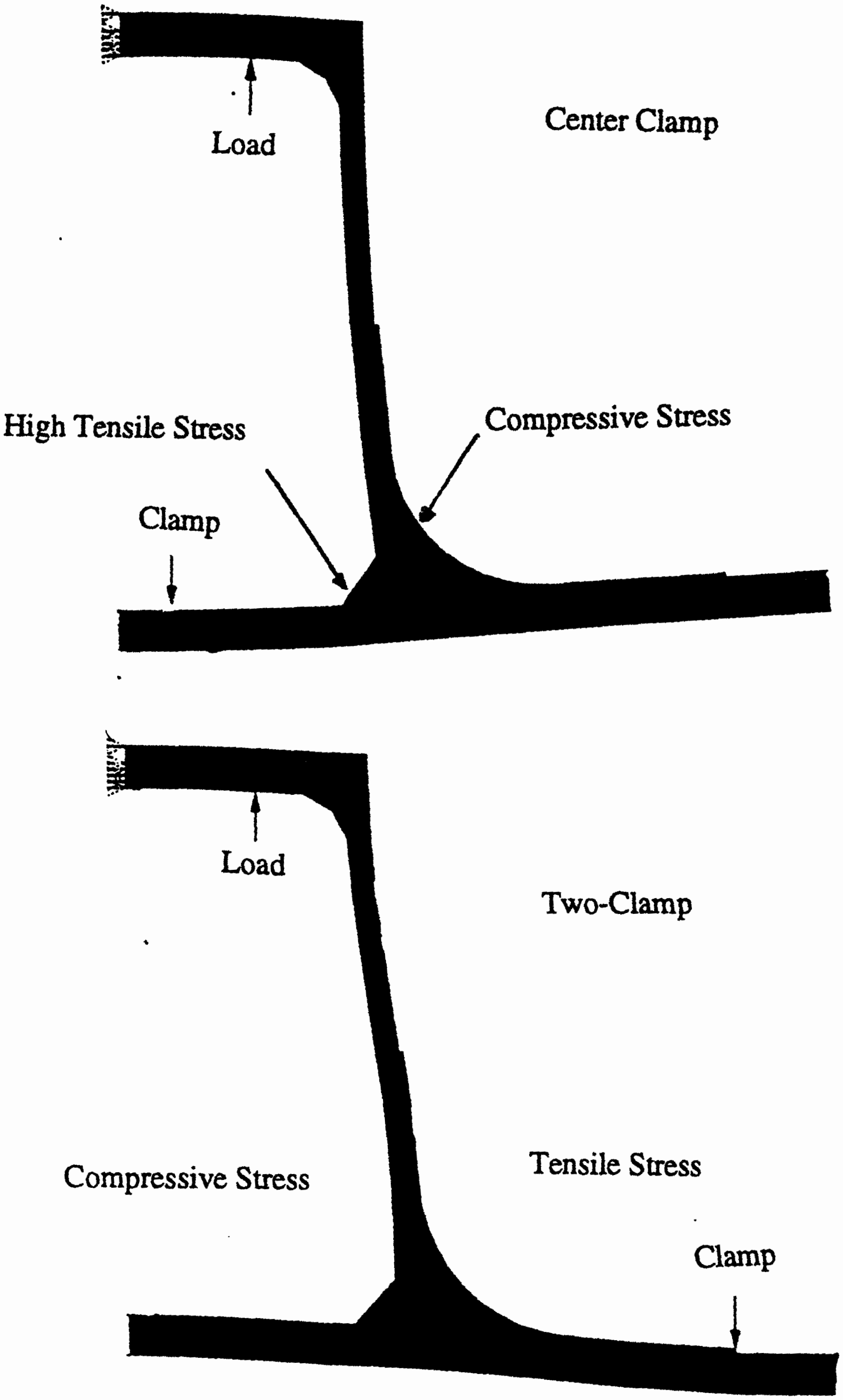
[3] G. Smith, A.K. Green and W.H. Bowyer, "The Fracture Toughness of Glass Fabric-Reinforced Polyester Resins," Proc Conf 'Fracture Mechanics in Engineering Practice', Sheffield 1976, ed. P. Stanley (London: Applied Science Publishers, 1977), p. 271.

[4] A.K. Green and W.H. Bowyer, "The Testing and Analysis of Novel Top-Hat Stiffener Fabrication Methods for Use in GRP Ships," Composite Structures, ed. I. H. Marshall (London: Applied Science Publishers, 1981) pp. 195-201.

[5] Unpublished MOD data



Elastic energy is stored in the side web, table and center of the base panel.

A.R. Dodkins, R.A. Shenoi and G.L. Hawkins, "Design of Joints and Attachments in FRP Ships' Structures," *Marine Structures*, 7 (1994), p.388.

Figure 3

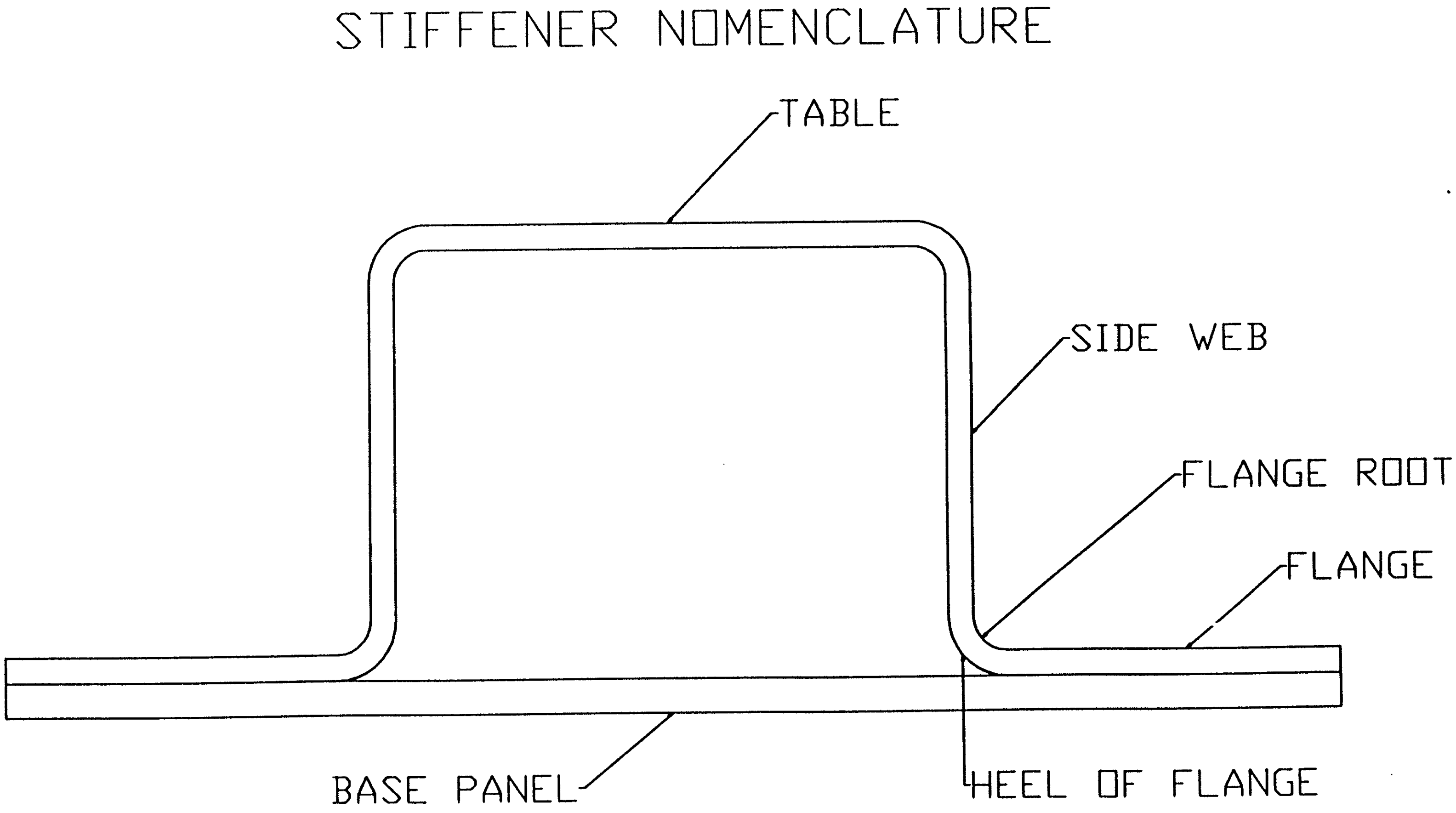
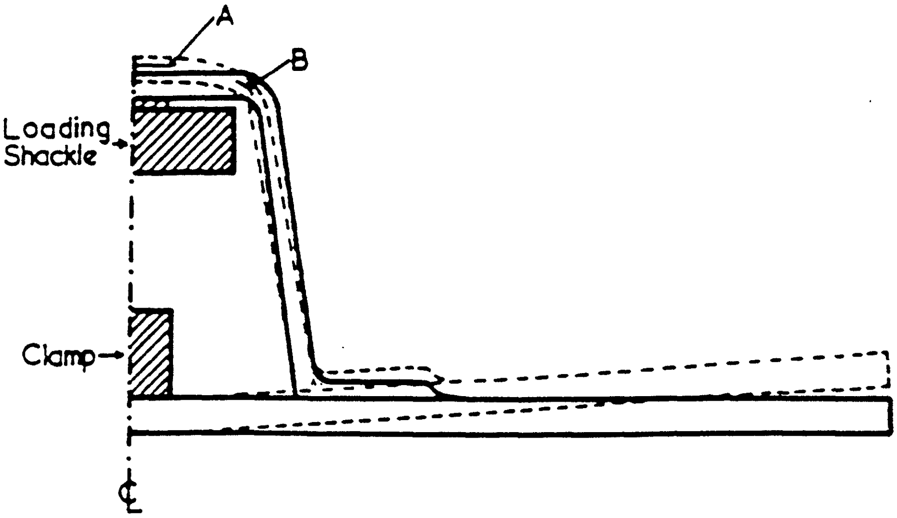

STIFFENER NOMENCLATURE
TABLE
SIDE WEB
FLANGE ROOT
FLANGE
BASE PANEL
HEEL OF FLANGE

Figure 4

The stresses in the bondline were evaluated with the load uniformly applied across the stiffener table for two clamp loading condition. The center clamp loading was applied as shown in Figure 5 in which an additional strip of steel was placed between the loading shackle and the stiffener table to concentrate the loading at the center of the stiffener table. The centre clamp results were compared to previous work in which the load was applied uniformly over the table. The results are presented in Figure 6.

It has been shown that failure of top-hat stiffened panels subjected to explosive loading occurred by complete separation of the hat stiffener from the base panel. Failure is caused by a crack originating at the heel of the side web which propagates through the secondary bondline. As a consequence, the larger the tensile stress concentration at the heel of the side web, the lower the load required to initiate failure. When the load was applied over the entire table in the centre clamped case, bending stresses were present in the side webs. These bending stresses are a result of a moment applied to the lower edge of the side web due to the curvature of the base panel under load. As a consequence, both tensile and compressive stresses were present in the secondary bondline. When the loading was concentrated towards the center of the table, the bending stresses in the side webs disappeared and only tensile stresses were present. This was due to the compensating moment applied to the top of the side web by the curvature of the stiffener table. When the bending stresses in the side webs are eliminated, the magnitude of the tensile stress concentration at the heel of the side web is reduced significantly. In the two clamp loading case, the stresses at the heel of the side web are compressive which indicates that the propagation of a crack through the bondline is not likely to be the initial failure event in this loading mode.

In order to determine the most severe loading case, it is important to know whether a crack will continue to propagate or arrest. The balance between overall system compliance and stored elastic energy in the deformed shape of the stiffener plate combination will determine the outcome. System compliance increases as the crack grows, along with a drop in the applied load, and stored elastic energy is released to propagate the crack. The drop in applied load during crack growth occurs because the



Schematic loading arrangement, centre clamped.

A.K. Green and W.H. Bowyer, "The Testing and Analysis of Novel Top-Hat Stiffenener Fabrication Methods for Use in GRP Ships," *Composites Structures*, ed. I.H. Marshall (London: Applied Science Publishers, 1981), p.187.

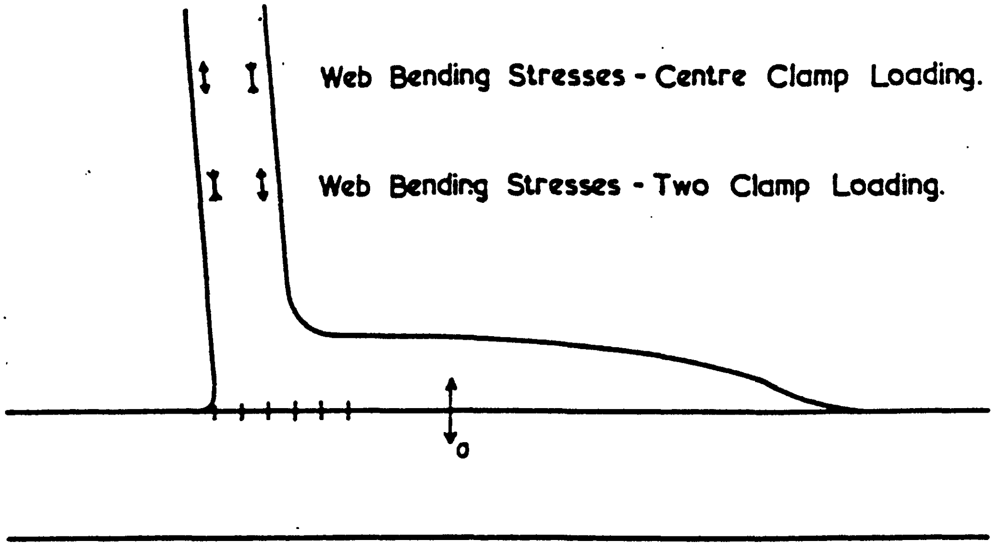
Figure 5

quasi-static pull-off process proceeds at such a slow rate that it is nearly analogous to applying a fixed displacement.

The centre clamp loading mode is shown in Figure 3a. In principle, the base panel acts as two cantilever beams fixed at the location of the centre clamp. The energy in the system which is available for continued crack propagation is stored in the region of the base panel under the top hat, the side webs, and the table. The initial system compliance is high, and as a result the load drops at a low rate as a function of crack length. As a result, a crack is likely to continue to propagate, and catastrophic failure in the form of the complete separation of the stiffener from the top-hat occurs. The catastrophic nature of the crack propagation and the high stress concentration at the heel of the side web result in a very low load to failure and an event which closely resembles the failure observed in panels subjected to explosive loads.

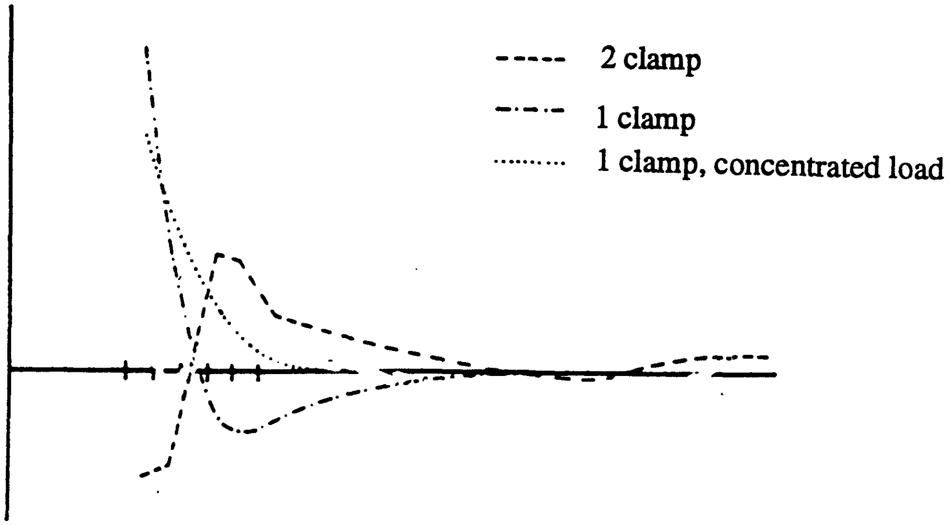
The failure sequence in the two clamped loading case is significantly than the centre clamp case. In this loading mode, the elastic energy in general is contained within the table (eliminated in this case due to the uniformly applied load), the side webs, and the base panel similar to the centre clamped case. The flexed portion of the base panel is, however, greater in this case. The initial failure event is the delamination of the outer ply in the region of the flange root to relieve the bending stresses in the side web. Progressive delamination occurs in subsequent plies, eventually forming a compliant hinge between the side web and the base panel. As this process continues, the compressive stress in the secondary bondline at the heel of the web disappears and a tensile stress builds up. When the tensile stress builds to a sufficient level, rapid failure occurs by a cleavage mechanism.

The study concluded that the centre clamp loading condition was the most severe and results in failure which most closely resembles what is observed in shock loaded panels. This can be seen in figure 7, which shows the representative load versus displacement curve for the two loading modes discussed. The centre clamp loading mode is the most severe case because of the large amount of stored elastic energy and the high tensile stress concentration at the heel of the side web. These factors initiated catastrophic failure at very low loads. It was also concluded that the bending of the side web has a profound effect on the stress distribution in the secondary bondline. This implies that



Stress distribution in stiffener flange region.

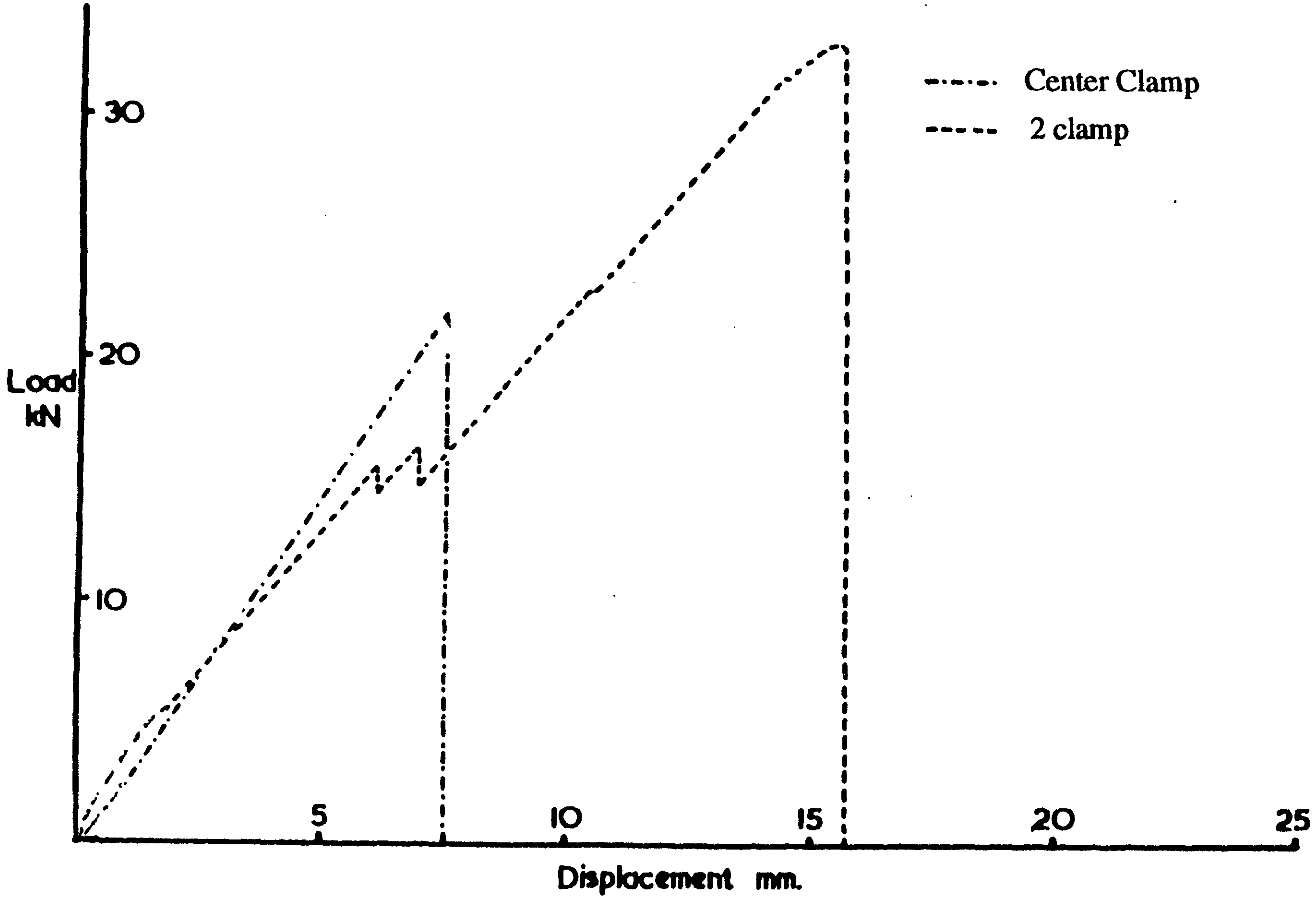
Figure 6A



Stress distribution in stiffener flange region.

A.K. Green and W.H. Bowyer, "The Testing and Analysis of Novel Top-Hat Stiffenener Fabrication Methods for Use in GRP Ships," *Composites Structures*, ed. I.H. Marshall (London: Applied Science Publishers, 1981), p.196.

Figure 6B



Top-hat pull-off tests, unreinforced specimens.

A.K. Green and W.H. Bowyer, "The Testing and Analysis of Novel Top-Hat Stiffenener Fabrication Methods for Use in GRP Ships," *Composites Structures*, ed. I.H. Marshall (London: Applied Science Publishers, 1981) , p.192.

Figure 7

increasing the compliance of the web/flange corner would reduce the bending stresses in the side web, thereby increasing the pull-off strength of the stiffener. Improvements may also be made by utilizing tough resins in the first few plies of the base panel in the most highly stressed regions.

Use of compliant resins:

Several studies were conducted in an attempt to utilize tough resins based on urethane acrylates as an alternative to the mechanical crack-stopping system and the other methods discussed previously.[6] The use of a fillet of resilient adhesive resin, with a strain to failure of up to 100%, in the critical region of the bondline (under the heel of the web) provided a frame/shell connection equivalent to or better than that provided by bolt-reinforcement. This was demonstrated both by static tests, as well as explosion tests on submerged panels and a large-scale floating hull-section. The details of the shock testing results were not given, and could not be found anywhere in the open literature.

In 1992, a study was conducted to improve stiffener to shell bonding by modification of the secondary bondline interface[7]. Several hull stiffener interfacial secondary bond arrangements were analyzed experimentally and compared. The bolted and screwed specimens are included for comparative purposes only.

a) Standard polyester resin only with woven roving glass (unmodified construction)

b) Standard polyester resin with titanium bolts

c) Standard polyester resin with stainless steel screws

d) Stitched cloth with polyester resin (internal flange)

e) Stitched cloth with acrylic matrix (internal flange)

f) Woven roving with chopped strand mat using polyester resin in bondline

g) Two-part elastomer modified acrylic system in bondline

h) Acrylic impregnated chopped strand mat in bondline

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[6] J. Bird and D. Bashford, "The Use of Flexible Resins to Improve Bond Connections in GRP Ship Construction," Proc. Composites-1988 World Conference on Composite Structures, Nice, France, June 1988.

[7] L.S. Norwood and C. Caulier, "Testing the Effectiveness of Tough Resin Systems for Improving Structural Performance of Joints in GRP Ships Composite Materials," Nautical Construction with Composite Materials: International Conference, Paris, ed. Peter Davies and Lional Lemoine (Paris: Ifremer,1992), pp. 246-255.

I) Urethane acrylate to wetout the woven roving reinforcement around the bond line

j) Standard polyester resin and lay-up, with a filled urethane acrylate fillet between the top-hat and the base panel at the heel of the web

k) Reinforced urethane-acrylate layers around the bondline plus a filled urethane-acrylate fillet at the heel of the web.

The slow pull-off testing of the above configurations was performed using center clamping, which was shown by Green and Bowyer to result in the most severe loading condition. The results of these tests are shown in figure 8-11. It should be noted that the results of items a-e are identical to those obtained by Green and Bowyer in 1981.

Several conclusions can be drawn from the results.

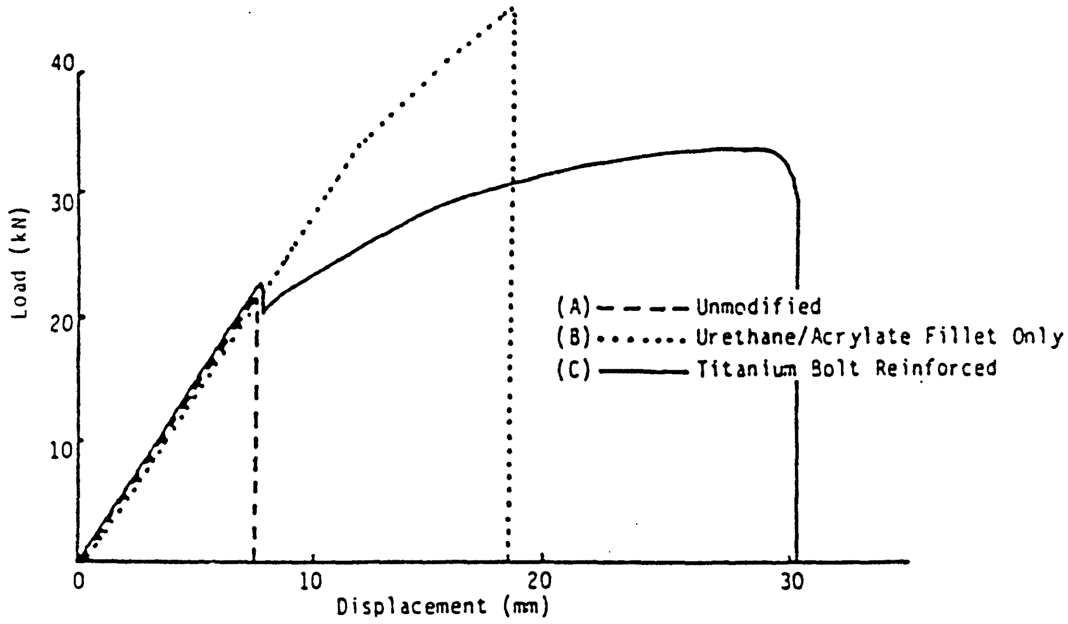
1. The work necessary to fail toughened resin top-hat sections is similar to that necessary to destroy bolted constructions, but the work input to initiate cracks is considerably higher in the toughened resin structure.

2. The use of stitched cloth-polyester resin lay-up in the secondary bondline region resulted in a load to failure 15% higher than the unmodified construction.

3. The use of chopped strand mat reinforcement with polyester resin at the bondline resulted in a load to failure 20% lower than the unmodified construction, and the use of acrylic impregnated chopped strand mat in the bondline resulted in inferior performance compared with the use of acrylic matrix alone.

4. The use of the tough two-part elastomer modified acrylic matrix increased the secondary bondline strength and the load for bondline crack initiation by allowing redistribution of stresses away from the regions of stress concentration. In addition, cracks starting in the bondline slowly, rather than catastrophically, opened up as the displacement increased.

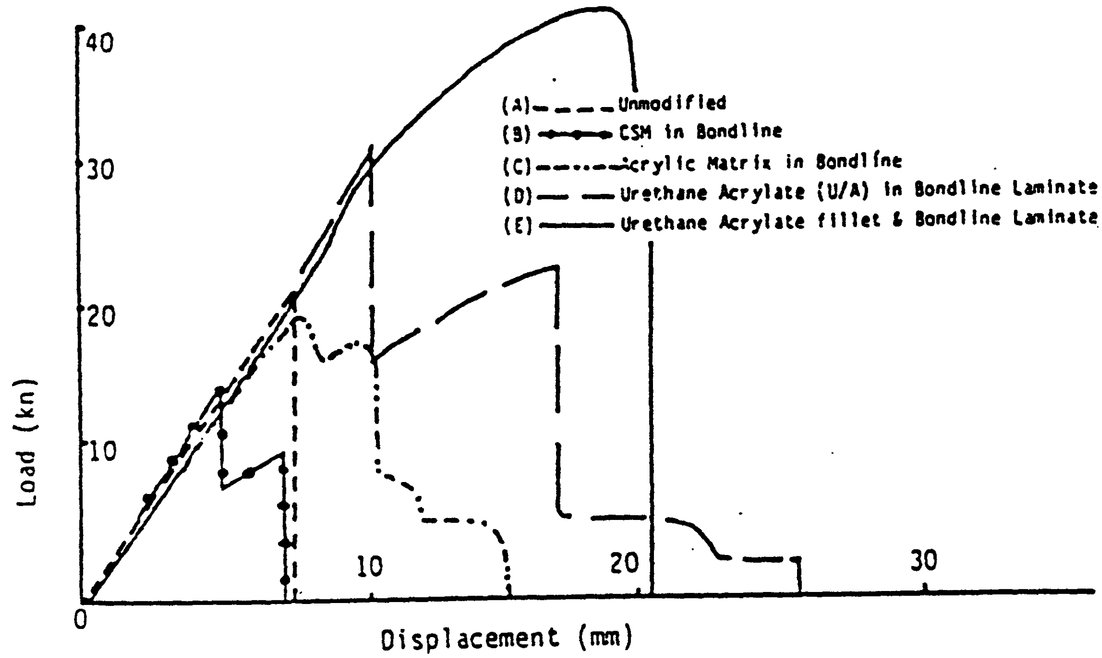
5. The use of a styrene crosslinked urethane acrylate resin capable of sustaining strains to failure in excess of 100%, in the bondline resulted in an increase in the linear load region of approximately 50% as compared to the unmodified structure. The use of urethane acrylate resulted in the need for three times the work input to initiate a crack as compared the standard unmodified structure. In addition, the web bending stresses are reduced as the tensile stress concentration under the web/flange heel is reduced. This is a result of the



Centre clamp top hat pull off tests, comparing standard bondlines with the adopted urethane acrylate system.

L.S. Norwood and C. Caulier, "Testing the Effectiveness of Tough Resin Systems for Improving Structural Performance of Joints in GRP Ship Composite Materials," *Nautical Contruction with Composite Materials: International Conference, Paris*, eds. Peter Davies and Lional Lemoine (Paris: Ifremer, 1992), p.255.

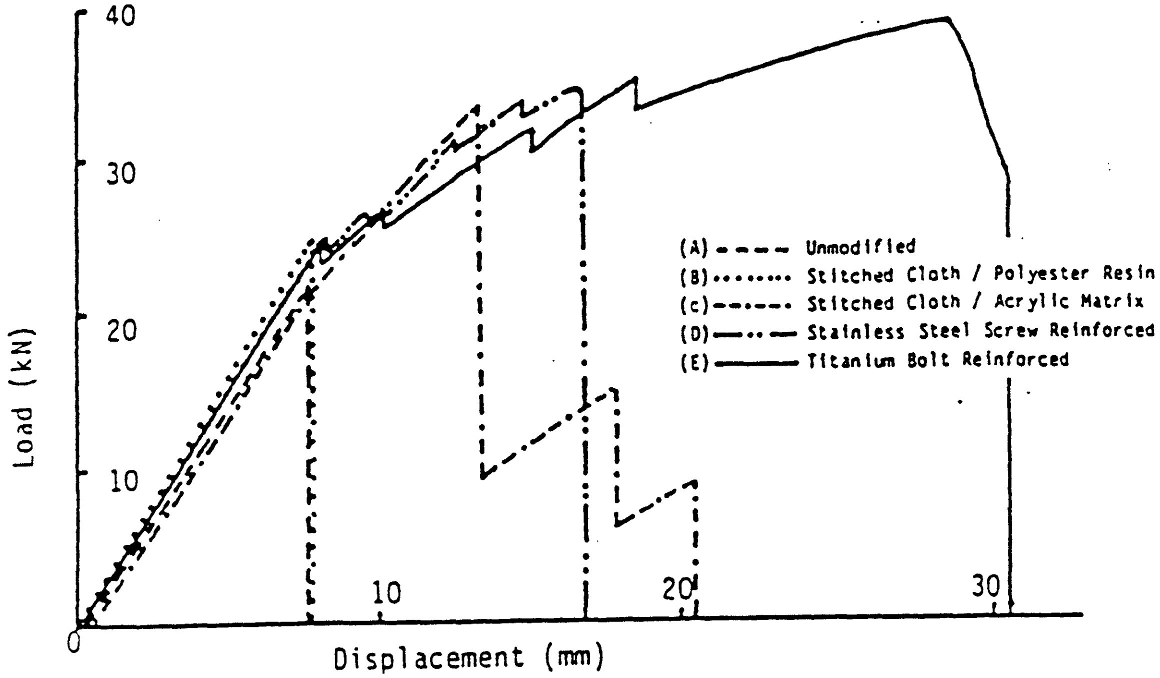
Figure 8



Centre clamp top hat pull off tests.

L.S. Norwood and C. Caulier, "Testing the Effectiveness of Tough Resin Systems for Improving Structural Performance of Joints in GRP Ship Composite Materials," *Nautical Contruction with Composite Materials: International Conference, Paris*, eds. Peter Davies and Lional Lemoine (Paris: Ifremer, 1992), p.255.

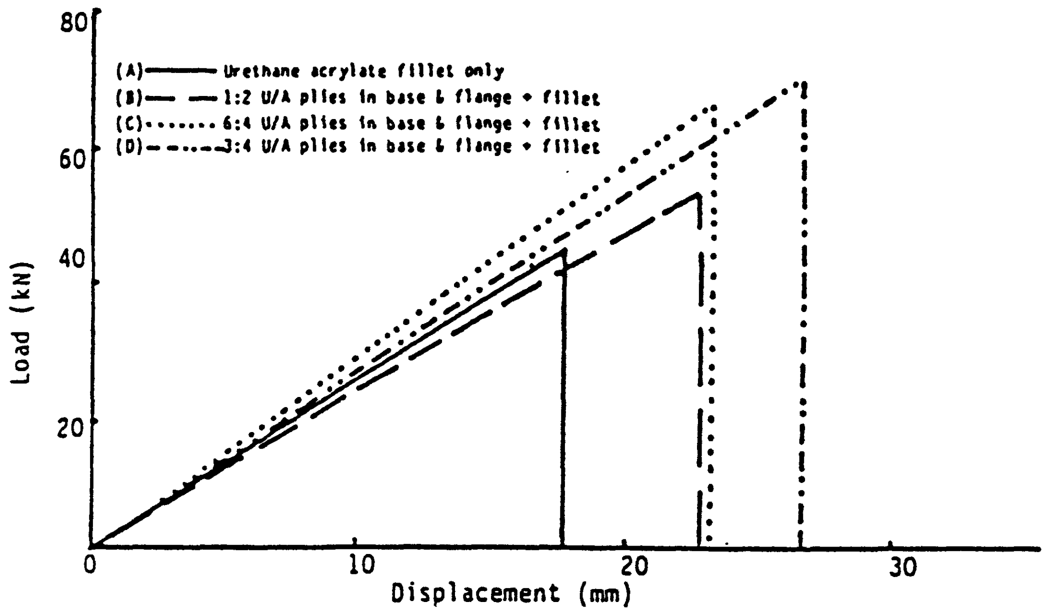
Figure 9



Centre clamp top hat pull off tests.

L.S. Norwood and C. Caulier, "Testing the Effectiveness of Tough Resin Systems for Improving Structural Performance of Joints in GRP Ship Composite Materials," *Nautical Contruction with Composite Materials: International Conference, Paris*, eds. Peter Davies and Lional Lemoine (Paris: Ifremer, 1992), p.255.

Figure 10



Centre clamp top hat pull off tests with different bondline conditions.

L.S. Norwood and C. Caulier, "Testing the Effectiveness of Tough Resin Systems for Improving Structural Performance of Joints in GRP Ship Composite Materials," *Nautical Contruction with Composite Materials: International Conference, Paris*, eds. Peter Davies and Lional Lemoine (Paris: Ifremer, 1992), p.255.

Figure 11

fact that the urethane acrylate resin at the secondary bondline has the capability of extending more than a rigid bond line, and can therefore store more elastic energy resulting in higher work input.

6. The presence of a flexible fillet as shown in figure 12, based on thixotropic urethane acrylate resin at the heel of the web significantly improved the performance of the bondline, but fillet design was not a major contributor to the overall performance of the bondline.

7. The use of a styrene crosslinked urethane acrylate in the last few layers of the main laminate, with woven roving glass, and the first few layers of the stiffener resulted in a significant increase in the linear load region as compared to the use of a urethane acrylate fillet alone.

Preformed stiffeners:

The current production practice involves forming the top-hat stiffeners over a foam former attached to the ship's hull. Often the hull shell is laminated many weeks prior to stiffener attachment. Using this production method, the flange thickness is the same as that of the stiffener web. The web thickness is determined by the stiffener's requirement to act as a beam with the web supporting the shear loading as the beam is placed in bending. This gives rise to an undesirable constraint that the web thickness drives the flange thickness. A parametric study was performed to assess the feasibility of using preformed top-hat stiffeners which would remove this constraint, and to determine the effect of several different design parameters.[8] [9] Figure 13 shows the typical configurations for commercial and naval applications, along with the proposed alternative configuration.

The preformed stiffeners are to be produced by a process known as SCRIMP (Seeman Composites Resin Injection Molding Process). It is essentially a modified form of the vacuum-assisted resin transfer molding (VARTM) process, which produces laminates of extremely high and consistent strength and quality. This process was conceived in the United States by Seemann Composites, Inc., and developed extensively for large scale

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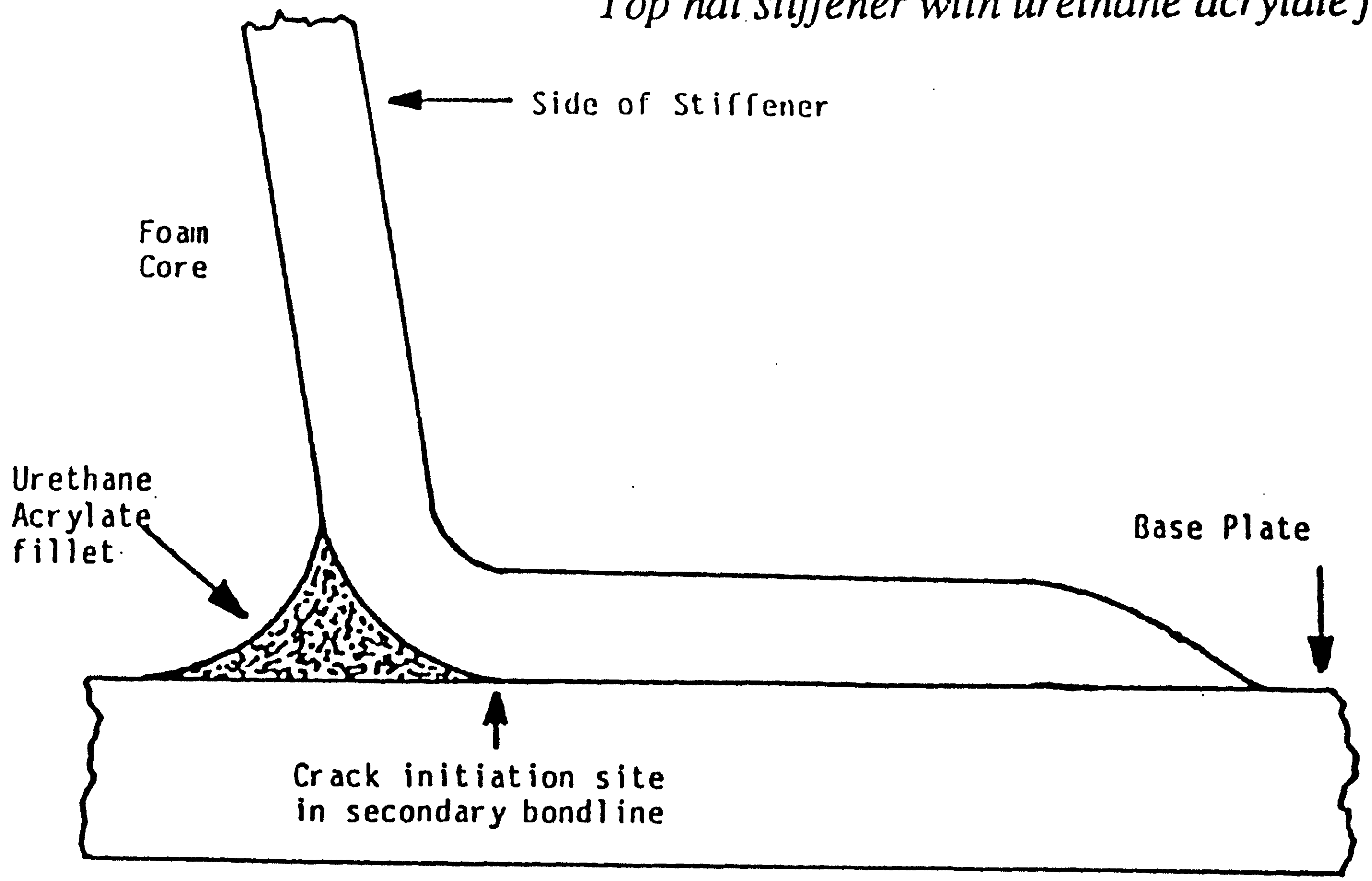
[8] A.R. Dodkins, R.A. Shenoi, and G.L. Hawkins, "Design of Joints and Attachments in FRP Ships' Structuresk," Marine Structures, 7 (1994), pp. 365-398.

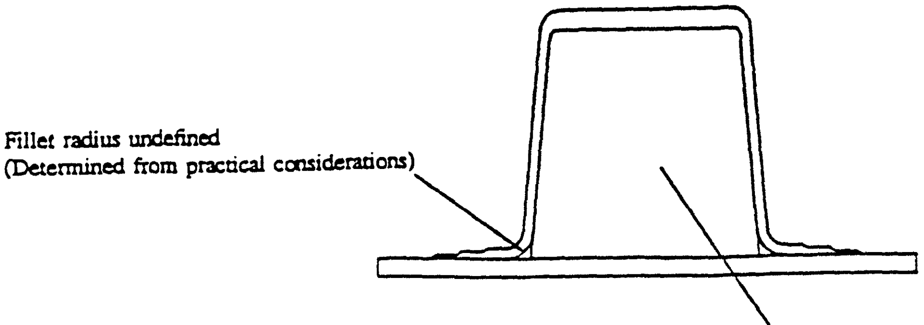
[9] R. A. Shenoi, and J. F. Wellicome, eds., Composite Materials in Marine Structures: Volume 2 Practical Considerations, (Cambridge: Cambridge University Press, 1993), pp. 77-82.

Top hat stiffener with urethane acrylate fillet

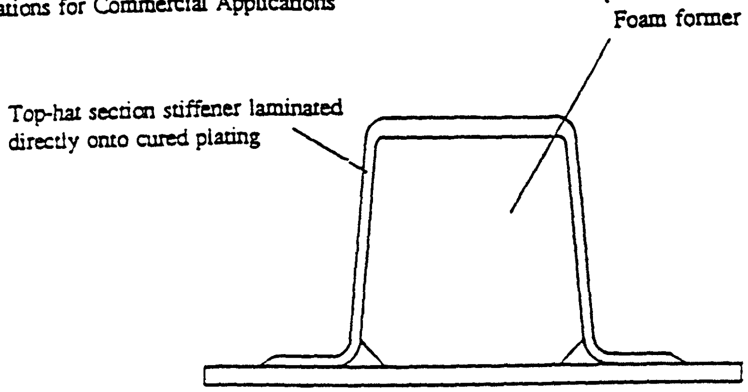


L.S. Norwood and C. Caulier, "Testing the Effectiveness of Tough Resin Systems for Improving Structural Performance of Joints in GRP Ship Composite Materials," *Nautical Contruction with Composite Materials: International Conference, Paris*, eds. Peter Davies and Lional Lemoine (Paris: Ifremer, 1992), p.254

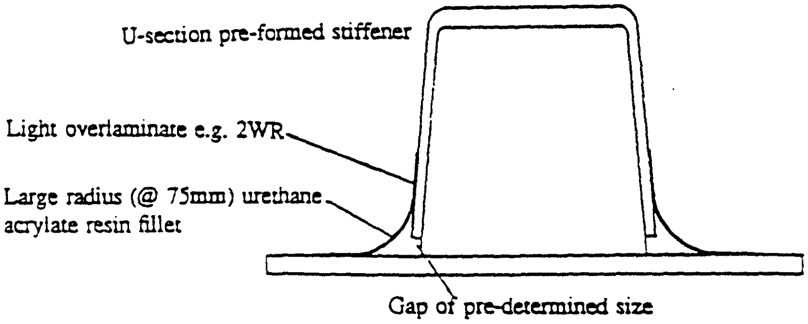
Figure 12



Typical Configurations for Commercial Applications



Typical Configurations for Naval Applications



"New" and Alternative Configurations.

A.R. Dodkins, R.A. Shenoi and G.L. Hawkins, "Design of Joints and Attachments in FRP Ships' Structures," *Marine Structures*, 7 (1994), p.367.

Figure 13

production by Vosper Thorncraft in the United Kingdom. It allows for the production of very large, high-quality FRP moldings in a cost effective manner.

This new production process has the following advantages over the traditional method:

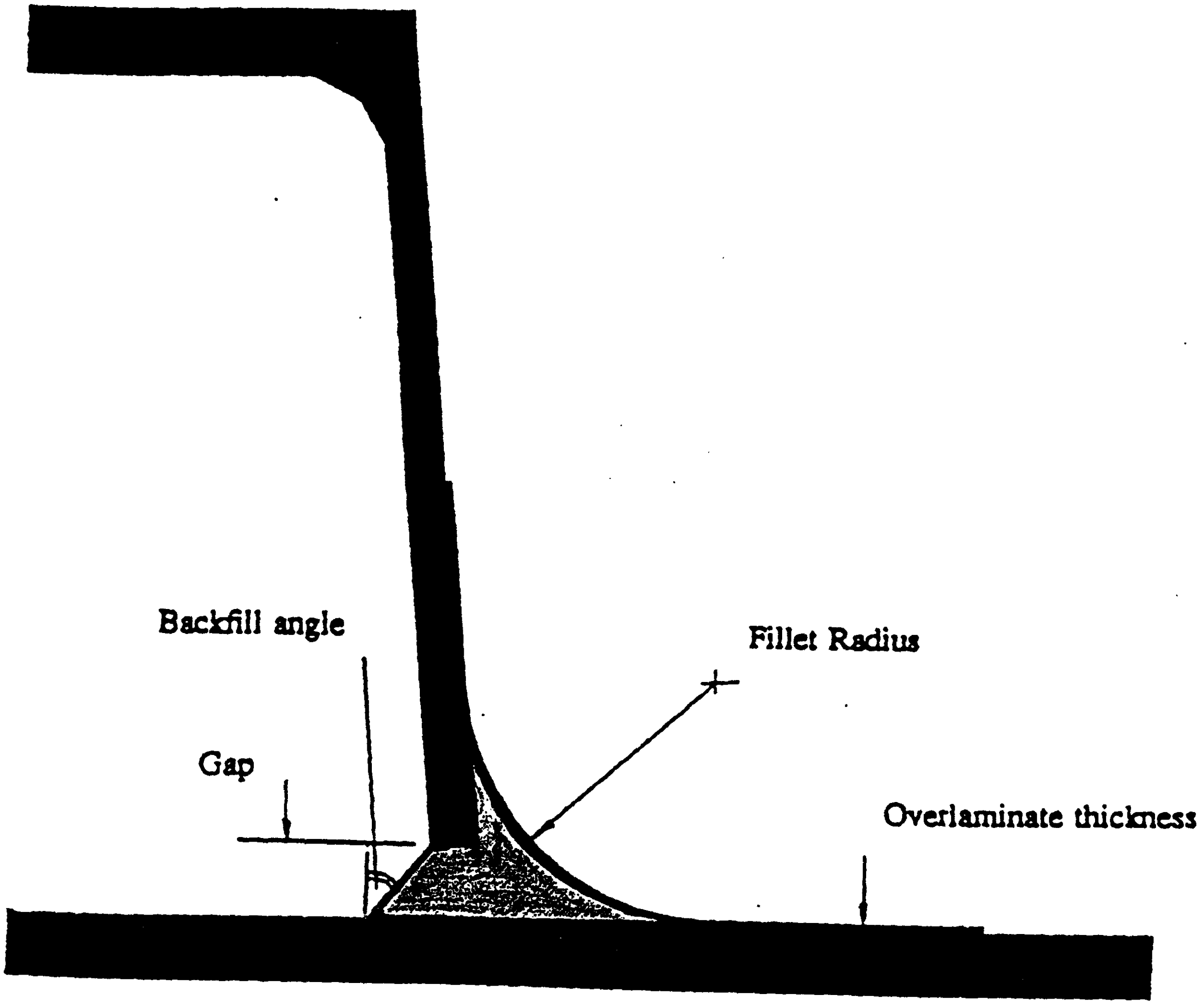
1. High fiber volume fraction and low void content laminates can be produced, resulting in stiffener laminate strength and modulus approaching twice that of the hand lay-up procedure, using the same constituent materials.
2. Producing stiffeners separately from plating means that most of the laminating work can take place in "ideal" workshop conditions rather than inside the hull where the access is more difficult.
3. It is estimated that, if this approach was applied throughout the ship, a saving of 30% in the weight of stiffeners could be achieved, including avoiding the need for foam formers which become redundant once the laminate cures, but nevertheless have to remain built-in the structure.
4. In this design, the compliance of the joint can be significantly improved over the traditional fabrication method. This will serve as a good follow-on to the work of Green and Bowyer who predicted that increasing the compliance of the web flange corner would significantly improve the performance of the joint.

In the design study of the new stiffener, the following design parameters were selected (see figure 14):

1. Radius of fillet (25-125 mm)
2. Backfill angle of fillet (0-45 degrees)
3. Fillet material (Urethane Acrylate only)
4. Thickness of overlaminate (1-12 laminates)
5. Overlaminate resin (Polyester resin only)
6. Gap between base panel and stiffener (10-50 mm)

Both the highest value of principal stress in the fillet and the through-thickness stress in the overlaminate were considered using both centre clamp and 2 clamp loading. The results are shown in figures 15-19.

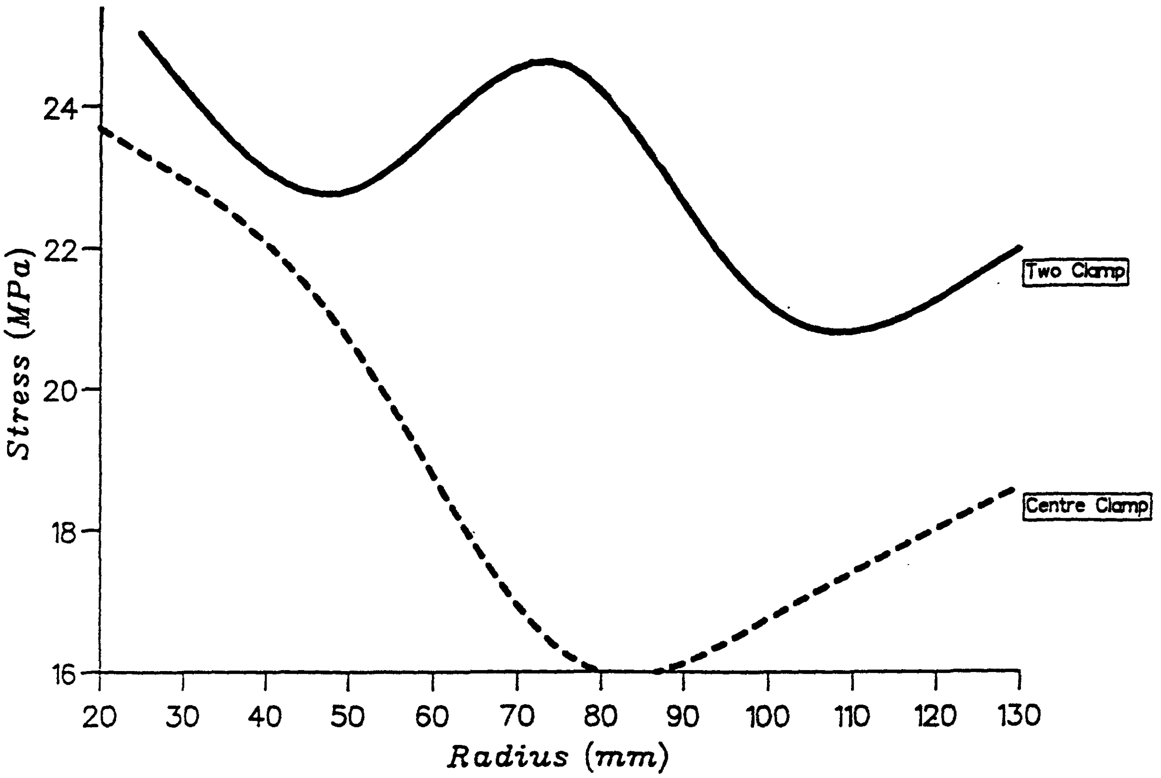
The stresses arising in the new stiffener design were compared with those obtained in a similar stiffener produced in the traditional way. In the new stiffener design under



Design variations considered for top-hat stiffeners.

A.R. Dodkins, R.A. Shenoi and G.L. Hawkins, "Design of Joints and Attachments in FRP Ships' Structures," *Marine Structures*, 7 (1994), p.387.

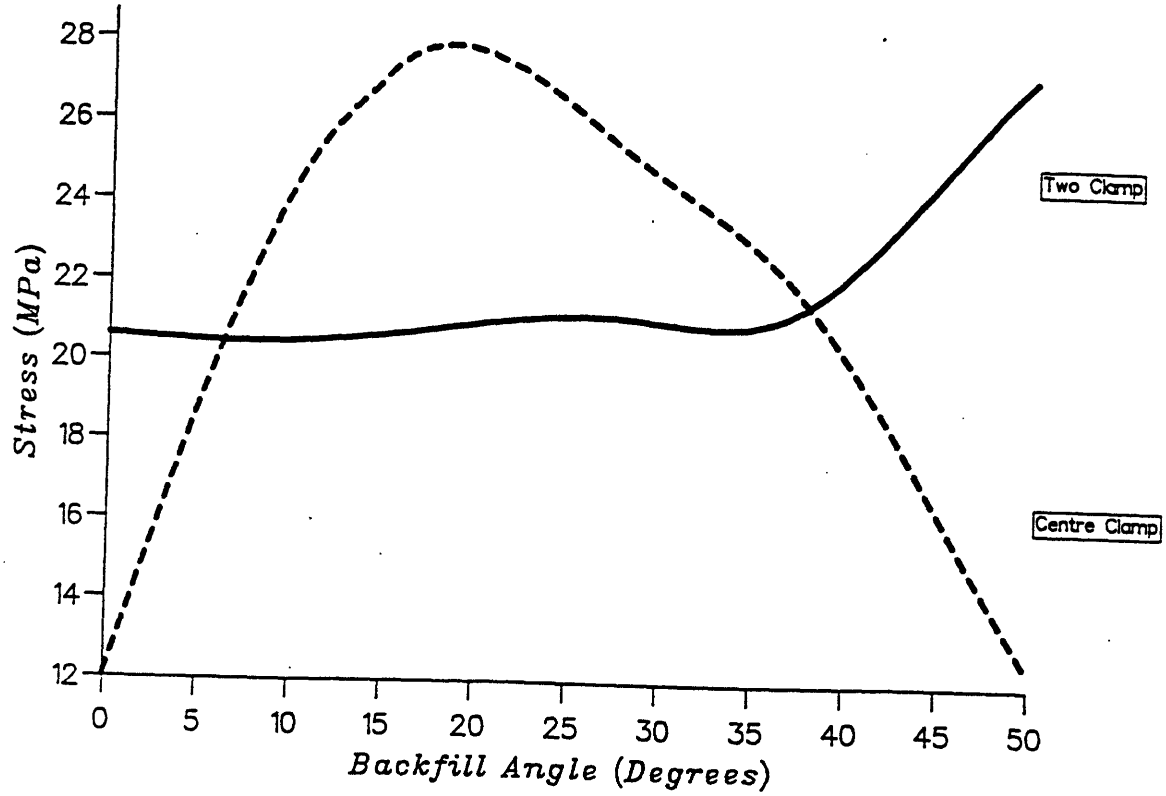
Figure 14



Effect of radius on stress in fillet.

R.A. Shenoi and J.F. Wellicome, eds., *Composite Materials in Marine Structures: Volume 2 Paractical Conciderations*, (Cambridge: Cambridge University Press, 1993), p. 80.

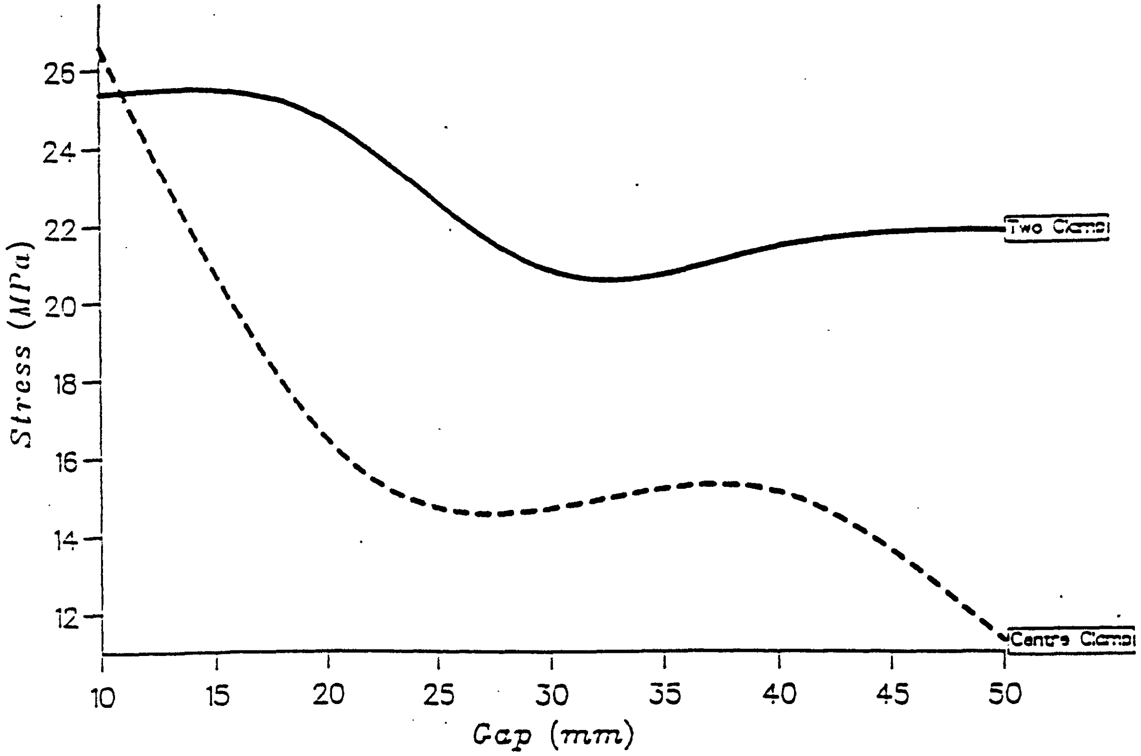
Figure 15



Effect of backfill angle on stress in fillet.

R.A. Shenoi and J.F. Wellicome, eds., *Composite Materials in Marine Structures: Volume 2 Paractical Conciderations,* (Cambridge: Cambridge University Press, 1993), p. 80.

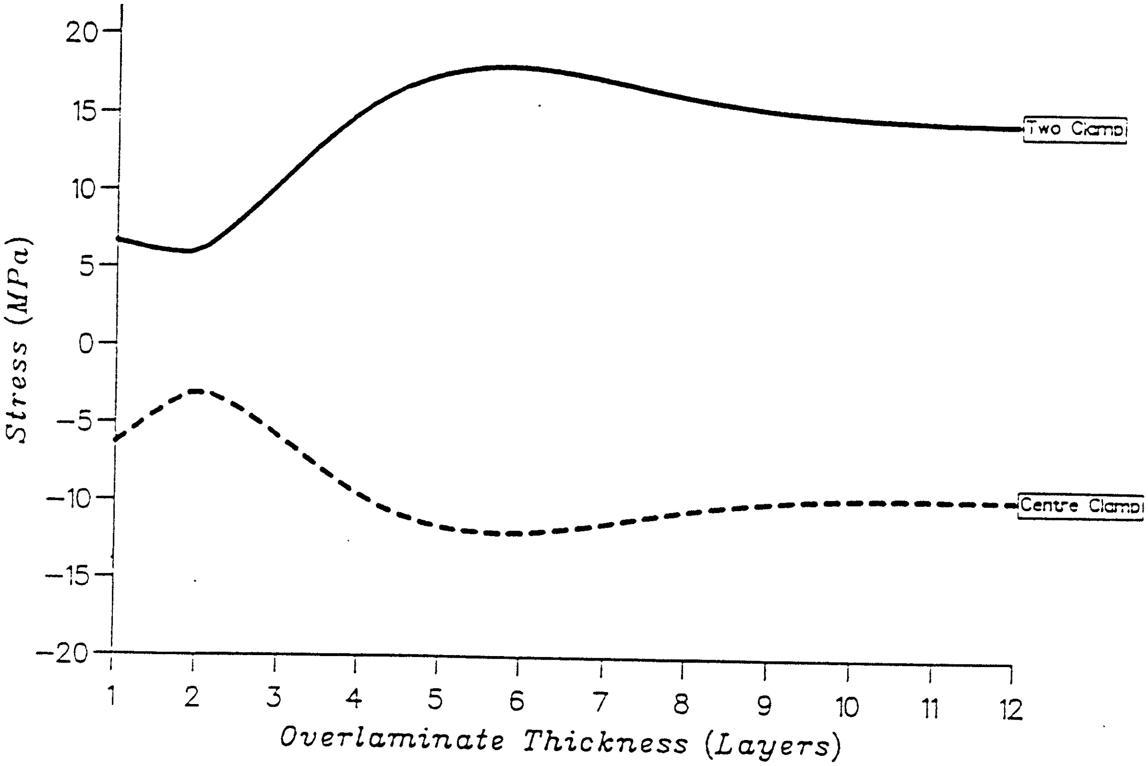
Figure 16



Effect of gap on stress in fillet.

R.A. Shenoi and J.F. Wellicome, eds., *Composite Materials in Marine Structures: Volume 2 Paractical Conciderations.* (Cambridge: Cambridge University Press, 1993), p. 81.

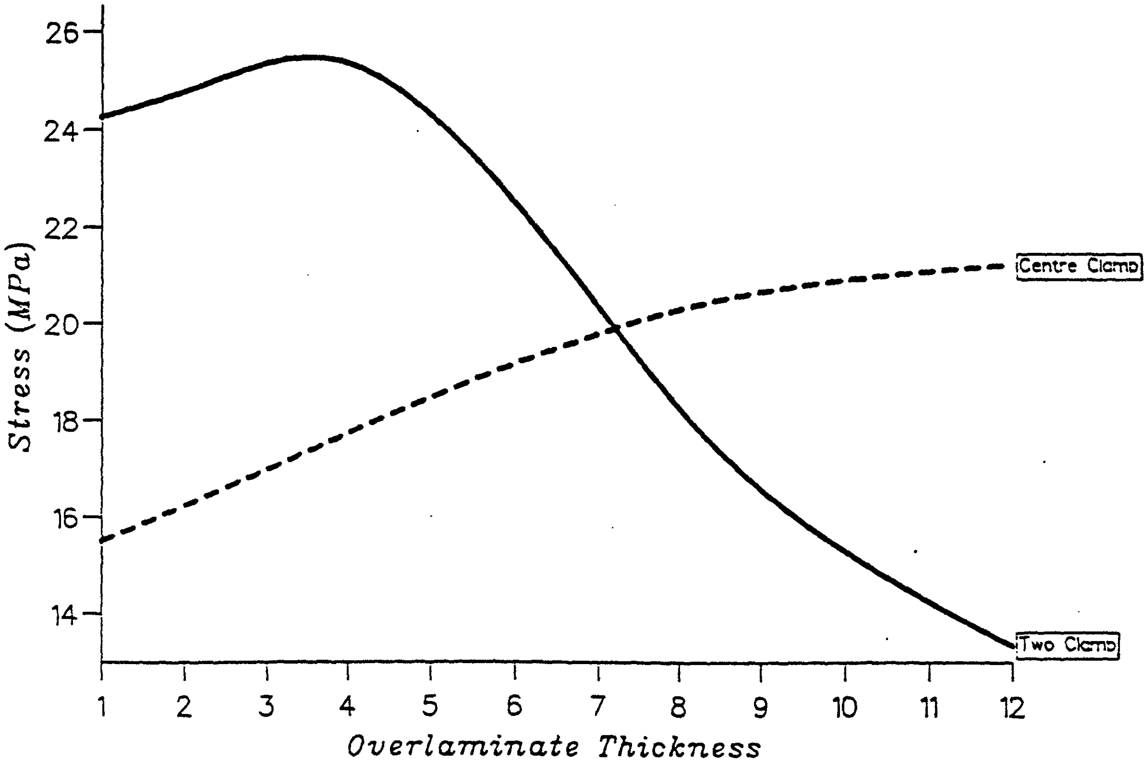
Figure 17



Effect of overlaminate thickness on through-thickness stress in overlaminate.

R.A. Shenoi and J.F. Wellicome, eds., *Composite Materials in Marine Structures: Volume 2 Paractical Conciderations*, (Cambridge: Cambridge University Press, 1993), p. 81.

Figure 18



Effect of overlaminate thickness on stress in fillet.

R.A. Shenoi and J.F. Wellicome, eds., *Composite Materials in Marine Structures: Volume 2 Paractical Conciderations*, (Cambridge: Cambridge University Press, 1993), p. 82.

Figure 19

centre clamp loading, the through-thickness stress concentration which was present in the traditional design is no longer in the region of the flange root, but are now in the web with reduced magnitude. Note that these through-thickness stresses are compressive and do not directly lead to any failure event. When the maximum principal stress in the fillet was compared with the those obtained by the analysis of the traditional design, it was found that the stresses were nearly the same. This would indicate that failure should occur at nearly the same load level for both the traditional and the new production method in this loading mode.

In the case of two-clamp loading, the through the thickness stresses in the new design are reduced over those in the traditional design, which will delay the failure in this mode until higher loads are applied. The load was redistributed to the fillet where the maximum value in the new design was about 50 percent higher than in the traditional design. This will likely lead to an increase in the applied load to failure over the traditional design, since the fillet can withstand a higher stress level than the overlaminate.

Conclusion:

There have been many advances in composite secondary bondline interfaces in the past 25 years. The evolution from bolted connections to adhesively bonded connections has come about as a result of the rapidly growing interest in special purpose polymers. With all the progress that has been made, there is a great deal which is still unknown about these materials. As an example, the behavior of polymers under conditions of dynamic loading is a complex subject which requires further study. Continued research in this and other related areas continue to improve the design of structures made of polymers and advanced composite materials.

Appendix B
Glass Transistion Temperature Calculation File

Input the observed values of Tg (Degrees C)

$i := 0..100$
$j := 0, 25..100$
$k := 25, 50..75$
$l := 0, 10..100$

Note: The index is the equal to the weight percentage of Crestomer 1080

$Tgobs_0 := 114.0 \quad Tgobs_{25} := 101.0 \quad Tgobs_{50} := 89.0 \quad Tgobs_{75} := 65.0 \quad Tgobs_{100} := 27$

$Tgobs2_0 := 113.0 \quad Tgobs2_{25} := 102.0 \quad Tgobs2_{50} := 90.0 \quad Tgobs2_{75} := 66.0 \quad Tgobs2_{100} := 27$

Define an expression for the glass transtion temperature of the copolymer:

$Tg_{1080} := Tgobs_{100} \qquad Tg_{8084} := Tgobs_0 \qquad W_{1080_i} := \frac{i}{100} \qquad W_{8084_i} := 1 - W_{1080_i}$

$$Tg_i := \frac{B \cdot W_{1080_i} + W_{8084_i}}{\left(\frac{B \cdot W_{1080_i}}{Tg_{1080}} + \frac{W_{8084_i}}{Tg_{8084}}\right)} \qquad Tg_{typ_i} := \frac{W_{1080_i} + W_{8084_i}}{\left(\frac{W_{1080_i}}{Tg_{1080}} + \frac{W_{8084_i}}{Tg_{8084}}\right)}$$

Calculate the percent difference between the observed Tg and the fitted value:

$$diffTg_k := \frac{Tg_k - Tgobs_k}{Tgobs_k} \cdot 100 \qquad diffTg_0 := 0 \qquad diffTg_{100} := 0 \qquad B \equiv 0.098$$

< B is the fitting factor >

j	Tg_j	$Tgobs_j$	$diffTg_j$
0	114	114	0
25	103.455	101	2.431
50	88.537	89	-0.52
75	65.816	65	1.256
100	27	27	0

Use WLF equation to predict Tg at higher frequency:

$f_{test} := 10 \cdot Hz$ < enter test frequency >

$n \equiv 1$ < enter desired increase in frequency over test frequency in decades >

$f_{des} := 10^n \cdot f_{test} \qquad f_{des} = 100 \cdot Hz$

Calculate the shift factor (aT) and constants for use in WLF equation:

$$a_T := \left(\frac{f_{test}}{f_{des}}\right) \qquad a_T = 0.1 \qquad C_2 := 51.6 \quad C_1 := 17.44$$

Calculate the change in Tg (Degrees C):

$$\Delta Tg := \frac{-\log(a_T) \cdot C_2}{C_1 + \log(a_T)} \qquad \Delta Tg = 3.139$$

< Note an increase in frequency results in a increase in Tg >

Appendix C
Instron Testing Data

Instron Test Data Log

The symbols used in this log are defined below.

[ID] The identification number given to each specimen. The coding is as follows, sample#-batch#-sector#.

[(I)] The Comp file location number of the specimen length.

COMP1 100% 8084/0% 1080
COMP2 75% 8084/25%1080
COMP3 50% 8084/50% 1080
COMP4 25% 8084/75% 1080
COMP5 0% 8084/100% 1080

[Comp.] The weight % composition of the specimen. D is Derakane 8084 and C is Crestomer 1080

[R.H.] Relative Humidity

[Test] The Load Rate that was used to test the specimen.

LR1 = 0.01 mm/sec.
LR2 = 0.10 mm/sec.
LR3 = 1.00 mm/sec.
LR4 = 2.50 mm/sec.
LR5 = 5.00 mm/sec.

[File#] Name given to the file containing the raw data. The format is IC (composition identification), LR (load rate identification) and T (trial #).

[Data] An X indicates files which were reduced and included in this report. The other data was excluded due to irregularities resulting from the fabrication process.

All Specimens:

Made at Boatex Inc., Natick, MA
Catalyst: 3 wt.% Lupersol Delta-X-9 MEKP (9% active oxygen)
Promoter: 0.6 wt.% Cobalt Napthenate (6% cobalt)

			Fabrication			Testing						
ID	(I)	Comp.	Date	Temp.^{0}F	R.H.	Date	Temp.^{0}F	R.H.	Test	Comments	File #	Data Inc.
11-3-A	1	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 1	Shattered, Bad Data		
11-3-B	2	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 1	Head Jerked, Bad Data		
11-3-C	3	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 1		IC1LR1T3	
11-3-D	4	100D/0C	3/6/95	62	28.0%					Glass Transition Testing (DSC)		
11-3-E	5	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 2		IC1LR2T1	
11-3-F	6	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 2		IC1LR2T2	
11-3-G	7	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 2		IC1LR2T3	
12-3-A	1	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR1		IC2LR1T1	
12-3-B	2	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR1		IC2LR1T2	
12-3-C	3	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR1		IC2LR1T3	
12-3-D	4	75D/25C	3/6/95	62	28.0%					Glass Transition Testing (DSC)		
12-3-E	5	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR2		IC2LR2T1	
12-3-F	6	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR2		IC2LR2T2	
12-3-G	7	75D/25C	3/6/95	62	28.0%					To Short To be Used		
13-3-A	1	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR1		IC3LR1T1	
13-3-B	2	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR1		IC3LR1T2	
13-3-C	3	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR1		IC3LR1T3	
13-3-D	4	50D/50C	3/6/95	62	28.0%					Glass Transition Testing (DSC)		
13-3-E	5	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR2	Compare to 26-4-C(24)	IC3LR2T1	
13-3-F	6	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR2	Compare to 26-4-C(25)	IC3LR2T2	
13-3-G	7	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR2		IC3LR2T3	
14-3-A	1	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR1		IC3LR1T1	X
14-3-B	2	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR1		IC4LR1T2	X
14-3-C	3	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR1		IC4LR1T3	X
14-3-D	4	25D/75C	3/6/95	62	28.0%					Glass Transition Testing (DSC)		
14-3-E	5	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR2		IC4LR2T1	X
14-3-F	6	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR2		IC4LR2T2	X
14-3-G	7	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR2		IC4LR2T3	X
15-3-A	1	0D/100C	3/6/95	62	28.0%	3/10/95	71.8	28.4%	LR1		IC5LR1T1	X
15-3-B	2	0D/100C	3/6/95	62	28.0%	3/10/95	71.8	28.4%	LR1		IC5LR1T2	X
15-3-C	3	0D/100C	3/6/95	62	28.0%	3/10/95	71.8	28.4%	LR2	Compare to 27-4-A (14)	IC5LR2T1	X
15-3-D	4	0D/100C	3/6/95	62	28.0%					Glass Transition Testing (DSC)		
15-3-E	5	0D/100C	3/6/95	62	28.0%	3/10/95	71.8	28.4%	LR2		IC5LR2T2	X
15-3-F	6	0D/100C	3/6/95	62	28.0%	3/10/95	71.8	28.4%	LR3		IC5LR3T1	X
15-3-G	7	0D/100C	3/6/95	62	28.0%	3/10/95	71.8	28.4%	LR3		IC5LR3T2	X

16-3-A	8	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR3		IC4LR3T1	X
16-3-B	9	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR3		IC4LR3T2	X
16-3-C	10	25D/75C	3/6/95	62	28.0%	3/10/95	74.1	17.7%	LR3		IC4LR3T3	X
16-3-D	11	25D/75C	3/6/95	62	28.0%					Retained		
16-3-E	12	25D/75C	3/6/95	62	28.0%					Retained		
16-3-F	13	25D/75C	3/6/95	62	28.0%					Retained		
16-3-G	14	25D/75C	3/6/95	62	28.0%					Retained		
17-3-A	15	25D/75C	3/6/95	62	28.0%					Retained		
17-3-B	16	25D/75C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR3		IC4LR3T4	X
17-3-C	17	25D/75C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR4		IC4LR4T1	X
17-3-D	18	25D/75C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR4		IC4LR4T2	X
17-3-E	19	25D/75C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR4		IC4LR4T3	X
17-3-F	20	25D/75C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR4		IC4LR4T4	X
17-3-G	21	25D/75C	3/6/95	62	28.0%					Retained		
18-3-A	8	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR2		IC3LR2T4	
18-3-B	9	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR3		IC3LR3T1	
18-3-C	10	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR3		IC3LR3T2	
18-3-D	11	50D/50C	3/6/95	62	28.0%	3/10/95	74.4	13.1%	LR3		IC3LR3T3	
18-3-E	12	50D/50C	3/6/95	62	28.0%	3/14/95	66.4	31.1%	LR1		IC3LR1T4	
18-3-F	13	50D/50C	3/6/95	62	28.0%	3/15/95	74.4	25.0%	LR4		IC3LR4T1	
18-3-G	14	50D/50C	3/6/95	62	28.0%	3/15/95	74.4	25.0%	LR4		IC3LR4T2	
19-3-A	15	50D/50C	3/6/95	62	28.0%	3/15/95	74.4	25.0%	LR4		IC3LR4T3	
19-3-B	16	50D/50C	3/6/95	62	28.0%	3/15/95	74.4	25.0%	LR4	Collection Error-No Data		
19-3-C	17	50D/50C	3/6/95	62	28.0%	3/15/95	74.4	25.0%	LR4		IC3LR4T4	
19-3-D	18	50D/50C	3/6/95	62	28.0%					Retained		
19-3-E	19	50D/50C	3/6/95	62	28.0%					Retained		
19-3-F	20	50D/50C	3/6/95	62	28.0%					Retained		
19-3-G	21	50D/50C	3/6/95	62	28.0%					Retained		
20-3-A	8	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR2		IC2LR2T3	
20-3-B	9	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR3		IC2LR3T1	
20-3-C	10	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR3		IC2LR3T2	
20-3-D	11	75D/25C	3/6/95	62	28.0%					Crack-Not Tested		
20-3-E	12	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR3		IC2LR3T3	
20-3-F	13	75D/25C	3/6/95	62	28.0%	3/10/95	70.6	16.4%	LR3		IC2LR3T4	
20-3-G	14	75D/25C	3/6/95	62	28.0%					Crack-Not Tested		
21-3-A	15	75D/25C	3/6/95	62	28.0%	3/15/95	74.2	24.6%	LR4		IC2LR4T1	
21-3-B	16	75D/25C	3/6/95	62	28.0%	3/14/95	74.8	26.3%	LR1		IC2LR1T4	

21-3-C	17	75D/25C	3/6/95	62	28.0%	3/15/95	74.2	24.6%	LR4		IC2LR4T2
21-3-D	18	75D/25C	3/6/95	62	28.0%	3/15/95	74.2	24.6%	LR4		IC2LR4T3
21-3-E	19	75D/25C	3/6/95	62	28.0%					Retained	
21-3-F	20	75D/25C	3/6/95	62	28.0%					Retained	
21-3-G	21	75D/25C	3/6/95	62	28.0%					Retained	
22-3-A	8	100D/0C	3/6/95	62	28.0%	3/14/95	66.3	31.1%	LR 1	Replace 11-3-B	IC1LR1T6
22-3-B	9	100D/0C	3/6/95	62	28.0%					Crack-Not Tested	
22-3-C	10	100D/0C	3/6/95	62	28.0%					Crack-Not Tested	
22-3-D	11	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 3		IC1LR3T1
22-3-E	12	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 3		IC1LR3T2
22-3-F	13	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 3		IC1LR3T3
22-3-G	14	100D/0C	3/6/95	62	28.0%	3/14/95	66.3	31.1%	LR 1	Replace 11-3-A	IC1LR1T5
23-3-A	15	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 1		IC1LR1T4
23-3-B	16	100D/0C	3/6/95	62	28.0%	3/10/95	69.7	20.9%	LR 2		IC1LR2T4
23-3-C	17	100D/0C	3/6/95	62	28.0%	3/15/95	74.2	24.6%	LR5	Shattered, data lost	
23-3-D	18	100D/0C	3/6/95	62	28.0%	3/15/95	74.2	24.6%	LR5		IC1LR4T1
23-3-E	19	100D/0C	3/6/95	62	28.0%	3/15/95	74.2	24.6%	LR5		IC1LR5T2
23-3-F	20	100D/0C	3/6/95	62	28.0%					Retained	
23-3-G	21	100D/0C	3/6/95	62	28.0%					Retained	
24-3-A	8	0D/100C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR1		IC5LR1T3
24-3-B	9	0D/100C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR1		IC5LR1T4
24-3-C	10	0D/100C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR2		IC5LR2T4
24-3-D	11	0D/100C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR2		IC5LR2T5
24-3-E		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
24-3-F		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
24-3-G		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
25-3-A	12	0D/100C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR3		IC5LR3T3
25-3-B	13	0D/100C	3/6/95	62	28.0%	3/18/95	72.4	26.9%	LR3		IC5LR3T4
25-3-C		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
25-3-D		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
25-3-E		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
25-3-F		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
25-3-G		0D/100C	3/6/95	62	28.0%					Poor Specimen-Not Tested	
26-4-A	22	100D/0C	3/16/95	64	30.4%	3/17/95	70.9	27.9%	LR5		IC1LR5T1
26-4-B	23	100D/0C	3/16/95	64	30.4%	3/17/95	70.9	27.9%	LR5		IC1LR5T2
26-4-C	24	100D/0C	3/16/95	64	30.4%	3/17/95	70.9	27.9%	LR2	To Confirm Batch Continuity	IC1LR2T5
26-4-D	25	100D/0C	3/16/95	64	30.4%	3/17/95	70.9	27.9%	LR2	To Confirm Batch Continuity	IC1LR2T6

26-4-E	26	100D/0C	3/16/95	64	30.4%					Retained		
26-4-F	27	100D/0C	3/16/95	64	30.4%					Retained		
26-4-G	28	100D/0C	3/16/95	64	30.4%					Retained		
27-4-A	14	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR2	To Confirm Batch Continuity	IC5LR2T1	
27-4-B	15	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR4		IC5LR4T1	
27-4-C	16	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR4		IC5LR4T5	X
27-4-D	17	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR4		IC5LR4T6	X
27-4-E	18	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR5		IC5LR5T1	
27-4-F	19	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR5		IC5LR5T2	
27-4-G	20	0D/100C	3/16/95	64	30.4%	3/18/95	72.4	26.9%	LR5		IC5LR5T3	
28-5-A	1	50D/50C	3/30/95	60.5	27.3%	4/2/95	70.5	17.6%	LR1		IC3LR1T5	X
28-5-B	2	50D/50C	3/30/95	60.5	27.3%	4/2/95	70.5	17.6%	LR1		IC3LR1T6	X
28-5-C	3	50D/50C	3/30/95	60.5	27.3%	4/2/95	70.5	17.6%	LR1		IC3LR1T7	X
28-5-D	4	50D/50C	3/30/95	60.5	27.3%					Retained		
28-5-E	5	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC3LR2T6	X
28-5-F	6	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC3LR2T7	X
28-5-G	7	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC3LR2T8	X
29-5-A	8	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR3		IC3LR3T4	X
29-5-B	9	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR3		IC3LR3T5	X
29-5-C	10	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR3		IC3LR3T6	X
29-5-D	11	50D/50C	3/30/95	60.5	27.3%					Retained		
29-5-E	12	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR4		IC3LR4T5	X
29-5-F	13	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR4		IC3LR4T6	X
29-5-G	14	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR4		IC3LR4T7	X
30-5-A	15	50D/50C	3/30/95	60.5	27.3%					Retained		
30-5-B	16	50D/50C	3/30/95	60.5	27.3%					Retained		
30-5-C	17	50D/50C	3/30/95	60.5	27.3%					Retained		
30-5-D	18	50D/50C	3/30/95	60.5	27.3%					Retained		
30-5-E	19	50D/50C	3/30/95	60.5	27.3%					Retained		
30-5-F	20	50D/50C	3/30/95	60.5	27.3%					Retained		
30-5-G	21	50D/50C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC3LR2T5	X
31-5-A	1	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR1		IC2LR1T5	X
31-5-B	2	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR1		IC2LR1T6	X
31-5-C	3	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR1		IC2LR1T7	X
31-5-D	4	75D/25C	3/30/95	60.5	27.3%					Cracked-Not Tested		
31-5-E	5	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC2LR2T5	X
31-5-F	6	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC2LR2T4	X

31-5-G	7	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR2		IC2LR2T6	X
32-5-A	8	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR3		IC2LR3T5	X
32-5-B	9	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR3		IC2LR3T6	X
32-5-C	10	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR3		IC2LR3T7	X
32-5-D	11	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR4		IC2LR4T4	X
32-5-E	12	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR4		IC2LR4T5	X
32-5-F	13	75D/25C	3/30/95	60.5	27.3%	4/2/95	69.5	17.6%	LR4	Retained	IC2LR4T6	X
32-5-G	14	75D/25C	3/30/95	60.5	27.3%					Retained		
33-5-A	15	75D/25C	3/30/95	60.5	27.3%					Retained		
33-5-B	16	75D/25C	3/30/95	60.5	27.3%					Cracked-Not Tested		
33-5-C	17	75D/25C	3/30/95	60.5	27.3%					Retained		
33-5-D	18	75D/25C	3/30/95	60.5	27.3%					Retained		
33-5-E	19	75D/25C	3/30/95	60.5	27.3%					Retained		
33-5-F	20	75D/25C	3/30/95	60.5	27.3%					Retained		
33-5-G	21	75D/25C	3/30/95	60.5	27.3%					Retained		
34-4-A	1	100D/0C	3/16/95	64	30.4%							
34-4-B	2	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR1		IC1LR1T7	X
34-4-C	3	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR1		IC1LR1T8	X
34-4-D	4	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR1		IC1LR1T9	X
34-4-E	5	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR3		IC1LR3T4	X
34-4-F	6	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR3		IC1LR3T5	X
34-4-G	7	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR3		IC1LR3T6	X
35-4-A	8	100D/0C	3/16/95	64	30.4%					Retained		
35-4-B	9	100D/0C	3/16/95	64	30.4%					Retained		
35-4-C	10	100D/0C	3/16/95	64	30.4%					Retained		
35-4-D	11	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR4		IC1LR4T3	X
35-4-E	12	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR4		IC1LR4T4	X
35-4-F	13	100D/0C	3/16/95	64	30.4%	4/2/95	70.5	14.2%	LR4		IC1LR4T5	X
35-4-G	14	100D/0C	3/16/95	64	30.4%					Retained		

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1 .. 4$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP1B})$ $\quad$ $\mathrm{SPEC}_A := 3$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP1A})$ $\quad$ $\mathrm{SPEC}_B := 24$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP1B})$ $\quad$ $\mathrm{SPEC}_C := 5$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP1B})$ $\quad$ $\mathrm{SPEC}_D := 11$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1\rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$ $\quad$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2\rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 191.38 \\ 192.854 \\ 193.346 \\ 189.911 \end{bmatrix} \cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC1LR1T8})$

$B := \mathrm{READPRN}(\mathrm{IC1LR2T5})$

$C := \mathrm{READPRN}(\mathrm{IC1LR3T4})$

$D := \mathrm{READPRN}(\mathrm{IC1LR4T3})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000 \cdot \mathrm{newton}$

$\mathrm{load}_A := A^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_A := A^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\mathrm{load}_D := D^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_D := D^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_B := B^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_B := B^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_C := C^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_C := C^{\langle 2 \rangle} \cdot \mathrm{mm}$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$ $d := 1 .. \mathrm{rows}(D)$

$b := 1 .. \mathrm{rows}(B)$

$c := 1 .. \mathrm{rows}(C)$

Calculate true stress and true strain (assume incompressibility): $\mathrm{MPa} := 1 \cdot 10^{6} \cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a}$ $A_{iA_a} := \dfrac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}}$ $L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b}$ $A_{iB_b} := \dfrac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$

$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c}$ $A_{iC_c} := \dfrac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}}$ $L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d}$ $A_{iD_d} := \dfrac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$

Calculate true stress (MPa) and true strain:

$\sigma_{trueA_a} := \dfrac{\mathrm{load}_{A_a}}{A_{iA_a}}$ $\sigma_{trueB_b} := \dfrac{\mathrm{load}_{B_b}}{A_{iB_b}}$ $\sigma_{trueC_c} := \dfrac{\mathrm{load}_{C_c}}{A_{iC_c}}$ $\sigma_{trueD_d} := \dfrac{\mathrm{load}_{D_d}}{A_{iD_d}}$

$\varepsilon_{trueA_a} := -\ln\left(\dfrac{L_{iA_a}}{L_{o_1}}\right)$ $\varepsilon_{trueB_b} := -\ln\left(\dfrac{L_{iB_b}}{L_{o_2}}\right)$ $\varepsilon_{trueC_c} := -\ln\left(\dfrac{L_{iC_c}}{L_{o_3}}\right)$ $\varepsilon_{trueD_d} := -\ln\left(\dfrac{L_{iD_d}}{L_{o_4}}\right)$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

