Flow induced vibrations on a cable caused by waves plus current

by

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Submitted to the Department of Ocean Engineering in partial fulfillment of the requirements for
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Abstract

The objective of this thesis is the prediction of a cable oscillating motion under wave plus current
flow-induced vibrations. The purpose of this thesis was to get a better experimental understanding of this
subject, in order to produce a relevant model capable of predicting the motion of underwater cables placed
under those conditions. After an introduction to the problem, this thesis presents a simple mathematical
model and its predictions, in the case of waves plus constant current. This mathematical model is
inspired by the concept of a single degree of freedom oscillator. It assumes mainly that the oscillation
motion is unidirectional and that in the lock-in range the force excitation frequency is at the natural
frequency of the cable.

Some experiments are presented which were conducted at the MIT Ocean Engineering Tow Tank
on a flexible cylinder under tension with pin-pin ends. This modeled cable was towed first with
constant current only and then with the addition of waves. Data obtained experimentally were compared
to the simple model we had used to run a computer simulation. This model was then tested, improved
and oriented to a process that would meet our first objective. The experimental results showed a lot of
similarity with the predictions drawn with our mathematical model.

We observed especially that the waves do not change the oscillation frequency which is only
driven by the mean flow velocity (due to towing only). Yet, the waves modulate the amplitude
of the oscillation by adding an oscillatory component to the reduced velocity which results in
moving in and out the lock-in window.

In conclusion, the improved mathematical model can quite accurately predict the main
oscillatory motion of an underwater cable, under certain assumptions. More especially we only
expect this model to be valid for a natural frequency (in water) to wave frequency ratio greater
than 2.5.

Thesis Supervisor: J. Kim Vandiver
Title: Professor of Ocean Engineering
The purpose of this thesis was to develop a simple model capable of predicting the oscillatory motion of a cable under waves plus constant current. These vibrations, also called flow-induced vibrations, are due to vortex shedding. The study of this very precise subject is quite recent and has been developed in the past century.

Since flow-induced vibrations are linked to vortex shedding which is related to the study of turbulent flow, there are still some large unknown areas in both the practical and the theoretical field. Yet, it is a significant issue with many applications in the industry. Hence, in the field of ocean engineering, it is relevant for the design of underwater structures which have to face waves and currents: as for instance, an offshore platform or communication underwater cables.

The focus will be on the study of flow-induced vibrations caused by waves plus current on a cable. In fact, flow-induced vibrations caused by constant current is a well known phenomenon where one would find sufficient data, both theoretical and experimental. The study of flow-induced vibrations, when the flow is not unidirectional and constant, is a much less known and understood topic.

Up to now there have been several approaches to this problem. We will refer to two:

1. One possible approach is to use numerical methods and computers in order to solve the Navier Stokes equation. This is the kind of method one would use in the study of turbulent flows, and really reflects today's approach to fluid dynamics. Yet, one can not expect too much because current methods are restricted to low Reynolds number flows.
2. The other approach is an experimental one, but is actually very similar to the former one. It consists of towing a cylinder (or creating an incoming flow towards a cylinder) and measuring several parameters,
mainly the forces on the cylinder and the oscillatory motion. Some parameters have to be set similar to the first approach. For example, a pattern of oscillatory motion for the cylinder is set, the cylinder is towed, and the force that is put on the cylinder is measured in order to effectively follow the forced motion (it requires a mechanically computerized feedback system).

However, in both cases we are not able to answer the following question: what are the motion and the lift force for a cylinder facing a 2-dimensional non-constant flow? In fact, these approaches require some knowledge we do not have, in order to be complete.

Let’s take the example of wave plus current excitation. If we impose a vertical oscillatory motion to the cylinder (sinusoidal) and then tow it at constant speed, what we will really simulate is a fixed cylinder facing waves plus current, but certainly not a free cylinder (free to oscillate according to the flow-induced excitation only).

The only way to simulate a free cable facing flow-induced vibration caused by wave plus current would be actually to know in advance the resulting motion of the cable and then simulate it mechanically, and there are poor chances that it would be a purely sinusoidal motion. But if we knew the resulting motion from experience, we would also know the resulting constraints on the cable.

That is the very reason why it was decided to lead our study to a pure experiment where we would effectively tow a free cable towards wave plus current. It showed convincingly that this is the compulsory step to a better understanding of flow-induced vibrations caused by waves plus current.
Ch II Review of flow-induced vibrations

The basic problem, as far as flow-induced vibrations are concerned, is the one represented by a cylinder facing an incoming, unidirectional flow. See figure 1.

Constant flow causes oscillatory motion of the cylinder. The cylinder vibrates along the X axis.

The significant dimensionless parameters which matter in flow-induced vibration problems are the reduced velocity, \( V_r \), and the Strouhal number, \( S_t \), as defined below:

\[
V_r = \frac{U}{f n \cdot D}
\]

\[
S_t = \frac{f s \cdot D}{U}
\]
where,
. \( U \) is the velocity of the incoming flow
. \( f_n \) is the natural frequency of the cylinder (in still water)

**Note:** It is important to note that we define \( V_r \) with reference to natural frequency in still water and not in air as some people usually do.

. \( D \) is the outside diameter of the cylinder
. \( f_s \) is the shedding frequency, from a stationary cylinder

\( \text{St} \) is defined for a non-moving cylinder. \( V_r \) is defined for a moving cylinder, in terms of \( f_n \). \( V_r \) is used as the relevant parameter in our case. \( St \) is useful for making an initial estimate of the excitation frequency. It is a function of Reynolds number as shown below.
Thus, a flow at $U$ is capable of making a cylinder vibrate at the vortex shedding frequency.

In this case, the cylinder is an underwater cable and we are mainly concerned with the frequency and amplitude of the vibrating motion, in order to design more reliable ocean systems. Consequently, we need information on the motion amplitude, and therefore we introduce the dimensionless number $X/D$, where $X$ is the amplitude of the vibrating motion. The response is a function of reduced velocity and damping as well as mass ratio.

In fact it is a function of $b$, where $b$ is defined as follow:

$$b = \frac{R \cdot \omega}{\rho_{\text{fluid}} \cdot U^2}$$

From experiments, we know that in reality $X/D$ is at maximum of the order of one, and more importantly typically varies with $V_r$ as described above. This leads us to the problem of the lock-in phenomenon. Indeed for a certain range of $V_r$, which we will call from now the “lock-in window”, the amplitude of the induced motion becomes really significant. Outside the response is small.
and depending on damping and mass ratio:

\[ \frac{|X|}{D} \]

\[ V_r \]

low b

high b
Let’s consider the following simple model for the cylinder: it is a single degree of freedom system (mass, spring, damper) subject to unidirectional flow-induced excitations. The cylinder is assumed only able to oscillate vertically and, more importantly, it is assumed that the oscillation amplitude is small enough so that flow-induced excitation is the same as if the cylinder was fixed (i.e. sinusoidal).

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \]

and \( R \) represents hydrodynamic and structural damping.
From experiments it is known that the induced motion is approximately sinusoidal, which is to say that the force on the cylinder is of a sinusoidal form too. One may model the motion as forced excitation of a single degree of freedom system (SDOF).

\[ X = X_0 \cos(w_s t) \]

\[ w_s = 2\pi f_s \]

\[ F = F_0 \cos(w_s t) \]

Then the experiments show that the amplitude of the motion is a function of the reduced velocity which is to say that the amplitude of the excitation force is also a function of the reduced velocity.

\[ X_0 = \text{function}(w_s) = \text{function}(Vr) \]

\[ F(t) = f(Vr) F_0 \cos(w_s t) \]

where \( f \) can be typically modeled as follows:

![Diagram](image)
The experiments also show that $f_s$ is not always proportional to $V_r$.

