CALCULATION OF THE SECOND ORDER MEAN FORCE ON A SHIP IN OBLIQUE SEAS

by

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ABSTRACT

A design tool is developed for use by engineers for the calculation of the mean added resistance or drift force on an elongated body such as a ship in a seaway. Only forces arising from wave-ship motion interaction or wave reflection are considered and developed in a form suitable for a computer program. This procedure allows computation of the mean second order force from first order quantities already known in strip theory of ship motions. Regular wave computer results generated by the MIT 5-D motions program for the Mariner cargo vessel are presented and compared with experiment, including a set of new beam seas experiments. Comparison is also made with published results from two other programs. The extension of the regular wave theory to an irregular long-crested seaway and then to a short-crested seaway is outlined. Finally, six representative sea spectra are used in a brief design analysis for the Mariner at service speed.

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LIST OF SYMBOLS

$a(x)$ or $a_{jk}(x)$	- Sectional added mass coefficient
A_x	- Sectional area
A_{jk}	- Added mass matrix of the ship
b	- Sectional beam
$b(x)$ or $b_{jk}(x)$	- Sectional damping coefficient
$b^*(x)$	- Gerritsma and Beukelman sectional quantity (3a)
B	- Ship beam
B_{jk}	- Damping matrix of the ship
C_A	- After cross-section
C_{jk}	- Hydrostatic restoring force matrix of the ship
d	- Sectional draft
E	- Radiated energy from ship
$E(\omega)$	- Energy spectrum of the sea based on full wave amplitude
f	- Functional relation
$f_j(x)$	- Sectional Froude-Kriloff force
$\bar{F}(\beta)$	- Mean second order force in long-crested irregular seas
$\bar{F}(\theta)$	- Mean second order force in short-crested irregular seas
F_j	- Complex exciting force vector
F_j^D	- Diffraction portion of exciting force

F_j^I	- Froude-Kriloff portion of exciting force
$\vec{F}$	- Unsteady hydrodynamic force vector
$\vec{\mathfrak{F}}$	- Mean value of $\vec{F}$
$\mathfrak{F}$	- Magnitude of $\vec{\mathfrak{F}}$
$\mathfrak{F}_D$	- Component of $\mathfrak{F}$ related to the diffraction potential
$\mathfrak{F}_x$	- Added resistance component of second order force
$\mathfrak{F}_y$	- Drift force component of second order force
$\mathfrak{F}_j^I$	- Component of $\mathfrak{F}$ related to the Froude-Kriloff excitation
$\mathfrak{F}_j^D$	- Component of $\mathfrak{F}$ analogous to the diffraction excitation
g	- Gravitational acceleration
G_j	- Added mass and damping part of H_j
$h_D(x)$	- Sectional quantity, integrand of $\mathfrak{F}_D$ (81)
$\hat{h}_j(x)$	- Sectional diffraction force
$\bar{h}_{1/3}$	- Significant wave height
$h_j(x)$	- Sectional quantity, integrand of $\mathfrak{F}_j^D$ (78d)
H_j	- Total hydrodynamic force vector
i	- $\sqrt{-1}$
I_D	- Integral used to develop $\mathfrak{F}_D$ expression (82)
I_{jk}	- Moment or product of inertia
j	- Subscript (1...6; surge, sway, heave, roll, pitch, yaw)
k	- similar to j

K	- Wave number
L	- Ship length (L.B.P.)
m_k	- Related to ship normal, n_k
M_{jk}	- Mass matrix of the ship
n_k	- Ship hull normal (generalized)
N_j	- Two-dimensional normal
p	- Pressure in fluid
Q	- Represents either A or B
S	- Ship surface
S_F	- Free surface
S_∞	- Control surface at infinity
t_{jk}	- Sectional integrand of T_{jk}
T_{jk}	- Complex quantity giving the A_{jk} and B_{jk} elements
U	- Forward velocity of ship
V	- Volume within S, S_F, S_∞
$V_{Za}(x)$	- Relative sectional velocity
x	- Longitudinal axis of the ship
x_b	- Distance from center of gravity to center of buoyancy
y	- Transverse axis of the ship
z	- Vertical axis of the ship
Z_c	- Coordinate of the center of gravity
α	- Wave amplitude (1/2 height)
β	- Heading angle of the ship

ε_k	- Phase angle between motion and excitation
ζ	- Free surface position
ζ_k	- Complex amplitude of ship motion
$\dot{\eta}^*$	- Corrected free surface velocity
θ	- Principal wind direction for short-crested seas
λ	- Wave length
μ	- Difference between heading angle β and wind direction θ
ρ	- Water density
σ	- Sectional area coefficient (A_X/bd)
ϕ_B	- Full body potential amplitude
ϕ_D	- Diffraction potential amplitude
ϕ_{DC}	- Non-zero mean higher order potential
ϕ_I	- Incident wave potential amplitude
ϕ_k	- Motion potential amplitude
ϕ_k^O	- Speed independent motion potential
ϕ_k^U	- Speed dependent motion potential
ϕ_S	- Steady state potential
ϕ_T	- Time dependent potential amplitude
Φ	- Overall velocity potential
Φ_B	- Full body potential
Φ_I	- Full incident wave potential
$\Phi_{\zeta\zeta}^+(\omega)$	- Sea spectrum based on half the wave amplitude

Ψ_k	- Two-dimensional (sectional potential
ω	- Encounter frequency (rad/sec)
ω_o	- Incident wave frequency (rad/sec)
ω_p	- Spectral peak frequency (rad/sec)
ω_s	- Spectral frequency ordinate (rad/sec)

I. INTRODUCTION

The forces that act on a ship in a seaway vary widely both in their nature and their relative importance. In general, they represent random processes which can only be quantified meaningfully through statistical analysis. Because of this complexity, naval architects have traditionally been forced to make many design decisions by combining judgement and experience with the results of comparatively simple, idealized analysis or experiments. One example of this process is the use of the ship-beam analogy and the quasistatic trochoidal wave profile for ship structural design. Another illustration can be found in the procedure used to determine ship power requirements. Common practice has been to obtain the results of calm water resistance and self-propulsion model tests, then, to account for the actual operating conditions by applyinga power increase of fifteen to thirty percent. This service margin must be appropriate if the ship is to fulfill her owner's requirements consistently, efficiently, and economically in the real ocean environment.

Recently, great progress has been made in the effort to include rigorously in the design process some of the factors that influence a ship at sea. The first order strip theory of ship motions, due originally to Korvin-Kroukovsky and Jacobs[13], has been extended to the point where motions can be calculated acceptably for a wide variety of ship types, sea states, and heading angles. This makes possible the calculation of dynamic loadings imposed on the ship by a seaway (Salvesen, et. al[27], and others). Statistical methods developed for systems subject to random excitation now provide useful probabilistic statements concerning significant design events (e.g., the chance of slamming or the highest likely bending moment).

Due in part to these advances, the problem of environmental influence on ship power requirements and the related problem of the sideways drift of a ship in a seaway can now be much more fully addressed. The forces involved can be traced to a wide variety of sources, for example:

1. The motions of the ship interact with the ocean waves to produce a net drift or added resistance.
2. The ocean waves reflect off the ship hull causing a net force.
3. Wind present at sea acts on the superstructure to cause a drift force and/or an extra resistance.
4. Marine fouling causes an increase in resistance due to surface roughness.
5. Involved interactions will also be present (e.g., the propeller may operate less efficiently due to

the ship motion, rudder motions necessary for coursekeeping might cause induced drag and side force, or the side slip of the whole vessel might similarly cause drag).

The complexity of the problem outlined still precludes complete analysis. However, it can be appreciated that even a partial solution, which reduces the significance of purely judgemental factors like the service margin, will greatly increase the confidence and capability of the naval architect. This is particularly true for design decisions that break new ground for the profession such as powering for ultra-large tankers, vessels on new trade routes, or even dynamically positioned drilling ships.

In order to develop a useful, flexible design tool of this nature, the following report addresses the portion of the drift force/added resistance problem included in the first two points above, namely the ship motion-wave interaction and the reflection. From this point, 'drift force' and 'added resistance' refer strictly to mean forces averaged during one wave period of encounter and caused by wave-ship hull interaction. 'Added resistance' will be the term for the mean force component parallel to the longitudinal, ship axis, positive toward the stern. 'Drift force' will signify the mean force component perpendicular to the longitudinal ship axis, positive when directed toward the same half-plane as the wave propagation.

To place the method to be presented in a historical perspective, the paper begins with a discussion of past analytical efforts on the problem. Then a brief description of first order strip theory of ship motions is presented, and the method of calculation of second order mean wave force developed by Salvesen[28] is detailed. As will be seen, it is the mean second order wave force which is the drift force/added resistance referred to above. After an investigation of the characteristics of the final Salvesen formulation, an outline of past experimental work done on the problem is given, and an experimental effort devised to test certain areas of applicability is described. Following presentation of the experimental results, some numerical predictions for regular waves are compared with the results shown by Salvesen[28] and Loukakis[16]. The theoretical basis for a probabalistic extension of the regular wave theory to irregular ocean conditions is described, and a brief design analysis for several sea states is given. Finally, some general conclusions and recommendations are offered. The appendices contain documentation for the computer program developed in the analysis.

II. HISTORICAL BACKGROUND

Kreitner[14] was one of the earliest researchers (1939) to investigate the problem of wave forces on a ship hull. He concluded that the force was caused primarily by the reflection of the incident waves from the hull. However, the real pioneer in the second order force problem was Havelock[9] who proposed a theory in 1942 for a ship with no forward speed in regular head seas that was allowed to pitch and heave. He found that Kreitner's reflection force was unrealistic for a pitching and heaving ship in waves of reasonable length. Instead it became clear that the phase difference between the ship motion and wave excitation was the primary source of added resistance. Following this line of thought, Havelock proposed the following:

$$\mathcal{F}_x = \frac{1}{2} K \left\{ |F_3^I| |\zeta_3| \sin \varepsilon_3 + |F_5^I| |\zeta_5| \sin \varepsilon_5 \right\} \quad (1)$$

where F_j^I is the exciting force or moment due to the wave (assuming that the ship does not affect the wave flow field); this force (moment) is referred to as the Froude-Kriloff excitation force (moment). Subscript 3 refers to heave, 5 to pitch. ζ_j is the motion amplitude, ε_j is the phase angle between the motion and the excitation, and K is the wave number (ω_o^2/g).

Hanaoka[6] examined the case of a moving ship in calm water that was externally forced to heave and pitch. He gave an expression for the wave resistance of the ship which predicted a considerable increase over the unforced case. This increase was related to the damping in the radiated waves produced by the moving hull. The damping could, in turn, be associated with the phase lag of Havelock, so the two theories were definitely supportive.

During the time when Hanaoka was working on his formulation (1953), Haskind[8] employed a potential flow method to combine the efforts of Havelock and Kreitner. He proposed that the net wave force was the sum of two parts, one wave reflection and the other wave-ship motion interaction. The integral equation that resulted was complicated, involving Kochin H-functions (surface ingegrals dependent on frequency and form).

The next big step in the field came with Maruo's[17] potential flow solution, which was presented in 1957. He divided the velocity potential into three parts: the incident waves, the steady-state body potential, and the time-dependent (oscillatory) body motion potentials. For practical purposes, the body potential was evaluated for each section of the ship separately in the conventional strip formulation. The end result was a solution for the added resistance consisting of a sum of six terms, one each for heave, pitch, and reflection and one each for their interactions. Maruo's work was very important in that it combined and refined the ideas of Kreitner and Havelock, added a consideration of forward velocity and considered interactions between the motion related and reflection related resistance that had been earlier ignored. Further information on Maruo's theory for head seas can be found in Ref. 7. The extension of Maruo's work to seas approaching the ship from any angle (oblique seas) was accomplished recently by Hosoda[10], but it is extremely complex since it involves twenty-five components for all five degrees of freedom.