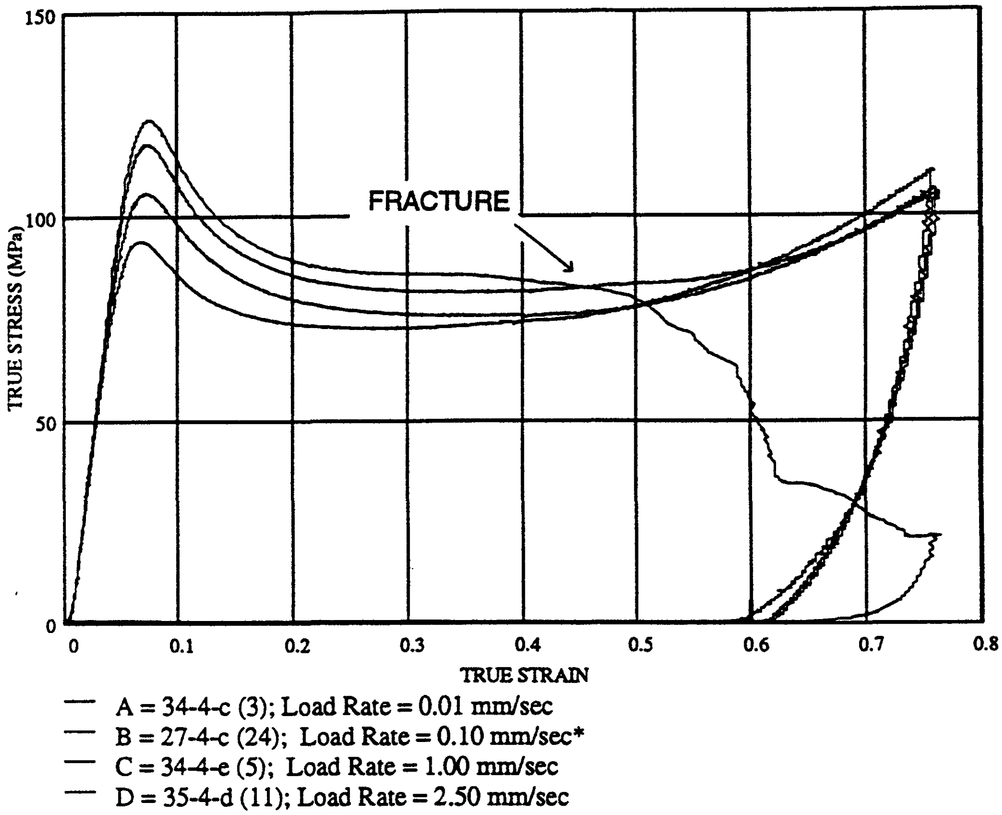
$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

Generate a plot of true stress (MPa) versus true strain for 100% 8084 / 0% 1080 at various load rates:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.759$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 110.58 \cdot \mathrm{MPa}$ $\sigma_{yA} = 93.767 \cdot \mathrm{MPa}$ $\varepsilon_{yA} = 0.065$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.761$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 106.084 \cdot \mathrm{MPa}$ $\sigma_{yB} = 105.442 \cdot \mathrm{MPa}$ $\varepsilon_{yB} = 0.071$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 0.763$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 117.63 \cdot \mathrm{MPa}$ $\sigma_{yC} = 117.63 \cdot \mathrm{MPa}$ $\varepsilon_{yC} = 0.073$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 0.763$ $\max\left(\sigma_{\mathrm{trueD}}\right) = 123.683 \cdot \mathrm{MPa}$ $\sigma_{yD} = 123.683 \cdot \mathrm{MPa}$ $\varepsilon_{yD} = 0.073$

Select specimens to be used: $z := 1..8$ ORIGIN $:= 1$

$\mathrm{DIM} := \mathrm{READPRN}(\mathrm{COMP1B})$

$$\mathrm{SPEC} := \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \\ 15 \\ 16 \\ 1 \\ 1 \end{bmatrix} \qquad L_{0_z} := \left(\mathrm{DIM}^{<1>}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm} \qquad D_{0_z} := \left(\mathrm{DIM}^{<2>}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm}$$

Show initial length (mm): Show initial diameter (mm):

$$L_0 = \begin{bmatrix} 15.7 \\ 15.8 \\ 15.92 \\ 15.37 \\ 15.54 \\ 15.67 \\ 15.37 \\ 15.37 \end{bmatrix} \cdot \mathrm{mm} \qquad D_0 = \begin{bmatrix} 15.61 \\ 15.61 \\ 15.68 \\ 15.73 \\ 15.67 \\ 15.63 \\ 15.73 \\ 15.73 \end{bmatrix} \cdot \mathrm{mm}$$

Length/Diameter ratio(LDR): Calculate the initial cross sectional area (mm^2):

$$\mathrm{LDR}_z := \frac{L_{0_z}}{D_{0_z}} \qquad A_{\mathrm{cso}_z} := \frac{\pi}{4} \cdot \left(D_{0_z}\right)^2$$

$$\mathrm{LDR} = \begin{bmatrix} 1.006 \\ 1.012 \\ 1.015 \\ 0.977 \\ 0.992 \\ 1.003 \\ 0.977 \\ 0.977 \end{bmatrix} \qquad A_{\mathrm{cso}} = \begin{bmatrix} 191.38 \\ 191.38 \\ 193.1 \\ 194.333 \\ 192.854 \\ 191.87 \\ 194.333 \\ 194.333 \end{bmatrix} \cdot \mathrm{mm}^2$$

Input maximum true strain (compressive values negative): Input loading head actuator speed (mm/sec):

$$\varepsilon_{\max} := \begin{bmatrix} -0.75 \\ -0.75 \\ -0.75 \\ 1 \\ -0.75 \\ -0.75 \\ 1 \\ 1 \end{bmatrix} \qquad \mathrm{Load}_{\mathrm{rate}} := \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 1 \\ 0.10 \\ 0.10 \\ 1 \\ 1 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{0_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 7.416 \\ 7.463 \\ 7.52 \\ 41.78 \\ 7.341 \\ 7.402 \\ 41.78 \\ 41.78 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the machine stroke (mm):

$$\mathrm{Stroke}_z := L_{f_z} - L_{0_z}$$

$$\mathrm{Stroke} = \begin{bmatrix} -8.284 \\ -8.337 \\ -8.4 \\ 26.41 \\ -8.199 \\ -8.268 \\ 26.41 \\ 26.41 \end{bmatrix} \cdot \mathrm{mm}$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $\mathrm{Stroke}_{tot_z} := 2 \cdot \mathrm{Stroke}_z$

$$\mathrm{Stroke}_{tot} = \begin{bmatrix} -16.568 \\ -16.673 \\ -16.8 \\ 52.82 \\ -16.399 \\ -16.536 \\ 52.82 \\ 52.82 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the total time required for test:

$$\mathrm{time}_{tot_z} := \left| \frac{\mathrm{Stroke}_{tot_z}}{\mathrm{Load}_{rate_z}} \right|$$

$$\mathrm{time}_{tot} = \begin{bmatrix} 1656.769 \\ 1667.322 \\ 1679.985 \\ 52.82 \\ 163.988 \\ 165.36 \\ 52.82 \\ 52.82 \end{bmatrix} \cdot \mathrm{sec} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 27.613 \\ 27.789 \\ 28 \\ 0.88 \\ 2.733 \\ 2.756 \\ 0.88 \\ 0.88 \end{bmatrix} \cdot \mathrm{min} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 0.46 \\ 0.463 \\ 0.467 \\ 0.015 \\ 0.046 \\ 0.046 \\ 0.015 \\ 0.015 \end{bmatrix} \cdot \mathrm{hr}$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$\mathrm{Freq}_{data} := \begin{bmatrix} 5 \\ 5 \\ 5 \\ 1 \\ 50 \\ 50 \\ 1 \\ 1 \end{bmatrix} \cdot \mathrm{Hz}$$

Calculate approximate number of data points:

$$\mathrm{NBR}_{data_z} := \mathrm{Freq}_{data_z} \cdot \mathrm{time}_{tot_z}$$

$$\mathrm{NBR}_{data} = \begin{bmatrix} 8283.845 \\ 8336.608 \\ 8399.924 \\ 52.82 \\ 8199.424 \\ 8268.016 \\ 52.82 \\ 52.82 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC1LR1T7})$ $E := \mathrm{READPRN}(\mathrm{IC1LR2T5})$
$B := \mathrm{READPRN}(\mathrm{IC1LR1T8})$ $F := \mathrm{READPRN}(\mathrm{IC1LR2T6})$
$C := \mathrm{READPRN}(\mathrm{IC1LR1T9})$ $G := \mathrm{READPRN}(\mathrm{IC0LR1T1})$
$D := \mathrm{READPRN}(\mathrm{IC0LR1T1})$ $H := \mathrm{READPRN}(\mathrm{IC0LR1T1})$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000 \cdot \mathrm{newton}$

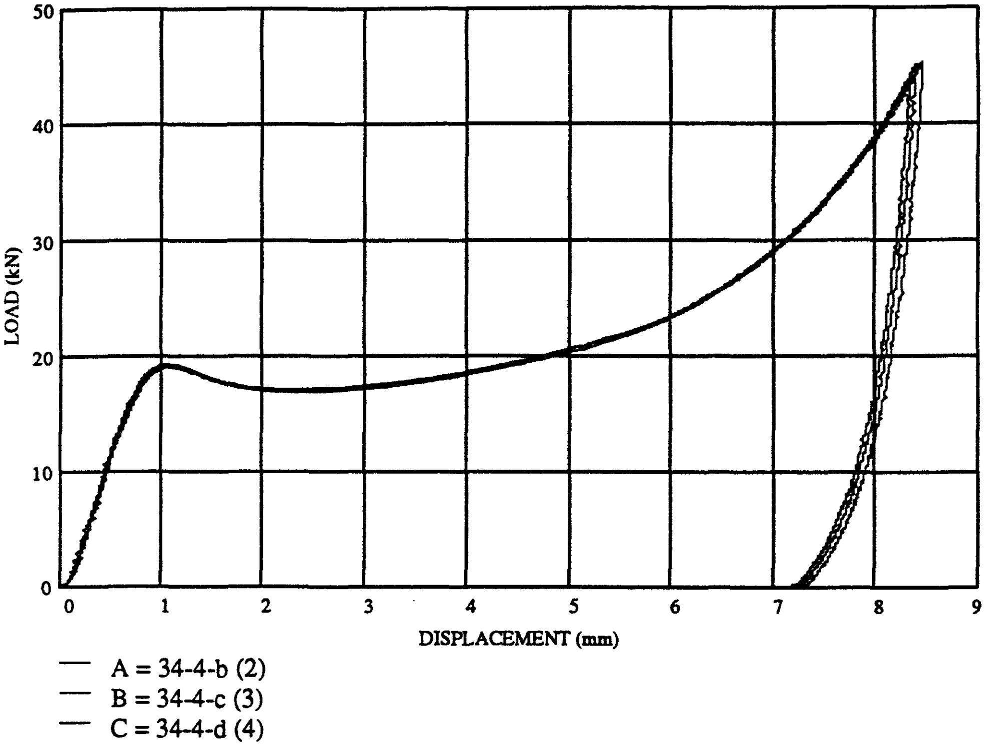
$\mathrm{load}_A := A^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_A := A^{\langle 2\rangle} \cdot \mathrm{mm}$ $\mathrm{load}_E := E^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_E := E^{\langle 2\rangle} \cdot \mathrm{mm}$
$\mathrm{load}_B := B^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_B := B^{\langle 2\rangle} \cdot \mathrm{mm}$ $\mathrm{load}_F := F^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_F := F^{\langle 2\rangle} \cdot \mathrm{mm}$
$\mathrm{load}_C := C^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_C := C^{\langle 2\rangle} \cdot \mathrm{mm}$ $\mathrm{load}_G := G^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_G := G^{\langle 2\rangle} \cdot \mathrm{mm}$
$\mathrm{load}_D := D^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_D := D^{\langle 2\rangle} \cdot \mathrm{mm}$ $\mathrm{load}_H := H^{\langle 1\rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_H := H^{\langle 2\rangle} \cdot \mathrm{mm}$

Look at matrices and calculate number of data points:

$a := 1 .. \mathrm{rows}(A)$ $e := 1 .. \mathrm{rows}(E)$
$b := 1 .. \mathrm{rows}(B)$ $f := 1 .. \mathrm{rows}(F)$
$c := 1 .. \mathrm{rows}(C)$ $g := 1 .. \mathrm{rows}(G)$
$d := 1 .. \mathrm{rows}(D)$ $h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 1 (0.01 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

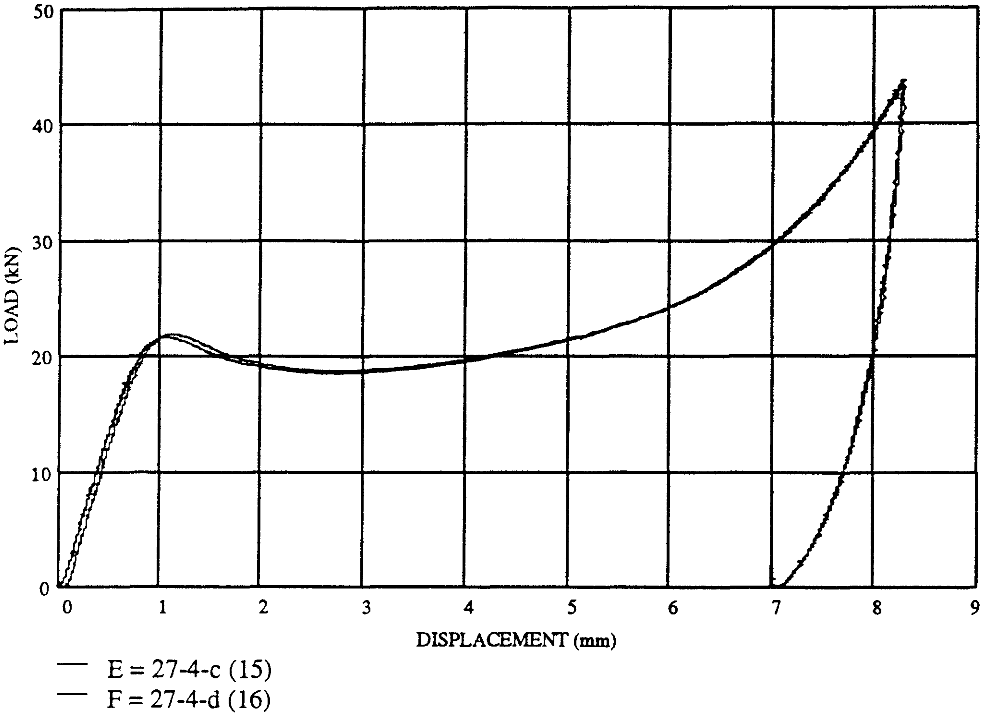
$\max\left(\mathrm{load}_A\right) = 43.82 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_A\right) = 8.327 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_B\right) = 45.041 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_B\right) = 8.401 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_C\right) = 45.335 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_C\right) = 8.451 \cdot \mathrm{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 2 (0.10 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max(\text{load}_E) = 43.674 \cdot \text{kN}$ $\max(\text{displ}_E) = 8.277 \cdot \text{mm}$

$\max(\text{load}_F) = 43.722 \cdot \text{kN}$ $\max(\text{displ}_F) = 8.302 \cdot \text{mm}$

Calculate true stress and true strain (assume incompressibility):

$\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}} \qquad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f} \qquad A_{iF_f} := \frac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$$

$$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g} \qquad A_{iG_g} := \frac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}} \qquad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h} \qquad A_{iH_h} := \frac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{\mathrm{load}_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{\mathrm{load}_{E_e}}{A_{iE_e}} \qquad \sigma_{trueF_f} := \frac{\mathrm{load}_{F_f}}{A_{iF_f}} \qquad \sigma_{trueG_g} := \frac{\mathrm{load}_{G_g}}{A_{iG_g}} \qquad \sigma_{trueH_h} := \frac{\mathrm{load}_{H_h}}{A_{iH_h}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right) \qquad \varepsilon_{trueF_f} := -\ln\left(\frac{L_{iF_f}}{L_{o_6}}\right) \qquad \varepsilon_{trueG_g} := -\ln\left(\frac{L_{iG_g}}{L_{o_7}}\right) \qquad \varepsilon_{trueH_h} := -\ln\left(\frac{L_{iH_h}}{L_{o_8}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$$

$$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot MPa\right) \quad \sigma_{yF} := \max\left(\sigma_{yFcalc}\right) \quad NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$$

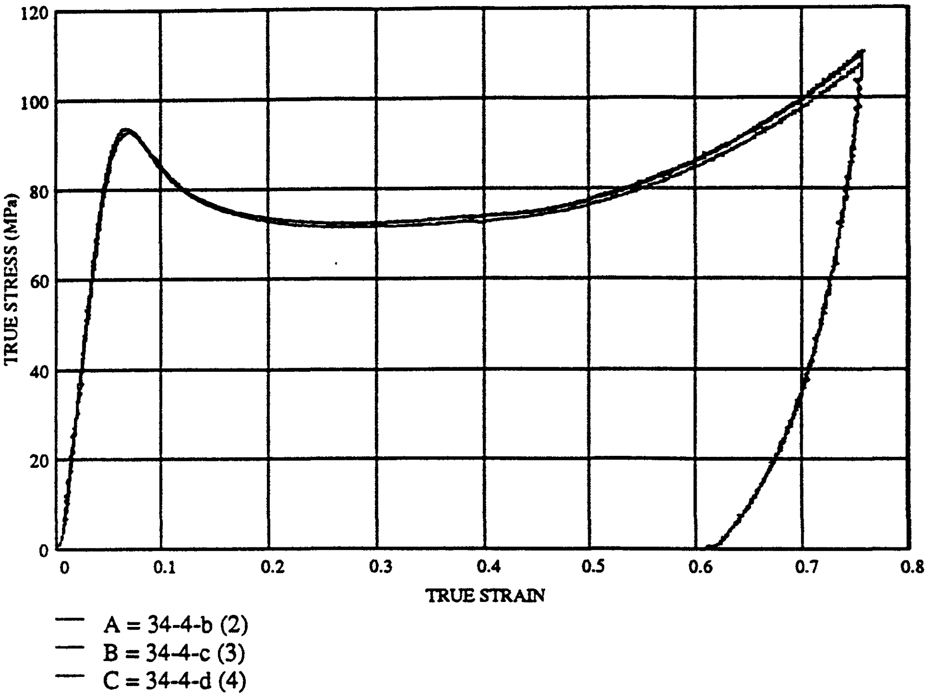
$$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 1200, \sigma_{trueG_g}, 0 \cdot MPa\right) \quad \sigma_{yG} := \max\left(\sigma_{yGcalc}\right) \quad NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$$

$$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot MPa\right) \quad \sigma_{yH} := \max\left(\sigma_{yHcalc}\right) \quad NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

$$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}} \quad \varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}} \quad \varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}} \quad \varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$$

Generate a plot of true stress (MPa) versus true strain for load rate 1 (0.01 mm/sec):



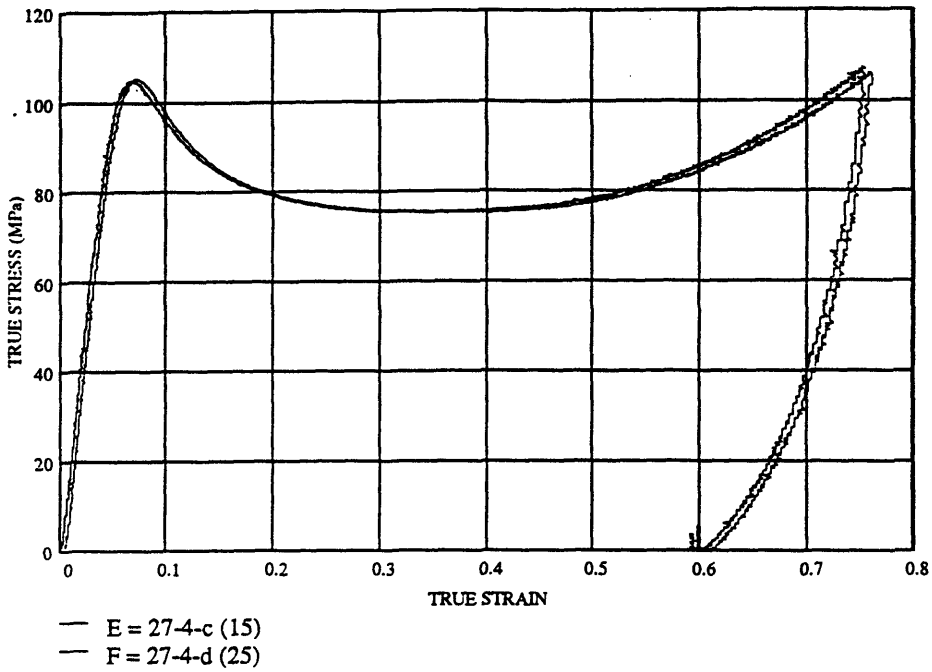
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\text{trueA}}) = 0.756$ $\max(\sigma_{\text{trueA}}) = 107.533 \cdot \text{MPa}$ $\sigma_{yA} = 92.851 \cdot \text{MPa}$ $\varepsilon_{yA} = 0.067$

$\max(\varepsilon_{\text{trueB}}) = 0.759$ $\max(\sigma_{\text{trueB}}) = 110.58 \cdot \text{MPa}$ $\sigma_{yB} = 93.767 \cdot \text{MPa}$ $\varepsilon_{yB} = 0.065$

$\max(\varepsilon_{\text{trueC}}) = 0.757$ $\max(\sigma_{\text{trueC}}) = 110.146 \cdot \text{MPa}$ $\sigma_{yC} = 93.608 \cdot \text{MPa}$ $\varepsilon_{yC} = 0.063$

Generate a plot of true stress (MPa) versus true strain for load rate 2 (0.10 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueE}}\right) = 0.761$ $\max\left(\sigma_{\mathrm{trueE}}\right) = 106.084 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yE}} = 105.442 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yE}} = 0.071$

$\max\left(\varepsilon_{\mathrm{trueF}}\right) = 0.755$ $\max\left(\sigma_{\mathrm{trueF}}\right) = 107.51 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yF}} = 104.98 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yF}} = 0.067$

Select specimens to be used: $z := 1..8$ ORIGIN := 1

$\mathrm{DIM} := \mathrm{READPRN}(\mathrm{COMP1B})$

$$\mathrm{SPEC} := \begin{bmatrix} 5 \\ 6 \\ 7 \\ 1 \\ 11 \\ 12 \\ 13 \\ 1 \end{bmatrix} \qquad L_{o_z} := \left(\mathrm{DIM}^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm} \qquad D_{o_z} := \left(\mathrm{DIM}^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm}$$

Show initial length (mm):

$$L_o = \begin{bmatrix} 15.46 \\ 15.42 \\ 15.68 \\ 15.37 \\ 15.27 \\ 15.37 \\ 15.21 \\ 15.37 \end{bmatrix} \cdot \mathrm{mm}$$

Show initial diameter (mm):

$$D_o = \begin{bmatrix} 15.69 \\ 15.74 \\ 15.76 \\ 15.73 \\ 15.55 \\ 15.57 \\ 15.6 \\ 15.73 \end{bmatrix} \cdot \mathrm{mm}$$

Length/Diameter ratio(LDR):

$$\mathrm{LDR}_z := \frac{L_{o_z}}{D_{o_z}} \qquad \mathrm{LDR} = \begin{bmatrix} 0.985 \\ 0.98 \\ 0.995 \\ 0.977 \\ 0.982 \\ 0.987 \\ 0.975 \\ 0.977 \end{bmatrix}$$

Calculate the initial cross sectional area (mm^2):

$$A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2 \qquad A_{cso} = \begin{bmatrix} 193.346 \\ 194.581 \\ 195.075 \\ 194.333 \\ 189.911 \\ 190.4 \\ 191.134 \\ 194.333 \end{bmatrix} \cdot \mathrm{mm}^2$$

Input maximum true strain (compressive values negative):

$$\varepsilon_{max} := \begin{bmatrix} -0.75 \\ -0.75 \\ -0.75 \\ 1 \\ -0.75 \\ -0.75 \\ -0.75 \\ 1 \end{bmatrix}$$

Input loading head actuator speed (mm/sec):

$$\mathrm{Load}_{rate} := \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1 \\ 2.50 \\ 2.50 \\ 2.50 \\ 1 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm): **Calculate the machine stroke (mm):**

$L_{f_z} := L_{0_z} \cdot e^{\varepsilon_{max_z}}$

$$L_f = \begin{bmatrix} 7.303 \\ 7.284 \\ 7.407 \\ 41.78 \\ 7.213 \\ 7.26 \\ 7.185 \\ 41.78 \end{bmatrix} \cdot mm$$

$\text{Stroke}_z := L_{f_z} - L_{0_z}$

$$\text{Stroke} = \begin{bmatrix} -8.157 \\ -8.136 \\ -8.273 \\ 26.41 \\ -8.057 \\ -8.11 \\ -8.025 \\ 26.41 \end{bmatrix} \cdot mm$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $\text{Stroke}_{tot_z} := 2 \cdot \text{Stroke}_z$

Calculate the total time required for test:

$$\text{time}_{tot_z} := \left| \frac{\text{Stroke}_{tot_z}}{\text{Load}_{rate_z}} \right|$$

$$\text{Stroke}_{tot} = \begin{bmatrix} -16.314 \\ -16.272 \\ -16.547 \\ 52.82 \\ -16.114 \\ -16.219 \\ -16.051 \\ 52.82 \end{bmatrix} \cdot mm$$

$$\text{time}_{tot} = \begin{bmatrix} 16.314 \\ 16.272 \\ 16.547 \\ 52.82 \\ 6.446 \\ 6.488 \\ 6.42 \\ 52.82 \end{bmatrix} \cdot sec \qquad \text{time}_{tot} = \begin{bmatrix} 0.272 \\ 0.271 \\ 0.276 \\ 0.88 \\ 0.107 \\ 0.108 \\ 0.107 \\ 0.88 \end{bmatrix} \cdot min \qquad \text{time}_{tot} = \begin{bmatrix} 0.005 \\ 0.005 \\ 0.005 \\ 0.015 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.015 \end{bmatrix} \cdot hr$$

Input data aq. frequency rate (Hz), range: 0-500 Hz: **Calculate approximate number of data points:**

$$\text{Freq}_{data} := \begin{bmatrix} 400 \\ 400 \\ 400 \\ 1 \\ 400 \\ 400 \\ 400 \\ 1 \end{bmatrix} \cdot Hz$$

$\text{NBR}_{data_z} := \text{Freq}_{data_z} \cdot \text{time}_{tot_z}$

$$\text{NBR}_{data} = \begin{bmatrix} 6525.77 \\ 6508.886 \\ 6618.634 \\ 52.82 \\ 2578.228 \\ 2595.112 \\ 2568.098 \\ 52.82 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(IC1LR3T4)$ $E := \mathrm{READPRN}(IC1LR4T3)$

$B := \mathrm{READPRN}(IC1LR3T5)$ $F := \mathrm{READPRN}(IC1LR4T4)$

$C := \mathrm{READPRN}(IC1LR3T6)$ $G := \mathrm{READPRN}(IC1LR4T5)$

$D := \mathrm{READPRN}(IC0LR1T1)$ $H := \mathrm{READPRN}(IC0LR1T1)$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{\langle 1 \rangle} \cdot kN$ $displ_A := A^{\langle 2 \rangle} \cdot mm$ $load_E := E^{\langle 1 \rangle} \cdot kN$ $displ_E := E^{\langle 2 \rangle} \cdot mm$

$load_B := B^{\langle 1 \rangle} \cdot kN$ $displ_B := B^{\langle 2 \rangle} \cdot mm$ $load_F := F^{\langle 1 \rangle} \cdot kN$ $displ_F := F^{\langle 2 \rangle} \cdot mm$

$load_C := C^{\langle 1 \rangle} \cdot kN$ $displ_C := C^{\langle 2 \rangle} \cdot mm$ $load_G := G^{\langle 1 \rangle} \cdot kN$ $displ_G := G^{\langle 2 \rangle} \cdot mm$

$load_D := D^{\langle 1 \rangle} \cdot kN$ $displ_D := D^{\langle 2 \rangle} \cdot mm$ $load_H := H^{\langle 1 \rangle} \cdot kN$ $displ_H := H^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$ $e := 1 .. \mathrm{rows}(E)$

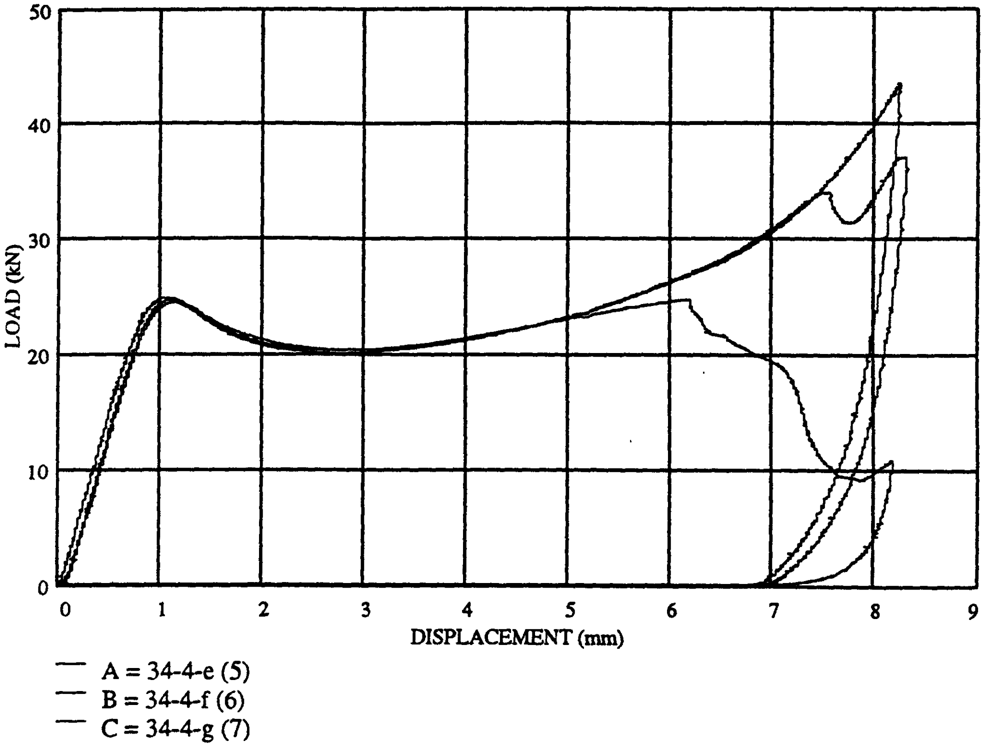
$b := 1 .. \mathrm{rows}(B)$ $f := 1 .. \mathrm{rows}(F)$

$c := 1 .. \mathrm{rows}(C)$ $g := 1 .. \mathrm{rows}(G)$

$d := 1 .. \mathrm{rows}(D)$ $h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 3 (1.00 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

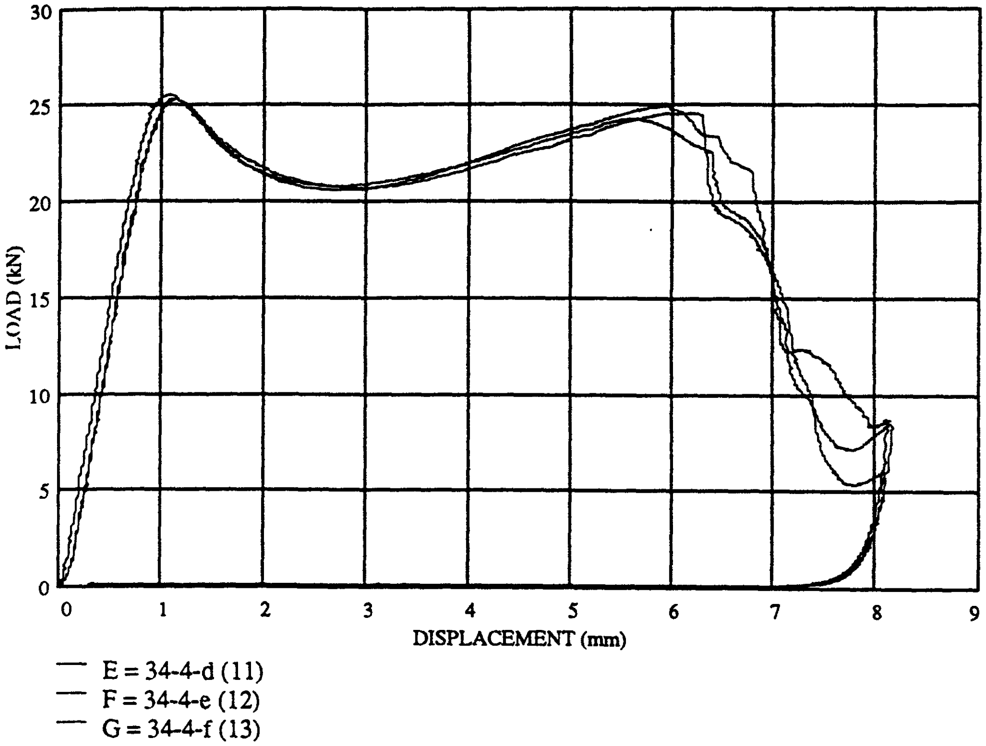
$\max\left(\mathrm{load}_A\right) = 43.527 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_A\right) = 8.252 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_B\right) = 24.768 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_B\right) = 8.178 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_C\right) = 37.03 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_C\right) = 8.327 \cdot \mathrm{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 4 (2.5 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max\left(\mathrm{load}_E\right) = 25.305 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_E\right) = 8.153 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_F\right) = 25.256 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_F\right) = 8.178 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_G\right) = 25.501 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_G\right) = 8.078 \cdot \mathrm{mm}$

Calculate true stress and true strain (assume incompressibility):

$\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a}$ $\quad A_{iA_a} := \dfrac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}}$ $\quad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b}$ $\quad A_{iB_b} := \dfrac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$

$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c}$ $\quad A_{iC_c} := \dfrac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}}$ $\quad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d}$ $\quad A_{iD_d} := \dfrac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$

$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e}$ $\quad A_{iE_e} := \dfrac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}}$ $\quad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f}$ $\quad A_{iF_f} := \dfrac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$

$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g}$ $\quad A_{iG_g} := \dfrac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}}$ $\quad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h}$ $\quad A_{iH_h} := \dfrac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$

Calculate true stress (MPa) and true strain:

$\sigma_{trueA_a} := \dfrac{\mathrm{load}_{A_a}}{A_{iA_a}}$ $\quad \sigma_{trueB_b} := \dfrac{\mathrm{load}_{B_b}}{A_{iB_b}}$ $\quad \sigma_{trueC_c} := \dfrac{\mathrm{load}_{C_c}}{A_{iC_c}}$ $\quad \sigma_{trueD_d} := \dfrac{\mathrm{load}_{D_d}}{A_{iD_d}}$

$\varepsilon_{trueA_a} := -\ln\left(\dfrac{L_{iA_a}}{L_{o_1}}\right)$ $\quad \varepsilon_{trueB_b} := -\ln\left(\dfrac{L_{iB_b}}{L_{o_2}}\right)$ $\quad \varepsilon_{trueC_c} := -\ln\left(\dfrac{L_{iC_c}}{L_{o_3}}\right)$ $\quad \varepsilon_{trueD_d} := -\ln\left(\dfrac{L_{iD_d}}{L_{o_4}}\right)$

$\sigma_{trueE_e} := \dfrac{\mathrm{load}_{E_e}}{A_{iE_e}}$ $\quad \sigma_{trueF_f} := \dfrac{\mathrm{load}_{F_f}}{A_{iF_f}}$ $\quad \sigma_{trueG_g} := \dfrac{\mathrm{load}_{G_g}}{A_{iG_g}}$ $\quad \sigma_{trueH_h} := \dfrac{\mathrm{load}_{H_h}}{A_{iH_h}}$

$\varepsilon_{trueE_e} := -\ln\left(\dfrac{L_{iE_e}}{L_{o_5}}\right)$ $\quad \varepsilon_{trueF_f} := -\ln\left(\dfrac{L_{iF_f}}{L_{o_6}}\right)$ $\quad \varepsilon_{trueG_g} := -\ln\left(\dfrac{L_{iG_g}}{L_{o_7}}\right)$ $\quad \varepsilon_{trueH_h} := -\ln\left(\dfrac{L_{iH_h}}{L_{o_8}}\right)$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$$

$$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yF} := \max\left(\sigma_{yFcalc}\right) \quad NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$$

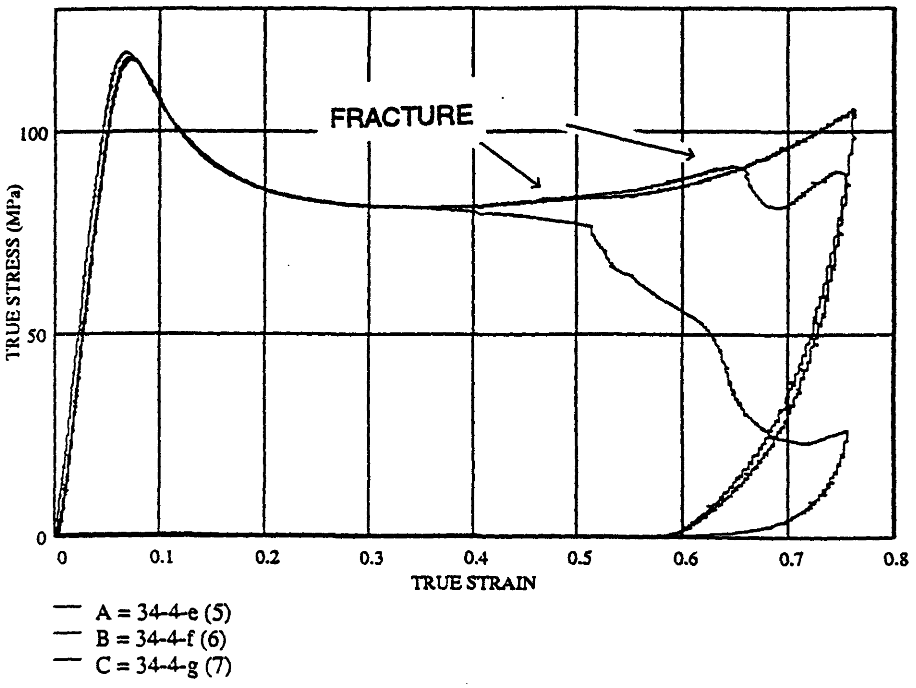
$$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 1200, \sigma_{trueG_g}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yG} := \max\left(\sigma_{yGcalc}\right) \quad NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$$

$$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yH} := \max\left(\sigma_{yHcalc}\right) \quad NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

$$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}} \quad \varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}} \quad \varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}} \quad \varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$$

Generate a plot of true stress (MPa) versus true strain for load rate 3 (1.00 mm/sec):



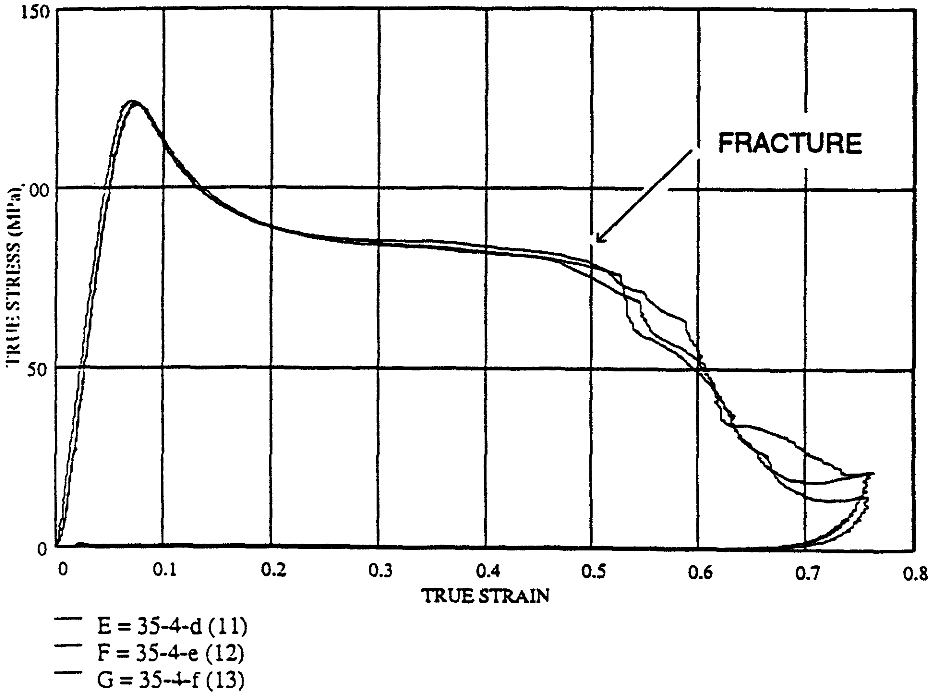
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{trueA}\right) = 0.763$ $\max\left(\sigma_{trueA}\right) = 117.63 \cdot \mathrm{MPa}$ $\sigma_{yA} = 117.63 \cdot \mathrm{MPa}$ $\varepsilon_{yA} = 0.073$

$\max\left(\varepsilon_{trueB}\right) = 0.756$ $\max\left(\sigma_{trueB}\right) = 118.261 \cdot \mathrm{MPa}$ $\sigma_{yB} = 118.261 \cdot \mathrm{MPa}$ $\varepsilon_{yB} = 0.074$

$\max\left(\varepsilon_{trueC}\right) = 0.757$ $\max\left(\sigma_{trueC}\right) = 119.417 \cdot \mathrm{MPa}$ $\sigma_{yC} = 119.417 \cdot \mathrm{MPa}$ $\varepsilon_{yC} = 0.067$

Generate a plot of true stress (MPa) versus true strain for load rate 4 (2.50 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\mathrm{trueE}}) = 0.763$ $\max(\sigma_{\mathrm{trueE}}) = 123.683 \cdot \mathrm{MPa}$ $\sigma_{yE} = 123.683 \cdot \mathrm{MPa}$ $\varepsilon_{yE} = 0.073$

$\max(\varepsilon_{\mathrm{trueF}}) = 0.759$ $\max(\sigma_{\mathrm{trueF}}) = 122.997 \cdot \mathrm{MPa}$ $\sigma_{yF} = 122.997 \cdot \mathrm{MPa}$ $\varepsilon_{yF} = 0.076$

$\max(\varepsilon_{\mathrm{trueG}}) = 0.757$ $\max(\sigma_{\mathrm{trueG}}) = 124.261 \cdot \mathrm{MPa}$ $\sigma_{yG} = 124.261 \cdot \mathrm{MPa}$ $\varepsilon_{yG} = 0.071$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..4$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP2B})$ $\quad$ $\mathrm{SPEC}_A := 1$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A}\cdot \mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A}\cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP2B})$ $\quad$ $\mathrm{SPEC}_B := 5$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B}\cdot \mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B}\cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP2B})$ $\quad$ $\mathrm{SPEC}_C := 8$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C}\cdot \mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C}\cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP2B})$ $\quad$ $\mathrm{SPEC}_D := 12$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1\rangle}\right)_{\mathrm{SPEC}_D}\cdot \mathrm{mm}$ $\quad$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2\rangle}\right)_{\mathrm{SPEC}_D}\cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4}\cdot\left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 183.374 \\ 183.374 \\ 185.782 \\ 186.023 \end{bmatrix}\cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(\mathrm{IC2LR1T5})$

$B := \mathrm{READPRN}(\mathrm{IC2LR2T5})$

$C := \mathrm{READPRN}(\mathrm{IC2LR3T5})$

$D := \mathrm{READPRN}(\mathrm{IC2LR4T5})$

Note: Files A-D are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000 \cdot \mathrm{newton}$

$\mathrm{load}_A := A^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_A := A^{<2>} \cdot \mathrm{mm}$ $\mathrm{load}_D := D^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_D := D^{<2>} \cdot \mathrm{mm}$

$\mathrm{load}_B := B^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_B := B^{<2>} \cdot \mathrm{mm}$

$\mathrm{load}_C := C^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_C := C^{<2>} \cdot \mathrm{mm}$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$ $d := 1 .. \mathrm{rows}(D)$

$b := 1 .. \mathrm{rows}(B)$

$c := 1 .. \mathrm{rows}(C)$

Calculate true stress and true strain (assume incompressibility): $\mathrm{MPa} := 1 \cdot 10^6 \cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \quad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \quad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \quad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \quad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \quad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d} \quad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \quad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \quad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}} \quad \sigma_{trueD_d} := \frac{\mathrm{load}_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \quad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \quad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \quad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \text{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \text{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad \text{NumA}_a := \text{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

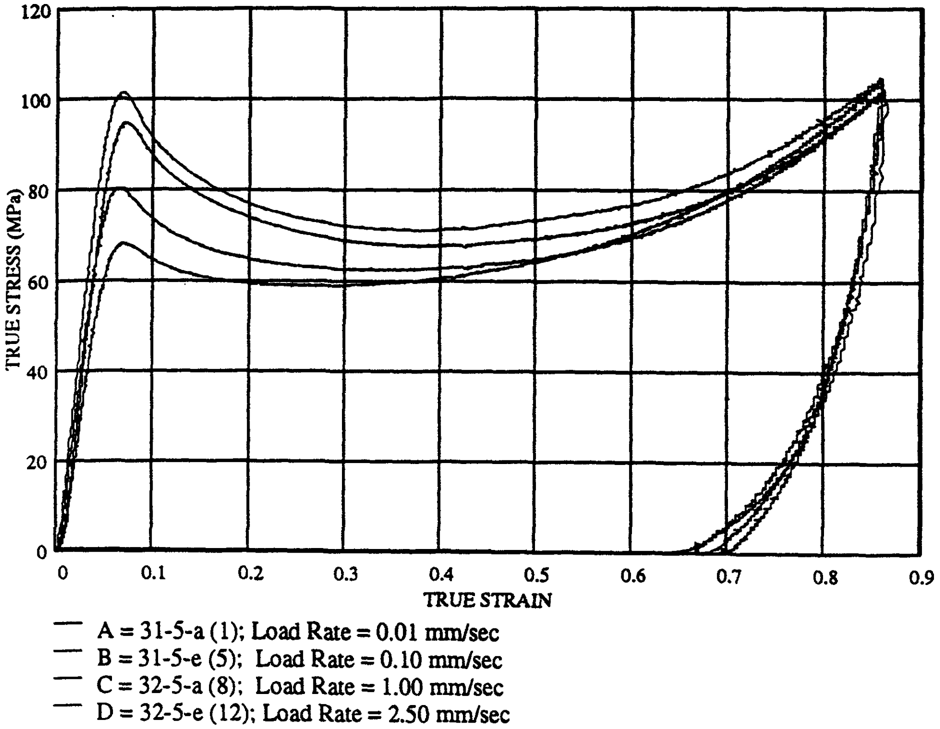
$$\sigma_{yBcalc_b} := \text{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \text{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad \text{NumB}_b := \text{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \text{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot \text{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad \text{NumC}_c := \text{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \text{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot \text{MPa}\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad \text{NumD}_d := \text{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(\text{NumA})}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(\text{NumB})}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(\text{NumC})}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(\text{NumD})}}$$

Generate a plot of true stress (MPa) versus true strain for 75% 8084 / 25% 1080 at various load rates:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{trueA}\right) = 0.858$ $\max\left(\sigma_{trueA}\right) = 104.291 \cdot MPa$ $\sigma_{yA} = 68.214 \cdot MPa$ $\varepsilon_{yA} = 0.071$

$\max\left(\varepsilon_{trueB}\right) = 0.861$ $\max\left(\sigma_{trueB}\right) = 102.532 \cdot MPa$ $\sigma_{yB} = 80.456 \cdot MPa$ $\varepsilon_{yB} = 0.064$

$\max\left(\varepsilon_{trueC}\right) = 0.862$ $\max\left(\sigma_{trueC}\right) = 101.544 \cdot MPa$ $\sigma_{yC} = 94.798 \cdot MPa$ $\varepsilon_{yC} = 0.071$

$\max\left(\varepsilon_{trueD}\right) = 0.864$ $\max\left(\sigma_{trueD}\right) = 104.907 \cdot MPa$ $\sigma_{yD} = 101.388 \cdot MPa$ $\varepsilon_{yD} = 0.07$

Select specimens to be used: $z := 1 .. 8$ $\quad$ ORIGIN := 1

DIM := READPRN(COMP2B)

$$SPEC := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 5 \\ 6 \\ 7 \\ 1 \end{bmatrix} \qquad L_{0_z} := \left(DIM^{\langle 1 \rangle}\right)_{SPEC_{(z,1)}} \cdot mm \qquad D_{0_z} := \left(DIM^{\langle 2 \rangle}\right)_{SPEC_{(z,1)}} \cdot mm$$

Show initial length (mm): $\qquad$ **Show initial diameter (mm):**

$$L_0 = \begin{bmatrix} 15.15 \\ 15.09 \\ 14.83 \\ 15.15 \\ 14.81 \\ 14.87 \\ 15.19 \\ 15.15 \end{bmatrix} \cdot mm \qquad D_0 = \begin{bmatrix} 15.28 \\ 15.27 \\ 15.22 \\ 15.28 \\ 15.28 \\ 15.29 \\ 15.36 \\ 15.28 \end{bmatrix} \cdot mm$$

Length/Diameter ratio(LDR): $\qquad$ **Calculate the initial cross sectional area (mm^2):**

$$LDR_z := \frac{L_{0_z}}{D_{0_z}} \qquad A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{0_z}\right)^2$$

$$LDR = \begin{bmatrix} 0.991 \\ 0.988 \\ 0.974 \\ 0.991 \\ 0.969 \\ 0.973 \\ 0.989 \\ 0.991 \end{bmatrix} \qquad A_{cso} = \begin{bmatrix} 183.374 \\ 183.134 \\ 181.936 \\ 183.374 \\ 183.374 \\ 183.614 \\ 185.299 \\ 183.374 \end{bmatrix} \cdot mm^2$$

Input maximum true strain (compressive values negative): $\qquad$ **Input loading head actuator speed (mm/sec):**

$$\varepsilon_{max} := \begin{bmatrix} -0.85 \\ -0.85 \\ -0.85 \\ 1 \\ -0.85 \\ -0.85 \\ -0.85 \\ 1 \end{bmatrix} \qquad Load_{rate} := \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 1 \\ 0.10 \\ 0.10 \\ 0.10 \\ 1 \end{bmatrix} \cdot \frac{mm}{sec}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{o_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 6.475 \\ 6.45 \\ 6.339 \\ 41.182 \\ 6.33 \\ 6.356 \\ 6.492 \\ 41.182 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the machine stroke (mm):

$$Stroke_z := L_{f_z} - L_{o_z}$$

$$Stroke = \begin{bmatrix} -8.675 \\ -8.64 \\ -8.491 \\ 26.032 \\ -8.48 \\ -8.514 \\ -8.698 \\ 26.032 \end{bmatrix} \cdot \mathrm{mm}$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $Stroke_{tot_z} := 2 \cdot Stroke_z$

$$Stroke_{tot} = \begin{bmatrix} -17.349 \\ -17.281 \\ -16.983 \\ 52.064 \\ -16.96 \\ -17.029 \\ -17.395 \\ 52.064 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the total time required for test:

$$time_{tot_z} := \left| \frac{Stroke_{tot_z}}{Load_{rate_z}} \right|$$

$$time_{tot} = \begin{bmatrix} 1734.933 \\ 1728.062 \\ 1698.287 \\ 52.064 \\ 169.6 \\ 170.287 \\ 173.951 \\ 52.064 \end{bmatrix} \cdot \mathrm{sec} \qquad time_{tot} = \begin{bmatrix} 28.916 \\ 28.801 \\ 28.305 \\ 0.868 \\ 2.827 \\ 2.838 \\ 2.899 \\ 0.868 \end{bmatrix} \cdot \mathrm{min} \qquad time_{tot} = \begin{bmatrix} 0.482 \\ 0.48 \\ 0.472 \\ 0.014 \\ 0.047 \\ 0.047 \\ 0.048 \\ 0.014 \end{bmatrix} \cdot \mathrm{hr}$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$Freq_{data} := \begin{bmatrix} 5 \\ 5 \\ 5 \\ 1 \\ 50 \\ 50 \\ 50 \\ 1 \end{bmatrix} \cdot \mathrm{Hz}$$

Calculate approximate number of data points:

$$NBR_{data_z} := Freq_{data_z} \cdot time_{tot_z}$$

$$NBR_{data} = \begin{bmatrix} 8674.664 \\ 8640.309 \\ 8491.437 \\ 52.064 \\ 8479.985 \\ 8514.34 \\ 8697.567 \\ 52.064 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \text{READPRN}(\text{IC2LR1T5})$ $E := \text{READPRN}(\text{IC2LR2T5})$

$B := \text{READPRN}(\text{IC2LR1T6})$ $F := \text{READPRN}(\text{IC2LR2T4})$

$C := \text{READPRN}(\text{IC2LR1T7})$ $G := \text{READPRN}(\text{IC2LR2T6})$

$D := \text{READPRN}(\text{ICOLR1T1})$ $H := \text{READPRN}(\text{ICOLR1T1})$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file 'ICOLR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$\text{load}_A := A^{<1>} \cdot kN$ $\text{displ}_A := A^{<2>} \cdot mm$ $\text{load}_E := E^{<1>} \cdot kN$ $\text{displ}_E := E^{<2>} \cdot mm$

$\text{load}_B := B^{<1>} \cdot kN$ $\text{displ}_B := B^{<2>} \cdot mm$ $\text{load}_F := F^{<1>} \cdot kN$ $\text{displ}_F := F^{<2>} \cdot mm$

$\text{load}_C := C^{<1>} \cdot kN$ $\text{displ}_C := C^{<2>} \cdot mm$ $\text{load}_G := G^{<1>} \cdot kN$ $\text{displ}_G := G^{<2>} \cdot mm$

$\text{load}_D := D^{<1>} \cdot kN$ $\text{displ}_D := D^{<2>} \cdot mm$ $\text{load}_H := H^{<1>} \cdot kN$ $\text{displ}_H := H^{<2>} \cdot mm$

Look at matrices and calculate number of data points:

$a := 1 .. \text{rows}(A)$ $e := 1 .. \text{rows}(E)$

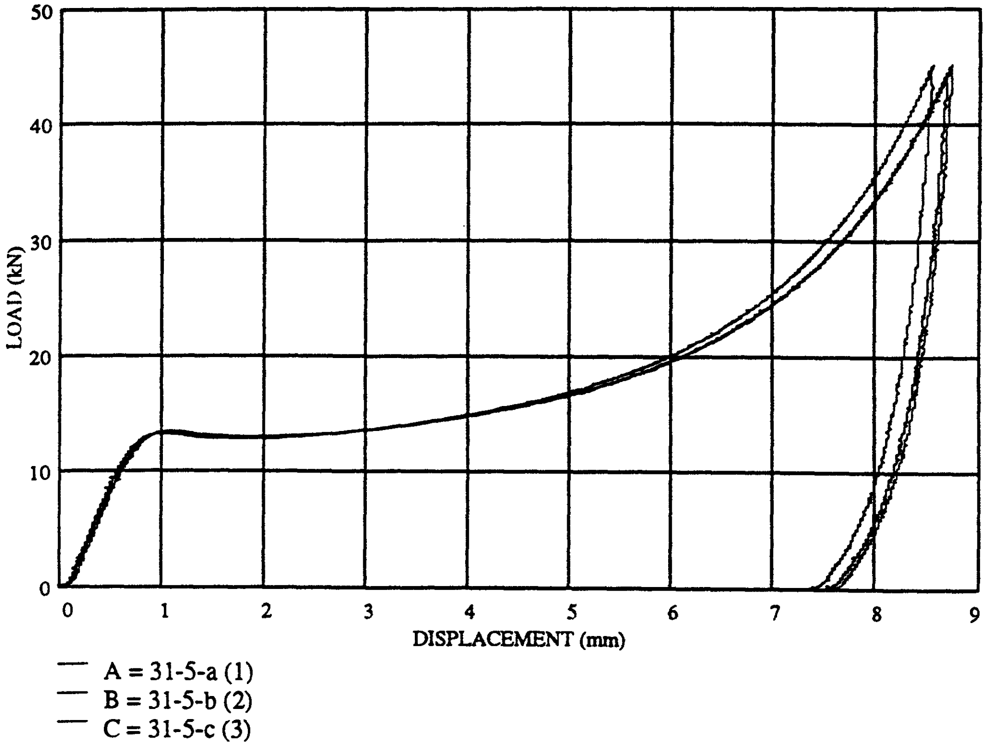
$b := 1 .. \text{rows}(B)$ $f := 1 .. \text{rows}(F)$

$c := 1 .. \text{rows}(C)$ $g := 1 .. \text{rows}(G)$

$d := 1 .. \text{rows}(D)$ $h := 1 .. \text{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 1 (0.01 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

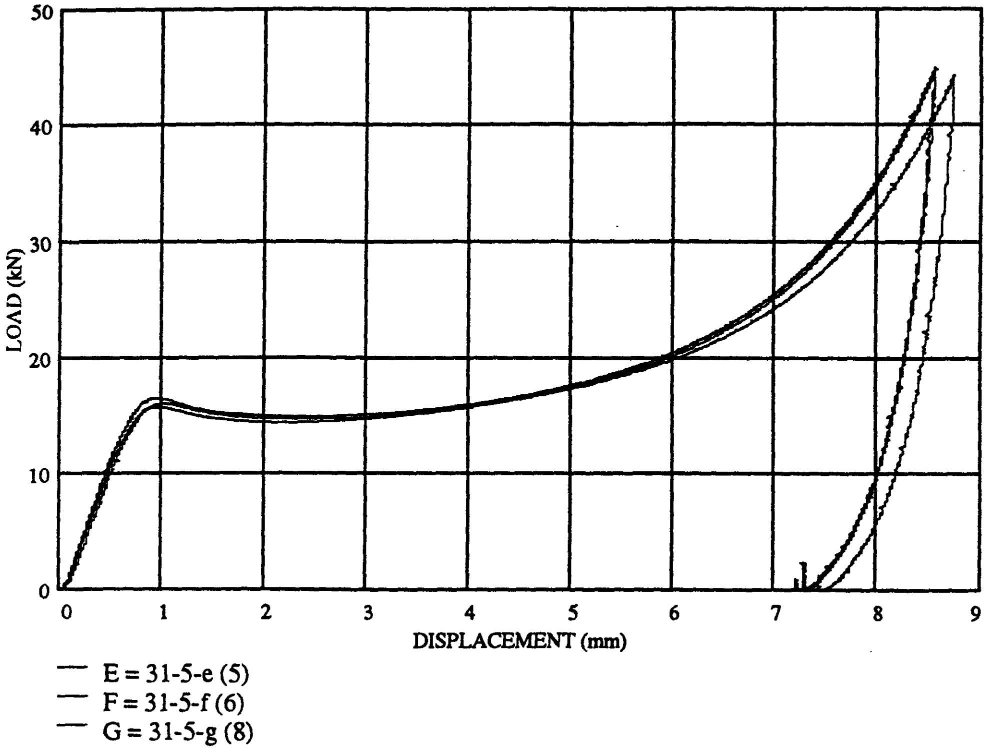
$\max\left(\text{load}_A\right) = 45.09\cdot\text{kN}$ $\max\left(\text{displ}_A\right) = 8.724\cdot\text{mm}$

$\max\left(\text{load}_B\right) = 44.455\cdot\text{kN}$ $\max\left(\text{displ}_B\right) = 8.675\cdot\text{mm}$

$\max\left(\text{load}_C\right) = 45.139\cdot\text{kN}$ $\max\left(\text{displ}_C\right) = 8.55\cdot\text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 2 (0.10 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max\left(\text{load}_E\right) = 44.309 \cdot \text{kN}$ $\max\left(\text{displ}_E\right) = 8.55 \cdot \text{mm}$

$\max\left(\text{load}_F\right) = 45.041 \cdot \text{kN}$ $\max\left(\text{displ}_F\right) = 8.575 \cdot \text{mm}$

$\max\left(\text{load}_G\right) = 44.211 \cdot \text{kN}$ $\max\left(\text{displ}_G\right) = 8.749 \cdot \text{mm}$

Calculate true stress and true strain (assume incompressibility):

$MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$L_{iA_a} := L_{o_1} - displ_{A_a}$ $A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}}$ $L_{iB_b} := L_{o_2} - displ_{B_b}$ $A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$

$L_{iC_c} := L_{o_3} - displ_{C_c}$ $A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}}$ $L_{iD_d} := L_{o_4} - displ_{D_d}$ $A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$

$L_{iE_e} := L_{o_5} - displ_{E_e}$ $A_{iE_e} := \frac{A_{cso_5} \cdot L_{o_5}}{L_{iE_e}}$ $L_{iF_f} := L_{o_6} - displ_{F_f}$ $A_{iF_f} := \frac{A_{cso_6} \cdot L_{o_6}}{L_{iF_f}}$

$L_{iG_g} := L_{o_7} - displ_{G_g}$ $A_{iG_g} := \frac{A_{cso_7} \cdot L_{o_7}}{L_{iG_g}}$ $L_{iH_h} := L_{o_8} - displ_{H_h}$ $A_{iH_h} := \frac{A_{cso_8} \cdot L_{o_8}}{L_{iH_h}}$

Calculate true stress (MPa) and true strain:

$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}}$ $\sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}}$ $\sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}}$ $\sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$

$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right)$ $\varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right)$ $\varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right)$ $\varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$

$\sigma_{trueE_e} := \frac{load_{E_e}}{A_{iE_e}}$ $\sigma_{trueF_f} := \frac{load_{F_f}}{A_{iF_f}}$ $\sigma_{trueG_g} := \frac{load_{G_g}}{A_{iG_g}}$ $\sigma_{trueH_h} := \frac{load_{H_h}}{A_{iH_h}}$

$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right)$ $\varepsilon_{trueF_f} := -\ln\left(\frac{L_{iF_f}}{L_{o_6}}\right)$ $\varepsilon_{trueG_g} := -\ln\left(\frac{L_{iG_g}}{L_{o_7}}\right)$ $\varepsilon_{trueH_h} := -\ln\left(\frac{L_{iH_h}}{L_{o_8}}\right)$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right)$ $\sigma_{yE} := \max\left(\sigma_{yEcalc}\right)$ $NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$

$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot MPa\right)$ $\sigma_{yF} := \max\left(\sigma_{yFcalc}\right)$ $NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$

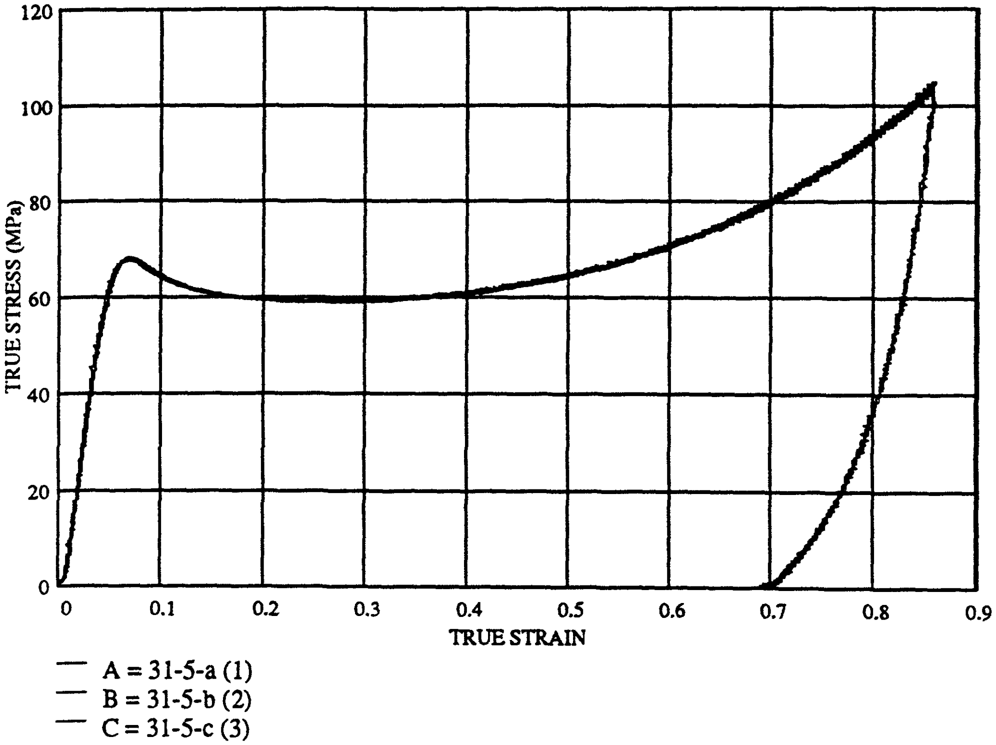
$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 1200, \sigma_{trueG_g}, 0 \cdot MPa\right)$ $\sigma_{yG} := \max\left(\sigma_{yGcalc}\right)$ $NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$

$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot MPa\right)$ $\sigma_{yH} := \max\left(\sigma_{yHcalc}\right)$ $NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$ $\varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}}$ $\varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}}$ $\varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for load rate 1 (0.01 mm/sec):



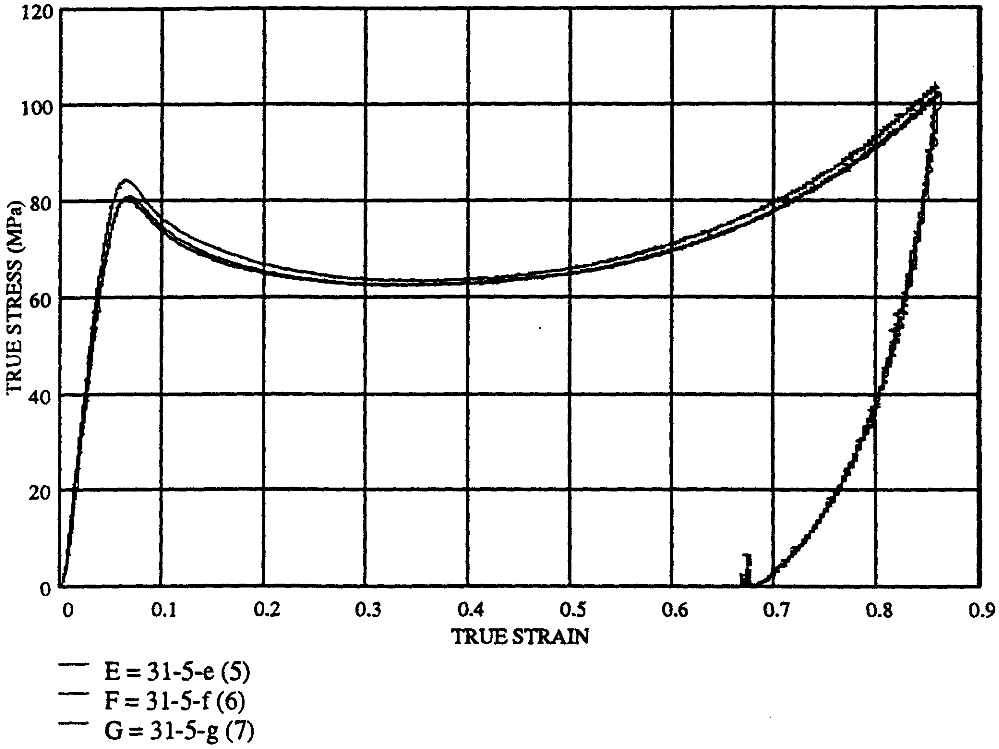
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.858$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 104.291 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 68.214 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.071$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.855$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 103.201 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 67.896 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.066$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 0.859$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 105.057 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 67.881 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.066$

Generate a plot of true stress (MPa) versus true strain for load rate 2 (0.10 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueE}}\right) = 0.861$ $\max\left(\sigma_{\text{trueE}}\right) = 102.532 \cdot \text{MPa}$ $\sigma_{\text{yE}} = 80.456 \cdot \text{MPa}$ $\varepsilon_{\text{yE}} = 0.064$

$\max\left(\varepsilon_{\text{trueF}}\right) = 0.86$ $\max\left(\sigma_{\text{trueF}}\right) = 104.253 \cdot \text{MPa}$ $\sigma_{\text{yF}} = 84.366 \cdot \text{MPa}$ $\varepsilon_{\text{yF}} = 0.064$

$\max\left(\varepsilon_{\text{trueG}}\right) = 0.858$ $\max\left(\sigma_{\text{trueG}}\right) = 101.445 \cdot \text{MPa}$ $\sigma_{\text{yG}} = 81.06 \cdot \text{MPa}$ $\varepsilon_{\text{yG}} = 0.068$

Select specimens to be used: $z := 1..8$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM} := \mathrm{READPRN}(\mathrm{COMP2B})$

$$\mathrm{SPEC} := \begin{bmatrix} 8 \\ 9 \\ 10 \\ 1 \\ 11 \\ 12 \\ 13 \\ 1 \end{bmatrix}$$

$$L_{o_z} := \left(\mathrm{DIM}^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm} \qquad D_{o_z} := \left(\mathrm{DIM}^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm}$$

Show initial length (mm):

$$L_o = \begin{bmatrix} 14.89 \\ 15.17 \\ 14.84 \\ 15.15 \\ 14.61 \\ 14.74 \\ 14.69 \\ 15.15 \end{bmatrix} \cdot \mathrm{mm}$$

Show initial diameter (mm):

$$D_o = \begin{bmatrix} 15.38 \\ 15.37 \\ 15.38 \\ 15.28 \\ 15.41 \\ 15.39 \\ 15.43 \\ 15.28 \end{bmatrix} \cdot \mathrm{mm}$$

Length/Diameter ratio(LDR):

$$\mathrm{LDR}_z := \frac{L_{o_z}}{D_{o_z}}$$

$$\mathrm{LDR} = \begin{bmatrix} 0.968 \\ 0.987 \\ 0.965 \\ 0.991 \\ 0.948 \\ 0.958 \\ 0.952 \\ 0.991 \end{bmatrix}$$

Calculate the initial cross sectional area (mm^2):

$$A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$$

$$A_{cso} = \begin{bmatrix} 185.782 \\ 185.54 \\ 185.782 \\ 183.374 \\ 186.507 \\ 186.023 \\ 186.991 \\ 183.374 \end{bmatrix} \cdot \mathrm{mm}^2$$

Input maximum true strain (compressive values negative):

$$\varepsilon_{max} := \begin{bmatrix} -0.85 \\ -0.85 \\ -0.85 \\ 1 \\ -0.85 \\ -0.85 \\ -0.85 \\ 1 \end{bmatrix}$$

Input loading head actuator speed (mm/sec):

$$\mathrm{Load}_{rate} := \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1 \\ 2.50 \\ 2.50 \\ 2.50 \\ 1 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{o_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 6.364 \\ 6.484 \\ 6.343 \\ 41.182 \\ 6.245 \\ 6.3 \\ 6.279 \\ 41.182 \end{bmatrix} \cdot mm$$

Calculate the machine stroke (mm):

$$Stroke_z := L_{f_z} - L_{o_z}$$

$$Stroke = \begin{bmatrix} -8.526 \\ -8.686 \\ -8.497 \\ 26.032 \\ -8.365 \\ -8.44 \\ -8.411 \\ 26.032 \end{bmatrix} \cdot mm$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $Stroke_{tot_z} := 2 \cdot Stroke_z$

Calculate the total time required for test:

$$time_{tot_z} := \left| \frac{Stroke_{tot_z}}{Load_{rate_z}} \right|$$

$$Stroke_{tot} = \begin{bmatrix} -17.052 \\ -17.372 \\ -16.994 \\ 52.064 \\ -16.731 \\ -16.88 \\ -16.823 \\ 52.064 \end{bmatrix} \cdot mm$$

$$time_{tot} = \begin{bmatrix} 17.052 \\ 17.372 \\ 16.994 \\ 52.064 \\ 6.692 \\ 6.752 \\ 6.729 \\ 52.064 \end{bmatrix} \cdot sec \qquad time_{tot} = \begin{bmatrix} 0.284 \\ 0.29 \\ 0.283 \\ 0.868 \\ 0.112 \\ 0.113 \\ 0.112 \\ 0.868 \end{bmatrix} \cdot min \qquad time_{tot} = \begin{bmatrix} 0.005 \\ 0.005 \\ 0.005 \\ 0.014 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.014 \end{bmatrix} \cdot hr$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$Freq_{data} := \begin{bmatrix} 400 \\ 400 \\ 400 \\ 1 \\ 400 \\ 400 \\ 400 \\ 1 \end{bmatrix} \cdot Hz$$

Calculate approximate number of data points:

$$NBR_{data_z} := Freq_{data_z} \cdot time_{tot_z}$$

$$NBR_{data} = \begin{bmatrix} 6820.633 \\ 6948.892 \\ 6797.73 \\ 52.064 \\ 2676.95 \\ 2700.769 \\ 2691.608 \\ 52.064 \end{bmatrix}$$

Select a buffer size which is larger than the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(IC2LR3T5)$ $E := \mathrm{READPRN}(IC2LR4T4)$

$B := \mathrm{READPRN}(IC2LR3T6)$ $F := \mathrm{READPRN}(IC2LR4T5)$

$C := \mathrm{READPRN}(IC2LR3T7)$ $G := \mathrm{READPRN}(IC2LR4T6)$

$D := \mathrm{READPRN}(IC0LR1T1)$ $H := \mathrm{READPRN}(IC0LR1T1)$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$\mathrm{load}_A := A^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_A := A^{\langle 2 \rangle} \cdot mm$ $\mathrm{load}_E := E^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_E := E^{\langle 2 \rangle} \cdot mm$

$\mathrm{load}_B := B^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_B := B^{\langle 2 \rangle} \cdot mm$ $\mathrm{load}_F := F^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_F := F^{\langle 2 \rangle} \cdot mm$

$\mathrm{load}_C := C^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_C := C^{\langle 2 \rangle} \cdot mm$ $\mathrm{load}_G := G^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_G := G^{\langle 2 \rangle} \cdot mm$

$\mathrm{load}_D := D^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_D := D^{\langle 2 \rangle} \cdot mm$ $\mathrm{load}_H := H^{\langle 1 \rangle} \cdot kN$ $\mathrm{displ}_H := H^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$ $e := 1 .. \mathrm{rows}(E)$

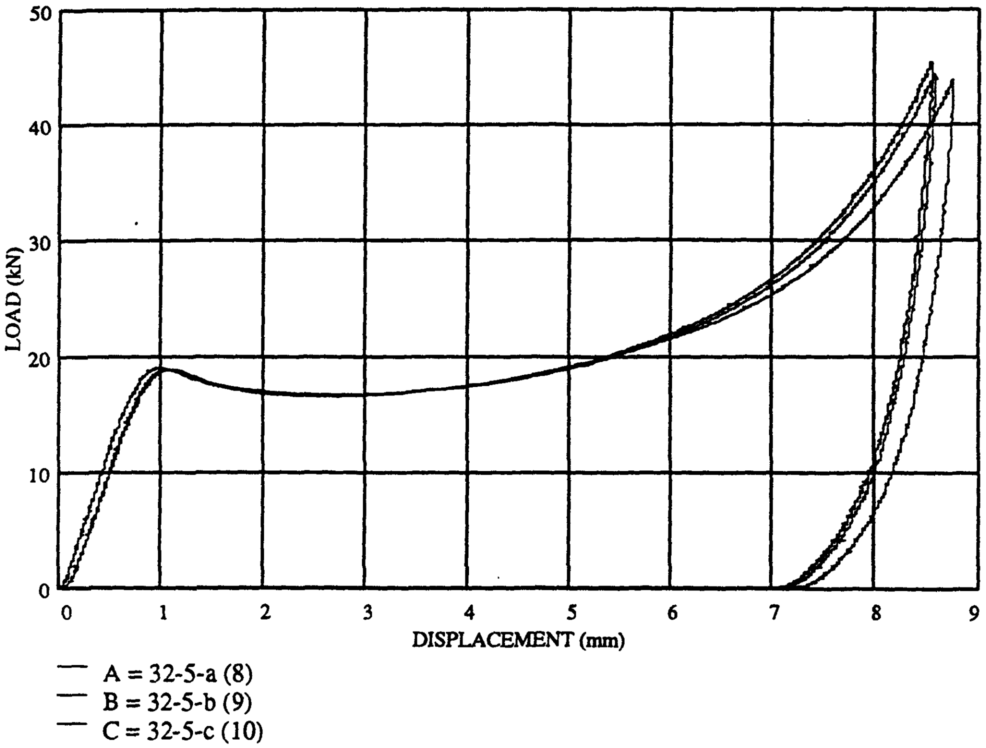
$b := 1 .. \mathrm{rows}(B)$ $f := 1 .. \mathrm{rows}(F)$

$c := 1 .. \mathrm{rows}(C)$ $g := 1 .. \mathrm{rows}(G)$

$d := 1 .. \mathrm{rows}(D)$ $h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 3 (1.00 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

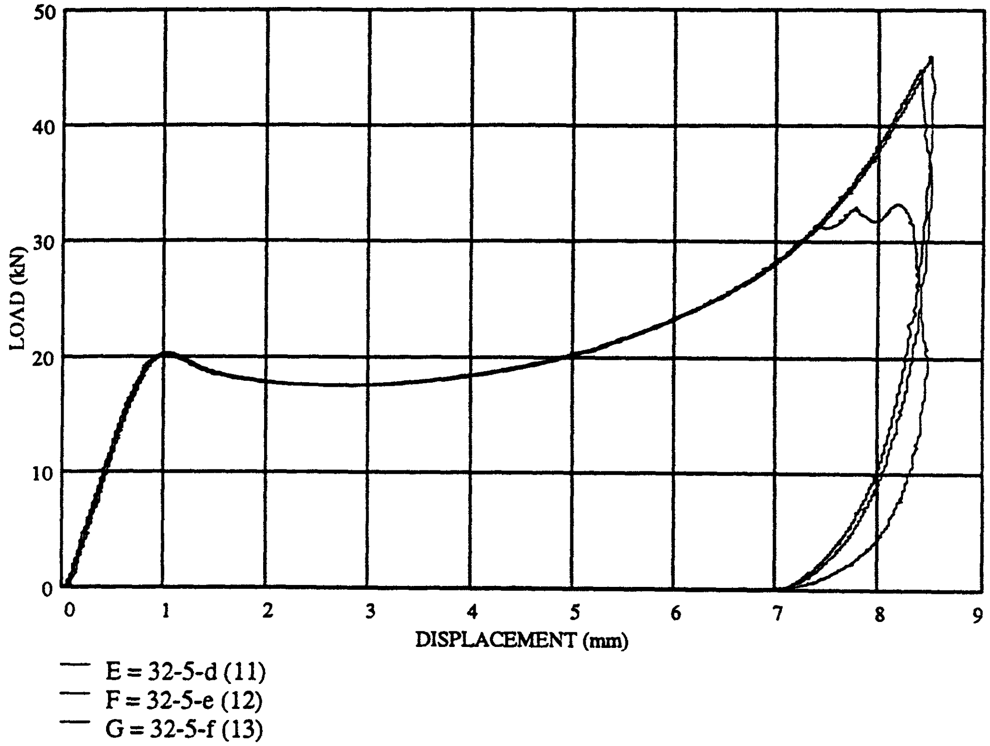
$\max(\text{load}_A) = 44.455 \cdot \text{kN}$ $\max(\text{displ}_A) = 8.6 \cdot \text{mm}$

$\max(\text{load}_B) = 43.918 \cdot \text{kN}$ $\max(\text{displ}_B) = 8.749 \cdot \text{mm}$

$\max(\text{load}_C) = 45.383 \cdot \text{kN}$ $\max(\text{displ}_C) = 8.55 \cdot \text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 4 (2.50 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max(\text{load}_E) = 33.268 \cdot \text{kN}$ $\max(\text{displ}_E) = 8.476 \cdot \text{mm}$

$\max(\text{load}_F) = 45.921 \cdot \text{kN}$ $\max(\text{displ}_F) = 8.526 \cdot \text{mm}$

$\max(\text{load}_G) = 44.846 \cdot \text{kN}$ $\max(\text{displ}_G) = 8.501 \cdot \text{mm}$

Calculate true stress and true strain (assume incompressibility):

$\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a}$ $\quad A_{iA_a} := \dfrac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}}$ $\quad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b}$ $\quad A_{iB_b} := \dfrac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$

$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c}$ $\quad A_{iC_c} := \dfrac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}}$ $\quad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d}$ $\quad A_{iD_d} := \dfrac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$

$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e}$ $\quad A_{iE_e} := \dfrac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}}$ $\quad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f}$ $\quad A_{iF_f} := \dfrac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$

$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g}$ $\quad A_{iG_g} := \dfrac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}}$ $\quad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h}$ $\quad A_{iH_h} := \dfrac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$

Calculate true stress (MPa) and true strain:

$\sigma_{trueA_a} := \dfrac{\mathrm{load}_{A_a}}{A_{iA_a}}$ $\quad \sigma_{trueB_b} := \dfrac{\mathrm{load}_{B_b}}{A_{iB_b}}$ $\quad \sigma_{trueC_c} := \dfrac{\mathrm{load}_{C_c}}{A_{iC_c}}$ $\quad \sigma_{trueD_d} := \dfrac{\mathrm{load}_{D_d}}{A_{iD_d}}$

$\varepsilon_{trueA_a} := -\ln\left(\dfrac{L_{iA_a}}{L_{o_1}}\right)$ $\quad \varepsilon_{trueB_b} := -\ln\left(\dfrac{L_{iB_b}}{L_{o_2}}\right)$ $\quad \varepsilon_{trueC_c} := -\ln\left(\dfrac{L_{iC_c}}{L_{o_3}}\right)$ $\quad \varepsilon_{trueD_d} := -\ln\left(\dfrac{L_{iD_d}}{L_{o_4}}\right)$

$\sigma_{trueE_e} := \dfrac{\mathrm{load}_{E_e}}{A_{iE_e}}$ $\quad \sigma_{trueF_f} := \dfrac{\mathrm{load}_{F_f}}{A_{iF_f}}$ $\quad \sigma_{trueG_g} := \dfrac{\mathrm{load}_{G_g}}{A_{iG_g}}$ $\quad \sigma_{trueH_h} := \dfrac{\mathrm{load}_{H_h}}{A_{iH_h}}$

$\varepsilon_{trueE_e} := -\ln\left(\dfrac{L_{iE_e}}{L_{o_5}}\right)$ $\quad \varepsilon_{trueF_f} := -\ln\left(\dfrac{L_{iF_f}}{L_{o_6}}\right)$ $\quad \varepsilon_{trueG_g} := -\ln\left(\dfrac{L_{iG_g}}{L_{o_7}}\right)$ $\quad \varepsilon_{trueH_h} := -\ln\left(\dfrac{L_{iH_h}}{L_{o_8}}\right)$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$$

$$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot MPa\right) \quad \sigma_{yF} := \max\left(\sigma_{yFcalc}\right) \quad NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$$

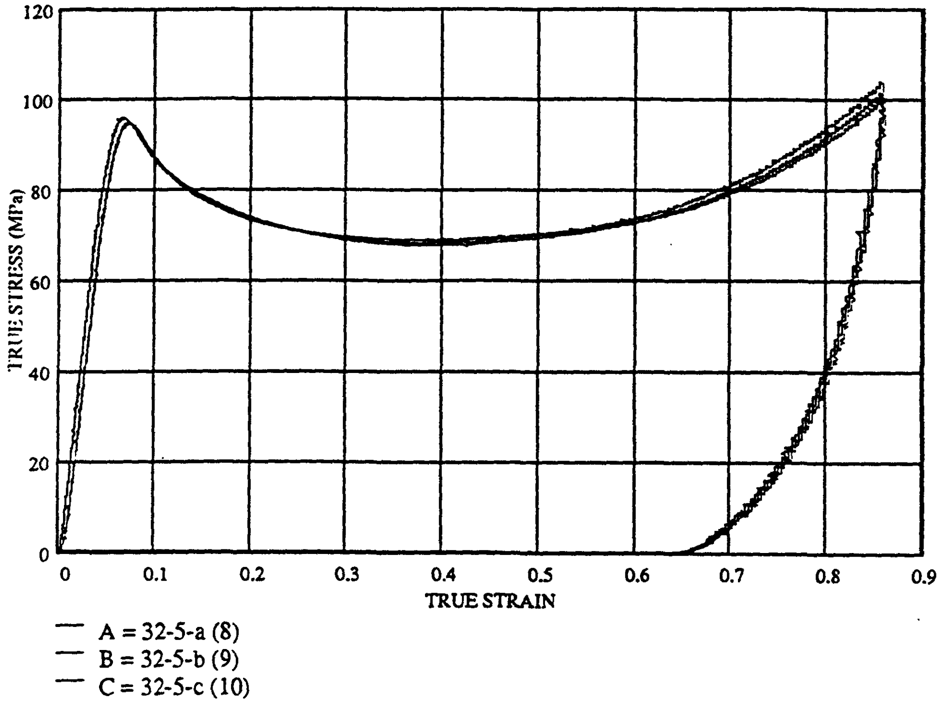
$$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 1200, \sigma_{trueG_g}, 0 \cdot MPa\right) \quad \sigma_{yG} := \max\left(\sigma_{yGcalc}\right) \quad NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$$

$$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot MPa\right) \quad \sigma_{yH} := \max\left(\sigma_{yHcalc}\right) \quad NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

$$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}} \quad \varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}} \quad \varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}} \quad \varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$$

Generate a plot of true stress (MPa) versus true strain for load rate 3 (1.00 mm/sec):



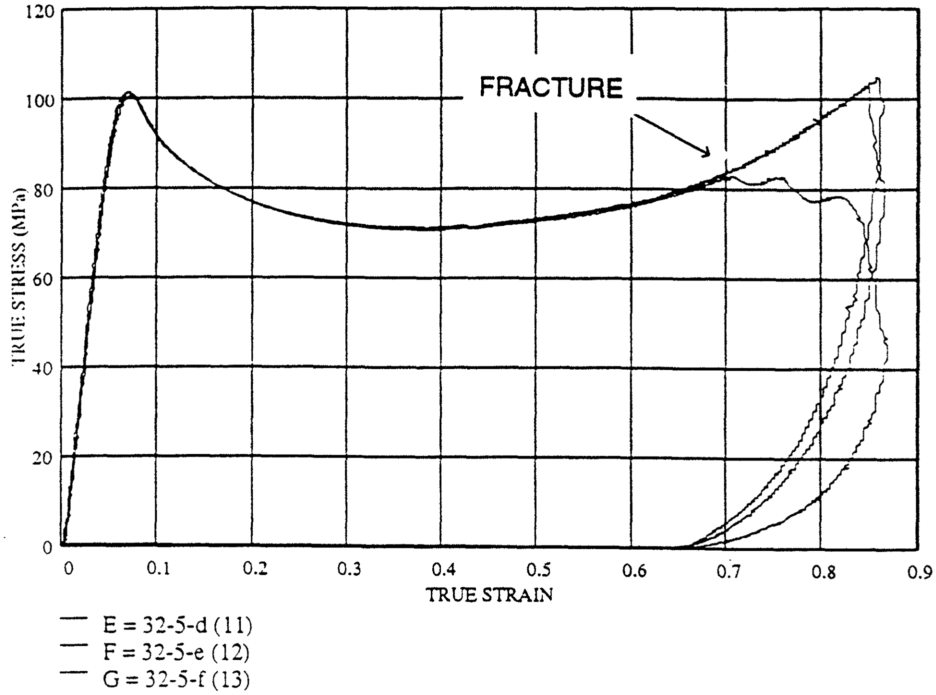
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 0.862$ $\max\left(\sigma_{\text{trueA}}\right) = 101.544 \cdot \text{MPa}$ $\sigma_{yA} = 94.798 \cdot \text{MPa}$ $\varepsilon_{yA} = 0.071$