Indeed in the very lock-in window, $f_s$ sticks to $f_n$. Since the response is only significant in this lock-in window, we can assume an excitation of the form:

$$F(t) = f(V_r)F_0 \cos(w_n t)$$

(as in the lock-in window $W_s=W_n$)

where $F_0$ depends on $|U|$ only.

Modeled this way, the problem can be analyzed as a special case of an SDOF system at resonance under forced excitation for which the excitation $|F|$ and the response $|X|$ take on a particularly simple relationship.
In this case we are interested in the flow-induced vibration caused by waves plus current. The incoming flow is two dimensional and consequently we should expect two dimensional vibration motion. Yet, in order to start with some simple model, we will consider first a one dimensional horizontal flow.

\[ U(t) = U_0 + A \cos (W \text{wave}. \ t) \]
where the first term corresponds to the constant current and the second to the wave influence (we assume deep waves, so that wave particles draw perfect circles). In fact what we are intentionally ignoring here is the vertical oscillatory flow caused by waves. Hence, we have:

\[ U(t) = U_0 + A \cos(w_{\text{wave}} t) \]

\[ V_r(t) = \frac{U_0 + A \cos(w_{\text{wave}} t)}{fn.D} \]

\[ V_{r0} = \frac{U_0}{fn.D} \]

The idea behind this simple model is the following: the natural vibration of cables under resonant flow-induced vibration conditions are only significant under lock-in.

Here is our mathematical model: an SDOF (single degree of freedom) system which is used to model the first mode motion of the cable under flow-induced vibration conditions.
Let's compare the reality (cable in water) and the model (SDOF system).

A review of the assumptions for the model shows that:

We assumed that the excitation force is at a constant frequency, namely the natural frequency of the cable; and, that flow velocity is slowly changing compared to the vibration period.

Thus, while considering the model as described above, we run a Matlab program to simulate flow-induced vibrations on a cable caused by waves plus current.
The Cable

Parameters:

- $L$: length of the cable
- $D$: diameter
- $\mu$: mass per unit length
- $R$: damping, per length of cable
- $T$: tension

For mode 1:

\[ w_1 = \sqrt{\frac{K_1}{M_1}} \text{ with } M_1 \text{ and } K_1 \text{ from modal analysis} \]

Mode shape: \( W(y) = \sin\left(\frac{\pi}{L}y\right) \)

Modal analysis/normalization:

\[ Q_1(t) = \int_0^L f(y,t)W(y)dy = \int_0^L \cos(w_1 t)\int_0^L \sin\left(\frac{\pi}{L}y\right)dy \]
\[ M_1 = \int_0^\xi \mu(y)W^2(y)dy = \int_0^L \sin^2\left(\frac{\pi}{L}y\right)dy \]

The Model

\[ \ddot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x = F(t) \]

\[ F(t) = F_0 f(V_r) \cos(w_1 t) \]

\[ 2\xi v_1 = \frac{c}{m} \]

\[ |x|_{\text{max}} = \left(\frac{F_0 f(V_r)}{K}\right)\left(\frac{1}{2\xi}\right) \]

(static response)(dynamic amplification)

For the cable, we choose:

\( \xi = 1\% \)

And assumed \( |x|_{\text{max}} \equiv 1D \)

\( D = 2\text{cm} \)

To simplify,

\( K = 1N/m \text{ and } m = 1Kg \), so that \( w_1 = 1 \text{ rad/s} \)

So,

\( |x| = \frac{F_0}{K}\frac{1}{2\xi} \) gives \( F_0 = K|x|_{\text{max}} 2\xi \equiv 2\xi KD = 4.10^{-4}N \)

And \( R = .2 \)
\[ K_i = \int_0^L T(y) \left( \frac{dW(y)}{dy} \right)^2 dy = T \int_0^L \frac{\pi^2}{L^2} \cos \left( \frac{\pi y}{L} \right) dy \]

\[ M_i = \frac{\mu L}{2} \]

\[ K_i = \frac{T \pi^2}{2L} \]

\[ C_i = \alpha M_i \text{ (Rayleigh hypothesis)} \]

so, \( \xi_i = \frac{\alpha}{2w_1} \)

\[ Q_i = \frac{2L}{\pi} f_0 \cos(w_1 t) \]

\[ M_i \ddot{q}_i + C_i \dot{q}_i + K_i q_i = Q_i(t) \]

**if we consider the response due to mode 1 only, then:**

\[ w(y, t) = W_1(y) q_1(t) \]

at \( w = w_1, \) \( w(y, t)_{\text{max}} = \frac{2L f_0}{\pi K_1} \frac{1}{2\xi_1} \)

at \( \omega = \omega_1 \)

\[ w(y, t) = \frac{2L f_0}{\pi K_1} \frac{1}{2\xi} \sin \left( \frac{\pi y}{L} \right) \sin(\omega t) \]

**example of simulation:**

we need according to previous experiments to have

\[ \frac{w_1}{w_{\text{max}}} = \frac{1}{0} \text{ rad/s} \]

so we take \( \omega = \omega_1 \)

and thus \( U_0 = 6 f_0 D = 0.19 \text{ m/s} \)
In order to simplify, we chose simple parameters in our model:

\[ K = 1 \text{ N/m} \]

\[ M = 1 \text{ kg} \quad \text{so that} \quad f_n = \frac{1}{2\pi} Hz \]

\[ w_n = 1 \text{ rad/s} \]

\[ f_w = 0.1 \text{ fn} \]

\[ R = 0.2 \text{ N/(m/s)} \quad \text{(damping = 10\%)} \]

\[ D = 0.02 \text{ m} \quad \text{(diameter of the cylinder)} \]

So that,

\[ F_0 = 4 \times 10^{-4} \text{ N} \quad \text{(maximum force excitation amplitude, at the lock-in peak)} \]

Yet, the most relevant parameters in this model are the following:

1. the ratio \( r = \frac{f_n}{f_w} \) which compares the natural frequency in water and the wave frequency.

For the following simulation we chose \( r = 20 \)

2. the mean reduced velocity \( Vr_0 = \frac{U_0}{fn.D} \)

3. the variation in reduce velocity \( \Delta Vr \)
\[ U(t) = U_0 + A \cdot \cos(w_a \cdot t) \]

where,

\[ Vr(t) = \frac{U_0 + A \cdot \cos(w_a \cdot t)}{fn \cdot D} \]

\[ \Delta Vr = \frac{A}{fn \cdot D} \]

Basically, this Matlab program (see appendix) computes the response of the cylinder to the wave plus current excitation, through a convolution integral. It includes the definition of the modulation function \( f(Vr) \) as shown in fig. 9.

![fig 9](image)

Hence, we made several runs with the program, while varying the mean reduced velocity and the variation in reduced velocity.

**Conclusions**

First of all we will describe two cases that really characterize the results we obtained:
1. for Vro = 6 and dVr = 2, so that Vr.max = 8 and Vr.min = 4
   In this case the excitation covers the whole lock-in window
   and within the window. The amplitude response is
   significant (1.1 diameter peak to peak) and clearly shows
   a beating phenomenon: the cylinder vibrates at fn, but the
   amplitude of the oscillations is driven by the wave
   frequency. In fact, as Vr moves up and down from 4 to 8,
   the excitation force follows the modulation function f(Vr).

2. for Vro = 4 and dVr = 2, so that Vr.max = 6 and Vr.min = 2
   (the only difference from the previous case is that the
   mean reduced velocity was lowered so that it is not
   centered any more in the lock-in window)

   In this case the excitation covers only partially the lock-in
   window, as it comes and goes. The result is that the
   amplitude of the motion is even more clearly subjected to
   modulation due to the waves. The modulation envelope is
   even more pronounced. Yet, the peak to peak maximum
   amplitude is roughly 1.5 which is greater that the one
   observed in the first case.