Joosen[11] offered a new result for the case of head seas in 1966. Joosen's equation resembled Havelock's original work, but included heave and pitch interaction. Newman presented an oblique seas theory in 1967[20] which was abstract in that it utilized pure slender body theory and a long wave approximation. Boese[29] derived a method for head seas similar to Havelock's in 1970. It was based on the determination of the pressure distribution around the hull in the wave.

Before describing the most recent work done on the second order wave force problem, some conclusions regarding the nature of the drift force/added resistance can be drawn. These general principles can be inferred from and are supported by the theoretical and experimental work that has been done on the problem.

1. The source of most of the drift force/added resistance (except for very short waves) is the phase lag between ship motions and wave excitations.[9, 17, 29]

A phase lag exists only in the presence of damping and, for ship motions, damping is almost exclusively by radiated waves. Furthermore, the energy loss through damping is directly related to the work necessary to maintain constant phasing. Therefore, the following can be concluded:

2. The drift force/added resistance problem can be formulated in terms of the waves radiated from the hull[5].
3. The second order force is a wave energy phenomenon which must be proportional in magnitude to the incident wave amplitude squared[17, 29]

The case for the last point above is particularly strong in that experimental data supports it well (See, for example, Ref. 29). This assertion is also crucial for the application of statistical methods. Two more statements of importance can also be made:

4. Since the source of the second order force is the seaway, and the associated ship motions, added resistance will be independent of calm water resistance. If some variation affects both, then it will do so through two distinct mechanisms.[17, 29]

5. Being totally a surface wave phenomenon, the drift force/added resistance can be measured in experiments using Froude scaling and fairly small models. (Ref. 29).

Gerritsma and Beukelman presented a theory in 1972[5] based on the idea that the added resistance in head waves could be calculated from the radiated energy contained in the outgoing damping and reflection waves around the ship. Their result continues to be extremely useful, due to its reliability and the fact that it is much easier to combine with a strip theory ship motions computer program than many of the other methods mentioned. In their development, they postulate that the energy radiated during one wave encounter period is,

$$E = \frac{\pi}{\omega} \int_0^L b^*(x)\, V_{za}^2(x)\, dx \qquad (2)$$

where ω is the encounter frequency and $b^*(x)$ and $V_{za}(x)$ are a sectional coefficient and a relative sectional velocity defined as follows:

$$b^*(x) = b(x) - U\left[\frac{da(x)}{dx}\right] \qquad (3a)$$

$$V_{za} = \dot{\zeta}_3 - x_b\dot{\zeta}_5 + U\zeta_5 - \dot{\eta}^* \qquad (3b)$$

where $b(x)$ is the sectional damping coefficient for an oscillating two-dimensional cylinder shaped like the section and $a(x)$ is the sectional added mass for the same problem. U is the forward velocity of the ship, x_b is measured from the center of gravity and $\dot{\eta}^*$ represents the velocity of the free surface. Referring to the work of Hanaoka et al.[7], it can be shown that the proportionality constant relating radiated energy and added resistance is the wave length, λ ($E = \lambda\mathcal{F}_x$). Applying this fact together with (3) in Equation (2) gives:

$$\mathcal{F}_x = \frac{\pi}{\omega}\int_0^L \left\{ b_{33}(x) - U\frac{da_{33}(x)}{dx}\right\}|V_{za}|^2 dx \qquad (4)$$

where $b_{33}(x)$ and $a_{33}(x)$ are now strictly interpreted as the sectional heave damping and added mass coefficients.

Through the use of energy flux consideration, Loukakis and Sclavounos[16] 1977, have succeeded in extending Gerritsmas's theory to computation of drift force and added resistance (and yaw moment) in oblique seas. This represents one of the three newest general methods for computation of second order mean force in oblique seas that seem well suited to inclusion in modern ship motions calculation schemes (the current goal being to obtain good second order mean force estimations from quantities known in the first order motion calculation, thereby minimizing computer time).

The second method that seems to hold promise in this sense is a potential flow theory due to Ankudinov[1]. It gives the second order force in a form similar to Havelock except that the total first order exciting force appears instead of just the Froude-Kriloff portion. It would be very easy to incorporate in a motions program, but it neglects wave reflection. Furthermore, there is some question about the validity of the final results presented (Salvesen[28]).

J. N. Newman was the originator of the third theory which will be derived and employed in this paper. He has worked on various aspects of the second order force problem

(Ref. 18, 19, 20, 22) but the particular paper that is most relevant to this report is Ref. 21. The method presented there can best be described as a potential flow formulation which determines a net pressure on the hull of the vessel due to higher order wave effects. Newman's equation was in surface integral form, and strict validity was assumed only for a submerged body beneath waves. Salvesen (Ref. 28) extended the analysis to surface vessels and applied the methods of strip theory to make numerical solution for a given ship within the context of a first order ship motions program possible.

III. THEORETICAL DEVELOPMENT

In order to generate a formula for the prediction of the mean second order force on a ship in oblique waves, it will first be necessary to develop the general potential flow problem that applies. Consider, then, a ship moving at speed U oriented arbitrarily to regular sinusoidal waves, as shown in Figure 1. The oscillatory motions that result will be assumed to be linear and harmonic. The coordinate system is illustrated in Figure 1. It is a right-handed orthogonal (x,y,z) system with the x-y plane coinciding with the plane of the undisturbed free surface, x along the longitudinal centerline, positive in the direction of forward motion, y positive to port and z positive upward through the ship center of gravity.** The waves are shown to have their direction of propagation at an angle, β, to the ship x-axis (β = 180°, head waves).

It is presumed that the ship oscillates as a rigid body in six degrees of freedom with complex motion amplitudes, ξ_k , $k=1 \ldots 6$, which refer to surge, sway, heave, roll, pitch, and yaw, respectively, as shown in Figure 1. Disregarding

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**The computer program has been generalized so the user can choose the longitudinal position of the origin arbitrarily. The choice of origin at the center of gravity is for the simplicity of this derivation only.

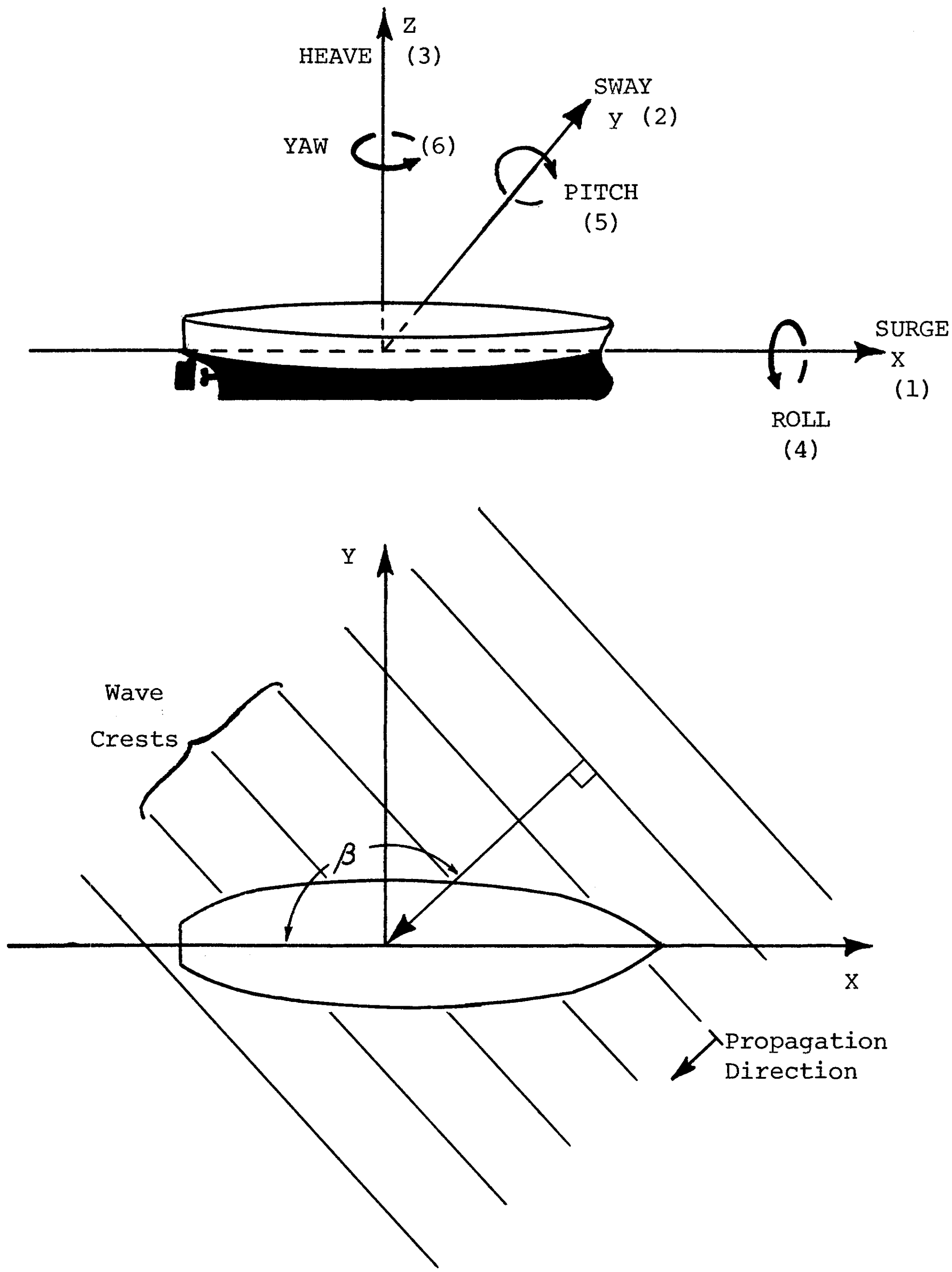


FIGURE 1: COORDINATE SYSTEM

viscosity and assuming irrotationality, potential flow theory can be applied. Thus, there is an overall velocity potential for the problem, $\Phi(x,y,z,t)$. This potential must satisfy Laplace's equation and the following exact boundary conditions:

On the ship surface ($f(x,y,z)=0$):

$$\frac{\partial f}{\partial t} + \nabla\Phi\cdot\nabla f = \frac{Df}{Dt} = 0 \tag{5}$$

Equation (5) expresses the condition that there must be zero flow velocity normal to the hull at the hull position.

On the free surface ($\zeta=\zeta(x,y,t)$ the following condition must apply:

$$-\rho\frac{D}{Dt}\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2 + g\zeta\right) = 0 \tag{6}$$

which expresses a pressure and gravitational force equilibrium at the boundary. Also, a radiation condition must be applied at $z = -\infty$ guaranteeing that the motion will be zero ($\nabla\Phi\to 0$).

The first step toward a solution is to separate the velocity potential into its time independent part due to the steady forward motion of the ship and its time dependent part associated with the incident waves and the harmonic motions,

$$\Phi(x,y,z,t) = \left[-Ux + \phi_s(x,y,z)\right] + \phi_T(x,y,z)\, e^{i\omega t} \tag{7}$$

where ω is the frequency of encounter, related to the incident wave frequency, ω_o, by.

$$\omega = \omega_o\left(1 - \frac{\omega_o}{g} U \cos\beta \right) \tag{8}$$

It should be understood that only the real part is to be taken in expressions involving $e^{i\omega t}$ that appear in this derivation.