$\max\left(\varepsilon_{\text{trueB}}\right) = 0.86$ $\max\left(\sigma_{\text{trueB}}\right) = 100.185 \cdot \text{MPa}$ $\sigma_{yB} = 94.716 \cdot \text{MPa}$ $\varepsilon_{yB} = 0.073$

$\max\left(\varepsilon_{\text{trueC}}\right) = 0.858$ $\max\left(\sigma_{\text{trueC}}\right) = 103.943 \cdot \text{MPa}$ $\sigma_{yC} = 96.025 \cdot \text{MPa}$ $\varepsilon_{yC} = 0.066$

Generate a plot of true stress (MPa) versus true strain for load rate 4 (2.50 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{trueE}) = 0.868$ $\max(\sigma_{trueE}) = 100.572 \cdot \mathrm{MPa}$ $\sigma_{yE} = 100.572 \cdot \mathrm{MPa}$ $\varepsilon_{yE} = 0.07$

$\max(\varepsilon_{trueF}) = 0.864$ $\max(\sigma_{trueF}) = 104.907 \cdot \mathrm{MPa}$ $\sigma_{yF} = 101.388 \cdot \mathrm{MPa}$ $\varepsilon_{yF} = 0.07$

$\max(\varepsilon_{trueG}) = 0.864$ $\max(\sigma_{trueG}) = 102.739 \cdot \mathrm{MPa}$ $\sigma_{yG} = 101.569 \cdot \mathrm{MPa}$ $\varepsilon_{yG} = 0.07$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..4$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN(COMP3B)}$ $\mathrm{SPEC}_A := 2$

$L_{o_1} := \left(\mathrm{DIM}_A^{<1>}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$ $D_{o_1} := \left(\mathrm{DIM}_A^{<2>}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN(COMP3B)}$ $\mathrm{SPEC}_B := 6$

$L_{o_2} := \left(\mathrm{DIM}_B^{<1>}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$ $D_{o_2} := \left(\mathrm{DIM}_B^{<2>}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN(COMP3B)}$ $\mathrm{SPEC}_C := 10$

$L_{o_3} := \left(\mathrm{DIM}_C^{<1>}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$ $D_{o_3} := \left(\mathrm{DIM}_C^{<2>}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN(COMP3B)}$ $\mathrm{SPEC}_D := 13$

$L_{o_4} := \left(\mathrm{DIM}_D^{<1>}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$ $D_{o_4} := \left(\mathrm{DIM}_D^{<2>}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 181.697 \\ 184.335 \\ 183.374 \\ 183.134 \end{bmatrix} \cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN(IC3LR1T6)}$

$B := \mathrm{READPRN(IC3LR2T7)}$

$C := \mathrm{READPRN(IC3LR3T6)}$

$D := \mathrm{READPRN(IC3LR4T6)}$

Note: Files A-D are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000 \cdot \mathrm{newton}$

$\mathrm{load}_A := A^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_A := A^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\mathrm{load}_D := D^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_D := D^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_B := B^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_B := B^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_C := C^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_C := C^{\langle 2 \rangle} \cdot \mathrm{mm}$

Look at matrices and calculate number of data points:

$a := 1 .. \mathrm{rows}(A)$ $d := 1 .. \mathrm{rows}(D)$

$b := 1 .. \mathrm{rows}(B)$

$c := 1 .. \mathrm{rows}(C)$

Calculate true stress and true strain (assume incompressibility): $\mathrm{MPa} := 1 \cdot 10^{6} \cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{\mathrm{load}_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

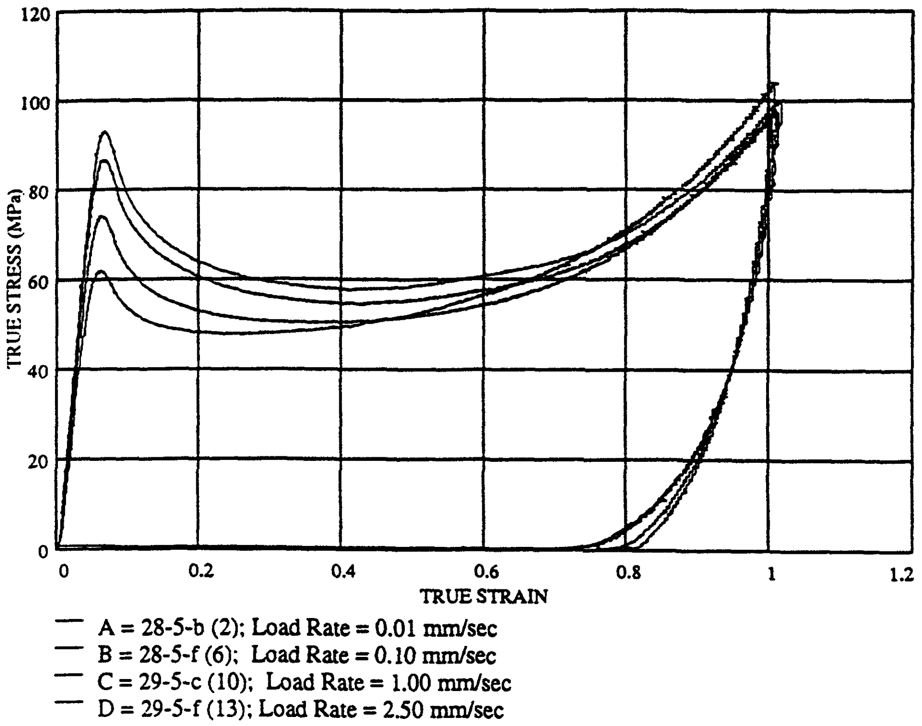
$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

Generate a plot of true stress (MPa) versus true strain for 50% 8084 / 50% 1080 at various load rates:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{trueA}\right) = 1.012$ $\max\left(\sigma_{trueA}\right) = 104.044 \cdot MPa$ $\sigma_{yA} = 62.089 \cdot MPa$ $\varepsilon_{yA} = 0.059$

$\max\left(\varepsilon_{trueB}\right) = 1.01$ $\max\left(\sigma_{trueB}\right) = 98.384 \cdot MPa$ $\sigma_{yB} = 74.187 \cdot MPa$ $\varepsilon_{yB} = 0.063$

$\max\left(\varepsilon_{trueC}\right) = 1.013$ $\max\left(\sigma_{trueC}\right) = 97.772 \cdot MPa$ $\sigma_{yC} = 86.821 \cdot MPa$ $\varepsilon_{yC} = 0.066$

$\max\left(\varepsilon_{trueD}\right) = 1.017$ $\max\left(\sigma_{trueD}\right) = 100 \cdot MPa$ $\sigma_{yD} = 93.068 \cdot MPa$ $\varepsilon_{yD} = 0.067$

Select specimens to be used: $z := 1 .. 8$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM} := \mathrm{READPRN}(\mathrm{COMP3B})$

$$\mathrm{SPEC} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 5 \\ 6 \\ 7 \\ 21 \end{bmatrix} \qquad L_{o_z} := \left(\mathrm{DIM}^{<1>}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm} \qquad D_{o_z} := \left(\mathrm{DIM}^{<2>}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm}$$

Show initial length (mm):

$$L_o = \begin{bmatrix} 14.67 \\ 15.15 \\ 14.78 \\ 14.67 \\ 14.53 \\ 14.35 \\ 15.58 \\ 15.26 \end{bmatrix} \cdot \mathrm{mm}$$

Show initial diameter (mm):

$$D_o = \begin{bmatrix} 15.35 \\ 15.21 \\ 15.28 \\ 15.35 \\ 15.29 \\ 15.32 \\ 15.35 \\ 15.29 \end{bmatrix} \cdot \mathrm{mm}$$

Length/Diameter ratio(LDR):

$$\mathrm{LDR}_z := \frac{L_{o_z}}{D_{o_z}} \qquad \mathrm{LDR} = \begin{bmatrix} 0.956 \\ 0.996 \\ 0.967 \\ 0.956 \\ 0.95 \\ 0.937 \\ 1.015 \\ 0.998 \end{bmatrix}$$

Calculate the initial cross sectional area (mm^2):

$$A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2 \qquad A_{cso} = \begin{bmatrix} 185.057 \\ 181.697 \\ 183.374 \\ 185.057 \\ 183.614 \\ 184.335 \\ 185.057 \\ 183.614 \end{bmatrix} \cdot \mathrm{mm}^2$$

Input maximum true strain (compressive values negative):

$$\varepsilon_{max} := \begin{bmatrix} -1.00 \\ -1.00 \\ -1.00 \\ 1 \\ -1.00 \\ -1.00 \\ -1.00 \\ -1.00 \end{bmatrix}$$

Input loading head actuator speed (mm/sec):

$$\mathrm{Load}_{rate} := \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 1 \\ 0.10 \\ 0.10 \\ 0.10 \\ 0.10 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{0_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 5.397 \\ 5.573 \\ 5.437 \\ 39.877 \\ 5.345 \\ 5.279 \\ 5.732 \\ 5.614 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the machine stroke (mm):

$$\mathrm{Stroke}_z := L_{f_z} - L_{0_z}$$

$$\mathrm{Stroke} = \begin{bmatrix} -9.273 \\ -9.577 \\ -9.343 \\ 25.207 \\ -9.185 \\ -9.071 \\ -9.848 \\ -9.646 \end{bmatrix} \cdot \mathrm{mm}$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $\mathrm{Stroke}_{tot_z} := 2 \cdot \mathrm{Stroke}_z$

Calculate the total time required for test:

$$\mathrm{time}_{tot_z} := \left| \frac{\mathrm{Stroke}_{tot_z}}{\mathrm{Load}_{rate_z}} \right|$$

$$\mathrm{Stroke}_{tot} = \begin{bmatrix} -18.546 \\ -19.153 \\ -18.685 \\ 50.414 \\ -18.369 \\ -18.142 \\ -19.697 \\ -19.292 \end{bmatrix} \cdot \mathrm{mm}$$

$$\mathrm{time}_{tot} = \begin{bmatrix} 1854.642 \\ 1915.325 \\ 1868.548 \\ 50.414 \\ 183.694 \\ 181.419 \\ 196.969 \\ 192.923 \end{bmatrix} \cdot \mathrm{sec} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 30.911 \\ 31.922 \\ 31.142 \\ 0.84 \\ 3.062 \\ 3.024 \\ 3.283 \\ 3.215 \end{bmatrix} \cdot \mathrm{min} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 0.515 \\ 0.532 \\ 0.519 \\ 0.014 \\ 0.051 \\ 0.05 \\ 0.055 \\ 0.054 \end{bmatrix} \cdot \mathrm{hr}$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$\mathrm{Freq}_{data} := \begin{bmatrix} 5 \\ 5 \\ 5 \\ 1 \\ 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} \cdot \mathrm{Hz}$$

Calculate approximate number of data points:

$$\mathrm{NBR}_{data_z} := \mathrm{Freq}_{data_z} \cdot \mathrm{time}_{tot_z}$$

$$\mathrm{NBR}_{data} = \begin{bmatrix} 9273.209 \\ 9576.626 \\ 9342.742 \\ 50.414 \\ 9184.712 \\ 9070.93 \\ 9848.438 \\ 9646.16 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(IC3LR1T5)$ $E := \mathrm{READPRN}(IC3LR2T6)$

$B := \mathrm{READPRN}(IC3LR1T6)$ $F := \mathrm{READPRN}(IC3LR2T7)$

$C := \mathrm{READPRN}(IC3LR1T7)$ $G := \mathrm{READPRN}(IC3LR2T8)$

$D := \mathrm{READPRN}(IC0LR1T1)$ $H := \mathrm{READPRN}(IC3LR2T5)$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000 \cdot \mathrm{newton}$

$\mathrm{load}_A := A^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_A := A^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\mathrm{load}_E := E^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_E := E^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_B := B^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_B := B^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\mathrm{load}_F := F^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_F := F^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_C := C^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_C := C^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\mathrm{load}_G := G^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_G := G^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_D := D^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_D := D^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\mathrm{load}_H := H^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\mathrm{displ}_H := H^{\langle 2 \rangle} \cdot \mathrm{mm}$

Look at matrices and calculate number of data points:

$a := 1 .. \mathrm{rows}(A)$ $e := 1 .. \mathrm{rows}(E)$

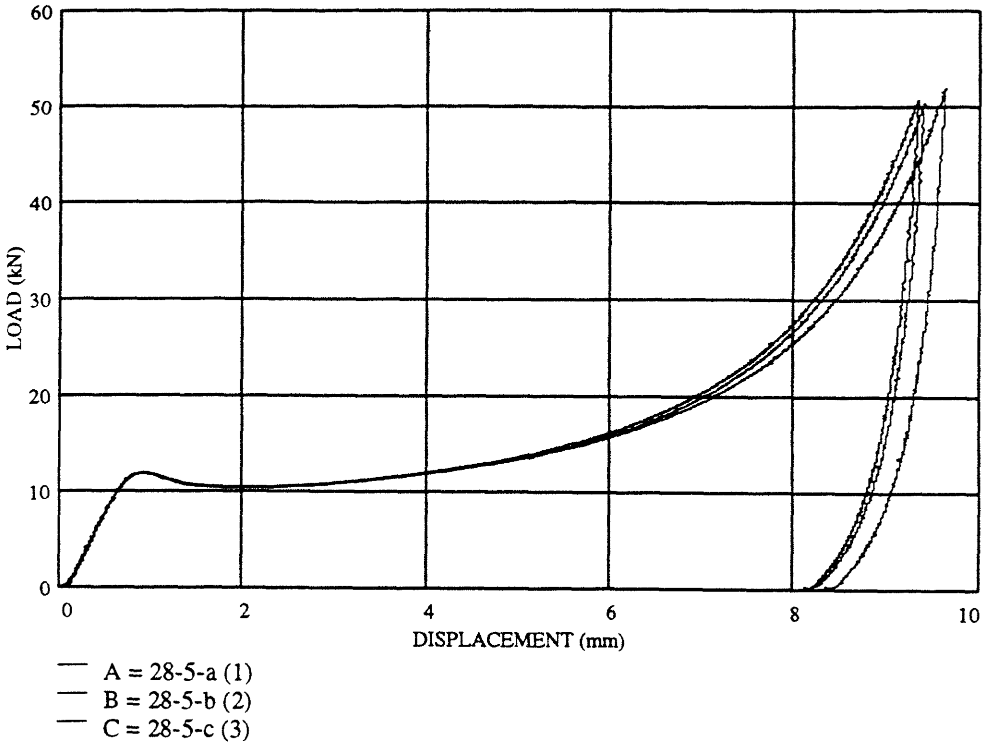
$b := 1 .. \mathrm{rows}(B)$ $f := 1 .. \mathrm{rows}(F)$

$c := 1 .. \mathrm{rows}(C)$ $g := 1 .. \mathrm{rows}(G)$

$d := 1 .. \mathrm{rows}(D)$ $h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 1 (0.01 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

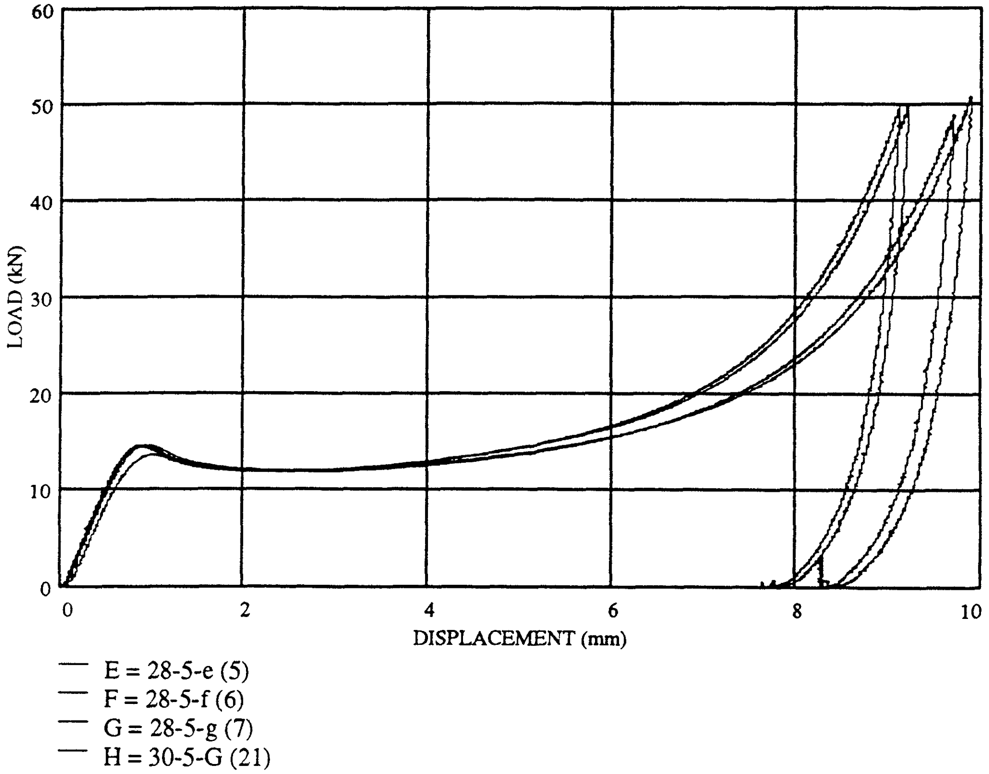
$\max\left(\text{load}_A\right) = 50.904 \cdot \text{kN}$ $\quad \max\left(\text{displ}_A\right) = 9.346 \cdot \text{mm}$

$\max\left(\text{load}_B\right) = 51.93 \cdot \text{kN}$ $\quad \max\left(\text{displ}_B\right) = 9.644 \cdot \text{mm}$

$\max\left(\text{load}_C\right) = 50.318 \cdot \text{kN}$ $\quad \max\left(\text{displ}_C\right) = 9.42 \cdot \text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 2 (0.10 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max\left(\mathrm{load}_E\right) = 49.976 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_E\right) = 9.222 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_F\right) = 49.78 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_F\right) = 9.122 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_G\right) = 50.708 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_G\right) = 9.893 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_H\right) = 48.852 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_H\right) = 9.719 \cdot \mathrm{mm}$

Calculate true stress and true strain (assume incompressibility):

$$\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}} \qquad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f} \qquad A_{iF_f} := \frac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$$

$$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g} \qquad A_{iG_g} := \frac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}} \qquad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h} \qquad A_{iH_h} := \frac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{\mathrm{load}_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{\mathrm{load}_{E_e}}{A_{iE_e}} \qquad \sigma_{trueF_f} := \frac{\mathrm{load}_{F_f}}{A_{iF_f}} \qquad \sigma_{trueG_g} := \frac{\mathrm{load}_{G_g}}{A_{iG_g}} \qquad \sigma_{trueH_h} := \frac{\mathrm{load}_{H_h}}{A_{iH_h}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right) \qquad \varepsilon_{trueF_f} := -\ln\left(\frac{L_{iF_f}}{L_{o_6}}\right) \qquad \varepsilon_{trueG_g} := -\ln\left(\frac{L_{iG_g}}{L_{o_7}}\right) \qquad \varepsilon_{trueH_h} := -\ln\left(\frac{L_{iH_h}}{L_{o_8}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right)$ $\sigma_{yE} := \max\left(\sigma_{yEcalc}\right)$ $NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$

$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot MPa\right)$ $\sigma_{yF} := \max\left(\sigma_{yFcalc}\right)$ $NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$

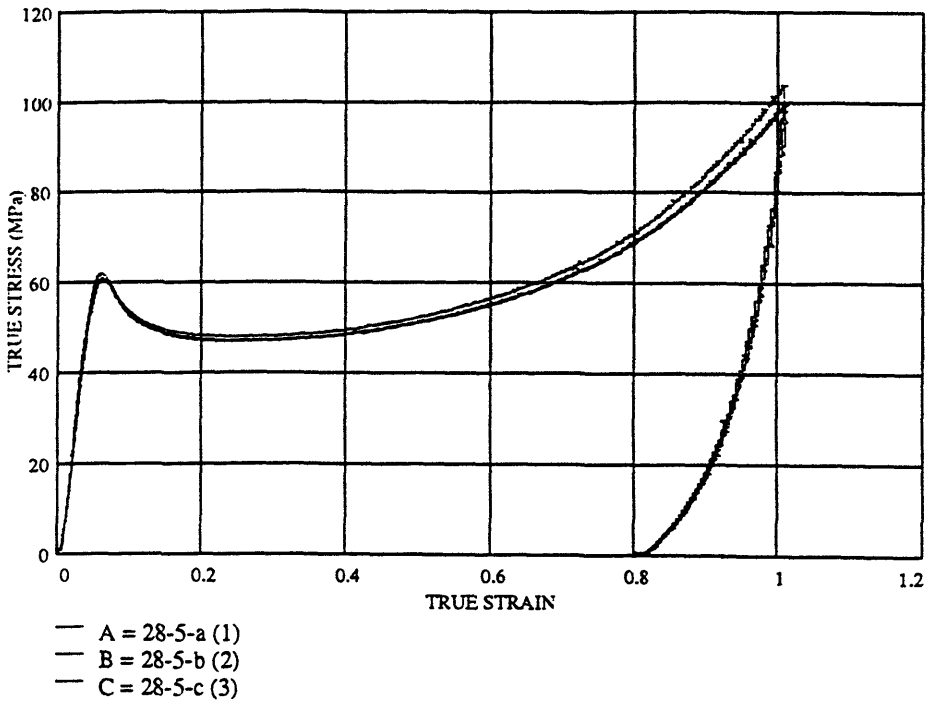
$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 1200, \sigma_{trueG_g}, 0 \cdot MPa\right)$ $\sigma_{yG} := \max\left(\sigma_{yGcalc}\right)$ $NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$

$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot MPa\right)$ $\sigma_{yH} := \max\left(\sigma_{yHcalc}\right)$ $NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$ $\varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}}$ $\varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}}$ $\varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for load rate 1 (0.01 mm/sec):



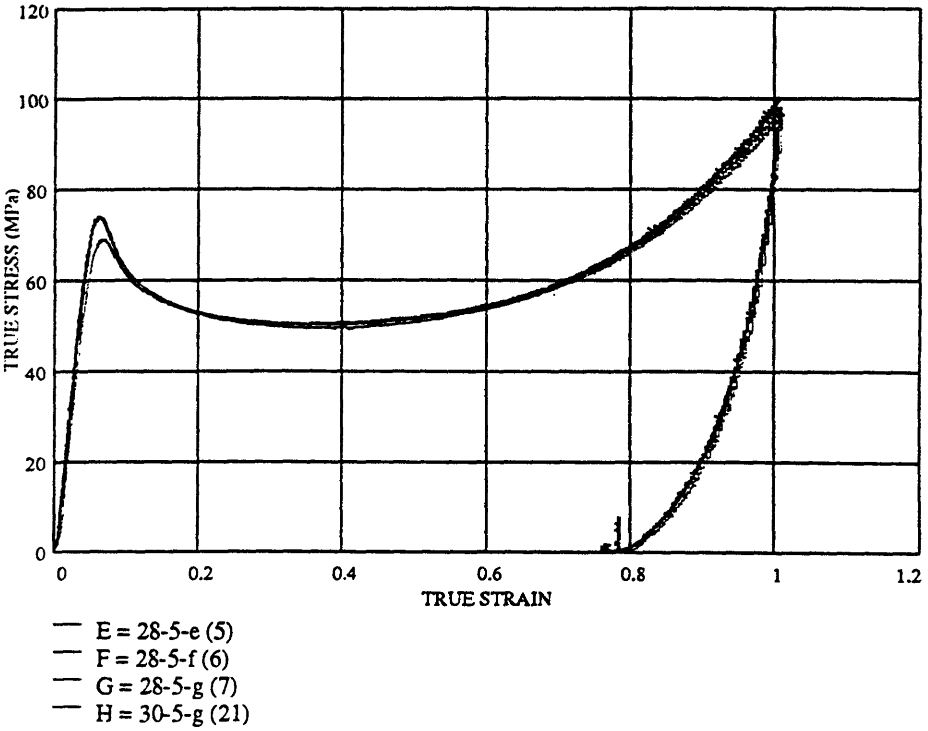
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{trueA}) = 1.014$ $\max(\sigma_{trueA}) = 99.832 \cdot \mathrm{MPa}$ $\sigma_{yA} = 60.483 \cdot \mathrm{MPa}$ $\varepsilon_{yA} = 0.063$

$\max(\varepsilon_{trueB}) = 1.012$ $\max(\sigma_{trueB}) = 104.044 \cdot \mathrm{MPa}$ $\sigma_{yB} = 62.089 \cdot \mathrm{MPa}$ $\varepsilon_{yB} = 0.059$

$\max(\varepsilon_{trueC}) = 1.014$ $\max(\sigma_{trueC}) = 99.966 \cdot \mathrm{MPa}$ $\sigma_{yC} = 61.068 \cdot \mathrm{MPa}$ $\varepsilon_{yC} = 0.062$

Generate a plot of true stress (MPa) versus true strain for load rate 2 (0.10 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{trueE}) = 1.007$ $\max(\sigma_{trueE}) = 99.44 \cdot MPa$ $\sigma_{yE} = 74.038 \cdot MPa$ $\varepsilon_{yE} = 0.062$

$\max(\varepsilon_{trueF}) = 1.01$ $\max(\sigma_{trueF}) = 98.384 \cdot MPa$ $\sigma_{yF} = 74.187 \cdot MPa$ $\varepsilon_{yF} = 0.063$

$\max(\varepsilon_{trueG}) = 1.008$ $\max(\sigma_{trueG}) = 100.367 \cdot MPa$ $\sigma_{yG} = 74.271 \cdot MPa$ $\varepsilon_{yG} = 0.061$

$\max(\varepsilon_{trueH}) = 1.013$ $\max(\sigma_{trueH}) = 97.047 \cdot MPa$ $\sigma_{yH} = 69.146 \cdot MPa$ $\varepsilon_{yH} = 0.067$

Select specimens to be used: $z := 1 .. 8$ $\quad$ $ORIGIN := 1$

$DIM := READPRN(COMP3B)$

$$SPEC := \begin{bmatrix} 8 \\ 9 \\ 10 \\ 1 \\ 12 \\ 13 \\ 14 \\ 1 \end{bmatrix}$$

$$L_{0_z} := \left(DIM^{\langle 1 \rangle}\right)_{SPEC_{(z,1)}} \cdot mm \qquad D_{0_z} := \left(DIM^{\langle 2 \rangle}\right)_{SPEC_{(z,1)}} \cdot mm$$

Show initial length (mm):

$$L_0 = \begin{bmatrix} 14.23 \\ 14.88 \\ 14.87 \\ 14.67 \\ 15.28 \\ 15.38 \\ 15.19 \\ 14.67 \end{bmatrix} \cdot mm$$

Show initial diameter (mm):

$$D_0 = \begin{bmatrix} 15.31 \\ 15.29 \\ 15.28 \\ 15.35 \\ 15.24 \\ 15.27 \\ 15.34 \\ 15.35 \end{bmatrix} \cdot mm$$

Length/Diameter ratio(LDR):

$$LDR_z := \frac{L_{0_z}}{D_{0_z}}$$

$$LDR = \begin{bmatrix} 0.929 \\ 0.973 \\ 0.973 \\ 0.956 \\ 1.003 \\ 1.007 \\ 0.99 \\ 0.956 \end{bmatrix}$$

Calculate the initial cross sectional area (mm^2):

$$A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{0_z}\right)^2$$

$$A_{cso} = \begin{bmatrix} 184.094 \\ 183.614 \\ 183.374 \\ 185.057 \\ 182.415 \\ 183.134 \\ 184.816 \\ 185.057 \end{bmatrix} \cdot mm^2$$

Input maximum true strain (compressive values negative):

$$\varepsilon_{max} := \begin{bmatrix} -1.00 \\ -1.00 \\ -1.00 \\ 1 \\ -1.00 \\ -1.00 \\ -1.00 \\ 1 \end{bmatrix}$$

Input loading head actuator speed (mm/sec):

$$Load_{rate} := \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1 \\ 2.50 \\ 2.50 \\ 2.50 \\ 1 \end{bmatrix} \cdot \frac{mm}{sec}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{o_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 5.235 \\ 5.474 \\ 5.47 \\ 39.877 \\ 5.621 \\ 5.658 \\ 5.588 \\ 39.877 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the machine stroke (mm):

$$\mathrm{Stroke}_z := L_{f_z} - L_{o_z}$$

$$\mathrm{Stroke} = \begin{bmatrix} -8.995 \\ -9.406 \\ -9.4 \\ 25.207 \\ -9.659 \\ -9.722 \\ -9.602 \\ 25.207 \end{bmatrix} \cdot \mathrm{mm}$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $\mathrm{Stroke}_{tot_z} := 2 \cdot \mathrm{Stroke}_z$

Calculate the total time required for test:

$$\mathrm{time}_{tot_z} := \left| \frac{\mathrm{Stroke}_{tot_z}}{\mathrm{Load}_{rate_z}} \right|$$

$$\mathrm{Stroke}_{tot} = \begin{bmatrix} -17.99 \\ -18.812 \\ -18.799 \\ 50.414 \\ -19.318 \\ -19.444 \\ -19.204 \\ 50.414 \end{bmatrix} \cdot \mathrm{mm}$$

$$\mathrm{time}_{tot} = \begin{bmatrix} 17.99 \\ 18.812 \\ 18.799 \\ 50.414 \\ 7.727 \\ 7.778 \\ 7.682 \\ 50.414 \end{bmatrix} \cdot \mathrm{sec} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 0.3 \\ 0.314 \\ 0.313 \\ 0.84 \\ 0.129 \\ 0.13 \\ 0.128 \\ 0.84 \end{bmatrix} \cdot \mathrm{min} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 0.005 \\ 0.005 \\ 0.005 \\ 0.014 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.014 \end{bmatrix} \cdot \mathrm{hr}$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$\mathrm{Freq}_{data} := \begin{bmatrix} 400 \\ 400 \\ 400 \\ 1 \\ 400 \\ 400 \\ 400 \\ 1 \end{bmatrix} \cdot \mathrm{Hz}$$

Calculate approximate number of data points:

$$\mathrm{NBR}_{data_z} := \mathrm{Freq}_{data_z} \cdot \mathrm{time}_{tot_z}$$

$$\mathrm{NBR}_{data} = \begin{bmatrix} 7196.06 \\ 7524.763 \\ 7519.706 \\ 50.414 \\ 3090.817 \\ 3111.045 \\ 3072.612 \\ 50.414 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(\mathrm{IC3LR3T4})$ $E := \mathrm{READPRN}(\mathrm{IC3LR4T5})$

$B := \mathrm{READPRN}(\mathrm{IC3LR3T5})$ $F := \mathrm{READPRN}(\mathrm{IC3LR4T6})$

$C := \mathrm{READPRN}(\mathrm{IC3LR3T6})$ $G := \mathrm{READPRN}(\mathrm{IC3LR4T7})$

$D := \mathrm{READPRN}(\mathrm{IC0LR1T1})$ $H := \mathrm{READPRN}(\mathrm{IC0LR1T1})$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000\cdot\mathrm{newton}$

$\mathrm{load}_A := A^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_A := A^{\langle 2\rangle}\cdot\mathrm{mm}$ $\mathrm{load}_E := E^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_E := E^{\langle 2\rangle}\cdot\mathrm{mm}$

$\mathrm{load}_B := B^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_B := B^{\langle 2\rangle}\cdot\mathrm{mm}$ $\mathrm{load}_F := F^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_F := F^{\langle 2\rangle}\cdot\mathrm{mm}$

$\mathrm{load}_C := C^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_C := C^{\langle 2\rangle}\cdot\mathrm{mm}$ $\mathrm{load}_G := G^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_G := G^{\langle 2\rangle}\cdot\mathrm{mm}$

$\mathrm{load}_D := D^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_D := D^{\langle 2\rangle}\cdot\mathrm{mm}$ $\mathrm{load}_H := H^{\langle 1\rangle}\cdot\mathrm{kN}$ $\mathrm{displ}_H := H^{\langle 2\rangle}\cdot\mathrm{mm}$

Look at matrices and calculate number of data points:

$a := 1..\mathrm{rows}(A)$ $e := 1..\mathrm{rows}(E)$

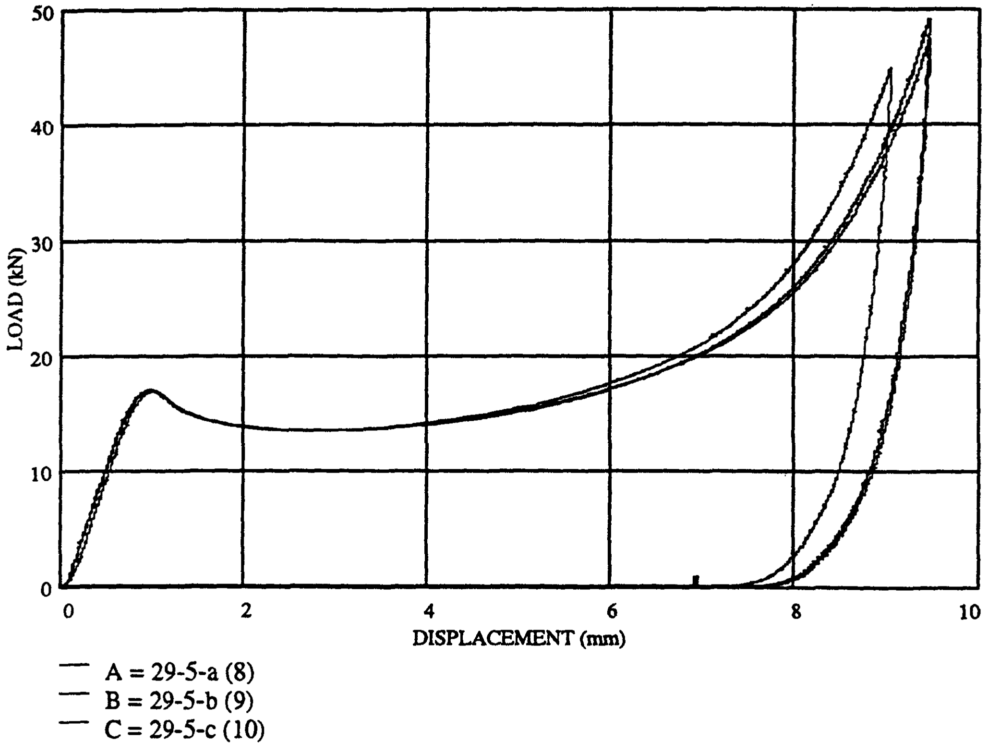
$b := 1..\mathrm{rows}(B)$ $f := 1..\mathrm{rows}(F)$

$c := 1..\mathrm{rows}(C)$ $g := 1..\mathrm{rows}(G)$

$d := 1..\mathrm{rows}(D)$ $h := 1..\mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 3 (1.00 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

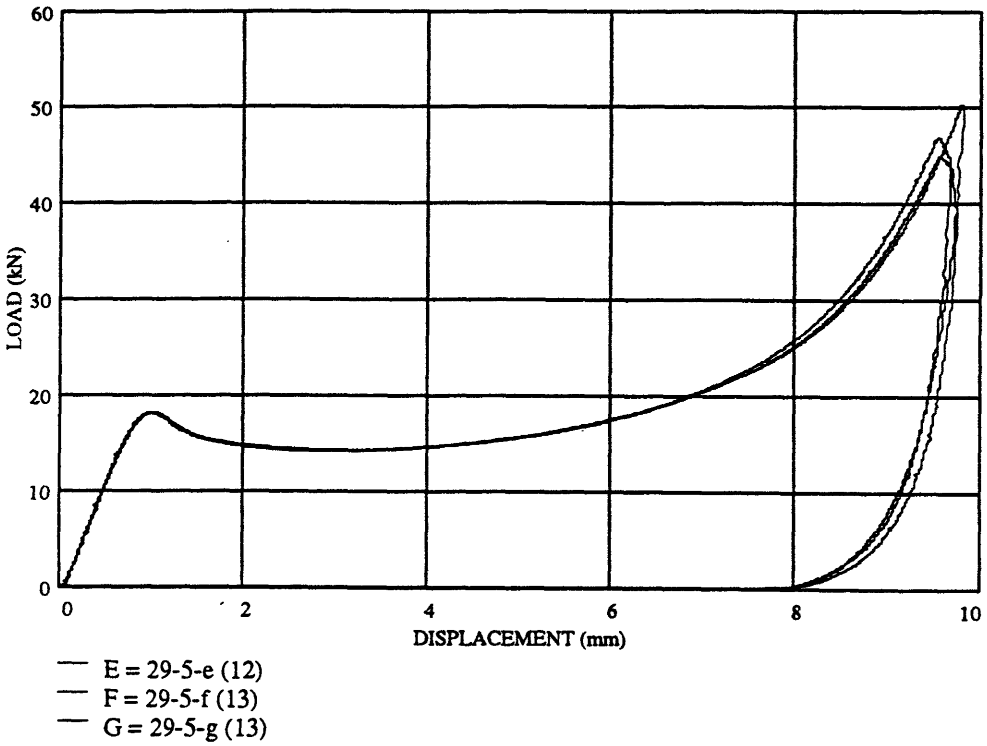
$\max\left(\mathrm{load}_A\right) = 44.944 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_A\right) = 9.047 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_B\right) = 47.386 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_B\right) = 9.47 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_C\right) = 49.145 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_C\right) = 9.47 \cdot \mathrm{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 4 (2.50 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max\left(\mathrm{load}_E\right) = 44.797 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_E\right) = 9.743 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_F\right) = 50.415 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_F\right) = 9.818 \cdot \mathrm{mm}$

$\max\left(\mathrm{load}_G\right) = 46.849 \cdot \mathrm{kN}$ $\max\left(\mathrm{displ}_G\right) = 9.669 \cdot \mathrm{mm}$

Calculate true stress and true strain (assume incompressibility):

$$\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}} \qquad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f} \qquad A_{iF_f} := \frac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$$

$$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g} \qquad A_{iG_g} := \frac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}} \qquad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h} \qquad A_{iH_h} := \frac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{\mathrm{load}_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{\mathrm{load}_{E_e}}{A_{iE_e}} \qquad \sigma_{trueF_f} := \frac{\mathrm{load}_{F_f}}{A_{iF_f}} \qquad \sigma_{trueG_g} := \frac{\mathrm{load}_{G_g}}{A_{iG_g}} \qquad \sigma_{trueH_h} := \frac{\mathrm{load}_{H_h}}{A_{iH_h}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right) \qquad \varepsilon_{trueF_f} := -\ln\left(\frac{L_{iF_f}}{L_{o_6}}\right) \qquad \varepsilon_{trueG_g} := -\ln\left(\frac{L_{iG_g}}{L_{o_7}}\right) \qquad \varepsilon_{trueH_h} := -\ln\left(\frac{L_{iH_h}}{L_{o_8}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right)$ $\sigma_{yE} := \max\left(\sigma_{yEcalc}\right)$ $NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$

$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot MPa\right)$ $\sigma_{yF} := \max\left(\sigma_{yFcalc}\right)$ $NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$

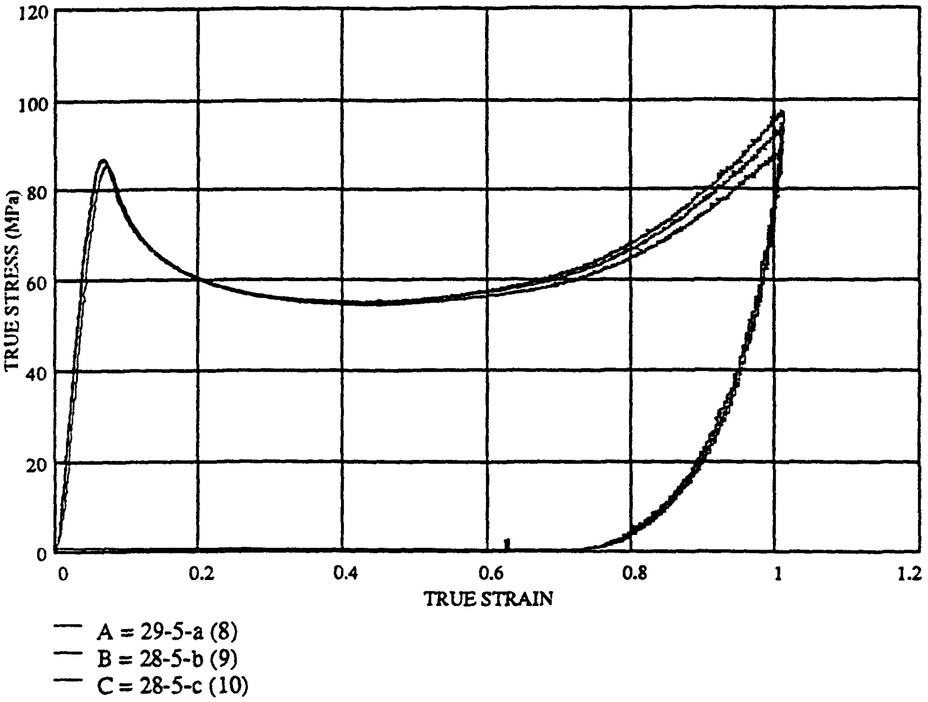
$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 1200, \sigma_{trueG_g}, 0 \cdot MPa\right)$ $\sigma_{yG} := \max\left(\sigma_{yGcalc}\right)$ $NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$

$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot MPa\right)$ $\sigma_{yH} := \max\left(\sigma_{yHcalc}\right)$ $NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$ $\varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}}$ $\varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}}$ $\varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for load rate 3 (1.00 mm/sec):



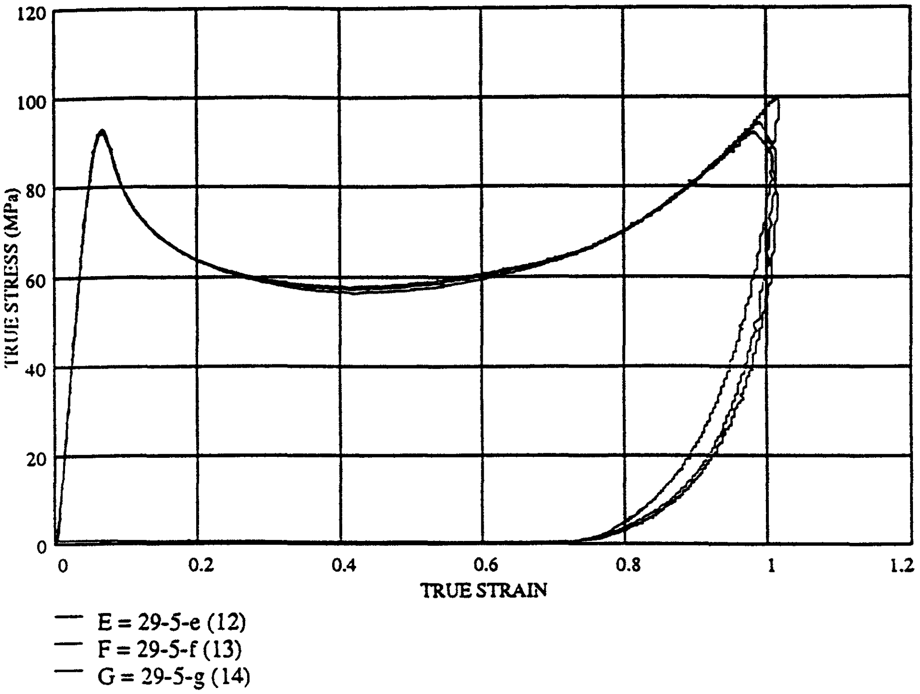
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\mathrm{trueA}}) = 1.01$ $\max(\sigma_{\mathrm{trueA}}) = 88.95 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 85.648 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.072$

$\max(\varepsilon_{\mathrm{trueB}}) = 1.012$ $\max(\sigma_{\mathrm{trueB}}) = 94.066 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 87.054 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.067$

$\max(\varepsilon_{\mathrm{trueC}}) = 1.013$ $\max(\sigma_{\mathrm{trueC}}) = 97.772 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 86.821 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.066$

Generate a plot of true stress (MPa) versus true strain for load rate 4 (2.50 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueE}}\right) = 1.015$ $\max\left(\sigma_{\text{trueE}}\right) = 92.802 \cdot \text{MPa}$ $\sigma_{yE} = 92.802 \cdot \text{MPa}$ $\varepsilon_{yE} = 0.066$

$\max\left(\varepsilon_{\text{trueF}}\right) = 1.017$ $\max\left(\sigma_{\text{trueF}}\right) = 100 \cdot \text{MPa}$ $\sigma_{yF} = 93.068 \cdot \text{MPa}$ $\varepsilon_{yF} = 0.067$

$\max\left(\varepsilon_{\text{trueG}}\right) = 1.012$ $\max\left(\sigma_{\text{trueG}}\right) = 94.208 \cdot \text{MPa}$ $\sigma_{yG} = 92.141 \cdot \text{MPa}$ $\varepsilon_{yG} = 0.068$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1 .. 4$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP4})$ $\qquad$ $\mathrm{SPEC}_A := 2$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$ $\qquad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP4})$ $\qquad$ $\mathrm{SPEC}_B := 6$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$ $\qquad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP4})$ $\qquad$ $\mathrm{SPEC}_C := 9$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$ $\qquad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP4})$ $\qquad$ $\mathrm{SPEC}_D := 18$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$ $\qquad$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 190.645 \\ 191.134 \\ 190.645 \\ 191.625 \end{bmatrix} \cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(\mathrm{IC4LR1T2})$

$B := \mathrm{READPRN}(\mathrm{IC4LR2T2})$

$C := \mathrm{READPRN}(\mathrm{IC4LR3T2})$

$D := \mathrm{READPRN}(\mathrm{IC4LR4T2})$

Note: Files A-D are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{<1>} \cdot kN$ $displ_A := A^{<2>} \cdot mm$ $load_D := D^{<1>} \cdot kN$ $displ_D := D^{<2>} \cdot mm$

$load_B := B^{<1>} \cdot kN$ $displ_B := B^{<2>} \cdot mm$

$load_C := C^{<1>} \cdot kN$ $displ_C := C^{<2>} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \quad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \quad L_{iB_b} := L_{o_2} - displ_{B_b} \quad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \quad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \quad L_{iD_d} := L_{o_4} - displ_{D_d} \quad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \quad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \quad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \quad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \quad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \quad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \quad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \text{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \text{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \text{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

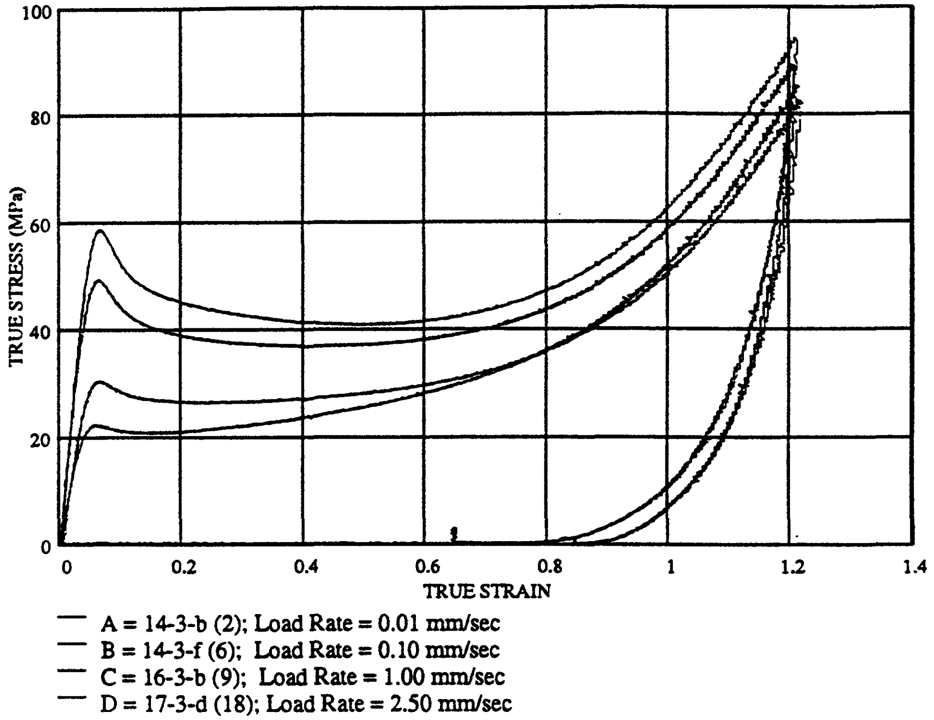
$$\sigma_{yBcalc_b} := \text{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \text{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \text{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \text{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot \text{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \text{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \text{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot \text{MPa}\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \text{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

Generate a plot of true stress (MPa) versus true strain for 25% 8084 / 75% 1080 at various load rates:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\mathrm{trueA}}) = 1.216$ $\max(\sigma_{\mathrm{trueA}}) = 85.539 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 22.46 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.059$

$\max(\varepsilon_{\mathrm{trueB}}) = 1.222$ $\max(\sigma_{\mathrm{trueB}}) = 82.846 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 30.551 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.068$

$\max(\varepsilon_{\mathrm{trueC}}) = 1.211$ $\max(\sigma_{\mathrm{trueC}}) = 89.4 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 49.394 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.066$

$\max(\varepsilon_{\mathrm{trueD}}) = 1.212$ $\max(\sigma_{\mathrm{trueD}}) = 94.333 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yD}} = 58.77 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yD}} = 0.069$

Select specimens to be used: $z := 1..8$ $\quad$ $ORIGIN := 1$

$DIM := READPRN(COMP4)$

$$SPEC := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 5 \\ 6 \\ 7 \\ 1 \end{bmatrix} \qquad L_{o_z} := \left(DIM^{<1>}\right)_{SPEC_{(z,1)}} \cdot mm \qquad D_{o_z} := \left(DIM^{<2>}\right)_{SPEC_{(z,1)}} \cdot mm$$

Show initial length (mm):

$$L_o = \begin{bmatrix} 15.66 \\ 15.12 \\ 16.18 \\ 15.66 \\ 15.87 \\ 15.4 \\ 15.75 \\ 15.66 \end{bmatrix} \cdot mm$$

Show initial diameter (mm):

$$D_o = \begin{bmatrix} 15.6 \\ 15.58 \\ 15.6 \\ 15.6 \\ 15.62 \\ 15.6 \\ 15.6 \\ 15.6 \end{bmatrix} \cdot mm$$

Length/Diameter ratio(LDR):

$$LDR_z := \frac{L_{o_z}}{D_{o_z}} \qquad LDR = \begin{bmatrix} 1.004 \\ 0.97 \\ 1.037 \\ 1.004 \\ 1.016 \\ 0.987 \\ 1.01 \\ 1.004 \end{bmatrix}$$

Calculate the initial cross sectional area (mm^2):

$$A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2 \qquad A_{cso} = \begin{bmatrix} 191.134 \\ 190.645 \\ 191.134 \\ 191.134 \\ 191.625 \\ 191.134 \\ 191.134 \\ 191.134 \end{bmatrix} \cdot mm^2$$

Input maximum true strain (compressive values negative):

$$\varepsilon_{max} := \begin{bmatrix} -1.20 \\ -1.20 \\ -1.20 \\ 1 \\ -1.20 \\ -1.20 \\ -1.20 \\ 1 \end{bmatrix}$$

Input loading head actuator speed (mm/sec):

$$Load_{rate} := \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 1 \\ 0.10 \\ 0.10 \\ 0.10 \\ 1 \end{bmatrix} \cdot \frac{mm}{sec}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{o_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 4.717 \\ 4.554 \\ 4.873 \\ 42.568 \\ 4.78 \\ 4.638 \\ 4.744 \\ 42.568 \end{bmatrix} \cdot \mathrm{mm}$$

Calculate the machine stroke (mm):

$$\mathrm{Stroke}_z := L_{f_z} - L_{o_z}$$

$$\mathrm{Stroke} = \begin{bmatrix} -10.943 \\ -10.566 \\ -11.307 \\ 26.908 \\ -11.09 \\ -10.762 \\ -11.006 \\ 26.908 \end{bmatrix} \cdot \mathrm{mm}$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $\mathrm{Stroke}_{tot_z} := 2 \cdot \mathrm{Stroke}_z$

Calculate the total time required for test:

$$\mathrm{time}_{tot_z} := \left| \frac{\mathrm{Stroke}_{tot_z}}{\mathrm{Load}_{rate_z}} \right|$$

$$\mathrm{Stroke}_{tot} = \begin{bmatrix} -21.887 \\ -21.132 \\ -22.613 \\ 53.817 \\ -22.18 \\ -21.523 \\ -22.012 \\ 53.817 \end{bmatrix} \cdot \mathrm{mm}$$

$$\mathrm{time}_{tot} = \begin{bmatrix} 2188.66 \\ 2113.189 \\ 2261.336 \\ 53.817 \\ 221.801 \\ 215.232 \\ 220.124 \\ 53.817 \end{bmatrix} \cdot \mathrm{sec} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 36.478 \\ 35.22 \\ 37.689 \\ 0.897 \\ 3.697 \\ 3.587 \\ 3.669 \\ 0.897 \end{bmatrix} \cdot \mathrm{min} \qquad \mathrm{time}_{tot} = \begin{bmatrix} 0.608 \\ 0.587 \\ 0.628 \\ 0.015 \\ 0.062 \\ 0.06 \\ 0.061 \\ 0.015 \end{bmatrix} \cdot \mathrm{hr}$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$\mathrm{Freq}_{data} := \begin{bmatrix} 5 \\ 5 \\ 5 \\ 1 \\ 50 \\ 50 \\ 50 \\ 1 \end{bmatrix} \cdot \mathrm{Hz}$$

Calculate approximate number of data points:

$$\mathrm{NBR}_{data_z} := \mathrm{Freq}_{data_z} \cdot \mathrm{time}_{tot_z}$$

$$\mathrm{NBR}_{data} = \begin{bmatrix} 10943.299 \\ 10565.944 \\ 11306.678 \\ 53.817 \\ 11090.048 \\ 10761.609 \\ 11006.191 \\ 53.817 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(IC4LR1T1)$ $E := \mathrm{READPRN}(IC4LR2T1)$

$B := \mathrm{READPRN}(IC4LR1T2)$ $F := \mathrm{READPRN}(IC4LR2T2)$

$C := \mathrm{READPRN}(IC4LR1T3)$ $G := \mathrm{READPRN}(IC4LR2T3)$

$D := \mathrm{READPRN}(IC0LR1T1)$ $H := \mathrm{READPRN}(IC0LR1T1)$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{\langle 1 \rangle} \cdot kN$ $displ_A := A^{\langle 2 \rangle} \cdot mm$ $load_E := E^{\langle 1 \rangle} \cdot kN$ $displ_E := E^{\langle 2 \rangle} \cdot mm$

$load_B := B^{\langle 1 \rangle} \cdot kN$ $displ_B := B^{\langle 2 \rangle} \cdot mm$ $load_F := F^{\langle 1 \rangle} \cdot kN$ $displ_F := F^{\langle 2 \rangle} \cdot mm$

$load_C := C^{\langle 1 \rangle} \cdot kN$ $displ_C := C^{\langle 2 \rangle} \cdot mm$ $load_G := G^{\langle 1 \rangle} \cdot kN$ $displ_G := G^{\langle 2 \rangle} \cdot mm$

$load_D := D^{\langle 1 \rangle} \cdot kN$ $displ_D := D^{\langle 2 \rangle} \cdot mm$ $load_H := H^{\langle 1 \rangle} \cdot kN$ $displ_H := H^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$ $e := 1 .. \mathrm{rows}(E)$

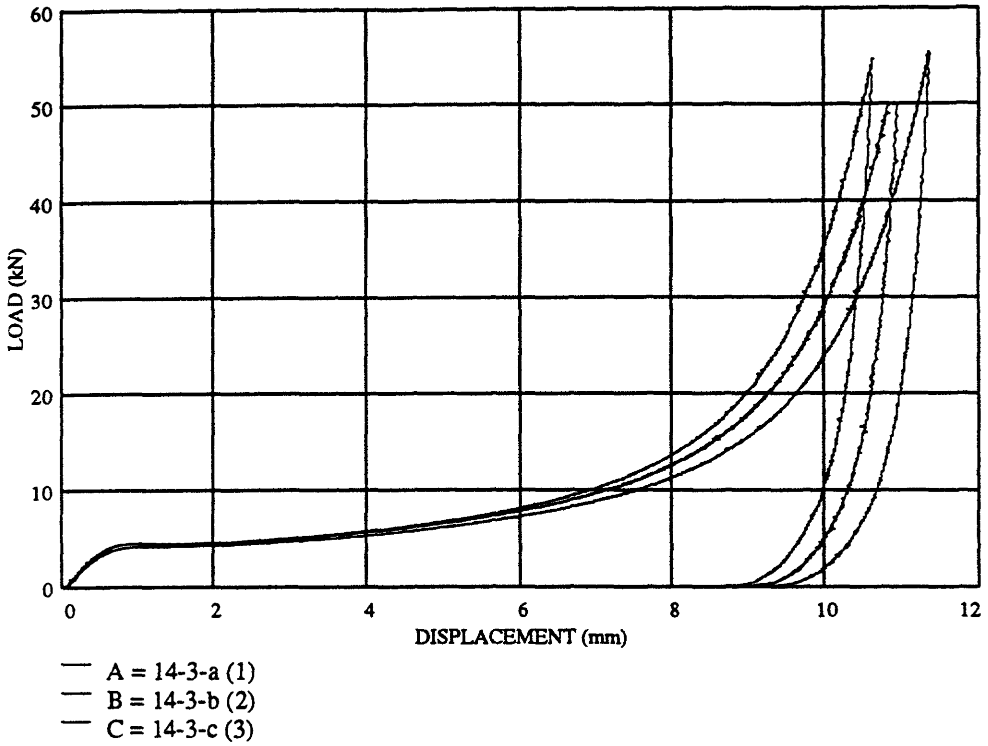
$b := 1 .. \mathrm{rows}(B)$ $f := 1 .. \mathrm{rows}(F)$

$c := 1 .. \mathrm{rows}(C)$ $g := 1 .. \mathrm{rows}(G)$

$d := 1 .. \mathrm{rows}(D)$ $h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 1 (0.01 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

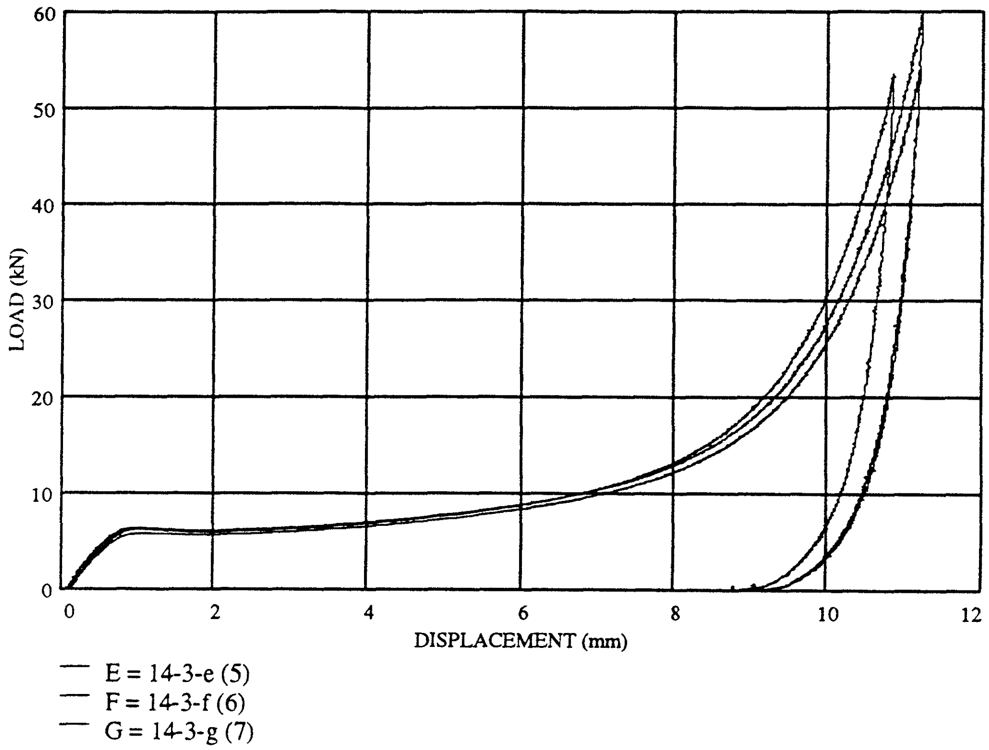
$\max(\text{load}_A) = 50.024 \cdot \text{kN}$ $\max(\text{displ}_A) = 10.986 \cdot \text{mm}$

$\max(\text{load}_B) = 54.714 \cdot \text{kN}$ $\max(\text{displ}_B) = 10.638 \cdot \text{mm}$

$\max(\text{load}_C) = 55.447 \cdot \text{kN}$ $\max(\text{displ}_C) = 11.384 \cdot \text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 2 (0.10 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max(\mathrm{load}_E) = 53.835 \cdot \mathrm{kN}$ $\quad$ $\max(\mathrm{displ}_E) = 11.185 \cdot \mathrm{mm}$

$\max(\mathrm{load}_F) = 53.444 \cdot \mathrm{kN}$ $\quad$ $\max(\mathrm{displ}_F) = 10.862 \cdot \mathrm{mm}$

$\max(\mathrm{load}_G) = 59.99 \cdot \mathrm{kN}$ $\quad$ $\max(\mathrm{displ}_G) = 11.21 \cdot \mathrm{mm}$

Calculate true stress and true strain (assume incompressibility):

$$MPa := 1\cdot 10^{6}\cdot Pa$$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - displ_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - displ_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - displ_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}} \qquad L_{iF_f} := L_{o_6} - displ_{F_f} \qquad A_{iF_f} := \frac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$$

$$L_{iG_g} := L_{o_7} - displ_{G_g} \qquad A_{iG_g} := \frac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}} \qquad L_{iH_h} := L_{o_8} - displ_{H_h} \qquad A_{iH_h} := \frac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{load_{E_e}}{A_{iE_e}} \qquad \sigma_{trueF_f} := \frac{load_{F_f}}{A_{iF_f}} \qquad \sigma_{trueG_g} := \frac{load_{G_g}}{A_{iG_g}} \qquad \sigma_{trueH_h} := \frac{load_{H_h}}{A_{iH_h}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right) \qquad \varepsilon_{trueF_f} := -\ln\left(\frac{L_{iF_f}}{L_{o_6}}\right) \qquad \varepsilon_{trueG_g} := -\ln\left(\frac{L_{iG_g}}{L_{o_7}}\right) \qquad \varepsilon_{trueH_h} := -\ln\left(\frac{L_{iH_h}}{L_{o_8}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 600, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$

$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot MPa\right) \quad \sigma_{yF} := \max\left(\sigma_{yFcalc}\right) \quad NumF_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$

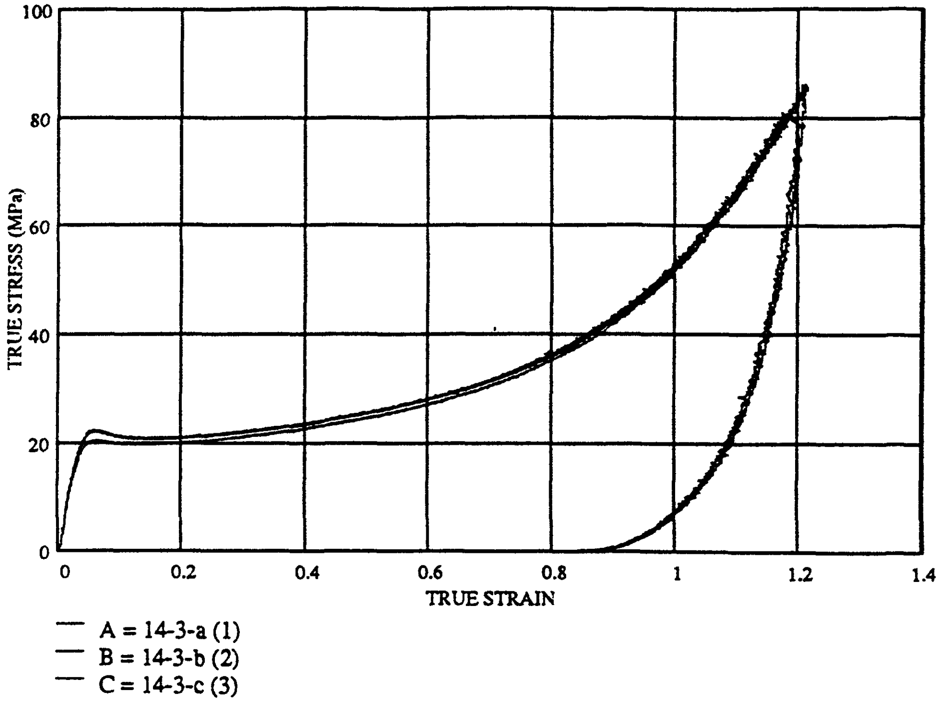
$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 600, \sigma_{trueG_g}, 0 \cdot MPa\right) \quad \sigma_{yG} := \max\left(\sigma_{yGcalc}\right) \quad NumG_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$

$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot MPa\right) \quad \sigma_{yH} := \max\left(\sigma_{yHcalc}\right) \quad NumH_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}} \quad \varepsilon_{yF} := \varepsilon_{trueF_{\max(NumF)}} \quad \varepsilon_{yG} := \varepsilon_{trueG_{\max(NumG)}} \quad \varepsilon_{yH} := \varepsilon_{trueH_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for load rate 1 (0.01 mm/sec):



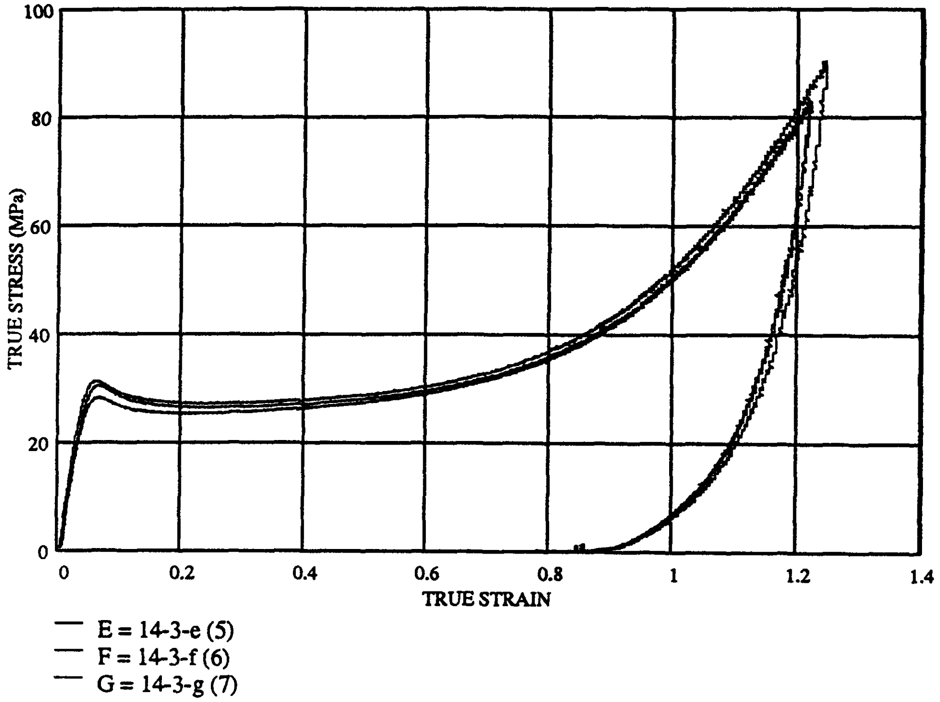
List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{trueA}\right) = 1.209$ $\max\left(\sigma_{trueA}\right) = 80.979 \cdot \mathrm{MPa}$ $\sigma_{yA} = 22.43 \cdot \mathrm{MPa}$ $\varepsilon_{yA} = 0.058$

$\max\left(\varepsilon_{trueB}\right) = 1.216$ $\max\left(\sigma_{trueB}\right) = 85.539 \cdot \mathrm{MPa}$ $\sigma_{yB} = 22.46 \cdot \mathrm{MPa}$ $\varepsilon_{yB} = 0.059$

$\max\left(\varepsilon_{trueC}\right) = 1.216$ $\max\left(\sigma_{trueC}\right) = 86.435 \cdot \mathrm{MPa}$ $\sigma_{yC} = 20.63 \cdot \mathrm{MPa}$ $\varepsilon_{yC} = 0.063$

Generate a plot of true stress (MPa) versus true strain for load rate 2 (0.10 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{trueE}\right) = 1.22$ $\max\left(\sigma_{trueE}\right) = 83.298 \cdot MPa$ $\sigma_{yE} = 28.342 \cdot MPa$ $\varepsilon_{yE} = 0.068$

$\max\left(\varepsilon_{trueF}\right) = 1.222$ $\max\left(\sigma_{trueF}\right) = 82.846 \cdot MPa$ $\sigma_{yF} = 30.551 \cdot MPa$ $\varepsilon_{yF} = 0.068$

$\max\left(\varepsilon_{trueG}\right) = 1.244$ $\max\left(\sigma_{trueG}\right) = 90.472 \cdot MPa$ $\sigma_{yG} = 31.369 \cdot MPa$ $\varepsilon_{yG} = 0.065$

Select specimens to be used: $z := 1..8$ $\quad$ ORIGIN := 1

DIM := READPRN(COMP4)

$$\mathrm{SPEC} := \begin{bmatrix} 8 \\ 9 \\ 10 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{bmatrix} \qquad L_{o_z} := \left(\mathrm{DIM}^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm} \qquad D_{o_z} := \left(\mathrm{DIM}^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm}$$

Show initial length (mm): $\qquad$ **Show initial diameter (mm):**

$$L_o = \begin{bmatrix} 15.92 \\ 15.47 \\ 15.45 \\ 15.5 \\ 15.51 \\ 16.03 \\ 15.3 \\ 16.08 \end{bmatrix} \cdot \mathrm{mm} \qquad D_o = \begin{bmatrix} 15.59 \\ 15.58 \\ 15.63 \\ 15.62 \\ 15.6 \\ 15.62 \\ 15.6 \\ 15.62 \end{bmatrix} \cdot \mathrm{mm}$$

Length/Diameter ratio(LDR): $\qquad$ **Calculate the initial cross sectional area (mm^2):**

$$\mathrm{LDR}_z := \frac{L_{o_z}}{D_{o_z}} \qquad A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$$

$$\mathrm{LDR} = \begin{bmatrix} 1.021 \\ 0.993 \\ 0.988 \\ 0.992 \\ 0.994 \\ 1.026 \\ 0.981 \\ 1.029 \end{bmatrix} \qquad A_{cso} = \begin{bmatrix} 190.89 \\ 190.645 \\ 191.87 \\ 191.625 \\ 191.134 \\ 191.625 \\ 191.134 \\ 191.625 \end{bmatrix} \cdot \mathrm{mm}^2$$

Input maximum true strain (compressive values negative): $\qquad$ **Input loading head actuator speed (mm/sec):**

$$\varepsilon_{max} := \begin{bmatrix} -1.20 \\ -1.20 \\ -1.20 \\ -1.20 \\ -1.20 \\ -1.20 \\ -1.20 \\ -1.20 \end{bmatrix} \qquad \mathrm{Load}_{rate} := \begin{bmatrix} 1.00 \\ 1.00 \\ 1.00 \\ 1.00 \\ 2.50 \\ 2.50 \\ 2.50 \\ 2.50 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{o_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 4.795 \\ 4.659 \\ 4.653 \\ 4.669 \\ 4.672 \\ 4.828 \\ 4.608 \\ 4.843 \end{bmatrix} \cdot mm$$

Calculate the machine stroke (mm):

$$Stroke_z := L_{f_z} - L_{o_z}$$

$$Stroke = \begin{bmatrix} -11.125 \\ -10.811 \\ -10.797 \\ -10.831 \\ -10.838 \\ -11.202 \\ -10.692 \\ -11.237 \end{bmatrix} \cdot mm$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $Stroke_{tot_z} := 2 \cdot Stroke_z$

$$Stroke_{tot} = \begin{bmatrix} -22.25 \\ -21.621 \\ -21.593 \\ -21.663 \\ -21.677 \\ -22.404 \\ -21.383 \\ -22.474 \end{bmatrix} \cdot mm$$

Calculate the total time required for test:

$$time_{tot_z} := \left| \frac{Stroke_{tot_z}}{Load_{rate_z}} \right|$$

$$time_{tot} = \begin{bmatrix} 22.25 \\ 21.621 \\ 21.593 \\ 21.663 \\ 8.671 \\ 8.961 \\ 8.553 \\ 8.989 \end{bmatrix} \cdot sec \qquad time_{tot} = \begin{bmatrix} 0.371 \\ 0.36 \\ 0.36 \\ 0.361 \\ 0.145 \\ 0.149 \\ 0.143 \\ 0.15 \end{bmatrix} \cdot min \qquad time_{tot} = \begin{bmatrix} 0.006 \\ 0.006 \\ 0.006 \\ 0.006 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \end{bmatrix} \cdot hr$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$Freq_{data} := \begin{bmatrix} 400 \\ 400 \\ 400 \\ 400 \\ 500 \\ 500 \\ 500 \\ 500 \end{bmatrix} \cdot Hz$$

Calculate approximate number of data points:

$$NBR_{data_z} := Freq_{data_z} \cdot time_{tot_z}$$

$$NBR_{data} = \begin{bmatrix} 8899.991 \\ 8648.42 \\ 8637.24 \\ 8665.192 \\ 4335.391 \\ 4480.743 \\ 4276.691 \\ 4494.719 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(IC4LR3T1)$ $\quad E := \mathrm{READPRN}(IC4LR4T1)$
$B := \mathrm{READPRN}(IC4LR3T2)$ $\quad F := \mathrm{READPRN}(IC4LR4T2)$
$C := \mathrm{READPRN}(IC4LR3T3)$ $\quad G := \mathrm{READPRN}(IC4LR4T3)$
$D := \mathrm{READPRN}(IC4LR3T4)$ $\quad H := \mathrm{READPRN}(IC4LR4T4)$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\quad \mathrm{kN} := 1000 \cdot \mathrm{newton}$

$\mathrm{load}_A := A^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_A := A^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\quad \mathrm{load}_E := E^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_E := E^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_B := B^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_B := B^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\quad \mathrm{load}_F := F^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_F := F^{\langle 2 \rangle} \cdot \mathrm{mm}$

$\mathrm{load}_C := C^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_C := C^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\quad \mathrm{load}_G := G^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_G := G^{\langle 2 \rangle} \cdot \mathrm{mm}$

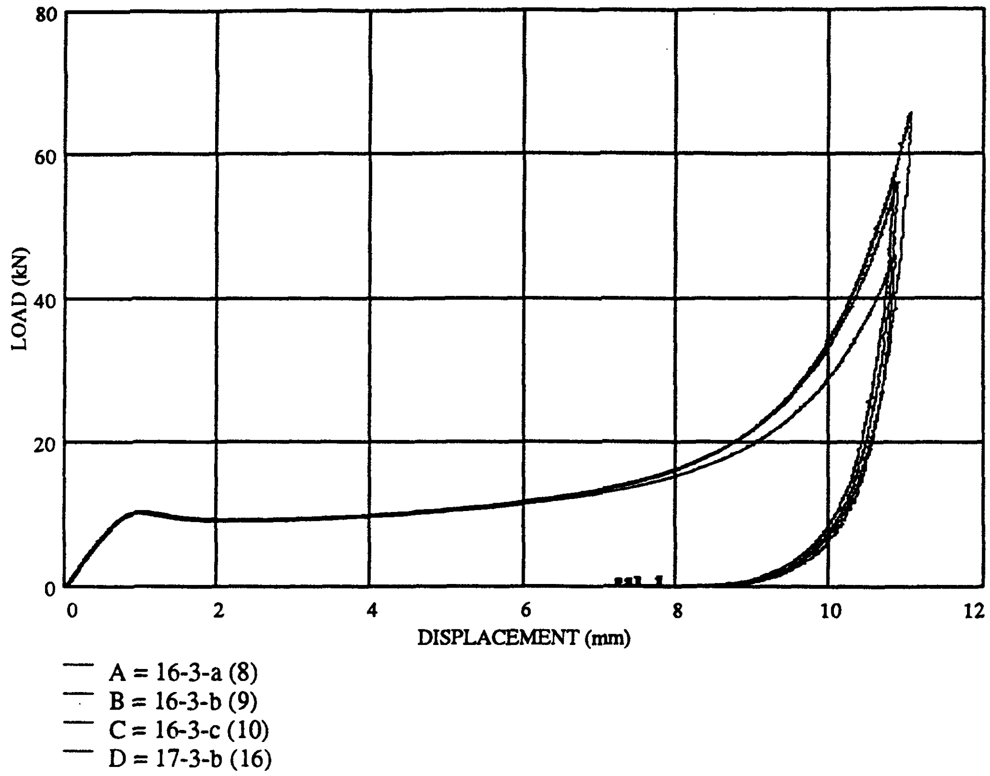
$\mathrm{load}_D := D^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_D := D^{\langle 2 \rangle} \cdot \mathrm{mm}$ $\quad \mathrm{load}_H := H^{\langle 1 \rangle} \cdot \mathrm{kN}$ $\quad \mathrm{displ}_H := H^{\langle 2 \rangle} \cdot \mathrm{mm}$

Look at matrices and calculate number of data points:

$a := 1 .. \mathrm{rows}(A)$ $\quad e := 1 .. \mathrm{rows}(E)$
$b := 1 .. \mathrm{rows}(B)$ $\quad f := 1 .. \mathrm{rows}(F)$
$c := 1 .. \mathrm{rows}(C)$ $\quad g := 1 .. \mathrm{rows}(G)$
$d := 1 .. \mathrm{rows}(D)$ $\quad h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 3 (1.00 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max\left(\text{load}_A\right) = 45.872 \cdot \text{kN}$ $\quad \max\left(\text{displ}_A\right) = 10.862 \cdot \text{mm}$

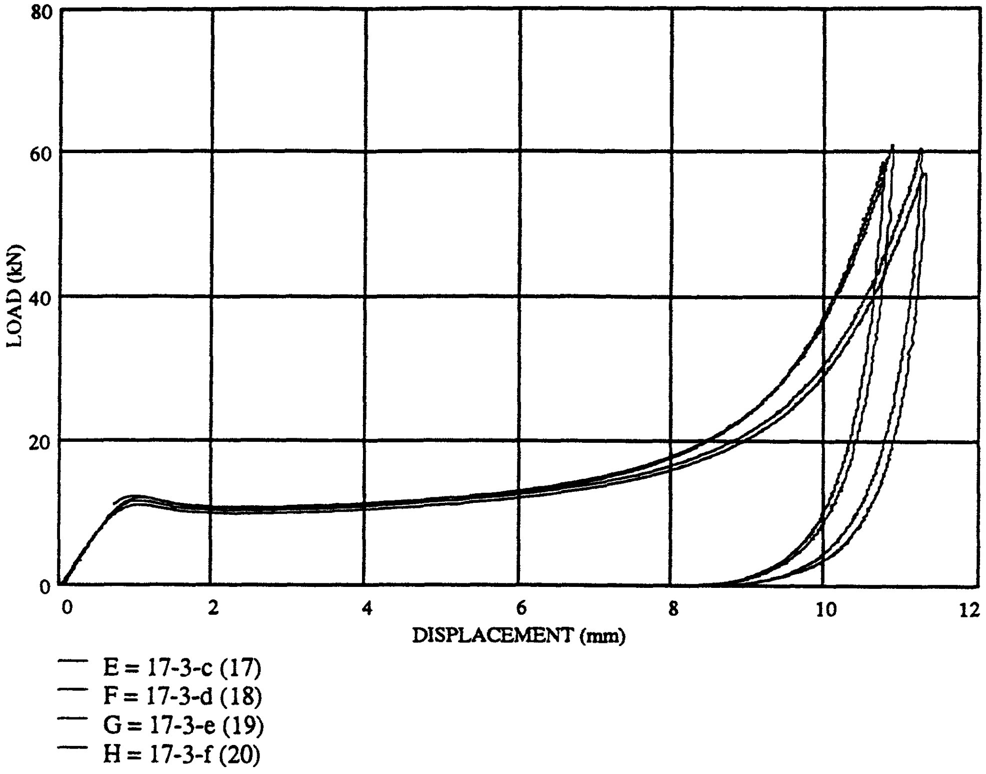
$\max\left(\text{load}_B\right) = 56.913 \cdot \text{kN}$ $\quad \max\left(\text{displ}_B\right) = 10.862 \cdot \text{mm}$

$\max\left(\text{load}_C\right) = 65.804 \cdot \text{kN}$ $\quad \max\left(\text{displ}_C\right) = 11.061 \cdot \text{mm}$

$\max\left(\text{load}_D\right) = 55.984 \cdot \text{kN}$ $\quad \max\left(\text{displ}_D\right) = 10.912 \cdot \text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 4 (2.50 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max(\text{load}_E) = 60.918 \cdot \text{kN}$ $\quad \max(\text{displ}_E) = 10.887 \cdot \text{mm}$

$\max(\text{load}_F) = 60.43 \cdot \text{kN}$ $\quad \max(\text{displ}_F) = 11.26 \cdot \text{mm}$

$\max(\text{load}_G) = 58.525 \cdot \text{kN}$ $\quad \max(\text{displ}_G) = 10.763 \cdot \text{mm}$

$\max(\text{load}_H) = 57.206 \cdot \text{kN}$ $\quad \max(\text{displ}_H) = 11.309 \cdot \text{mm}$

Calculate true stress and true strain (assume incompressibility):

$\mathrm{MPa} := 1 \cdot 10^{6} \cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a}$ $\quad A_{iA_a} := \dfrac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}}$ $\quad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b}$ $\quad A_{iB_b} := \dfrac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$

$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c}$ $\quad A_{iC_c} := \dfrac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}}$ $\quad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d}$ $\quad A_{iD_d} := \dfrac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$

$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e}$ $\quad A_{iE_e} := \dfrac{A_{cso_5} \cdot L_{o_5}}{L_{iE_e}}$ $\quad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f}$ $\quad A_{iF_f} := \dfrac{A_{cso_6} \cdot L_{o_6}}{L_{iF_f}}$

$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g}$ $\quad A_{iG_g} := \dfrac{A_{cso_7} \cdot L_{o_7}}{L_{iG_g}}$ $\quad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h}$ $\quad A_{iH_h} := \dfrac{A_{cso_8} \cdot L_{o_8}}{L_{iH_h}}$

Calculate true stress (MPa) and true strain:

$\sigma_{trueA_a} := \dfrac{\mathrm{load}_{A_a}}{A_{iA_a}}$ $\quad \sigma_{trueB_b} := \dfrac{\mathrm{load}_{B_b}}{A_{iB_b}}$ $\quad \sigma_{trueC_c} := \dfrac{\mathrm{load}_{C_c}}{A_{iC_c}}$ $\quad \sigma_{trueD_d} := \dfrac{\mathrm{load}_{D_d}}{A_{iD_d}}$

$\varepsilon_{trueA_a} := -\ln\left(\dfrac{L_{iA_a}}{L_{o_1}}\right)$ $\quad \varepsilon_{trueB_b} := -\ln\left(\dfrac{L_{iB_b}}{L_{o_2}}\right)$ $\quad \varepsilon_{trueC_c} := -\ln\left(\dfrac{L_{iC_c}}{L_{o_3}}\right)$ $\quad \varepsilon_{trueD_d} := -\ln\left(\dfrac{L_{iD_d}}{L_{o_4}}\right)$

$\sigma_{trueE_e} := \dfrac{\mathrm{load}_{E_e}}{A_{iE_e}}$ $\quad \sigma_{trueF_f} := \dfrac{\mathrm{load}_{F_f}}{A_{iF_f}}$ $\quad \sigma_{trueG_g} := \dfrac{\mathrm{load}_{G_g}}{A_{iG_g}}$ $\quad \sigma_{trueH_h} := \dfrac{\mathrm{load}_{H_h}}{A_{iH_h}}$

$\varepsilon_{trueE_e} := -\ln\left(\dfrac{L_{iE_e}}{L_{o_5}}\right)$ $\quad \varepsilon_{trueF_f} := -\ln\left(\dfrac{L_{iF_f}}{L_{o_6}}\right)$ $\quad \varepsilon_{trueG_g} := -\ln\left(\dfrac{L_{iG_g}}{L_{o_7}}\right)$ $\quad \varepsilon_{trueH_h} := -\ln\left(\dfrac{L_{iH_h}}{L_{o_8}}\right)$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad \mathrm{NumA}_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad \mathrm{NumB}_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 600, \sigma_{trueC_c}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad \mathrm{NumC}_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad \mathrm{NumD}_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad \mathrm{NumE}_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$$

$$\sigma_{yFcalc_f} := \mathrm{if}\left(f \le 1200, \sigma_{trueF_f}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yF} := \max\left(\sigma_{yFcalc}\right) \quad \mathrm{NumF}_f := \mathrm{if}\left(\sigma_{trueF_f} = \sigma_{yF}, f, 0\right)$$

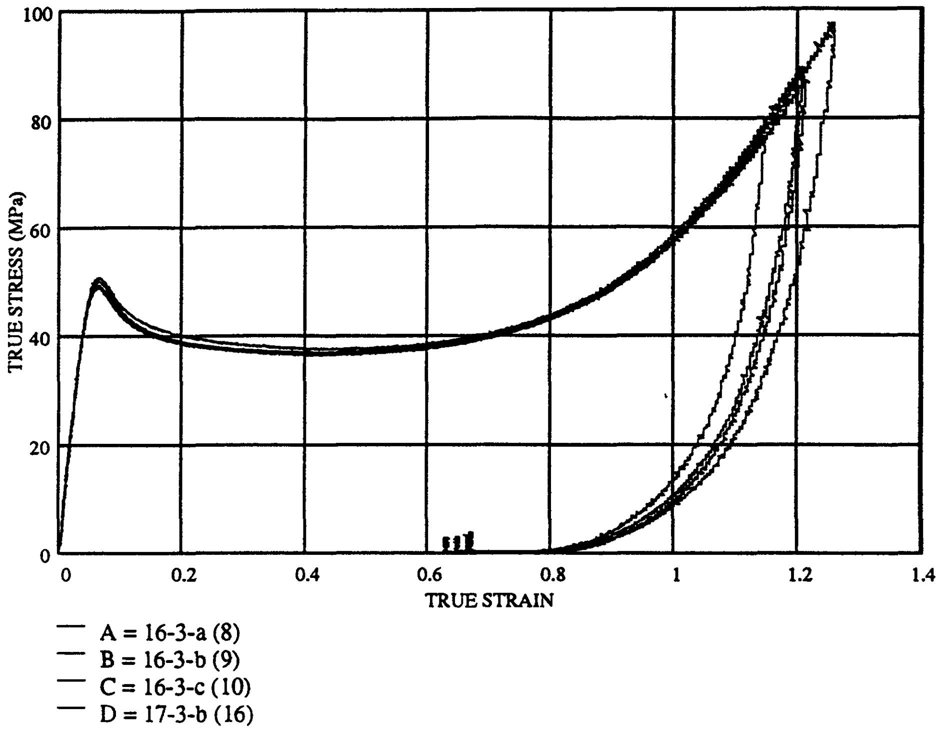
$$\sigma_{yGcalc_g} := \mathrm{if}\left(g \le 600, \sigma_{trueG_g}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yG} := \max\left(\sigma_{yGcalc}\right) \quad \mathrm{NumG}_g := \mathrm{if}\left(\sigma_{trueG_g} = \sigma_{yG}, g, 0\right)$$

$$\sigma_{yHcalc_h} := \mathrm{if}\left(h \le 1200, \sigma_{trueH_h}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yH} := \max\left(\sigma_{yHcalc}\right) \quad \mathrm{NumH}_h := \mathrm{if}\left(\sigma_{trueH_h} = \sigma_{yH}, h, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(\mathrm{NumA})}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(\mathrm{NumB})}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(\mathrm{NumC})}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(\mathrm{NumD})}}$$

$$\varepsilon_{yE} := \varepsilon_{trueE_{\max(\mathrm{NumE})}} \quad \varepsilon_{yF} := \varepsilon_{trueF_{\max(\mathrm{NumF})}} \quad \varepsilon_{yG} := \varepsilon_{trueG_{\max(\mathrm{NumG})}} \quad \varepsilon_{yH} := \varepsilon_{trueH_{\max(\mathrm{NumH})}}$$

Generate a plot of true stress (MPa) versus true strain for load rate 3 (1.00 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

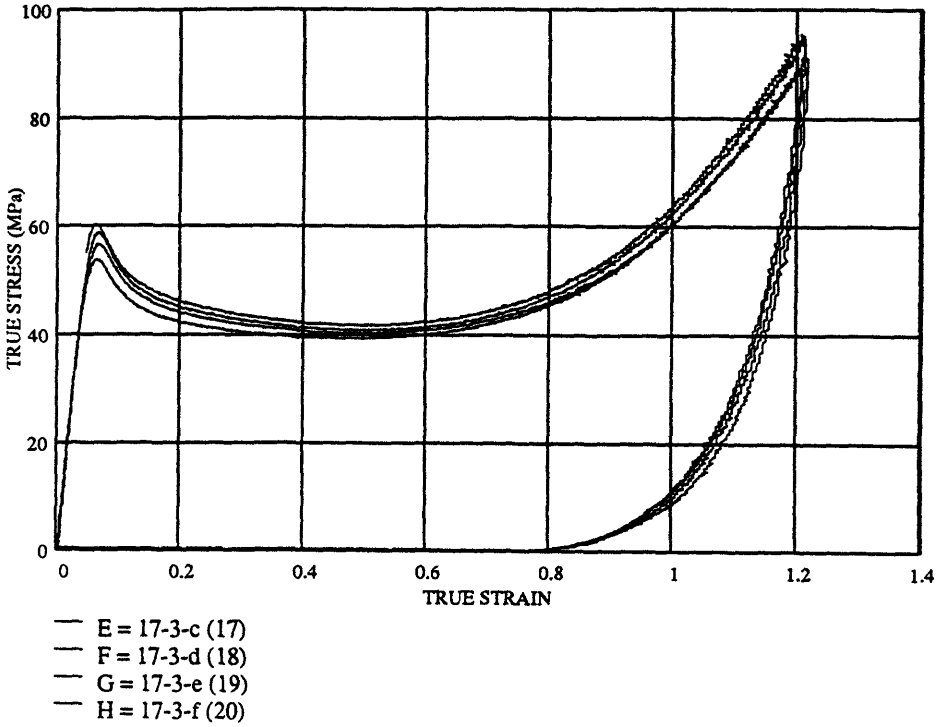
$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 1.147$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 76.723 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 50.147 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.064$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 1.211$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 89.4 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 49.394 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.066$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.258$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 97.545 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 49.004 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.063$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.217$ $\max\left(\sigma_{\mathrm{trueD}}\right) = 87.116 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yD}} = 50.818 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yD}} = 0.066$

Generate a plot of true stress (MPa) versus true strain for load rate 4 (2.50 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{trueE}) = 1.21$ $\max(\sigma_{trueE}) = 95.36 \cdot MPa$ $\sigma_{yE} = 60.246 \cdot MPa$ $\varepsilon_{yE} = 0.063$

$\max(\varepsilon_{trueF}) = 1.212$ $\max(\sigma_{trueF}) = 94.333 \cdot MPa$ $\sigma_{yF} = 58.77 \cdot MPa$ $\varepsilon_{yF} = 0.069$

$\max(\varepsilon_{trueG}) = 1.215$ $\max(\sigma_{trueG}) = 91.305 \cdot MPa$ $\sigma_{yG} = 56.638 \cdot MPa$ $\varepsilon_{yG} = 0.067$

$\max(\varepsilon_{trueH}) = 1.215$ $\max(\sigma_{trueH}) = 89.032 \cdot MPa$ $\sigma_{yH} = 53.814 \cdot MPa$ $\varepsilon_{yH} = 0.064$

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Select specimens to be used: $z := 1 .. 3$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP5})$ $\quad$ $\mathrm{SPEC}_A := 1$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A}\cdot \mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A}\cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP5})$ $\quad$ $\mathrm{SPEC}_B := 3$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B}\cdot \mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B}\cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP5})$ $\quad$ $\mathrm{SPEC}_C := 6$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C}\cdot \mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C}\cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4}\cdot\left(D_{o_z}\right)^2$

$$A_{cso} = \begin{pmatrix} 194.828 \\ 193.346 \\ 193.839 \end{pmatrix}\cdot \mathrm{mm}^2$$

<u>Data file information:</u>

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC5LR1T1})$

$B := \mathrm{READPRN}(\mathrm{IC5LR2T1})$

$C := \mathrm{READPRN}(\mathrm{IC5LR3T1})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000\cdot \mathrm{newton}$

$\mathrm{load}_A := A^{\langle 1\rangle}\cdot \mathrm{kN}$ $\quad \mathrm{displ}_A := A^{\langle 2\rangle}\cdot \mathrm{mm}$

$\mathrm{load}_B := B^{\langle 1\rangle}\cdot \mathrm{kN}$ $\quad \mathrm{displ}_B := B^{\langle 2\rangle}\cdot \mathrm{mm}$

$\mathrm{load}_C := C^{\langle 1\rangle}\cdot \mathrm{kN}$ $\quad \mathrm{displ}_C := C^{\langle 2\rangle}\cdot \mathrm{mm}$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$

$b := 1 .. \mathrm{rows}(B)$

$c := 1 .. \mathrm{rows}(C)$

Calculate true stress and true strain (assume incompressibility): $\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

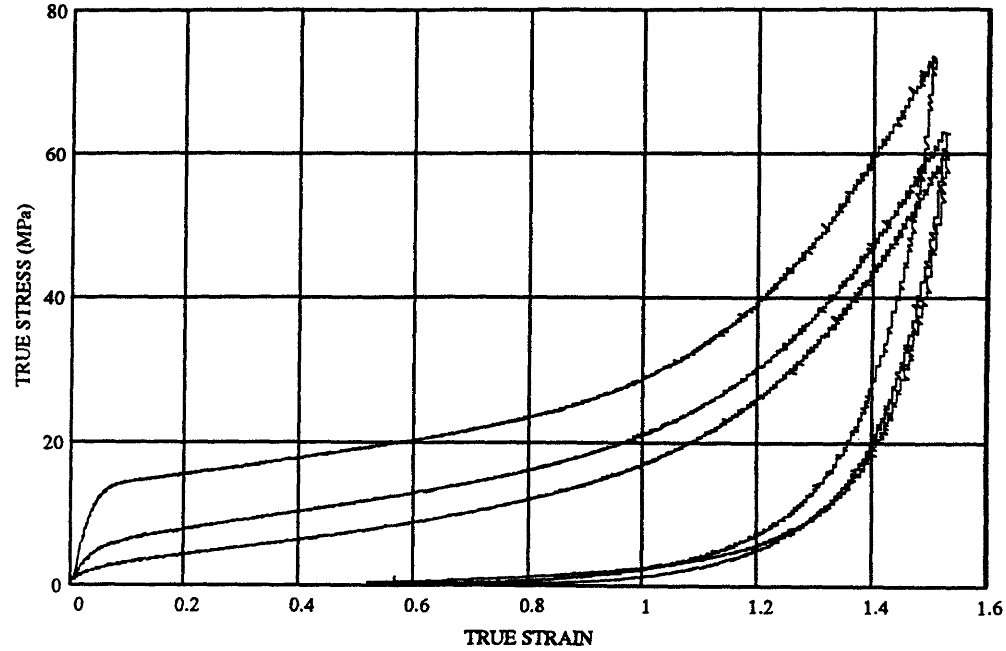
$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad \mathrm{NumA}_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad \mathrm{NumB}_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0\cdot \mathrm{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad \mathrm{NumC}_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(\mathrm{NumA})}} \qquad \varepsilon_{yB} := \varepsilon_{trueB_{\max(\mathrm{NumB})}} \qquad \varepsilon_{yC} := \varepsilon_{trueC_{\max(\mathrm{NumC})}}$$

Generate a plot of true stress (MPa) versus true strain for 0% 8084 / 100% 1080 at various load rates:



— A = 15-3-a (1); Load Rate = 0.01 mm/sec
— B = 15-3-c (3); Load Rate = 0.10 mm/sec
— C = 15-3-f (6); Load Rate = 1.00 mm/sec

List maximum true strain obtained and corresponding stress (MPa), the yield stress could not be defined:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 1.529 \quad \max\left(\sigma_{\mathrm{trueA}}\right) = 60.495 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 1.53 \quad \max\left(\sigma_{\mathrm{trueB}}\right) = 63.194 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.506 \quad \max\left(\sigma_{\mathrm{trueC}}\right) = 73.66 \cdot \mathrm{MPa}$

Select specimens to be used: $z := 1..8$ $\quad$ ORIGIN := 1

DIM := READPRN(COMP5)

$$\mathrm{SPEC} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 6 \\ 7 \\ 1 \\ 1 \end{bmatrix} \qquad L_{o_z} := \left(\mathrm{DIM}^{<1>}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm} \qquad D_{o_z} := \left(\mathrm{DIM}^{<2>}\right)_{\mathrm{SPEC}_{(z,1)}} \cdot \mathrm{mm}$$

Show initial length (mm): $\qquad$ **Show initial diameter (mm):**

$$L_o = \begin{bmatrix} 14.85 \\ 16.02 \\ 15.83 \\ 15.88 \\ 15.94 \\ 16.49 \\ 14.85 \\ 14.85 \end{bmatrix} \cdot \mathrm{mm} \qquad D_o = \begin{bmatrix} 15.75 \\ 15.68 \\ 15.69 \\ 15.7 \\ 15.71 \\ 15.75 \\ 15.75 \\ 15.75 \end{bmatrix} \cdot \mathrm{mm}$$

Length/Diameter ratio(LDR): $\qquad$ **Calculate the initial cross sectional area (mm^2):**

$$\mathrm{LDR}_z := \frac{L_{o_z}}{D_{o_z}} \qquad A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$$

$$\mathrm{LDR} = \begin{bmatrix} 0.943 \\ 1.022 \\ 1.009 \\ 1.011 \\ 1.015 \\ 1.047 \\ 0.943 \\ 0.943 \end{bmatrix} \qquad A_{cso} = \begin{bmatrix} 194.828 \\ 193.1 \\ 193.346 \\ 193.593 \\ 193.839 \\ 194.828 \\ 194.828 \\ 194.828 \end{bmatrix} \cdot \mathrm{mm}^2$$

Input maximum true strain (compressive values negative): $\qquad$ Input loading head actuator speed (mm/sec):

$$\varepsilon_{max} := \begin{bmatrix} -1.50 \\ -1.50 \\ -1.50 \\ -1.50 \\ -1.50 \\ -1.50 \\ 1 \\ 1 \end{bmatrix} \qquad \mathrm{Load}_{rate} := \begin{bmatrix} 0.01 \\ 0.01 \\ 0.10 \\ 0.10 \\ 1.00 \\ 1.00 \\ 1 \\ 1 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}}$$

NOTE: A default value of 1 with no trailing zeros is used as a place filler for vector locations which were not used.