The oscillator gets energy when the increasing reduced velocity crosses into the lock-in
window. The excitation force amplitude reaches its maximum when Vr = 6 (f(Vr=6)=1)
and the oscillator gets energy. Then, when Vr drops down to 4 or less, the excitation
force is killed (f(Vr=4) = 0) and the amplitude of the vibrations decreases more or less
quickly depending on damping, until Vr increases again and passes Vr = 4.
EXAMPLE 1

MODEL I damping=10% Vr=6 dVr=2 Wn=1 fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
EXAMPLE 2

MODEL I  damping=10%  Vr=4  dVr=2  Wn=1  fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
From these various simulations we draw the following general conclusions.

1. When we set the mean reduced velocity and the reduced velocity variation so that the excitation is in (or comes and goes out of) the lock-in window, then a beating phenomenon is clearly observed between the natural frequency and the wave frequency. The cylinder still oscillates at $f_n$, but the amplitude of the oscillations is now also driven by the wave frequency.

2. If we are under or over the lock-in window but come and go into it, then there is still an amplitude response which is similar in both cases.

3. The variation in reduced velocity directly influences the amplitude response: in fact, the more area the excitation curve covers in the lock-in window, the more energy gets into the response and the higher the oscillation amplitude.
Ch IV Research on a first step experiment

Up to now we have only considered a very simple mathematical model which assumed that the effect of the incoming waves was only one dimensional (horizontal). But in fact the problem is far more complex. For instance, we do not know the answers to the following questions:

- what is the real motion of the cable?
- are 2nd and higher modes relevant?
- what is the influence of the vertical component of the incoming flow’s velocity?

Hence, while facing so many unknowns, it was not possible to build a more sophisticated model. And considering the general absence of theoretical and experimental data, the next step appeared clear: it was necessary to get experimental data to improve our understanding of the problem and improve the model.

Yet, it was not possible to wait until the final experiment, planned at the Offshore Technology Research Center at TEXAS A&M University. So it was necessary to find a way to do some preliminary experiments in the MIT Ocean Engineering towing tank. The main problem then was trying to match two types of constraints:

1. the practical constraints of the tow tank
   - limited towing speed (1.5 m/s)
   - limited wave length/height/frequency
   - limited response measurement capability

2. constraints of the proposed problem
   To stimulate lock-in we have to get $4 < V_r < 8$ lock-in window
   To avoid wave breaking $Wavelength/H > 7$
To achieve a fully developed wake

\[ K_c > 30 \quad \text{where } K_c \text{ is the Keulegan Carpenter number, defined as follow} \]

\[ K_c = \frac{U_m}{f_w\cdot D} \]

(\( U_m \): velocity amplitude of an oscillating flow)

**Note:** From previous experiments it is known that in order to observe quasi steady vortex-shedding of cables in oscillating flow, \( K_c \) has to be approximately 30 or higher. Yet, in our case we do not deal with a pure oscillating flow (wave only) but with wave plus current:

- in pure waves \( K_c \) is defined as above
- in pure current, with that same definition \( K_c \) tends to infinity
- in waves plus current the definition of \( K_c \) is not clear.

It is more appropriate to refer to dimensionless parameters that really fit the wave plus current case. We have chosen two: the ratio of natural frequency to wave frequency and the variation of reduced velocity

\[ r = \frac{f_n}{f_w} \]

\[ \Delta Vr \]

where \( f_n \) is the natural frequency in water and \( f_w \) is the wave frequency
Experiment design

The following set-up resulted: a pvc pipe (to mimic the cable) is held by an aluminum frame and towed along the tank. The cylinder is then fixed to a carriage which is towed along a rail:

Then the different parameters of the experiment were adjusted:

\[ V_r \ldots \text{Reduced Velocity} \]
\[ K_c \ldots \text{Carpenter number} \]
\[ f_n \ldots \text{Natural frequency} \]
\[ T \ldots \text{Tension of the cable} \]
\[ L \ldots \text{Length of the cable} \]
\[ \mu \ldots \text{Cable mass per unit length} \]
\[ d \ldots \text{Specific gravity of the cable} \]
\[ D \ldots \text{Diameter of the cable} \]
\[ U_m \ldots \text{Mean horizontal flow velocity} \]

In order to fit the constraints of the tow tank and the objectives at the same time, we made the following selections and determinations:
The wavelength/H>7 criterion would not be a problem in our case. Indeed we would never run waves at a higher frequency than 1 Hz and at a wave height larger than 0.15 cm (0.5 ft), as the wave maker could not do it anyway.

We chose a natural frequency in air of approximately 4 Hz so that we would be able to cover the whole range of the lock-in window at towing speeds between 0.2 and 1.3 m/s. We wanted $V_r=U/fn.D$ to be reached at a medium towing speed (0.6 m/s). This was achieved with a cylinder 2.19 cm in diameter.

We also want to have a fn / fw ratio as large as possible. We had to settle for fn/fw at approximately 4 to 6.
Once the basic parameters of the experiment we wanted to conduct were set, according to theory and previous data, it was necessary to check that the results were the ones expected. A few preliminary experiments were conducted at constant speed without waves. A few parameters were varied such as the density of the cable, its tension and natural frequency, and its diameter. It would not be useful to describe all these experiments and their results, as they mainly represent a phase of searching for the right parameters to use in our final experiment with waves, simulating current plus waves. However, we will emphasize on two of these experiments in order to show how they influenced our further choices.

1. general experimental setup, description

First of all, it is important to describe some different experiments, their setup, and the materials used in each case. For the three experiments which we are going to relate later, the basic setup was the same which allowed us to effectively compare them with one another. In each case the so-called cable was the same, and was held by an aluminum frame, and was attached on the towing tank running carriage.

Hence this frame was made of aluminum, to have high stiffness for the frame and more especially to have high natural frequencies (torsion of the main cylinder, bending of the struts ...), in order not to couple with the flow-induced vibration frequencies we intended to observe (less than 10 Hz). See figures 11 and 12.
steel rod supporting the carriage all along the tank

electronics

the carriage

Towing

water level

aluminium strut

cylinder depth

test cylinder

(main aluminium cylinder clamped to the carriage)

black light

underwater camera

( side view showing one strut)
PVC pipe (one end) stainless steel turnbuckle fixed to the aluminium strut

threaded into epoxy plug
thread in the epoxy plug

PVC pipe (one end) filled with,
- water: towtank test III
- sand: towtank tests V & VII

"The Cable"

OD=2.19cm
ID=1.74cm

PVC pipe density of PVC: 2800 Kg/cubic m

Fig 11

Fig 12

bolts spherical joint

water or sand

Epoxy

stainless steel turnbuckle fixed to the aluminium strut

Then, as we wanted to simulate the behavior of a cable, the following issues were dealt with:

- the ratio of the length of the cable to the diameter of the cable should be high because when $L/D < 20$ ends effects are dramatic. For instance at $L/D$ close to 10 the Strouhal number is lowered to 0.15, from 0.2 expected for this range of Reynolds number. Our design had an $L/D = 82$.

- the ends should be pin/pin, to provide low damping and a simple predictable natural frequency and mode shape.

Practically, we chose:

$L = 1.80 \text{ m}$

$D = 0.0219 \text{ m}$

so that,

$L/D = 82 >> 10$

and we mounted some spherical joints on the struts to hold the cable (see fig 15).