At this point it is necessary to linearize the problem by assuming that ϕ_S, the steady perturbation associated with wavemaking in calm water, is small and so are its derivatives. The potential ϕ_T and its derivatives must also be assumed small. In this way, higher order terms and cross products of the potential components can be neglected. Decomposing the complex amplitude of the time dependent potential gives,

$$\phi_T = \phi_I + \phi_D + \sum_{k=1}^{6} \mathcal{J}_k \phi_k \tag{9}$$

where ϕ_I is the incident wave potential, ϕ_D is the diffraction potential, and ϕ_k is the potential contribution from the kth mode of motion. It can be seen that the problem has been transformed into the solution and superposition of several simpler potentials. ϕ_I is the well-known potential for an infinite free surface of plane progressive gravity waves. ϕ_D and the ϕ_k's must be solved, since the first deals with the wave reflecting properties of the motionless ship and the rest, of course, represent the properties of the motion.

An application of a Taylor series expansion about the mean ship position in connection with the above simplifications will show that the boundary conditions can now be linearized as follows:

$$\frac{\partial}{\partial n}\left(-Ux + \phi_s\right) = 0 \quad \text{on the 'hull'} \tag{10}$$

which represents the time independent portion of the body boundary condition applied at the mean hull position

$$U^2 \frac{\partial^2 \phi_s}{\partial x^2} + g \frac{\partial \phi_s}{\partial z} = 0 \quad \text{on } \zeta = 0 \tag{11}$$

which represents the time independent part of the free surface boundary condition applied at the undistrubed free surface.

$$\frac{\partial \phi_I}{\partial n} + \frac{\partial \phi_D}{\partial n} = 0 \quad \text{on the 'hull'} \tag{12}$$

which represents the incident and diffraction portion of the time dependent part of the body boundary condition on the mean hull location.

$$\left[\left(i\omega - U\frac{\partial}{\partial x} \right)^2 + g\frac{\partial}{\partial z} \right] \left(\phi_I , \phi_D \right) = 0 \quad \text{on } \zeta = 0 \tag{13}$$

which gives the free-surface condition on ϕ_I and ϕ_D. In the above equations, n is the outward normal to the hull.

The oscillatory motion potentials must satisfy:

$$\frac{\partial \phi_k}{\partial n} = i\omega n_k + U m_k \quad \text{on the 'hull'} \tag{14}$$

and

$$\left(i\omega - U\frac{\partial}{\partial x} \right)^2 \phi_k + g\frac{\partial \phi_k}{\partial z} = 0 \quad \text{on } \zeta = 0 \tag{15}$$

In these equations, n_k is a generalized normal defined by,

$$(n_1, n_2, n_3) = \vec{n} \quad \text{and} \quad (n_4, n_5, n_6) = \vec{r} \times \vec{n} \tag{16}$$

where $\vec{n}$ is the ship hull normal and $\vec{r}$ is a position vector. Also, m_k is defined as follows,

$$m_k = 0 \quad \text{for } k = 1,2,3,4 \; ; \; m_5 = n_3 \; ; \; m_6 = -n_2 \tag{17}$$

Further simplification of the hull boundary condition on the body motion potentials can be achieved by separating the potentials into two parts,

$$\phi_k = \phi_k^{o} + \frac{U}{i\omega}\phi_k^{U} \tag{18}$$

where ϕ_k^{o} will be assumed speed-independent. Substituting (18) into (14) gives two hull conditions,

$$\frac{\partial \phi_k^{o}}{\partial n} = i\omega n_k \quad \text{and} \quad \frac{\partial \phi_k^{U}}{\partial n} = i\omega m_k \tag{19}$$

Since ϕ_k^{U} and ϕ_k^{o} must satisfy all the same conditions and the Laplace equation, it follows from the relations (17) and (19) that:

$$\phi_k^{U} = 0 \text{ for } k = 1 \ldots 4 \; ; \; \phi_5^{U} = \phi_3^{o} \; ; \; \phi_6^{U} = -\phi_2^{o} \tag{20}$$

Thus, the oscillatory motion potential components can be given from (18) in a form which includes speed only as a simple factor. (This 'speed-independence' is an involved assumption which will be clarified later.)

$$\phi_k = \phi_k^{o} \quad \text{for} \quad k = 1 \ldots 4 \tag{21a}$$

$$\phi_5 = \phi_5^{o} + \frac{U}{i\omega}\phi_3^{o} \tag{21b}$$

$$\phi_6 = \phi_6^o - \frac{U}{i\omega}\phi_2^o \tag{21c}$$

with the body boundary condition from (19) being

$$\frac{\partial \phi_k^o}{\partial n} = i\omega n_k \tag{22}$$

and the free surface condition becoming from (15)

$$(i\omega - U\frac{\partial}{\partial x})^2 \phi_k^o + g\frac{\partial \phi_k^o}{\partial z} = 0 \quad \text{on } \zeta = 0 \tag{23}$$

This completes the synthesis of the relevant conditions governing this problem. To summarize, the general potential is separated into several terms as per Equations (7) and (9). These linearized potentials must satisfy Laplace's equation, the linearized boundary conditions on the calm water surface. (11), (13) and (23), and the body boundary conditions (10), (12), and (22) at the hull position as well as the radiation conditions at infinity.

Having formulated the potential flow problem for a ship moving on an arbitrary heading in regular waves, the equations of motion of the vessel will now be developed. In this way specific quantities that must be extracted from the potential flow solution can be identified.

The ship is considered as a rigid body with six degrees of freedom. Under the assumption of linear harmonic motions already stated, the governing equations can be written in matirx form in the frequency domain,

$$\sum_{k=1}^{6} [-\omega^2(M_{jk}+A_{jk})+i\omega B_{jk}+C_{jk}]\zeta_k = F_j \qquad j=1\ldots 6 \tag{24}$$

where M_{jk} is the generalized mass matrix of the ship (25), A_{jk} and B_{jk} are the added-mass and damping coefficients (26), the C_{jk}'s are the hydrostatic restoring coefficients, and the F_j's are the complex exciting forces and moments. Note that the ship is idealized as a coupled spring-mass-dashpot system with harmonic forcing functions. Assuming that the ship possesses lateral symmetry, then the matrices above can be given as follows:

$$M_{jk} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_c & 0 \\ 0 & M & 0 & -Mz_c & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -Mz_c & 0 & I_{44} & 0 & -I_{46} \\ Mz_c & 0 & 0 & 0 & I_{55} & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_{66} \end{bmatrix} \tag{25}$$

where the center of gravity is located at $(0,0,z_c)$, M is the ship mass, and the I_{jk}'s are mass moments of inertia, when $j=k$ and products of inertia for $j\neq k$.

$$A_{jk} \text{ or } B_{jk} = \begin{bmatrix} Q_{11} & 0 & Q_{13} & 0 & Q_{15} & 0 \\ 0 & Q_{22} & 0 & Q_{24} & 0 & Q_{26} \\ Q_{31} & 0 & Q_{33} & 0 & Q_{35} & 0 \\ 0 & Q_{42} & 0 & Q_{44} & 0 & Q_{46} \\ Q_{51} & 0 & Q_{53} & 0 & Q_{55} & 0 \\ 0 & Q_{62} & 0 & Q_{64} & 0 & Q_{66} \end{bmatrix} \tag{26}$$

where **Q** stands for A or B.

These terms (26) are hydrodynamic in nature and must be developed from the potential flow problem outlined. For the hydrostatic coefficients the only nonzero terms are C_{33}, C_{44}, C_{55}, and C_{35}. All these are quantities well known to the naval architect from hydrostatics.

An examination of the matrices in Equations (25), (26), and the C-matrix will reveal that for the laterally symmetric ship there are two sets of three equations, one for surge, heave, and pitch, and another for sway, roll, and yaw. These two sets are decoupled. Furthermore, under the assumption that the hull form is slender , it has been shown that the forces associated with surge are small, and this mode can be disregarded leaving five degrees of freedom.

To solve these equations, it will be necessary to develop the added mass and damping matrix elements and the five exciting forces from the potential flow solution already outlined. These are all obtainable from the pressure in the fluid on the hull. By Bernoulli's equation,

$$p = -\rho\left(\frac{\partial \Phi}{\partial t} + \frac{1}{2}|\nabla \Phi|^2 + gz\right) \tag{27}$$

If this pressure is expanded in a Taylor series about the mean hull position and linearized consistently, it follows that the unsteady pressure is

$$p = -\rho\left(i\omega - U\frac{\partial}{\partial x}\right)\phi_T e^{i\omega t} - \rho g(\xi_3 + \xi_4 y - \xi_5 x)e^{i\omega t} \tag{28}$$

The last term is just the hydrostatic restoring force which was already separately included in the equations of motion (see above). Thus, the hydrodynamic force and moment acting on the ship will be given by

$$H_j = -\rho \iint_{S} n_j\left(i\omega - U\frac{\partial}{\partial x}\right)\phi_T \, ds \qquad j = 2, \ldots, 6 \tag{29}$$

where $\mathcal{S}$ is the hull surface at its mean position. The use of Equation (9) will allow the division of (29) into two parts, the exciting force and moment due to the wave (including the diffracting effects of the hull):

$$F_j = -\rho \iint_{\mathcal{S}} n_j \left(i\omega - U\frac{\partial}{\partial x}\right)(\phi_I + \phi_D)\, ds \tag{30}$$

and the force and moment due to the body motions (which physically represent both the added mass and damping).

$$G_j = -\rho \iint_{\mathcal{S}} n_j \left(i\omega - U\frac{\partial}{\partial x}\right) \sum_{k=2}^{6} \mathcal{J}_k \phi_k \, ds \tag{31}$$

$$= \sum_{j=2}^{6} T_{jk} \mathcal{J}_k \tag{32}$$

where

$$T_{jk} = -\rho \iint_{\mathcal{S}} n_j \left(i\omega - U\frac{\partial}{\partial x}\right) \phi_k \, ds \tag{33}$$

Equation (24) shows that the real part of (33) will be the added mass while the imaginary part will be the damping.

Therefore,

$$T_{jk} = \omega^2 A_{jk} - i\omega \tag{34}$$

The evaluation of Equation (33) is not currently possible because the motion potentials involved are three-dimensional in nature and associated with an arbitrary hull shape. Strip theory allows the solution of the problem through a simple lengthwise integration of two-dimensional potentials associated with each section. This is developed using a variant of Stokes Theorem derived in Ref. 27.

$$\iint_S n_j U \frac{\partial \phi}{\partial x} ds = U \iint_S m_j \phi \, ds - U \int_{C_A} n_j \phi \, dl \tag{35}$$

Applying this to (33), and assuming that the ship has zero after cross-section (C_A)** yields

$$T_{jk} = -\rho i \omega \iint_S n_j \phi_k \, ds + U\rho \iint_S m_j \phi_k \, ds \tag{36}$$

** In the original MIT motions program, this after section term was included in the calculation. For consistency with the second order force subroutine to follow, either all these terms must be removed from the motions calculation or a "zero" after section must be included in the input.

Using (36), the T_{jk}'s can be written for any desired jk combination by substituting from (21). Next, conventional strip theory approximations can be applied by assuming that the ship hull is long and slender. This allows the following transformation of the surface integral of (36) for the speed-independent potentials

$$T^{o}_{jk} = -\rho i \omega \int_L \int_{C_x} n_j \phi^{o}_k \, dl \, dx \equiv \int_L t_{jk} \, dx \qquad (37)$$

As a consequence of the slender-hull assumption, it can be seen that, along the hull, $\partial/\partial x \ll \partial/\partial y$ or $\partial/\partial z$ and $n_1 \ll n_2$ or n_3. The last of these allows substitution of a two-dimensional normal, N_j, (noting that now $n_5 = -xN_3$ and $n_6 = xN_2$).