Assume true strain, calculate final specimen length (mm):

$$L_{f_z} := L_{o_z} \cdot e^{\varepsilon_{max_z}}$$

$$L_f = \begin{bmatrix} 3.313 \\ 3.575 \\ 3.532 \\ 3.543 \\ 3.557 \\ 3.679 \\ 40.366 \\ 40.366 \end{bmatrix} \cdot mm$$

Calculate the machine stroke (mm):

$$Stroke_z := L_{f_z} - L_{o_z}$$

$$Stroke = \begin{bmatrix} -11.537 \\ -12.445 \\ -12.298 \\ -12.337 \\ -12.383 \\ -12.811 \\ 25.516 \\ 25.516 \end{bmatrix} \cdot mm$$

Input test type: Double-Ramp Loading

Calculate total machine stroke (mm): $Stroke_{tot_z} := 2 \cdot Stroke_z$

$$Stroke_{tot} = \begin{bmatrix} -23.073 \\ -24.891 \\ -24.596 \\ -24.673 \\ -24.767 \\ -25.621 \\ 51.033 \\ 51.033 \end{bmatrix} \cdot mm$$

Calculate the total time required for test:

$$time_{tot_z} := \left| \frac{Stroke_{tot_z}}{Load_{rate_z}} \right|$$

$$time_{tot} = \begin{bmatrix} 2307.303 \\ 2489.091 \\ 245.957 \\ 246.734 \\ 24.767 \\ 25.621 \\ 51.033 \\ 51.033 \end{bmatrix} \cdot sec \qquad time_{tot} = \begin{bmatrix} 38.455 \\ 41.485 \\ 4.099 \\ 4.112 \\ 0.413 \\ 0.427 \\ 0.851 \\ 0.851 \end{bmatrix} \cdot min \qquad time_{tot} = \begin{bmatrix} 0.641 \\ 0.691 \\ 0.068 \\ 0.069 \\ 0.007 \\ 0.007 \\ 0.014 \\ 0.014 \end{bmatrix} \cdot hr$$

Input data aq. frequency rate (Hz), range: 0-500 Hz:

$$Freq_{data} := \begin{bmatrix} 5 \\ 5 \\ 50 \\ 50 \\ 400 \\ 400 \\ 1 \\ 1 \end{bmatrix} \cdot Hz$$

Calculate approximate number of data points:

$$NBR_{data_z} := Freq_{data_z} \cdot time_{tot_z}$$

$$NBR_{data} = \begin{bmatrix} 11536.517 \\ 12445.455 \\ 12297.85 \\ 12336.693 \\ 9906.644 \\ 10248.467 \\ 51.033 \\ 51.033 \end{bmatrix}$$

Select a buffer size which is larger than
the number of data points !!

<u>Data file information:</u>

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(IC5LR1T1)$ $E := \mathrm{READPRN}(IC5LR3T1)$
$B := \mathrm{READPRN}(IC5LR1T2)$ $F := \mathrm{READPRN}(IC5LR3T2)$
$C := \mathrm{READPRN}(IC5LR2T1)$ $G := \mathrm{READPRN}(IC0LR1T1)$
$D := \mathrm{READPRN}(IC5LR2T2)$ $H := \mathrm{READPRN}(IC0LR1T1)$

Note: Files A-H are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

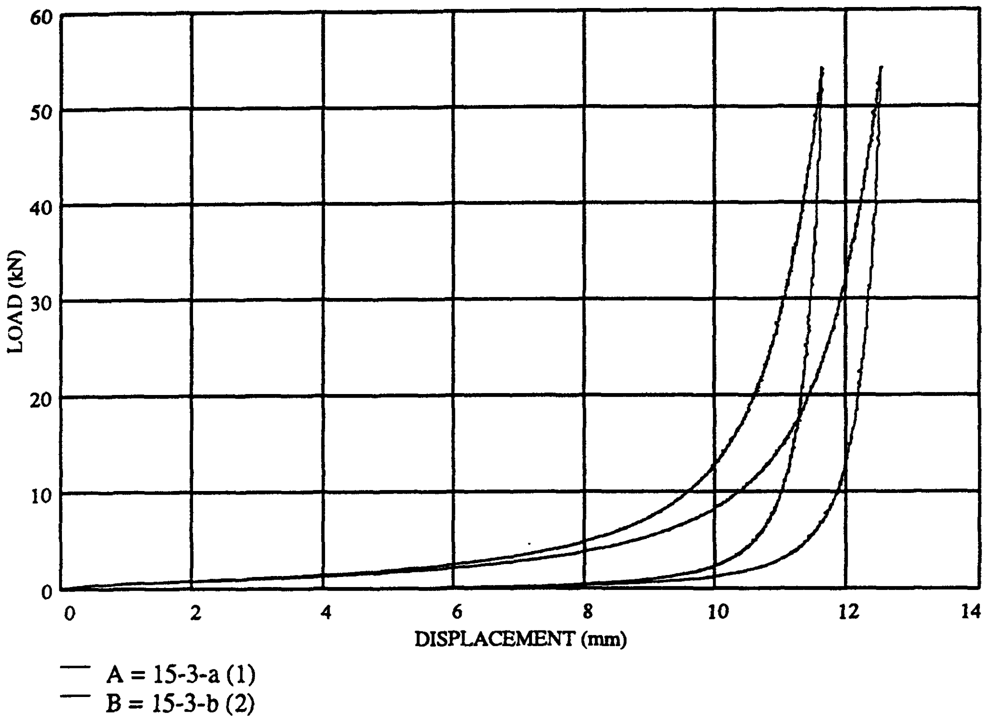
$load_A := A^{\langle 1 \rangle} \cdot kN$ $displ_A := A^{\langle 2 \rangle} \cdot mm$ $load_E := E^{\langle 1 \rangle} \cdot kN$ $displ_E := E^{\langle 2 \rangle} \cdot mm$
$load_B := B^{\langle 1 \rangle} \cdot kN$ $displ_B := B^{\langle 2 \rangle} \cdot mm$ $load_F := F^{\langle 1 \rangle} \cdot kN$ $displ_F := F^{\langle 2 \rangle} \cdot mm$
$load_C := C^{\langle 1 \rangle} \cdot kN$ $displ_C := C^{\langle 2 \rangle} \cdot mm$ $load_G := G^{\langle 1 \rangle} \cdot kN$ $displ_G := G^{\langle 2 \rangle} \cdot mm$
$load_D := D^{\langle 1 \rangle} \cdot kN$ $displ_D := D^{\langle 2 \rangle} \cdot mm$ $load_H := H^{\langle 1 \rangle} \cdot kN$ $displ_H := H^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate number of data points:

$a := 1 .. \mathrm{rows}(A)$ $e := 1 .. \mathrm{rows}(E)$
$b := 1 .. \mathrm{rows}(B)$ $f := 1 .. \mathrm{rows}(F)$
$c := 1 .. \mathrm{rows}(C)$ $g := 1 .. \mathrm{rows}(G)$
$d := 1 .. \mathrm{rows}(D)$ $h := 1 .. \mathrm{rows}(H)$

Generate a plot of load (kN) versus displacement (mm) for load rate 1 (0.01 mm/sec):

Load limit: max(load) < 100 kN



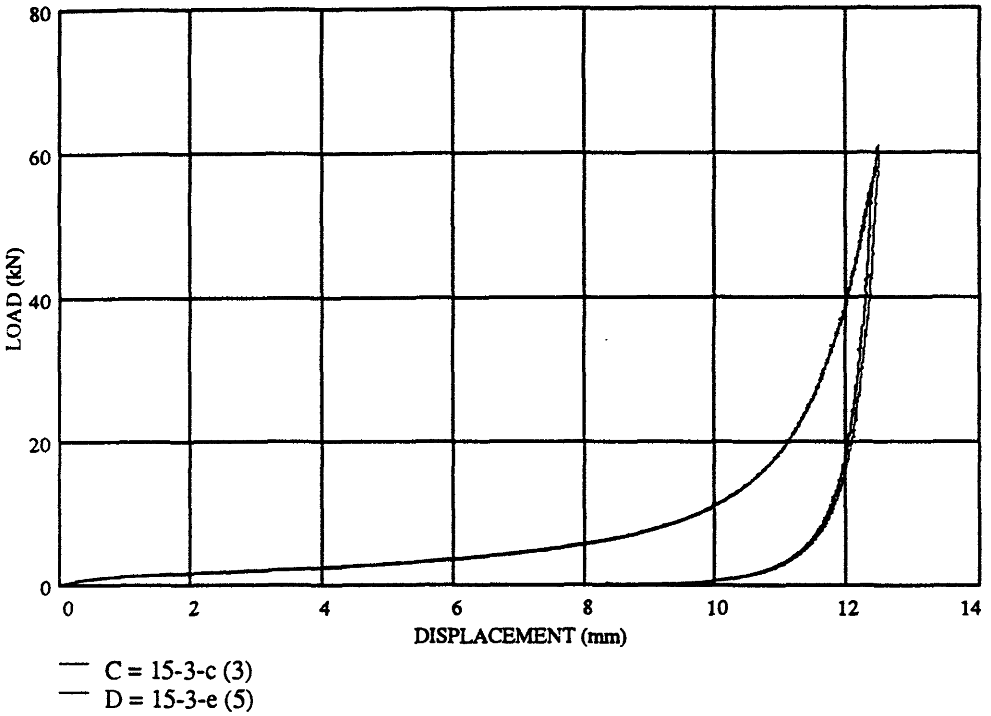
Maximum load (kN) and displacement (mm):

$\max\left(\text{load}_A\right) = 53.981 \cdot \text{kN}$ $\max\left(\text{displ}_A\right) = 11.633 \cdot \text{mm}$

$\max\left(\text{load}_B\right) = 54.03 \cdot \text{kN}$ $\max\left(\text{displ}_B\right) = 12.527 \cdot \text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 2 (0.10 mm/sec):

Load limit: max(load) < 100 kN



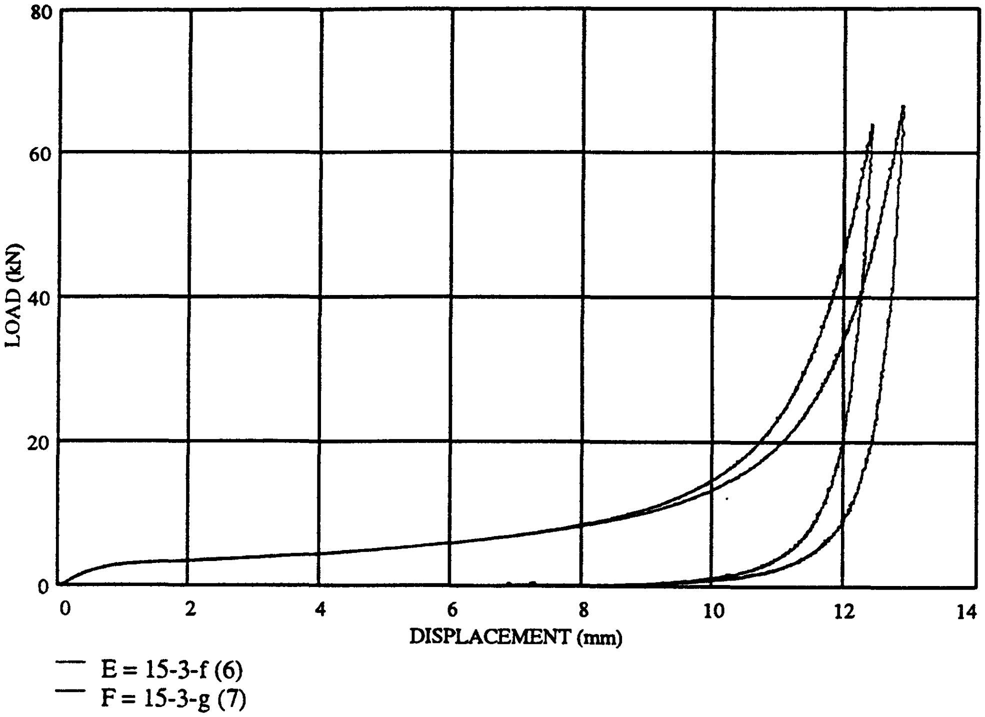
Maximum load (kN) and displacement (mm):

$\max\left(\text{load}_C\right) = 56.033 \cdot \text{kN}$ $\max\left(\text{displ}_C\right) = 12.403 \cdot \text{mm}$

$\max\left(\text{load}_D\right) = 61.114 \cdot \text{kN}$ $\max\left(\text{displ}_D\right) = 12.478 \cdot \text{mm}$

Generate a plot of load (kN) versus displacement (mm) for load rate 3 (1.00 mm/sec):

Load limit: max(load) < 100 kN



Maximum load (kN) and displacement (mm):

$\max(\text{load}_E) = 63.898 \cdot \text{kN}$ $\max(\text{displ}_E) = 12.403 \cdot \text{mm}$

$\max(\text{load}_F) = 66.487 \cdot \text{kN}$ $\max(\text{displ}_F) = 12.875 \cdot \text{mm}$

Calculate true stress and true strain (assume incompressibility):

$$\mathrm{MPa} := 1\cdot 10^{6}\cdot \mathrm{Pa}$$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - \mathrm{displ}_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - \mathrm{displ}_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}} \qquad L_{iF_f} := L_{o_6} - \mathrm{displ}_{F_f} \qquad A_{iF_f} := \frac{A_{cso_6}\cdot L_{o_6}}{L_{iF_f}}$$

$$L_{iG_g} := L_{o_7} - \mathrm{displ}_{G_g} \qquad A_{iG_g} := \frac{A_{cso_7}\cdot L_{o_7}}{L_{iG_g}} \qquad L_{iH_h} := L_{o_8} - \mathrm{displ}_{H_h} \qquad A_{iH_h} := \frac{A_{cso_8}\cdot L_{o_8}}{L_{iH_h}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{\mathrm{load}_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{\mathrm{load}_{E_e}}{A_{iE_e}} \qquad \sigma_{trueF_f} := \frac{\mathrm{load}_{F_f}}{A_{iF_f}} \qquad \sigma_{trueG_g} := \frac{\mathrm{load}_{G_g}}{A_{iG_g}} \qquad \sigma_{trueH_h} := \frac{\mathrm{load}_{H_h}}{A_{iH_h}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right) \qquad \varepsilon_{trueF_f} := -\ln\left(\frac{L_{iF_f}}{L_{o_6}}\right) \qquad \varepsilon_{trueG_g} := -\ln\left(\frac{L_{iG_g}}{L_{o_7}}\right) \qquad \varepsilon_{trueH_h} := -\ln\left(\frac{L_{iH_h}}{L_{o_8}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{\mathrm{yAcalc}_a} := \mathrm{if}\left(a \le 1200, \sigma_{\mathrm{trueA}_a}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yA}} := \max\left(\sigma_{\mathrm{yAcalc}}\right) \quad \mathrm{NumA}_a := \mathrm{if}\left(\sigma_{\mathrm{trueA}_a} = \sigma_{\mathrm{yA}}, a, 0\right)$$

$$\sigma_{\mathrm{yBcalc}_b} := \mathrm{if}\left(b \le 1200, \sigma_{\mathrm{trueB}_b}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yB}} := \max\left(\sigma_{\mathrm{yBcalc}}\right) \quad \mathrm{NumB}_b := \mathrm{if}\left(\sigma_{\mathrm{trueB}_b} = \sigma_{\mathrm{yB}}, b, 0\right)$$

$$\sigma_{\mathrm{yCcalc}_c} := \mathrm{if}\left(c \le 600, \sigma_{\mathrm{trueC}_c}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yC}} := \max\left(\sigma_{\mathrm{yCcalc}}\right) \quad \mathrm{NumC}_c := \mathrm{if}\left(\sigma_{\mathrm{trueC}_c} = \sigma_{\mathrm{yC}}, c, 0\right)$$

$$\sigma_{\mathrm{yDcalc}_d} := \mathrm{if}\left(d \le 1200, \sigma_{\mathrm{trueD}_d}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yD}} := \max\left(\sigma_{\mathrm{yDcalc}}\right) \quad \mathrm{NumD}_d := \mathrm{if}\left(\sigma_{\mathrm{trueD}_d} = \sigma_{\mathrm{yD}}, d, 0\right)$$

$$\sigma_{\mathrm{yEcalc}_e} := \mathrm{if}\left(e \le 1200, \sigma_{\mathrm{trueE}_e}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yE}} := \max\left(\sigma_{\mathrm{yEcalc}}\right) \quad \mathrm{NumE}_e := \mathrm{if}\left(\sigma_{\mathrm{trueE}_e} = \sigma_{\mathrm{yE}}, e, 0\right)$$

$$\sigma_{\mathrm{yFcalc}_f} := \mathrm{if}\left(f \le 1200, \sigma_{\mathrm{trueF}_f}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yF}} := \max\left(\sigma_{\mathrm{yFcalc}}\right) \quad \mathrm{NumF}_f := \mathrm{if}\left(\sigma_{\mathrm{trueF}_f} = \sigma_{\mathrm{yF}}, f, 0\right)$$

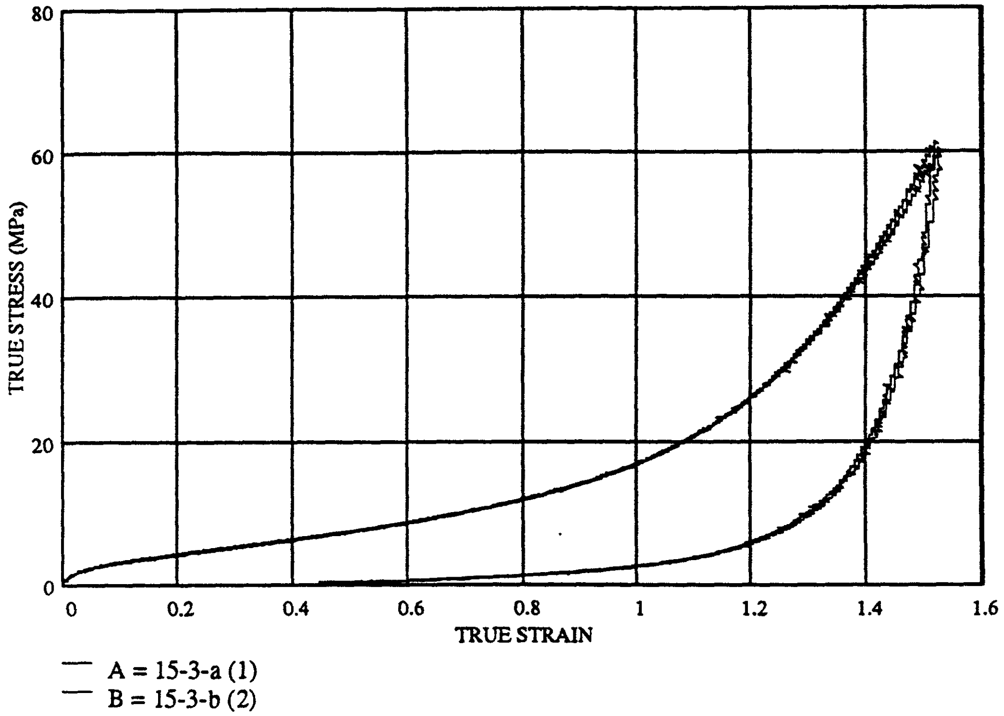
$$\sigma_{\mathrm{yGcalc}_g} := \mathrm{if}\left(g \le 600, \sigma_{\mathrm{trueG}_g}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yG}} := \max\left(\sigma_{\mathrm{yGcalc}}\right) \quad \mathrm{NumG}_g := \mathrm{if}\left(\sigma_{\mathrm{trueG}_g} = \sigma_{\mathrm{yG}}, g, 0\right)$$

$$\sigma_{\mathrm{yHcalc}_h} := \mathrm{if}\left(h \le 1200, \sigma_{\mathrm{trueH}_h}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{\mathrm{yH}} := \max\left(\sigma_{\mathrm{yHcalc}}\right) \quad \mathrm{NumH}_h := \mathrm{if}\left(\sigma_{\mathrm{trueH}_h} = \sigma_{\mathrm{yH}}, h, 0\right)$$

$$\varepsilon_{\mathrm{yA}} := \varepsilon_{\mathrm{trueA}_{\max(\mathrm{NumA})}} \quad \varepsilon_{\mathrm{yB}} := \varepsilon_{\mathrm{trueB}_{\max(\mathrm{NumB})}} \quad \varepsilon_{\mathrm{yC}} := \varepsilon_{\mathrm{trueC}_{\max(\mathrm{NumC})}} \quad \varepsilon_{\mathrm{yD}} := \varepsilon_{\mathrm{trueD}_{\max(\mathrm{NumD})}}$$

$$\varepsilon_{\mathrm{yE}} := \varepsilon_{\mathrm{trueE}_{\max(\mathrm{NumE})}} \quad \varepsilon_{\mathrm{yF}} := \varepsilon_{\mathrm{trueF}_{\max(\mathrm{NumF})}} \quad \varepsilon_{\mathrm{yG}} := \varepsilon_{\mathrm{trueG}_{\max(\mathrm{NumG})}} \quad \varepsilon_{\mathrm{yH}} := \varepsilon_{\mathrm{trueH}_{\max(\mathrm{NumH})}}$$

Generate a plot of true stress (MPa) versus true strain for load rate 1 (0.01 mm/sec):

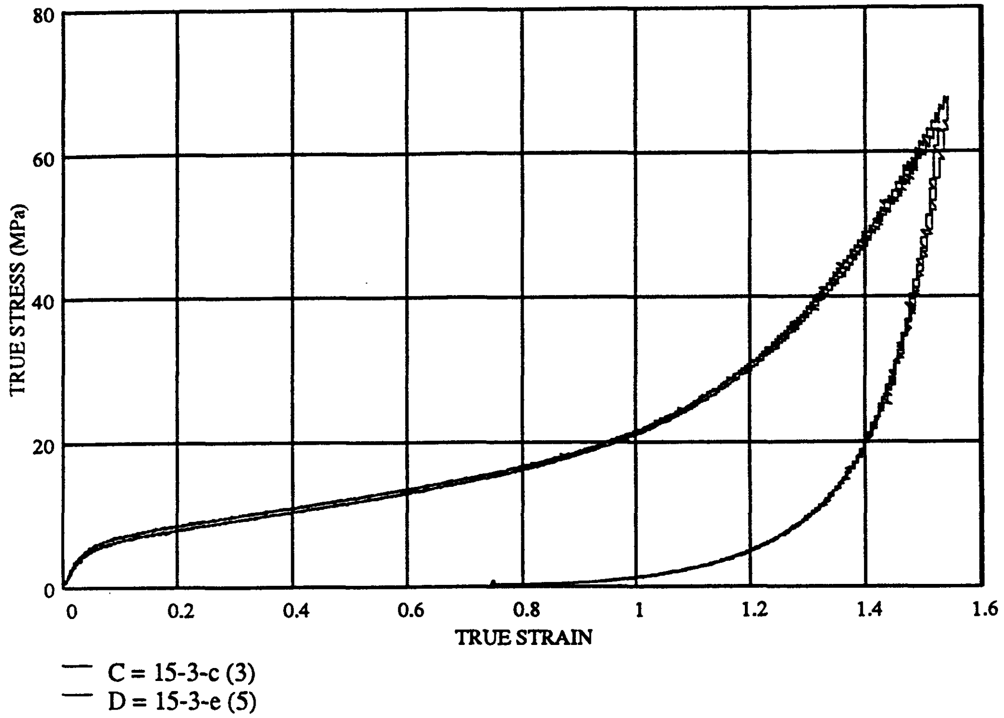


List maximum true strain obtained and corresponding stress (MPa), no yield stress can be defined:

$\max\left(\varepsilon_{\text{trueA}}\right) = 1.529 \qquad \max\left(\sigma_{\text{trueA}}\right) = 60.495 \cdot \text{MPa}$

$\max\left(\varepsilon_{\text{trueB}}\right) = 1.523 \qquad \max\left(\sigma_{\text{trueB}}\right) = 61.325 \cdot \text{MPa}$

Generate a plot of true stress (MPa) versus true strain for load rate 2 (0.10 mm/sec):

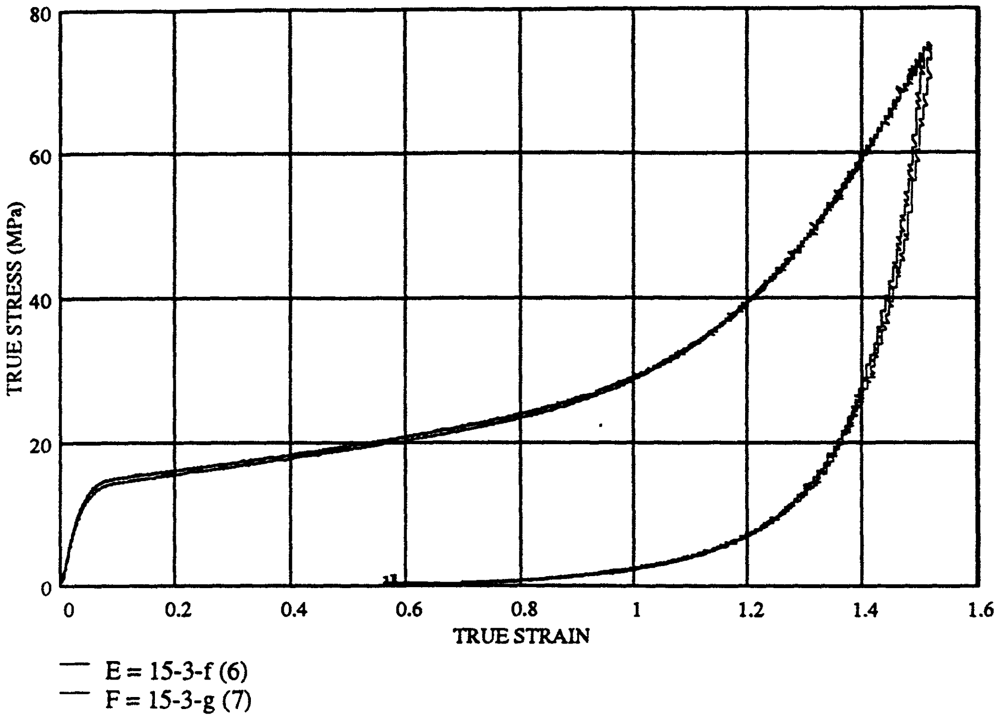


List maximum true strain obtained and corresponding stress (MPa), no yield stress can be defined:

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.53 \qquad \max\left(\sigma_{\mathrm{trueC}}\right) = 63.194 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.541 \qquad \max\left(\sigma_{\mathrm{trueD}}\right) = 67.635 \cdot \mathrm{MPa}$

Generate a plot of true stress (MPa) versus true strain for load rate 3 (1.00 mm/sec):



List maximum true strain obtained and corresponding stress (MPa), no yield stress can be defined:

$\max\left(\varepsilon_{\text{trueE}}\right) = 1.506 \qquad \max\left(\sigma_{\text{trueE}}\right) = 73.66 \cdot \text{MPa}$

$\max\left(\varepsilon_{\text{trueF}}\right) = 1.518 \qquad \max\left(\sigma_{\text{trueF}}\right) = 75.32 \cdot \text{MPa}$

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Select specimens to be used: $z := 1..5$ $\quad$ $ORIGIN := 1$

$DIM_A := READPRN(COMP1B)$ $\qquad$ $SPEC_A := 3$

$L_{o_1} := \left(DIM_A^{\langle 1\rangle}\right)_{SPEC_A}\cdot mm$ $\quad$ $D_{o_1} := \left(DIM_A^{\langle 2\rangle}\right)_{SPEC_A}\cdot mm$

$DIM_B := READPRN(COMP2B)$ $\qquad$ $SPEC_B := 2$

$L_{o_2} := \left(DIM_B^{\langle 1\rangle}\right)_{SPEC_B}\cdot mm$ $\quad$ $D_{o_2} := \left(DIM_B^{\langle 2\rangle}\right)_{SPEC_B}\cdot mm$

$DIM_C := READPRN(COMP3B)$ $\qquad$ $SPEC_C := 2$

$L_{o_3} := \left(DIM_C^{\langle 1\rangle}\right)_{SPEC_C}\cdot mm$ $\quad$ $D_{o_3} := \left(DIM_C^{\langle 2\rangle}\right)_{SPEC_C}\cdot mm$

$DIM_D := READPRN(COMP4)$ $\qquad$ $SPEC_D := 2$

$L_{o_4} := \left(DIM_D^{\langle 1\rangle}\right)_{SPEC_D}\cdot mm$ $\quad$ $D_{o_4} := \left(DIM_D^{\langle 2\rangle}\right)_{SPEC_D}\cdot mm$

$DIM_E := READPRN(COMP5)$ $\qquad$ $SPEC_E := 1$

$L_{o_5} := \left(DIM_E^{\langle 1\rangle}\right)_{SPEC_E}\cdot mm$ $\quad$ $D_{o_5} := \left(DIM_E^{\langle 2\rangle}\right)_{SPEC_E}\cdot mm$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4}\cdot\left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 191.38 \\ 183.134 \\ 181.697 \\ 190.645 \\ 194.828 \end{bmatrix}\cdot mm^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := READPRN(IC1LR1T8)$

$B := READPRN(IC2LR1T6)$

$C := READPRN(IC3LR1T6)$

$D := READPRN(IC4LR1T2)$

$E := READPRN(IC5LR1T1)$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{<1>} \cdot kN$ $displ_A := A^{<2>} \cdot mm$ $load_D := D^{<1>} \cdot kN$ $displ_D := D^{<2>} \cdot mm$

$load_B := B^{<1>} \cdot kN$ $displ_B := B^{<2>} \cdot mm$ $load_E := E^{<1>} \cdot kN$ $displ_E := E^{<2>} \cdot mm$

$load_C := C^{<1>} \cdot kN$ $displ_C := C^{<2>} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$ $e := 1 .. rows(E)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - displ_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - displ_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - displ_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5} \cdot L_{o_5}}{L_{iE_e}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{load_{E_e}}{A_{iE_e}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

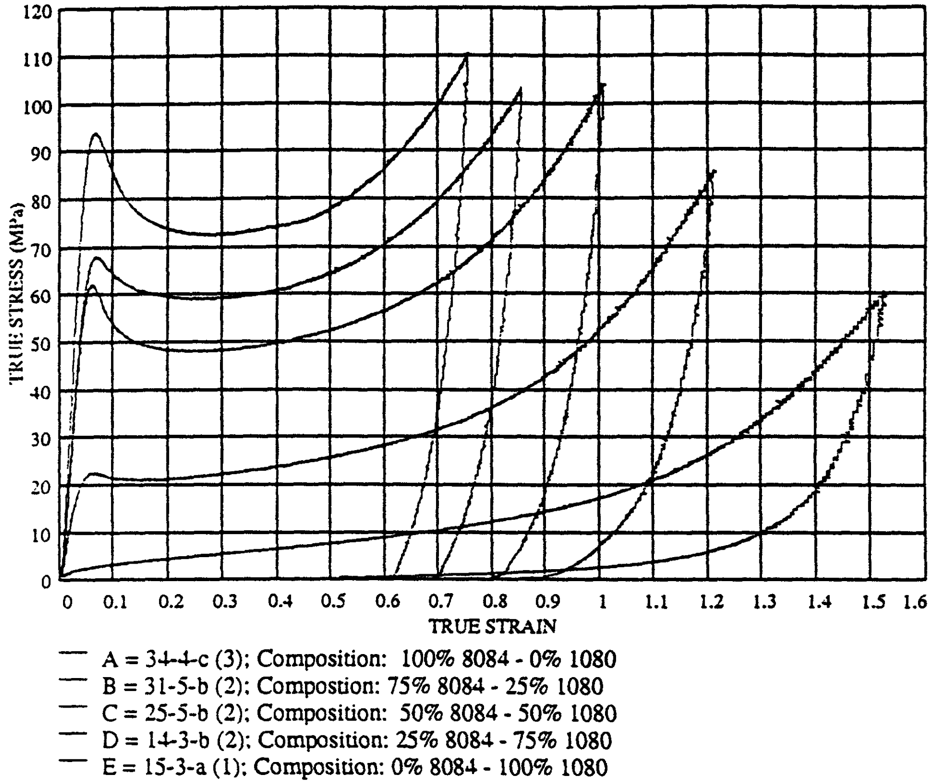
$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right)$ $\sigma_{yE} := \max\left(\sigma_{yEcalc}\right)$ $NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$

Generate a plot of true stress (MPa) versus true strain for a loading rate of 0.01 mm/sec for various compositions:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 0.759$ $\max\left(\sigma_{\text{trueA}}\right) = 110.58 \cdot \text{MPa}$ $\sigma_{yA} = 93.767 \cdot \text{MPa}$ $\varepsilon_{yA} = 0.065$

$\max\left(\varepsilon_{\text{trueB}}\right) = 0.855$ $\max\left(\sigma_{\text{trueB}}\right) = 103.201 \cdot \text{MPa}$ $\sigma_{yB} = 67.896 \cdot \text{MPa}$ $\varepsilon_{yB} = 0.066$

$\max\left(\varepsilon_{\text{trueC}}\right) = 1.012$ $\max\left(\sigma_{\text{trueC}}\right) = 104.044 \cdot \text{MPa}$ $\sigma_{yC} = 62.089 \cdot \text{MPa}$ $\varepsilon_{yC} = 0.059$

$\max\left(\varepsilon_{\text{trueD}}\right) = 1.216$ $\max\left(\sigma_{\text{trueD}}\right) = 85.539 \cdot \text{MPa}$ $\sigma_{yD} = 22.46 \cdot \text{MPa}$ $\varepsilon_{yD} = 0.059$

$\max\left(\varepsilon_{\text{trueE}}\right) = 1.529$ $\max\left(\sigma_{\text{trueE}}\right) = 60.495 \cdot \text{MPa}$

The yield stress could not be defined for the 100% Crestomer 1080 sample

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..5$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP1A})$ $\quad$ $\mathrm{SPEC}_A := 24$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A}\cdot \mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A}\cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP2B})$ $\quad$ $\mathrm{SPEC}_B := 6$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B}\cdot \mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B}\cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP3B})$ $\quad$ $\mathrm{SPEC}_C := 6$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C}\cdot \mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C}\cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP4})$ $\quad$ $\mathrm{SPEC}_D := 6$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1\rangle}\right)_{\mathrm{SPEC}_D}\cdot \mathrm{mm}$ $\quad$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2\rangle}\right)_{\mathrm{SPEC}_D}\cdot \mathrm{mm}$

$\mathrm{DIM}_E := \mathrm{READPRN}(\mathrm{COMP5})$ $\quad$ $\mathrm{SPEC}_E := 3$

$L_{o_5} := \left(\mathrm{DIM}_E^{\langle 1\rangle}\right)_{\mathrm{SPEC}_E}\cdot \mathrm{mm}$ $\quad$ $D_{o_5} := \left(\mathrm{DIM}_E^{\langle 2\rangle}\right)_{\mathrm{SPEC}_E}\cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4}\cdot\left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 192.854 \\ 183.614 \\ 184.335 \\ 191.134 \\ 193.346 \end{bmatrix}\cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC1LR2T5})$

$B := \mathrm{READPRN}(\mathrm{IC2LR2T4})$

$C := \mathrm{READPRN}(\mathrm{IC3LR2T7})$

$D := \mathrm{READPRN}(\mathrm{IC4LR2T2})$

$E := \mathrm{READPRN}(\mathrm{IC5LR2T1})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{\langle 1 \rangle} \cdot kN$ $displ_A := A^{\langle 2 \rangle} \cdot mm$ $load_D := D^{\langle 1 \rangle} \cdot kN$ $displ_D := D^{\langle 2 \rangle} \cdot mm$

$load_B := B^{\langle 1 \rangle} \cdot kN$ $displ_B := B^{\langle 2 \rangle} \cdot mm$ $load_E := E^{\langle 1 \rangle} \cdot kN$ $displ_E := E^{\langle 2 \rangle} \cdot mm$

$load_C := C^{\langle 1 \rangle} \cdot kN$ $displ_C := C^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$ $e := 1 .. rows(E)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - displ_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - displ_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - displ_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5} \cdot L_{o_5}}{L_{iE_e}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{load_{E_e}}{A_{iE_e}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

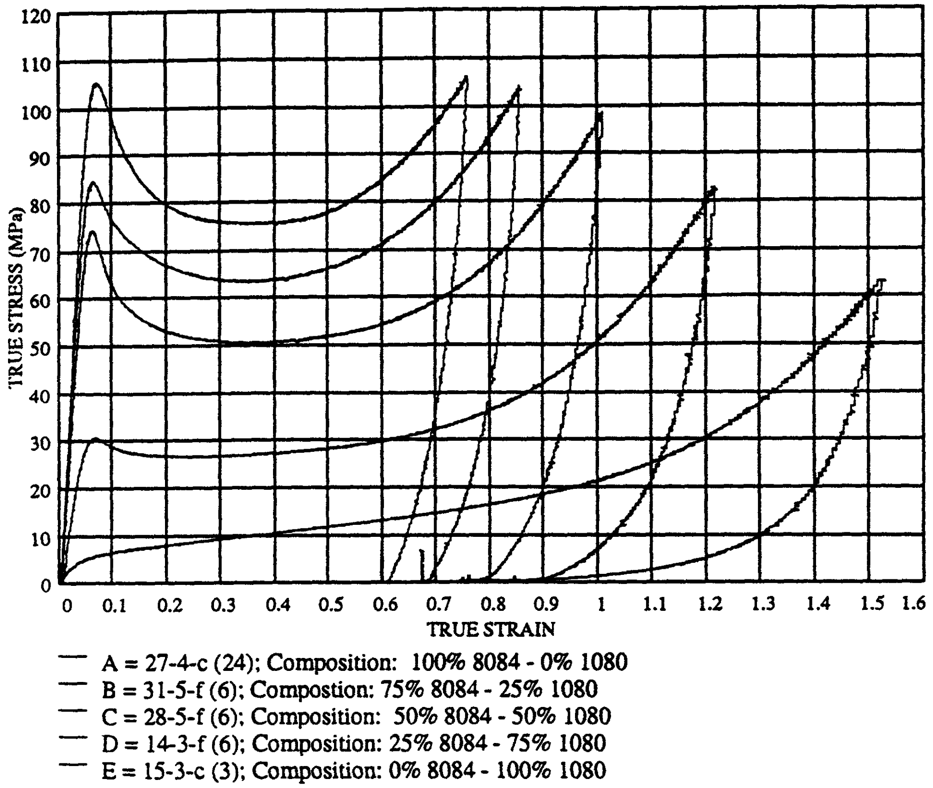
$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot \mathrm{MPa}\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

$$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$$

Generate a plot of true stress (MPa) versus true strain for a loading rate of 0.10 mm/sec for various compositions:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.761$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 106.084 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yA}} = 105.442 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yA}} = 0.071$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.86$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 104.253 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yB}} = 84.366 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yB}} = 0.064$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.01$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 98.384 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yC}} = 74.187 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yC}} = 0.063$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.222$ $\max\left(\sigma_{\mathrm{trueD}}\right) = 82.846 \cdot \mathrm{MPa}$ $\sigma_{\mathrm{yD}} = 30.551 \cdot \mathrm{MPa}$ $\varepsilon_{\mathrm{yD}} = 0.068$

$\max\left(\varepsilon_{\mathrm{trueE}}\right) = 1.53$ $\max\left(\sigma_{\mathrm{trueE}}\right) = 63.194 \cdot \mathrm{MPa}$

The yield stress could not be defined for the 100% Crestomer 1080 sample

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..5$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP1B})$ $\mathrm{SPEC}_A := 5$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP2B})$ $\mathrm{SPEC}_B := 8$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP3B})$ $\mathrm{SPEC}_C := 9$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP4})$ $\mathrm{SPEC}_D := 9$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$

$\mathrm{DIM}_E := \mathrm{READPRN}(\mathrm{COMP5})$ $\mathrm{SPEC}_E := 6$

$L_{o_5} := \left(\mathrm{DIM}_E^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_E} \cdot \mathrm{mm}$ $D_{o_5} := \left(\mathrm{DIM}_E^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_E} \cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 193.346 \\ 185.782 \\ 183.614 \\ 190.645 \\ 193.839 \end{bmatrix} \cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(\mathrm{IC1LR3T4})$

$B := \mathrm{READPRN}(\mathrm{IC2LR3T5})$

$C := \mathrm{READPRN}(\mathrm{IC3LR3T5})$

$D := \mathrm{READPRN}(\mathrm{IC4LR3T2})$

$E := \mathrm{READPRN}(\mathrm{IC5LR3T1})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{<1>} \cdot kN$ $displ_A := A^{<2>} \cdot mm$ $load_D := D^{<1>} \cdot kN$ $displ_D := D^{<2>} \cdot mm$

$load_B := B^{<1>} \cdot kN$ $displ_B := B^{<2>} \cdot mm$ $load_E := E^{<1>} \cdot kN$ $displ_E := E^{<2>} \cdot mm$

$load_C := C^{<1>} \cdot kN$ $displ_C := C^{<2>} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$ $e := 1 .. rows(E)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - displ_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - displ_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - displ_{E_e} \qquad A_{iE_e} := \frac{A_{cso_5} \cdot L_{o_5}}{L_{iE_e}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{load_{E_e}}{A_{iE_e}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

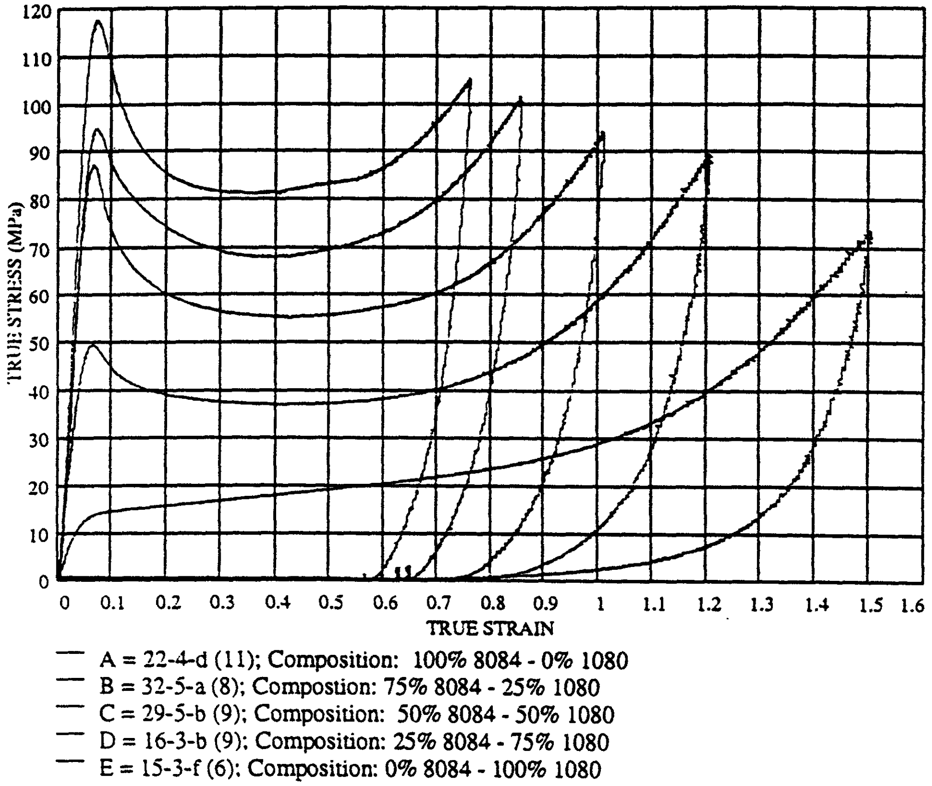
$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right)$ $\sigma_{yE} := \max\left(\sigma_{yEcalc}\right)$ $NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$

Generate a plot of true stress (MPa) versus true strain for a loading rate of 1.00 mm/sec for various compositions:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 0.763$ $\max\left(\sigma_{\text{trueA}}\right) = 117.63 \cdot \text{MPa}$ $\sigma_{yA} = 117.63 \cdot \text{MPa}$ $\varepsilon_{yA} = 0.073$

$\max\left(\varepsilon_{\text{trueB}}\right) = 0.862$ $\max\left(\sigma_{\text{trueB}}\right) = 101.544 \cdot \text{MPa}$ $\sigma_{yB} = 94.798 \cdot \text{MPa}$ $\varepsilon_{yB} = 0.071$

$\max\left(\varepsilon_{\text{trueC}}\right) = 1.012$ $\max\left(\sigma_{\text{trueC}}\right) = 94.066 \cdot \text{MPa}$ $\sigma_{yC} = 87.054 \cdot \text{MPa}$ $\varepsilon_{yC} = 0.067$

$\max\left(\varepsilon_{\text{trueD}}\right) = 1.211$ $\max\left(\sigma_{\text{trueD}}\right) = 89.4 \cdot \text{MPa}$ $\sigma_{yD} = 49.394 \cdot \text{MPa}$ $\varepsilon_{yD} = 0.066$

$\max\left(\varepsilon_{\text{trueE}}\right) = 1.506$ $\max\left(\sigma_{\text{trueE}}\right) = 73.66 \cdot \text{MPa}$

The yield stress could not be defined for the 100% Crestomer 1080 sample

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1 .. 5$ $\quad ORIGIN := 1$

$DIM_A := READPRN(COMP1B)$ $\quad SPEC_A := 11$

$L_{o_1} := \left(DIM_A^{\langle 1 \rangle}\right)_{SPEC_A} \cdot mm$ $\quad D_{o_1} := \left(DIM_A^{\langle 2 \rangle}\right)_{SPEC_A} \cdot mm$

$DIM_B := READPRN(COMP2B)$ $\quad SPEC_B := 12$

$L_{o_2} := \left(DIM_B^{\langle 1 \rangle}\right)_{SPEC_B} \cdot mm$ $\quad D_{o_2} := \left(DIM_B^{\langle 2 \rangle}\right)_{SPEC_B} \cdot mm$

$DIM_C := READPRN(COMP3B)$ $\quad SPEC_C := 13$

$L_{o_3} := \left(DIM_C^{\langle 1 \rangle}\right)_{SPEC_C} \cdot mm$ $\quad D_{o_3} := \left(DIM_C^{\langle 2 \rangle}\right)_{SPEC_C} \cdot mm$

$DIM_D := READPRN(COMP4)$ $\quad SPEC_D := 18$

$L_{o_4} := \left(DIM_D^{\langle 1 \rangle}\right)_{SPEC_D} \cdot mm$ $\quad D_{o_4} := \left(DIM_D^{\langle 2 \rangle}\right)_{SPEC_D} \cdot mm$

$DIM_E := READPRN(COMP5)$ $\quad SPEC_E := 20$

$L_{o_5} := \left(DIM_E^{\langle 1 \rangle}\right)_{SPEC_E} \cdot mm$ $\quad D_{o_5} := \left(DIM_E^{\langle 2 \rangle}\right)_{SPEC_E} \cdot mm$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 189.911 \\ 186.023 \\ 183.134 \\ 191.625 \\ 196.067 \end{bmatrix} \cdot mm^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := READPRN(IC1LR4T3)$

$B := READPRN(IC2LR4T5)$

$C := READPRN(IC3LR4T6)$

$D := READPRN(IC4LR4T2)$

$E := READPRN(IC5LR4T1)$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000\cdot newton$

$load_A := A^{<1>}\cdot kN$ $displ_A := A^{<2>}\cdot mm$ $load_D := D^{<1>}\cdot kN$ $displ_D := D^{<2>}\cdot mm$

$load_B := B^{<1>}\cdot kN$ $displ_B := B^{<2>}\cdot mm$ $load_E := E^{<1>}\cdot kN$ $displ_E := E^{<2>}\cdot mm$

$load_C := C^{<1>}\cdot kN$ $displ_C := C^{<2>}\cdot mm$

Look at matrices and calculate
number of data points:

$a := 1..rows(A)$ $d := 1..rows(D)$

$b := 1..rows(B)$ $e := 1..rows(E)$

$c := 1..rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1\cdot 10^6\cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \quad A_{iA_a} := \frac{A_{cso_1}\cdot L_{o_1}}{L_{iA_a}} \quad L_{iB_b} := L_{o_2} - displ_{B_b} \quad A_{iB_b} := \frac{A_{cso_2}\cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \quad A_{iC_c} := \frac{A_{cso_3}\cdot L_{o_3}}{L_{iC_c}} \quad L_{iD_d} := L_{o_4} - displ_{D_d} \quad A_{iD_d} := \frac{A_{cso_4}\cdot L_{o_4}}{L_{iD_d}}$$

$$L_{iE_e} := L_{o_5} - displ_{E_e} \quad A_{iE_e} := \frac{A_{cso_5}\cdot L_{o_5}}{L_{iE_e}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \quad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \quad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \quad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \quad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \quad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \quad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

$$\sigma_{trueE_e} := \frac{load_{E_e}}{A_{iE_e}}$$

$$\varepsilon_{trueE_e} := -\ln\left(\frac{L_{iE_e}}{L_{o_5}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$$

$$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$$

$$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$$

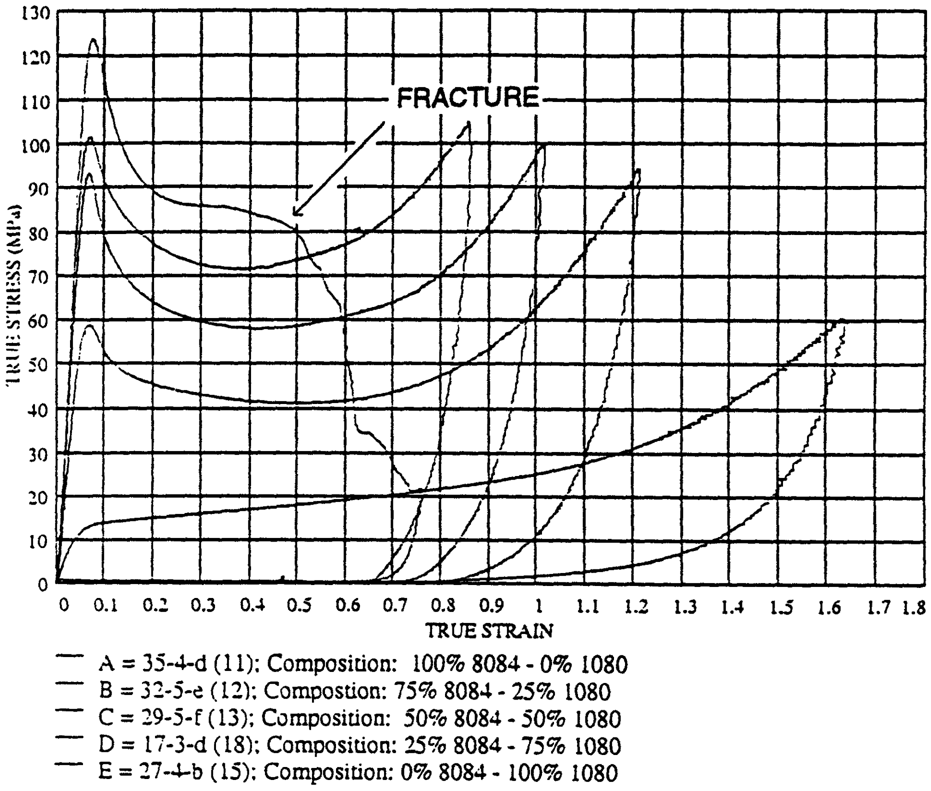
$$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$$

$$\sigma_{yEcalc_e} := \mathrm{if}\left(e \le 1200, \sigma_{trueE_e}, 0 \cdot MPa\right) \quad \sigma_{yE} := \max\left(\sigma_{yEcalc}\right) \quad NumE_e := \mathrm{if}\left(\sigma_{trueE_e} = \sigma_{yE}, e, 0\right)$$

$$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$$

$$\varepsilon_{yE} := \varepsilon_{trueE_{\max(NumE)}}$$

Generate a plot of true stress (MPa) versus true strain for a loading rate of 2.50 mm/sec for various compositions:



List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 0.763$ $\max\left(\sigma_{\mathrm{trueA}}\right) = 123.683 \cdot \mathrm{MPa}$ $\sigma_{yA} = 123.683 \cdot \mathrm{MPa}$ $\varepsilon_{yA} = 0.073$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 0.864$ $\max\left(\sigma_{\mathrm{trueB}}\right) = 104.907 \cdot \mathrm{MPa}$ $\sigma_{yB} = 101.388 \cdot \mathrm{MPa}$ $\varepsilon_{yB} = 0.07$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.017$ $\max\left(\sigma_{\mathrm{trueC}}\right) = 100 \cdot \mathrm{MPa}$ $\sigma_{yC} = 93.068 \cdot \mathrm{MPa}$ $\varepsilon_{yC} = 0.067$

$\max\left(\varepsilon_{\mathrm{trueD}}\right) = 1.212$ $\max\left(\sigma_{\mathrm{trueD}}\right) = 94.333 \cdot \mathrm{MPa}$ $\sigma_{yD} = 58.77 \cdot \mathrm{MPa}$ $\varepsilon_{yD} = 0.069$

$\max\left(\varepsilon_{\mathrm{trueE}}\right) = 1.643$ $\max\left(\sigma_{\mathrm{trueE}}\right) = 60.466 \cdot \mathrm{MPa}$

The yield stress could not be defined for the 100% Crestomer 1080 sample

Specimen dimension data file #1 (100% Derakane 8084 / 0 % Crestomer 1080) ORIGIN := 1

Note: The first column is the specimen length and the second is the specimen diameter.

File applies to specimen: 11, 22, 23, 26

$$\mathrm{COMP1A} := \begin{bmatrix} 14.03 & 15.53 \\ 15.27 & 15.56 \\ 13.99 & 15.60 \\ 15.20 & 15.55 \\ 14.98 & 15.57 \\ 15.02 & 15.60 \\ 15.91 & 15.60 \\ 15.96 & 15.55 \\ 0 & 0 \\ 0 & 0 \\ 15.94 & 15.58 \\ 15.63 & 15.56 \\ 15.31 & 15.57 \\ 15.94 & 15.56 \\ 16.16 & 15.53 \\ 15.59 & 15.55 \\ 15.54 & 15.55 \\ 15.83 & 15.58 \\ 15.89 & 15.56 \\ 15.84 & 15.53 \\ 15.83 & 15.54 \\ 15.60 & 15.69 \\ 15.99 & 15.63 \\ 15.54 & 15.67 \\ 15.67 & 15.63 \\ 15.53 & 15.65 \\ 15.21 & 15.64 \\ 16.06 & 15.69 \end{bmatrix} \qquad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ 27 \\ 28 \end{bmatrix}$$

$$\mathrm{WRITEPRN(COMP1A)} := \mathrm{COMP1A}$$

Specimen dimension data file #1 (100% Derakane 8084 / 0 % Crestomer 1080) ORIGIN := 1

Note: The first column is the specimen length and the second is the specimen diameter.

File applies to specimen: 34,35

$$\mathrm{COMP1B} := \begin{bmatrix} 15.37 & 15.73 \\ 15.70 & 15.61 \\ 15.80 & 15.61 \\ 15.92 & 15.68 \\ 15.46 & 15.69 \\ 15.42 & 15.74 \\ 15.68 & 15.76 \\ 15.52 & 15.69 \\ 15.65 & 15.63 \\ 15.89 & 15.50 \\ 15.27 & 15.55 \\ 15.37 & 15.57 \\ 15.21 & 15.60 \\ 14.94 & 15.64 \\ 15.54 & 15.67 \\ 15.67 & 15.63 \end{bmatrix} \qquad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \end{bmatrix}$$

$$\mathrm{WRITEPRN}(\mathrm{COMP1B}) := \mathrm{COMP1B}$$

Specimen dimension data file #2 (75% Derakane 8084 / 25 % Crestomer 1080) ORIGIN := 1

$$\mathrm{COMP2} := \begin{bmatrix} 15.90 & 15.63 \\ 15.03 & 15.69 \\ 15.68 & 15.65 \\ 15.61 & 15.68 \\ 15.71 & 15.68 \\ 16.54 & 15.67 \\ 11.69 & 15.72 \\ 15.51 & 15.71 \\ 16.16 & 15.68 \\ 15.52 & 15.64 \\ 0 & 0 \\ 15.89 & 15.66 \\ 15.79 & 15.63 \\ 0 & 0 \\ 15.71 & 15.67 \\ 15.95 & 15.67 \\ 15.57 & 15.70 \\ 15.73 & 15.68 \\ 15.54 & 15.69 \\ 14.59 & 15.71 \\ 15.97 & 15.65 \end{bmatrix} \qquad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{bmatrix}$$

Note: The first column iis the specimen length and the second is the specimen diameter.

File applies to specimen: 12, 20, 21

WRITEPRN(COMP2) := COMP2

Specimen dimension data file #2 (75% Derakane 8084 / 25 % Crestomer 1080) ORIGIN := 1

Note: The first column iis the specimen length and the second is the specimen diameter.

File applies to specimen: 31,32,33

$$\mathrm{COMP2B} := \begin{bmatrix} 15.15 & 15.28 \\ 15.09 & 15.27 \\ 14.83 & 15.22 \\ 0 & 0 \\ 14.81 & 15.28 \\ 14.87 & 15.29 \\ 15.19 & 15.36 \\ 14.89 & 15.38 \\ 15.17 & 15.37 \\ 14.84 & 15.38 \\ 14.61 & 15.41 \\ 14.74 & 15.39 \\ 14.69 & 15.43 \\ 15.09 & 15.49 \\ 15.02 & 15.36 \\ 15.37 & 15.36 \\ 0 & 0 \\ 15.05 & 15.36 \\ 14.92 & 15.36 \\ 14.84 & 15.36 \\ 15.05 & 15.36 \end{bmatrix} \quad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{bmatrix}$$

WRITEPRN(COMP2B) := COMP2B

Specimen dimension data file #3 (50% Derakane 8084 / 50 % Crestomer 1080) ORIGIN := 1

$$\mathrm{COMP3} := \begin{bmatrix} 14.91 & 15.62 \\ 15.61 & 15.68 \\ 14.96 & 15.64 \\ 15.63 & 15.68 \\ 16.12 & 15.72 \\ 15.97 & 15.67 \\ 15.64 & 15.70 \\ 15.71 & 15.70 \\ 15.47 & 15.70 \\ 15.88 & 15.73 \\ 15.80 & 15.72 \\ 16.16 & 15.69 \\ 15.38 & 15.69 \\ 15.74 & 15.70 \\ 15.92 & 15.63 \\ 15.39 & 15.64 \\ 15.80 & 15.69 \\ 15.01 & 15.67 \\ 15.70 & 15.66 \\ 16.19 & 15.67 \\ 15.01 & 15.68 \\ 15.26 & 15.39 \end{bmatrix} \qquad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \end{bmatrix}$$

Note: The first column iis the specimen length and the second is the specimen diameter.

File applies to specimen: 13, 18, 19, 30

WRITEPRN(COMP3) := COMP3

Specimen dimension data file #3 (50% Derakane 8084 / 50 % Crestomer 1080) ORIGIN := 1

$$\mathrm{COMP3B} := \begin{bmatrix} 14.67 & 15.35 \\ 15.15 & 15.21 \\ 14.78 & 15.28 \\ 15.10 & 15.28 \\ 14.53 & 15.29 \\ 14.35 & 15.32 \\ 15.58 & 15.35 \\ 14.23 & 15.31 \\ 14.88 & 15.29 \\ 14.87 & 15.28 \\ 15.28 & 15.22 \\ 15.28 & 15.24 \\ 15.38 & 15.27 \\ 15.19 & 15.34 \\ 15.33 & 15.32 \\ 15.24 & 15.31 \\ 14.93 & 15.33 \\ 15.23 & 15.33 \\ 14.50 & 15.34 \\ 15.45 & 15.33 \\ 15.26 & 15.29 \end{bmatrix} \quad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{bmatrix}$$

Note: The first column iis the specimen length and the second is the specimen diameter.

File applies to specimen: 28, 29 , 30

WRITEPRN(COMP3B) := COMP3B

Specimen dimension data file #4 (25% Derakane 8084 / 75 % Crestomer 1080) ORIGIN := 1

$$\mathrm{COMP4} := \begin{bmatrix} 15.66 & 15.60 \\ 15.12 & 15.58 \\ 16.18 & 15.60 \\ 15.88 & 15.59 \\ 15.87 & 15.62 \\ 15.40 & 15.60 \\ 15.75 & 15.60 \\ 15.92 & 15.59 \\ 15.47 & 15.58 \\ 15.45 & 15.63 \\ 15.77 & 15.61 \\ 15.36 & 15.60 \\ 15.60 & 15.65 \\ 16.02 & 15.62 \\ 15.68 & 15.61 \\ 15.50 & 15.62 \\ 15.51 & 15.60 \\ 16.03 & 15.62 \\ 15.30 & 15.60 \\ 16.08 & 15.62 \\ 15.73 & 15.69 \end{bmatrix} \qquad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \end{bmatrix}$$

Note: The first column iis the specimen length and the second is the specimen diameter.

File applies to specimen: 14, 16, 17

WRITEPRN(COMP4) := COMP4

Specimen dimension data file #5 (0% Derakane 8084 / 100 % Crestomer 1080) ORIGIN := 1

Note: The first column is the specimen length and the second is the specimen diameter.

File applies to specimen: 15, 24, 27

$$\mathrm{COMP5} := \begin{bmatrix} 14.85 & 15.75 \\ 16.02 & 15.68 \\ 15.83 & 15.69 \\ 15.91 & 15.71 \\ 15.88 & 15.70 \\ 15.94 & 15.71 \\ 16.49 & 15.75 \\ 14.92 & 15.75 \\ 15.45 & 15.75 \\ 14.91 & 15.75 \\ 15.45 & 15.75 \\ 15.47 & 15.70 \\ 15.60 & 15.70 \\ 15.57 & 15.65 \\ 15.57 & 15.67 \\ 15.58 & 15.68 \\ 15.15 & 15.71 \\ 15.39 & 15.71 \\ 15.47 & 15.74 \\ 15.13 & 15.80 \end{bmatrix} \qquad \mathrm{num} := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{bmatrix}$$

$$\mathrm{WRITEPRN}(\mathrm{COMP5}) := \mathrm{COMP5}$$

Appendix D

Prediction of the Relationship Between Strain Rate and Yield Stress

Upper Yield Stress Model

Lower Yield Stress Model

Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa} \cdot 10^6 \quad i := 1..4 \quad \mathrm{ORIGIN} := 1$

Compositon: 100% 8084, 0% 1080

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}} \quad L_o := \begin{bmatrix} 15.80 \\ 15.54 \\ 15.46 \\ 15.27 \end{bmatrix} \cdot \mathrm{mm} \quad \sigma_c := \begin{bmatrix} 93.409 \\ 105.201 \\ 118.436 \\ 123.647 \end{bmatrix} \cdot \mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.040 \\ 0.221 \\ 0.740 \\ 0.517 \end{bmatrix} \cdot \mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (v) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 2020 \cdot \mathrm{MPa} \quad \nu := 0.33 \quad \Theta := 298 \cdot \mathrm{K}$

$\alpha := 0.20$ **<< assume α = 0.20 >>**

$$\mu := \frac{E}{2 \cdot (1 + \nu)} \qquad \mu = 759.398 \cdot \mathrm{MPa}$$

$$s := \frac{0.077 \cdot \mu}{1 - \nu} \qquad s = 87.274 \cdot \mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_i := \frac{\sigma_{c_i}}{3} \qquad \tau_i := \frac{\sigma_{c_i}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	31.136	53.93
2	35.067	60.738
3	39.479	68.379
4	41.216	71.388

Define the applied plastic shear strain rate:

$$\gamma_{\mathrm{dotpl}_i} := \sqrt{3} \cdot \left[\frac{\mathrm{Vel}_i}{L_{o_i} \cdot \left(1 - \frac{\sigma_{c_i}}{E}\right)} \right]$$

$\gamma_{\mathrm{dotpl}_i}$
$0.001 \cdot \mathrm{sec}^{-1}$
$0.012 \cdot \mathrm{sec}^{-1}$
$0.119 \cdot \mathrm{sec}^{-1}$
$0.302 \cdot \mathrm{sec}^{-1}$

Input Hopkinson Bar data point:

$$\varepsilon_{\mathrm{hop}} := \begin{bmatrix} 3000 \\ 2500 \\ 3200 \\ 3200 \end{bmatrix} \cdot \mathrm{sec}^{-1} \quad \sigma_{\mathrm{hop}} := \begin{bmatrix} 189.8 \\ 200.1 \\ 208.5 \\ 203.6 \end{bmatrix} \cdot \mathrm{MPa} \quad \gamma_{\mathrm{hop}} := \sqrt{3} \cdot \varepsilon_{\mathrm{hop}}$$

$$\mathrm{MEAN}\varepsilon_{\mathrm{hop}} := \mathrm{mean}\left(\varepsilon_{\mathrm{hop}}\right) \quad \mathrm{MEAN}\sigma_{\mathrm{hop}} := \mathrm{mean}\left(\sigma_{\mathrm{hop}}\right)$$

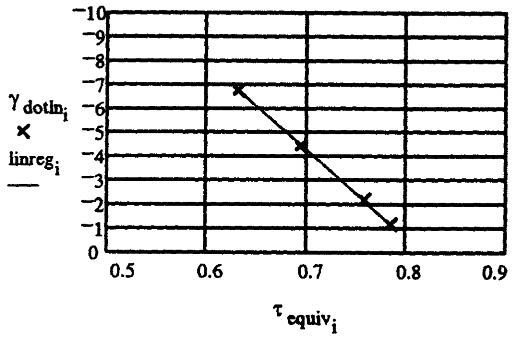
$$\mathrm{STDEV}\varepsilon_{\mathrm{hop}} := \mathrm{stdev}\left(\varepsilon_{\mathrm{hop}}\right) \quad \mathrm{STDEV}\sigma_{\mathrm{hop}} := \mathrm{stdev}\left(\sigma_{\mathrm{hop}}\right)$$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{\text{equiv}_i} := \left(\frac{\tau_i}{s + \alpha \cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{\text{equiv}} = \begin{bmatrix} 0.632 \\ 0.693 \\ 0.759 \\ 0.785 \end{bmatrix} \qquad \gamma_{\text{dotln}_i} := \ln\left(\frac{\gamma_{\text{dotpl}_i}}{\text{sec}^{-1}}\right) \qquad \gamma_{\text{dotln}} = \begin{bmatrix} -6.769 \\ -4.443 \\ -2.129 \\ -1.197 \end{bmatrix}$$

$$\text{linreg} := \text{slope}\left(\tau_{\text{equiv}}, \gamma_{\text{dotln}}\right) \cdot \tau_{\text{equiv}} + \text{intercept}\left(\tau_{\text{equiv}}, \gamma_{\text{dotln}}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$\text{corr}\left(\tau_{\text{equiv}}, \gamma_{\text{dotln}}\right) = 1$$

$$C := \text{slope}\left(\tau_{\text{equiv}}, \gamma_{\text{dotln}}\right) \qquad C = 36.391$$

$$B := \text{intercept}\left(\tau_{\text{equiv}}, \gamma_{\text{dotln}}\right) \qquad B = -29.736$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C \cdot \Theta}{s + \alpha \cdot p_i} \qquad \gamma_{\text{dot0}} := e^{B + C} \cdot \text{sec}^{-1} \qquad \chi = \begin{bmatrix} 115.981 \\ 115.014 \\ 113.948 \\ 113.533 \end{bmatrix} \cdot \frac{K}{\text{MPa}} \qquad \chi := \text{mean}(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{\text{dot0}} = 776.106 \cdot \text{sec}^{-1}$$

$$\chi = 114.619 \cdot \frac{K}{\text{MPa}}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 87.274 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 114.619 \cdot \frac{K}{\mathrm{MPa}} \quad \Theta = 298 \cdot K \quad \gamma_{\mathrm{dot0}} = 776.106 \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right]} \cdot \ln\left(\frac{\gamma_{\mathrm{dot}}}{\gamma_{\mathrm{dot0}}}\right)\right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{\mathrm{dot}}) := \mathrm{Find}(\sigma_{c2})$

$z := 1 .. 9$

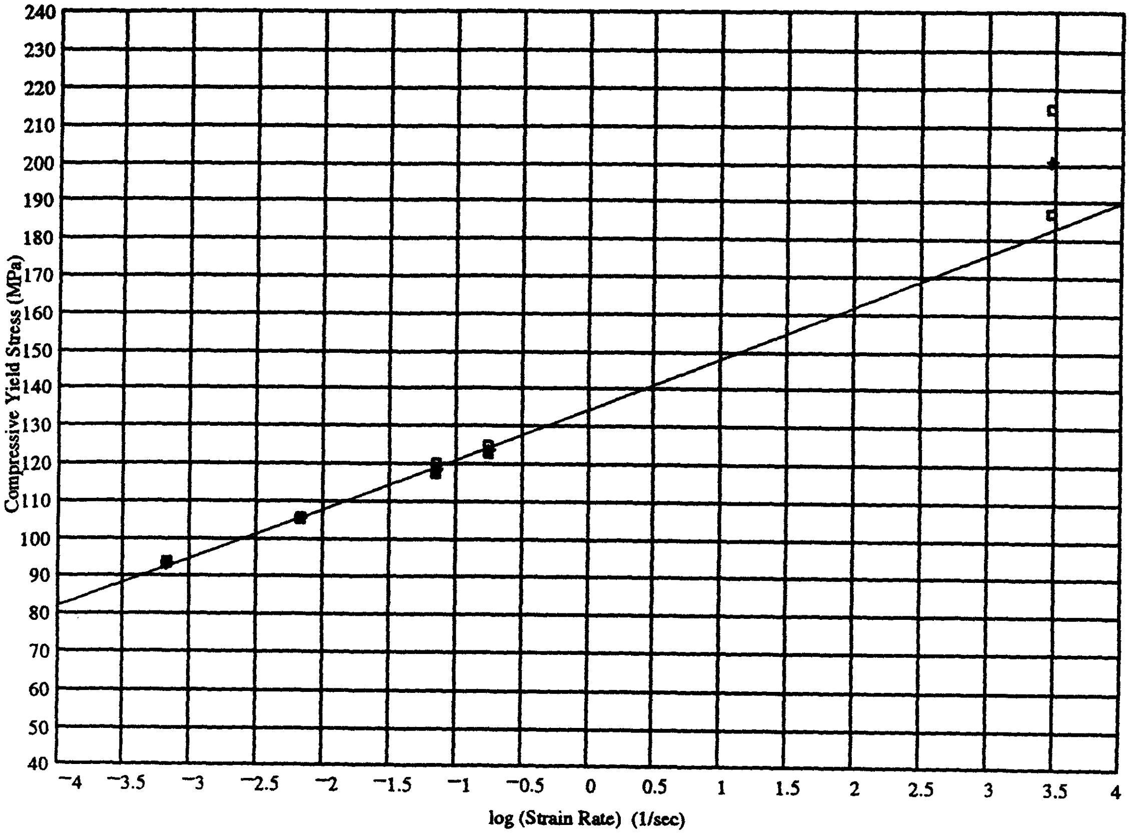
$\gamma_{\mathrm{dot}_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \quad \sigma_{c2_z} := \mathrm{Stress}(\gamma_{\mathrm{dot}_z})$

$\varepsilon_{\mathrm{dot}} := \frac{\gamma_{\mathrm{dot}}}{\sqrt{3}} \qquad \varepsilon_{\mathrm{dotpl}} := \frac{\gamma_{\mathrm{dotpl}}}{\sqrt{3}}$

Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN(YIELD6B)} := \mathrm{augment}\left(\frac{\varepsilon_{\mathrm{dot}}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}}\right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 100% 8084, 0% 1080



Compare predicted compressive yield stress and experimentally obtained value:

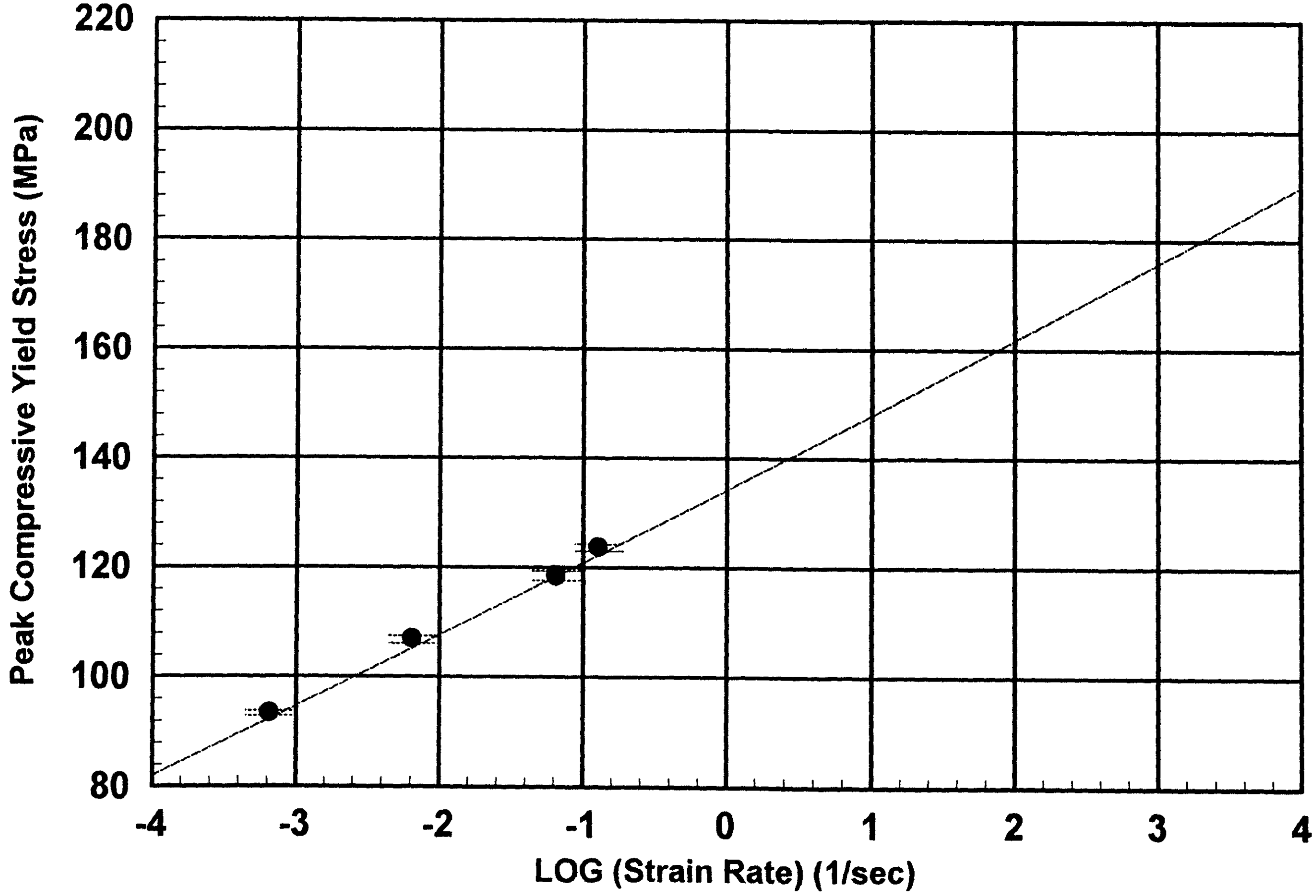
$\sigma_{\text{hoppred}_i} := \text{Stress}\left(\varepsilon_{\text{hop}_i}\right)$

$$\sigma_{\text{hoppred}} = \begin{bmatrix} 179.186 \\ 178.065 \\ 179.583 \\ 179.583 \end{bmatrix} \cdot \text{MPa} \qquad \sigma_{\text{hop}} = \begin{bmatrix} 189.8 \\ 200.1 \\ 208.5 \\ 203.6 \end{bmatrix} \cdot \text{MPa}$$