Then, as far as the cable is concerned, the one we used for these three experiments had the following properties:
Aluminium Frame (one half)

main aluminium cylinder pipe (OD=3 1/4 inch ID= 3 inch)

aluminium strut welded to the main cylinder

the "cable"

marks made on the cable with fluorescent paint

adjustable, to set the tension of the cable

at this point, the natural frequency of the aluminium strut only (in bending) is 210 Hz

Fig 13

Fig 14

aluminium strut accelerometer (vertical measurement)

water level

cable

1/4 of the length
in practice we did not use a cable but a PVC pipe

outside diameter OD = 0.0219 m
inside diameter ID = 0.0174 m
length L = 1.80 m

density of this PVC : 2800 Kg/ cubic meter

See fig 14.

To change the mass per unit length of the "cable", the PVC pipe was filled with different materials: air, water and sand. Tension was also varied to increase or decrease the natural frequency.

Acquisition of data :

In terms of data the following information was needed:
- flow-induced vibration frequency of the cable.
- amplitude of these vibration motions.
- some qualitative information on what the vibration motion looks like.

To collect this information we used an underwater camera in tow tank tests III, V & VII and for tow tank VII we also used accelerometers.
under water camera:

Marshall inc.
black & white
380 lines resolution
0.1 lux low light capability
12 volts
( waterproofed for the experiment )

accelerometers:

PCB Piezotronics inc.
- accelerometer model A353B16 Calibration 10.12 mv/g
- Amplification by model 480E09 Icp power unit ( gain on 10 )

See. fig 13.

video acquisition system:

The carriage is linked to the control room by a set of cables, so that the data can be collected in real time in the control room. The setup mainly uses bnc cables and allows use of up to 15 different channels.

Thus we made a special setup with the camera (power supply and signal) so that the video signal would go through a bnc cable plugged into the carriage’s panel, so that we could observe and record.

Consequently, we were able to record to a VCR in real time for each run.
Fig 15

phototransistor stuck to the TV screen

TV screen (adjusting contrast and brightness)

Cylinder appearing on the screen

VCR high precision slow motion

spectrum analyser HP

phototransistor probe power supply

Fig 16

Wave probe

main cylinder of the aluminium frame

aluminium strut

sinusoidal waves

towing

25 cm
We also used black light tubes mounted on the carriage to illuminate marks drawn on the cylinder with fluorescent paint. We were able to run the carriage in the dark, with the black light only and have a precise record of the cable oscillating motion.

Then, to get the oscillation frequencies and amplitudes, we used an appropriate VCR (capable of frame by frame) and an appropriate high resolution TV screen.

- **VCR**: Panasonic MTS AG 1960
- **TV screen**: Sony Trinitron PVM 1341

**phototransistor probe**:

Practically we made a special setup in order to be able to get the oscillation frequencies from the tape, played in real time.

Mainly, the idea was to use a phototransistor which could generate a signal while being stuck to the TV screen in an area where the cable image comes and goes. This signal was fed into a spectrum analyzer which revealed the different frequencies involved in the flow-induced vibrations of the cable.

- **phototransistor**: infrared phototransistor (radio shack cat 276.145A)
- **spectrum analyzer**: model HP 3582 A

This setup proved to be very useful, precise and reliable. Each run of the carriage along the towing tank took from 8 to 35 seconds, in the range of towing speeds we used. We were able to acquire at least 8 seconds of data which was enough to use the spectrum analyzer (in real time analysis mode) with adequate resolution. The frequencies we were interested in were lower than 10Hz which allowed us to use a maximum bandwidth of 10 Hz.
Summary of the different experiments:

**Towtank test III  video only**

- Cylinder: PVC pipe filled with water
- Natural frequencies:
  - \( f_{\text{air}} = 6.2 \text{Hz} \)
  - \( f_{\text{water}} = 5.2 \text{Hz} \)
- Mass per unit length/specific gravity: \( \mu = 0.65 \text{kg/m} \), \( d = 1.7 \)
- Damping:
  - \( \xi_{\text{air}} = 6\% \)
  - \( \xi_{\text{water}} = 9\% \)

**Towtank test V  video only**

- Cylinder: PVC pipe filled with sand
- Natural frequencies:
  - \( f_{\text{air}} = 4 \text{Hz} \)
  - \( f_{\text{water}} = 3.2 \text{Hz} \)
- Damping:
  - \( \xi_{\text{air}} = 6\% \)
  - \( \xi_{\text{water}} = 13\% \)
- Mass per unit length/specific gravity: \( \mu = 0.81 \text{kg/m} \), \( d = 2.1 \)

**Towtank test VII  video plus accelerometers**

- Same setup as in Towtank V
- Plus waves:
  - Series A: Wave frequency \( f_w = 0.7 \text{Hz} \)
  - Series B: Wave frequency \( f_w = 0.9 \text{Hz} \)
- In both cases the wave amplitude (1/2 wave height) varies a little bit, but remains around the average of 4 cm.
2. results and conclusions

In the first experiments the purpose was to observe the full lock-in window, with an expected narrow peak in oscillating amplitude at a critical reduced velocity (around 6 or 7) and a clear drop in response after exceeding an upper limit in reduced velocity.

Mainly, the purpose was to compare experimental results with theory and previous data, in order to check that our experimental setup was appropriate and to determine the proper parameters to get good data in combination with waves (complete/precise/reproducible).

During this experimental phase problems became evident:

. lock-in window too broad (in terms of \( V_r \) range)

  We could not get a “nice” lock-in window (cf fig 6) and mostly could not reach the point where the oscillation amplitude drops after the peak.

  Indeed we had a very broad lock-in window and were not able to see the end of it, even while towing at speeds up to 1 m/s, i.e. \( V_r = 12 \)!

. no clear lock-in range as indicated by a constant vibration frequency at \( f_n \).

In fact, with the first experiments, the focus was on plotting \( f_s \) as a function of \( V_r \) in order to observe the flat step of the curve which represents the very lock-in window. Yet, the curve did not really stick to \( f_n \) water as expected for reduced velocity between 4 and 8 (see fig 5).

These two issues can in fact be related to some known phenomena which will be described now in order to explain our results.
- the lock-in window bandwidth as a function of the cable's density

Previous experiments have shown that the so-called lock-in bandwidth (i.e. the range of Vr in which the oscillating motion amplitude is large) decreases as the density of the cable increases.

As a result, at low specific gravity (lower than 1.7), there is a broad flat peak: at around Vr=4 the amplitude of the oscillations dramatically increases up to roughly 1 diameter and supposedly drops at high Vr (>10), so that it could not be seen in our range of towing speeds.

Fig 18 is taken from experiments done in water and in air (Chung 1987 and Sarpkaya 1977): in air the specific gravity of the cable is defined with reference to air (instead of water as in our case). The data from Chung 1987 were obtained on experiments very similar to ours (free vibrating cylinder with pin-pin ends).

This plot shows in a very obvious way that we can only expect a nice and narrow lock-in peak at high cable specific gravity (higher than 5). To get a specific gravity (s.g.) of 8 for instance, we should use a solid metallic cable.

This plot also shows us that at a s.g. of 1 (s.g.=0.99, squares on the plot) we can expect to see the drop at Vr between 9 and 10 (which corresponds to a towing speed of 0.7 m/s) and can be reached at the towing tank.

- added mass coefficient as a function of reduced velocity

The other very important issue is that the added mass coefficient of the cylinder varies with reduced velocity and consequently the natural
frequency of the cable also varies with the reduced velocity. That is the very reason why we cannot observe a lock-in step for which \( f_s \) sticks to \( f_n \) in water, as the natural frequency in water changes with \( V_r \).