In view of the above assumptions, it is possible to quantify the limitations inherent in making the motion potentials 'speed independent' (See (21)). The free surface condition (15) gives, upon reduction, that w>>U2/2x for the speed-independence to be workable. This is equivalent to a wave-length on the order of the ship beam. Fortunately, this theoretical restriction on strip theory does not preclude very reasonable answers for fairly long waves, since the hydrostatic terms grow in importance for the smaller frequencies.

The above observations can now be used to infer that the speed-independent, three-dimensional motion potentials, ϕ_k^o, can be replaced as follows with sectional potentials

$$\phi_k^o = \Psi_k \quad \text{for} \quad k = 2,3,4 \tag{38a}$$

$$\phi_5^o = -x\Psi_3 \tag{38b}$$

$$\phi_6^o = x\Psi_2 \tag{38c}$$

where the Ψ_k's are the potentials for the two-dimensional problems of an infinite cylinder with the shape of the section oscillating in sway, heave, or roll. The problem of a cylinder oscillating in any of these modes is a classic one in hydrodynamics, and several techniques exist for mapping the solution to an arbitrary section shape. These methods include the Frank close-fit source-distribution Tsai-Porter close-fit mapping method, the Demanche bulb-form, and the Lewis form. The last two are used by the M.I.T. 5-D program.

In summary, the sectional potentials can be computed by known numerical methods, then combined with Equation (37) to give

$$t_{jk} = -\rho i\omega \int_{C_x} N_j \Psi_k \, d\ell = \omega^2 a_{jk} - i\omega b_{jk} \tag{39}$$

for $j=k=2,3,4$ and $j=2, k=4$

where a_{jk} and b_{jk} are the sectional added mass and damping inferred from (34). This makes possible the computation of all the speed-independent T_{jk}^{o}'s which can be used in (36) in conjunction with (21) to give the full T_{jk}'s. The real and imaginary parts of the T_{jk}'s then correspond (34) to the desired damping and added mass. The process is complicated in an algebraic sense, but further details, including results, are available in Reference 27. The final answers are given in terms of the sectional added mass and damping for the two-dimensional problem, for example,

$$A_{53} = -\int_L x\, a_{33}\, dx + \frac{U}{\omega^2} B_{33}^{o} \tag{40a}$$

or

$$B_{46} = \int_L x\, b_{24}\, dx - U A_{24}^{o}$$

(40b)

where $$B_{33}^{o} = \int_L b_{33}\, dx$$

and, $$A^{\circ}_{24} = \int_L a_{24}\,dx$$

The B_{44} damping term developed in this way does not give realistic answers when used in Equation (24). This is due to the fact that ship rolling is governed by viscous effects. The resulting non-linearity can be handled by defining a quasi-linear damping augmentation based on roll velocity and using an iterative scheme (See Ref. [27]).

The development of the exciting force and moment from (30) is also crucial to the understanding of the second-order force theory to be derived. Salvesen, ėt. al[27] show that the forces and moments can be expressed in terms of the sectional potentials discussed above. This method due to Haskin circumvents the solution of the diffraction potential, ϕ_D, a very important simplification.

To proceed, the exciting force (30) is separated into two parts,

$$F_j = F_j^I + F_j^D \tag{41}$$

where

$$F_j^I = -\rho \iint_S n_j \left(i\omega - U\frac{\partial}{\partial x}\right)\phi_I \, ds \tag{42}$$

and

$$F_j^D = -\rho \iint_S n_j \left(i\omega - U\frac{\partial}{\partial x}\right) \phi_D \, ds \qquad (43)$$

The potential for the incident waves is well known from classic linear gravity wave theory:

$$\phi_I = \frac{ig\alpha}{\omega_o} \exp\left[-iK(x\cos\beta - y\sin\beta) + Kz\right] \qquad (44)$$

(where α = wave amplitude, $K = \omega_o^2/g$ = wave number)

This quantity can be substituted into (42), giving,

$$F_j^I = -\rho i \iint_S n_j \omega_o \phi_I \, ds \qquad (45)$$

where ω_o is written as a consequence of (8). Equation (45) is the Froude-Kriloff exciting force which is easily computed without knowledge of the sectional potentials.

Continuing with Equation (43), the diffraction part of the exciting force, application of the special version of Stokes theorem used earlier (35) gives, for a ship with zero after cross-section,

$$F_j^D = -\rho \iint_S (i\omega n_j - U m_j) \phi_D \, ds \qquad (46)$$

Use of the hull boundary condition (19) yields:

$$F_j^D = -\rho \iint_{S'} \frac{\partial}{\partial n}\left(\phi_j^o - \frac{U}{i\omega}\phi_j^U\right)\phi_D \, ds \qquad (47)$$

A theorem of vector calculus known as 'Green's Second Identity' can be used with two functions (ϕ and Ψ) satisfying the same Laplace equation, free-surface condition, radiation condition, and bottom condition to yield the following identity

$$\iint_{S'} \phi \frac{\partial \Psi}{\partial n} ds = \iint_{S'} \Psi \frac{\partial \phi}{\partial n} ds \qquad (48)$$

Applying this to (47) gives the result

$$F_j^D = -\rho \iint_{S'} \left(\phi_j^o - \frac{U}{i\omega}\phi_j^U\right)\frac{\partial \phi_D}{\partial n} ds \qquad (49)$$

Now the use of the hull boundary condition (12) leads to

$$F_j^D = \rho \iint_{S'} \left(\phi_j^o - \frac{U}{i\omega}\phi_j^U\right)\frac{\partial \phi_I}{\partial n} ds \qquad (50)$$

It is clear from (50) that the diffraction potential has been extracted from the problem. The normal derivative of the incident wave potential (44) becomes, after use of strip theory assumptions:

$$\frac{\partial \phi_I}{\partial n} = (i n_2 \sin\beta + n_3) K \phi_I \tag{51}$$

Using (51) and the motion potential relations (20) in Equation (50) and invoking the standard strip theory assumptions to transform the surface integral gives,

$$F_j^D = \rho\alpha \int_L e^{-iKx\cos\beta} \int_{C_x} e^{iKy\sin\beta} e^{Kz} \Big\{ \omega_o (i n_3 - n_2 \sin\beta)\phi_j^o \pm \omega_o \frac{U}{i\omega} \left[(i n_3 - n_2 \sin\beta)\phi_{3,2}^o \right]_{j=5,6} \Big\} \, dl \, dx \tag{52}$$

Making use of the two-dimensional sectional normal, and the concept of a sectional force, allows the following simplifications:

From (45):

$$F_j^I = \rho\alpha \int_L f_j(x)\, dx \qquad j = 2,3,4 \tag{53a}$$

$$F_5^I = -\rho\alpha \int_L x\, f_3(x)\, dx \tag{53b}$$

$$F_6^{I} = \rho\alpha \int_L x\, f_2(x)\, dx \tag{53c}$$

where

$$f_j(x) = g\, e^{-iKx\cos\beta} \int_{C_x} N_j\, e^{iKy\sin\beta} e^{Kz}\, dl \qquad j = 2,3,4 \tag{53d}$$

from (52):

$$F_j^{D} = \rho\alpha \int_L h_j(x)\, dx \tag{54a}$$

$$F_5^{D} = \rho\alpha \int_L \left(x + \frac{U}{i\omega}\right) h_3(x)\, dx \tag{54b}$$

$$F_6^{D} = -\rho\alpha \int_L \left(x + \frac{U}{i\omega}\right) h_2(x)\, dx \tag{54c}$$

where

$$h_j(x) = \omega_0 e^{-iKx\cos\beta} \int_{C_x} (iN_3 - N_2 \sin\beta)\, e^{iKy\sin\beta} \cdots$$
$$\cdots\; e^{Kz}\, \Psi_j\, dl \tag{54d}$$

The strip theory formulation of the linearized ship motions problem has now been completed. Once the sectional, two-dimensional potentials for sway, heave, and roll are determined by one of the methods mentioned (See Ref. [27]), then these can be used in (39) to produce the various sectional damping and added mass coefficients. Equation (40) and other similar relationships detailed in Ref. [27] can then be applied to find the added mass and damping matrices for the whole ship. This means that the left side of (24) is known. The exciting force is then determined from (41), (53), and (54) since the sectional potentials are known. After this (24) is easily solved for the complex motion amplitudes, ζ_k. These complex quantities contain both the real motion amplitudes and the phasing. It should be reemphasized that these motions are, by assumption, linear and harmonic in the frequency of encounter.

In a nonlinear analysis of the problem, higher order terms in the incident wave potential would tend to interact with the other potentials, producing periodic higher order forces with non-zero means. Of course, these forces tend to be small in comparison with the simple harmonic first order excitations. They are, in fact, negligible in the pitch, roll, and heave modes because of the strong hydrostatic restoring forces that are acting. However, they can act to produce large displacements over time in the horizontal modes. It can be seen, then, that in order to force the ship to perform the assumed motion at constant speed, U, and constant heading angle, β, in the 'real' ocean, an extra time varying force will have to be applied both longitudinally and transversely (also a moment will be necessary). In the absence of these the ship will drift and its speed will vary from that expected in calm water.

A method by Newman, already mentioned in Section II, makes possible the calculation of the mean of the second order force which produces the speed loss and drift. As will be shown, this can be done using only quantities known from the first order analysis.

The unsteady hydrodynamic force on a body in an inviscid medium with a free surface is given by Equation (29). This can be expressed in a more general form as follows:

$$\vec{F} = \iint_{S} p\vec{n}\, ds \tag{55}$$

where S is the wetted surface of the body. Applying Gauss' theorem and utlizing the fact that $p = 0$ on the free surface,

$$\vec{F} = \iiint_{V} \nabla p\, dv - \iint_{S_\infty} p\vec{n}\, ds \tag{56}$$

where S_∞ is a control surface in the far field and V is the volume enclosed between S, S_∞ , and the free surface. Bernoulli's equation gives the pressure in terms of the total velocity potential:

$$\vec{F} = -\rho \iiint_V \left\{ \nabla \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla |\nabla \Phi|^2 \right\} dv + \rho \iint_{S_\infty} \left\{ \frac{\partial \Phi}{\partial t} + \frac{1}{2} |\nabla \Phi|^2 \right\} ds \quad (57)$$

The transport theorem[23] shows that

$$\frac{d}{dt} \iiint_V \nabla \Phi \, dv = \iiint_V \nabla \frac{\partial \Phi}{\partial t} dv + \iint_{S+S_F} \nabla \Phi \frac{\partial \Phi}{\partial n} ds \quad (58)$$

where S_F is the free surface. The truth of (58) depends on the fact that $\vec{V} \cdot \vec{n} = 0$ on S_∞ and $\vec{V} \cdot \vec{n} = \frac{\partial \phi}{\partial n}$ on S and S_F (where $\vec{V}$ is the surface velocity). Using (58) in (57) yields

$$\vec{F} = -\rho \frac{d}{dt} \iiint_V \nabla \Phi \, dv + \rho \iint_{S+S_F} \nabla \Phi \frac{\partial \Phi}{\partial n} ds - \frac{\rho}{2} \iiint_V \nabla |\nabla \Phi|^2 dv + \rho \iint_{S_\infty} \left\{ \frac{\partial \Phi}{\partial t} + \frac{1}{2} |\nabla \Phi|^2 \right\} \cdot \vec{n} \, ds \quad (59)$$