$\text{mean}\left(\sigma_{\text{hoppred}}\right) = 179.104 \cdot \text{MPa}$ $\quad \text{mean}\left(\sigma_{\text{hop}}\right) = 200.5 \cdot \text{MPa}$

$\text{stdev}\left(\sigma_{\text{hoppred}}\right) = 0.622 \cdot \text{MPa}$ $\quad \text{stdev}\left(\sigma_{\text{hop}}\right) = 6.86 \cdot \text{MPa}$

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS
LOG (STRAIN RATE) FOR THE 100% 8084 MATERIAL



Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$MPa := Pa\cdot 10^{6}$ $\quad i := 1..4$ $\quad ORIGIN := 1$

Compositon: 75% 8084, 25% 1080

$$Vel := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix}\cdot\frac{mm}{sec} \quad L_{0} := \begin{bmatrix} 15.15 \\ 14.81 \\ 14.89 \\ 14.74 \end{bmatrix}\cdot mm \quad \sigma_{c} := \begin{bmatrix} 67.997 \\ 81.961 \\ 95.180 \\ 100.843 \end{bmatrix}\cdot MPa \quad STDEV := \begin{bmatrix} 0.154 \\ 1.719 \\ 0.599 \\ 0.519 \end{bmatrix}\cdot MPa$$

Input the modulus of elasticity (E), the Poisson ration (ν) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 1880\cdot MPa \quad \nu := 0.33 \quad \Theta := 298\cdot K$ $\qquad \alpha := 0.20$ **<< assume α = 0.20 >>**

$$\mu := \frac{E}{2\cdot(1+\nu)} \qquad \mu = 706.767\cdot MPa$$

$$s := \frac{0.077\cdot\mu}{1-\nu} \qquad s = 81.225\cdot MPa$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_{i} := \frac{\sigma_{c_{i}}}{3} \qquad \tau_{i} := \frac{\sigma_{c_{i}}}{\sqrt{3}}$$

i	$\frac{p_i}{MPa}$	$\frac{\tau_i}{MPa}$
1	22.666	39.258
2	27.32	47.32
3	31.727	54.952
4	33.614	58.222

Define the applied plastic shear strain rate:

$$\gamma_{dotpl_{i}} := \sqrt{3}\cdot\left[\frac{Vel_{i}}{L_{0_{i}}\cdot\left(1-\frac{\sigma_{c_{i}}}{E}\right)}\right]$$

$\gamma_{dotpl_{i}}$
$0.001\cdot sec^{-1}$
$0.012\cdot sec^{-1}$
$0.123\cdot sec^{-1}$
$0.31\cdot sec^{-1}$

Input Hopkinson Bar data point:

$$\varepsilon_{hop} := \begin{pmatrix} 3000 \\ 3600 \\ 3000 \end{pmatrix}\cdot sec^{-1} \quad \sigma_{hop} := \begin{pmatrix} 179.29 \\ 187.32 \\ 182.35 \end{pmatrix}\cdot MPa \quad \gamma_{hop} := \sqrt{3}\cdot\varepsilon_{hop}$$

$MEAN\varepsilon_{hop} := mean(\varepsilon_{hop}) \qquad MEAN\sigma_{hop} := mean(\sigma_{hop})$

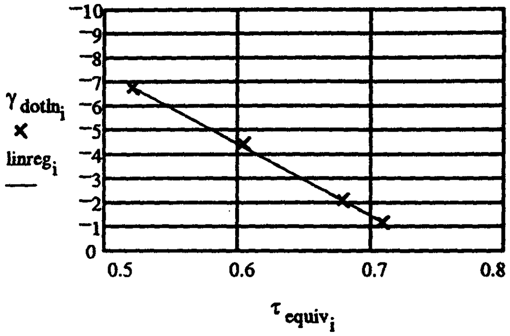
$STDEV\varepsilon_{hop} := stdev(\varepsilon_{hop}) \qquad STDEV\sigma_{hop} := stdev(\sigma_{hop})$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{\mathrm{equiv}_i} := \left(\frac{\tau_i}{s + \alpha \cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{\mathrm{equiv}} = \begin{bmatrix} 0.521 \\ 0.604 \\ 0.678 \\ 0.709 \end{bmatrix} \qquad \gamma_{\mathrm{dotln}_i} := \ln\left(\frac{\gamma_{\mathrm{dotpl1}_i}}{\mathrm{sec}^{-1}}\right) \qquad \gamma_{\mathrm{dotln}} = \begin{bmatrix} -6.737 \\ -4.404 \\ -2.099 \\ -1.17 \end{bmatrix}$$

$$\mathrm{linreg} := \mathrm{slope}\left(\tau_{\mathrm{equiv}}, \gamma_{\mathrm{dotln}}\right) \cdot \tau_{\mathrm{equiv}} + \mathrm{intercept}\left(\tau_{\mathrm{equiv}}, \gamma_{\mathrm{dotln}}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$\mathrm{corr}\left(\tau_{\mathrm{equiv}}, \gamma_{\mathrm{dotln}}\right) = 1$$

$$C := \mathrm{slope}\left(\tau_{\mathrm{equiv}}, \gamma_{\mathrm{dotln}}\right) \qquad C = 29.763$$

$$B := \mathrm{intercept}\left(\tau_{\mathrm{equiv}}, \gamma_{\mathrm{dotln}}\right) \qquad B = -22.298$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C \cdot \Theta}{s + \alpha \cdot p_i} \qquad \gamma_{\mathrm{dot0}} := e^{B + C} \cdot \mathrm{sec}^{-1} \qquad \chi = \begin{bmatrix} 103.424 \\ 102.314 \\ 101.284 \\ 100.849 \end{bmatrix} \cdot \frac{K}{\mathrm{MPa}} \qquad \chi := \mathrm{mean}(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{\mathrm{dot0}} = 1746.306 \cdot \mathrm{sec}^{-1}$$

$$\chi = 101.968 \cdot \frac{K}{\mathrm{MPa}}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 81.225 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 101.968 \cdot \frac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298 \cdot \mathrm{K} \quad \gamma_{\mathrm{dot0}} = 1746.306 \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right]} \cdot \ln\left(\frac{\gamma_{\mathrm{dot}}}{\gamma_{\mathrm{dot0}}}\right)\right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{\mathrm{dot}}) := \mathrm{Find}(\sigma_{c2})$

$z := 1 .. 9$

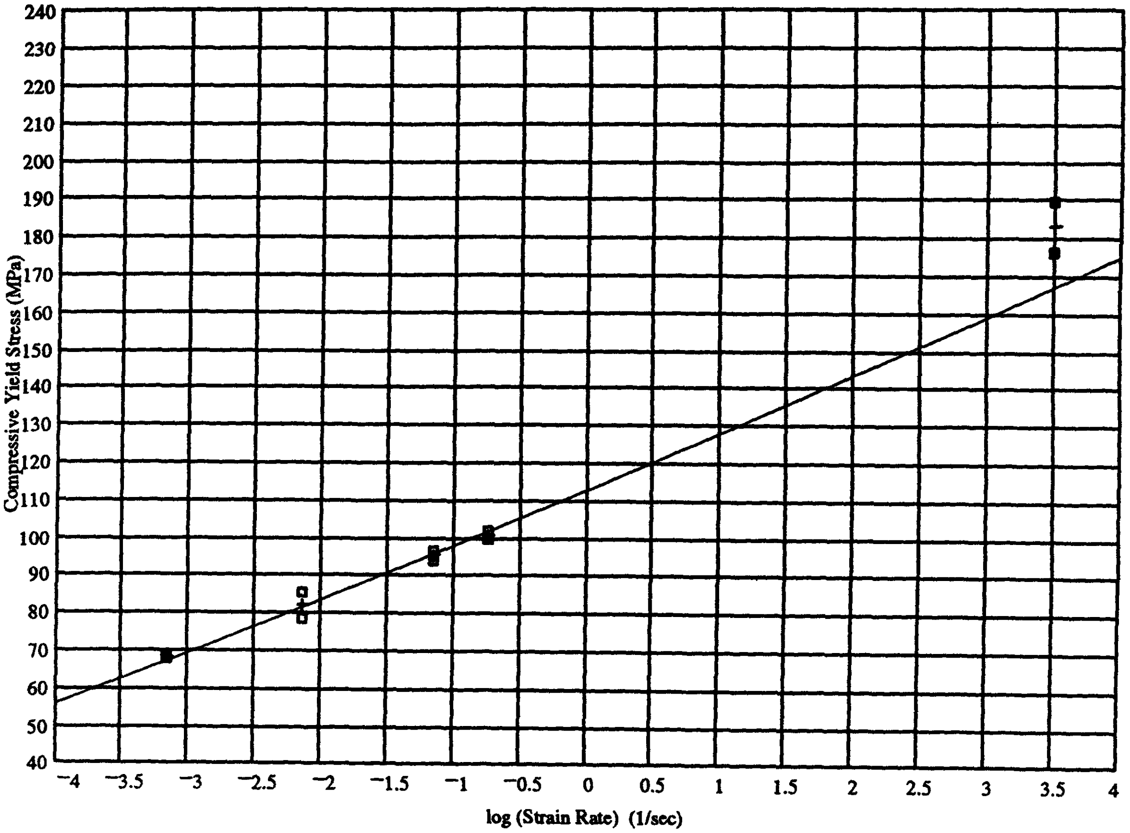
$\gamma_{\mathrm{dot}_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \quad \sigma_{c2_z} := \mathrm{Stress}(\gamma_{\mathrm{dot}_z})$

$\varepsilon_{\mathrm{dot}} := \frac{\gamma_{\mathrm{dot}}}{\sqrt{3}} \qquad \varepsilon_{\mathrm{dotpl}} := \frac{\gamma_{\mathrm{dotpl}}}{\sqrt{3}}$

Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN}(\mathrm{YIELD7B}) := \mathrm{augment}\left(\frac{\varepsilon_{\mathrm{dot}}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}}\right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 75% 8084, 25% 1080



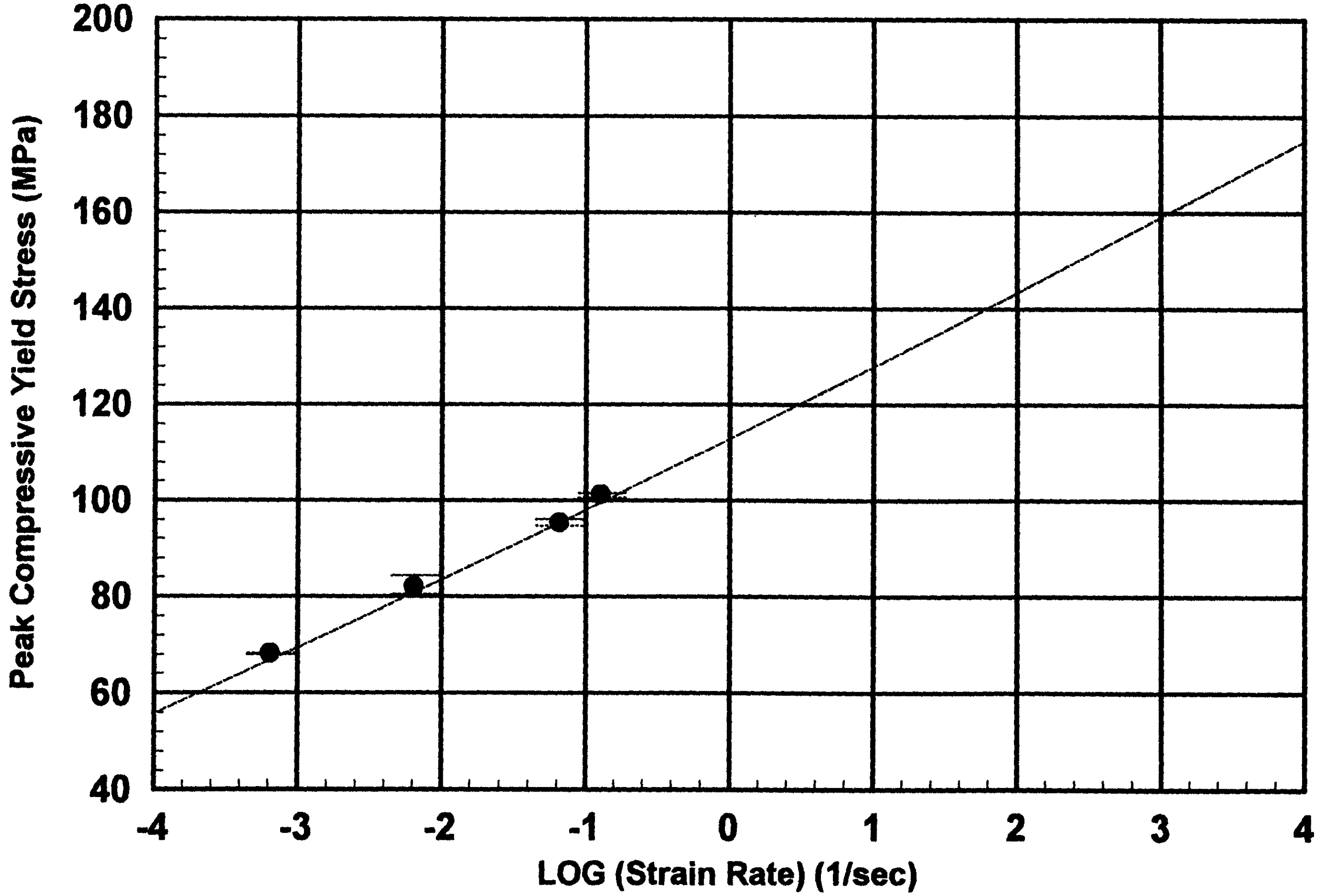
Compare predicted compressive yield stress and experimentally obtained value: $j := 1..3$

$$\sigma_{hoppred_j} := Stress\left(\varepsilon_{hop_j}\right) \qquad \sigma_{hoppred} = \begin{pmatrix} 162.775 \\ 164.032 \\ 162.775 \end{pmatrix} \cdot MPa \qquad \sigma_{hop} = \begin{pmatrix} 179.29 \\ 187.32 \\ 182.35 \end{pmatrix} \cdot MPa$$

$$mean\left(\sigma_{hoppred}\right) = 163.194 \cdot MPa \qquad mean\left(\sigma_{hop}\right) = 182.987 \cdot MPa$$

$$stdev\left(\sigma_{hoppred}\right) = 0.593 \cdot MPa \qquad stdev\left(\sigma_{hop}\right) = 3.309 \cdot MPa$$

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR THE 75% 8084 / 25% 1080 BLEND



Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$ $\quad i := 1..4 \quad$ $\mathrm{ORIGIN} := 1$

Compositon: 50% 8084, 50% 1080

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}} \quad L_o := \begin{bmatrix} 15.15 \\ 14.35 \\ 14.87 \\ 15.38 \end{bmatrix} \cdot \mathrm{mm} \quad \sigma_c := \begin{bmatrix} 61.213 \\ 74.165 \\ 86.508 \\ 92.670 \end{bmatrix} \cdot \mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.664 \\ 0.096 \\ 0.615 \\ 0.390 \end{bmatrix} \cdot \mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (v) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 1680\cdot\mathrm{MPa} \quad \nu := 0.33 \quad \Theta := 298\cdot\mathrm{K}$ $\qquad \alpha := 0.20$ **<< assume α = 0.20 >>**

$$\mu := \frac{E}{2\cdot(1+\nu)} \qquad \mu = 631.579\,\mathrm{MPa}$$

$$s := \frac{0.077\cdot\mu}{1-\nu} \qquad s = 72.584\,\mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_i := \frac{\sigma_{c_i}}{3} \qquad \tau_i := \frac{\sigma_{c_i}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	20.404	35.341
2	24.722	42.819
3	28.836	49.945
4	30.89	53.503

Define the applied plastic shear strain rate:

$$\gamma_{dotpl_i} := \sqrt{3}\cdot\left[\frac{\mathrm{Vel}_i}{L_{o_i}\cdot\left(1-\frac{\sigma_{c_i}}{E}\right)}\right]$$

γ_{dotpl_i}
$0.001\cdot\mathrm{sec}^{-1}$
$0.013\cdot\mathrm{sec}^{-1}$
$0.123\cdot\mathrm{sec}^{-1}$
$0.298\cdot\mathrm{sec}^{-1}$

Input Hopkinson Bar data point:

$$\varepsilon_{hop} := \begin{pmatrix} 2500 \\ 2500 \\ 2100 \end{pmatrix}\cdot\mathrm{sec}^{-1} \quad \sigma_{hop} := \begin{pmatrix} 171.79 \\ 174.61 \\ 176.76 \end{pmatrix}\cdot\mathrm{MPa} \quad \gamma_{hop} := \sqrt{3}\cdot\varepsilon_{hop}$$

$\mathrm{MEAN}\varepsilon_{hop} := \mathrm{mean}(\varepsilon_{hop}) \quad \mathrm{MEAN}\sigma_{hop} := \mathrm{mean}(\sigma_{hop})$

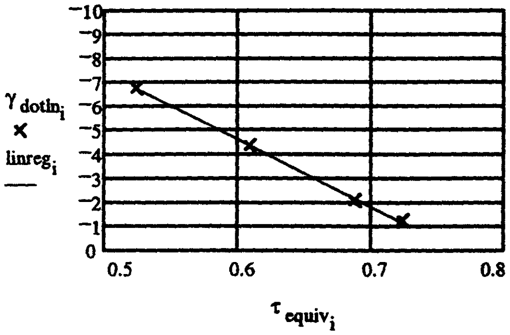
$\mathrm{STDEV}\varepsilon_{hop} := \mathrm{stdev}(\varepsilon_{hop}) \quad \mathrm{STDEV}\sigma_{hop} := \mathrm{stdev}(\sigma_{hop})$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{equiv_i} := \left(\frac{\tau_i}{s+\alpha\cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{equiv} = \begin{bmatrix}0.524\\0.61\\0.687\\0.725\end{bmatrix} \qquad \gamma_{dotln_i} := \ln\left(\frac{\gamma_{dotpl1_i}}{sec^{-1}}\right) \qquad \gamma_{dotln} = \begin{bmatrix}-6.737\\-4.372\\-2.097\\-1.211\end{bmatrix}$$

$$linreg := slope\left(\tau_{equiv}, \gamma_{dotln}\right)\cdot\tau_{equiv} + intercept\left(\tau_{equiv}, \gamma_{dotln}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$corr\left(\tau_{equiv}, \gamma_{dotln}\right) = 1$$

$$C := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad C = 27.93$$

$$B := intercept\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad B = -21.381$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C\cdot\Theta}{s+\alpha\cdot p_i} \qquad \gamma_{dot0} := e^{B+C}\cdot sec^{-1} \qquad \chi = \begin{bmatrix}108.564\\107.355\\106.227\\105.673\end{bmatrix}\cdot\frac{K}{MPa} \qquad \chi := mean(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{dot0} = 698.669\cdot sec^{-1}$$

$$\chi = 106.955\cdot\frac{K}{MPa}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 72.584 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 106.955 \cdot \dfrac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298 \cdot \mathrm{K} \quad \gamma_{\mathrm{dot0}} = 698.669 \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3} \right) \right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3} \right) \right]} \cdot \ln \left(\frac{\gamma_{\mathrm{dot}}}{\gamma_{\mathrm{dot0}}} \right) \right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{\mathrm{dot}}) := \mathrm{Find}(\sigma_{c2})$

$z := 1 .. 9$

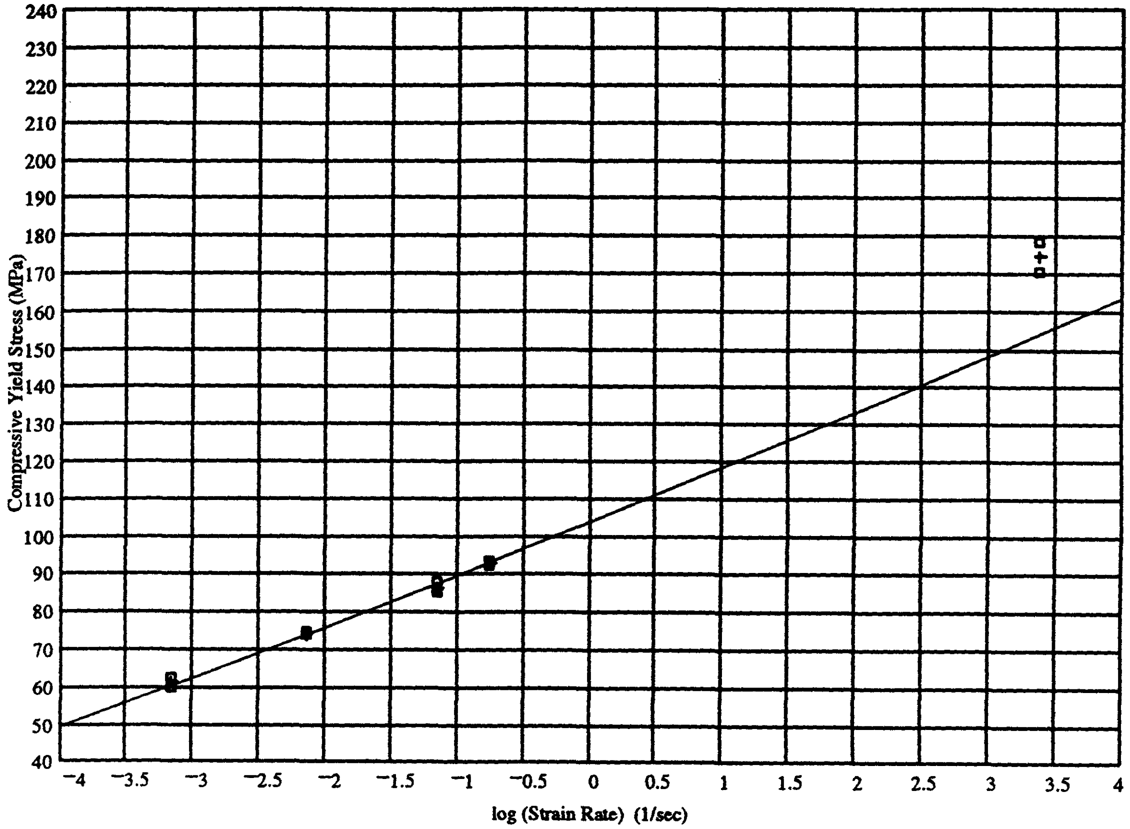
$\gamma_{\mathrm{dot}_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \quad \sigma_{c2_z} := \mathrm{Stress}\left(\gamma_{\mathrm{dot}_z}\right)$

$\varepsilon_{\mathrm{dot}} := \dfrac{\gamma_{\mathrm{dot}}}{\sqrt{3}} \qquad \varepsilon_{\mathrm{dotpl}} := \dfrac{\gamma_{\mathrm{dotpl}}}{\sqrt{3}}$

Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN(YIELD8B)} := \mathrm{augment}\left(\dfrac{\varepsilon_{\mathrm{dot}}}{\mathrm{sec}^{-1}}, \dfrac{\sigma_{c2}}{\mathrm{MPa}} \right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 50% 8084, 50% 1080



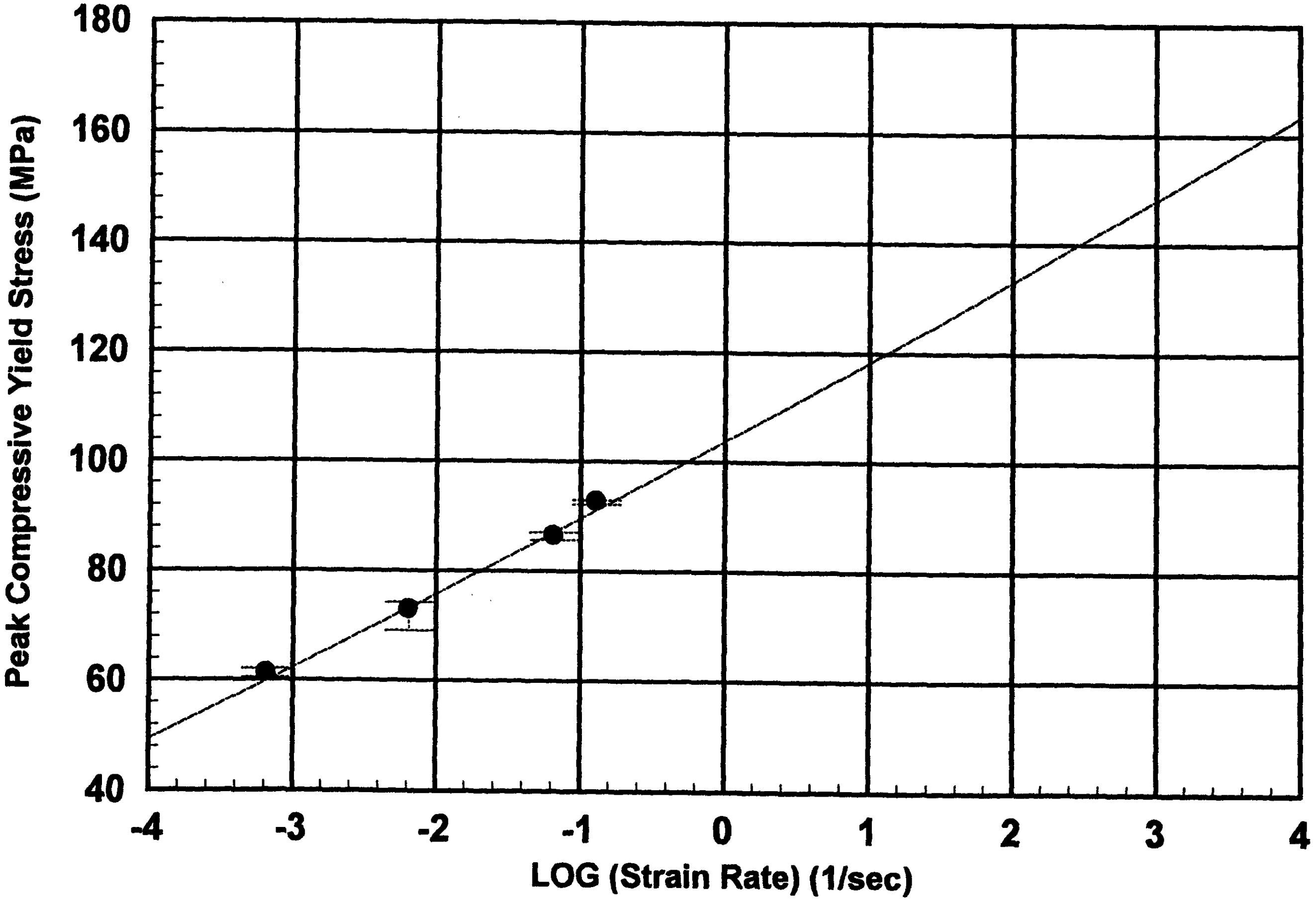
Compare predicted compressive yield stress and experimentally obtained value: $j := 1..3$

$\sigma_{hoppred_j} := \text{Stress}\left(\varepsilon_{hop_j}\right)$ $\sigma_{hoppred} = \begin{pmatrix} 150.514 \\ 150.514 \\ 149.364 \end{pmatrix} \cdot \text{MPa}$ $\sigma_{hop} = \begin{pmatrix} 171.79 \\ 174.61 \\ 176.76 \end{pmatrix} \cdot \text{MPa}$

$\text{mean}\left(\sigma_{hoppred}\right) = 150.131 \cdot \text{MPa}$ $\text{mean}\left(\sigma_{hop}\right) = 174.387 \cdot \text{MPa}$

$\text{stdev}\left(\sigma_{hoppred}\right) = 0.542 \cdot \text{MPa}$ $\text{stdev}\left(\sigma_{hop}\right) = 2.035 \cdot \text{MPa}$

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS LOG(STRAIN RATE) FOR THE 50% 8084 / 50% 1080 BLEND



Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$ $\quad i := 1..4$ $\quad \mathrm{ORIGIN} := 1$

Compositon: 25% 8084, 75% 1080

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix}\cdot\frac{\mathrm{mm}}{\mathrm{sec}} \quad L_o := \begin{bmatrix} 15.66 \\ 15.75 \\ 15.92 \\ 15.30 \end{bmatrix}\cdot\mathrm{mm} \quad \sigma_c := \begin{bmatrix} 22.445 \\ 30.087 \\ 49.841 \\ 57.367 \end{bmatrix}\cdot\mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.015 \\ 1.279 \\ 0.698 \\ 2.419 \end{bmatrix}\cdot\mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (v) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 1125\cdot\mathrm{MPa}$ $\quad \nu := 0.33$ $\quad \Theta := 298\cdot\mathrm{K}$ $\qquad \alpha := 0.20$ $\quad$ << assume $\alpha = 0.20$ >>

$$\mu := \frac{E}{2\cdot(1+\nu)} \qquad \mu = 422.932\cdot\mathrm{MPa}$$

$$s := \frac{0.077\cdot\mu}{1-\nu} \qquad s = 48.606\cdot\mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_i := \frac{\sigma_{c_i}}{3} \qquad \tau_i := \frac{\sigma_{c_i}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	7.482	12.959
2	10.029	17.371
3	16.614	28.776
4	19.122	33.121

Define the applied plastic shear strain rate:

$$\gamma_{\mathrm{dotpl}_i} := \sqrt{3}\cdot\left[\frac{\mathrm{Vel}_i}{L_{o_i}\cdot\left(1-\frac{\sigma_{c_i}}{E}\right)}\right]$$

$\gamma_{\mathrm{dotpl}_i}$
$0.001\cdot\mathrm{sec}^{-1}$
$0.011\cdot\mathrm{sec}^{-1}$
$0.114\cdot\mathrm{sec}^{-1}$
$0.298\cdot\mathrm{sec}^{-1}$

Input Hopkinson Bar data point:

$$\varepsilon_{\mathrm{hop}} := \begin{bmatrix} 2800 \\ 2600 \\ 2900 \\ 3000 \end{bmatrix}\cdot\mathrm{sec}^{-1} \quad \sigma_{\mathrm{hop}} := \begin{bmatrix} 145.16 \\ 140.00 \\ 137.79 \\ 147.00 \end{bmatrix}\cdot\mathrm{MPa} \quad \gamma_{\mathrm{hop}} := \sqrt{3}\cdot\varepsilon_{\mathrm{hop}}$$

$$\mathrm{MEAN}\varepsilon_{\mathrm{hop}} := \mathrm{mean}\left(\varepsilon_{\mathrm{hop}}\right) \qquad \mathrm{MEAN}\sigma_{\mathrm{hop}} := \mathrm{mean}\left(\sigma_{\mathrm{hop}}\right)$$

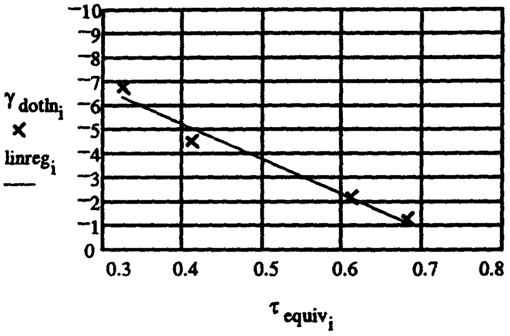
$$\mathrm{STDEV}\varepsilon_{\mathrm{hop}} := \mathrm{stdev}\left(\varepsilon_{\mathrm{hop}}\right) \qquad \mathrm{STDEV}\sigma_{\mathrm{hop}} := \mathrm{stdev}\left(\sigma_{\mathrm{hop}}\right)$$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{equiv_i} := \left(\frac{\tau_i}{s + \alpha \cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{equiv} = \begin{bmatrix} 0.324 \\ 0.41 \\ 0.611 \\ 0.682 \end{bmatrix} \qquad \gamma_{dotln_i} := \ln\left(\frac{\gamma_{dotpl_i}}{sec^{-1}}\right) \qquad \gamma_{dotln} = \begin{bmatrix} -6.787 \\ -4.483 \\ -2.173 \\ -1.21 \end{bmatrix}$$

$$linreg := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \cdot \tau_{equiv} + intercept\left(\tau_{equiv}, \gamma_{dotln}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$corr\left(\tau_{equiv}, \gamma_{dotln}\right) = 0.985$$

$$C := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad C = 14.647$$

$$B := intercept\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad B = -11.088$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C \cdot \Theta}{s + \alpha \cdot p_i} \qquad \gamma_{dot0} := e^{B + C} \cdot sec^{-1} \qquad \chi = \begin{bmatrix} 87.116 \\ 86.239 \\ 84.052 \\ 83.248 \end{bmatrix} \cdot \frac{K}{MPa} \qquad \chi := mean(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{dot0} = 35.127 \cdot sec^{-1}$$

$$\chi = 85.164 \cdot \frac{K}{MPa}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 48.606\cdot\mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 85.164\cdot\frac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298\cdot\mathrm{K} \quad \gamma_{dot0} = 35.127\cdot\mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50\cdot\mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha\cdot\left(\frac{\sigma_{c2}}{3}\right)\right]\cdot\left[1 + \frac{\Theta}{\chi\cdot\left[s + \alpha\cdot\left(\frac{\sigma_{c2}}{3}\right)\right]}\cdot\ln\left(\frac{\gamma_{dot}}{\gamma_{dot0}}\right)\right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{dot}) := \mathrm{Find}(\sigma_{c2})$

$z := 1 .. 9$

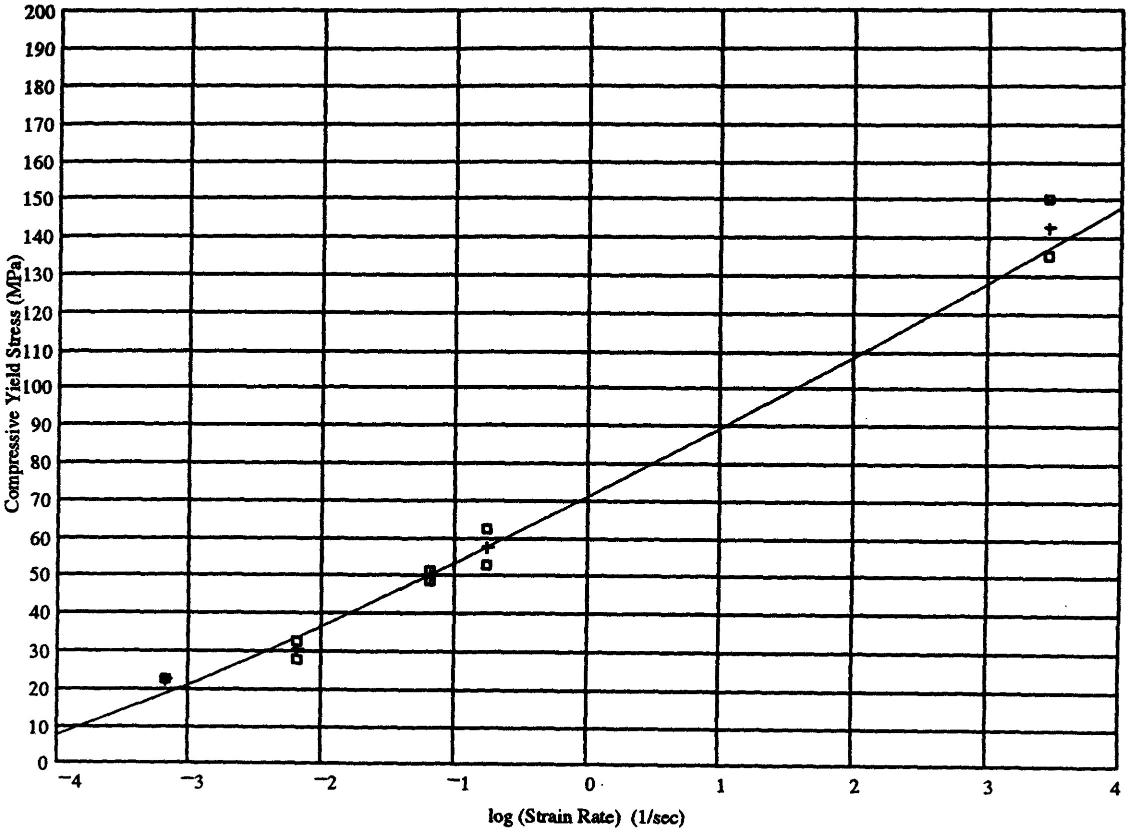
$\gamma_{dot_z} := \left(10^{z-5}\cdot\sqrt{3}\right)\cdot\mathrm{sec}^{-1} \qquad \sigma_{c2_z} := \mathrm{Stress}\left(\gamma_{dot_z}\right)$

$\varepsilon_{dot} := \frac{\gamma_{dot}}{\sqrt{3}} \qquad \varepsilon_{dotpl} := \frac{\gamma_{dotpl}}{\sqrt{3}}$

Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN}(\mathrm{YIELD10B}) := \mathrm{augment}\left(\frac{\varepsilon_{dot}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}}\right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 25% 8084, 75% 1080



<u>Compare predicted compressive yield stress and experimentally obtained value:</u> $j := 1 .. 3$

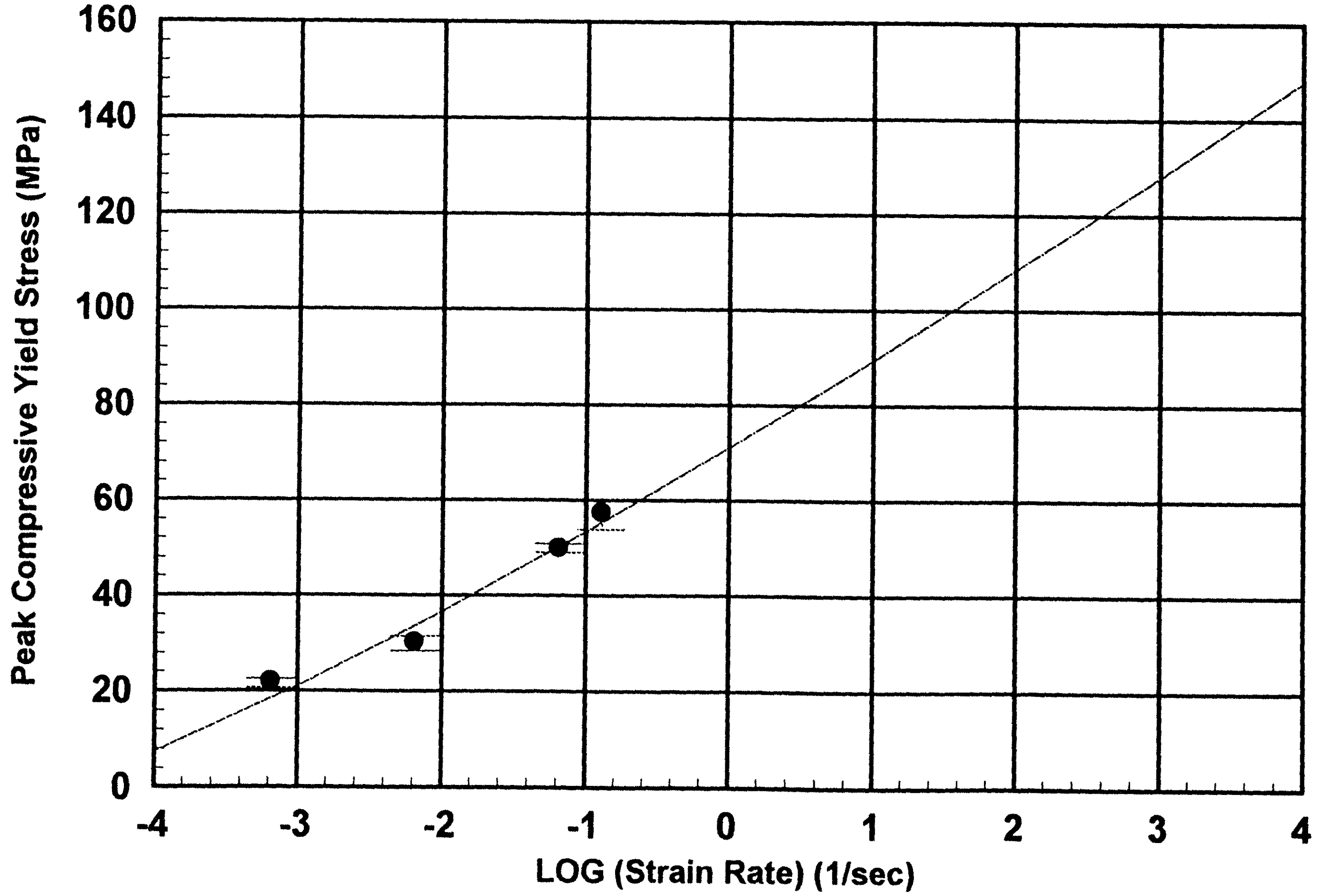
$\sigma_{\text{hoppred}_i} := \text{Stress}\left(\varepsilon_{\text{hop}_i}\right)$

$$\sigma_{\text{hoppred}} = \begin{bmatrix} 132.077 \\ 131.439 \\ 132.38 \\ 132.672 \end{bmatrix} \cdot \text{MPa} \qquad \sigma_{\text{hop}} = \begin{bmatrix} 145.16 \\ 140 \\ 137.79 \\ 147 \end{bmatrix} \cdot \text{MPa}$$

$\text{mean}\left(\sigma_{\text{hoppred}}\right) = 132.142 \cdot \text{MPa}$ $\qquad$ $\text{mean}\left(\sigma_{\text{hop}}\right) = 142.488 \cdot \text{MPa}$

$\text{stdev}\left(\sigma_{\text{hoppred}}\right) = 0.457 \cdot \text{MPa}$ $\qquad$ $\text{stdev}\left(\sigma_{\text{hop}}\right) = 3.734 \cdot \text{MPa}$

PLOT OF PEAK COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR THE 25% 8084 / 75% 1080 BLEND



Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa}\cdot 10^{6} \quad i := 1..4 \quad \mathrm{ORIGIN} := 1$ **Compositon: 100% 8084, 0% 1080**

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix}\cdot\frac{\mathrm{mm}}{\mathrm{sec}} \quad L_{0} := \begin{bmatrix} 15.80 \\ 15.54 \\ 15.46 \\ 15.27 \end{bmatrix}\cdot\mathrm{mm} \quad \sigma_{c} := \begin{bmatrix} 72.00 \\ 75.34 \\ 81.07 \\ 84.45 \end{bmatrix}\cdot\mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.51 \\ 0.06 \\ 0.11 \\ 0.74 \end{bmatrix}\cdot\mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (ν) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 2020\cdot\mathrm{MPa} \quad \nu := 0.33 \quad \Theta := 298\cdot\mathrm{K}$ $\alpha := 0.20$ << assume $\alpha = 0.20$ >>

$$\mu := \frac{E}{2\cdot(1+\nu)} \qquad \mu = 759.398\cdot\mathrm{MPa}$$

$$s := \frac{0.077\cdot\mu}{1-\nu} \qquad s = 87.274\cdot\mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_{i} := \frac{\sigma_{c_{i}}}{3} \qquad \tau_{i} := \frac{\sigma_{c_{i}}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	24	41.569
2	25.113	43.498
3	27.023	46.806
4	28.15	48.757

Define the applied plastic shear strain rate:

$$\gamma_{\mathrm{dotpl}_{i}} := \sqrt{3}\cdot\left[\frac{\mathrm{Vel}_{i}}{L_{0_{i}}\cdot\left(1-\frac{\sigma_{c_{i}}}{E}\right)}\right]$$

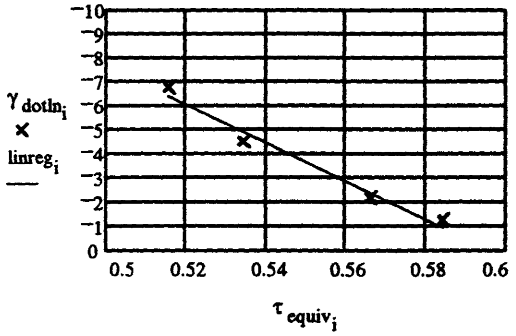
$\gamma_{\mathrm{dotpl}_{i}}$
$0.001\cdot\mathrm{sec}^{-1}$
$0.012\cdot\mathrm{sec}^{-1}$
$0.117\cdot\mathrm{sec}^{-1}$
$0.296\cdot\mathrm{sec}^{-1}$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{equiv_i} := \left(\frac{\tau_i}{s+\alpha\cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{equiv} = \begin{bmatrix}0.515\\0.534\\0.566\\0.584\end{bmatrix} \qquad \gamma_{dotln_i} := \ln\left(\frac{\gamma_{dotpl_i}}{sec^{-1}}\right) \qquad \gamma_{dotln} = \begin{bmatrix}-6.78\\-4.459\\-2.148\\-1.218\end{bmatrix}$$

$$linreg := slope\left(\tau_{equiv}, \gamma_{dotln}\right)\cdot\tau_{equiv} + intercept\left(\tau_{equiv}, \gamma_{dotln}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$corr\left(\tau_{equiv}, \gamma_{dotln}\right) = 0.987$$

$$C := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad C = 79.414$$

$$B := intercept\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad B = -47.328$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C\cdot\Theta}{s+\alpha\cdot p_i} \qquad \gamma_{dot0} := e^{B+C}\cdot sec^{-1} \qquad \chi = \begin{bmatrix}257.024\\256.404\\255.347\\254.728\end{bmatrix}\cdot\frac{K}{MPa} \qquad \chi := mean(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{dot0} = 8.603\cdot 10^{13}\cdot sec^{-1}$$

$$\chi = 255.876\cdot\frac{K}{MPa}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 87.274 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 255.876 \cdot \frac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298 \cdot \mathrm{K} \quad \gamma_{dot0} = 8.603 \cdot 10^{13} \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3} \right) \right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3} \right) \right]} \cdot \ln\left(\frac{\gamma_{dot}}{\gamma_{dot0}} \right) \right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{dot}) := \mathrm{Find}(\sigma_{c2})$

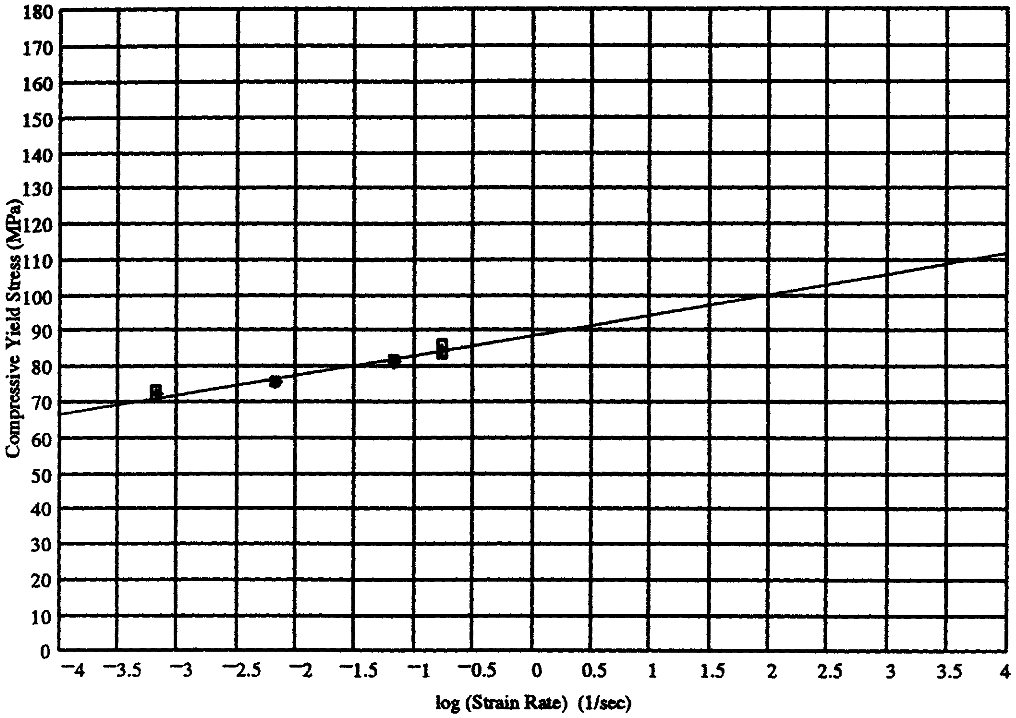
$z := 1 .. 9$

$\gamma_{dot_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \quad \sigma_{c2_z} := \mathrm{Stress}(\gamma_{dot_z})$

$\varepsilon_{dot} := \frac{\gamma_{dot}}{\sqrt{3}} \qquad \varepsilon_{dotpl} := \frac{\gamma_{dotpl}}{\sqrt{3}}$

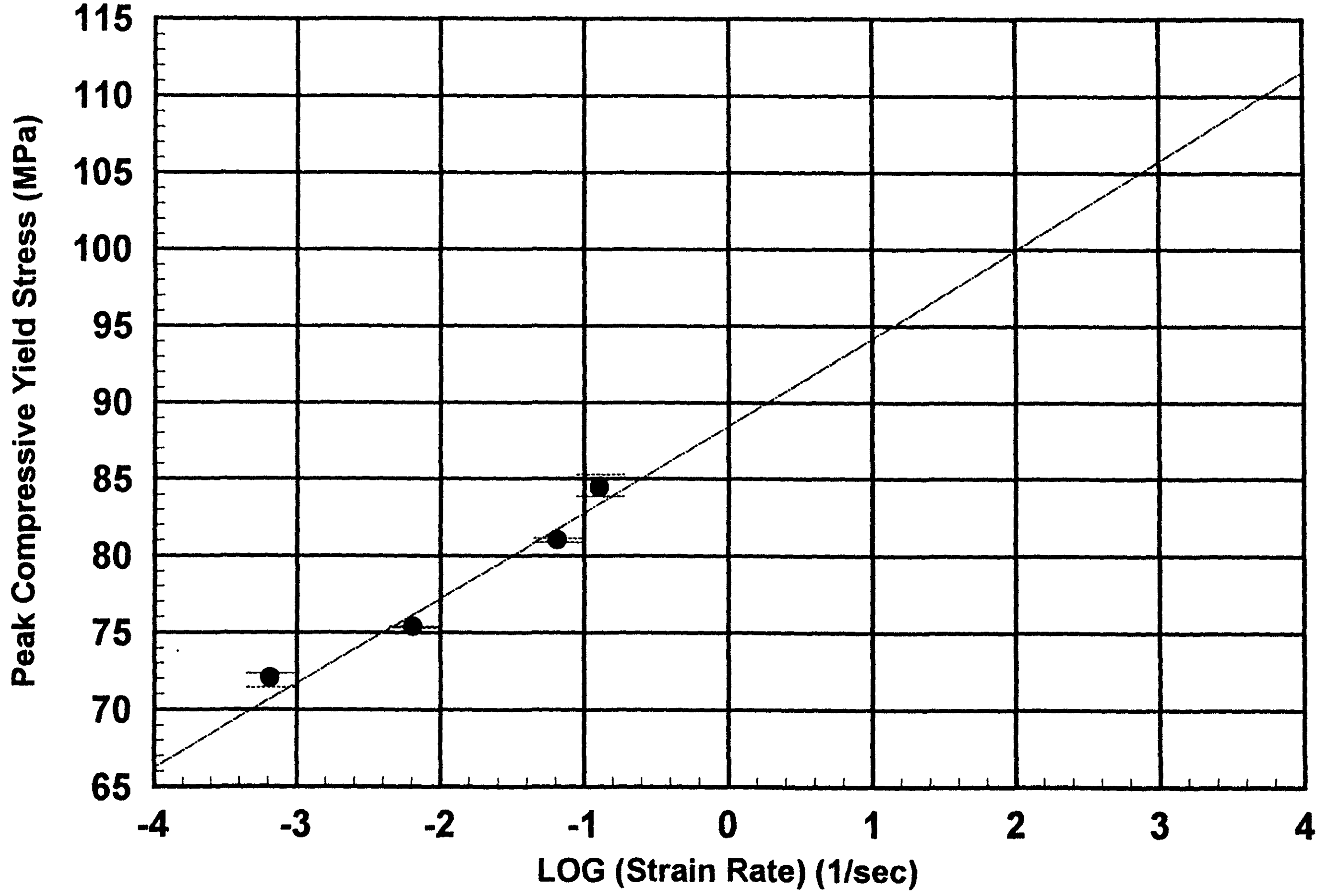
Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN(YIELD6A)} := \mathrm{augment}\left(\frac{\varepsilon_{dot}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}} \right)$



Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 100% 8084, 0% 1080

PLOT OF LOWER COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR THE 100% 8084 MATERIAL



Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa}\cdot 10^6 \quad i := 1..4 \quad \mathrm{ORIGIN} := 1$

Compositon: 75% 8084, 25% 1080

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix}\cdot\frac{\mathrm{mm}}{\mathrm{sec}} \quad L_o := \begin{bmatrix} 15.15 \\ 14.81 \\ 14.89 \\ 14.74 \end{bmatrix}\cdot\mathrm{mm} \quad \sigma_c := \begin{bmatrix} 59.09 \\ 62.84 \\ 68.34 \\ 71.18 \end{bmatrix}\cdot\mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.15 \\ 0.54 \\ 0.60 \\ 0.42 \end{bmatrix}\cdot\mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (ν) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 1880\cdot\mathrm{MPa} \quad \nu := 0.33 \quad \Theta := 298\cdot\mathrm{K}$

$\alpha := 0.20$ **<< assume α = 0.20 >>**

$$\mu := \frac{E}{2\cdot(1+\nu)} \qquad \mu = 706.767\cdot\mathrm{MPa}$$

$$s := \frac{0.077\cdot\mu}{1-\nu} \qquad s = 81.225\cdot\mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_i := \frac{\sigma_{c_i}}{3} \qquad \tau_i := \frac{\sigma_{c_i}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	19.697	34.116
2	20.947	36.281
3	22.78	39.456
4	23.727	41.096

Define the applied plastic shear strain rate:

$$\gamma_{\mathrm{dotpl}_i} := \sqrt{3}\cdot\left[\frac{\mathrm{Vel}_i}{L_{o_i}\cdot\left(1-\frac{\sigma_{c_i}}{E}\right)}\right]$$

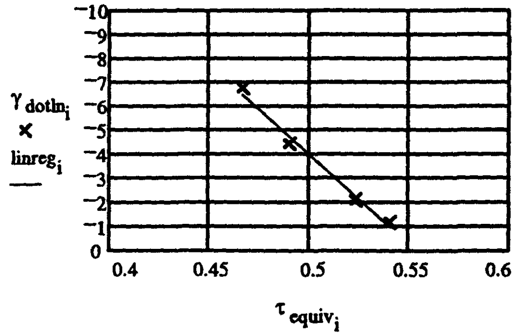
$\gamma_{\mathrm{dotpl}_i}$
$0.001\cdot\mathrm{sec}^{-1}$
$0.012\cdot\mathrm{sec}^{-1}$
$0.121\cdot\mathrm{sec}^{-1}$
$0.305\cdot\mathrm{sec}^{-1}$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{equiv_i} := \left(\frac{\tau_i}{s + \alpha \cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{equiv} = \begin{bmatrix} 0.467 \\ 0.49 \\ 0.524 \\ 0.541 \end{bmatrix} \qquad \gamma_{dotln_i} := \ln\left(\frac{\gamma_{dotpl1_i}}{sec^{-1}}\right) \qquad \gamma_{dotln} = \begin{bmatrix} -6.742 \\ -4.415 \\ -2.114 \\ -1.186 \end{bmatrix}$$

$$linreg := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \cdot \tau_{equiv} + intercept\left(\tau_{equiv}, \gamma_{dotln}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$corr\left(\tau_{equiv}, \gamma_{dotln}\right) = 0.994$$

$$C := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad C = 74.358$$

$$B := intercept\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad B = -41.176$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C \cdot \Theta}{s + \alpha \cdot p_i} \qquad \gamma_{dot0} := e^{B+C} \cdot sec^{-1} \qquad \chi = \begin{bmatrix} 260.187 \\ 259.425 \\ 258.316 \\ 257.748 \end{bmatrix} \cdot \frac{K}{MPa} \qquad \chi := mean(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{dot0} = 2.575 \cdot 10^{14} \cdot sec^{-1}$$

$$\chi = 258.919 \cdot \frac{K}{MPa}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 81.225 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 258.919 \cdot \frac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298 \cdot \mathrm{K} \quad \gamma_{dot0} = 2.575 \cdot 10^{14} \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right]} \cdot \ln\left(\frac{\gamma_{dot}}{\gamma_{dot0}}\right)\right]^{\frac{6}{5}}$$

$\mathrm{Stress}\left(\gamma_{dot}\right) := \mathrm{Find}\left(\sigma_{c2}\right)$

$z := 1 .. 9$

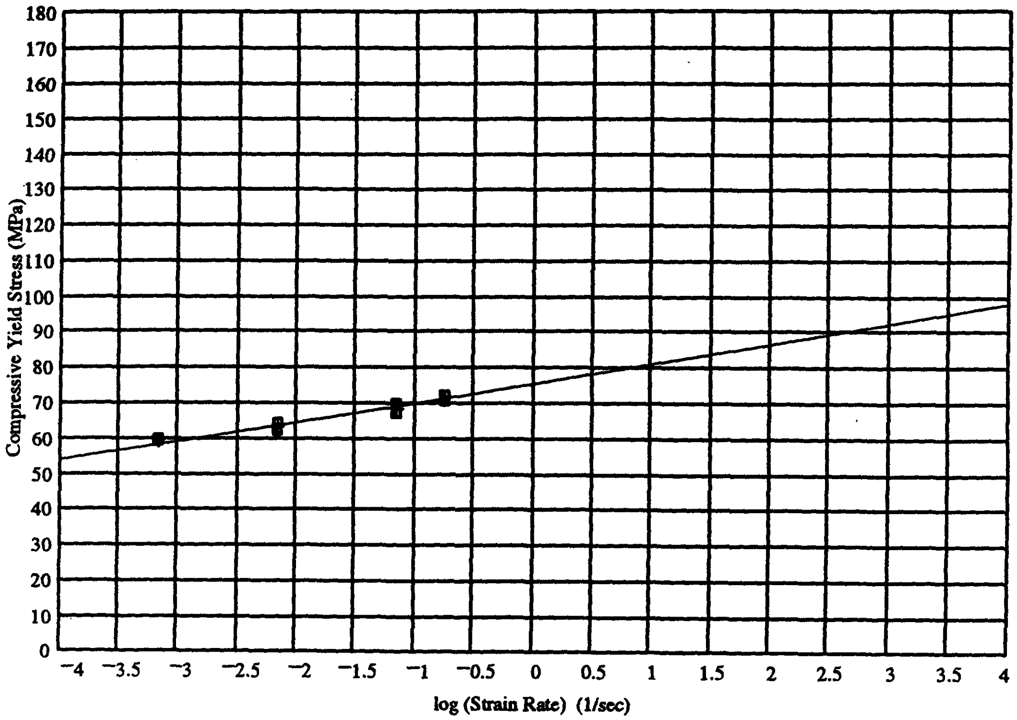
$\gamma_{dot_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \quad \sigma_{c2_z} := \mathrm{Stress}\left(\gamma_{dot_z}\right)$

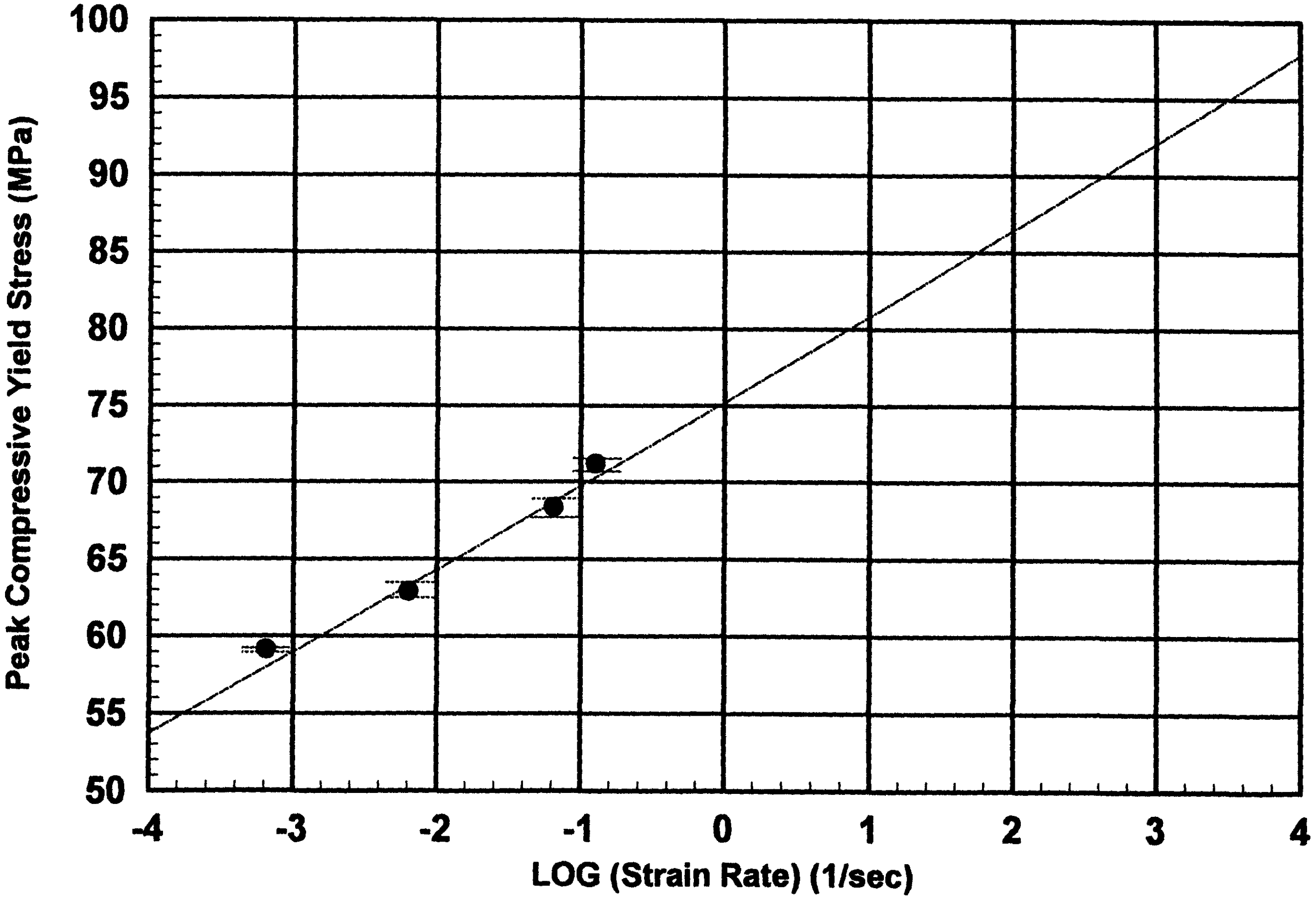
$\varepsilon_{dot} := \frac{\gamma_{dot}}{\sqrt{3}} \qquad \varepsilon_{dotpl} := \frac{\gamma_{dotpl}}{\sqrt{3}}$

Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN(YIELD7A)} := \mathrm{augment}\left(\frac{\varepsilon_{dot}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}}\right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 75% 8084, 25% 1080





PLOT OF LOWER COMPRESSIVE YIELD STRESS VERSUS LOG (STRAIN RATE) FOR THE 75% 8084 / 25% 1080 BLEND

Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa}\cdot 10^{6} \quad i := 1..4 \quad \mathrm{ORIGIN} := 1$

Compositon: 50% 8084, 50% 1080

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix} \cdot \frac{\mathrm{mm}}{\mathrm{sec}} \quad L_0 := \begin{bmatrix} 15.15 \\ 14.35 \\ 14.87 \\ 15.38 \end{bmatrix} \cdot \mathrm{mm} \quad \sigma_c := \begin{bmatrix} 47.40 \\ 50.30 \\ 54.77 \\ 57.22 \end{bmatrix} \cdot \mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.54 \\ 0.45 \\ 0.34 \\ 0.73 \end{bmatrix} \cdot \mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (v) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 1680\cdot\mathrm{MPa} \quad v := 0.33 \quad \Theta := 298\cdot K$

$\alpha := 0.20$ << assume $\alpha = 0.20$ >>

$$\mu := \frac{E}{2\cdot(1+v)} \qquad \mu = 631.579\cdot\mathrm{MPa}$$

$$s := \frac{0.077\cdot\mu}{1-v} \qquad s = 72.584\cdot\mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_i := \frac{\sigma_{c_i}}{3} \qquad \tau_i := \frac{\sigma_{c_i}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	15.8	27.366
2	16.767	29.041
3	18.257	31.621
4	19.073	33.036

Define the applied plastic shear strain rate:

$$\gamma_{\mathrm{dotpl}_i} := \sqrt{3}\cdot\left[\frac{\mathrm{Vel}_i}{L_{0_i}\cdot\left(1-\frac{\sigma_{c_i}}{E}\right)}\right]$$

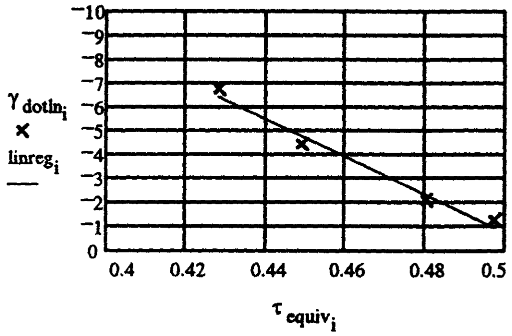
$\gamma_{\mathrm{dotpl}_i}$
$0.001\cdot\mathrm{sec}^{-1}$
$0.012\cdot\mathrm{sec}^{-1}$
$0.12\cdot\mathrm{sec}^{-1}$
$0.291\cdot\mathrm{sec}^{-1}$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{equiv_i} := \left(\frac{\tau_i}{s + \alpha \cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{equiv} = \begin{bmatrix} 0.428 \\ 0.449 \\ 0.48 \\ 0.497 \end{bmatrix} \qquad \gamma_{dotln_i} := \ln\left(\frac{\gamma_{dotpl_i}}{sec^{-1}}\right) \qquad \gamma_{dotln} = \begin{bmatrix} -6.745 \\ -4.387 \\ -2.117 \\ -1.233 \end{bmatrix}$$

$$linreg := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \cdot \tau_{equiv} + intercept\left(\tau_{equiv}, \gamma_{dotln}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$corr\left(\tau_{equiv}, \gamma_{dotln}\right) = 0.99$$

$$C := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad C = 78.82$$

$$B := intercept\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad B = -40.165$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C \cdot \Theta}{s + \alpha \cdot p_i} \qquad \gamma_{dot0} := e^{B + C} \cdot sec^{-1} \qquad \chi = \begin{bmatrix} 310.102 \\ 309.312 \\ 308.103 \\ 307.445 \end{bmatrix} \cdot \frac{K}{MPa} \qquad \chi := mean(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{dot0} = 6.138 \cdot 10^{16} \cdot sec^{-1}$$

$$\chi = 308.74 \cdot \frac{K}{MPa}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 72.584 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 308.74 \cdot \frac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298 \cdot \mathrm{K} \quad \gamma_{\mathrm{dot0}} = 6.138 \cdot 10^{16} \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right]} \cdot \ln\left(\frac{\gamma_{\mathrm{dot}}}{\gamma_{\mathrm{dot0}}}\right)\right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{\mathrm{dot}}) := \mathrm{Find}(\sigma_{c2})$

$z := 1 .. 9$

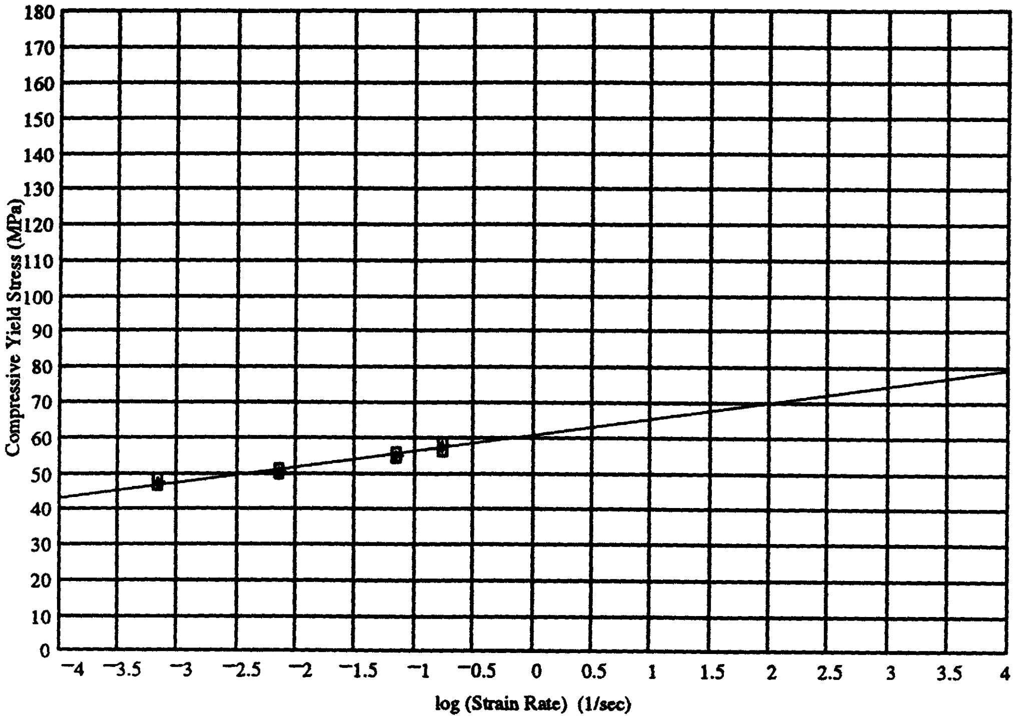
$\gamma_{\mathrm{dot}_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \quad \sigma_{c2_z} := \mathrm{Stress}(\gamma_{\mathrm{dot}_z})$

$\varepsilon_{\mathrm{dot}} := \frac{\gamma_{\mathrm{dot}}}{\sqrt{3}} \qquad \varepsilon_{\mathrm{dotpl}} := \frac{\gamma_{\mathrm{dotpl}}}{\sqrt{3}}$

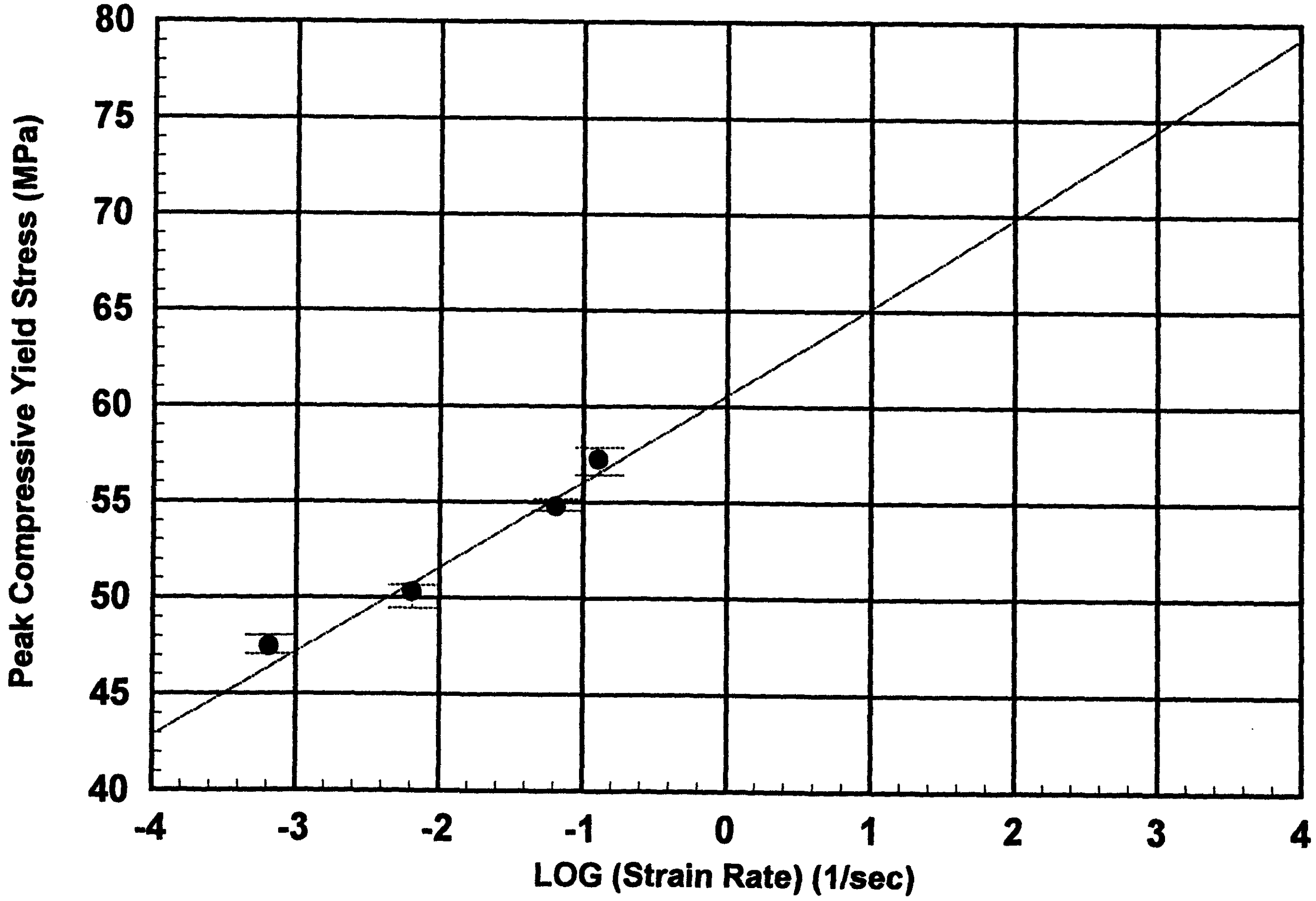
Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN}(\mathrm{YIELD8A}) := \mathrm{augment}\left(\frac{\varepsilon_{\mathrm{dot}}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}}\right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 50% 8084, 50% 1080



PLOT OF LOWER COMPRESSIVE YIELD STRESS VERSUS
LOG(STRAIN RATE) FOR THE 50% 8084 / 50% 1080 BLEND



Input the loading velocity of the Instron head (Vel), the inital length of the specimens (Lo), and the compressive flow stress obtained (σc):

$\mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$ $\quad i := 1..4$ $\quad \mathrm{ORIGIN} := 1$

Compositon: 25% 8084, 75% 1080

$$\mathrm{Vel} := \begin{bmatrix} 0.01 \\ 0.10 \\ 1.00 \\ 2.50 \end{bmatrix}\cdot\frac{\mathrm{mm}}{\mathrm{sec}} \quad L_0 := \begin{bmatrix} 15.66 \\ 15.75 \\ 15.92 \\ 15.30 \end{bmatrix}\cdot\mathrm{mm} \quad \sigma_c := \begin{bmatrix} 20.54 \\ 26.31 \\ 36.99 \\ 40.68 \end{bmatrix}\cdot\mathrm{MPa} \quad \mathrm{STDEV} := \begin{bmatrix} 0.60 \\ 1.05 \\ 0.45 \\ 1.03 \end{bmatrix}\cdot\mathrm{MPa}$$

Input the modulus of elasticity (E), the Poisson ration (ν) and the ambient temperature (Θ) at the time of the test. Using these values, calculate the shear modulus (μ) and athermal shear stress (s)

$E := 1125\cdot\mathrm{MPa}$ $\quad \nu := 0.33$ $\quad \Theta := 298\cdot\mathrm{K}$ $\qquad \alpha := 0.20$ $\quad$ << assume $\alpha = 0.20$ >>

$$\mu := \frac{E}{2\cdot(1+\nu)} \qquad \mu = 422.932\cdot\mathrm{MPa}$$

$$s := \frac{0.077\cdot\mu}{1-\nu} \qquad s = 48.606\cdot\mathrm{MPa}$$

Calculate the pressure (p) and the shear stress (τ) corresponding to each of the data points:

$$p_i := \frac{\sigma_{c_i}}{3} \qquad \tau_i := \frac{\sigma_{c_i}}{\sqrt{3}}$$

i	$\frac{p_i}{\mathrm{MPa}}$	$\frac{\tau_i}{\mathrm{MPa}}$
1	6.847	11.859
2	8.77	15.19
3	12.33	21.356
4	13.56	23.487

Define the applied plastic shear strain rate:

$$\gamma_{\mathrm{dotpl}_i} := \sqrt{3}\cdot\left[\frac{\mathrm{Vel}_i}{L_{0_i}\cdot\left(1-\frac{\sigma_{c_i}}{E}\right)}\right]$$

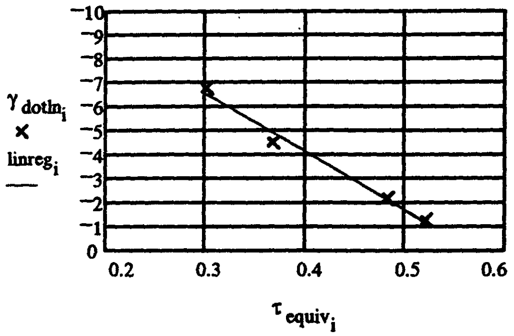
$\gamma_{\mathrm{dotpl}_i}$
$0.001\cdot\mathrm{sec}^{-1}$
$0.011\cdot\mathrm{sec}^{-1}$
$0.112\cdot\mathrm{sec}^{-1}$
$0.294\cdot\mathrm{sec}^{-1}$

Calculate γdot0 and A/k=χ using equation 6-17, assuming $\alpha = 0.2$

$$\tau_{equiv_i} := \left(\frac{\tau_i}{s + \alpha \cdot p_i}\right)^{\frac{5}{6}} \qquad \tau_{equiv} = \begin{bmatrix} 0.302 \\ 0.368 \\ 0.484 \\ 0.521 \end{bmatrix} \qquad \gamma_{dotln_i} := \ln\left(\frac{\gamma_{dotpl_i}}{sec^{-1}}\right) \qquad \gamma_{dotln} = \begin{bmatrix} -6.789 \\ -4.486 \\ -2.185 \\ -1.225 \end{bmatrix}$$

$$linreg := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \cdot \tau_{equiv} + intercept\left(\tau_{equiv}, \gamma_{dotln}\right)$$

Plot ln(shear strain rate) (1/sec) versus equivalent shear stress:



Calculate the slope, intercept, and correlation coefficient for predictive line:

$$corr\left(\tau_{equiv}, \gamma_{dotln}\right) = 0.993$$

$$C := slope\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad C = 24.312$$

$$B := intercept\left(\tau_{equiv}, \gamma_{dotln}\right) \qquad B = -13.851$$

Solve for γdot0 and χ:

$$\chi_i := \frac{C \cdot \Theta}{s + \alpha \cdot p_i} \qquad \gamma_{dot0} := e^{B+C} \cdot sec^{-1} \qquad \chi = \begin{bmatrix} 144.973 \\ 143.866 \\ 141.86 \\ 141.18 \end{bmatrix} \cdot \frac{K}{MPa} \qquad \chi := mean(\chi)$$

γdot0 and χ are as follows:

$$\gamma_{dot0} = 34934.045 \cdot sec^{-1}$$

$$\chi = 142.97 \cdot \frac{K}{MPa}$$

Using the values below, calculate the shear stress as a function of shear strain rate:

$s = 48.606 \cdot \mathrm{MPa} \quad \alpha = 0.2 \quad \chi = 142.97 \cdot \frac{\mathrm{K}}{\mathrm{MPa}} \quad \Theta = 298 \cdot \mathrm{K} \quad \gamma_{dot0} = 34934.045 \cdot \mathrm{sec}^{-1}$

Input a guess for solver: $\sigma_{c2} := 50 \cdot \mathrm{MPa}$

Use the constants to generate a plot of compressive stress (MPa) versus shear strain rate (1/sec):

Given

$$\frac{\sigma_{c2}}{\sqrt{3}} = \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right] \cdot \left[1 + \frac{\Theta}{\chi \cdot \left[s + \alpha \cdot \left(\frac{\sigma_{c2}}{3}\right)\right]} \cdot \ln\left(\frac{\gamma_{dot}}{\gamma_{dot0}}\right)\right]^{\frac{6}{5}}$$

$\mathrm{Stress}(\gamma_{dot}) := \mathrm{Find}(\sigma_{c2})$

$z := 1 .. 9$

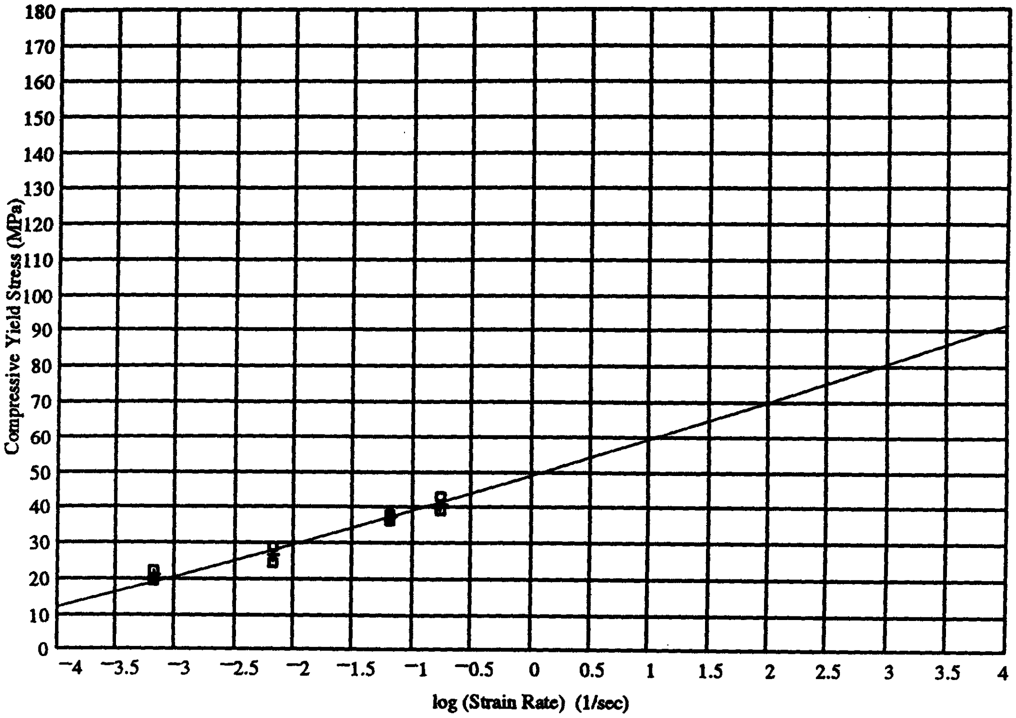
$\gamma_{dot_z} := 10^{z-5} \cdot \sqrt{3} \cdot \mathrm{sec}^{-1} \qquad \sigma_{c2_z} := \mathrm{Stress}\left(\gamma_{dot_z}\right)$

$\varepsilon_{dot} := \frac{\gamma_{dot}}{\sqrt{3}} \qquad \varepsilon_{dotpl} := \frac{\gamma_{dotpl}}{\sqrt{3}}$

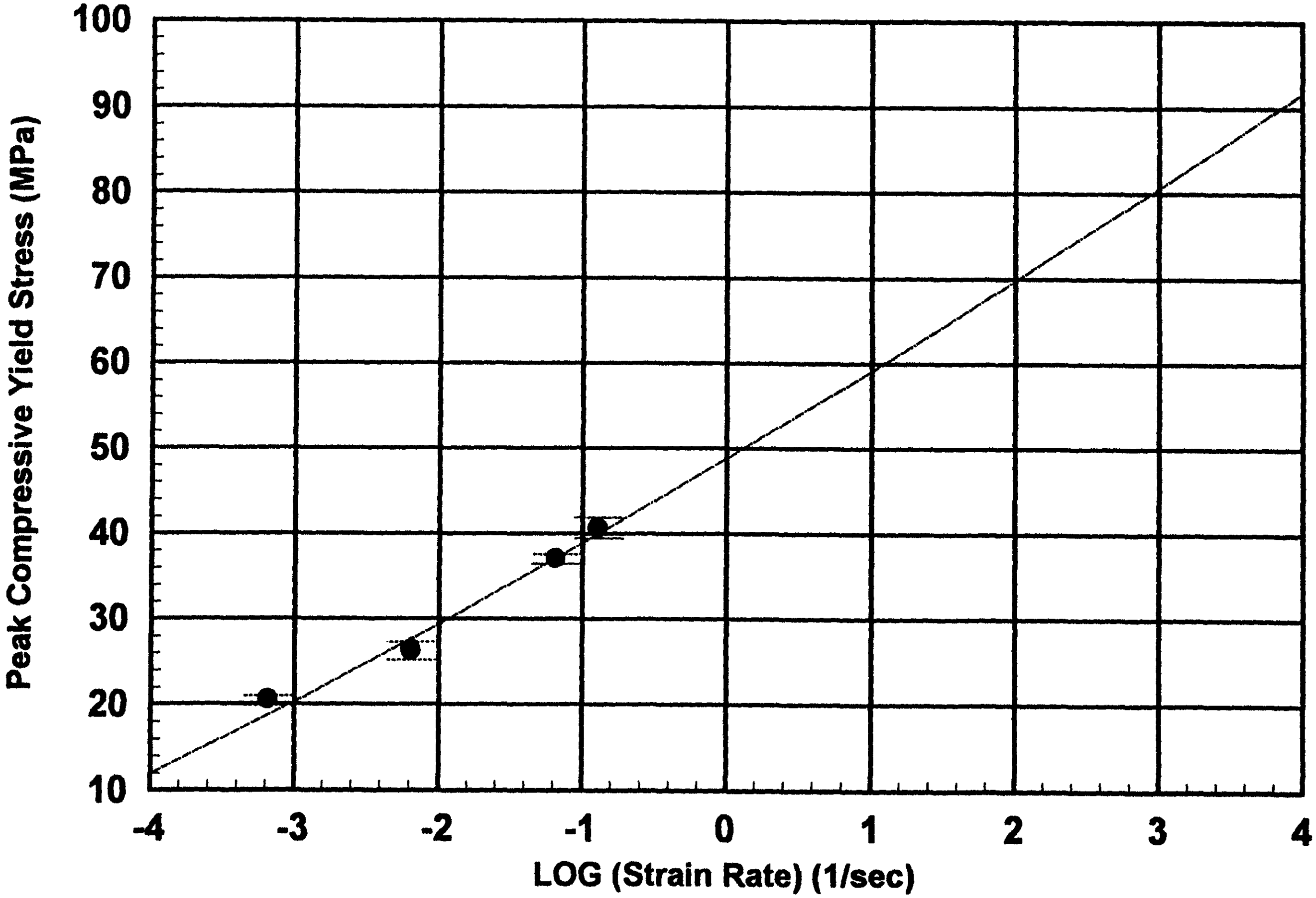
Input cofidence interval desired (1=67%, 2=95%, 3=99%): $r := 2$

$\mathrm{WRITEPRN}(\mathrm{YIELD10A}) := \mathrm{augment}\left(\frac{\varepsilon_{dot}}{\mathrm{sec}^{-1}}, \frac{\sigma_{c2}}{\mathrm{MPa}}\right)$

Compressive Flow Stress (MPa) versus Log(Strain Rate)(1/sec)
Composition: 25% 8084, 75% 1080