For the cable in air:

\[
f_{n,\text{air}} = \frac{1}{2L} \sqrt{\frac{T}{\mu}}
\]

\( \mu = \text{mass per unit length} \)

\( T = \text{tension of the cable} \)

\( L = \text{length of the cable} \)

then in water, with the contribution of added mass:

\[
f_{n,\text{water}} = \frac{1}{2L} \sqrt{\frac{T}{\mu + \mu_a}}
\]

\( \mu_a = \text{added mass per unit length} \)

\[
C_a = \frac{\mu_a}{\mu_{eq}}
\]

\( \mu_{eq} = \text{mass per unit length of an equivalent cylinder of water} \)

\( (\mu_{eq} = 0.39 \text{Kg/m for our cylinder}) \)

Thus when \( C_a \) decreased with \( V_r \), the added mass per unit length and so the \( f_n \) in water increased and consequently explains why in the lock-in window the \( f_s(V_r) \) curve is not flat at all and shows instead an increase: \( f_s \) sticks to \( f_n \) in water in the lock-in window, but \( f_n \) in water increases with \( V_r \).
Tow tank test III

Considering those two issues, we made some specific choices for tow tank test III:

We chose to fill the cylinder with water in order to increase the specific gravity up to 1.7.

The idea is to increase the mass per unit length of the cylinder while not changing the stiffness to keep the fn in water low. That is the very reason why we chose the PVC pipe instead of a plain cylinder or cable: we wanted low stiffness and still to be able to increase the mass per unit length. Then obviously we chose to express the results in an non-dimensional way so that we can easily compare the different tow tank experiments.

A/D is the ratio of the vertical oscillation amplitude (at mid span) to cylinder diameter

\[ \frac{fs}{fn} \] where fs is the vertical oscillating frequency and fn the natural frequency in still water (ie cylinder not towed)

For tow tank test III we went up to a reduced velocity of 12.

The following plots show A/D as a function of Vr and fs/fn as a function of Vr, on the same plot.

In terms of data acquisition, we collected fs with a precision of 0.02 Hz with the [spectrum analyzer + phototransistor + TV screen + VCR] setup. The data were very reproducible: two different runs at the same speed would give the same fs at 0.02 Hz precision.
We plotted both \( A/D \) and \( fs/fn \) versus the reduced velocity \( V_r \), where \( fn \) is the natural frequency in still water.

**Note**: Unless specified differently, \( A/D \) is taken at mid span of the cable.

Obviously the concept of \( fn \) raises a problem, considering that it changes with \( V_r \), so in this thesis \( fn \) will represent the natural frequency in still water: i.e. the natural frequency of the oscillations we get when we pull on the non-moving cylinder (not towed) and release it. It will be refereed to as \( f_{no} \) to eliminate confusion, when necessary.

Therefore, tow tank test III clearly shows two phenomena which were expected:

a. in the lock-in window \( 4 < V_r < 10 \) the oscillation amplitude dramatically increases, reaches a broad peak around \( V_r = 8 \) and then drops at \( V_r = 10 \).

b. \( fs/fn \) tends to 1 around \( V_r = 8 \) and then jumps when lock-in ends exactly when \( A/D \) drops, at \( V_r = 10 \).

This experiment shows also that with the proper parameters it is possible to reduce the lock-in bandwidth so that it fits into the limited reduced velocity range due to constraints of the tow tank. By increasing the specific gravity of the cable, we were able to lower the lock-in bandwidth in a quite predictable way.
towtank III

cylinder I filled with water

$fn=5.2 \text{ Hz (water)} \& fn.\text{air}=6.2\text{Hz}$
Tow tank test V

After having studied the results of the previous tow tank experiments, we were able to plan a more complete and detailed experiment, with a "nice" lock-in window which could then serve as a basis for the final experiment with waves. Thus, for tow tank test V we chose to slightly increase the density of the cable and lower the natural frequency in still water. To implement this, we chose to fill the PVC pipe with sand: the idea was to increase the mass per unit length while not changing the stiffness of the "cable". Then we lowered the tension till we reached $f_{n,\text{air}} = 4 \text{ Hz}$, which corresponds to $f_{n,\text{water}} = 3.2 \text{ Hz}$.

The sand:

We used some colored, extra thin sand (bought in a hobby store, originally made for landscape models) with a density of 1700 Kg/cubic meter.

Hence we had:

\begin{align*}
    f_{n,\text{air}} &= 4 \text{ Hz} \\
    f_{n,\text{water}} &= 3.2 \text{ Hz} \\
    \xi_{\text{air}} &= 6\% \\
    \xi_{\text{water}} &= 13\% \\
    s.\ g. &= 2.1 \\
    \mu &= 0.81Kg/m
\end{align*}

This experiment gave extremely satisfying results, as shown on the following plot.

In terms of $A/D = \text{function (Vr)}$, we got a clear lock-in peak at $Vr = 8$ and a clear drop just after this peak. At the same time, $f_s$ tends to $f_{n,\text{no}}$ and reaches it at $Vr = 8$ exactly when the peak occurs and then the oscillation frequency dramatically increases, precisely when the amplitude drops.
towtank V

Cylinder I filled with sand

$\text{fn.air}=4\text{Hz}$ & $\text{fn}=3.2\text{Hz}$ (water)

---

Graph showing:
- $fs/fn$ (circles)
- $A/D$ vertical and horizontal (squares)

Multiple lines representing different data sets.

Axes:
- Reduced velocity $V_r$
- $fs/fn$ (circles)
- $A/D$ vertical and horizontal (squares)
Thus, we obtained similar results to the ones of tow tank test III, but with a clearer and better defined peak.

With more than 24 different towing speeds, the $f_s/f_{no}$ ($V_r$) plot confirmed without a doubt a phenomenon that we could have predicted from the tow tank III data: in the lock-in window $f_s/f_{no}$ is a linear function of $V_r$.

This simple result is quite surprising and we then tried to take advantage of this unexpected information, in order to shed more light on the $C_a = \text{function} (V_r)$ issue we had detailed earlier: See appendix.

We obviously made an approximation, as in reality the “cable” is a pipe and behaves somewhat like a beam than as a string. However this hypothesis was useful, so we used it, while keeping in mind that our “cable” is only a model of a cable.

Fig 19 shows the results of this experimental law we found for $C_a (V_r)$ at s.g. = 2.1, and compares it with the previous data (Chung 1989) with s.g. = 2.0. The result is surprisingly good, considering the approximation we used and it shows that out of the $f_s (V_r)$ plot we are able to get back to $C_a (V_r)$ with good accuracy.
Tow tank VII

Considering the satisfying data we got in tow tank test V, we indeed choose to keep Tow tank test V as the basis for our experiment with waves. So, we used the same setup: same cylinder, same density, same natural frequencies (same tension).

Thus we had in tow tank VII the exact same parameters as in tow tank V.

But for this experiment we used accelerometers to measure the vibration of the cable and a wave gauge on the carriage to measure incoming waves. See fig 17

the waves:

The Ocean Engineering tow tank may be used to generate sinusoidal waves of fixed amplitude and frequency. We chose two types of waves and towed the cylinder into the waves.

- series A, at a frequency of 0.7 Hz
- series B, at a frequency of 0.9 Hz

As we towed the cylinder towards the incoming waves, the actual wave encounter frequency that the cylinder saw was slightly higher, depending on the towing speed. During each run, we recorded the accelerometer signal (vertical acceleration of the cable at 1/4 length from one end) and its spectrum, and the wave gauge signal which showed in real time the free surface level (see. fig 13). Then, after having recorded these data on a portable HP 3560 spectrum analyzer, we treated them with Viewdata (the software that comes with this HP device) and Matlab.

The purpose of each run was to get:

- the actual wave frequency
- the wave amplitude
- the oscillation frequency
- the oscillation amplitude (maximum oscill. amplitude at the middle of the cable)

To obtain the oscillation amplitude from the accelerometer signal, we used the following process:

We double integrated the acceleration spectrum from “right to left”, i.e. starting at high frequency and going down to a frequency \( f \) (down to a lower limit of 0.5 Hz for \( f \)) and plotted the value of the integral as a function of frequency. The idea was to get, at each frequency \( f \), the cumulative displacement contribution at all frequencies greater than \( f \).