Again, Gauss' theorem can be employed to change the first volume integral to a surface integral, with the result:

$$-\rho\frac{\partial}{\partial t}\iiint_V \nabla\Phi\, dv = -\rho\frac{\partial}{\partial t}\iint_{S+S_F} \Phi\cdot\vec{n}\, ds - \rho\iint_{S_\infty} \frac{\partial\Phi}{\partial t}\cdot\vec{n}\, ds \tag{60}$$

When this is included in (59) the result is

$$\vec{F} = -\rho\frac{\partial}{\partial t}\iint_{S+S_F} \Phi\cdot\vec{n}\, ds + \rho\iint_{S+S_F} \nabla\Phi\frac{\partial\Phi}{\partial n}\, ds - \frac{1}{2}\rho\iiint_V \nabla|\nabla\Phi|^2 dv + \frac{1}{2}\rho\iint_{S_\infty} |\nabla\Phi|^2\vec{n}\, ds \tag{61}$$

Once more invoking Gauss' theorem,

$$\frac{1}{2}\iiint_V \nabla|\nabla\Phi|^2 dv = \iint_{S+S_F+S_\infty} \frac{\partial\Phi}{\partial n}\nabla\Phi\, ds \tag{62}$$

Putting (62) in (61) gives as a final result,

$$\vec{F} = -\rho\frac{\partial}{\partial t}\iint_{S+S_F} \Phi\cdot\vec{n}\,ds - \rho\iint_{S_\infty}\left\{\frac{\partial\Phi}{\partial n}\nabla\Phi - \frac{1}{2}|\nabla\Phi|^2\vec{n}\,ds\right. \qquad (63)$$

The expression (63) provides the force on a body in a fluid with a free surface exactly (within the limits of the theory of potential flow). Using a first order potential in (63), would yield (after linearization) the oscillatory hydrodynamic force components (30) and (31). The simplest higher order potential that could be used in the problem would be of the form

$$\Phi = \left(\phi_S^{(1)} + \phi_S^{(2)} + \ldots\right) + \left(\phi_T^{(1)} e^{i\omega t} + \phi_T^{(2)} e^{2i\omega t} + \phi_{D.C.}^{(2)} + \ldots\right) \qquad (64)$$

where the numbers in paraentheses indicate the order of the terms and the 'D.C.' is responsible for the non-zero mean of the oscillatory potential. Putting (64) in (63) shows that there will be no net contribution from the first integral because the DC-potential has no time derivative. The second integral will give a steady state contribution which can be developed by writing $\Phi = \Phi_I + \Phi_B$, substituting in (63), and performing the vector calculus,

$$\vec{F} = \rho \iint_{S_\infty} \left(\Phi_B \frac{\partial}{\partial n} - \frac{\partial \Phi_B}{\partial n} \right) \left(\nabla \Phi_I + \frac{1}{2} \nabla \Phi_B \right) ds \qquad (65)$$

Newman applied a 'weak scatterer' assumption at this point ($\Phi_B << \Phi_I$). This is easily justified for a body submerged beneath the surface. Salvesen reasoned that this might also be true for a slender body like a ship, an assumption that will be more accurate for head than beam seas. The result is

$$\vec{F} = \rho \iint_{S_\infty} \left(\Phi_B \frac{\partial}{\partial n} - \frac{\partial \Phi_B}{\partial n} \right) \nabla \Phi_I \, ds \qquad (66)$$

Writing $\Phi_B = \phi_S + \phi_B e^{iwt}$ and $\Phi_I = \phi_I e^{iwt}$, taking the mean value of (66), and using Green's theorem to change the integration surface from the far-field to the body, gives the following:

$$\vec{\mathfrak{F}} = \frac{1}{T} \int_T \vec{F} dt = -\frac{1}{2} \rho \iint_S \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n} \right) \nabla \phi_I^* \, ds \qquad (67)$$

where ( )* refers to the complex conjugate and $\vec{\mathcal{F}}$ represents the mean force vector for one regular wave period.

Since only the horizontal component of $\vec{\mathcal{F}}$ is of interest, $\nabla\phi_I{}^*$ can be written,

$$\nabla\phi_I^* = iK\left(\cos\beta\,\hat{i} - \sin\beta\,\hat{j}\right)\phi_I^* \tag{68}$$

Substitution of (68) in (67) allows the magnitude of (67) to be given as,

$$|\vec{\mathcal{F}}| = \mathcal{F} = -\frac{i}{2}\rho K \iint_{S'} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n}\right)\phi_I^* \, ds \tag{69}$$

In the ship coordinate system, the beam and lengthwise components of this mean force will be,

$$\mathcal{F}_x = \mathcal{F}\cos\beta \qquad \mathcal{F}_y = \mathcal{F}\sin\beta \tag{70}$$

which follows because $\vec{\mathcal{F}}$ is in the direction of wave propagation. It should be noted that (69) is a form of the Kochin function, first developed in connection with this problem by Haskind.

As a consequence of the fact that the mean higher order force contribution has been reexpressed in terms of the full body potential, substitution of the first order body potential of strip theory will not give an approximation of the net force contributed by the higher order DC terms in the full potential (64). The extra mean force necessary to sustain the prescribed body motion in waves is given by substituting $\phi_B \approx \sum_{j=2}^{6} \phi_j \xi_j + \phi_D$ in (69).

$$\mathcal{F} = -\frac{i}{2}\rho K \sum_{j=2}^{6} \left\{ \xi_j \iint_{S} \left(\phi_j \frac{\partial}{\partial n} - \frac{\partial \phi_j}{\partial n} \right) \phi_I^* \, ds \right\} - \frac{i}{2}\rho K \iint_{S} \left(\phi_D \frac{\partial}{\partial n} - \frac{\partial \phi_D}{\partial n} \right) \phi_I^* \, ds \tag{71}$$

This can be rewritten for computational purposes as

$$\mathcal{F} = \sum_{j=2}^{6} \left\{ \mathcal{F}_j^I + \mathcal{F}_j^D \right\} + \mathcal{F}_D \tag{72a}$$

where

$$\mathcal{F}_j^I = \frac{i}{2}\rho K \xi_j \iint_{S} \frac{\partial \phi_j}{\partial n} \phi_I^* \, ds \tag{72b}$$

$$\mathcal{F}_j^D = -\frac{i}{2}\rho K \zeta_j \iint_{S'} \phi_j \frac{\partial \phi_I^*}{\partial n} ds \tag{72c}$$

and after application of the body boundary condition (12)

$$\mathcal{F}_D = -\frac{i}{2}\rho K \iint_{S'} \phi_D \frac{\partial \phi_I^*}{\partial n} ds \tag{72d}$$

The next problem is to express Equations (72) in strip theory terms. Examining (72b) first, a substitution of the boundary condition (14) gives

$$\mathcal{F}_j^I = \frac{i}{2}\rho K \zeta_j \iint_{S'} (i\omega n_j + U m_j)\, \phi_I^* ds \tag{73}$$

Applying the variant of Stokes theorem (35) and substituting for ω_0 from (8):

$$\mathcal{F}_j^I = -\frac{1}{2}\rho K \zeta_j \iint_{S'} \omega_0 n_j \phi_I^* ds \tag{74}$$

Comparing this with (45) reveals that:

$$\mathcal{F}_j^I = \frac{i}{2} K \mathcal{f}_j (F_j^I)^* \tag{75}$$

This can easily be shown by trigonometric identity to be the same as the formula developed by Havelock (1).

Continuing now to (72c), it follows from the strip theory assumption, $n_1 << n_2$ or n_3, and (44) that

$$\frac{\partial \phi_I^*}{\partial n} = K(-n_3 + i n_2 \sin\beta)\phi_I^* \tag{76}$$

Including (76) in (72c) and replacing ϕ_j using (21)

$$\mathcal{F}_j^D = \frac{i}{2} \rho K^2 \mathcal{f}_j \iint_S \phi_j^\circ (-n_3 + i n_2 \sin\beta)\phi_I^* ds \qquad j = 2,3,4 \tag{77a}$$

$$\mathcal{F}_{5,6}^D = \frac{i}{2} \rho K^2 \mathcal{f}_{5,6} \iint_S \left(\phi_{5,6}^\circ \pm \frac{U}{i\omega} \phi_{3,2}^\circ\right) \ldots \ldots (-n_3 + i n_2 \sin\beta)\phi_I^* ds \tag{77b}$$

The next step is to replace the unknown 3-D potentials with the sectional potentials according to (38) and transform the surface integral,

$$\mathcal{F}_j^D = \frac{i}{2} K \mathcal{S}_j \int_L \hat{h}_j(x)\, dx \quad j = 2,3,4 \tag{78a}$$

$$\mathcal{F}_{5,6}^D = \mp \frac{i}{2} K \mathcal{S}_{5,6} \int_L \left(x + \frac{iU}{\omega}\right) \hat{h}_{3,2}(x)\, dx \tag{78b}$$

where a sectional quantity $\hat{h}_j(x)$ has been defined as

$$\hat{h}_j(x) = \rho K \int_{C_x} \Psi_j (-N_3 + i N_2 \sin\beta)\, \phi_I^* \, dl \qquad j = 2,3,4 \tag{78c}$$

$$\hat{h}_j(x) = \rho \alpha \omega_o e^{iKx\cos\beta} \int_{C_x} (N_2 \sin\beta + i N_3) \ldots$$

$$\ldots e^{-iKy\sin\beta} e^{-Kz} \Psi_j \, dl \qquad j = 2,3,4 \tag{78d}$$

The similarity between (78) and (54) can be seen; however, the two are not algebraically related and must be computed separately. For simplicity, let:

$$\hat{F}_j^D = \int_L \hat{h}_j(x)\,dx \qquad j = 2,3,4 \tag{79a}$$

$$\hat{F}_{5,6}^D = \mp \int_L \left(x + \frac{iU}{\omega} \right) \hat{h}_{3,2}(x)\,dx \tag{79b}$$

giving

$$\mathcal{F}_j^D = \frac{i}{2} K \mathcal{S}_j \hat{F}_j^D \tag{79c}$$

Finally, consider the third contributor (72d). Substituting (76) and rewriting the expression:

$$\mathcal{F}_D = \frac{1}{2} \int_L h_D(x)\,dx \tag{80a}$$

$$h_D(x) = i\rho K^2 \int_{C_x} \phi_D (-n_3 + i n_2 \sin\beta) \phi_I^* \,dl \tag{80b}$$

Including the expression for ϕ_I^*:

$$h_D(x) = \alpha \rho K \omega_o e^{iKx\cos\beta} \int_{C_x} \phi_D(-n_3 + in_2\sin\beta) e^{-Kz} e^{iKy\sin\beta} d\ell \tag{81}$$

In order to simplify computation, it is consistent with the strip theory assumptions already made to replace e^{+Kz} with $e^{+Kd\sigma}$ and $e^{-iKy\sin\beta}$ with $e^{-iK(\pm\frac{1}{2}b)\sigma\sin\beta}$, where $d \equiv$ sectional draft, $\sigma \equiv$ sectional area coefficient (A_x/bd), and $b \equiv$ sectional beam. The first of these is conventional in strip theory and the second should be legitimate if $\lambda \gg \frac{1}{2}b$. All this gives:

$$h_D(x) = \alpha \rho K \omega_o e^{-Kd\sigma} e^{-iK(\pm\frac{1}{2}b)\sigma\sin\beta} e^{iKx\cos\beta} \cdots\cdots \underbrace{\int_{C_x} \phi_D(-n_3 + in_2\sin\beta) d\ell}_{I_D} \tag{82}$$

The integral in (82) can be rewritten by substituting for the hull normal from (22) and using Green's second identity (48) in two dimensions.

$$I_D = -\frac{i}{\omega}\int_{C_x}(-\phi_3^o + i\sin\beta\phi_2^o)\frac{\partial\phi_D}{\partial n}\,d\ell \qquad (83)$$

Using the now familiar hull condition (12), writing out the wave potential (44) and applying the same assumptions about e^{-KZ} and $e^{iKy\ \sin}$ outlined above in route to (82) gives:

$$h_D(x) = \alpha^2\rho K\frac{\omega_o^2}{\omega}e^{-2Kd\sigma}\int_{C_x}\left\{\phi_3^o n_3 + \sin^2\beta\,\phi_2^o n_2\right\}d\ell \qquad (84)$$

where the symmetric section assumption has been used to neglect two cross products involving the potentials and hull normals. Examination of (84) will reveal that when the two dimensional normals and sectional potentials are introduced (39) will be directly applicable. This will allow the reexpression of (84) in terms of the sectional added mass and damping already known.

$$h_D(x) = \alpha^2 K\frac{\omega_o^2}{\omega}e^{-2Kd\sigma}\left\{b_{33}(x) + \sin^2\beta\,b_{22}(x)\right\} \qquad (85)$$

Where (85) represents just the real part of (84) because the imaginary part is not needed in (72a). (The reader is reminded, "...only the real part is to be taken in expressions involving e^{iwt} that appear in this derivation.")