PLOT OF LOWER COMPRESSIVE YIELD STRESS VERSUS
LOG (STRAIN RATE) FOR THE 25% 8084 / 75% 1080 BLEND



Appendix E
Hopkinson Bar Test Results

Testing

All Load Rates

General Definitions: **% Derakane 8084: 100 %Crestomer 1080: 0 Trial: 1 of 4**

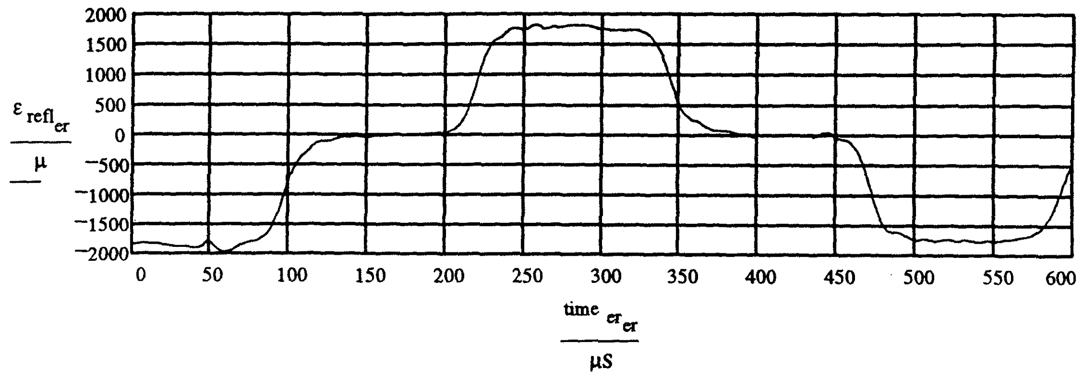
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa\cdot 10^{9} \quad MPa := Pa\cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot GPa \quad D_{bar} := 0.75\cdot in \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{2} \quad A_{bar} = 285.023\cdot mm^{2}$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(HD1C0ER) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \quad time_{er} := \varepsilon_{reflected}^{<2>}\cdot \sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4\cdot\varepsilon_{refl}}{GF\cdot V_O}$ << apply calibration >>

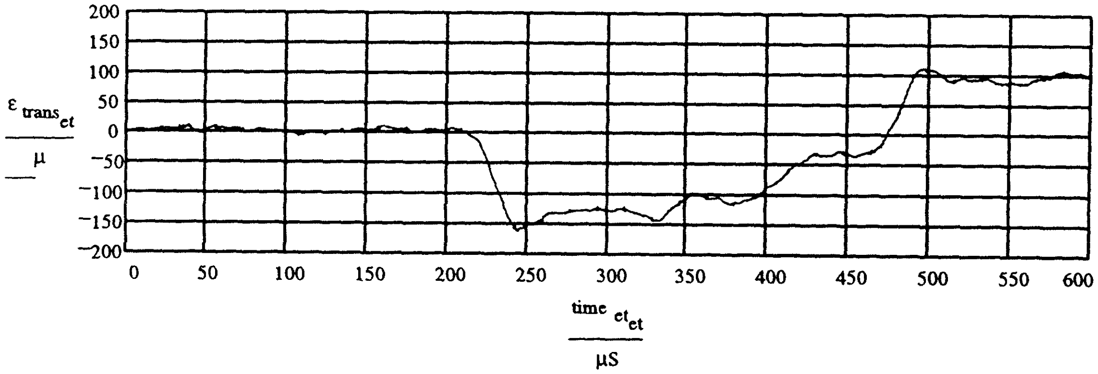


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(HD1C0ET) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \quad time_{et} := \varepsilon_{transmitted}^{<2>}\cdot \sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4\cdot\varepsilon_{trans}}{GF\cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{1800}}$ << shift pre-reflected signal to a strain of zero >>

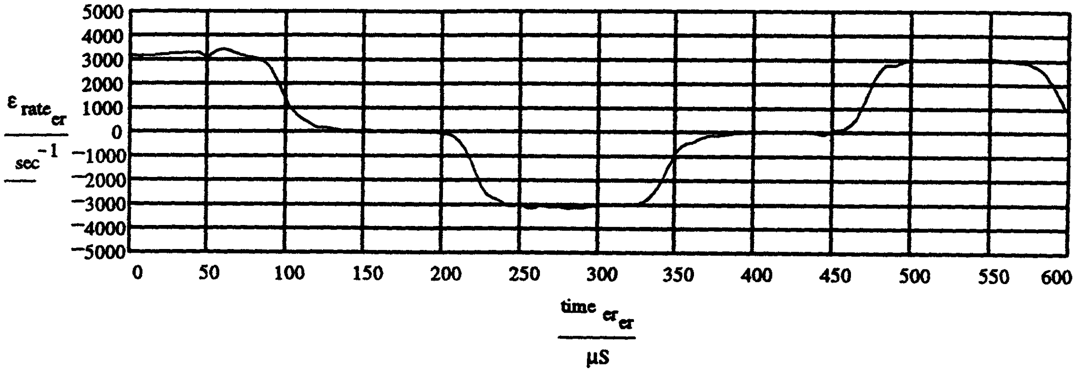


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 5.78 \cdot mm$ $D_0 := 7.77 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\;2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

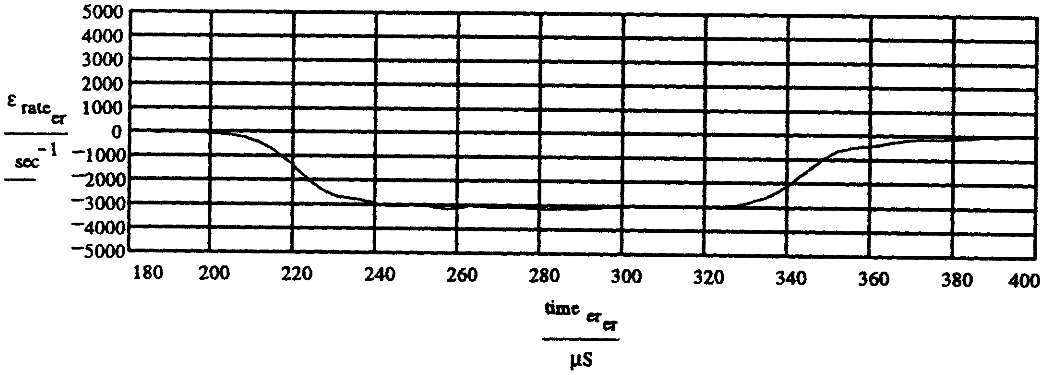


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 180 fin := 400 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\text{start} := \text{start} \cdot 10 \quad \text{fin} := \text{fin} \cdot 10$

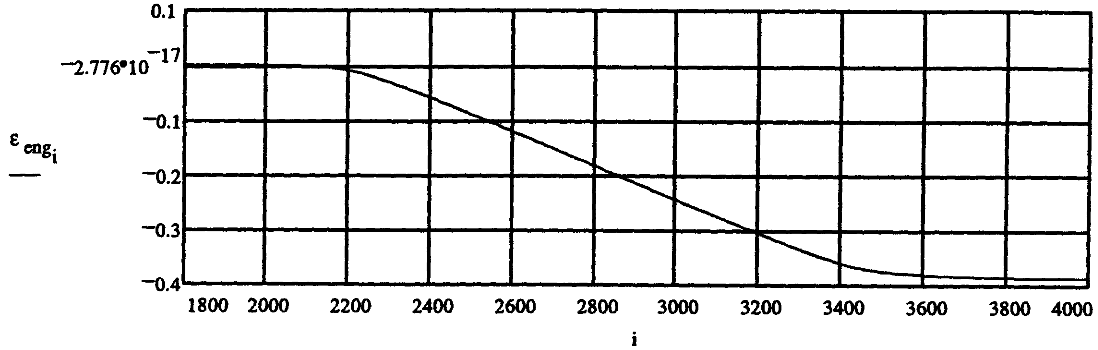
$\varepsilon_{maxeng} := \sum_{n = start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.386$ << max eng. strain >>

Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \text{start}, \text{start} + \text{index} .. \text{fin}$

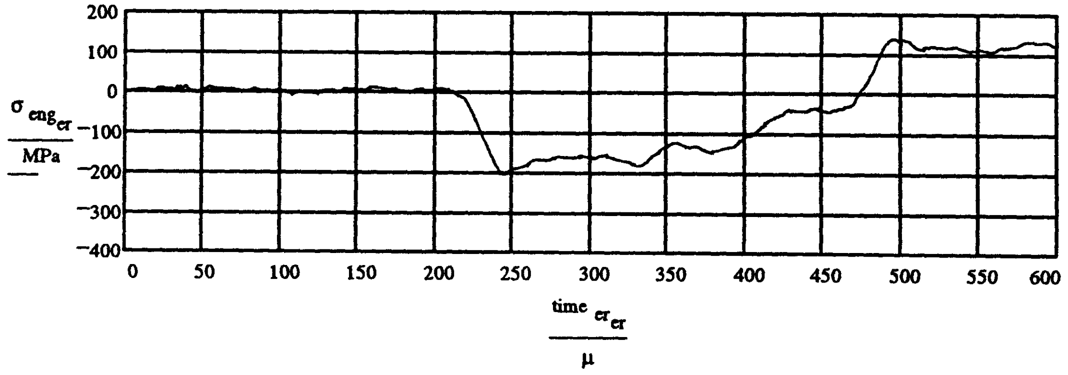
$\varepsilon_{eng_i} := \sum_{n = start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2}$ << numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



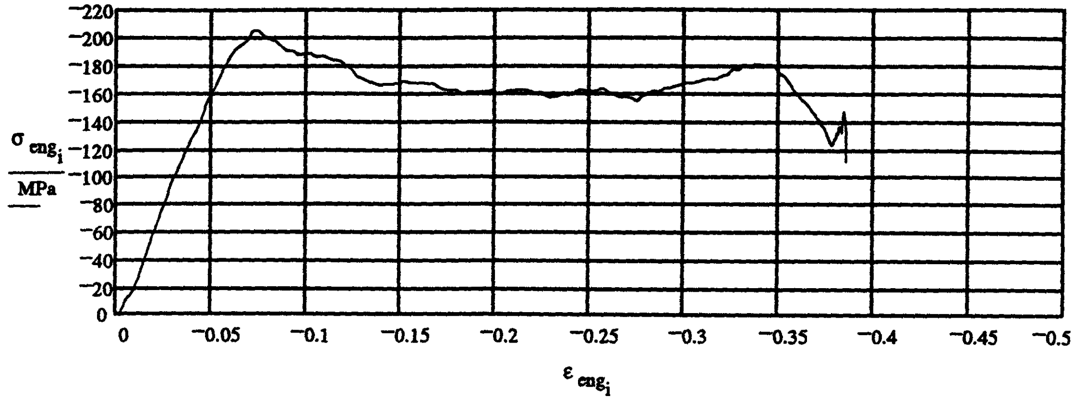
Calculate the axial compressive engineering stress in the sample:

$\sigma_{eng} := \frac{A_{bar}}{A_0} \cdot E_{bar} \cdot \varepsilon_{trans}$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \dfrac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index≡10



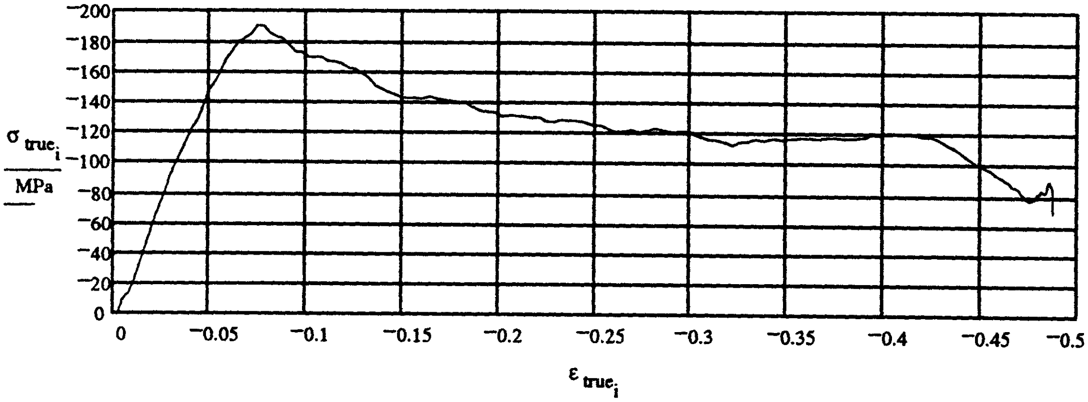
The maximum compresssive stress is: $\min(\sigma_{eng}) = -205.176 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.488$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -189.806 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 100% Dow Chemical Derakane 8084 rubber-toughened vinyl ester. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0054.WFT is the reflected pulse file.
2. WAVE0055.WFT is the transmitted pulse file.
3. WAVE0056.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D1C0T1} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D1C0T1)} := \mathrm{D1C0T1}$$

General Definitions: **% Derakane 8084: 100 %Crestomer 1080: 0 Trial: 2 of 4**

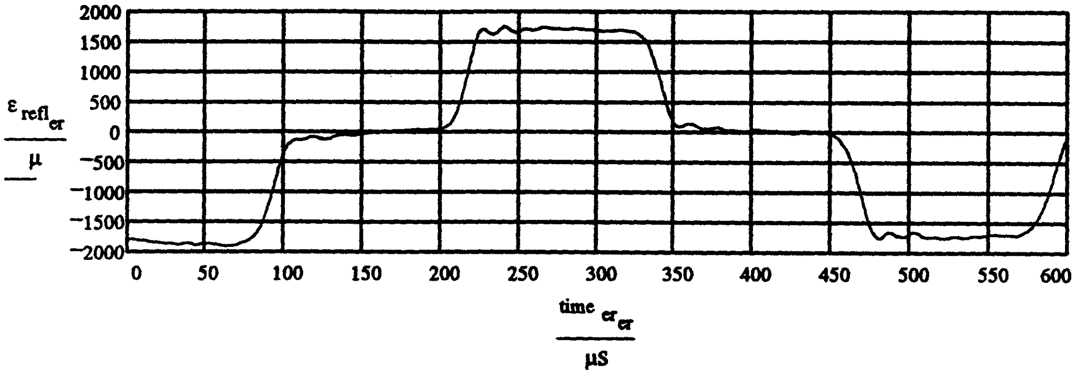
$\mathrm{ORIGIN} := 1 \quad \mu S := \mathrm{sec}\cdot 10^{-6} \quad \mu := 10^{-6} \quad \mathrm{GPa} := \mathrm{Pa}\cdot 10^{9} \quad \mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$

$\mathrm{GF} := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot \mathrm{GPa} \quad D_{bar} := 0.75\cdot \mathrm{in} \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\;2} \quad A_{bar} = 285.023 \cdot \mathrm{mm}^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD1C0ER2)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1\rangle} \quad time_{er} := \varepsilon_{reflected}^{\langle 2\rangle}\cdot \mathrm{sec}$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4\cdot \varepsilon_{refl}}{\mathrm{GF}\cdot V_O}$ << apply calibration >>

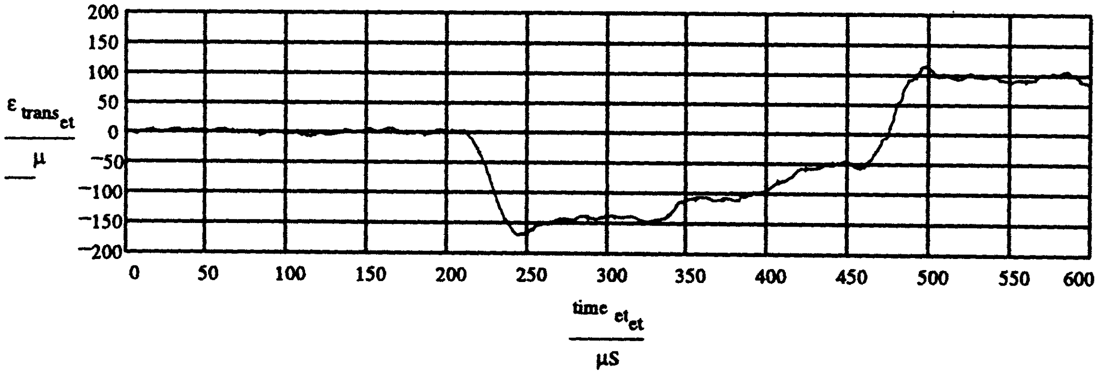


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD1C0ET2)} \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1\rangle} \quad time_{et} := \varepsilon_{transmitted}^{\langle 2\rangle}\cdot \mathrm{sec}$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4\cdot \varepsilon_{trans}}{\mathrm{GF}\cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{1800}}$ << shift pre-reflected signal to a strain of zero >>

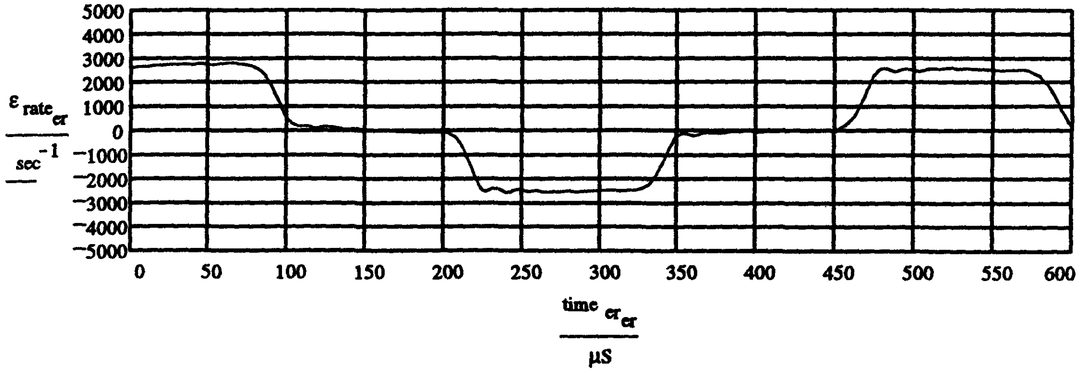


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ **<< longitudinal wave speed for steel >>**

$L_0 := 6.80 \cdot mm$ $D_0 := 7.77 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\,2}$ **<< inital specimen length, diameter, and Acs >>**

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

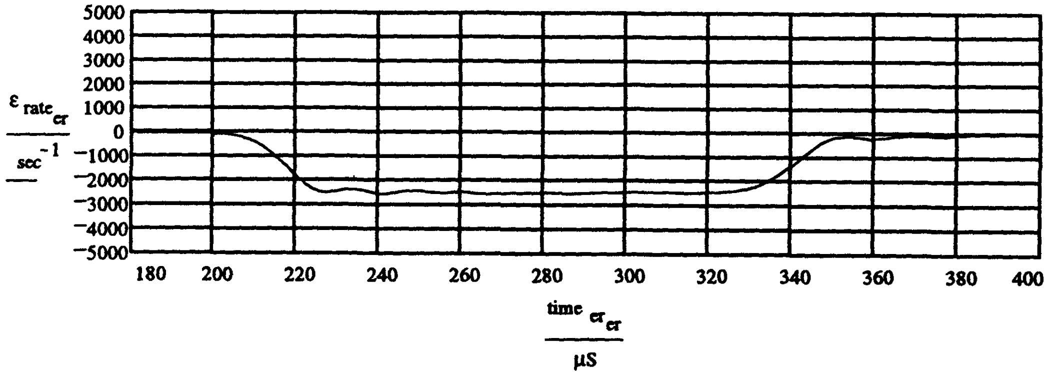


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

$start := 180$ $fin := 400$ $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\text{start} := \text{start} \cdot 10 \qquad \text{fin} := \text{fin} \cdot 10$

$$\varepsilon_{\text{maxeng}} := \sum_{n = \text{start}}^{\text{fin}} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2} \qquad \varepsilon_{\text{maxeng}} = -0.317$$

<< max eng. strain >>

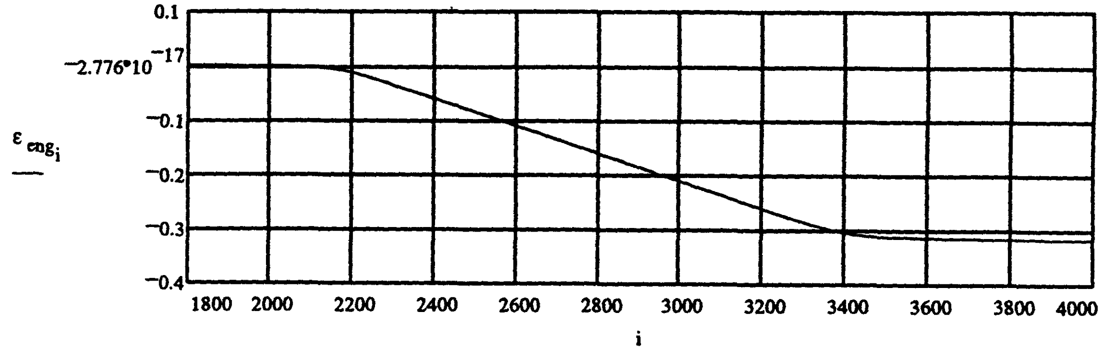
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \text{start}, \text{start} + \text{index} .. \text{fin}$

$$\varepsilon_{\text{eng}_i} := \sum_{n = \text{start}}^{i} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2}$$

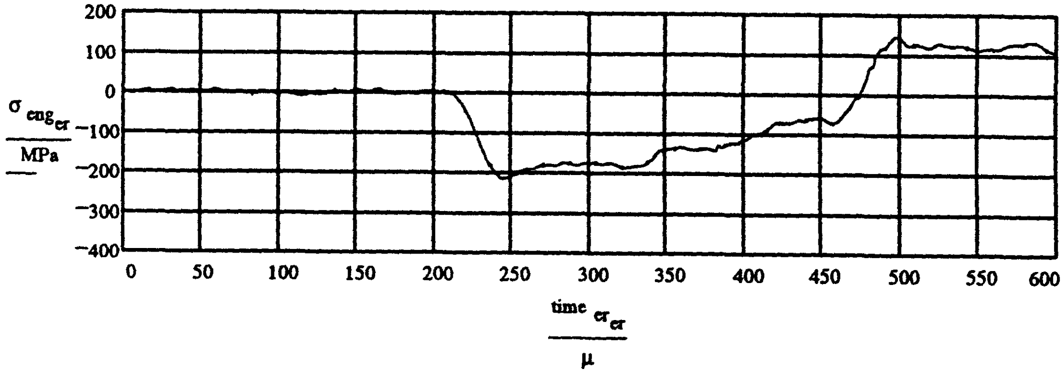
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



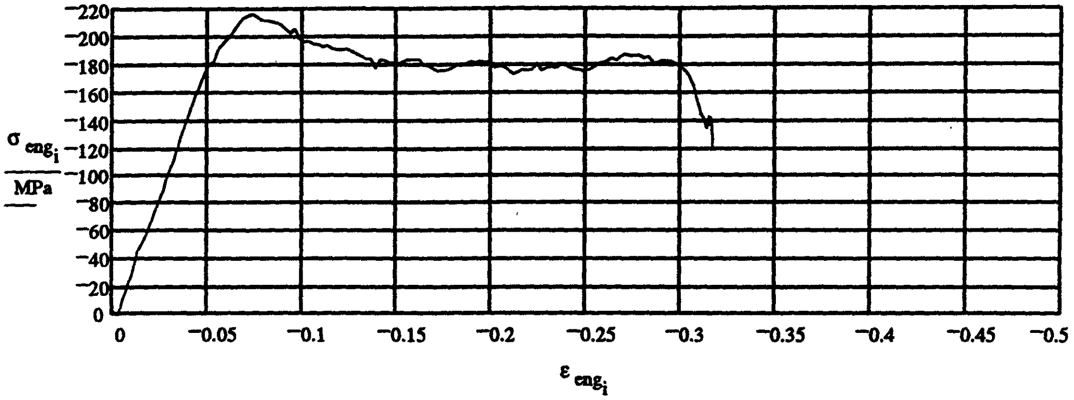
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\text{eng}} := \frac{A_{\text{bar}}}{A_o} \cdot E_{\text{bar}} \cdot \varepsilon_{\text{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index $\equiv$ 10



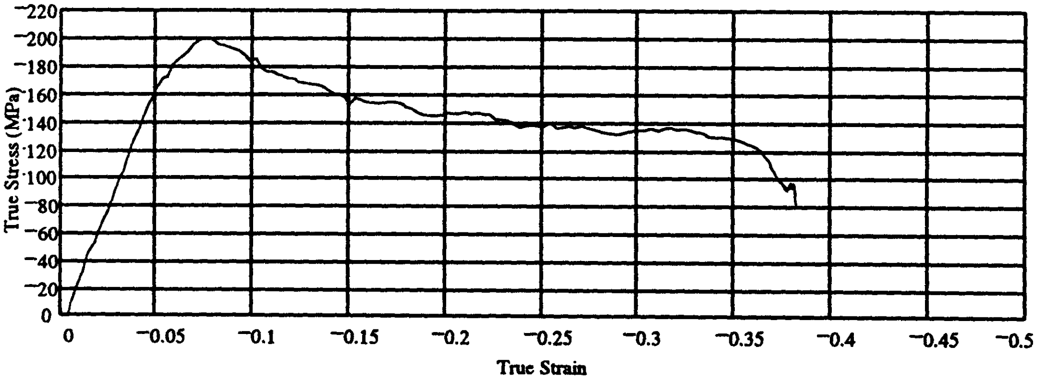
The maximum compresssive stress is: $\min(\sigma_{eng}) = -216.076 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.382$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -200.044 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 100% Dow Chemical Derakane 8084 rubber-toughened vinyl ester. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0057.WFT is the reflected pulse file.
2. WAVE0058.WFT is the transmitted pulse file.
3. WAVE0059.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D1C0T2} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D1C0T2)} := \mathrm{D1C0T2}$$

General Definitions: **% Derakane 8084: 100 %Crestomer 1080: 0 Trial: 3 of 4**

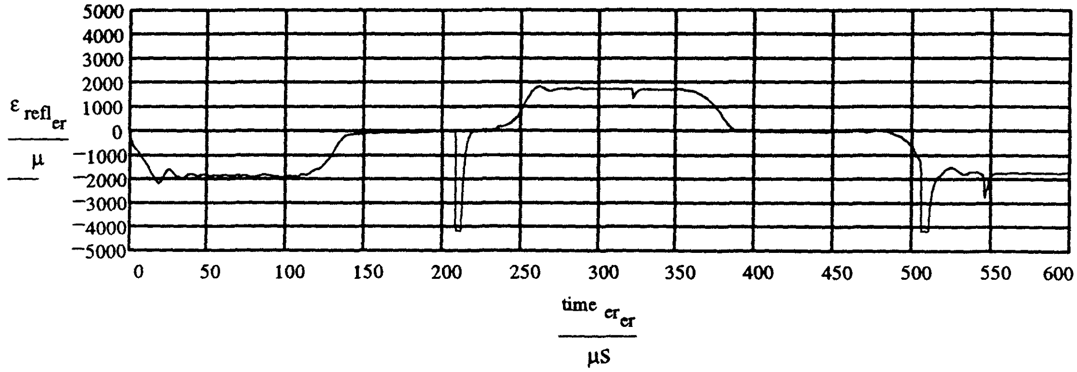
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa \cdot 10^{9} \quad MPa := Pa \cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot GPa \quad D_{bar} := 0.75 \cdot in \quad A_{bar} := \frac{\pi}{4} \cdot D_{bar}^{\;2} \quad A_{bar} = 285.023 \cdot mm^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(BD1C0ER2) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \quad time_{er} := \varepsilon_{reflected}^{<2>} \cdot \sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ << apply calibration >>

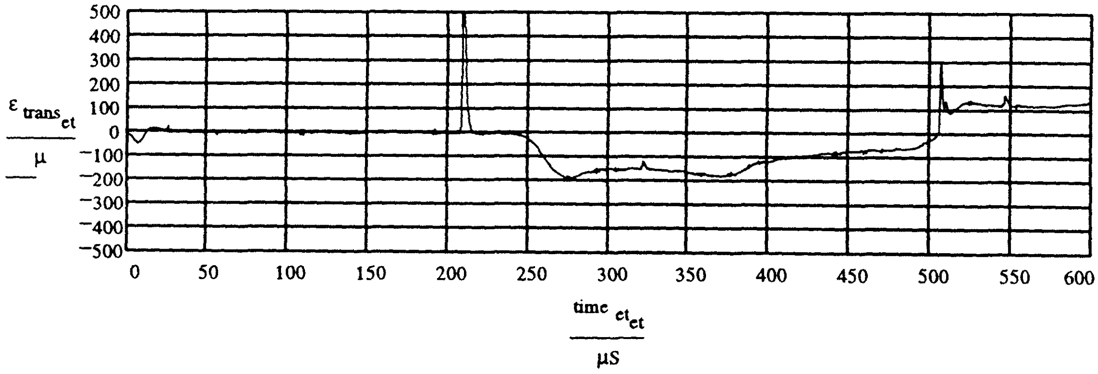


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(BD1C0ET2) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \quad time_{et} := \varepsilon_{transmitted}^{<2>} \cdot \sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{1800}}$ << shift pre-reflected signal to a strain of zero >>

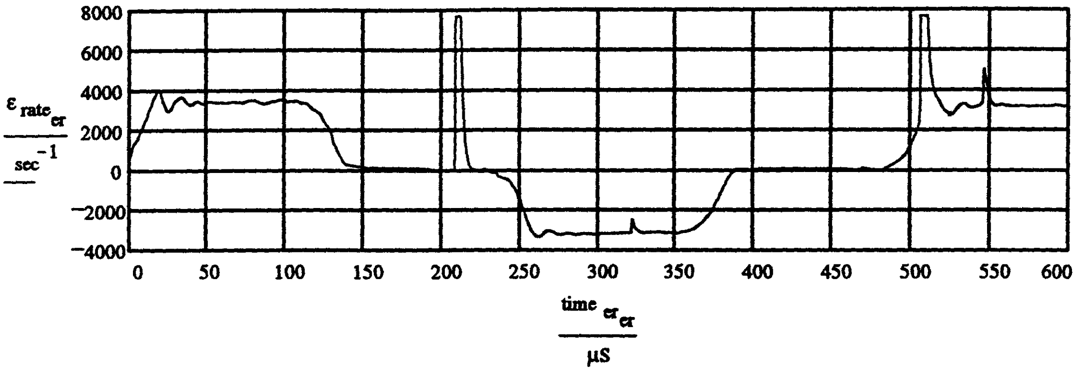


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 5.43 \cdot mm$ $D_0 := 8.02 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\,2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

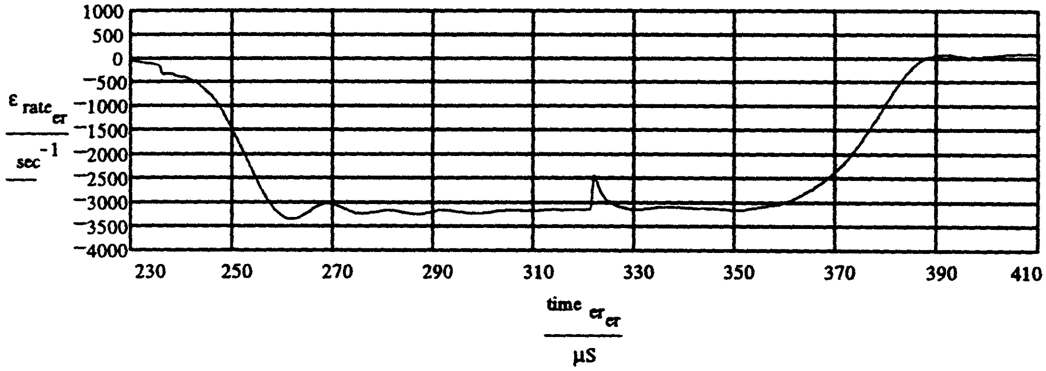


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start $:= 230$ fin $:= 410$ $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1µS intervals) and calculate max strain:

$\mathrm{start} := \mathrm{start} \cdot 10 \quad \mathrm{fin} := \mathrm{fin} \cdot 10$

$$\varepsilon_{\mathrm{maxeng}} := \sum_{n=\mathrm{start}}^{\mathrm{fin}} \Delta t \cdot \frac{\varepsilon_{\mathrm{rate}_n} + \varepsilon_{\mathrm{rate}_{n+1}}}{2} \qquad \varepsilon_{\mathrm{maxeng}} = -0.396$$

<< max eng. strain >>

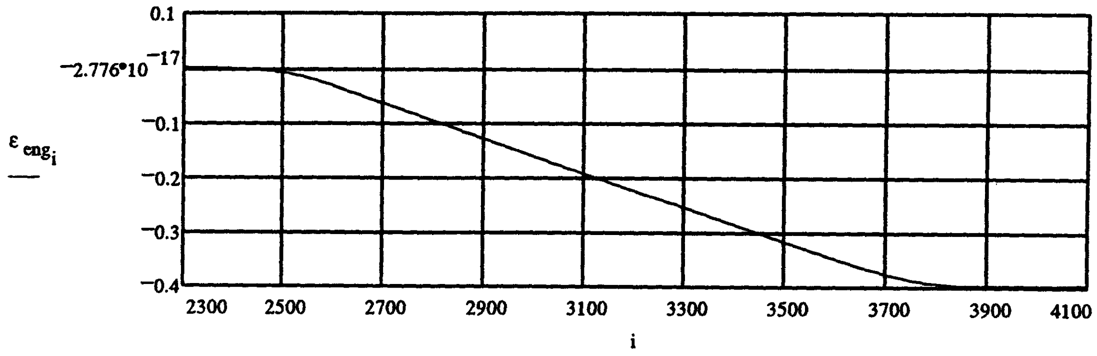
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \mathrm{start}, \mathrm{start} + \mathrm{index} .. \mathrm{fin}$

$$\varepsilon_{\mathrm{eng}_i} := \sum_{n=\mathrm{start}}^{i} \Delta t \cdot \frac{\varepsilon_{\mathrm{rate}_n} + \varepsilon_{\mathrm{rate}_{n+1}}}{2}$$

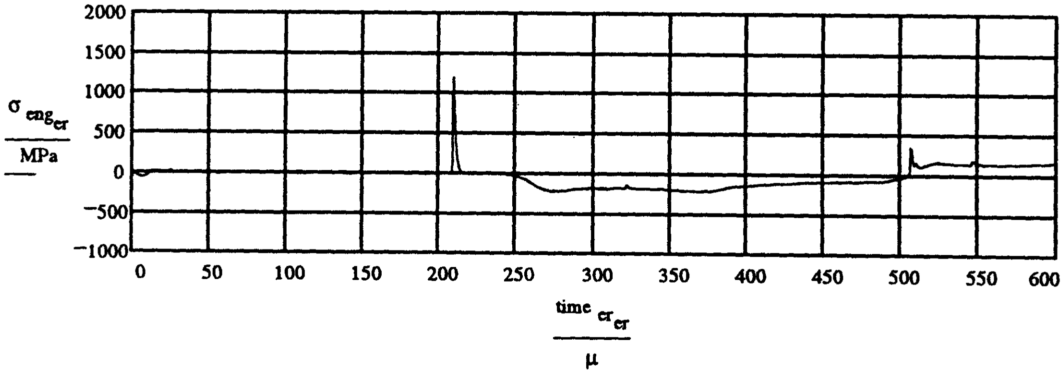
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1µS:



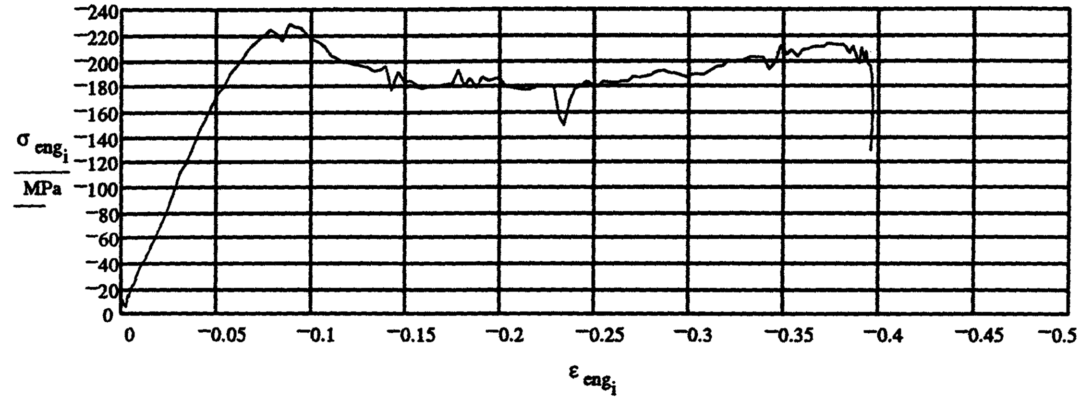
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\mathrm{eng}} := \frac{A_{\mathrm{bar}}}{A_o} \cdot E_{\mathrm{bar}} \cdot \varepsilon_{\mathrm{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index≡10



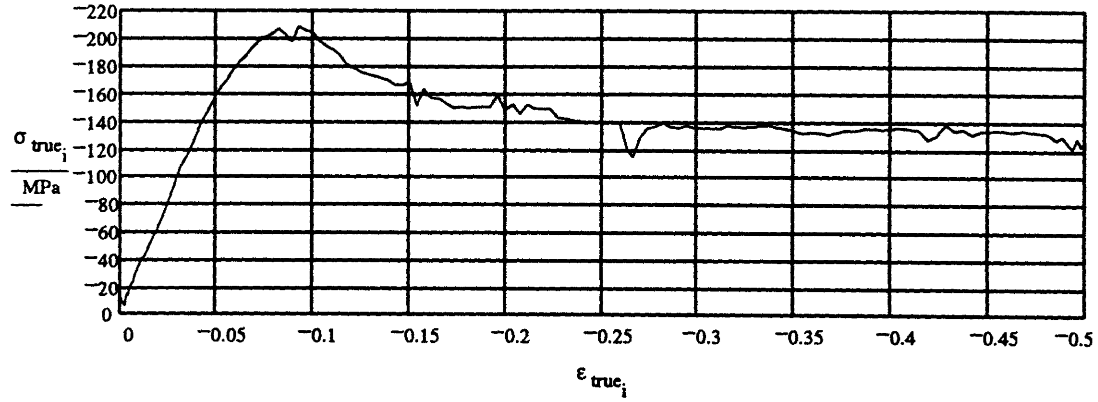
The maximum compresssive stress is: $\min(\sigma_{eng}) = -235.313 \cdot \text{MPa}$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.506$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -208.491 \cdot \text{MPa}$

Supplemental note: This stress strain curve represents a specimen composed of 100% Dow Chemical Derakane 8084 rubber-toughened vinyl ester. The raw data was obtained on a commmpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0015.WFT is the reflected pulse file.
2. WAVE0016.WFT is the transmitted pulse file.
3. WAVE0017.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D1C0T3} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D1C0T3)} := \mathrm{D1C0T3}$$

General Definitions: **% Derakane 8084: 100 %Crestomer 1080: 0 Trial: 4 of 4**

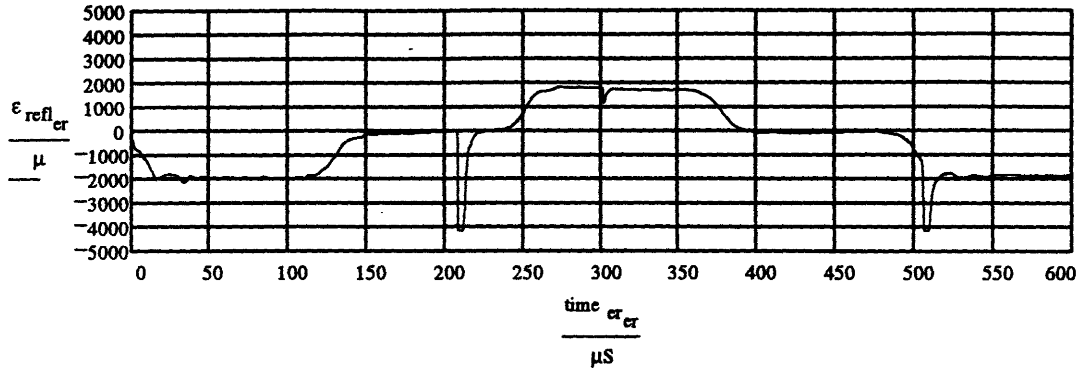
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad \mathrm{GPa} := \mathrm{Pa}\cdot 10^{9} \quad \mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$

$\mathrm{GF} := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot\mathrm{GPa} \quad D_{bar} := 0.75\cdot\mathrm{in} \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\ 2} \quad A_{bar} = 285.023\cdot\mathrm{mm}^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(\mathrm{BD1C0ER3}) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1\rangle} \quad \mathrm{time}_{er} := \varepsilon_{reflected}^{\langle 2\rangle}\cdot\sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ **<< set up range >>** $\varepsilon_{refl} := \frac{4\cdot\varepsilon_{refl}}{\mathrm{GF}\cdot V_O}$ **<< apply calibration >>**

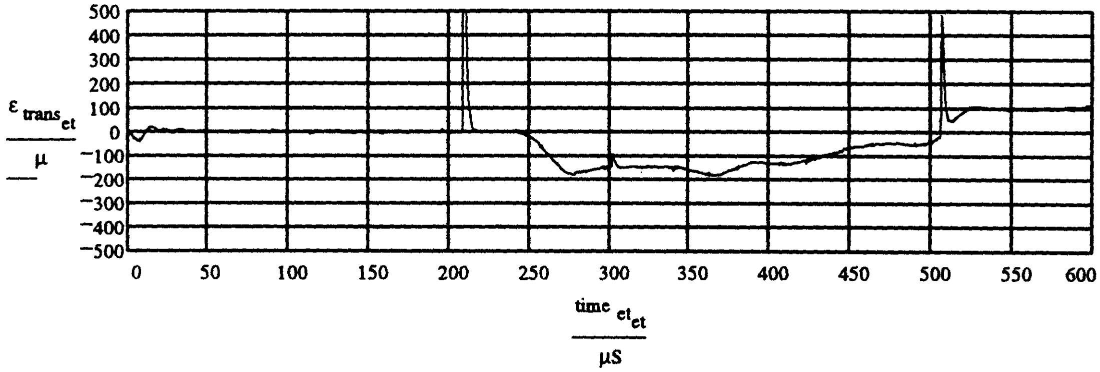


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(\mathrm{BD1C0ET3}) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1\rangle} \quad \mathrm{time}_{et} := \varepsilon_{transmitted}^{\langle 2\rangle}\cdot\sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ **<< set up range >>** $\varepsilon_{trans} := \frac{4\cdot\varepsilon_{trans}}{\mathrm{GF}\cdot V_O}$ **<< apply calibration >>**

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2300}}$ **<< shift pre-reflected signal to a strain of zero >>**

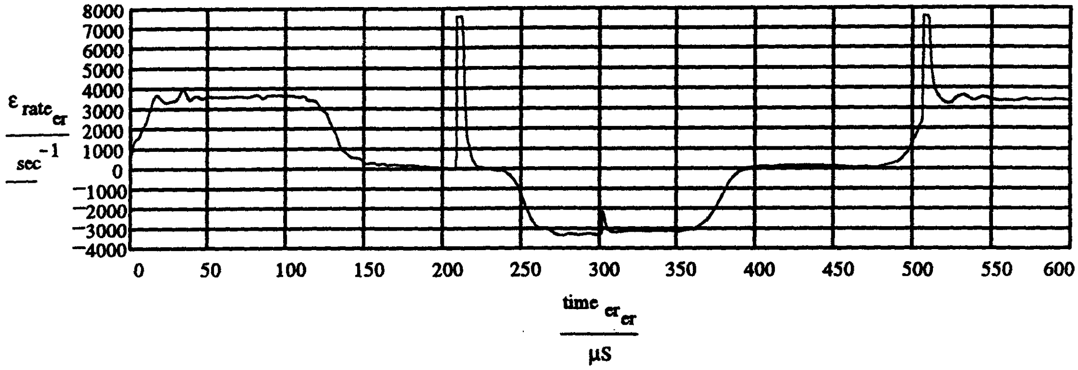


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 5.46 \cdot mm$ $D_0 := 7.87 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\,2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

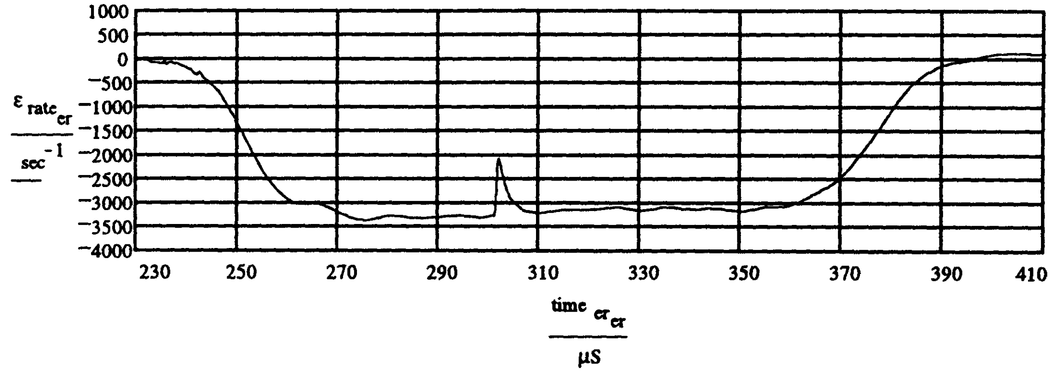


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

$start := 230$ $fin := 410$ $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\text{start} := \text{start} \cdot 10 \quad \text{fin} := \text{fin} \cdot 10$

$$\varepsilon_{\text{maxeng}} := \sum_{n=\text{start}}^{\text{fin}} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2} \qquad \varepsilon_{\text{maxeng}} = -0.396$$

<< max eng. strain >>

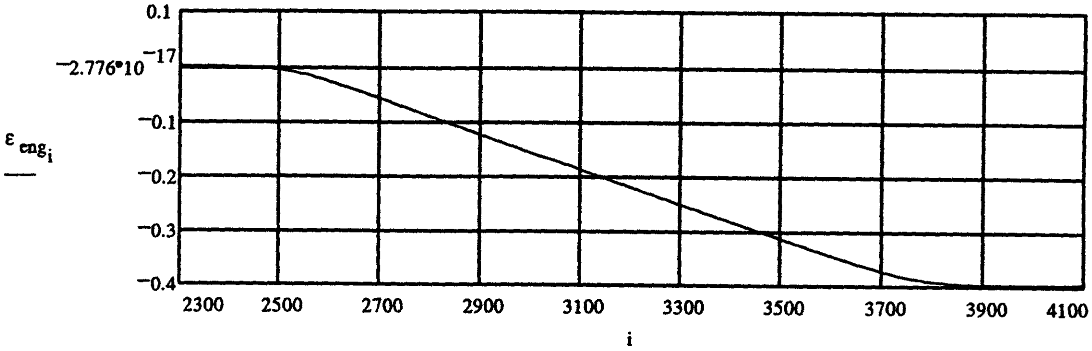
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \text{start}, \text{start} + \text{index} .. \text{fin}$

$$\varepsilon_{\text{eng}_i} := \sum_{n=\text{start}}^{i} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2}$$

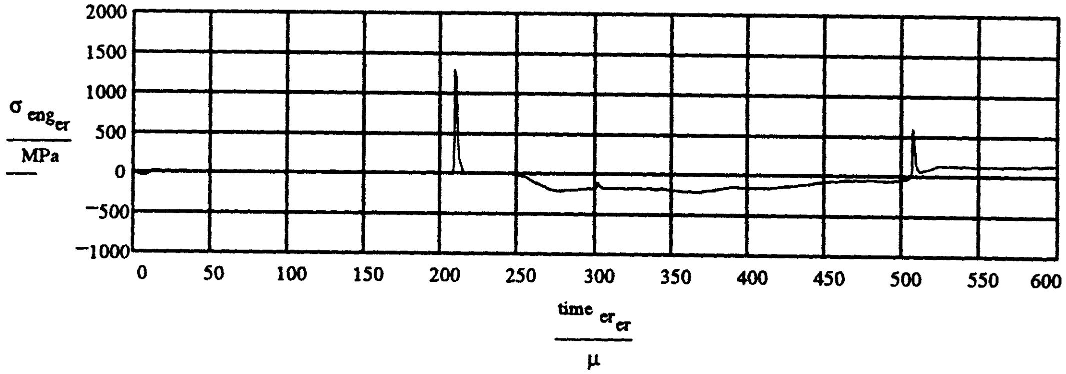
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



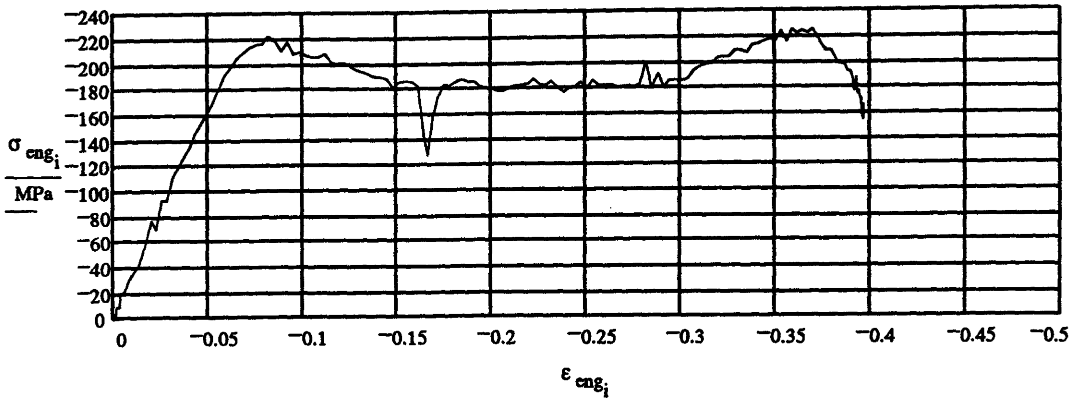
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\text{eng}} := \frac{A_{\text{bar}}}{A_0} \cdot E_{\text{bar}} \cdot \varepsilon_{\text{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_0} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index≡10



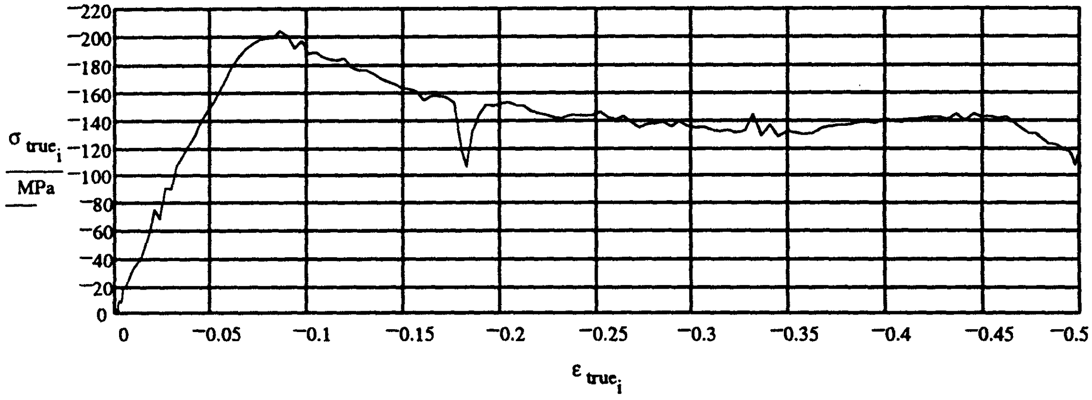
The maximum compresssive stress is: $\min(\sigma_{eng}) = -226.244 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.506$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -203.56 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 100% Dow Chemical Derakane 8084 rubber-toughened vinyl ester. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0018.WFT is the reflected pulse file.
2. WAVE0019.WFT is the transmitted pulse file.
3. WAVE0020.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D1C0T4} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D1C0T4)} := \mathrm{D1C0T4}$$

General Definitions: **% Derakane 8084: 75 %Crestomer 1080: 25 Trial: 1 of 3**

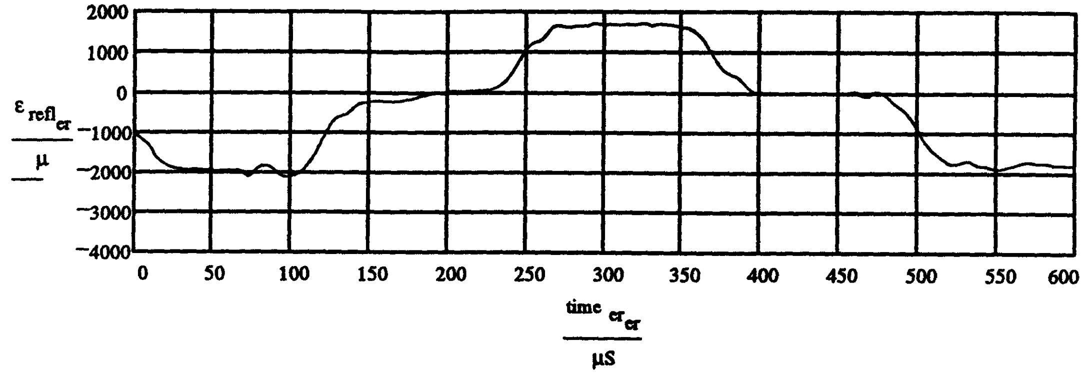
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad \mathrm{GPa} := \mathrm{Pa}\cdot 10^{9} \quad \mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$

$\mathrm{GF} := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot\mathrm{GPa} \quad D_{bar} := 0.75\cdot\mathrm{in} \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\;2} \quad A_{bar} = 285.023\cdot\mathrm{mm}^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD7C2ER1)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \qquad \mathrm{time}_{er} := \varepsilon_{reflected}^{<2>}\cdot\sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \dfrac{4\cdot\varepsilon_{refl}}{\mathrm{GF}\cdot V_O}$ << apply calibration >>

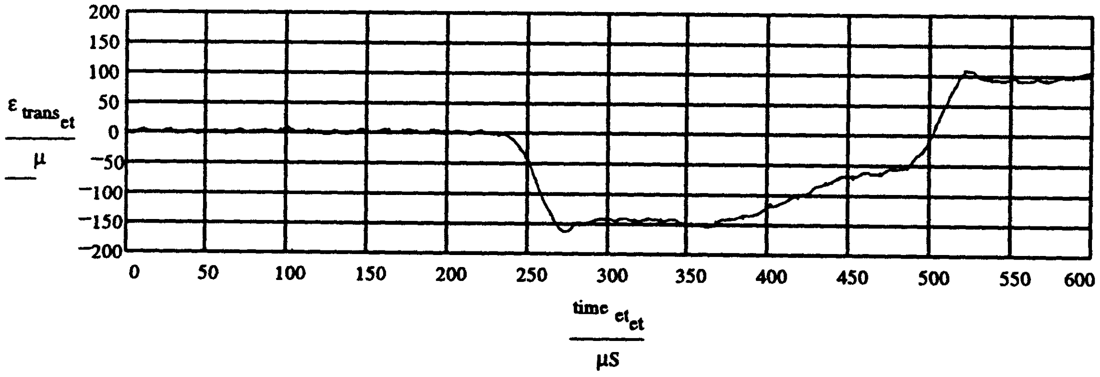


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD7C2ET1)} \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \qquad \mathrm{time}_{et} := \varepsilon_{transmitted}^{<2>}\cdot\sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \dfrac{4\cdot\varepsilon_{trans}}{\mathrm{GF}\cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2000}}$ << shift pre-reflected signal to a strain of zero >>

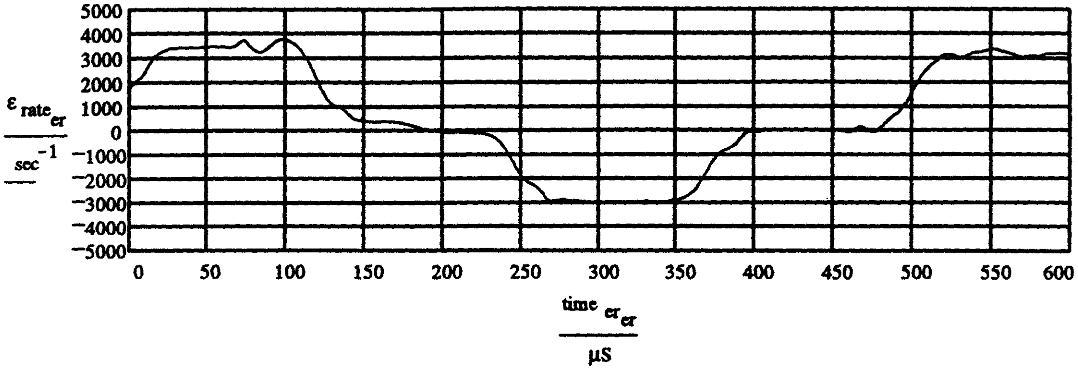


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 5.63 \cdot mm$ $D_0 := 8.00 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

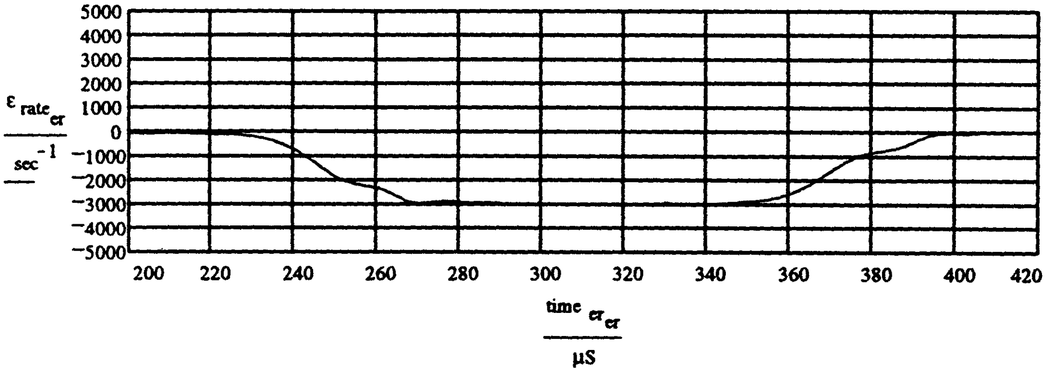


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

$start := 200$ $fin := 420$ $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

start := start·10   fin := fin·10

$$\varepsilon_{maxeng} := \sum_{n = start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.375$$

<< max eng. strain >>

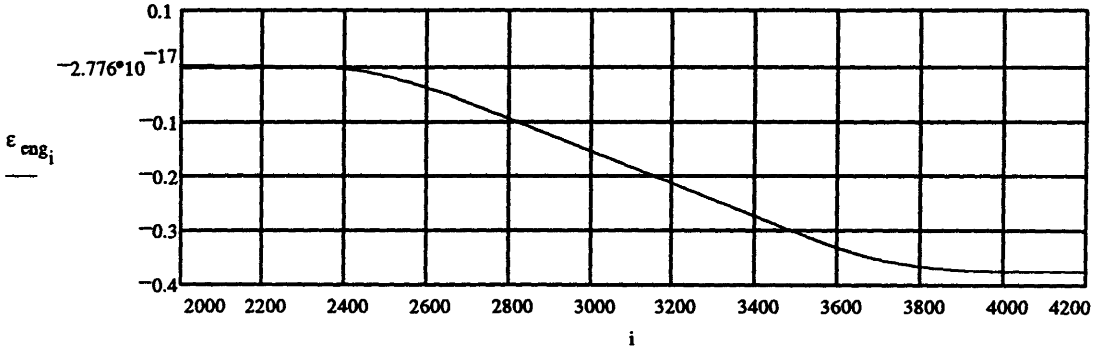
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := start, start + index .. fin$

$$\varepsilon_{eng_i} := \sum_{n = start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2}$$

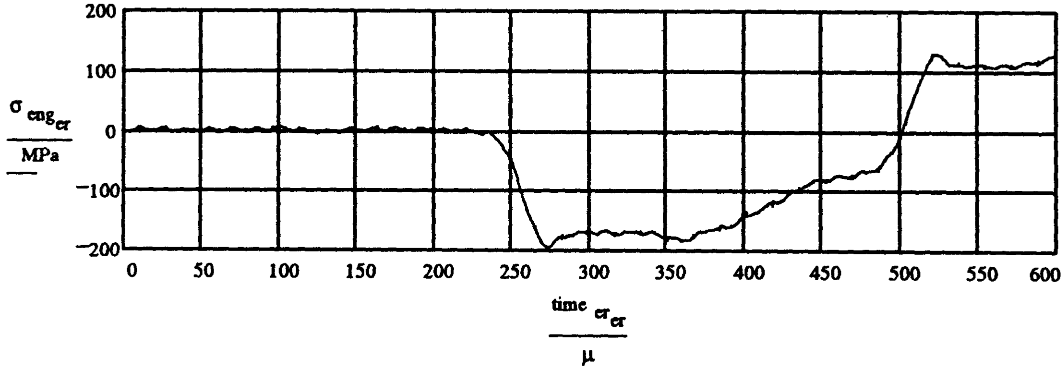
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



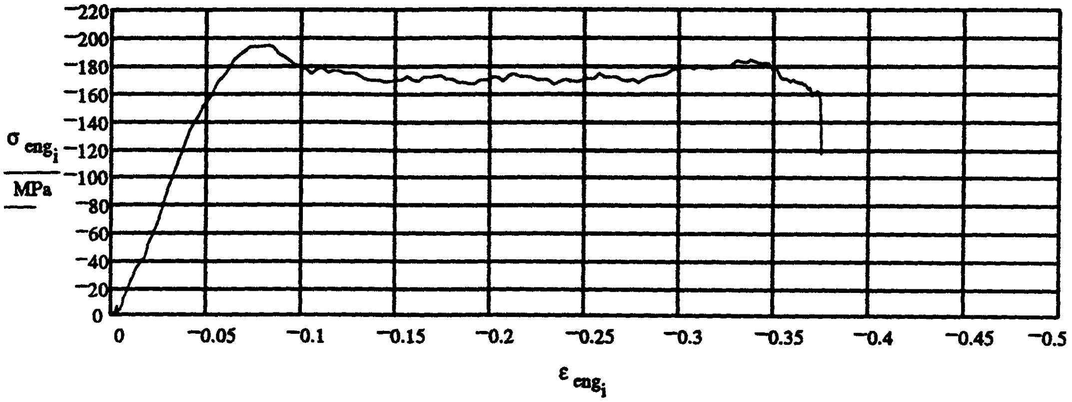
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index ≡ 10



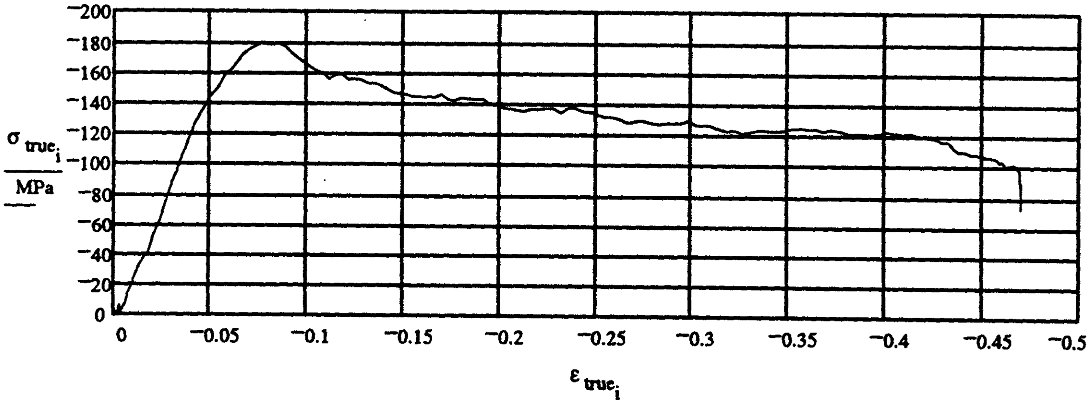
The maximum compresssive stress is: $\min(\sigma_{eng}) = -195.363 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.47$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -179.287 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 75% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 25% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0048.WFT is the reflected pulse file.
2. WAVE0049.WFT is the transmitted pulse file.
3. WAVE0050.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D7C2T1} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D7C2T1)} := \mathrm{D7C2T1}$$

General Definitions: **% Derakane 8084: 75 %Crestomer 1080: 25 Trial: 2 of 3**

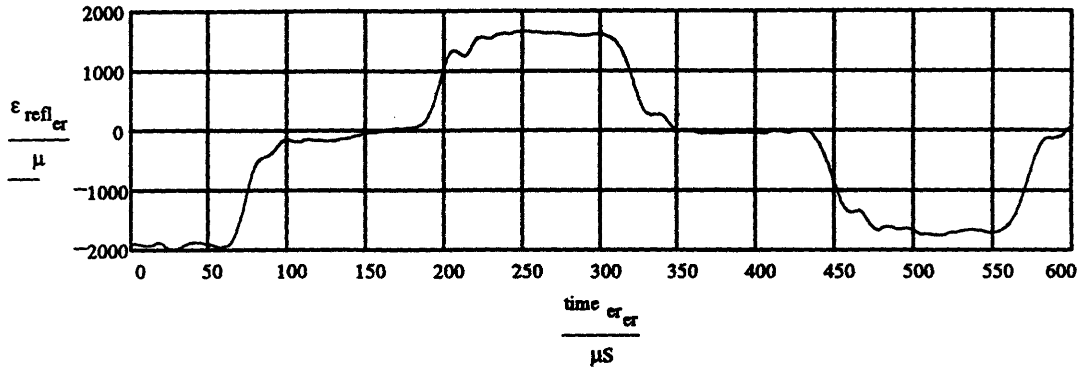
$\mathrm{ORIGIN} := 1 \quad \mu S := sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa \cdot 10^{9} \quad MPa := Pa \cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot GPa \quad D_{bar} := 0.75 \cdot in \quad A_{bar} := \frac{\pi}{4} \cdot D_{bar}^{2} \quad A_{bar} = 285.023 \cdot mm^{2}$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(HD7C2ER2) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1 \rangle} \quad time_{er} := \varepsilon_{reflected}^{\langle 2 \rangle} \cdot sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ << apply calibration >>

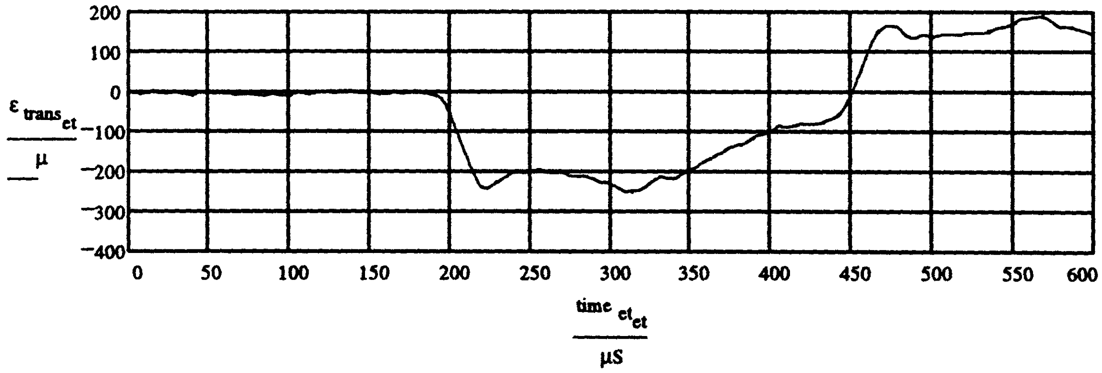


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(HD7C2ET2) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1 \rangle} \quad time_{et} := \varepsilon_{transmitted}^{\langle 2 \rangle} \cdot sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{1500}}$ << shift pre-reflected signal to a strain of zero >>

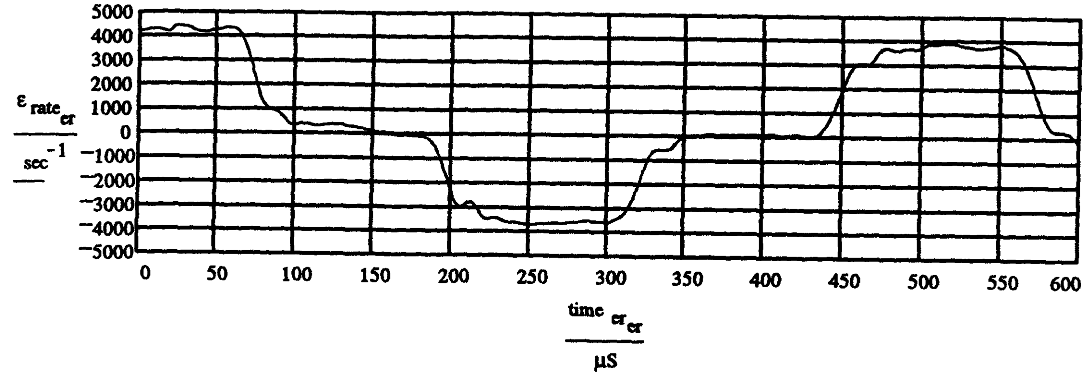


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 4.49 \cdot mm$ $D_0 := 9.54 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

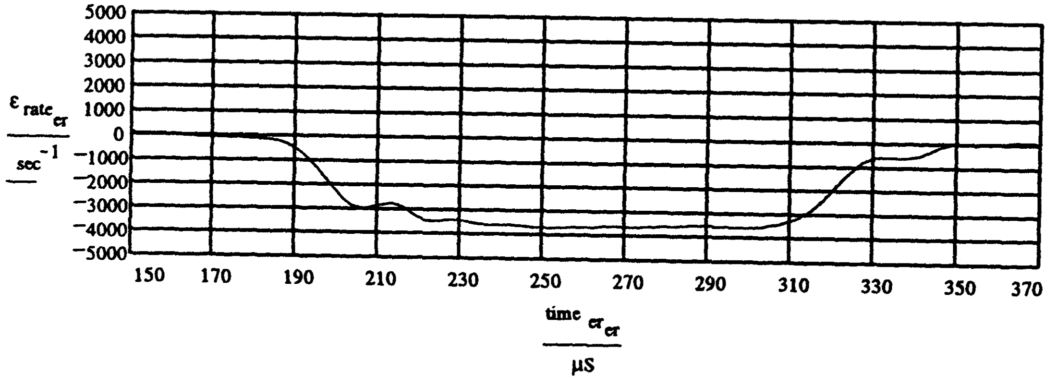


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 150 fin := 370 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

start := start·10   fin := fin·10

$$\varepsilon_{\text{maxeng}} := \sum_{n=\text{start}}^{\text{fin}} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2} \qquad \varepsilon_{\text{maxeng}} = -0.444$$

<< max eng. strain >>

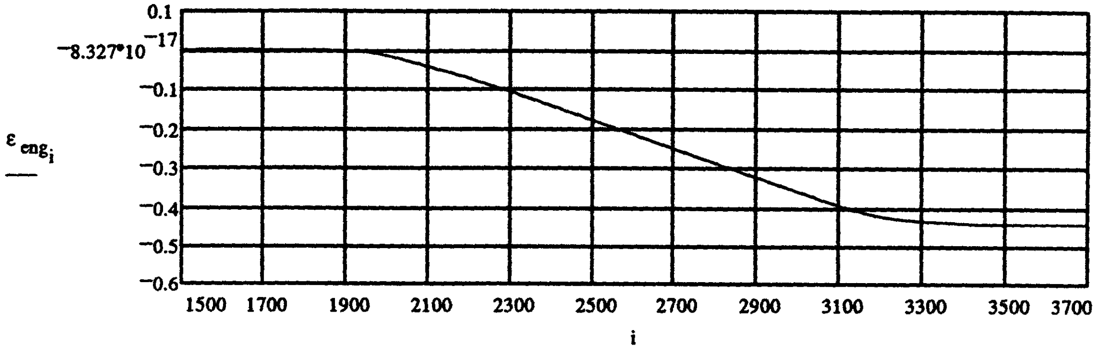
Calculate the engineering strain vector by numerical integration of the strain rate:

i := start, start + index.. fin

$$\varepsilon_{\text{eng}_i} := \sum_{n=\text{start}}^{i} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2}$$

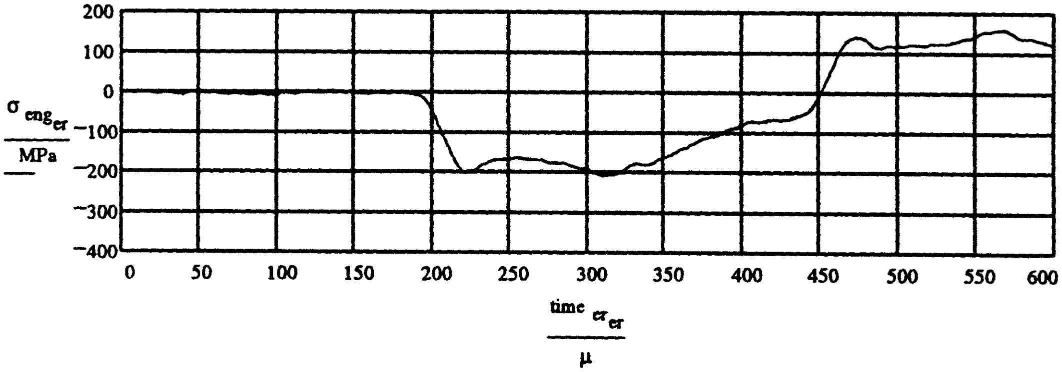
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



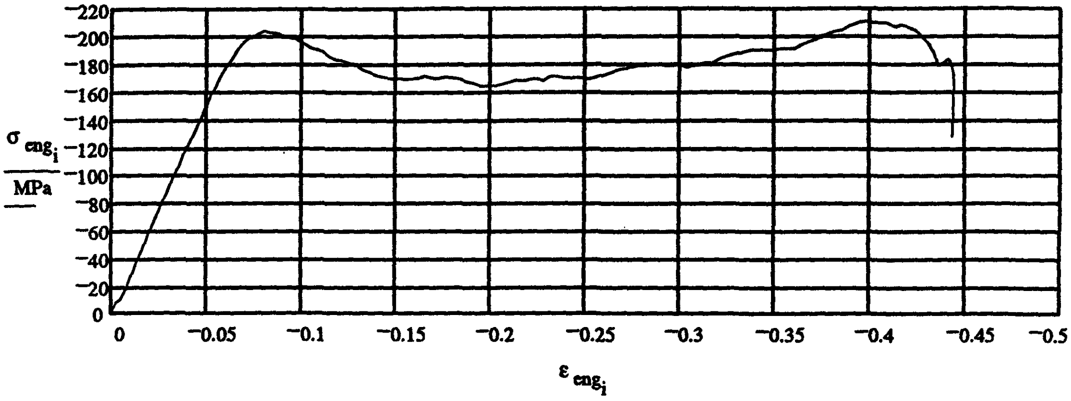
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\text{eng}} := \frac{A_{\text{bar}}}{A_o} \cdot E_{\text{bar}} \cdot \varepsilon_{\text{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ $\qquad$ $index \equiv 10$



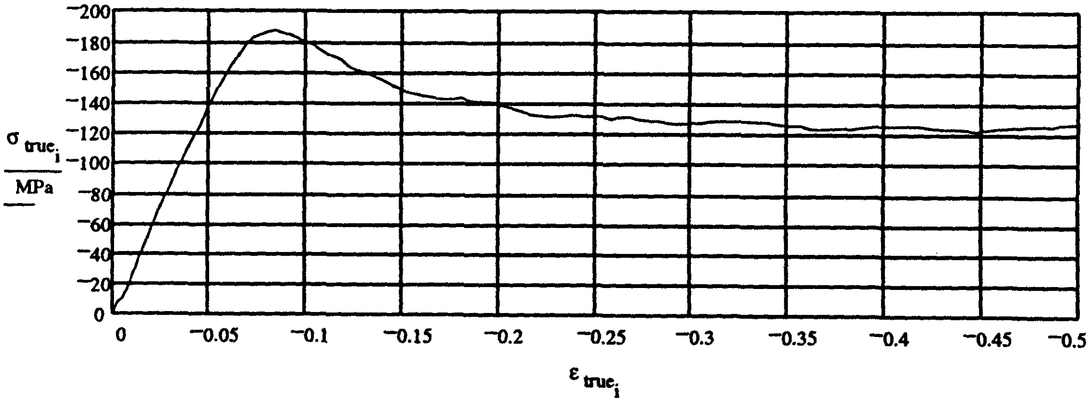
The maximum compresssive stress is: $\min(\sigma_{eng}) = -211.812 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\qquad$ $\min(\varepsilon_{true}) = -0.588$ $\qquad$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\qquad$ $\min(\sigma_{true}) = -187.317 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 75% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 25% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0051.WFT is the reflected pulse file.
2. WAVE0052.WFT is the transmitted pulse file.
3. WAVE0053.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$D7C2T2 := \mathrm{augment}\left(\varepsilon_{true}, \frac{\sigma_{true}}{MPa}\right)$$

$$\mathrm{WRITEPRN}(D7C2T2) := D7C2T2$$

General Definitions: **% Derakane 8084: 75 %Crestomer 1080: 25 Trial: 3 of 3**

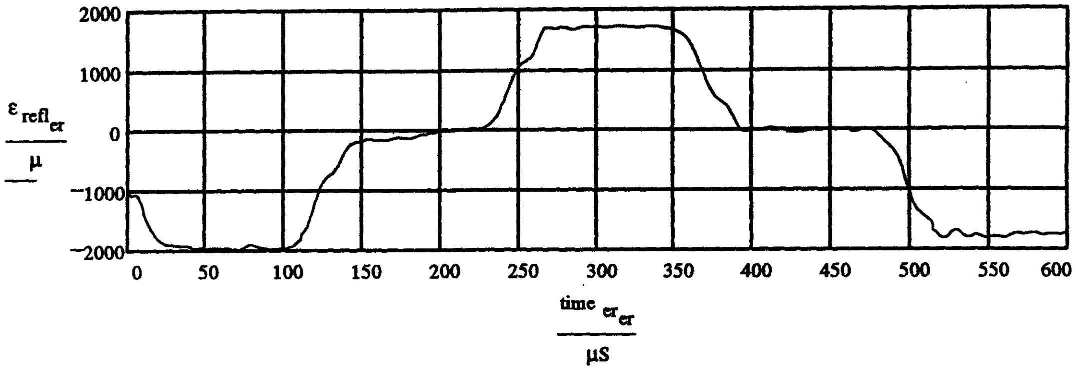
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad \mathrm{GPa} := \mathrm{Pa} \cdot 10^{9} \quad \mathrm{MPa} := \mathrm{Pa} \cdot 10^{6}$

$\mathrm{GF} := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot \mathrm{GPa} \quad D_{bar} := 0.75 \cdot \mathrm{in} \quad A_{bar} := \frac{\pi}{4} \cdot D_{bar}^{\ 2} \quad A_{bar} = 285.023 \cdot \mathrm{mm}^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(\mathrm{BD7C2ER3}) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1 \rangle} \quad \mathrm{time}_{er} := \varepsilon_{reflected}^{\langle 2 \rangle} \cdot \sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{\mathrm{GF} \cdot V_O}$ << apply calibration >>

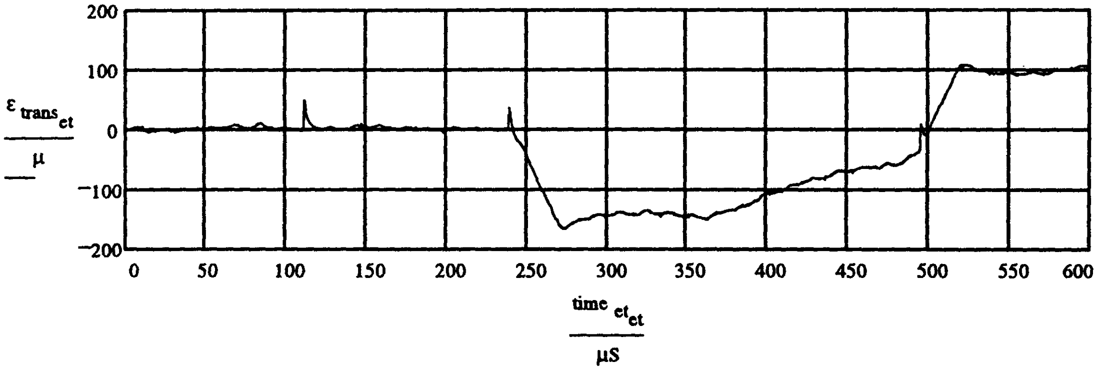


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(\mathrm{BD7C2ET3}) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1 \rangle} \quad \mathrm{time}_{et} := \varepsilon_{transmitted}^{\langle 2 \rangle} \cdot \sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{\mathrm{GF} \cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2100}}$ << shift pre-reflected signal to a strain of zero >>

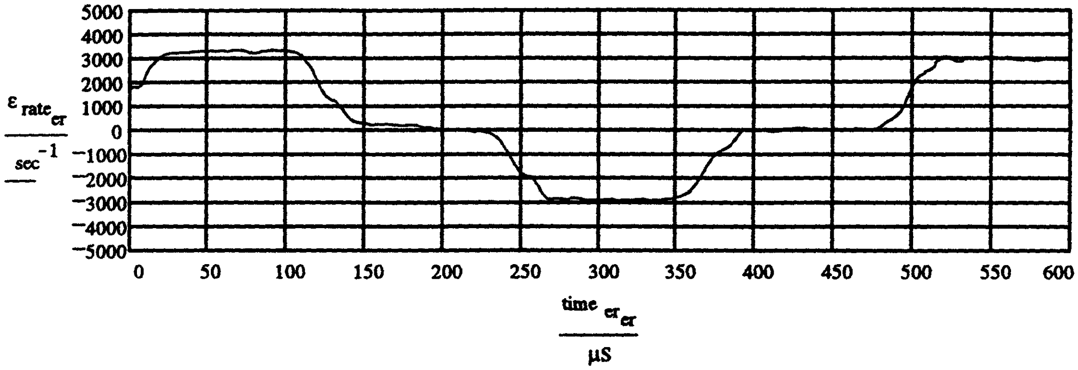


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ **<< longitudinal wave speed for steel >>**

$L_0 := 5.94 \cdot mm$ $D_0 := 8.03 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ **<< inital specimen length, diameter, and Acs >>**

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

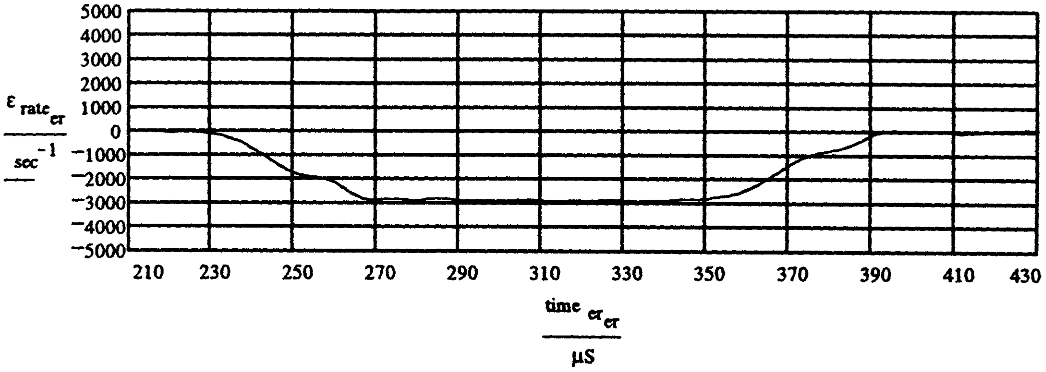


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

$start := 210$ $fin := 430$ $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$start := start \cdot 10 \qquad fin := fin \cdot 10$

$$\varepsilon_{maxeng} := \sum_{n = start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.355$$

<< max eng. strain >>

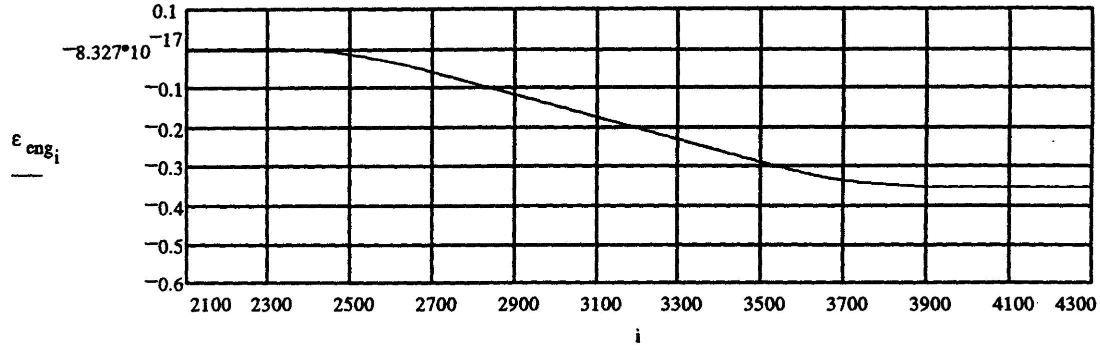
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := start, start + index .. fin$

$$\varepsilon_{eng_i} := \sum_{n = start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2}$$

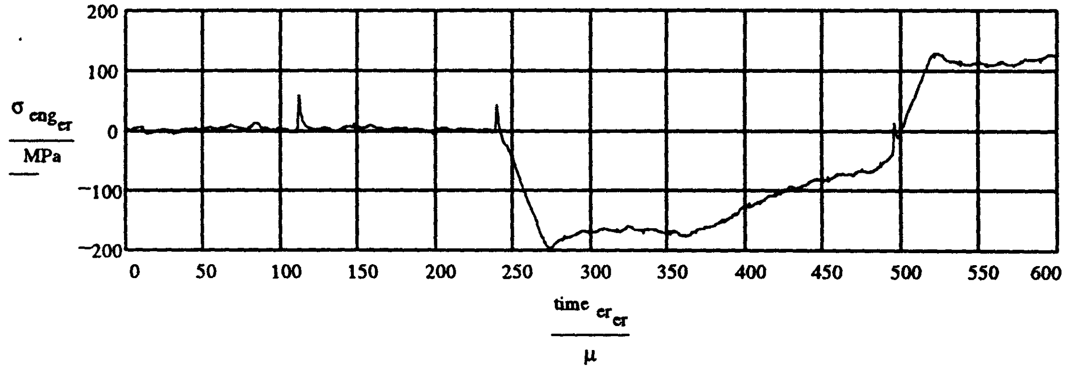
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



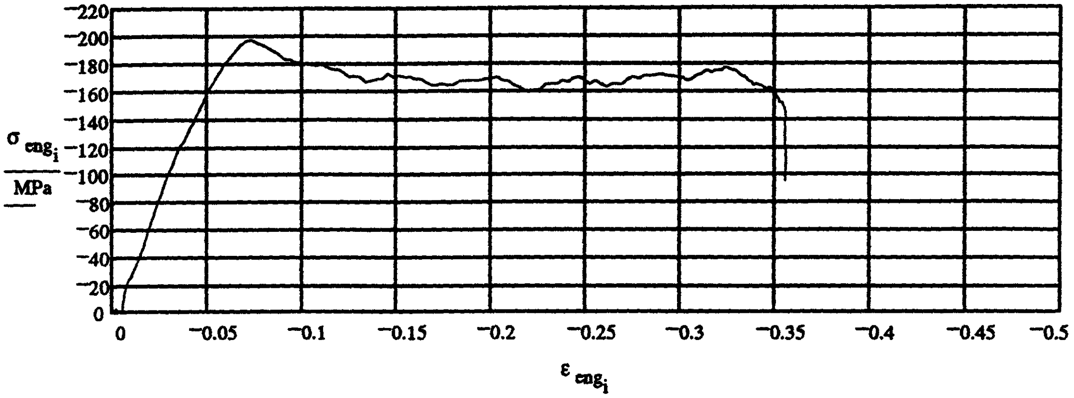
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index≡10