The lower limit for \( f \) is only made to avoid low frequency noise expansion problems around zero. So we limited it to 0.5 Hz as the lowest frequency we were interested in was the wave frequency at 0.7 or 0.9 Hz.

This way, in theory, if the oscillating motion is at a precise frequency \( \omega_0 \), the spectrum would show a peak at \( \omega_0 \):

. for \( f > \omega_0 \) : 0 (no contribution of higher frequencies)

. for \( f < \omega_0 \) : constant step (no new contribution by lower frequencies than the one given at \( \omega_0 \))

. and the value of the step is precisely the displacement amplitude at \( \omega_0 \).

On the following page is an example:

at 0.3 m/s towing speed with 0.7 Hz incoming waves
The reverse cumulative RMS displacement plot clearly shows a jump (reading from higher to lower frequencies) at $f_s = 2.5$ Hz, and also a jump at around 4 Hz.

This is to be interpreted that: most of the oscillating motion occurs at $f_s = 2.5$ Hz and has an amplitude equal to the value of the jump (i.e. 0.4 Diameter here).
Example: Reverse cumulative RMS displacement

Series B at \( U = 0.5 \text{ m/s} \)
Towtank V & VII at 0.7 Hz
Comparison $f_s(V_r)$
Towtank V & VII at 0.9 Hz
Comparison fs(Vr)
To get $A/D$ (at the middle of the cylinder), we multiply our displacement amplitude at 1/4 length by $\sqrt{2}/D$, to account for the first mode shape being $\sin(\pi x/L)$.

Thus the following data was obtained for the two series of waves $A(0.7 \text{ Hz}) \& B(0.9 \text{ Hz})$.

In order to sum them up in an easy visual way, we gathered all plots for each run into one sheet: each sheet contains the spectrum of the oscillations and the spectrum of the incoming waves, and in parallel the time series of the wave, the excitation in terms of horizontal reduced velocity, and the oscillation response of the cable.

We will describe three examples of data sheets:

1. **Series B**  \hspace{1cm} $U=0.3 \text{ m/s}$ \hspace{1cm} $f_w=0.99 \text{ Hz}$ \hspace{1cm} $V_{ro}=4.3$ \hspace{1cm} $dV_{r}=2$

   The wave spectrum shows a narrow peak which means that the waves are quite pure in frequency. Yet, the value of the peak is slightly higher than 0.99 Hz (this is the encounter frequency), the more reliable value obtained from the time series (we measured the time it takes to get 10 periods, computed the period and then the frequency). That is the reason we only use the spectrum to have a qualitative idea of how “pure” the waves were in terms of frequency, but we used the time series to get precisely the frequency in each case. In fact the resolution for the spectrum is .125 Hz, i.e. $1/T=1/8s$.

   The plot from the wave probe signal shows clearly the incoming waves on the carriage, with a wave height of 8 cm. The wave plus current excitation (in terms of reduced velocity) comes and goes into the low side of the lock-in window. The resulting oscillation (accelerometer signal) shows a beating phenomenon. The cylinder oscillates at $f_s$ and the amplitude is modulated at the frequency of the waves. The maximum peak to peak acceleration is 0.7 g.
Serie B  U=0.4 m/s & \( \text{fw}=1.04 \ \text{Hz} \)

Wave spectrum \( f \) in Hz

INCOMING WAVES

Acc. spectrum \( f \) in Hz

displacement in m

WAVE + CURRENT EXCITATION

LOCK-IN WINDOW \( V_r=5.7 \) \& \( dV_r=1.87 \)

FLOW INDUCED OSCILLATIONS

acceleration in g

A/D = 0.6
Serie B  U=0.6 m/s & f_w=1.12 Hz

Wave spectrum f in Hz

INCOMING WAVES

Acc. spectrum f in Hz

Displacement in m

WAVE + CURRENT EXCITATION

V_r reduced velocity

LOCK-IN WINDOW  \( V_r=8.6 \) & \( dV_r=2.02 \)

FLOW INDUCED OSCILLATIONS

Acceleration in g

ATD = 0.07
2. Series B  \( U = 0.4 \text{ m/s} \quad f_w = 1.04 \text{ Hz} \quad V_{ro} = 5.7 \quad dV_r = 2 \)

In this case the excitation is right in the lock-in window and varies from \( V_r = 4 \) to \( V_r = 8 \).

The acceleration spectrum shows a nice narrow peak at \( f_s = 2.62 \) Hz.

The response amplitude is roughly 1 g, peak to peak, and the beating phenomenon is extremely clear.

3. Series B  \( U = 0.6 \text{ m/s} \quad f_w = 1.12 \text{ Hz} \quad V_{ro} = 8.6 \quad dV_r = 2.2 \)

This case is quite similar to the first one except that the excitation comes and goes in the lock-in window from the high \( V_r \) side. As a result, the response also shows the beating phenomenon, but in that case the peak-to-peak maximum amplitude is 3 g.
Conclusions

From the data we obtained in tow tank test VII, the following observations are made:

1. Series A and B show very similar results and these results look surprisingly close to the ones obtained with the model (Matlab simulation): we can observe similar beating phenomena, consisting of a modulation at the frequency of the incoming waves.

2. If we look at the oscillation frequency, it is clear that with or without waves $f_s$ is driven by the towing speed (i.e. mean horizontal flow velocity) (see comparison between Tow tank tests V & VII).

3. As far as the maximum amplitude of the oscillations is concerned, it also appears that it is extremely similar to the case without waves (tow tank V). The only slight difference is that the drop occurs in both cases with waves (series A & B) at a little bit higher $V_r$, but so slightly that it does not seem relevant.

4. In both cases, Series A & B, we observe that when $V_{ro}$ is out of the lock-in window and the variation of $V_r$ is such that $V_r(t)$ comes and goes in and out of the lock-in window, there are clearly two cases. If $V_{ro}$ is under the lock-in window ($V_{ro} < 4.2$, ex.cf. series B at 0.2 m/s), the amplitude is very small and the oscillating motion not very regular. Yet, when $V_{ro}$ is above the lock-in window ($V_{ro} > 8.3$, ex.cf. series B at 0.6 m/s), the amplitude response is important and significantly show the beating phenomenon described above. In addition, the maximum amplitude response in that case is greater than when the excitation always remains in the lock-in window with a similar $dV_r$ (c.f. Series B U=.6m/s / Series B U=.4m/s).

This last point surprised us and we tried to find an explanation, all the more that this difference between the “low mean $V_r$” and “high mean $V_r$” cases had not appeared in the SDOF model simulation.
Consequently, we tried to find an explanation for this nonsymmetric behavior, and returned to the Matlab simulation and changed the \( f(V_r) \) function (lock-in modulation window) to look more like the pattern observed in tow tank \( V \) (A/D as a function of \( V_r \)) : non symmetric peak, with a steady increase followed by a sudden drop at the end of lock-in (\( V_r = 8.3 \)). With this new simulation, for which we also changed \( r \) to 4 (closer to our experimental case \( r = f_{n}/f_{w} = 4 \) (series A) & 3 (series B)), we got the following results, which simulate the difference between low and high \( V_r \) lock-in window excitation.