In summary, the mean second order 'DC' force on a ship in regular waves is available from the real part of the following expression,

$$\mathfrak{F} = \sum_{j=2}^{6} \frac{i}{2} K \, \mathfrak{f}_j \left[(F_j^I)^* + \hat{F}_j^D \right] + \mathfrak{F}_D \qquad (86)$$

where the motions are available from the first order computation already outlined, $(F_j^I)^*$ is also available from that process, $\hat{F}_j^D$ can be developed from (79) using strictly quantities known from strip theory, and $\mathfrak{F}_D$ comes from (80) in combination with (85). Then the added resistance and drift force can be given by simply applying (70). The M.I.T. 5-D motions program has been modified to properly extract and recombine these quantities (See Appendices for details).

IV. VERIFICATION OF THEORY

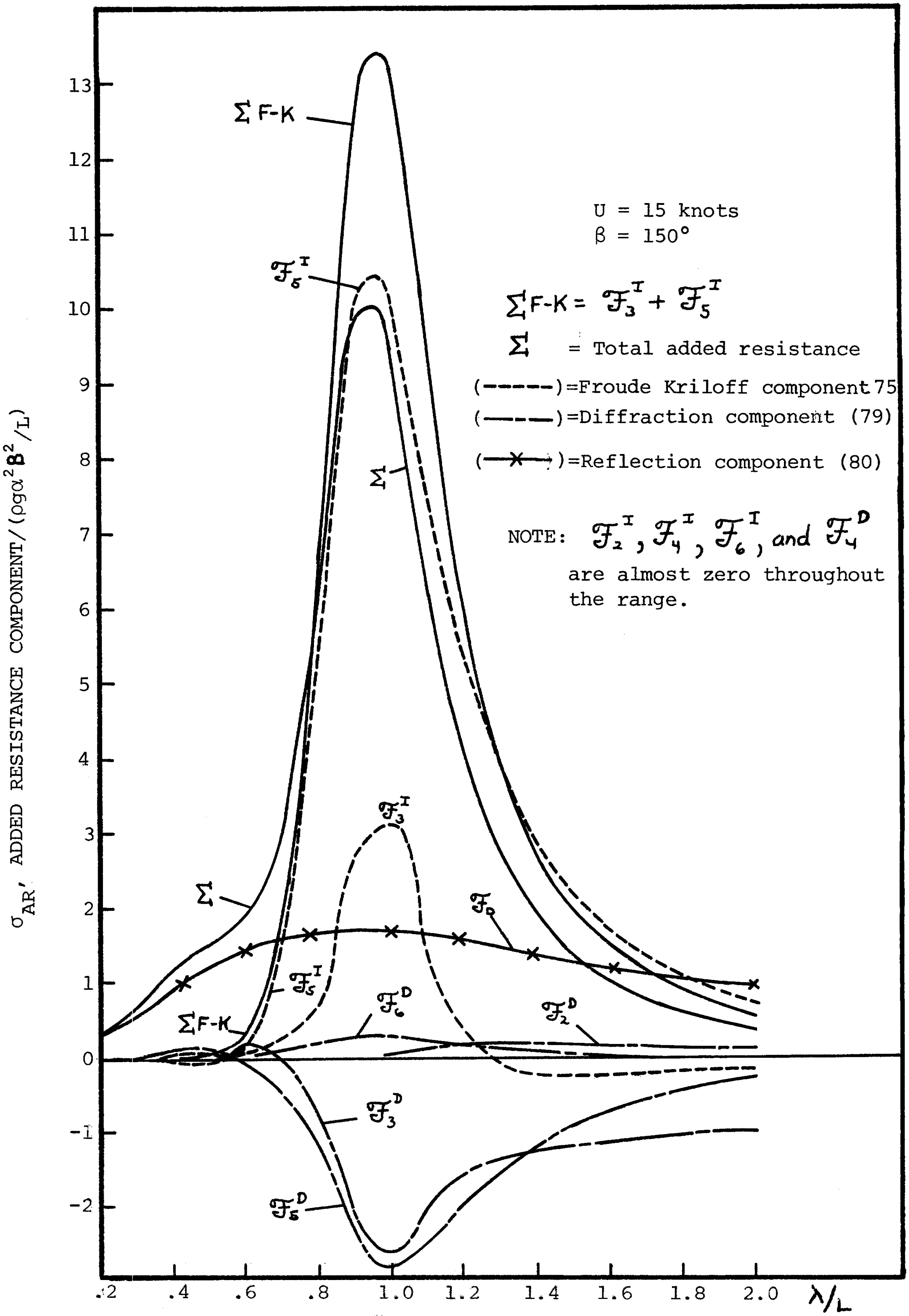
In Chapter III it was shown how the added resistance and drift force could be computed for any ship using Equations (86) and (70). This capability was incorporated in the M.I.T. 5-D motions program in the form of two subroutines (ADDRES and RESIST). The first of these is primarily an output organizer; the second does the actual calculation outlined earlier. As mentioned before, all the input necessary is directly available from the first-order motions calculation except the sectional quantity given by (78d). This quantity is generated from the sectional potentials within subroutine INTRPL. A user's manual for the program, briefly describing these routines, is presented in the appendices, together with input and output samples.

In order to maximize the possibilities for comparison with existing second-order force results, the Mariner-type fast cargo vessel was used in all the examples that follow. The Mariner was designed about 1950 at the Bethlehem Steel yard in Quincy, MA[25]. She has a length-between-perpendiculars of 528 feet, a beam of 76 feet, and a service speed of 20 knots. For this study, a full load draft of 29.75 feet was chosen giving a displacement of 21,000 tons. The pitch radius of gyration was set at about 25% of the L.B.P.

Further details on the ship can be obtained from Appendix D which gives a copy of the input used for this study in the M.I.T. 5-D program.

As a first step in a consideration of the theory, Equation (86) will be examined in detail for one particular oblique regular seas case ($\beta = 150°$, $U = 15$ knots). From the form of the equation, it is clear that the second-order force prediction is generated as the sum of eleven components: Froude-Kriloff and Diffraction terms for each of the five modes of motion and the wave reflection term. These components are all plotted separately in Figure 2. The non-dimensionalized added resistance component (defined by σ_{AR} = Added Resistance/$\rho g \alpha^2 B^2/L$, where B is the ship beam, L is the L.B.P., and α is the wave amplitude) is given as a function of wave-length to ship length ratio. For this case, the Froude-Kriloff pitch term ($\mathfrak{F}_5^I$, Eq. (75), $j = 5$) makes a large contribution as does the Froude-Kriloff heave term ($\mathfrak{F}_3^I$, Eq. (75), $j = 3$). The wave reflection term ($\mathfrak{F}_D$ in Eq. (80)) also makes a contribution, particularly for very short waves, where it is totally dominant. The heave and pitch 'wave diffraction' terms ($\mathfrak{F}_3^D$, $\mathfrak{F}_5^D$, Eq. (77), $j = 3,5$) act to reduce the added resistance considerably, while all the terms involving roll, sway, and yaw are practically insignificant. These relative magnitudes persist

 FIGURE 2: MARINER ADDED RESISTANCE COMPONENTS (Terms in Eq. (86))



in general in all near-head sea conditions, but in near beam seas, the pitch terms, $\mathfrak{F}_5^I$ and $\mathfrak{F}_5^D$, play a much less important role while the sway, yaw, and reflection terms, ($\mathfrak{F}_2^D$, $\mathfrak{F}_2^I$, $\mathfrak{F}_6^D$, $\mathfrak{F}_6^I$, and $\mathfrak{F}_D$) increase their relative magnitudes. In following waves, the contributions of the various components are heavily dependent on encounter frequency and the associated accuracy of the strip theory, a question which will be discussed later at greater length.

It can be seen that all the significant motion-related second-order force components peak at one place (generally near the heave or pitch resonance, respectively) producing a sharp peak in the total force (marked Σ). This sharp peak is present for all heading angles, although its location varies, depending primarily on the location of the heave and pitch peaks in bow and beam waves. One final observation that can be made is that, for near head seas, the original formula of Havelock((2), marked ΣF-k) can be applied with some success. Since all the force components peak in the same area, and Havelock's formula contains two of the most significant positive terms, it will predict the peak location for the second-order force. Several investigators have found the magnitude of its predictions to be within a factor of two in most cases[5,28]. This has engineering significance because the Froude-Kriloff force (See (75) and (45)) can be computed easily without knowledge of the sectional potentials. In fact, the calculations can be done by hand if a first order motions printout giving sectional Froude-Kriloff force is available.

Having generated a working second-order force computational scheme based on (86), the next step is to ascertain its validity. This will be done by comparison with experimental results and analytical predictions generated by other methods.

A. Comparison With Experimental Results

Experimental efforts on the second-order force problem are historically very complex and not too repeatable. The main reason for most of the difficulties can be easily identified. The periodic forces involved are extremely small,making measuremnt very difficult, especially in the presence of friction in the mechanical equipment, vibration in the towing carriage, and electronic noise. Most of the towing tank work that has been done has been directed toward finding the added resistance in regular head seas. There are two methods for carrying out these measurements: constant velocity (where the model is free only to heave and pitch) and constant thrust (where a self-propelled model is attached to a movable sub-carriage and allowed to surge, as well). All the experimental work presented in this report was obtained using the constant velocity method This allows a more effective comparison with the computer program, which neglects surge.

As mentioned before, the second-order force is primarily a wave phenomenon and can be scaled by Froude number so that the force on the real ship will be proportional to the model force times the cube of the scale ratio. Pure Froude scaling also implies that the use of small models is justifiable, but this must be tempered with the realization that the forces involved must remain measurable. It is also important that the wave height be kept fairly small to be consistent with the linear strip theory.

The paper by Strom-Tejsen, et. al.[29] contains a detailed description of an experimental program carried out at NSRDC to determine the added resistance in head seas of a range of Series 60 models, a destroyer, and a high-speed form. Other extensive head sea experiments have been done by Gerritsma and Beukelman[5], Beck and Wang at M.I.T.[2,31], and the University of Osaka[10]. Sibul has also done a great deal of work with Series 60 models at the University of California [28], and some of his results are shown in Figure 3.