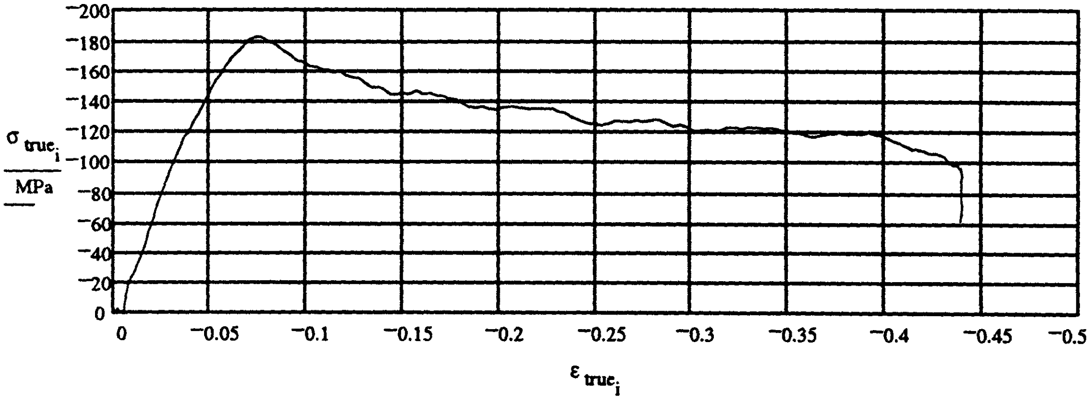
The maximum compresssive stress is: $\min(\sigma_{eng}) = -196.907 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.44$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -182.353 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 75% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 25% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0045.WFT is the reflected pulse file.
2. WAVE0046.WFT is the transmitted pulse file.
3. WAVE0047.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$D7C2T3 := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN}(D7C2T3) := D7C2T3$$

General Definitions: **% Derakane 8084: 50 %Crestomer 1080: 50 Trial: 1 of 3**

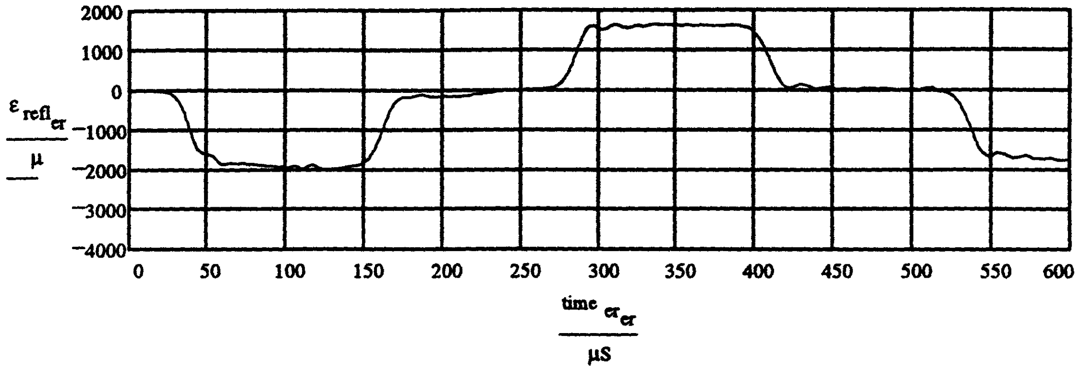
$\mathrm{ORIGIN} := 1 \quad \mu S := \mathrm{sec}\cdot 10^{-6} \quad \mu := 10^{-6} \quad \mathrm{GPa} := \mathrm{Pa}\cdot 10^{9} \quad \mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$

$\mathrm{GF} := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot \mathrm{GPa} \quad D_{bar} := 0.75\cdot \mathrm{in} \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\;2} \quad A_{bar} = 285.023\,\mathrm{mm}^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD5C5ER1)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1\rangle} \quad time_{er} := \varepsilon_{reflected}^{\langle 2\rangle}\cdot \mathrm{sec}$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ **<< set up range >>** $\varepsilon_{refl} := \dfrac{4\cdot \varepsilon_{refl}}{\mathrm{GF}\cdot V_O}$ **<< apply calibration >>**

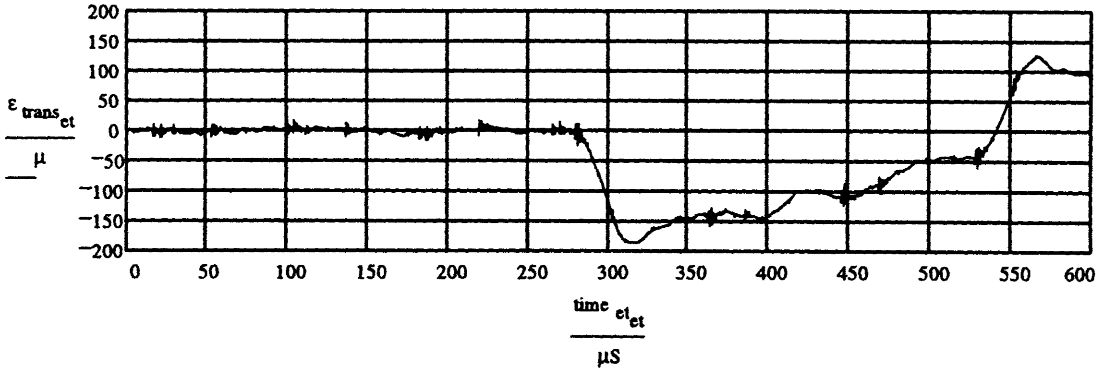


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD5C5ET1)} \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1\rangle} \quad time_{et} := \varepsilon_{transmitted}^{\langle 2\rangle}\cdot \mathrm{sec}$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ **<< set up range >>** $\varepsilon_{trans} := \dfrac{4\cdot \varepsilon_{trans}}{\mathrm{GF}\cdot V_O}$ **<< apply calibration >>**

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2400}}$ **<< shift pre-reflected signal to a strain of zero >>**

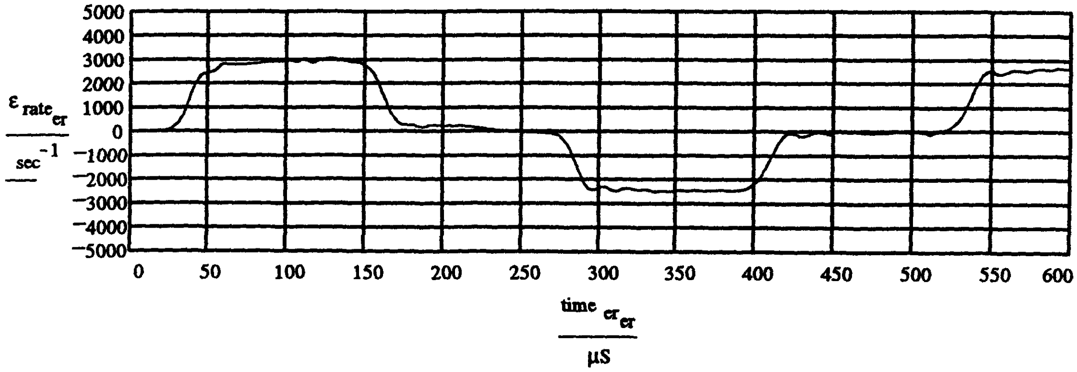


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 6.56 \cdot mm \quad D_0 := 8.80 \cdot mm \quad A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

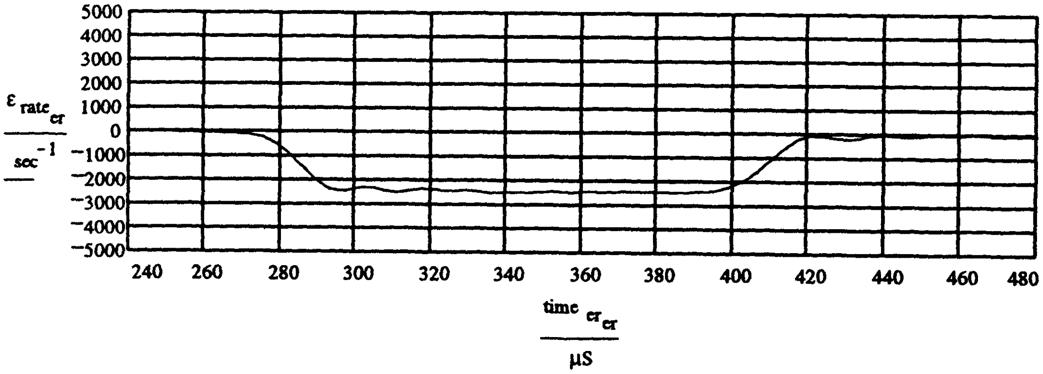


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 240 fin := 480 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$start := start \cdot 10 \qquad fin := fin \cdot 10$

$$\varepsilon_{maxeng} := \sum_{n=start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.311$$

<< max eng. strain >>

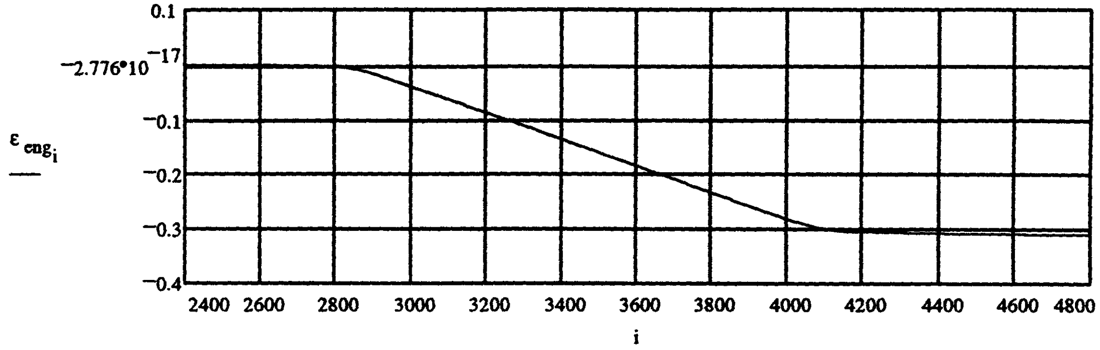
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := start, start + index .. fin$

$$\varepsilon_{eng_i} := \sum_{n=start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2}$$

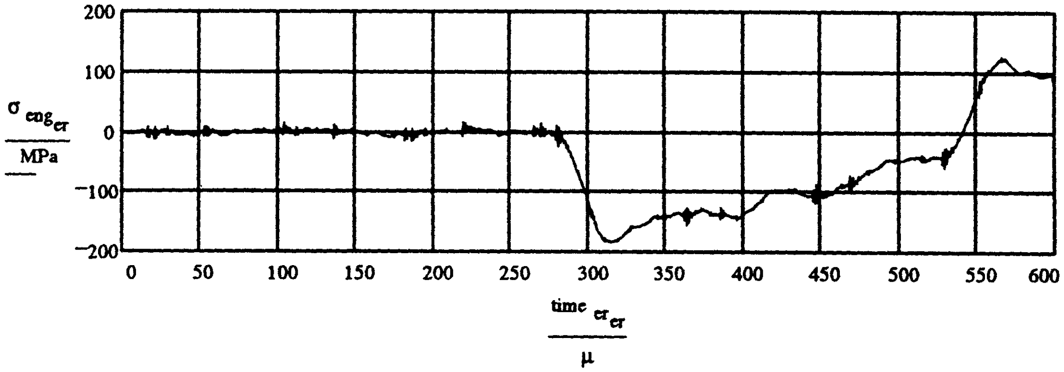
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



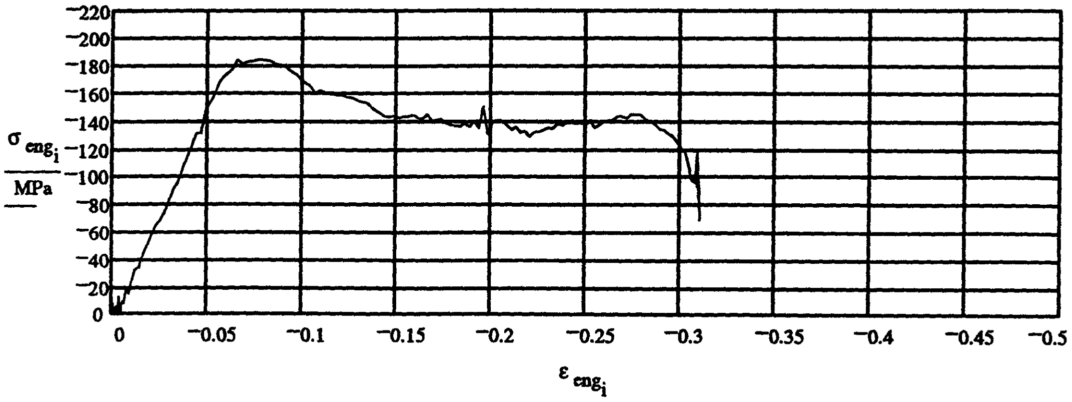
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index ≡ 10



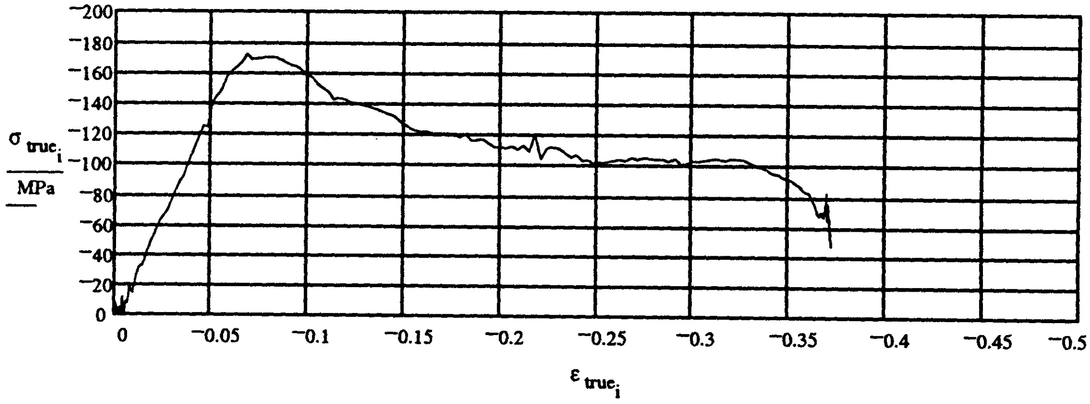
The maximum compresssive stress is: $\min(\sigma_{eng}) = -184.45 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.372$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -171.791 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 50% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 50% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0060.WFT is the reflected pulse file.
2. WAVE0061.WFT is the transmitted pulse file.
3. WAVE0062.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D5C5T1} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D5C5T1)} := \mathrm{D5C5T1}$$

General Definitions: **% Derakane 8084: 50 %Crestomer 1080: 50 Trial: 2 of 3**

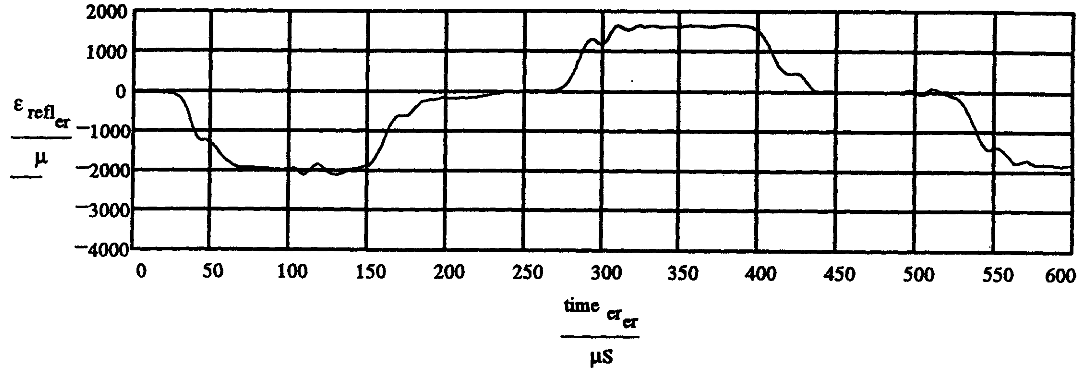
$\text{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa\cdot 10^{9} \quad MPa := Pa\cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot GPa \quad D_{bar} := 0.75\cdot in \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{2} \quad A_{bar} = 285.023\cdot mm^{2}$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \text{READPRN}(HD5C5ER2) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \quad time_{er} := \varepsilon_{reflected}^{<2>}\cdot \sec$

$er := 1 .. \text{rows}(\varepsilon_{refl})$ **<< set up range >>** $\varepsilon_{refl} := \frac{4\cdot \varepsilon_{refl}}{GF\cdot V_O}$ **<< apply calibration >>**

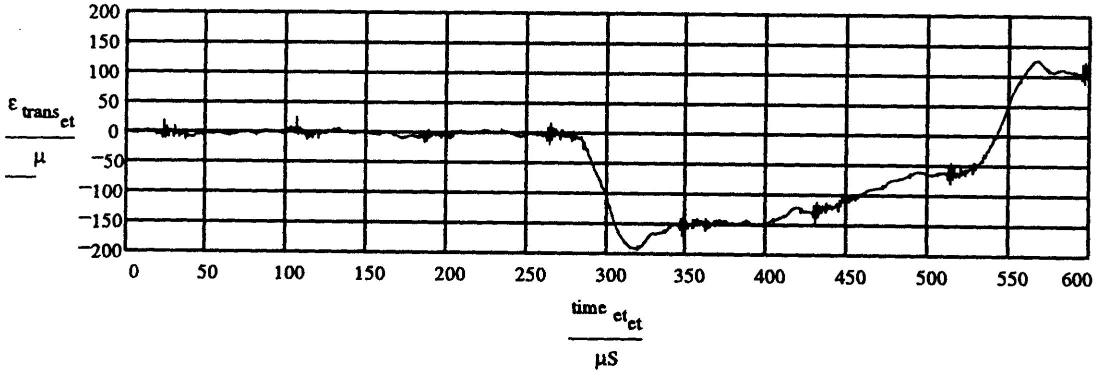


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \text{READPRN}(HD5C5ET2) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \quad time_{et} := \varepsilon_{transmitted}^{<2>}\cdot \sec$

$et := 1 .. \text{rows}(\varepsilon_{trans})$ **<< set up range >>** $\varepsilon_{trans} := \frac{4\cdot \varepsilon_{trans}}{GF\cdot V_O}$ **<< apply calibration >>**

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2400}}$ **<< shift pre-reflected signal to a strain of zero >>**

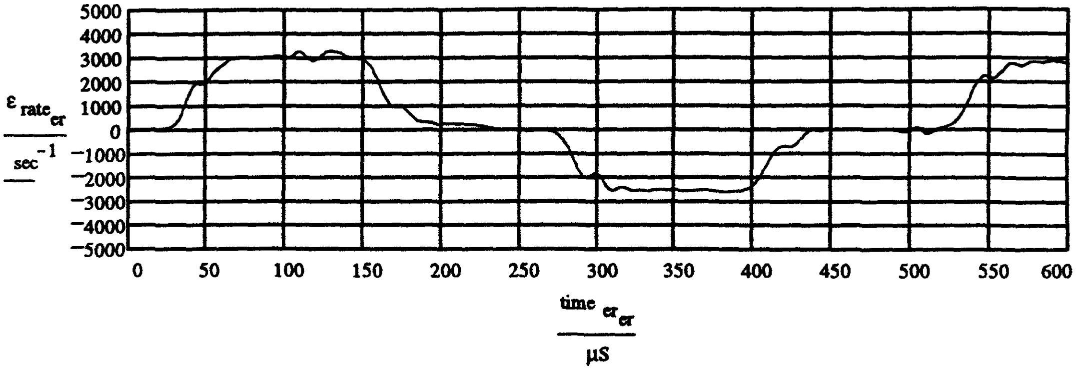


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 6.44 \cdot mm$ $D_0 := 8.80 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

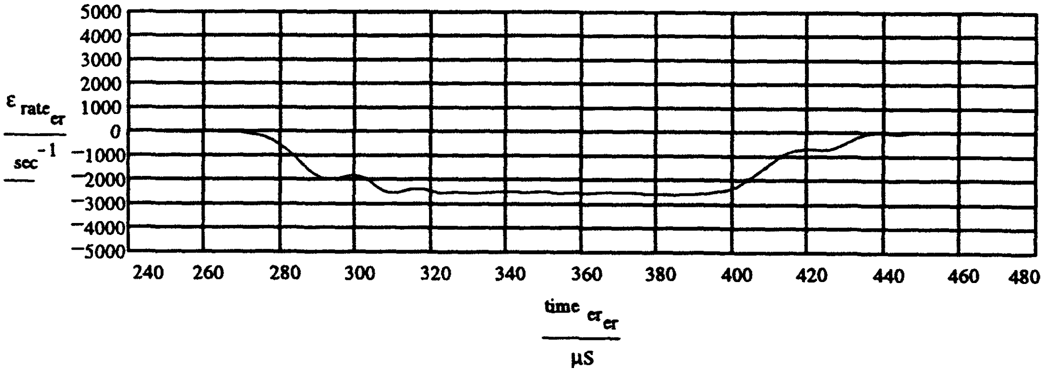


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

$start := 240$ $fin := 480$ $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$start := start \cdot 10 \qquad fin := fin \cdot 10$

$$\varepsilon_{maxeng} := \sum_{n = start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.319$$

<< max eng. strain >>

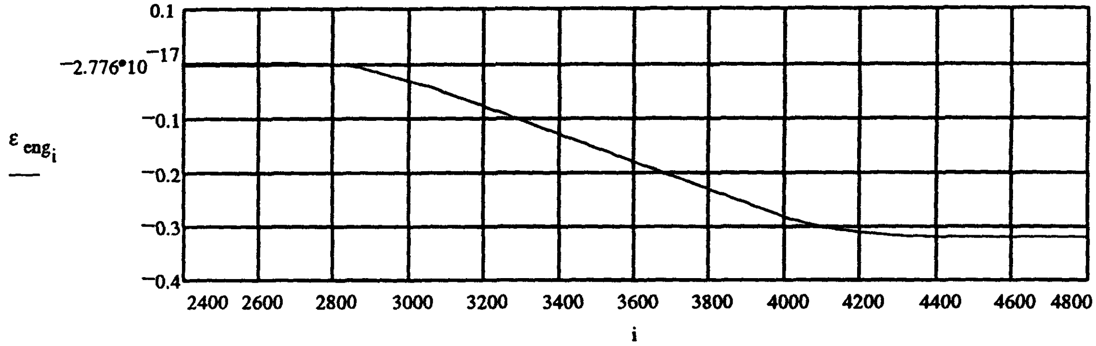
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := start, start + index .. fin$

$$\varepsilon_{eng_i} := \sum_{n = start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2}$$

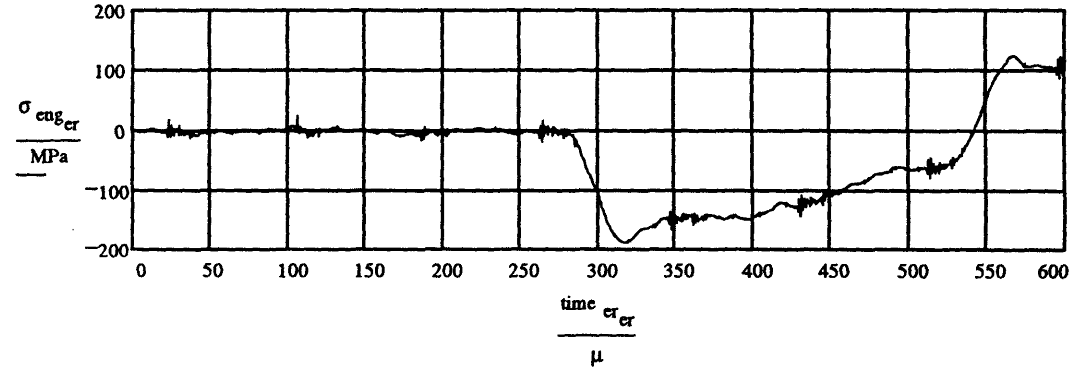
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



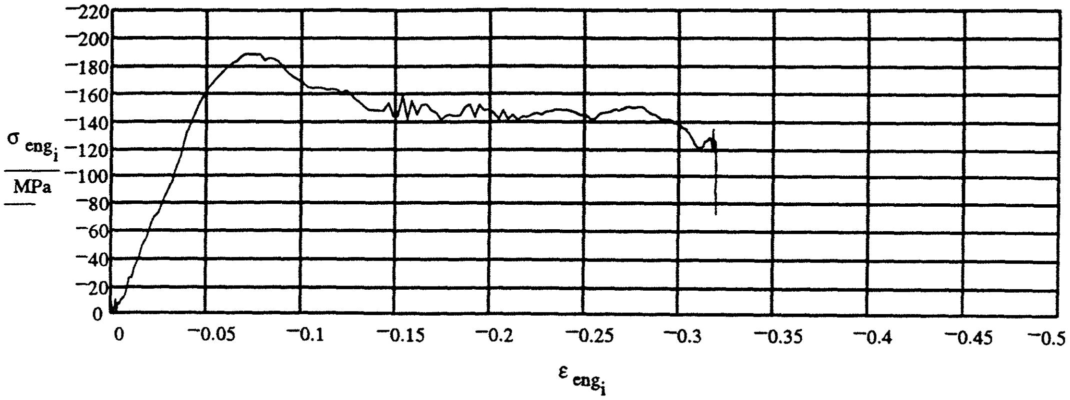
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i} \qquad index \equiv 10$$



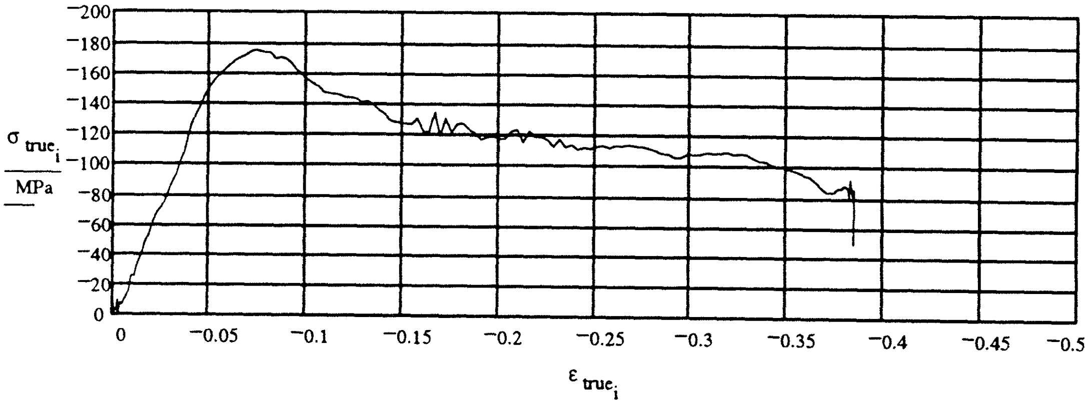
The maximum compresssive stress is: $\min(\sigma_{eng}) = -190.949 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.385$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -174.609 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 50% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 50% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0063.WFT is the reflected pulse file.
2. WAVE0064.WFT is the transmitted pulse file.
3. WAVE0065.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$D5C5T2 := \mathrm{augment}\left(\varepsilon_{true}, \frac{\sigma_{true}}{MPa}\right)$$

$$\mathrm{WRITEPRN}(D5C5T2) := D5C5T2$$

General Definitions: **% Derakane 8084: 50 %Crestomer 1080: 50 Trial: 3 of 3**

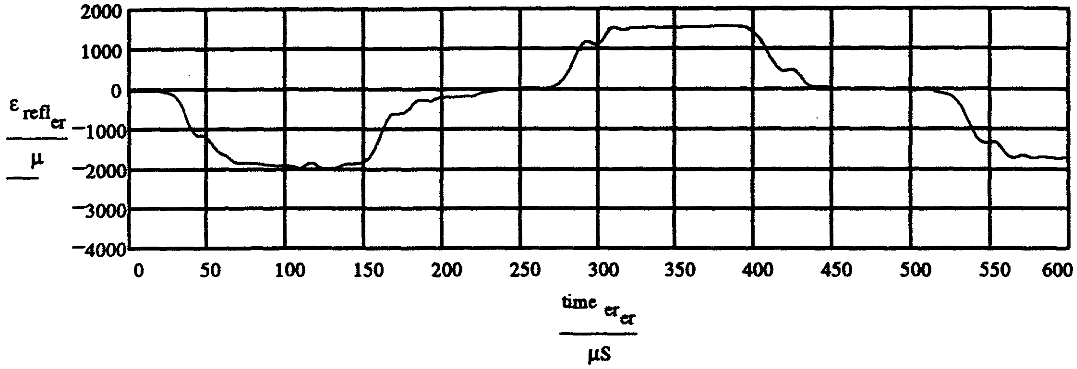
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa\cdot 10^{9} \quad MPa := Pa\cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot GPa \quad D_{bar} := 0.75\cdot in \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\;2} \quad A_{bar} = 285.023\cdot mm^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(HD5C5ER3) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1\rangle} \qquad time_{er} := \varepsilon_{reflected}^{\langle 2\rangle}\cdot \sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4\cdot\varepsilon_{refl}}{GF\cdot V_O}$ << apply calibration >>

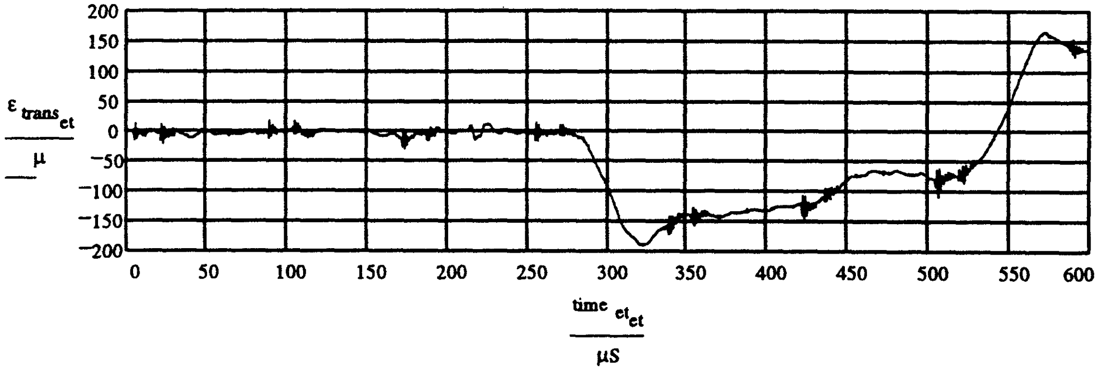


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(HD5C5ET3) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1\rangle} \qquad time_{et} := \varepsilon_{transmitted}^{\langle 2\rangle}\cdot \sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4\cdot\varepsilon_{trans}}{GF\cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2400}}$ << shift pre-reflected signal to a strain of zero >>

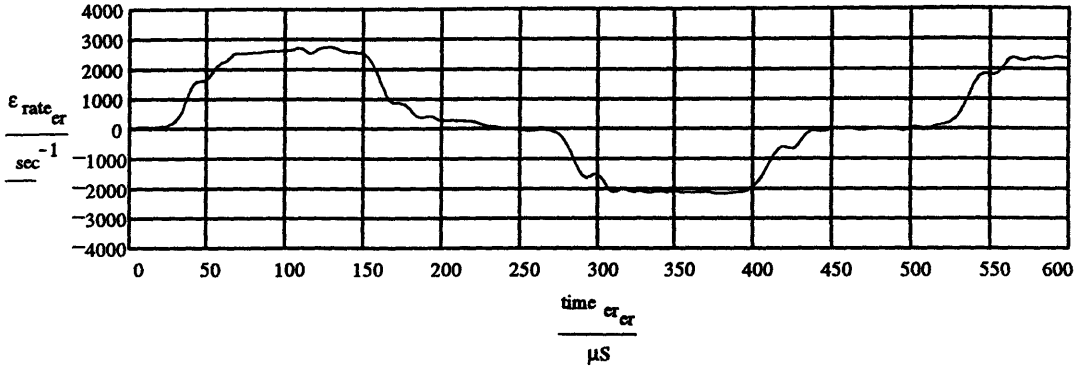


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 7.29 \cdot mm$ $D_0 := 8.73 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\;2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

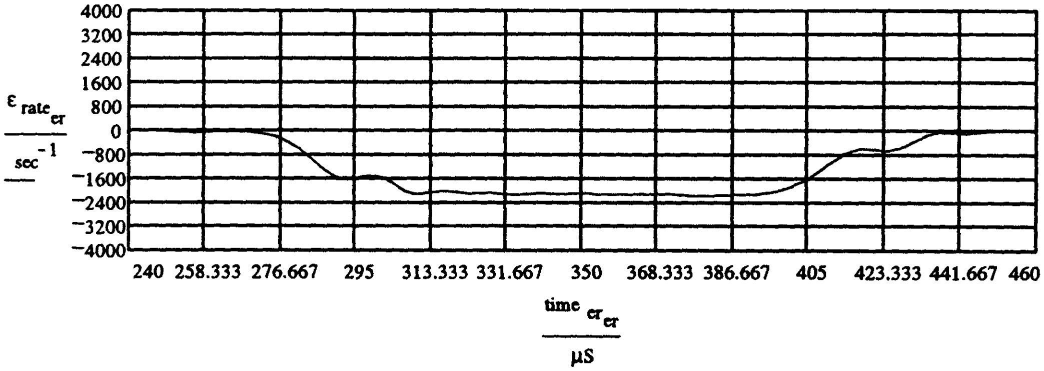


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 240 fin := 460 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\mathrm{start} := \mathrm{start}\cdot 10 \quad \mathrm{fin} := \mathrm{fin}\cdot 10$

$$\varepsilon_{\mathrm{maxeng}} := \sum_{n=\mathrm{start}}^{\mathrm{fin}} \Delta t\cdot\frac{\varepsilon_{\mathrm{rate}_n}+\varepsilon_{\mathrm{rate}_{n+1}}}{2} \qquad \varepsilon_{\mathrm{maxeng}} = -0.268$$

<< max eng. strain >>

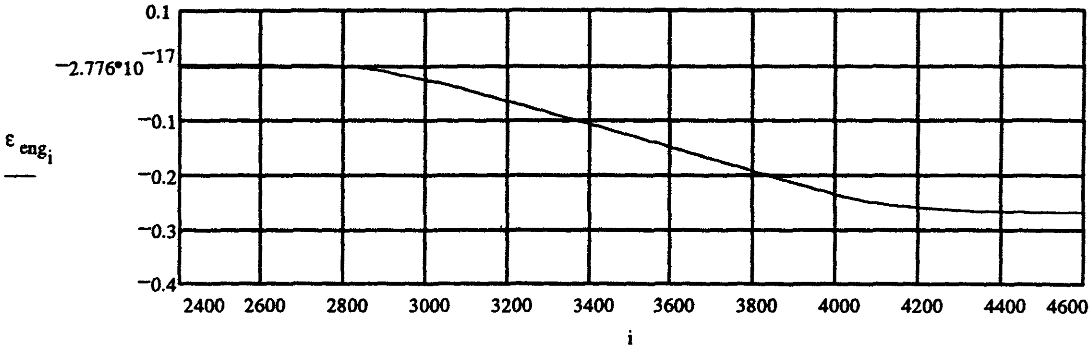
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \mathrm{start}, \mathrm{start}+\mathrm{index} .. \mathrm{fin}$

$$\varepsilon_{\mathrm{eng}_i} := \sum_{n=\mathrm{start}}^{i} \Delta t\cdot\frac{\varepsilon_{\mathrm{rate}_n}+\varepsilon_{\mathrm{rate}_{n+1}}}{2}$$

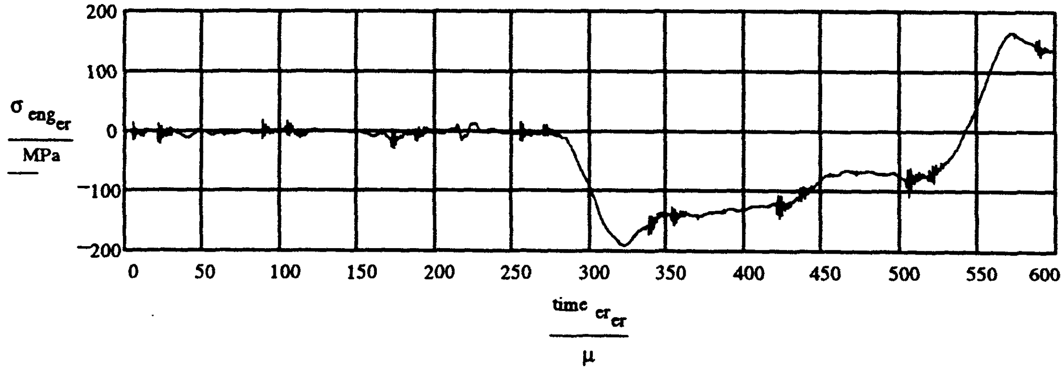
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



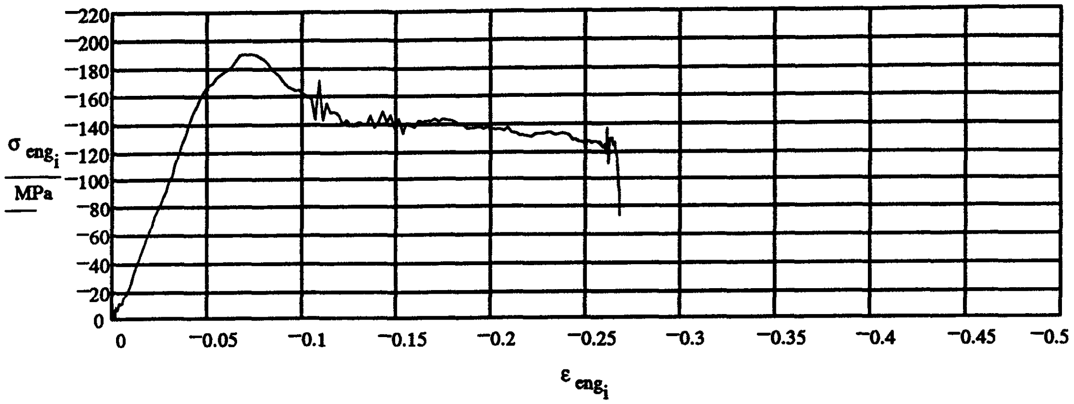
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\mathrm{eng}} := \frac{A_{\mathrm{bar}}}{A_0}\cdot E_{\mathrm{bar}}\cdot\varepsilon_{\mathrm{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index≡10



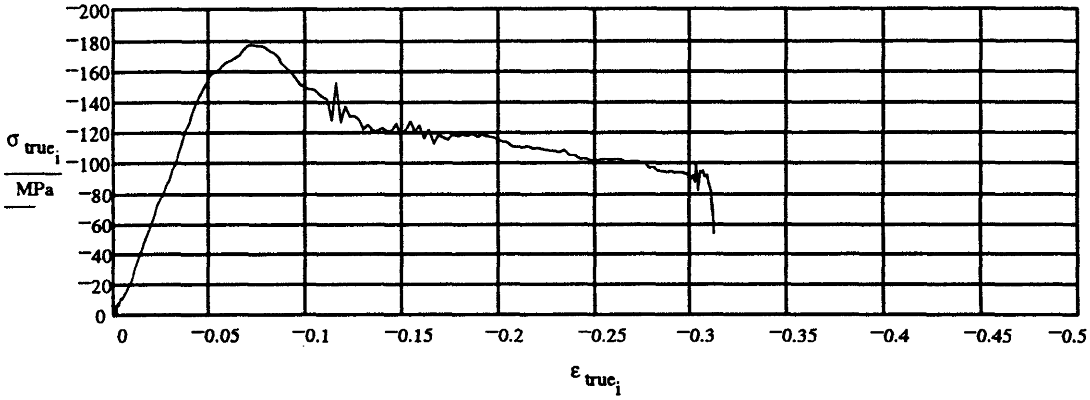
The maximum compresssive stress is: $\min(\sigma_{eng}) = -191.991 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.312$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -176.762 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 50% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 50% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0066.WFT is the reflected pulse file.
2. WAVE0067.WFT is the transmitted pulse file.
3. WAVE0068.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$D5C5T3 := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

WRITEPRN(D5C5T3) := D5C5T3

General Definitions: **% Derakane 8084: 25 %Crestomer 1080: 75 Trial: 1 of 4**

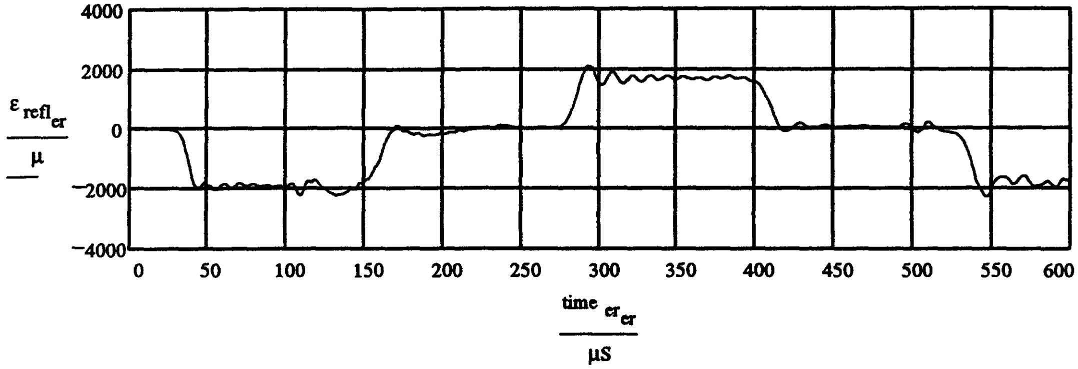
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa \cdot 10^{9} \quad MPa := Pa \cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot GPa \quad D_{bar} := 0.75 \cdot in \quad A_{bar} := \frac{\pi}{4} \cdot D_{bar}^{\;2} \quad A_{bar} = 285.023 \cdot mm^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD2C7ER1)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1 \rangle} \qquad time_{er} := \varepsilon_{reflected}^{\langle 2 \rangle} \cdot sec$

$er := 1 .. \mathrm{rows}\left(\varepsilon_{refl}\right)$ << set up range >> $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ << apply calibration >>

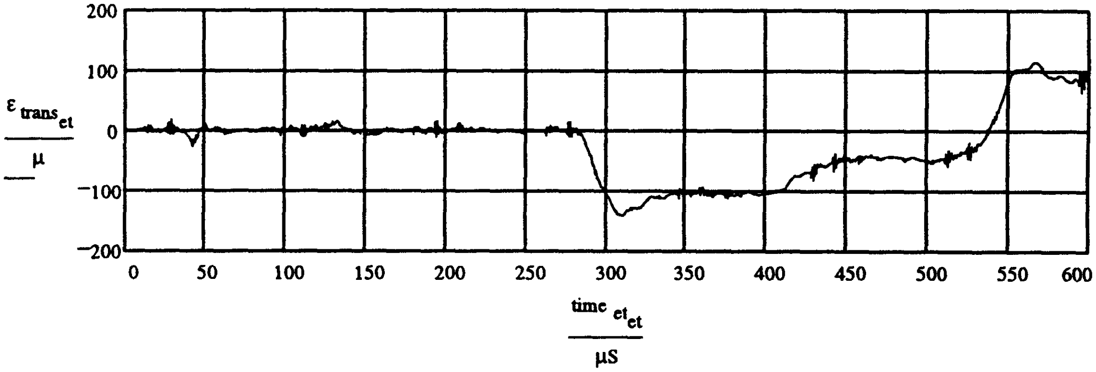


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD2C7ET1)} \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1 \rangle} \qquad time_{et} := \varepsilon_{transmitted}^{\langle 2 \rangle} \cdot sec$

$et := 1 .. \mathrm{rows}\left(\varepsilon_{trans}\right)$ << set up range >> $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2500}}$ << shift pre-reflected signal to a strain of zero >>

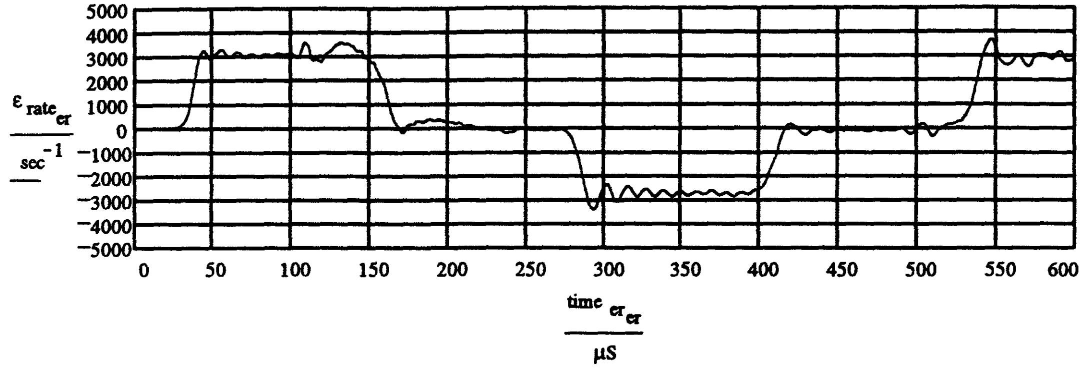


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 6.22 \cdot mm \quad D_0 := 8.29 \cdot mm \quad A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

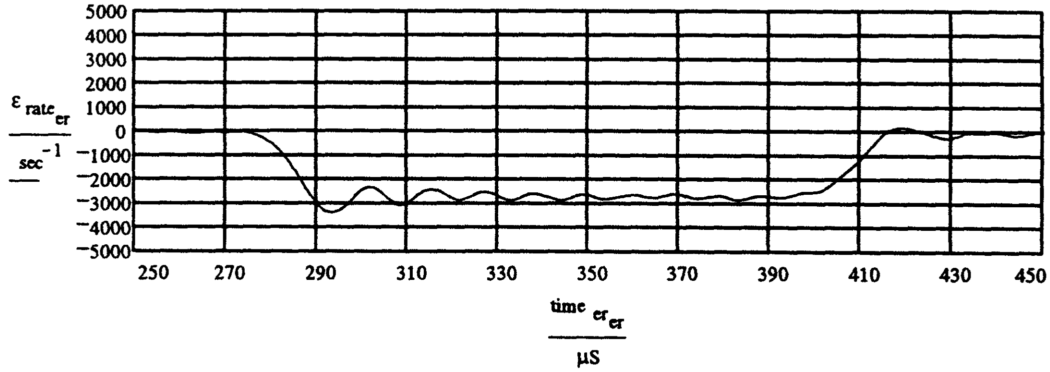


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

$start := 250 \quad fin := 450 \quad \Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\mathrm{start} := \mathrm{start}\cdot 10 \qquad \mathrm{fin} := \mathrm{fin}\cdot 10$

$$\varepsilon_{\mathrm{maxeng}} := \sum_{n=\mathrm{start}}^{\mathrm{fin}} \Delta t\cdot\frac{\varepsilon_{\mathrm{rate}_n}+\varepsilon_{\mathrm{rate}_{n+1}}}{2} \qquad \varepsilon_{\mathrm{maxeng}} = -0.346$$

<< max eng. strain >>

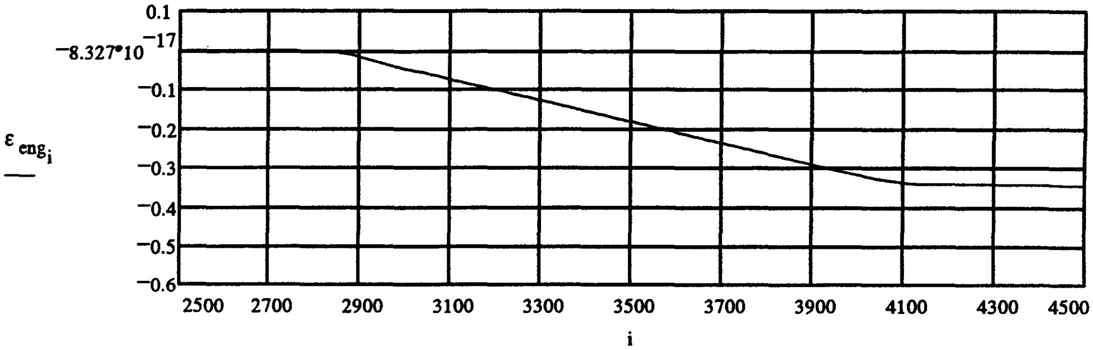
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \mathrm{start}, \mathrm{start}+\mathrm{index}\,..\,\mathrm{fin}$

$$\varepsilon_{\mathrm{eng}_i} := \sum_{n=\mathrm{start}}^{i} \Delta t\cdot\frac{\varepsilon_{\mathrm{rate}_n}+\varepsilon_{\mathrm{rate}_{n+1}}}{2}$$

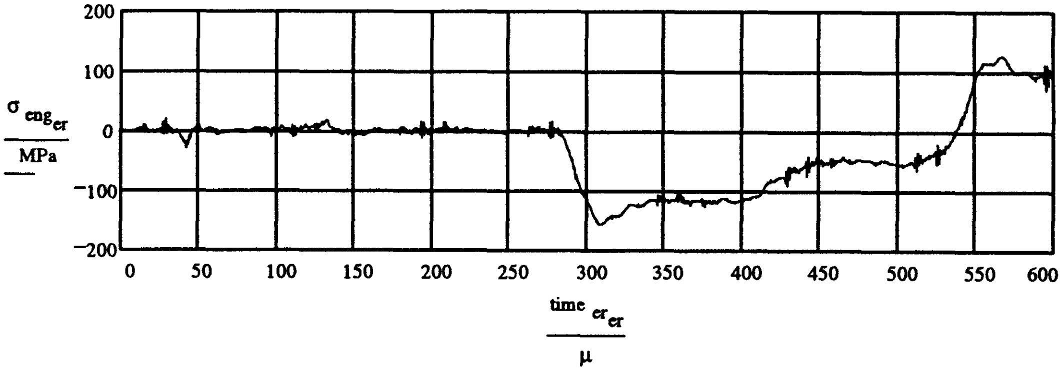
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



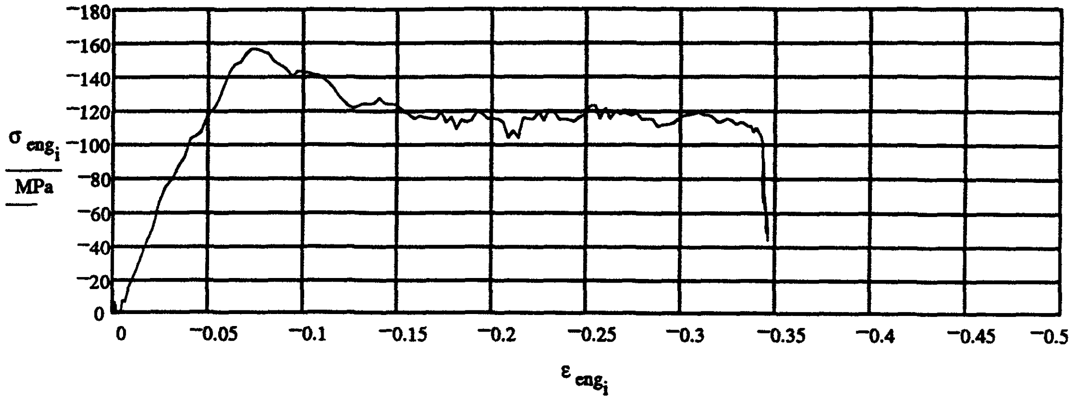
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\mathrm{eng}} := \frac{A_{\mathrm{bar}}}{A_o}\cdot E_{\mathrm{bar}}\cdot\varepsilon_{\mathrm{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index ≡ 10



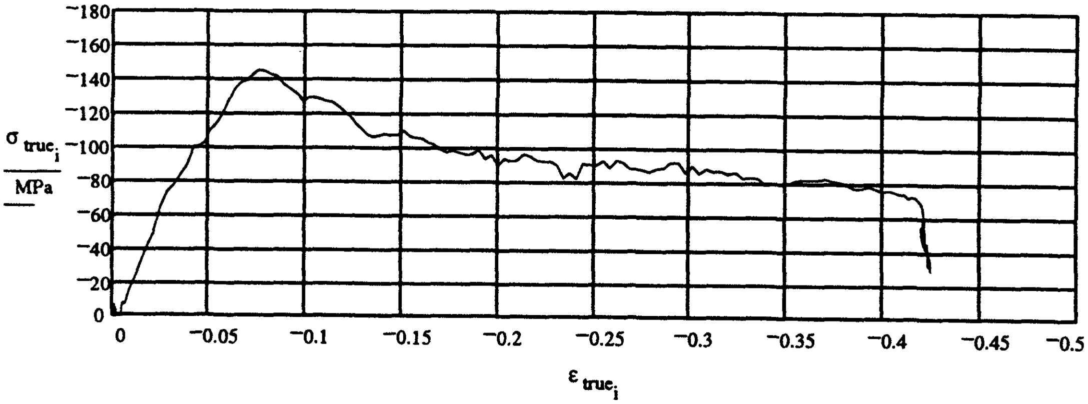
The maximum compresssive stress is: $\min(\sigma_{eng}) = -157.15 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.425$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -145.159 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 25% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 75% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0069.WFT is the reflected pulse file.
2. WAVE0070.WFT is the transmitted pulse file.
3. WAVE0071.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D2C7T1} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D2C7T1)} := \mathrm{D2C7T1}$$

General Definitions: **% Derakane 8084: 25 %Crestomer 1080: 75 Trial: 2 of 4**

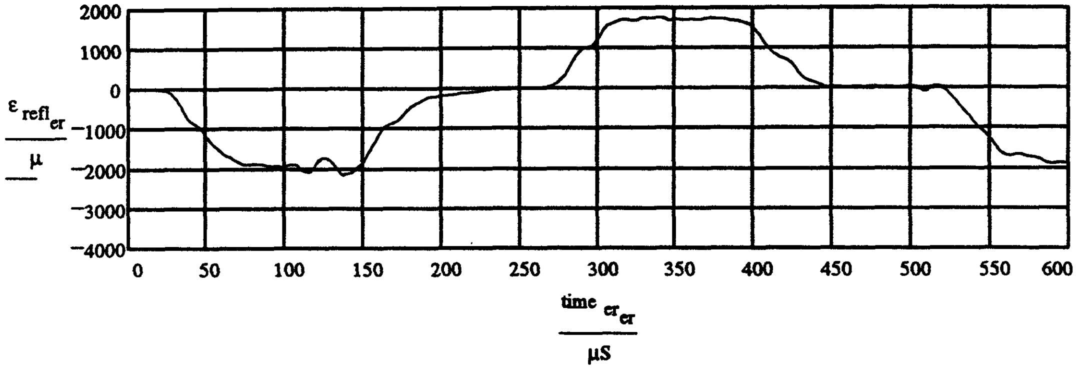
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad \mathrm{GPa} := \mathrm{Pa}\cdot 10^{9} \quad \mathrm{MPa} := \mathrm{Pa}\cdot 10^{6}$

$\mathrm{GF} := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot\mathrm{GPa} \quad D_{bar} := 0.75\cdot\mathrm{in} \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\ 2} \quad A_{bar} = 285.023\,\mathrm{mm}^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD2C7ER2)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{\langle 1\rangle} \qquad \mathrm{time}_{er} := \varepsilon_{reflected}^{\langle 2\rangle}\cdot\sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ **<< set up range >>** $\varepsilon_{refl} := \frac{4\cdot\varepsilon_{refl}}{\mathrm{GF}\cdot V_O}$ **<< apply calibration >>**

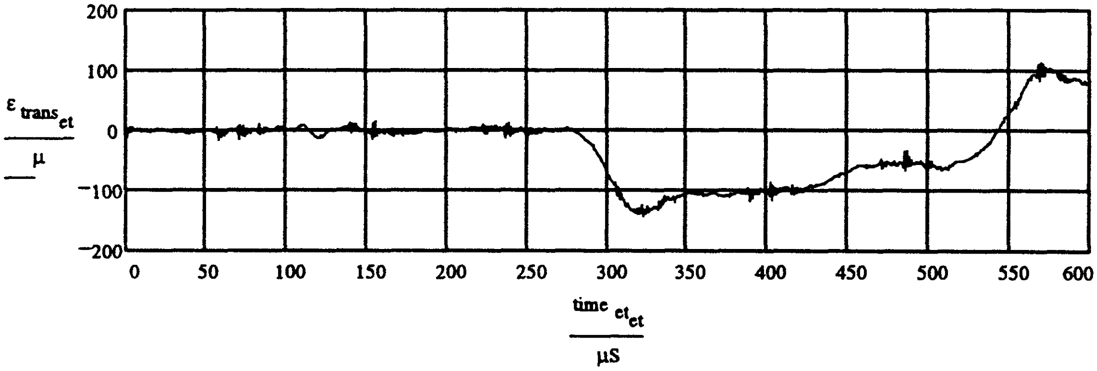


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD2C7ET2)} \ \varepsilon_{trans} := \varepsilon_{transmitted}^{\langle 1\rangle} \qquad \mathrm{time}_{et} := \varepsilon_{transmitted}^{\langle 2\rangle}\cdot\sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ **<< set up range >>** $\varepsilon_{trans} := \frac{4\cdot\varepsilon_{trans}}{\mathrm{GF}\cdot V_O}$ **<< apply calibration >>**

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2600}}$ **<< shift pre-reflected signal to a strain of zero >>**

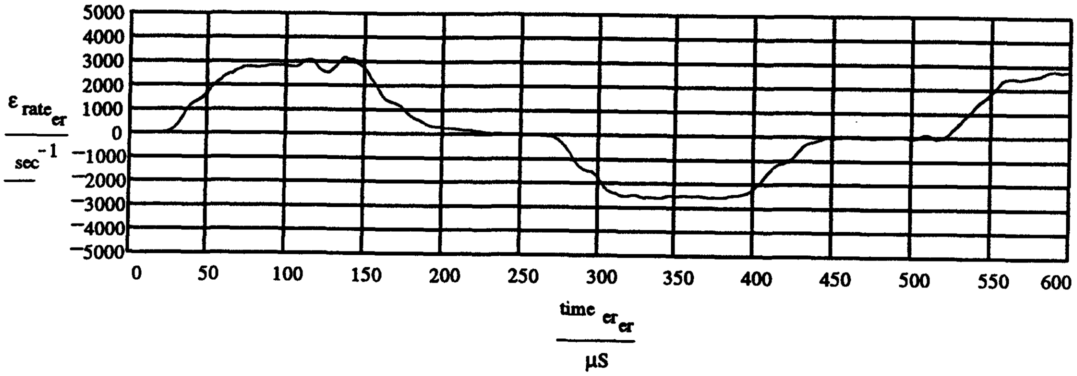


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 6.78 \cdot mm$ $D_0 := 8.22 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

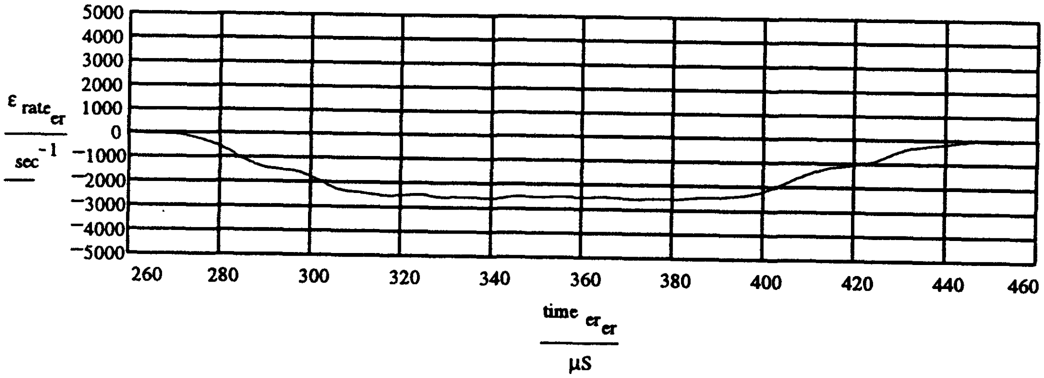


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 260 fin := 460 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

start := start·10   fin := fin·10

$$\varepsilon_{maxeng} := \sum_{n=start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.318$$

<< max eng. strain >>

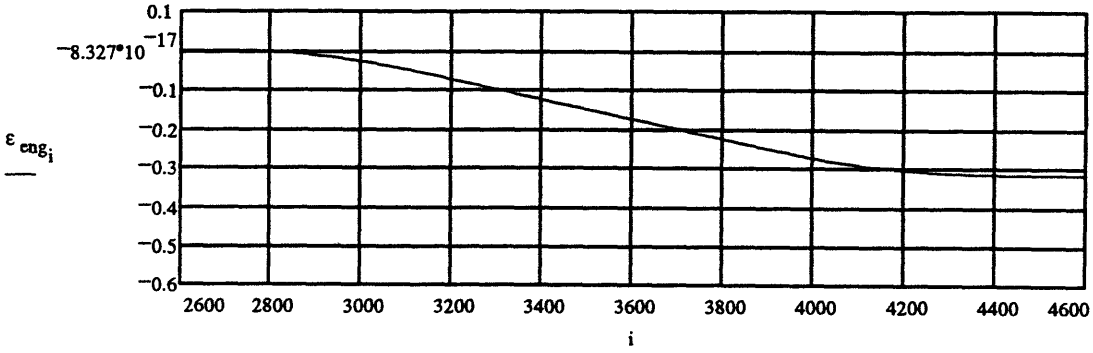
Calculate the engineering strain vector by numerical integration of the strain rate:

i := start, start + index.. fin

$$\varepsilon_{eng_i} := \sum_{n=start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_n} + \varepsilon_{rate_{n+1}}}{2}$$

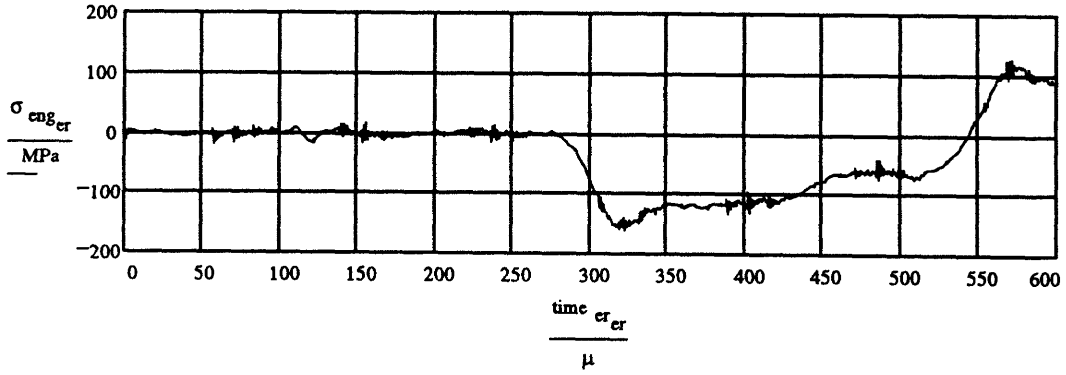
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



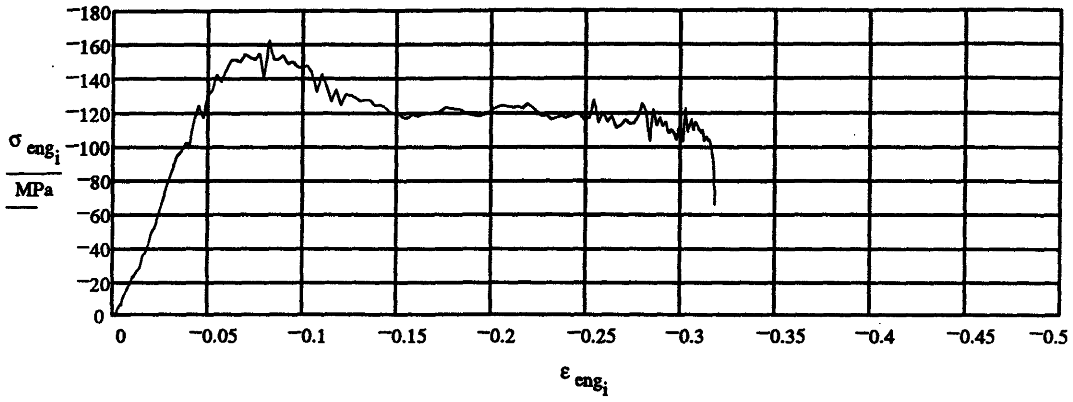
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i} \qquad index \equiv 10$$



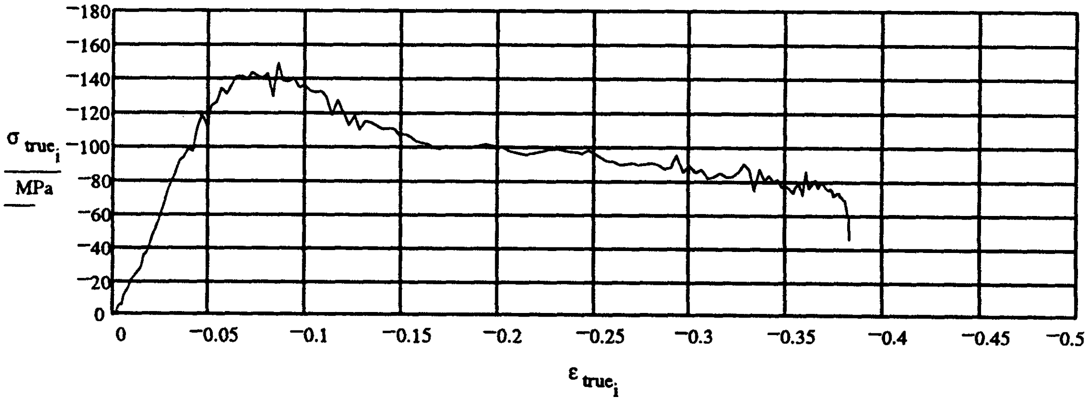
The maximum compresssive stress is: $\min(\sigma_{eng}) = -162.129 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.383$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -148.8 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 25% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 75% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0072.WFT is the reflected pulse file.
2. WAVE0073.WFT is the transmitted pulse file.
3. WAVE0074.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D2C7T2} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D2C7T2)} := \mathrm{D2C7T2}$$

General Definitions: **% Derakane 8084: 25 %Crestomer 1080: 75 Trial: 3 of 4**

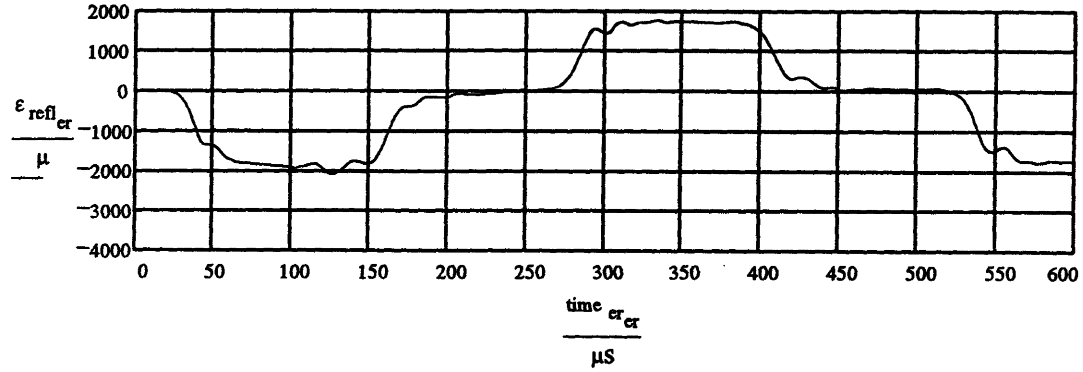
$ORIGIN := 1$ $\mu S := sec \cdot 10^{-6}$ $\mu := 10^{-6}$ $GPa := Pa \cdot 10^{9}$ $MPa := Pa \cdot 10^{6}$

$GF := 2.1$ $V_O := 30$ $E_{bar} := 210 \cdot GPa$ $D_{bar} := 0.75 \cdot in$ $A_{bar} := \frac{\pi}{4} \cdot D_{bar}^{2}$ $A_{bar} = 285.023 \cdot mm^{2}$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := READPRN(HD2C7ER3)$ $\varepsilon_{refl} := \varepsilon_{reflected}^{<1>}$ $time_{er} := \varepsilon_{reflected}^{<2>} \cdot sec$

$er := 1 .. rows(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ << apply calibration >>

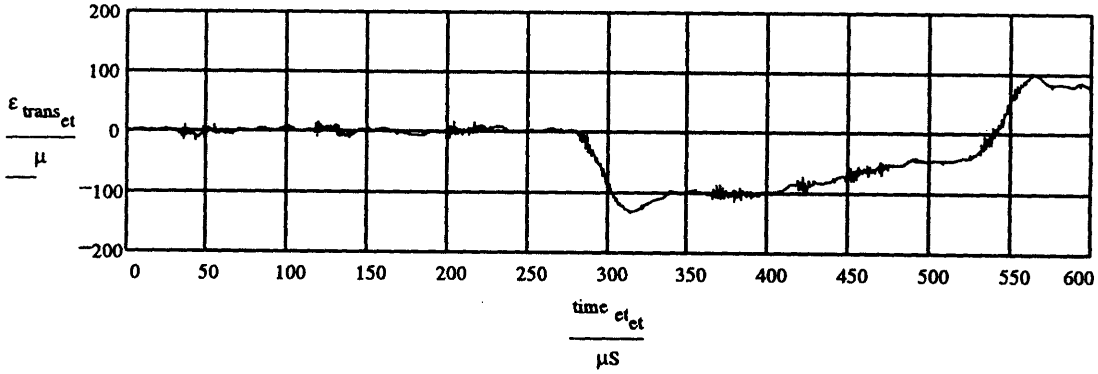


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := READPRN(HD2C7ET3)$ $\varepsilon_{trans} := \varepsilon_{transmitted}^{<1>}$ $time_{et} := \varepsilon_{transmitted}^{<2>} \cdot sec$

$et := 1 .. rows(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2550}}$ << shift pre-reflected signal to a strain of zero >>

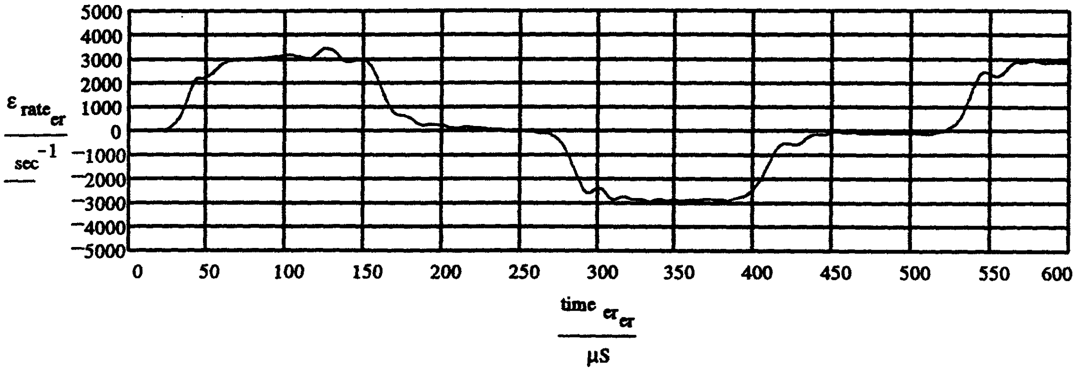


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 6.03 \cdot mm$ $D_0 := 8.20 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$

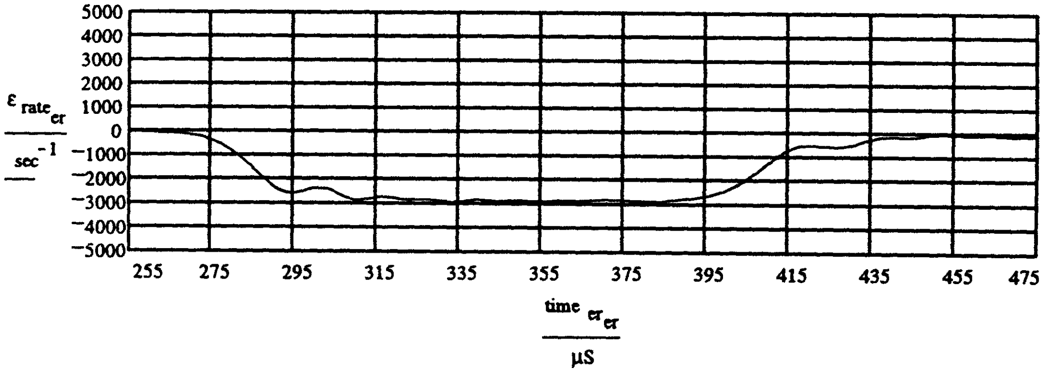


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 255 fin := 475 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$start := start\cdot 10 \quad fin := fin\cdot 10$

$$\varepsilon_{maxeng} := \sum_{n=start}^{fin} \Delta t\cdot\frac{\varepsilon_{rate_n}+\varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.366$$

<< max eng. strain >>

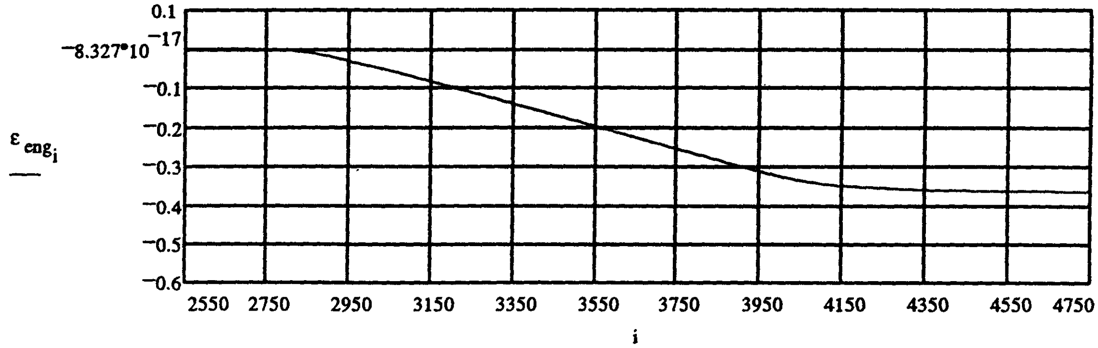
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := start, start + index .. fin$

$$\varepsilon_{eng_i} := \sum_{n=start}^{i} \Delta t\cdot\frac{\varepsilon_{rate_n}+\varepsilon_{rate_{n+1}}}{2}$$

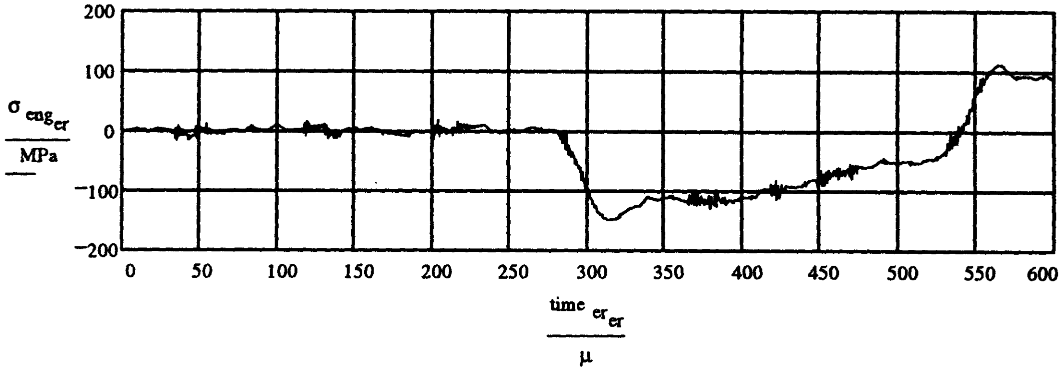
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



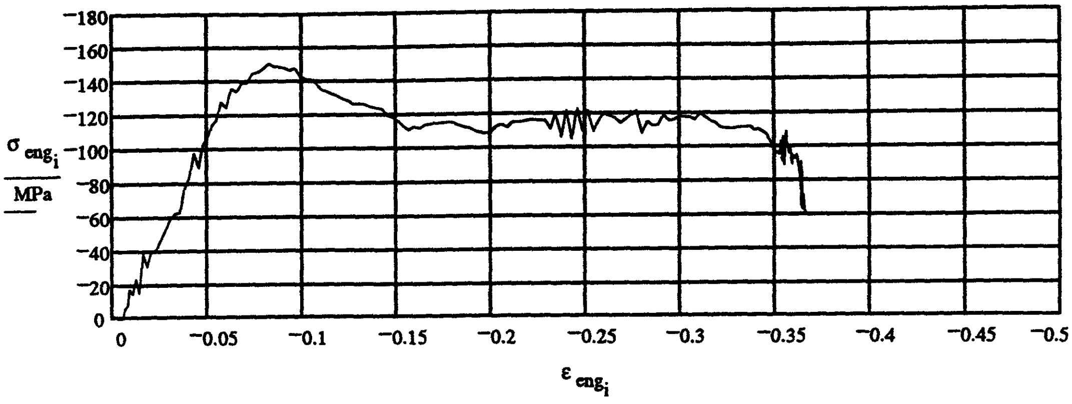
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_o}\cdot E_{bar}\cdot\varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index$\equiv$10



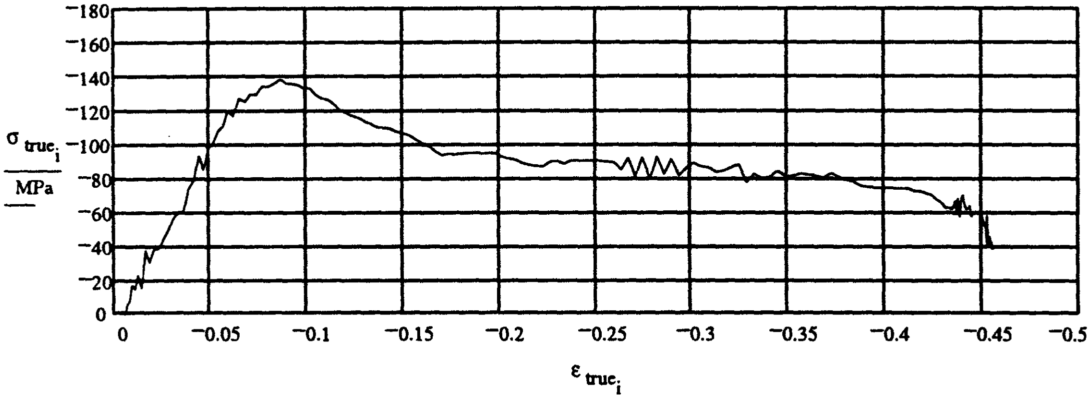
The maximum compresssive stress is: $\min\left(\sigma_{eng}\right) = -150.256 \cdot \text{MPa}$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln\left(\varepsilon_{eng_i} + 1\right)$ $\min\left(\varepsilon_{true}\right) = -0.456$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot \left(\varepsilon_{eng_i} + 1\right)$



The maximum compresssive true stress is: $\min\left(\sigma_{true}\right) = -137.791 \cdot \text{MPa}$

Supplemental note: This stress strain curve represents a specimen composed of 25% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 75% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0075.WFT is the reflected pulse file.
2. WAVE0076.WFT is the transmitted pulse file.
3. WAVE0077.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D2C7T3} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN}(\mathrm{D2C7T3}) := \mathrm{D2C7T3}$$

General Definitions: **% Derakane 8084: 25 %Crestomer 1080: 75 Trial: 4 of 4**

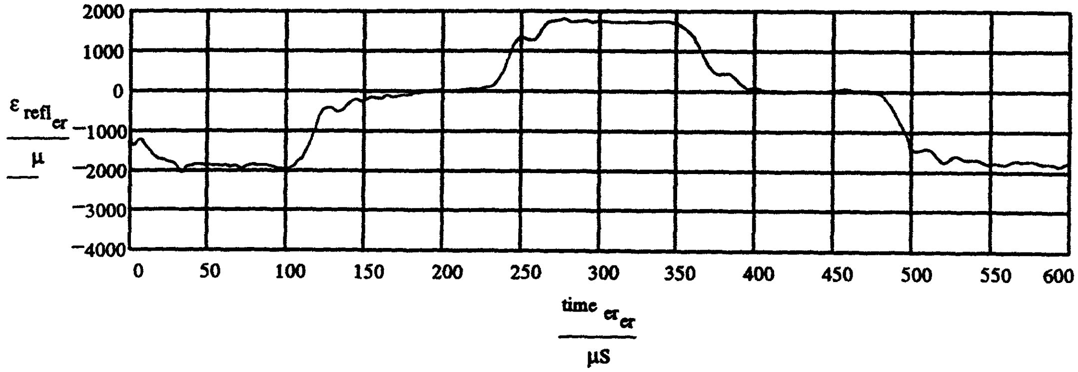
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa \cdot 10^{9} \quad MPa := Pa \cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot GPa \quad D_{bar} := 0.75 \cdot in \quad A_{bar} := \frac{\pi}{4} \cdot D_{bar}^{\;2} \quad A_{bar} = 285.023 \cdot mm^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN}(BD2C7ER1) \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \quad time_{er} := \varepsilon_{reflected}^{<2>} \cdot \sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ **<< set up range >>** $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ **<< apply calibration >>**

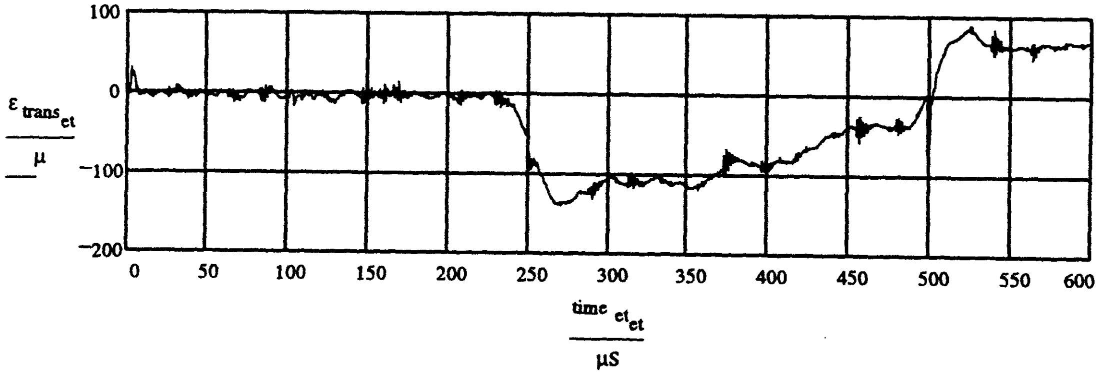


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN}(BD2C7ET1) \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \quad time_{et} := \varepsilon_{transmitted}^{<2>} \cdot \sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ **<< set up range >>** $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ **<< apply calibration >>**

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2100}}$ **<< shift pre-reflected signal to a strain of zero >>**

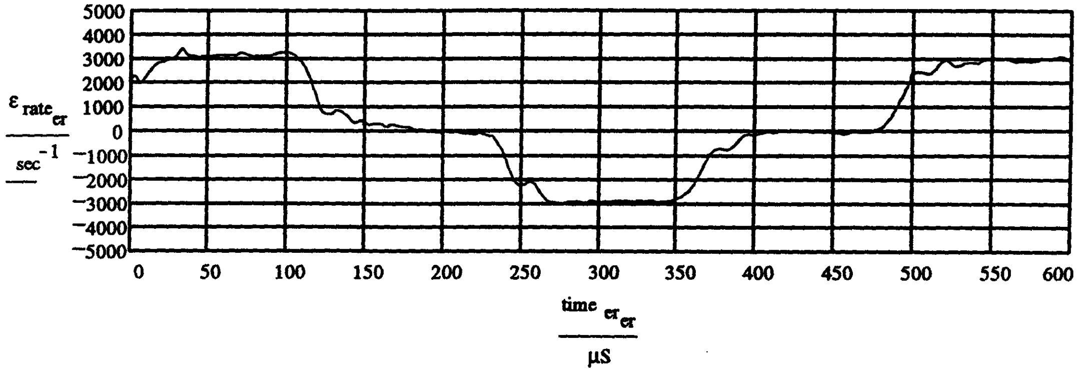


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 5.98 \cdot mm$ $D_0 := 8.19 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

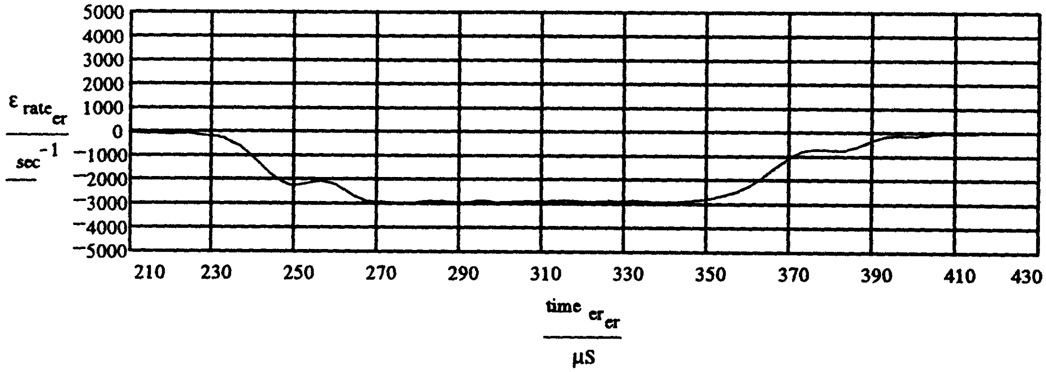


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 210 fin := 430 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\text{start} := \text{start}\cdot 10 \quad \text{fin} := \text{fin}\cdot 10$

$$\varepsilon_{\text{maxeng}} := \sum_{n=\text{start}}^{\text{fin}} \Delta t\cdot\frac{\varepsilon_{\text{rate}_n}+\varepsilon_{\text{rate}_{n+1}}}{2} \qquad \varepsilon_{\text{maxeng}} = -0.366$$

<< max eng. strain >>

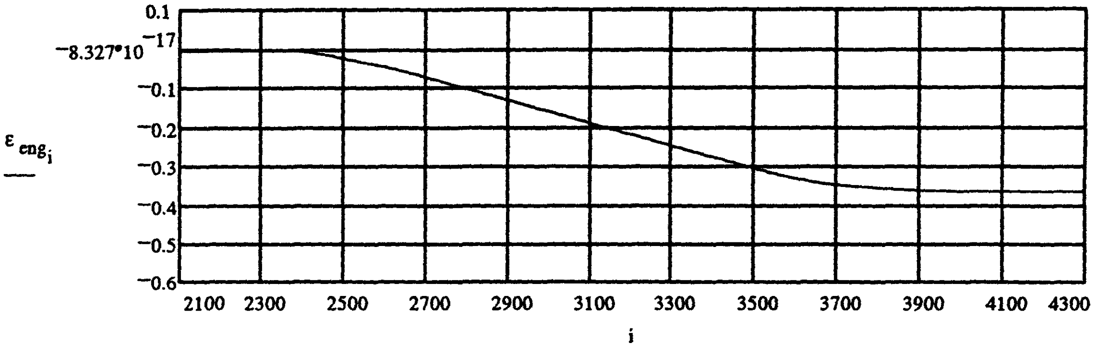
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \text{start}, \text{start} + \text{index} .. \text{fin}$

$$\varepsilon_{\text{eng}_i} := \sum_{n=\text{start}}^{i} \Delta t\cdot\frac{\varepsilon_{\text{rate}_n}+\varepsilon_{\text{rate}_{n+1}}}{2}$$