**fig 20**

- First model simulation (simulation)
- New model

![Figure 20](image-url)
Comparison between

Towtank VII Series B

&

Model II SDOF

in a precise case

Towtank VII

towing speed 0.4 m/s

\[ V_r = V_{ro} \pm dV_r \]

\[ V_{ro} = 5.7 \]

\[ dV_r = 1.5 \]

\[ A/D = 0.6 \]

Model II

damping = 10%

\[ V_{ro} = 6 \]

\[ dV_r = 2 \]

\[ A/D = 0.6 \]
Serie B  $U=0.4 \text{ m/s} \& \text{fw}=1.04 \text{ Hz}$

- Wave spectrum $f$ in Hz
- INCOMING WAVES
- Acc. spectrum $f$ in Hz
- Displacement in m
- WAVE + CURRENT EXCITATION
- $Vr$ reduced velocity
- LOCK-IN WINDOW $V_0=5.7$ & $dV_r=1.87$
- FLOW INDUCED OSCILLATIONS
- Acceleration in g
- Time in s
Hence, from these results we draw the following conclusions:

1. It is surprising that the flow-induced vibrations caused by wave plus current on a free to vibrate cable (real conditions) can be accurately modeled by a single degree of freedom system for which we had made some strong assumptions:
   - we only considered the horizontal component of the flow, and
   - we did not account for any horizontal oscillating motion
Besides the fact that, when we went back to the model and changed the $f(V_r)$ pattern so that it looks more like what we observed in tow tank V (towing only), we simulated what happens in reality, demonstrating the reliability of the model.

2. Waves only change the amplitude of the oscillations and do not change the value of the oscillating frequency $f_s$ which is only driven by the mean flow velocity (constant current). What waves do is clearly to create a modulation in amplitude by making the reduced velocity vary in the lock-in window.

3. For a same $dV_r$, the maximum amplitude response is greater when $V_{ro}$ is close to the lock-in range upper limit ($V_{r}=8$ in our case): i.e., when the excitation comes and goes into the lock-in window with a high mean $V_r$.

4. Consequently, it seems that the best way to predict the behavior of a real cable under wave plus constant current would be to follow this process:

   a. determine the important parameters of the cable (mass per unit length, stiffness, modes)
   b. determine from experimentation the shape of the $f(V_r)$ modulation lock-in function. It only requires constant towing and can be obtained from experiments on reduced size models.
c. run an SDOF simulation (similar to the one we used) where $f(V_r)$ is mimicking the real modulation function ($A/D(V_r)$), the shape of which is obtained from b.

**Recommendations:**

Considering the surprisingly good results of that SDOF model, we would recommend continuing with that model. It would be interesting for instance to simulate varying current (which happens in reality) by just adding an oscillating component to $V_r$. More significantly, one possible experiment would be to place a small test cable under sea water with good conditions of wave plus current horizontal flow (shallow water in an estuary), record the excitation and the response, and finally compare this response with the one predicted by an SDOF model facing the same excitation.

Yet, this simple model has its limitations. For instance, one should not expect to get any good results when $f_s/f_w$ is lower that 2.5. We can indeed consider that we “pushed” the model to its limit with $f_s/f_w = 2.62$. In addition, the model has to be “calibrated” if quantitative information is required, like $A/D(t)$ for example. In that case, the excitation force has to be set which can be done the way we did: we assumed $A/D_{\text{max.}} = 1 \text{ D}$ and obtained $F_0$ from

$$F_0 = 2K|x|_{\text{max.}} \cdot \text{damping}.$$  

Considering the result it gave in the comparison case we developed (Series B $U = 0.4 \text{ m/s}$), it seems an appropriate method. The best would certainly be to use the computed $A/D$ and put it back into the formula, to get a new $F_0$, and then iterate this process until we get $A/D$ constant with the required precision.
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Appendix

1. the added mass coefficient as a function of the reduced velocity
2. Model I simulation
3. Model II simulation
4. Towtank test VII series A
5. Towtank test VII series B
**The added mass coefficient as a function of the reduced velocity \( Ca (Vr) \)**

The theory says that in the lock-in window \( f_s \) sticks to the natural frequency in water. So we know that \( f_s = f_n \) for \( Vr \) in \([4.2; 8.3]\) (which is the lock-in window for towtank \( V \) data), with \( f_n = \text{function (Vr)} \).

Towtank \( V \) data give :
- at \( Vr = 4.2 \) \( f_s = 2.14 \text{ Hz} = f_n (4.2) \)
- at \( Vr = 8.3 \) \( f_s = 3.2 \text{ Hz} = f_n (8.3) \)

we notice that at the peak \( f_n (8.3) = f_{no} \)

Then according to the \( f_s/f_n (Vr) \) plot,
\[
\frac{f_s}{f_{no}} = \alpha Vr \quad \text{in the lock-in window}
\]

but in the lock-in window we also have
\[
f_s = f_n (Vr)
\]

so,
\[
\frac{f_s}{f_{no}} = \frac{f_n (Vr)}{f_{no}} = \alpha Vr
\]

\[
f_n (Vr) = (\alpha f_{no}).Vr \quad \ldots \ldots (1)
\]

and we know that
\[
f_n = \text{function (Ca)}
\]
\[
Ca = \text{function (Vr)}
\]

We differentiate (1):
\[
df_n (Ca) = (\alpha f_{no}) dVr
\]

\[
\frac{df_n (Ca)}{dCa} \cdot \frac{dCa}{dVr} = \alpha f_{no} = \text{Cst} = \delta \quad \ldots \ldots (2)
\]

Besides, we know the relation between \( f_n \) and \( Ca \):
\[ f_n = \frac{\sqrt{T}}{2L} \frac{1}{\sqrt{\mu + \frac{\pi}{4} Ca \rho_w D^2}} \]

(from... \[ f_n = \frac{1}{2L} \sqrt{\frac{T}{\mu + \mu_0}} \] so,

\[ f_n(Ca) = K (\mu + \beta Ca)^{3/2} \]

\[ \frac{\partial f_n(Ca)}{\partial Ca} = -\frac{K\beta}{2} \cdot \frac{1}{\mu + \beta Ca} \]

with,

\[ \delta = \alpha f_n \]

\[ \beta = \frac{\pi}{4} \rho_w D^2 \]

\[ K = \frac{\sqrt{T}}{2L} \]

So (2) gives:

\[ \frac{dCa}{dV_r} = -\frac{2\delta}{K\beta} (\mu + \beta Ca)^{3/2} \]

\[ \frac{dCa}{(\mu + \beta Ca)^{3/2}} = -\frac{2\delta}{K\beta} dV_r \]

which we integrate,

\[ \frac{1}{\sqrt{\mu + \beta Ca(V_r)}} - \frac{1}{\sqrt{\mu + \beta Ca(8.3)}} = \frac{\delta}{K} (V_r - 8.3) \]

We chose \( V_r = 8.3 \) as a reference, and in fact we made the following reasoning: at \( V_r = 8.3 \), \( f_n = f_{no} = \) natural frequency in still water, so that we assumed \( Ca(8.3) = Ca \) still water =1.17 (cf. note below).