It is intended that this figure will be representative

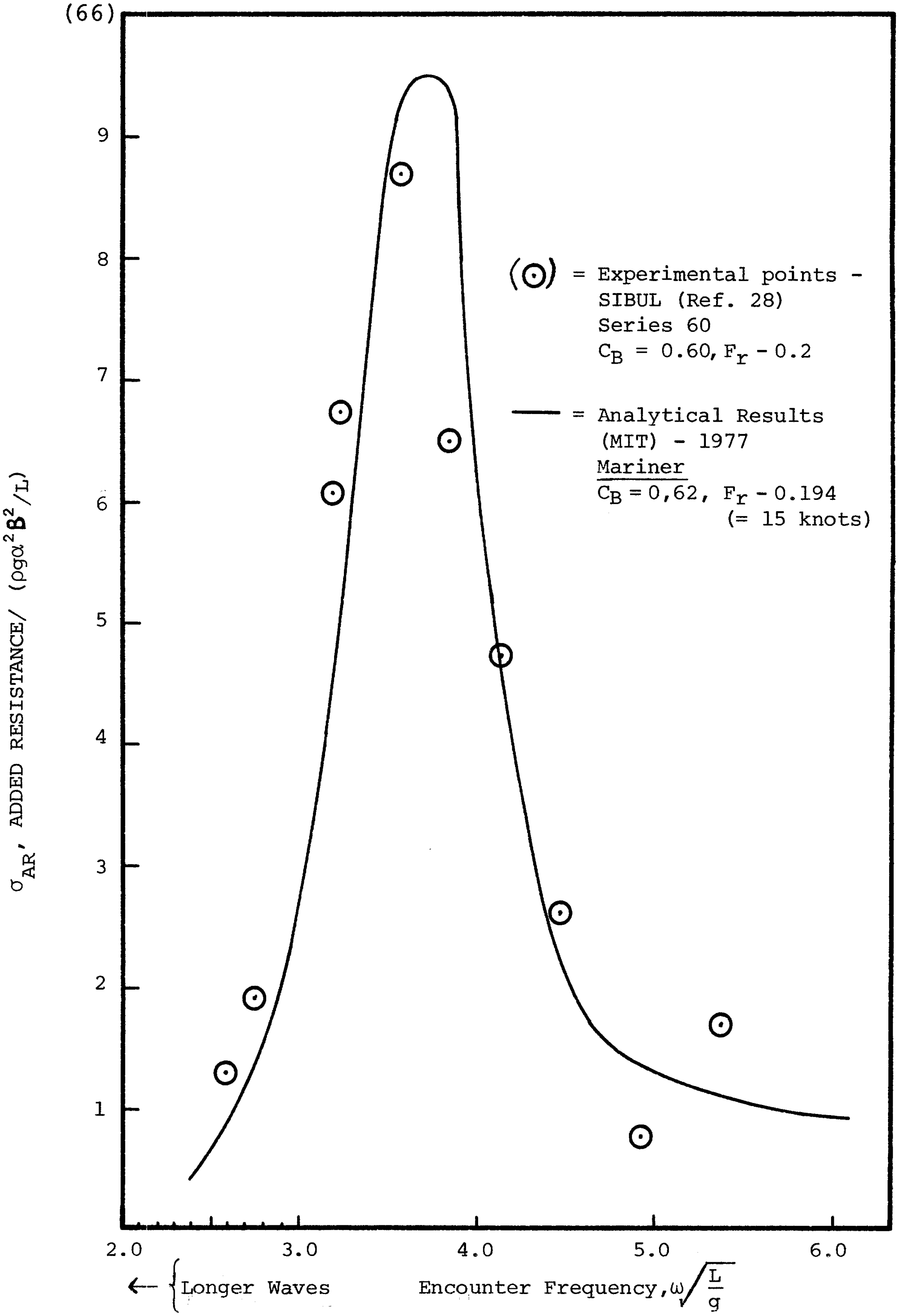


FIGURE 3: ADDED RESISTANCE/HEAD WAVES

of the fact that Equation (86) provides a very good correlation with experiments for the familiar case of head seas. The plot shows experimental results for a Series 60 model very similar to the Mariner at nearly the same Froude number. The agreement with (86) is well within experimental error throughout most of the range of practical wavelengths. However, acceptable theoretical prediction methods for added resistance in head seas have been available for several years, and this report is primarily intended to address the more general problem of second-order force in oblique seas.

As soon as the head seas restriction is lifted, the experimental difficulties involved become almost insurmountable. A large basin must be available to achieve the desired range of heading angles, furthermore the model must now ideally be allowed complete freedom of motion, making measurement very difficult. Many other complexities might be enumerated, but it is perhaps sufficient to say that very little oblique seas second-order force data is presently available. Spens and Lalangas have done some work on a Series 60 model at Stevens, measuring drift force and yawing moment[28]. Also, Hosoda presents a few results for a container ship[10]. Journee[12] did work at Delft with another container ship in following seas.

The work done at Stevens has already been employed by

Salvesen for comparison in his paper[28]. Since comparison with Salvesen's calculations will later be presented, the Stevens results are not reproduced here. Furthermore, ship characteristics were not readily available for the other cases. Consequently, it was decided that an oblique seas experiment should be carried out in order to provide a basis for theoretical comparison in this report. The only work of this nature that could be done in the M.I.T. ship model towing tank involved near beam sea cases at zero speed. The zero speed restriction is obvious since the carriage and the generated waves must run parallel to the long axis of the conventionally shaped tank. The beam seas restriction was intended to minimize interference between the outgoing damping waves of the ship and the tank walls.

An existing fiberglass Mariner model (L.B.P. = 5.42') was employed for the experiment. It was mounted beamwise in the tank and connected to the carriage by a heave staff and a roll bearing. All other modes of motion were thus restrained. The M.I.T. tank is 108' long, 8-1/2' wide, and 4' deep, and the model was positioned at approximately the halfway point. Waves were generated by a pivoted hydraulically driven paddle. Only regular sinusoidal waves of varying length were used.

Instrumentation consisted of a dynamometer-force block fixed transversely in the model to measure drift force, a transducer on the heave rod, a transducer on the roll bearing, and an electrical resistance wave probe for measuring incident wave height. All the data was taken to a Sanborn multi-channel chart recorder which provided a real-time visualization of the signals. Data was taken at regular wavelengths ranging from about one-half to about four times the ship length. Smaller waves could not be generated, and longer ones were pointless because of the 'deep water wave' assumption inherent to the theory. Newman states[23] that if the depth over wavelength ratio becomes less than 1/2, the 'deep water wave' assumption begins to break down. This affects the incident wave potential formulation (44). For a wave in the tank four times the ship length, the ratio is less than 1/4.

Throughout the experiment wave height was kept constant at about 1.25 inches. This represented a compromise between the desires for small wave heights for linearity and large wave heights for measurable forces. At the most, the mean forces measured represented a few tenths of a pound, and the motions were attimes unavoidably large. The experimental procedure consisted of sending a regular wave train toward the model and taking data until a steady state was obtained in all the responses. The data record was stopped when the incident regular wave train became contaminated (e.g. by reflected waves from the towing tank beach.) The mean drift force was obtained by graphical measurement using the output from the chart recorder. The wave amplitude (necessary for nondimensionalization) was similarly estimated. The resulting experimental points are shown as circled-dots on Figures 4, 5, and 6 for the three heading angles investigated ($\beta = 90°$, 105°, and 75°, respectively).

Two drift force computations are also shown on each of the graphs. The solid line corresponds to the full five degrees of freedom prediction, (Eq. (86), $j=2,..6$) while the starred line represents a calculation in which the computer program was artificially restrained* to represent a roll-heave two-degree of freedom, system. (Eq. (86), $j = 3,4$). The computer was run using a higher metacentric height (Scale, 8') and a smaller roll radius of gyration (Scale, 11') than the real ship because it was very impractical to adjust the measured model characteristics. At any rate, this made little difference in the predictions, because these changes impacted most heavily on roll response, and roll is always a small contributor to the second-order force calculated in (86).

Examination of the three graphs will show that the expected level of agreement is obtained for all wavelengths

*see Appendix C.

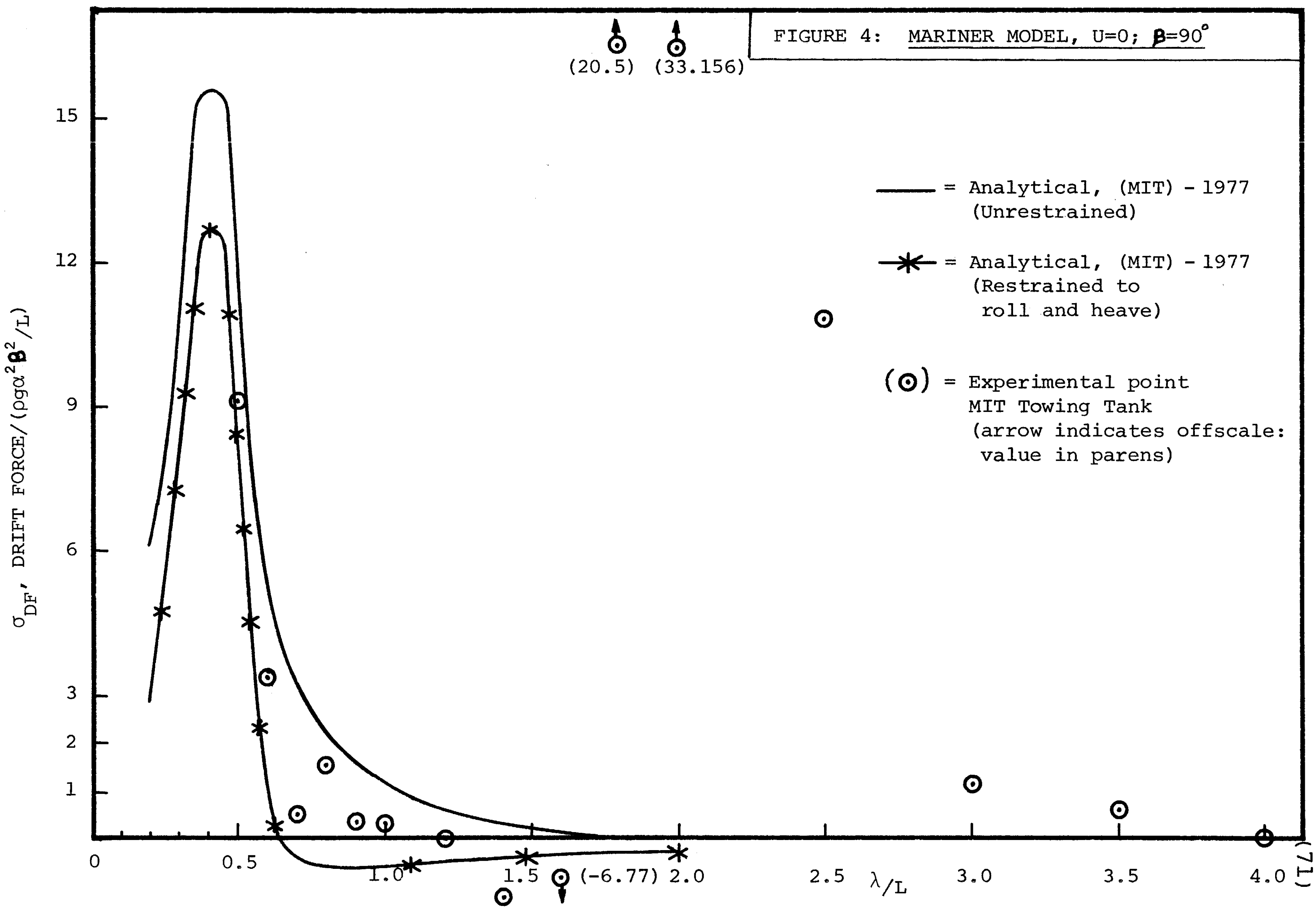


FIGURE 4: MARINER MODEL, U=0; β=90°
(20.5)
(33.156)
(-6.77)
= Analytical, (MIT) - 1977 (Unrestrained)
= Analytical, (MIT) - 1977 (Restrained to roll and heave)
(⊙) = Experimental point MIT Towing Tank (arrow indicates offscale: value in parens)
σDF, DRIFT FORCE/(ρgα²B²/L)
λ/L
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1
2
3
6
9
12
15