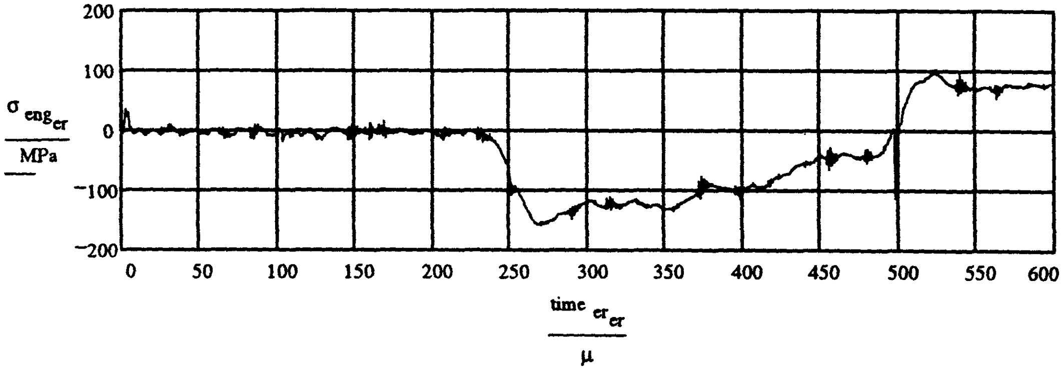
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



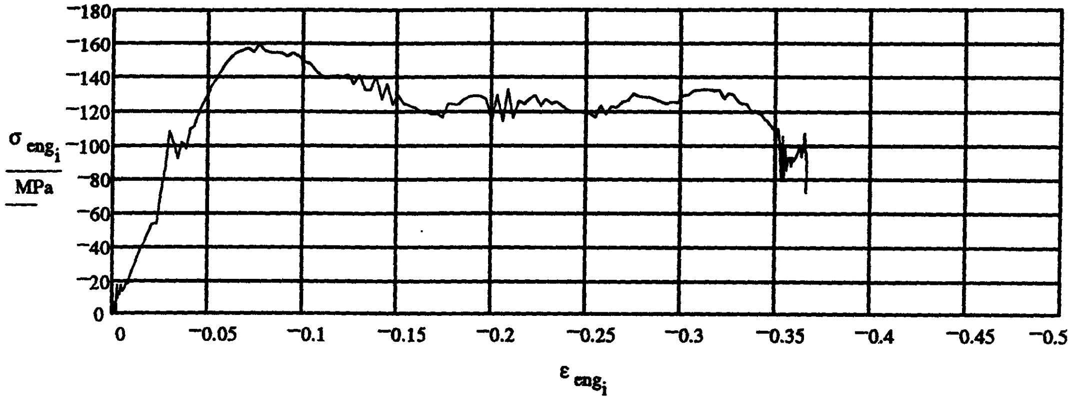
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\text{eng}} := \frac{A_{\text{bar}}}{A_o}\cdot E_{\text{bar}}\cdot\varepsilon_{\text{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index ≡ 10



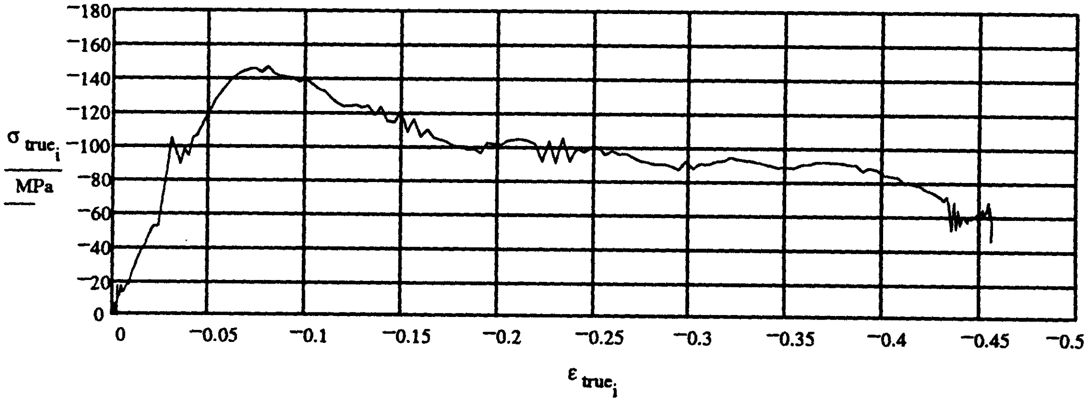
The maximum compresssive stress is: $\min(\sigma_{eng}) = -159.28 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.457$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -147.004 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 25% Dow Chemical Derakane 8084 rubber-toughened vinyl ester and 75% Scott Bader Crestomer 1080. The raw data was obtained on a commmression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0030.WFT is the reflected pulse file.
2. WAVE0031.WFT is the transmitted pulse file.
3. WAVE0032.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D2C7T4} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN}(\mathrm{D2C7T4}) := \mathrm{D2C7T4}$$

General Definitions: **% Derakane 8084: 0 %Crestomer 1080: 100 Trial: 1 of 3**

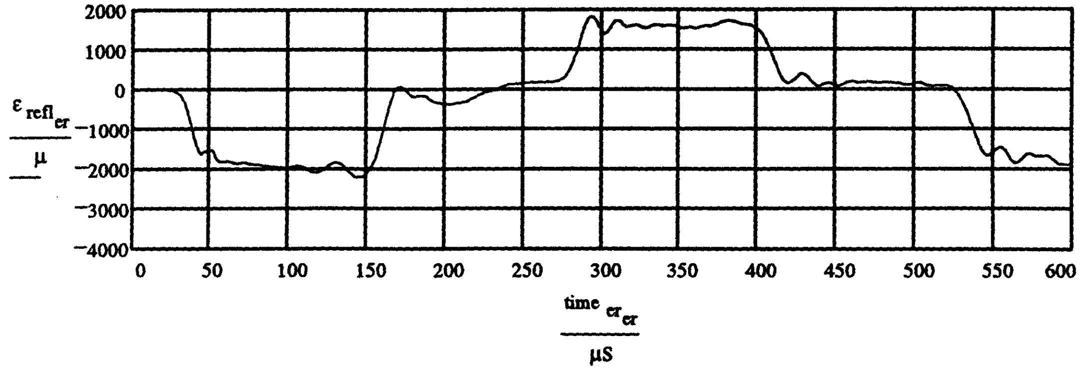
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa \cdot 10^{9} \quad MPa := Pa \cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot GPa \quad D_{bar} := 0.75 \cdot in \quad A_{bar} := \frac{\pi}{4} \cdot {D_{bar}}^2 \quad A_{bar} = 285.023 \cdot mm^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD0C1ER1)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \quad time_{er} := \varepsilon_{reflected}^{<2>} \cdot sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ **<< set up range >>** $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ **<< apply calibration >>**

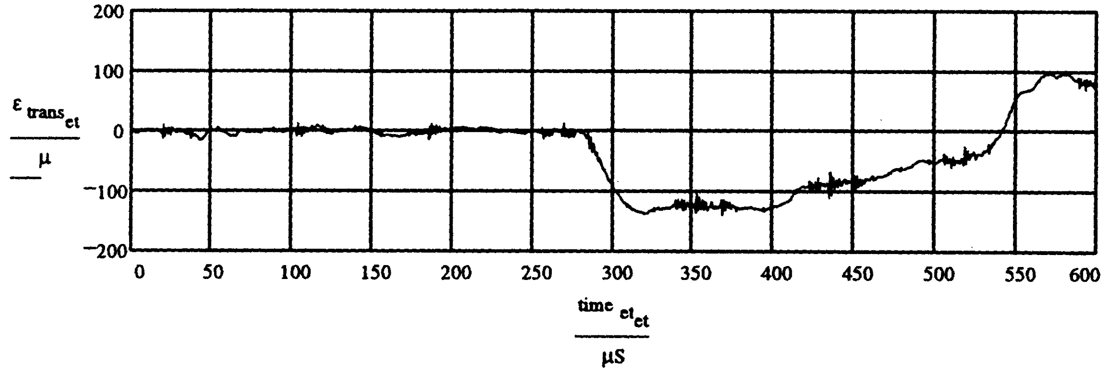


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD0C1ET1)} \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \quad time_{et} := \varepsilon_{transmitted}^{<2>} \cdot sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ **<< set up range >>** $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ **<< apply calibration >>**

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2500}}$ **<< shift pre-reflected signal to a strain of zero >>**

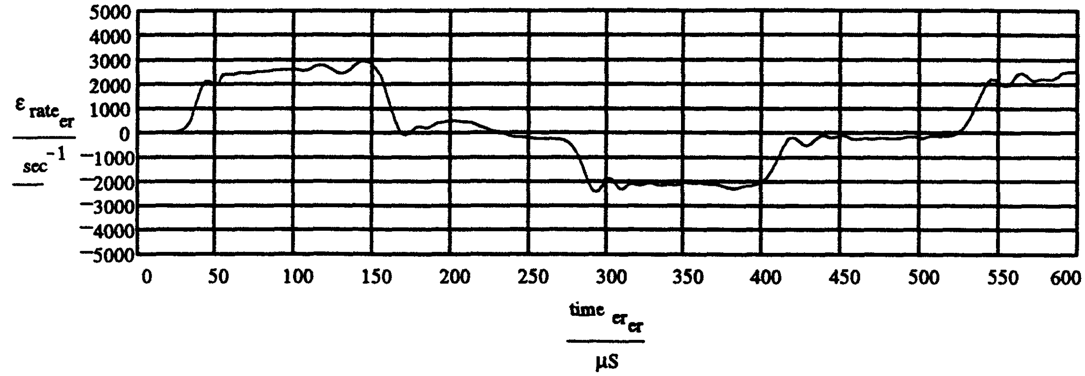


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ **<< longitudinal wave speed for steel >>**

$L_0 := 7.53 \cdot mm$ $D_0 := 10.46 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\ 2}$ **<< inital specimen length, diameter, and Acs >>**

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

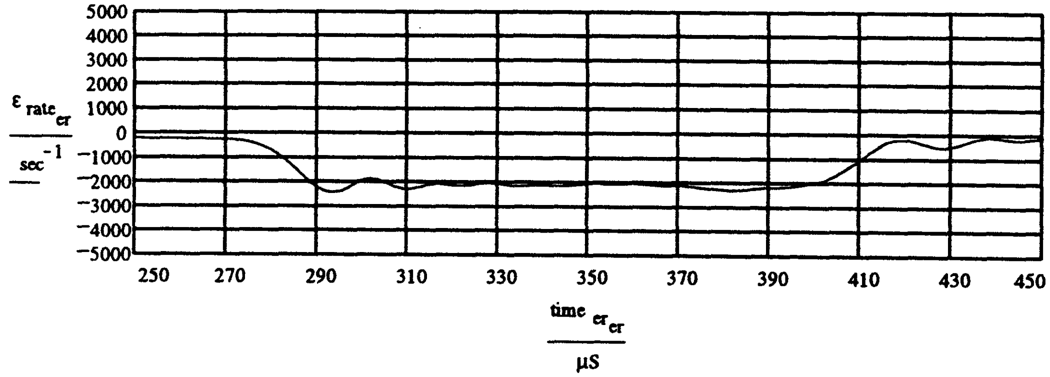


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 250 fin := 450 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\mathrm{start} := \mathrm{start} \cdot 10 \quad \mathrm{fin} := \mathrm{fin} \cdot 10$

$$\varepsilon_{maxeng} := \sum_{n = start}^{fin} \Delta t \cdot \frac{\varepsilon_{rate_{n}} + \varepsilon_{rate_{n+1}}}{2} \qquad \varepsilon_{maxeng} = -0.287$$

<< max eng. strain >>

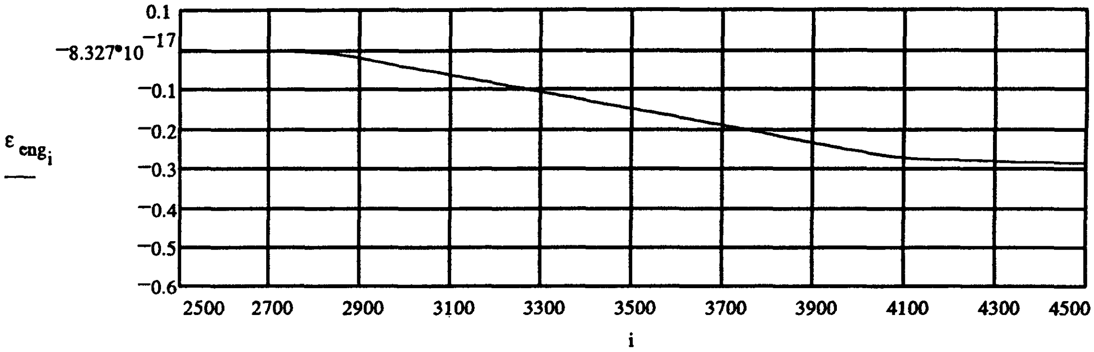
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \mathrm{start}, \mathrm{start} + \mathrm{index} .. \mathrm{fin}$

$$\varepsilon_{eng_{i}} := \sum_{n = start}^{i} \Delta t \cdot \frac{\varepsilon_{rate_{n}} + \varepsilon_{rate_{n+1}}}{2}$$

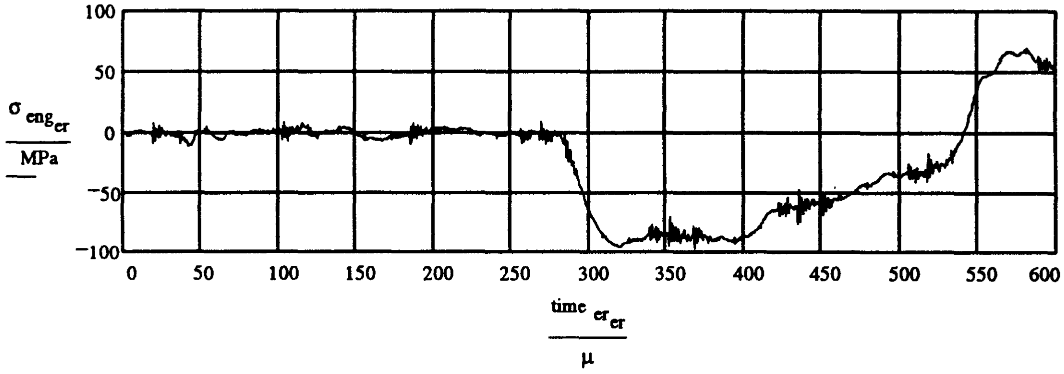
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



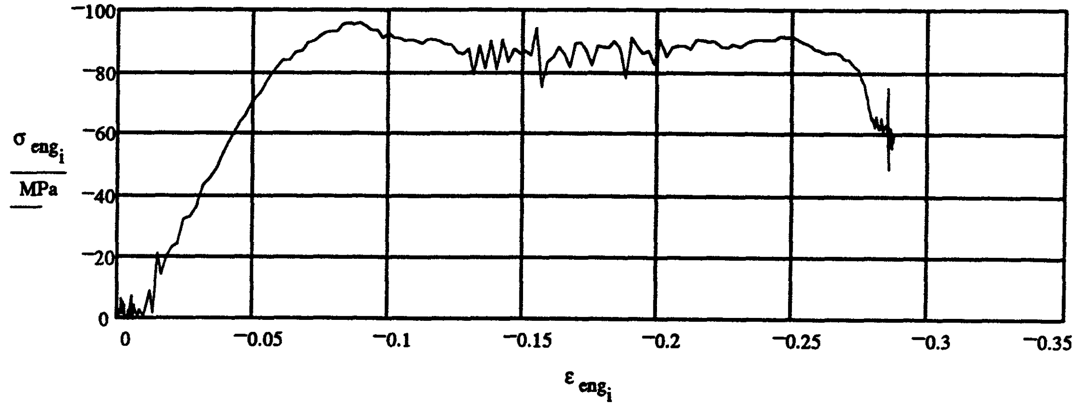
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{eng} := \frac{A_{bar}}{A_{o}} \cdot E_{bar} \cdot \varepsilon_{trans}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \dfrac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index ≡ 10



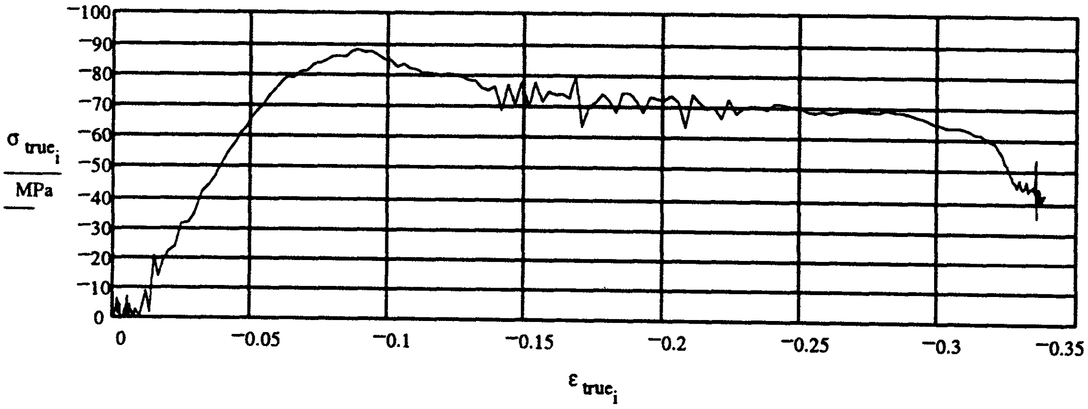
The maximum compresssive stress is: $\min(\sigma_{eng}) = -97.294 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.339$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -87.72 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 100% Scott Bader Crestomer 1080. The raw data was obtained on a commmpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0078.WFT is the reflected pulse file.
2. WAVE0079.WFT is the transmitted pulse file.
3. WAVE0080.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D0C1T1} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D0C1T1)} := \mathrm{D0C1T1}$$

General Definitions: **% Derakane 8084: 0 %Crestomer 1080: 100 Trial: 2 of 3**

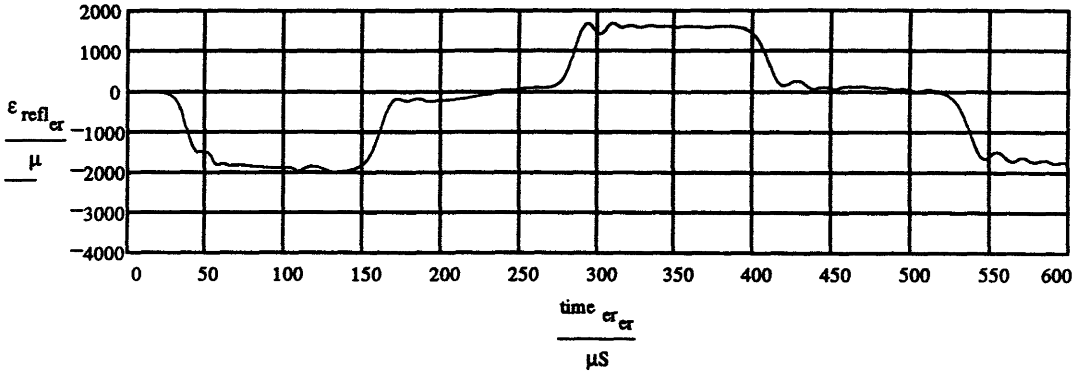
$ORIGIN := 1 \quad \mu S := sec \cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa \cdot 10^{9} \quad MPa := Pa \cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210 \cdot GPa \quad D_{bar} := 0.75 \cdot in \quad A_{bar} := \frac{\pi}{4} \cdot D_{bar}{}^{2} \quad A_{bar} = 285.023 \cdot mm^{2}$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := READPRN(HD0C1ER2) \quad \varepsilon_{refl} := \varepsilon_{reflected}{}^{<1>} \quad time_{er} := \varepsilon_{reflected}{}^{<2>} \cdot sec$

$er := 1 .. rows(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4 \cdot \varepsilon_{refl}}{GF \cdot V_O}$ << apply calibration >>

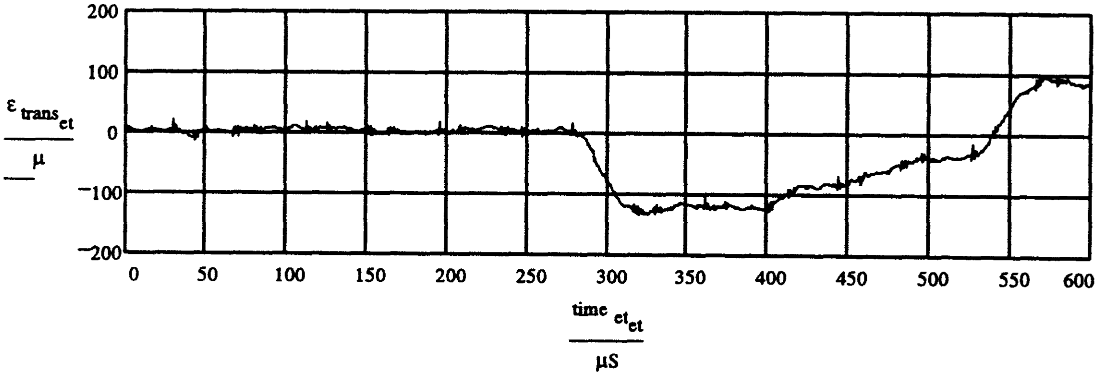


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := READPRN(HD0C1ET2) \quad \varepsilon_{trans} := \varepsilon_{transmitted}{}^{<1>} \quad time_{et} := \varepsilon_{transmitted}{}^{<2>} \cdot sec$

$et := 1 .. rows(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4 \cdot \varepsilon_{trans}}{GF \cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2500}}$ << shift pre-reflected signal to a strain of zero >>

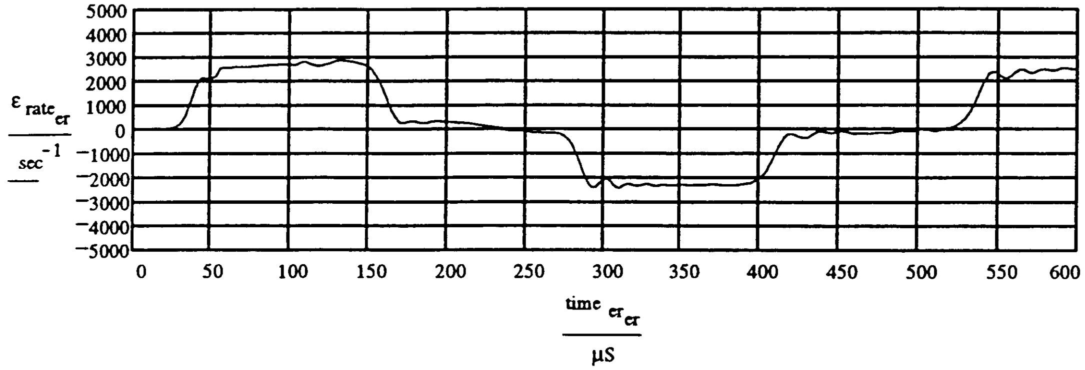


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ << longitudinal wave speed for steel >>

$L_0 := 6.99 \cdot mm$ $D_0 := 10.46 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{\;2}$ << inital specimen length, diameter, and Acs >>

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

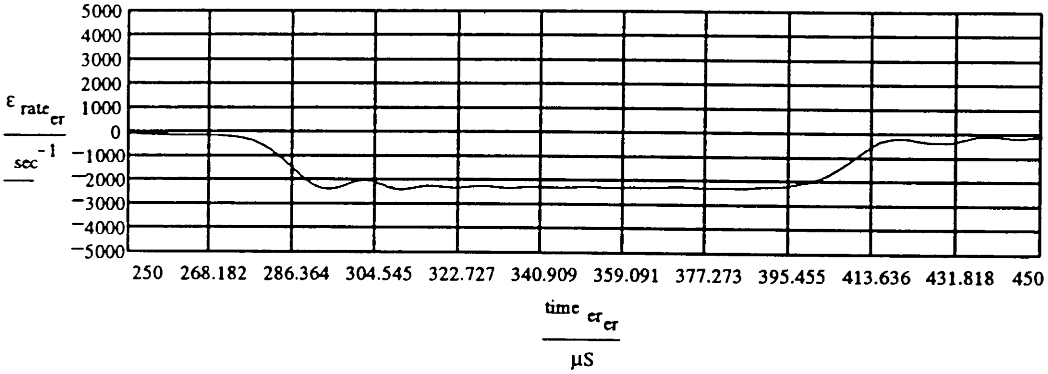


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 250 fin := 450 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\text{start} := \text{start}\cdot 10 \qquad \text{fin} := \text{fin}\cdot 10$

$$\varepsilon_{\text{maxeng}} := \sum_{n=\text{start}}^{\text{fin}} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2} \qquad \varepsilon_{\text{maxeng}} = -0.297$$

<< max eng. strain >>

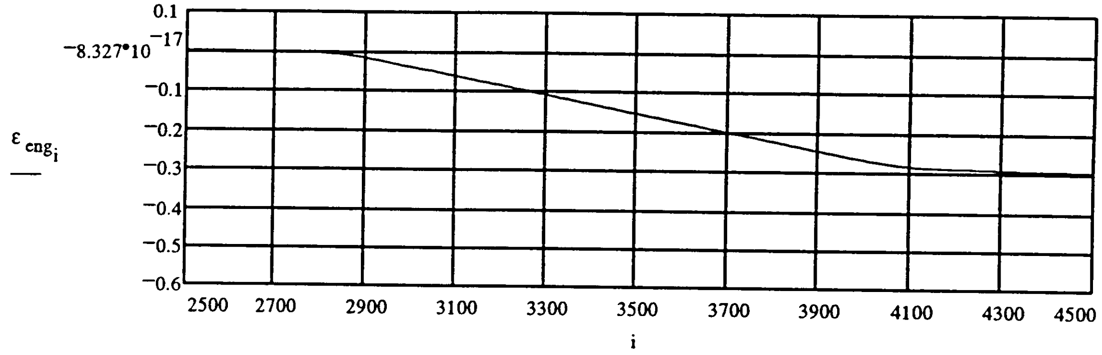
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \text{start}, \text{start} + \text{index} .. \text{fin}$

$$\varepsilon_{\text{eng}_i} := \sum_{n=\text{start}}^{i} \Delta t \cdot \frac{\varepsilon_{\text{rate}_n} + \varepsilon_{\text{rate}_{n+1}}}{2}$$

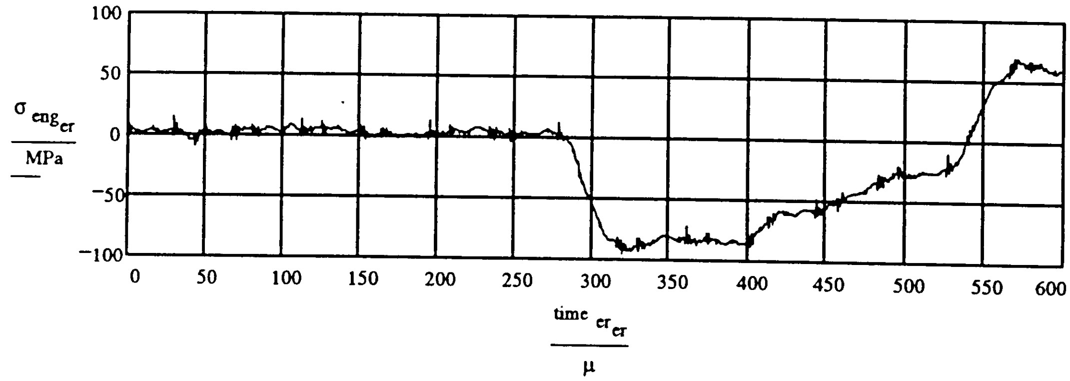
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



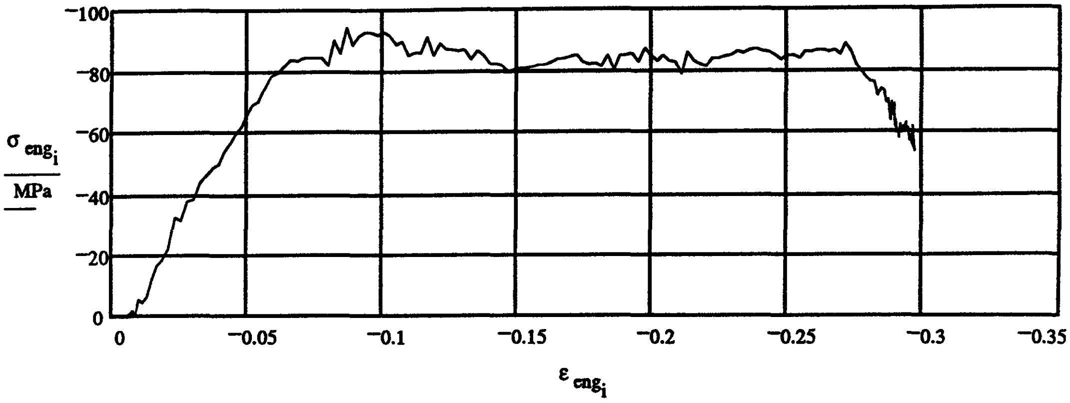
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\text{eng}} := \frac{A_{\text{bar}}}{A_o} \cdot E_{\text{bar}} \cdot \varepsilon_{\text{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i}$ index≡10



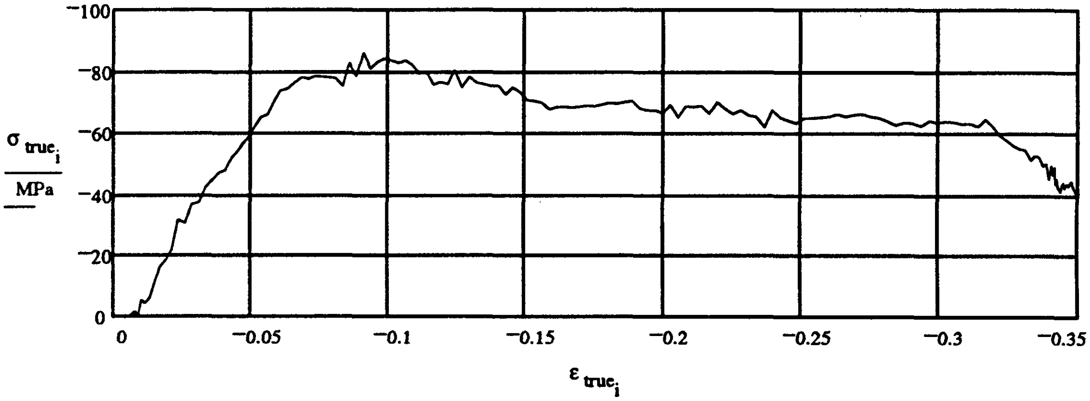
The maximum compresssive stress is: $\min(\sigma_{eng}) = -95.172 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.352$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -85.937 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 100% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0081.WFT is the reflected pulse file.
2. WAVE0082.WFT is the transmitted pulse file.
3. WAVE0083.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D0C1T2} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN}(\mathrm{D0C1T2}) := \mathrm{D0C1T2}$$

General Definitions: **% Derakane 8084: 0 %Crestomer 1080: 100 Trial: 3 of 3**

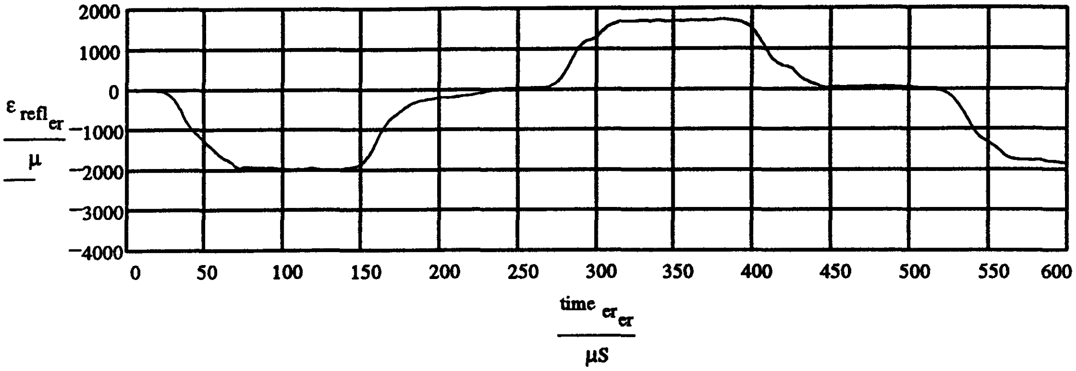
$\mathrm{ORIGIN} := 1 \quad \mu S := \sec\cdot 10^{-6} \quad \mu := 10^{-6} \quad GPa := Pa\cdot 10^{9} \quad MPa := Pa\cdot 10^{6}$

$GF := 2.1 \quad V_O := 30 \quad E_{bar} := 210\cdot GPa \quad D_{bar} := 0.75\cdot in \quad A_{bar} := \frac{\pi}{4}\cdot D_{bar}^{\ 2} \quad A_{bar} = 285.023\cdot mm^2$

Read and calibrate reflected strain file:

$\varepsilon_{reflected} := \mathrm{READPRN(HD0C1ER3)} \quad \varepsilon_{refl} := \varepsilon_{reflected}^{<1>} \quad time_{er} := \varepsilon_{reflected}^{<2>}\cdot sec$

$er := 1 .. \mathrm{rows}(\varepsilon_{refl})$ << set up range >> $\varepsilon_{refl} := \frac{4\cdot\varepsilon_{refl}}{GF\cdot V_O}$ << apply calibration >>

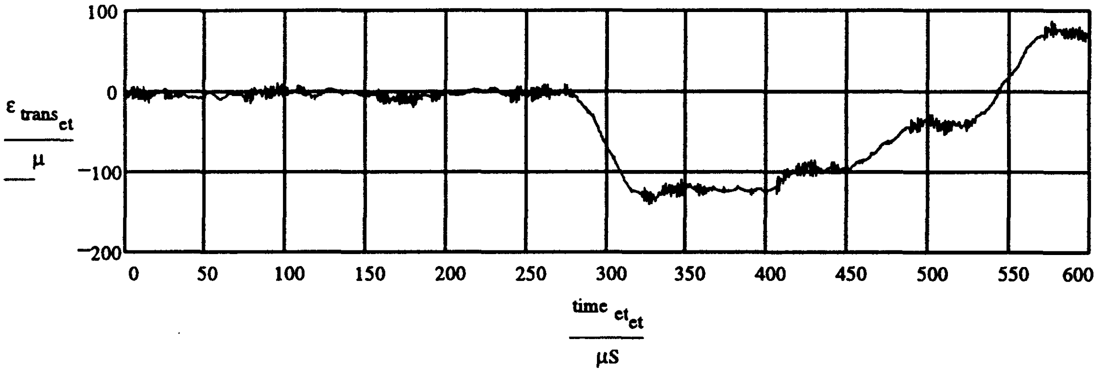


Read and calibrate transmitted strain file:

$\varepsilon_{transmitted} := \mathrm{READPRN(HD0C1ET3)} \quad \varepsilon_{trans} := \varepsilon_{transmitted}^{<1>} \quad time_{et} := \varepsilon_{transmitted}^{<2>}\cdot sec$

$et := 1 .. \mathrm{rows}(\varepsilon_{trans})$ << set up range >> $\varepsilon_{trans} := \frac{4\cdot\varepsilon_{trans}}{GF\cdot V_O}$ << apply calibration >>

$\varepsilon_{trans} := \varepsilon_{trans} - \varepsilon_{trans_{2500}}$ << shift pre-reflected signal to a strain of zero >>

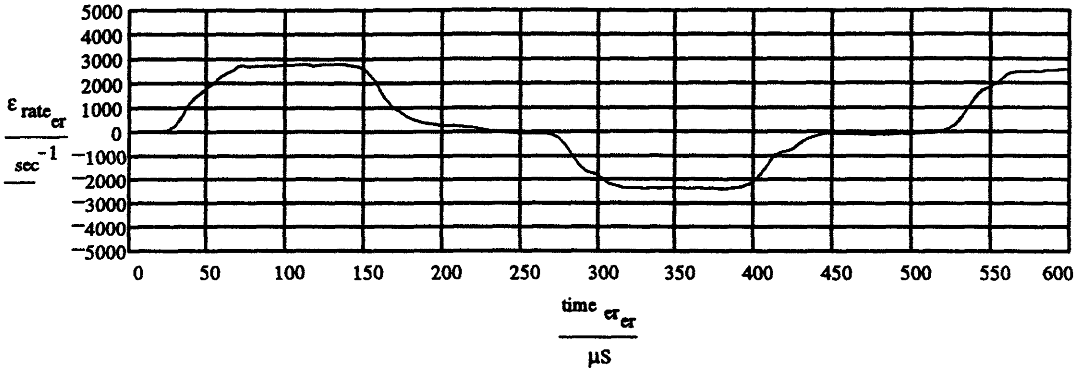


Calculate the strain rate in the sample:

$C_0 := 5000 \cdot \frac{m}{sec}$ **<< longitudinal wave speed for steel >>**

$L_0 := 7.15 \cdot mm$ $D_0 := 10.18 \cdot mm$ $A_0 := \frac{\pi}{4} \cdot D_0^{2}$ **<< inital specimen length, diameter, and Acs >>**

$$\varepsilon_{rate_{er}} := \frac{-2 \cdot C_0}{L_0} \cdot \varepsilon_{refl_{er}}$$

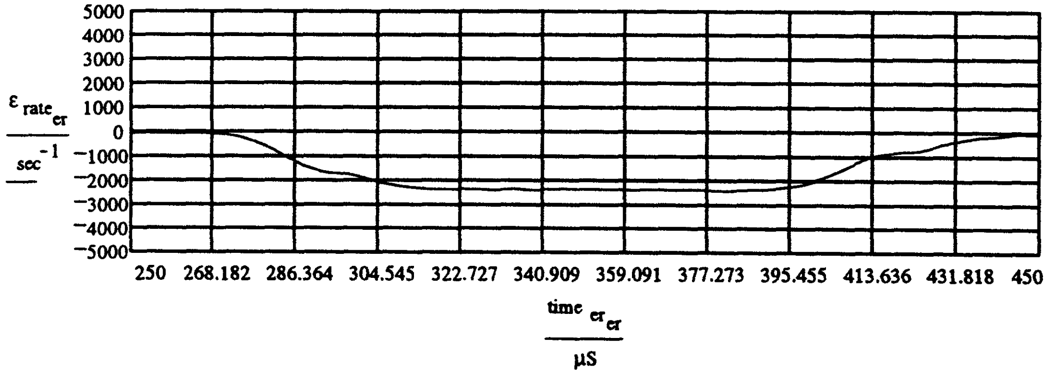


Calculate the strain in the sample:

Input start and finish time of reflected pulse as well as the time interval for numerical integration in units of microseconds:

start := 250 fin := 450 $\Delta t := 0.1 \cdot \mu S$

Plot the selected reflected pulse below:



Convert above times to vector indices (data collected at 0.1μS intervals) and calculate max strain:

$\mathrm{start} := \mathrm{start}\cdot 10 \qquad \mathrm{fin} := \mathrm{fin}\cdot 10$

$$\varepsilon_{\mathrm{maxeng}} := \sum_{n=\mathrm{start}}^{\mathrm{fin}} \Delta t\cdot\frac{\varepsilon_{\mathrm{rate}_n}+\varepsilon_{\mathrm{rate}_{n+1}}}{2} \qquad \varepsilon_{\mathrm{maxeng}} = -0.301$$

<< max eng. strain >>

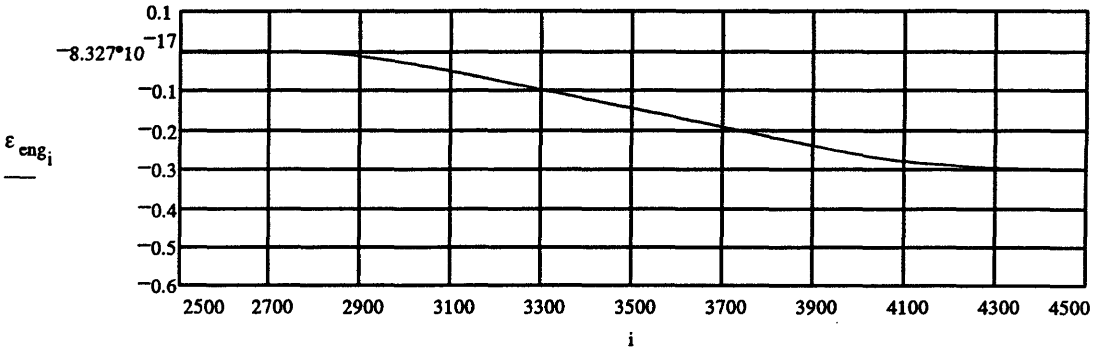
Calculate the engineering strain vector by numerical integration of the strain rate:

$i := \mathrm{start}, \mathrm{start}+\mathrm{index}\,..\,\mathrm{fin}$

$$\varepsilon_{\mathrm{eng}_i} := \sum_{n=\mathrm{start}}^{i} \Delta t\cdot\frac{\varepsilon_{\mathrm{rate}_n}+\varepsilon_{\mathrm{rate}_{n+1}}}{2}$$

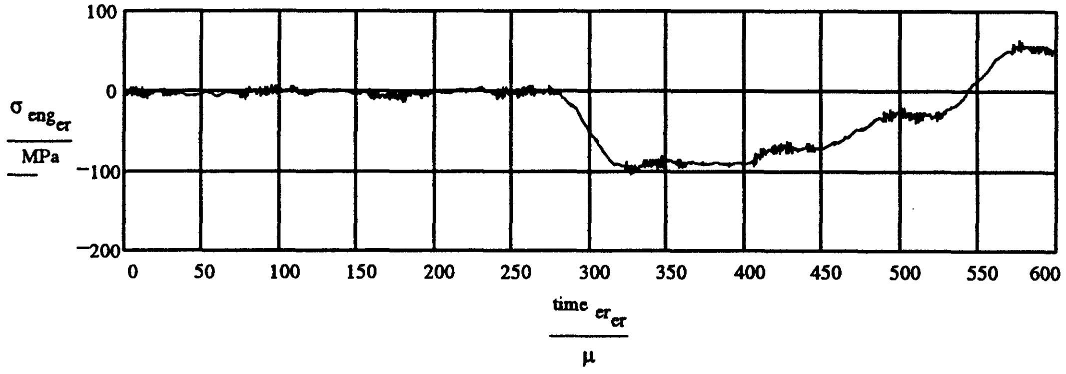
<< numerical integration by rectangle rule >>

Plot the engineering strain with respect to the selected time index (i). Each time index (i) represents 0.1μS:



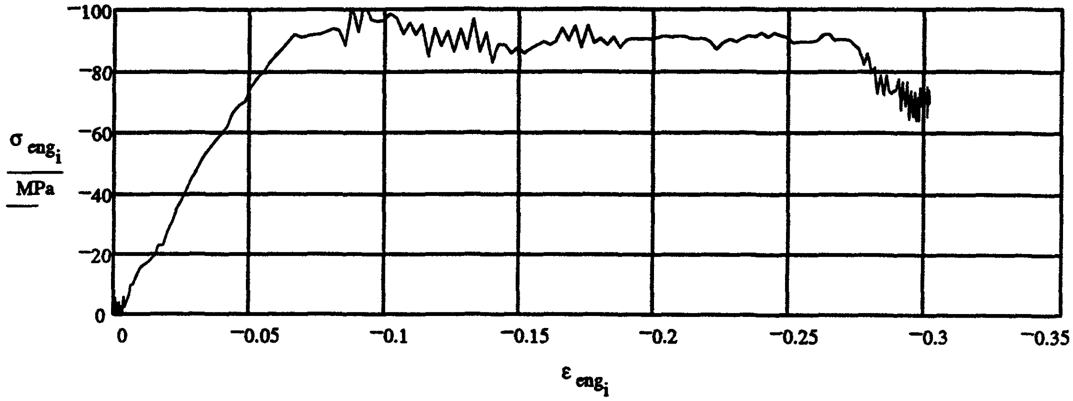
Calculate the axial compressive engineering stress in the sample:

$$\sigma_{\mathrm{eng}} := \frac{A_{\mathrm{bar}}}{A_{\mathrm{o}}}\cdot E_{\mathrm{bar}}\cdot\varepsilon_{\mathrm{trans}}$$



Using the same index (i) from the engineering strain calculation, draw a plot of engineering stress versus engineering strain:

$$\sigma_{eng_i} := \frac{A_{bar}}{A_o} \cdot E_{bar} \cdot \varepsilon_{trans_i} \qquad \text{index} \equiv 10$$



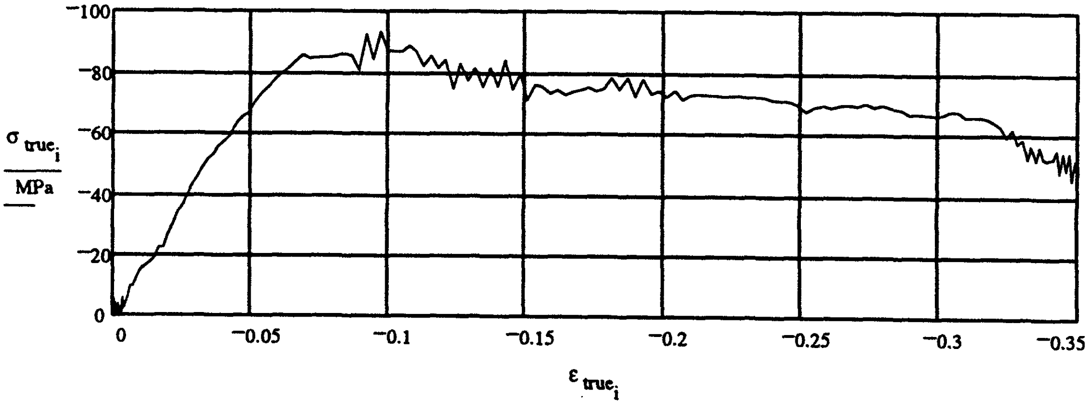
The maximum compresssive stress is: $\min(\sigma_{eng}) = -103.841 \cdot MPa$

Calculate true (logaritmic) strain in the specimen:

$\varepsilon_{true_i} := \ln(\varepsilon_{eng_i} + 1)$ $\min(\varepsilon_{true}) = -0.359$ << maximum compressive true strain >>

Calculate true stress in the specimen:

$$\sigma_{true_i} := \sigma_{eng_i} \cdot (\varepsilon_{eng_i} + 1)$$



The maximum compresssive true stress is: $\min(\sigma_{true}) = -93.227 \cdot MPa$

Supplemental note: This stress strain curve represents a specimen composed of 100% Scott Bader Crestomer 1080. The raw data was obtained on a commpression split Hopkinson bar located at the California Institute of Technology in a testing session from 4/20/95 to 4/21/95. The temperature at the time of the test was 71.6 F and the relative humidity was 32.3%. The raw data files have been archived in compressed form under the following names:

1. WAVE0084.WFT is the reflected pulse file.
2. WAVE0085.WFT is the transmitted pulse file.
3. WAVE0086.WFT is the measured temperature file.

Write true stress and strain data to a .prn file:

$$\mathrm{D0C1T3} := \mathrm{augment}\left(\varepsilon_{\mathrm{true}}, \frac{\sigma_{\mathrm{true}}}{\mathrm{MPa}}\right)$$

$$\mathrm{WRITEPRN(D0C1T3)} := \mathrm{D0C1T3}$$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..4$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP1B})$ $\quad$ $\mathrm{SPEC}_A := 3$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A}\cdot\mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A}\cdot\mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP1A})$ $\quad$ $\mathrm{SPEC}_B := 24$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B}\cdot\mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B}\cdot\mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP1B})$ $\quad$ $\mathrm{SPEC}_C := 5$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C}\cdot\mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C}\cdot\mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP1B})$ $\quad$ $\mathrm{SPEC}_D := 11$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1\rangle}\right)_{\mathrm{SPEC}_D}\cdot\mathrm{mm}$ $\quad$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2\rangle}\right)_{\mathrm{SPEC}_D}\cdot\mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4}\cdot\left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 191.38 \\ 192.854 \\ 193.346 \\ 189.911 \end{bmatrix}\cdot\mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC1LR1T8})$

$B := \mathrm{READPRN}(\mathrm{IC1LR2T5})$

$C := \mathrm{READPRN}(\mathrm{IC1LR3T4})$

$D := \mathrm{READPRN}(\mathrm{IC1LR4T3})$

Note: Files A-D are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{\langle 1 \rangle} \cdot kN$ $displ_A := A^{\langle 2 \rangle} \cdot mm$ $load_D := D^{\langle 1 \rangle} \cdot kN$ $displ_D := D^{\langle 2 \rangle} \cdot mm$

$load_B := B^{\langle 1 \rangle} \cdot kN$ $displ_B := B^{\langle 2 \rangle} \cdot mm$

$load_C := C^{\langle 1 \rangle} \cdot kN$ $displ_C := C^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility):

$MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$L_{iA_a} := L_{o_1} - displ_{A_a}$ $A_{iA_a} := \dfrac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}}$ $L_{iB_b} := L_{o_2} - displ_{B_b}$ $A_{iB_b} := \dfrac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$

$L_{iC_c} := L_{o_3} - displ_{C_c}$ $A_{iC_c} := \dfrac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}}$ $L_{iD_d} := L_{o_4} - displ_{D_d}$ $A_{iD_d} := \dfrac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$

Calculate true stress (MPa) and true strain:

$\sigma_{trueA_a} := \dfrac{load_{A_a}}{A_{iA_a}}$ $\sigma_{trueB_b} := \dfrac{load_{B_b}}{A_{iB_b}}$ $\sigma_{trueC_c} := \dfrac{load_{C_c}}{A_{iC_c}}$ $\sigma_{trueD_d} := \dfrac{load_{D_d}}{A_{iD_d}}$

$\varepsilon_{trueA_a} := -\ln\left(\dfrac{L_{iA_a}}{L_{o_1}}\right)$ $\varepsilon_{trueB_b} := -\ln\left(\dfrac{L_{iB_b}}{L_{o_2}}\right)$ $\varepsilon_{trueC_c} := -\ln\left(\dfrac{L_{iC_c}}{L_{o_3}}\right)$ $\varepsilon_{trueD_d} := -\ln\left(\dfrac{L_{iD_d}}{L_{o_4}}\right)$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

Read the stress strain curve data from the Hopkinson Bar analysis:

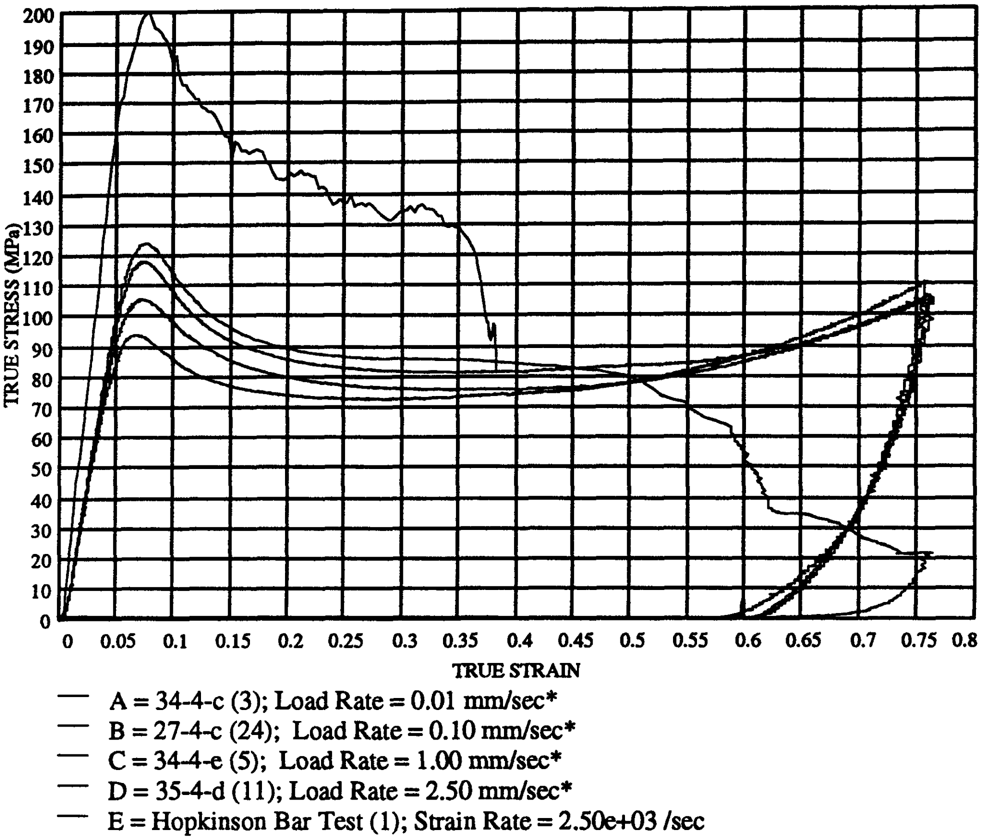
$i := 1800, 1810 .. 4000$

$TRIAL := READPRN(D1C0T2)$

$\varepsilon_{trueh} := (-TRIAL)^{<1>} \quad \sigma_{trueh} := (-TRIAL)^{<2>} \cdot MPa$

$\sigma_{yh} := \max\left(\sigma_{trueh}\right) \quad NumH_i := \mathrm{if}\left(\sigma_{trueh_i} = \sigma_{yh}, i, 0\right) \quad \varepsilon_{yh} := \varepsilon_{trueh_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for 100% 8084 / 0% 1080 at various load rates:



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate/initial specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right)=0.759$ $\max\left(\sigma_{\mathrm{trueA}}\right)=110.58\cdot\mathrm{MPa}$ $\sigma_{\mathrm{yA}}=93.767\cdot\mathrm{MPa}$ $\varepsilon_{\mathrm{yA}}=0.065$

$\max\left(\varepsilon_{\mathrm{trueB}}\right)=0.761$ $\max\left(\sigma_{\mathrm{trueB}}\right)=106.084\cdot\mathrm{MPa}$ $\sigma_{\mathrm{yB}}=105.442\cdot\mathrm{MPa}$ $\varepsilon_{\mathrm{yB}}=0.071$

$\max\left(\varepsilon_{\mathrm{trueC}}\right)=0.763$ $\max\left(\sigma_{\mathrm{trueC}}\right)=117.63\cdot\mathrm{MPa}$ $\sigma_{\mathrm{yC}}=117.63\cdot\mathrm{MPa}$ $\varepsilon_{\mathrm{yC}}=0.073$

$\max\left(\varepsilon_{\mathrm{trueD}}\right)=0.763$ $\max\left(\sigma_{\mathrm{trueD}}\right)=123.683\cdot\mathrm{MPa}$ $\sigma_{\mathrm{yD}}=123.683\cdot\mathrm{MPa}$ $\varepsilon_{\mathrm{yD}}=0.073$

$\max\left(\varepsilon_{\mathrm{trueh}}\right)=0.382$ $\max\left(\sigma_{\mathrm{trueh}}\right)=200\cdot\mathrm{MPa}$ $\sigma_{\mathrm{yh}}=200\cdot\mathrm{MPa}$ $\varepsilon_{\mathrm{yh}}=0.077$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..4$ ORIGIN := 1

$DIM_A := READPRN(COMP2B)$ $SPEC_A := 2$

$L_{o_1} := \left(DIM_A^{<1>}\right)_{SPEC_A} \cdot mm$ $D_{o_1} := \left(DIM_A^{<2>}\right)_{SPEC_A} \cdot mm$

$DIM_B := READPRN(COMP2B)$ $SPEC_B := 6$

$L_{o_2} := \left(DIM_B^{<1>}\right)_{SPEC_B} \cdot mm$ $D_{o_2} := \left(DIM_B^{<2>}\right)_{SPEC_B} \cdot mm$

$DIM_C := READPRN(COMP2B)$ $SPEC_C := 8$

$L_{o_3} := \left(DIM_C^{<1>}\right)_{SPEC_C} \cdot mm$ $D_{o_3} := \left(DIM_C^{<2>}\right)_{SPEC_C} \cdot mm$

$DIM_D := READPRN(COMP2B)$ $SPEC_D := 12$

$L_{o_4} := \left(DIM_D^{<1>}\right)_{SPEC_D} \cdot mm$ $D_{o_4} := \left(DIM_D^{<2>}\right)_{SPEC_D} \cdot mm$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 183.134 \\ 183.614 \\ 185.782 \\ 186.023 \end{bmatrix} \cdot mm^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

A := READPRN(IC2LR1T6)
B := READPRN(IC2LR2T4)
C := READPRN(IC2LR3T5)
D := READPRN(IC2LR4T5)

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{<1>} \cdot kN$ $displ_A := A^{<2>} \cdot mm$ $load_D := D^{<1>} \cdot kN$ $displ_D := D^{<2>} \cdot mm$

$load_B := B^{<1>} \cdot kN$ $displ_B := B^{<2>} \cdot mm$

$load_C := C^{<1>} \cdot kN$ $displ_C := C^{<2>} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \quad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \quad L_{iB_b} := L_{o_2} - displ_{B_b} \quad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \quad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \quad L_{iD_d} := L_{o_4} - displ_{D_d} \quad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \quad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \quad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \quad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \quad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \quad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \quad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

Read the stress strain curve data from the Hopkinson Bar analysis:

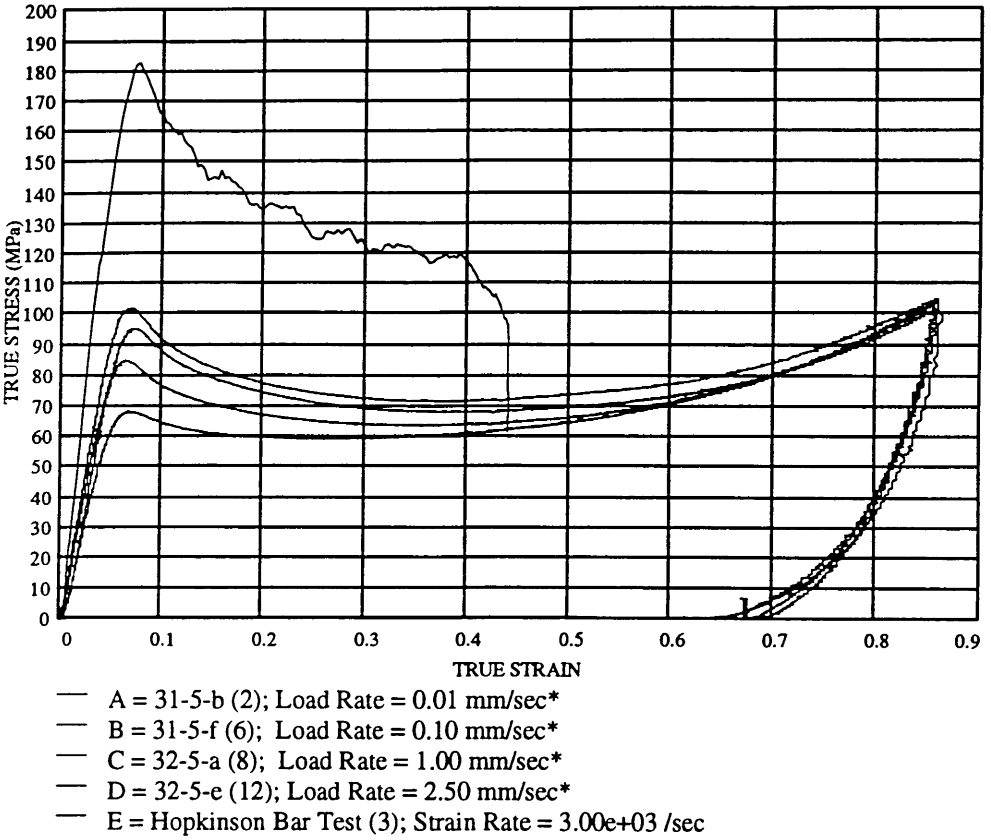
$i := 2100, 2110 .. 4300$

$TRIAL := READPRN(D7C2T3)$

$\varepsilon_{trueh} := (-TRIAL)^{\langle 1 \rangle} \quad \sigma_{trueh} := (-TRIAL)^{\langle 2 \rangle} \cdot MPa$

$\sigma_{yh} := \max\left(\sigma_{trueh}\right) \quad NumH_i := \mathrm{if}\left(\sigma_{trueh_i} = \sigma_{yh}, i, 0\right) \quad \varepsilon_{yh} := \varepsilon_{trueh_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for 75% 8084 / 25% 1080 at various load rates:



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{\mathrm{trueA}}) = 0.855$ $\max(\sigma_{\mathrm{trueA}}) = 103.201 \cdot \mathrm{MPa}$ $\sigma_{yA} = 67.896 \cdot \mathrm{MPa}$ $\varepsilon_{yA} = 0.066$

$\max(\varepsilon_{\mathrm{trueB}}) = 0.86$ $\max(\sigma_{\mathrm{trueB}}) = 104.253 \cdot \mathrm{MPa}$ $\sigma_{yB} = 84.366 \cdot \mathrm{MPa}$ $\varepsilon_{yB} = 0.064$

$\max(\varepsilon_{\mathrm{trueC}}) = 0.862$ $\max(\sigma_{\mathrm{trueC}}) = 101.544 \cdot \mathrm{MPa}$ $\sigma_{yC} = 94.798 \cdot \mathrm{MPa}$ $\varepsilon_{yC} = 0.071$

$\max(\varepsilon_{\mathrm{trueD}}) = 0.864$ $\max(\sigma_{\mathrm{trueD}}) = 104.907 \cdot \mathrm{MPa}$ $\sigma_{yD} = 101.388 \cdot \mathrm{MPa}$ $\varepsilon_{yD} = 0.07$

$\max(\varepsilon_{\mathrm{trueh}}) = 0.44$ $\max(\sigma_{\mathrm{trueh}}) = 182.4 \cdot \mathrm{MPa}$ $\sigma_{yh} = 182.4 \cdot \mathrm{MPa}$ $\varepsilon_{yh} = 0.077$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1 .. 4$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP3B})$ $\mathrm{SPEC}_A := 2$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP3B})$ $\mathrm{SPEC}_B := 6$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP3B})$ $\mathrm{SPEC}_C := 9$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP3B})$ $\mathrm{SPEC}_D := 13$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1 \rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2 \rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{cso} = \begin{bmatrix} 181.697 \\ 184.335 \\ 183.614 \\ 183.134 \end{bmatrix} \cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC3LR1T6})$

$B := \mathrm{READPRN}(\mathrm{IC3LR2T7})$

$C := \mathrm{READPRN}(\mathrm{IC3LR3T5})$

$D := \mathrm{READPRN}(\mathrm{IC3LR4T6})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{<1>} \cdot kN$ $displ_A := A^{<2>} \cdot mm$ $load_D := D^{<1>} \cdot kN$ $displ_D := D^{<2>} \cdot mm$

$load_B := B^{<1>} \cdot kN$ $displ_B := B^{<2>} \cdot mm$

$load_C := C^{<1>} \cdot kN$ $displ_C := C^{<2>} \cdot mm$

Look at matrices and calculate number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility):

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$MPa := 1 \cdot 10^6 \cdot Pa$

$$L_{iA_a} := L_{o_1} - displ_{A_a} \quad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \quad L_{iB_b} := L_{o_2} - displ_{B_b} \quad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \quad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \quad L_{iD_d} := L_{o_4} - displ_{D_d} \quad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \quad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \quad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \quad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \quad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \quad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \quad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right)$ $\sigma_{yD} := \max\left(\sigma_{yDcalc}\right)$ $NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$ $\varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

Read the stress strain curve data from the Hopkinson Bar analysis:

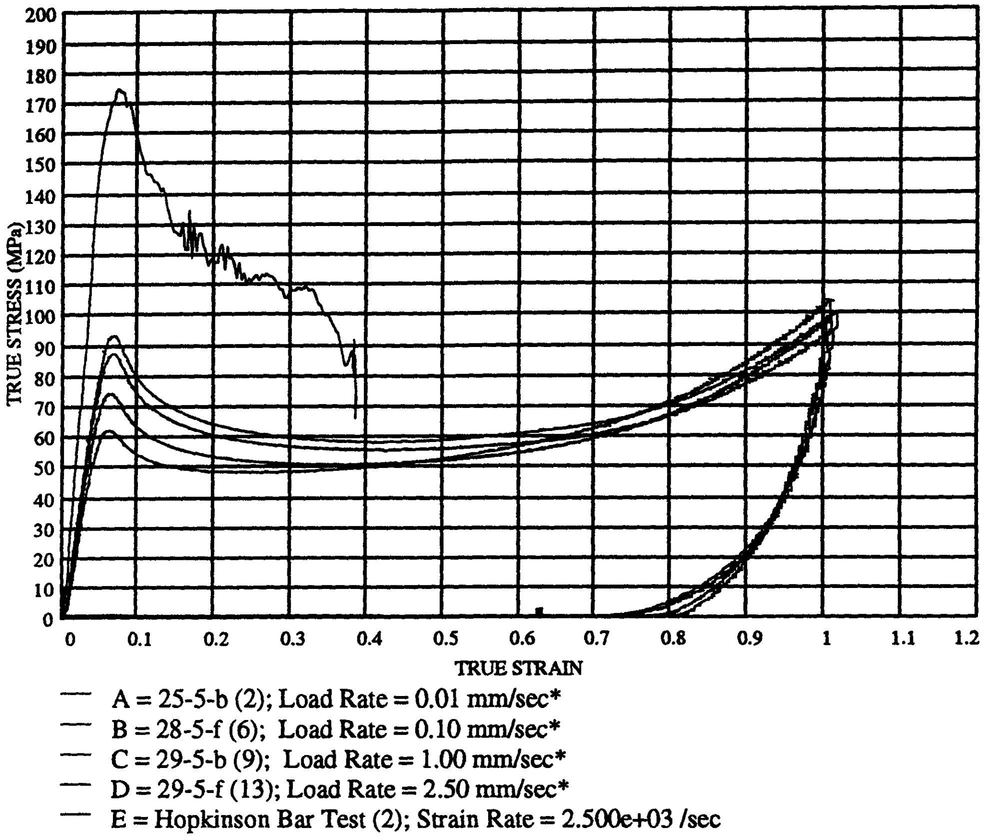
$i := 2650, 2660 .. 4600$

$TRIAL := READPRN(D5C5T2)$

$\varepsilon_{trueh} := (-TRIAL)^{\langle 1 \rangle}$ $\sigma_{trueh} := (-TRIAL)^{\langle 2 \rangle} \cdot MPa$

$\sigma_{yh} := \max\left(\sigma_{trueh}\right)$ $NumH_i := \mathrm{if}\left(\sigma_{trueh_i} = \sigma_{yh}, i, 0\right)$ $\varepsilon_{yh} := \varepsilon_{trueh_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for 50% 8084 / 50% 1080 at various load rates:



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\text{trueA}}\right) = 1.012$	$\max\left(\sigma_{\text{trueA}}\right) = 104.044\cdot\text{MPa}$	$\sigma_{\text{yA}} = 62.089\cdot\text{MPa}$	$\varepsilon_{\text{yA}} = 0.059$
$\max\left(\varepsilon_{\text{trueB}}\right) = 1.01$	$\max\left(\sigma_{\text{trueB}}\right) = 98.384\cdot\text{MPa}$	$\sigma_{\text{yB}} = 74.187\cdot\text{MPa}$	$\varepsilon_{\text{yB}} = 0.063$
$\max\left(\varepsilon_{\text{trueC}}\right) = 1.012$	$\max\left(\sigma_{\text{trueC}}\right) = 94.066\cdot\text{MPa}$	$\sigma_{\text{yC}} = 87.054\cdot\text{MPa}$	$\varepsilon_{\text{yC}} = 0.067$
$\max\left(\varepsilon_{\text{trueD}}\right) = 1.017$	$\max\left(\sigma_{\text{trueD}}\right) = 100\cdot\text{MPa}$	$\sigma_{\text{yD}} = 93.068\cdot\text{MPa}$	$\varepsilon_{\text{yD}} = 0.067$
$\max\left(\varepsilon_{\text{trueh}}\right) = 0.385$	$\max\left(\sigma_{\text{trueh}}\right) = 174.6\cdot\text{MPa}$	$\sigma_{\text{yh}} = 174.6\cdot\text{MPa}$	$\varepsilon_{\text{yh}} = 0.074$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..4$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP4})$ $\quad$ $\mathrm{SPEC}_A := 2$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A} \cdot \mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP4})$ $\quad$ $\mathrm{SPEC}_B := 6$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B} \cdot \mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP4})$ $\quad$ $\mathrm{SPEC}_C := 9$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C} \cdot \mathrm{mm}$

$\mathrm{DIM}_D := \mathrm{READPRN}(\mathrm{COMP4})$ $\quad$ $\mathrm{SPEC}_D := 18$

$L_{o_4} := \left(\mathrm{DIM}_D^{\langle 1\rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$ $\quad$ $D_{o_4} := \left(\mathrm{DIM}_D^{\langle 2\rangle}\right)_{\mathrm{SPEC}_D} \cdot \mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{\mathrm{cso}_z} := \frac{\pi}{4} \cdot \left(D_{o_z}\right)^2$

$$A_{\mathrm{cso}} = \begin{bmatrix} 190.645 \\ 191.134 \\ 190.645 \\ 191.625 \end{bmatrix} \cdot \mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.prn) extension

$A := \mathrm{READPRN}(\mathrm{IC4LR1T2})$

$B := \mathrm{READPRN}(\mathrm{IC4LR2T2})$

$C := \mathrm{READPRN}(\mathrm{IC4LR3T2})$

$D := \mathrm{READPRN}(\mathrm{IC4LR4T2})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.prn.

Split matrix matrix into two vectors, load (kN) and displ (mm): $kN := 1000 \cdot newton$

$load_A := A^{\langle 1 \rangle} \cdot kN$ $displ_A := A^{\langle 2 \rangle} \cdot mm$ $load_D := D^{\langle 1 \rangle} \cdot kN$ $displ_D := D^{\langle 2 \rangle} \cdot mm$

$load_B := B^{\langle 1 \rangle} \cdot kN$ $displ_B := B^{\langle 2 \rangle} \cdot mm$

$load_C := C^{\langle 1 \rangle} \cdot kN$ $displ_C := C^{\langle 2 \rangle} \cdot mm$

Look at matrices and calculate
number of data points:

$a := 1 .. rows(A)$ $d := 1 .. rows(D)$

$b := 1 .. rows(B)$

$c := 1 .. rows(C)$

Calculate true stress and true strain (assume incompressibility): $MPa := 1 \cdot 10^6 \cdot Pa$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - displ_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - displ_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - displ_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}} \qquad L_{iD_d} := L_{o_4} - displ_{D_d} \qquad A_{iD_d} := \frac{A_{cso_4} \cdot L_{o_4}}{L_{iD_d}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{trueA_a} := \frac{load_{A_a}}{A_{iA_a}} \qquad \sigma_{trueB_b} := \frac{load_{B_b}}{A_{iB_b}} \qquad \sigma_{trueC_c} := \frac{load_{C_c}}{A_{iC_c}} \qquad \sigma_{trueD_d} := \frac{load_{D_d}}{A_{iD_d}}$$

$$\varepsilon_{trueA_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{trueB_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{trueC_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right) \qquad \varepsilon_{trueD_d} := -\ln\left(\frac{L_{iD_d}}{L_{o_4}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot MPa\right) \quad \sigma_{yA} := \max\left(\sigma_{yAcalc}\right) \quad NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot MPa\right) \quad \sigma_{yB} := \max\left(\sigma_{yBcalc}\right) \quad NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot MPa\right) \quad \sigma_{yC} := \max\left(\sigma_{yCcalc}\right) \quad NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\sigma_{yDcalc_d} := \mathrm{if}\left(d \le 1200, \sigma_{trueD_d}, 0 \cdot MPa\right) \quad \sigma_{yD} := \max\left(\sigma_{yDcalc}\right) \quad NumD_d := \mathrm{if}\left(\sigma_{trueD_d} = \sigma_{yD}, d, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}} \quad \varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}} \quad \varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}} \quad \varepsilon_{yD} := \varepsilon_{trueD_{\max(NumD)}}$

Read the stress strain curve data from the Hopkinson Bar analysis:

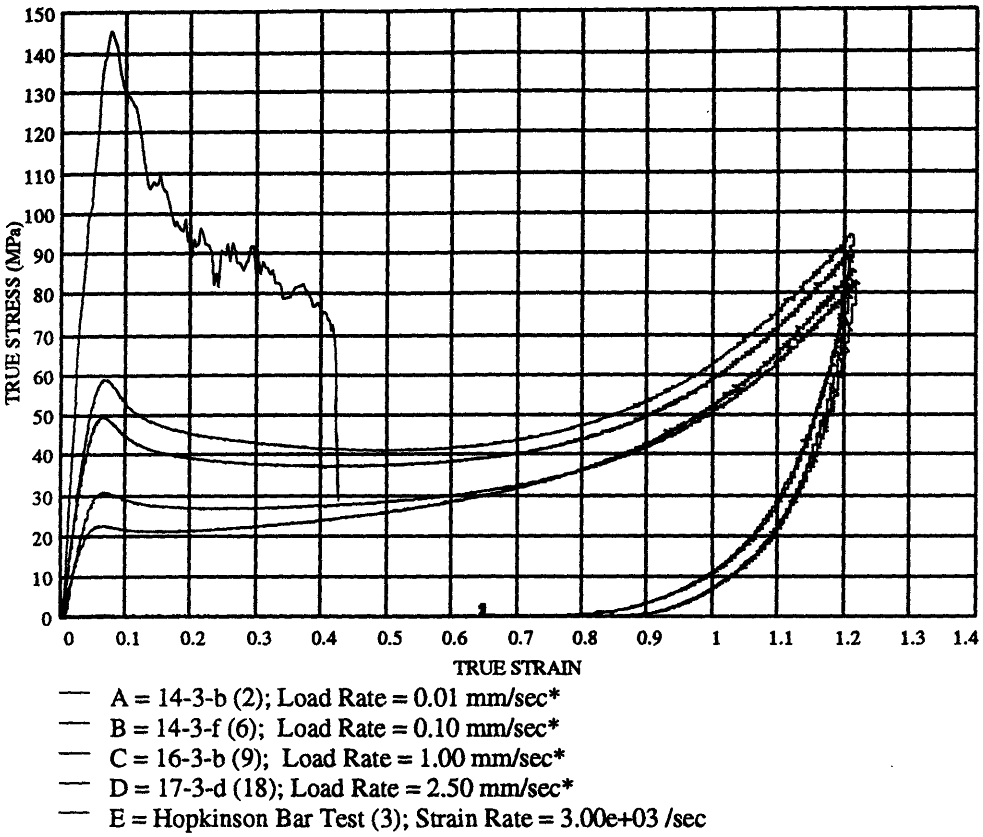
$i := 2500, 2510 .. 4500$

$TRIAL := READPRN(D2C7T1)$

$\varepsilon_{trueh} := (-TRIAL)^{<1>} \quad \sigma_{trueh} := (-TRIAL)^{<2>} \cdot MPa$

$\sigma_{yh} := \max\left(\sigma_{trueh}\right) \quad NumH_i := \mathrm{if}\left(\sigma_{trueh_i} = \sigma_{yh}, i, 0\right) \quad \varepsilon_{yh} := \varepsilon_{trueh_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for 25% 8084 / 75% 1080 at various load rates:



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - D:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec
4. A load rate of 2.50 mm/sec corresponds to an initial strain rate of 1.61e-01 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max(\varepsilon_{trueA}) = 1.216$ $\max(\sigma_{trueA}) = 85.539 \cdot MPa$ $\sigma_{yA} = 22.46 \cdot MPa$ $\varepsilon_{yA} = 0.059$

$\max(\varepsilon_{trueB}) = 1.222$ $\max(\sigma_{trueB}) = 82.846 \cdot MPa$ $\sigma_{yB} = 30.551 \cdot MPa$ $\varepsilon_{yB} = 0.068$

$\max(\varepsilon_{trueC}) = 1.211$ $\max(\sigma_{trueC}) = 89.4 \cdot MPa$ $\sigma_{yC} = 49.394 \cdot MPa$ $\varepsilon_{yC} = 0.066$

$\max(\varepsilon_{trueD}) = 1.212$ $\max(\sigma_{trueD}) = 94.333 \cdot MPa$ $\sigma_{yD} = 58.77 \cdot MPa$ $\varepsilon_{yD} = 0.069$

$\max(\varepsilon_{trueh}) = 0.425$ $\max(\sigma_{trueh}) = 145.2 \cdot MPa$ $\sigma_{yh} = 145.2 \cdot MPa$ $\varepsilon_{yh} = 0.076$

MICHAEL ZIV
THESIS DATA REDUCTION PROGRAM
UNIAXIAL COMPRESSION TEST

Select specimens to be used: $z := 1..3$ $\quad$ $\mathrm{ORIGIN} := 1$

$\mathrm{DIM}_A := \mathrm{READPRN}(\mathrm{COMP5})$ $\qquad$ $\mathrm{SPEC}_A := 1$

$L_{o_1} := \left(\mathrm{DIM}_A^{\langle 1\rangle}\right)_{\mathrm{SPEC}_A}\cdot\mathrm{mm}$ $\quad$ $D_{o_1} := \left(\mathrm{DIM}_A^{\langle 2\rangle}\right)_{\mathrm{SPEC}_A}\cdot\mathrm{mm}$

$\mathrm{DIM}_B := \mathrm{READPRN}(\mathrm{COMP5})$ $\qquad$ $\mathrm{SPEC}_B := 3$

$L_{o_2} := \left(\mathrm{DIM}_B^{\langle 1\rangle}\right)_{\mathrm{SPEC}_B}\cdot\mathrm{mm}$ $\quad$ $D_{o_2} := \left(\mathrm{DIM}_B^{\langle 2\rangle}\right)_{\mathrm{SPEC}_B}\cdot\mathrm{mm}$

$\mathrm{DIM}_C := \mathrm{READPRN}(\mathrm{COMP5})$ $\qquad$ $\mathrm{SPEC}_C := 6$

$L_{o_3} := \left(\mathrm{DIM}_C^{\langle 1\rangle}\right)_{\mathrm{SPEC}_C}\cdot\mathrm{mm}$ $\quad$ $D_{o_3} := \left(\mathrm{DIM}_C^{\langle 2\rangle}\right)_{\mathrm{SPEC}_C}\cdot\mathrm{mm}$

Calculate the initial cross sectional area (mm^2): $A_{cso_z} := \frac{\pi}{4}\cdot\left(D_{o_z}\right)^2$

$$A_{cso} = \begin{pmatrix} 194.828 \\ 193.346 \\ 193.839 \end{pmatrix}\cdot\mathrm{mm}^2$$

Data file information:

The data file was created on the directory: D:\MIKEDAT1 (on 1.307 hard drive), transfered to directory C:\winmcad (on home pc), file must have a (.pm) extension

$A := \mathrm{READPRN}(\mathrm{IC5LR1T1})$

$B := \mathrm{READPRN}(\mathrm{IC5LR2T1})$

$C := \mathrm{READPRN}(\mathrm{IC5LR3T1})$

Note: Files A-E are only used for tracking purposes. There is no relation to the specimen position indicator. Analysis slots not used read the default file IC0LR1T1.pm.

Split matrix matrix into two vectors, load (kN) and displ (mm): $\mathrm{kN} := 1000 \cdot \mathrm{newton}$

$\mathrm{load}_A := A^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_A := A^{<2>} \cdot \mathrm{mm}$

$\mathrm{load}_B := B^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_B := B^{<2>} \cdot \mathrm{mm}$

$\mathrm{load}_C := C^{<1>} \cdot \mathrm{kN}$ $\mathrm{displ}_C := C^{<2>} \cdot \mathrm{mm}$

Look at matrices and calculate
number of data points:

$a := 1 .. \mathrm{rows}(A)$

$b := 1 .. \mathrm{rows}(B)$

$c := 1 .. \mathrm{rows}(C)$

Calculate true stress and true strain (assume incompressibility): $\mathrm{MPa} := 1 \cdot 10^6 \cdot \mathrm{Pa}$

Calculate instantaneous length (mm) and cross sectional area (mm^2):

$$L_{iA_a} := L_{o_1} - \mathrm{displ}_{A_a} \qquad A_{iA_a} := \frac{A_{cso_1} \cdot L_{o_1}}{L_{iA_a}} \qquad L_{iB_b} := L_{o_2} - \mathrm{displ}_{B_b} \qquad A_{iB_b} := \frac{A_{cso_2} \cdot L_{o_2}}{L_{iB_b}}$$

$$L_{iC_c} := L_{o_3} - \mathrm{displ}_{C_c} \qquad A_{iC_c} := \frac{A_{cso_3} \cdot L_{o_3}}{L_{iC_c}}$$

Calculate true stress (MPa) and true strain:

$$\sigma_{\mathrm{true}A_a} := \frac{\mathrm{load}_{A_a}}{A_{iA_a}} \qquad \sigma_{\mathrm{true}B_b} := \frac{\mathrm{load}_{B_b}}{A_{iB_b}} \qquad \sigma_{\mathrm{true}C_c} := \frac{\mathrm{load}_{C_c}}{A_{iC_c}}$$

$$\varepsilon_{\mathrm{true}A_a} := -\ln\left(\frac{L_{iA_a}}{L_{o_1}}\right) \qquad \varepsilon_{\mathrm{true}B_b} := -\ln\left(\frac{L_{iB_b}}{L_{o_2}}\right) \qquad \varepsilon_{\mathrm{true}C_c} := -\ln\left(\frac{L_{iC_c}}{L_{o_3}}\right)$$

Calculate the yield stress (MPa) and corresponding true strain:

$\sigma_{yAcalc_a} := \mathrm{if}\left(a \le 1200, \sigma_{trueA_a}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yA} := \max\left(\sigma_{yAcalc}\right)$ $NumA_a := \mathrm{if}\left(\sigma_{trueA_a} = \sigma_{yA}, a, 0\right)$

$\sigma_{yBcalc_b} := \mathrm{if}\left(b \le 1200, \sigma_{trueB_b}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yB} := \max\left(\sigma_{yBcalc}\right)$ $NumB_b := \mathrm{if}\left(\sigma_{trueB_b} = \sigma_{yB}, b, 0\right)$

$\sigma_{yCcalc_c} := \mathrm{if}\left(c \le 1200, \sigma_{trueC_c}, 0 \cdot \mathrm{MPa}\right)$ $\sigma_{yC} := \max\left(\sigma_{yCcalc}\right)$ $NumC_c := \mathrm{if}\left(\sigma_{trueC_c} = \sigma_{yC}, c, 0\right)$

$\varepsilon_{yA} := \varepsilon_{trueA_{\max(NumA)}}$ $\varepsilon_{yB} := \varepsilon_{trueB_{\max(NumB)}}$ $\varepsilon_{yC} := \varepsilon_{trueC_{\max(NumC)}}$

Read the stress strain curve data from the Hopkinson Bar analysis:

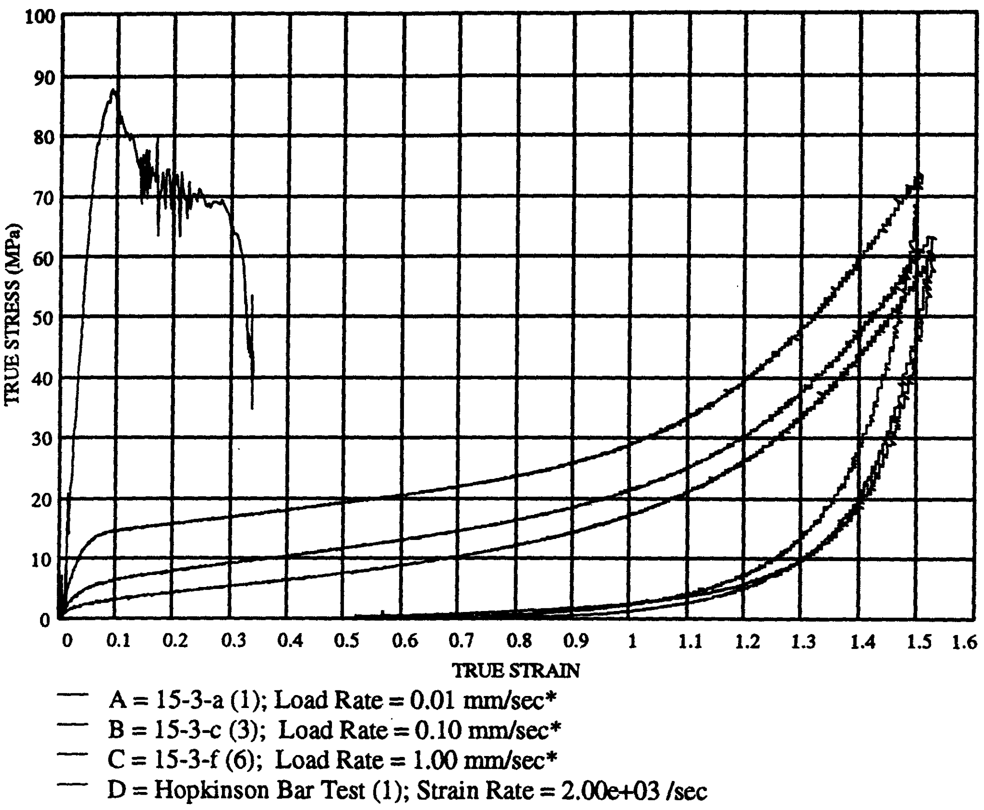
$i := 2500, 2510 .. 4500$

$TRIAL := READPRN(D0C1T1)$

$\varepsilon_{trueh} := (-TRIAL)^{\langle 1 \rangle}$ $\sigma_{trueh} := (-TRIAL)^{\langle 2 \rangle} \cdot \mathrm{MPa}$

$\sigma_{yh} := \max\left(\sigma_{trueh}\right)$ $NumH_i := \mathrm{if}\left(\sigma_{trueh_i} = \sigma_{yh}, i, 0\right)$ $\varepsilon_{yh} := \varepsilon_{trueh_{\max(NumH)}}$

Generate a plot of true stress (MPa) versus true strain for 0% 8084 / 100% 1080 at various load rates:



* The Instron tests were conducted at a constant loading rate. As a consequence, the strain rate is not constant throughout the test. As a basis of comparison with the Hopkinson Bar test, the inital strain rate can be estimated as the loading rate / inital specimen length. Assuming an average specimen length of 15.5 mm, the following inital strain rates apply to specimens A - C:

1. A load rate of 0.01 mm/sec corresponds to an initial strain rate of 6.45e-04 / sec
2. A load rate of 0.10 mm/sec corresponds to an initial strain rate of 6.45e-03 / sec
3. A load rate of 1.00 mm/sec corresponds to an initial strain rate of 6.45e-02 / sec

List maximum true strain obtained and corresponding stress (MPa), yield stress (MPa) and true strain at yield:

$\max\left(\varepsilon_{\mathrm{trueA}}\right) = 1.529 \quad \max\left(\sigma_{\mathrm{trueA}}\right) = 60.495 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueB}}\right) = 1.53 \quad \max\left(\sigma_{\mathrm{trueB}}\right) = 63.194 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueC}}\right) = 1.506 \quad \max\left(\sigma_{\mathrm{trueC}}\right) = 73.66 \cdot \mathrm{MPa}$

$\max\left(\varepsilon_{\mathrm{trueh}}\right) = 0.339 \quad \max\left(\sigma_{\mathrm{trueh}}\right) = 87.72 \cdot \mathrm{MPa} \qquad \sigma_{\mathrm{yh}} = 87.72 \cdot \mathrm{MPa} \qquad \varepsilon_{\mathrm{yh}} = 0.089$

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