Finally, we obtained with the appropriate numerical values (all in IS units):
\[
\begin{align*}
K &= 3.26 \\
Ca(V_r) &= \frac{2.63}{(0.28 + 0.0733 \times V_r)^2} - 2.13
\end{align*}
\]

Note: we had to estimate Ca in still water and T tension, which was done the following way:

\[
\frac{f_{n, \text{air}}}{f_{n, \text{water}}} = \frac{\mu + \mu_a}{\mu}
\]

\[
\left(\frac{f_{n, \text{air}}}{f_{n, \text{water}}}\right)^2 = 1 + \frac{\mu_a}{\mu}
\]

\[
\mu_a = 0.456 \text{Kg / m}
\]

\[
Ca = 0.46 / 0.39,
\]

\[
Ca = 1.17
\]

then,

\[
f_{n, \text{air}} = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \rightarrow T \approx 170N
\]
MODEL I

Symmetric modulation function $f(V_r)$

- damping = 10%
- $W_n = 1 \text{ rad/s}$
- $W_w = 0.1 \text{ rad/s}$
- $f_n / f_w = 10$
MODEL I  damping=10% Vro=6 dVr=2 Wn=1 fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL I  damping=10%  $V_{ro}=4$  $dV_r=2$  $W_n=1$  $f_n/f_w=10$

Force Excitation $W_n/W_w=10$

Response to the above excitation
MODEL I: damping=10% Vro=8 dVr=2 Wn=1 fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL I  damping=10%  Vro=6  dVr=1  Wn=1  fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL I damping=10% Vro=4 dVr=1 Wn=1 fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL 1: damping=10%  Vro=8  dVr=1  Wn=1  fn/fw =10

LOCK-IN WINDOW

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL II

Non Symmetric modulation function $f(V_r)$

damping = 10 %
$W_n = 1$ rad/s
$W_w = 0.1$ rad/s
$fn / fw = 10$
MODEL II damping=10%  Vro=6  dVr=2  Wn=1  fn/fw =10

LOCK-IN WINDOW

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL II damping=10% Vr=4 dVr=2 Wn=1 fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL II  damping=10%  Vro=8 dVr=2 Wn=1 fn/fw =10

Response to the above excitation
MODEL II  damping=10%  Vrc=6  dVr=0.5  Wn=1  fn/fw =10

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL II damping=10% Vro=4 dVr=0.5 Wn=1 fn/fw =10

LOCK-IN WINDOW

Force Excitation Wn/Ww=10

Response to the above excitation
MODEL II damping=10% \( V_0=8 \) \( dV=0.5 \) \( W_n=1 \) \( f_n/f_w=10 \)

\[ \frac{V_r(t)}{\sqrt{m^*}} \]

\[ \text{LOCK-IN WINDOW} \]

\[ \text{Force Excitation } \frac{W_n}{W_w}=10 \]

\[ \frac{F(t)}{F_{\text{max}}} \]

\[ \text{Response to the above excitation} \]

\[ \frac{x(t)}{d} \]
TOWTANK VII

Series A

waves at 0.7 Hz
<table>
<thead>
<tr>
<th>U m/s</th>
<th>fs Hz</th>
<th>fw Hz</th>
<th>A/D rms</th>
<th>Vr</th>
<th>d Vr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>?</td>
<td>0.72</td>
<td>?</td>
<td>2.86</td>
<td>1.41</td>
</tr>
<tr>
<td>0.3</td>
<td>?</td>
<td>0.75</td>
<td>0.27</td>
<td>4.29</td>
<td>1.47</td>
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<tr>
<td>0.4</td>
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<td>0.78</td>
<td>0.58</td>
<td>5.71</td>
<td>1.53</td>
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<tr>
<td>0.5</td>
<td>3</td>
<td>0.81</td>
<td>0.79</td>
<td>7.14</td>
<td>1.59</td>
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<tr>
<td>0.6</td>
<td>3.37</td>
<td>0.84</td>
<td>0.88</td>
<td>8.57</td>
<td>1.65</td>
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<tr>
<td>0.65</td>
<td>3.75</td>
<td>0.855</td>
<td>0.6</td>
<td>9.29</td>
<td>1.68</td>
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<tr>
<td>0.7</td>
<td>4.69</td>
<td>0.87</td>
<td>0.46</td>
<td>10</td>
<td>1.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>U m/s</th>
<th>fs Hz</th>
<th>fw Hz</th>
<th>A/D rms</th>
<th>Vr</th>
<th>d Vr</th>
</tr>
</thead>
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<td>0.94</td>
<td>?</td>
<td>2.86</td>
<td>1.84</td>
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<td>1.06</td>
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<td>6.43</td>
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<tr>
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<td>4.75</td>
<td>1.17</td>
<td>0.45</td>
<td>10</td>
<td>2.29</td>
</tr>
</tbody>
</table>
Serie A  \( U = 0.3 \text{ m/s} \) & \( f_w = 0.75 \text{ Hz} \)

**WAVE + CURRENT EXCITATION**

**FLOW INDUCED OSCILLATIONS**

\( V_{re} = 4.3 \) & \( dV_r = 1.35 \)
Serie A  U=0.4 m/s & \( f_w=0.78 \) Hz

- Wave spectrum f in Hz
- Incoming waves
- Acc. spectrum f in Hz
- Displacement in m
- Wave + Current Excitation
- \( V_r \) reduced velocity
- Lock-in window \( V_r=5.7 \) & \( dV_r=1.4 \)
- Flow induced oscillations
- Acceleration in g
- Time in s
Serie A  U=0.5 m/s & f_w=0.81 Hz

Wave spectrum f in Hz

INCOMING WAVES

Acc. spectrum f in Hz

displacement in m

WAVE + CURRENT EXCITATION

V_r reduced velocity

LOCK-IN WINDOW  V_ro=7.1 & dV_r=1.46

FLOW INDUCED OSCILLATIONS

acceleration in g

Time in s
Serie A  U=0.6 m/s & fw=0.84 Hz

Wave spectrum f in Hz

Incoming Waves

Acc. spectrum f in Hz

Displacement in m

Wave + Current Excitation

V_r reduced velocity

Lock-in Window  V_0=8.6 & dV_r=1.51

Flow induced oscillations

Acceleration in g

Time in s
Serie A  U=0.65 m/s & fw=0.855 Hz
TOWTANK VII

Series B

waves at 0.9 Hz
Serie B \( U = 0.2 \text{ m/s} \) & \( f_w = 0.94 \text{ Hz} \)
Serie B U=0.35 m/s & f_w=1.01 Hz

Wave spectrum f in Hz

Acc. spectrum f in Hz

displacement in m

WAVE + CURRENT EXCITATION

Vr reduced velocity

LOCK-IN WINDOW V_ro=5 & dVr=1.82

FLOW INDUCED OSCILLATIONS

acceleration in g
Serie B  $U=0.4\, \text{m/s} \& \, f_w=1.04\, \text{Hz}$

- Wave spectrum $f$ in Hz
- INCOMING WAVES
- Acc. spectrum $f$ in Hz
- Displacement in m
- WAVE + CURRENT EXCITATION
- $V_r$ reduced velocity
- LOCK-IN WINDOW $V_{ro}=5.7$ & $dV_{r}=1.87$
- FLOW INDUCED OSCILLATIONS
- Acceleration in g
- $A/D=0.6$
Serie B  \( U = 0.5 \text{ m/s} \) & \( f_w = 1.08 \text{ Hz} \)

Wave spectrum \( f \) in Hz

Incoming Waves

Acc. spectrum \( f \) in Hz

Displacement in m

\( 0 \quad 0.05 \quad 0 \quad -0.05 \)

\( 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16 \)

Wave + Current Excitation

\( V_r \) reduced velocity

\( 4 \quad 6 \quad 8 \quad 10 \)

\( 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16 \)

Flow Induced Oscillations

\( -2 \quad -1 \quad 0 \quad 1 \quad 2 \)

\( 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16 \)

\( V_D = 0.8 \)
Serie B  U=0.65 m/s &  \( f_w=1.15 \) Hz

Wave spectrum \( f \) in Hz

Incoming Waves

Acc. spectrum \( f \) in Hz

Displacement in m

WAVE + CURRENT EXCITATION

\( V_{\text{r reduced velocity}} \)

LOCK-\text{IN WINDOW}  \( V_{\text{r}}=9.3 \) &  \( \Delta V_{\text{r}}=2.07 \)

FLOW INDUCED OSCILLATIONS

Acceleration in g

\( \Delta V_{\text{r}} = 0.95 \)