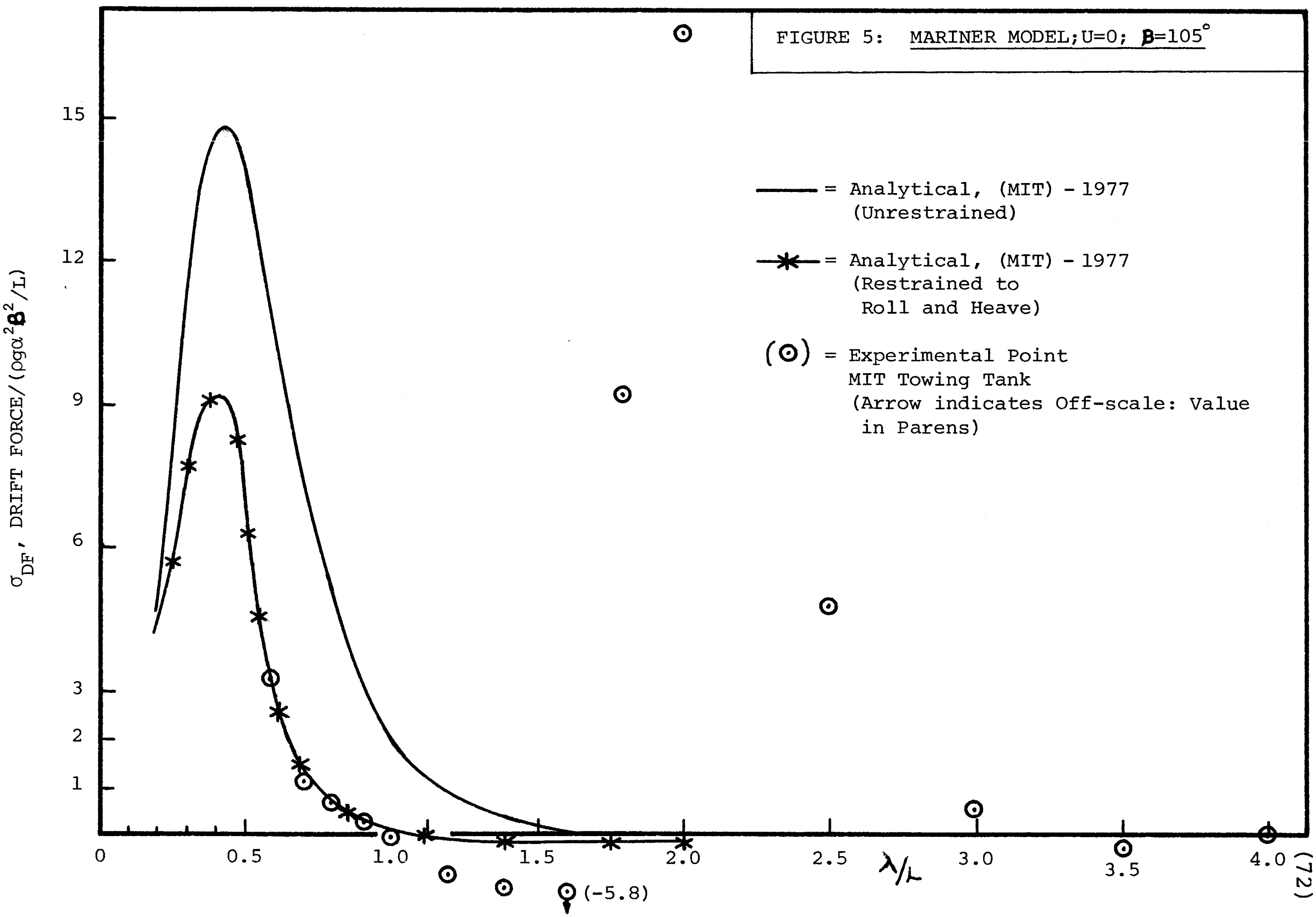


FIGURE 5: MARINER MODEL; U=0; β=105°

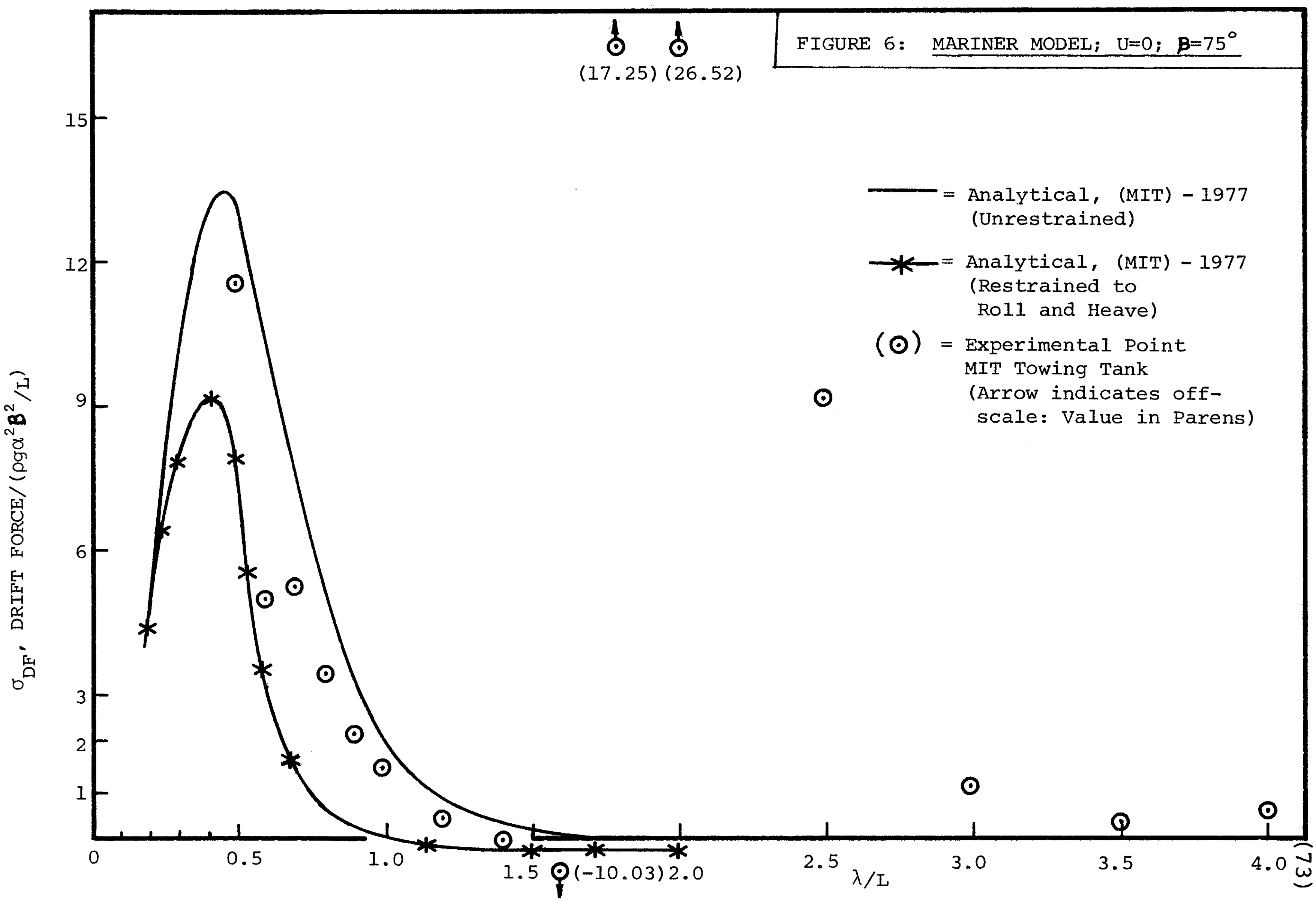


FIGURE 6: MARINER MODEL; U=0; β=75°
(17.25) (26.52)
= Analytical, (MIT) - 1977 (Unrestrained)
= Analytical, (MIT) - 1977 (Restrained to Roll and Heave)
(⊙) = Experimental Point MIT Towing Tank (Arrow indicates off-scale: Value in Parens)
(-10.03)
σDF, DRIFT FORCE/(ρgα²B²/L)
λ/L
15
12
9
6
3
2
1
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

between 0.5 and about 1.4 times the ship length. This is particularly true for the $\beta = 105°$ case (Figure 5), where the points fall nicely on the line predicted by the computer using the two-degree of freedom assumption. It can also be seen that for wavelengths greater than about 3.5 times the ship length, the mean forces go to zero as the computer predicts.

However, a serious surprise is contained in the data for the wavelengths between about 1.5 and 3.5 times the ship length. In this area on all three graphs, the computer predicts near zero mean force, but the measured results 'blow-up' as the wavelength increases. A solution or a full explanation for this theoretical discrepancy is, as yet, unavailable.

It can be noted that the model attains roll resonance in the area of $\lambda/L = 1.7$ and sustains a very large roll angle (no less than plus or minus 15 degrees). Large roll amplitudes persist up to $\lambda/L = 3.0$. Furthermore, when the roll natural period of the ship was shifted by using outrigger weights, the location of the problem area shifted with it. Currently, it is felt that the observed behavior does not represent a problem with the measurement equipment. Rather, it is supposed that this represents a nonlinear interaction associated with the large roll angle. This would explain the lack of correlation with any linear, small motion theory. An appreciation for the experimental difficulties involved in these measurements can be obtained, when it is realized that the measurements described above as 'blowing-up' represent a few tenths of a pound on model scale. In closing, it should be emphasized that good agreement was obtained over a sizable portion of the range where linear theory would be expected to apply.

B. Comparison With Other Theoretical Results

In the preceding portion of this chapter, Salvesen's second-order force was examined in some detail in order to gain insight into the behavior of its various components. Then it was compared with some existing and some new experimental results. The outcome was generally quite favorable in the area where linear theory would be expected to work, but some serious questions were also raised concerning applications in the area of roll resonance.

The next available step is to compare the results of the theory in the M.I.T. 5-D program with the results from other programs and linear theories. To date, the only second order force computational results published for oblique seas have been offered by Salvesen[28] and Loukakis (Ref. [16]).

Salvesen also programmed Equation (86), but he did so within the framework of the Naval Ship Research and Development Center Ship Motion and Sea Load Program. This program uses a different scheme for creating the two-dimensional sectional potentials. It also incorporates several other differences in numerical techniques, so his results (called 'Salvesen' and labeled NAV) can be used to assess the impact of computational procedure on the theoretical predictions.

Loukakis presents results called 'Loukakis' and labeled (NTUA) which are based on a different theory. His method was briefly mentioned in Chapter II. It is based on the original work of Gerritsma and Beukelman, and it represents a totally different "radiated energy" formulation of the problem. However, this new theory was implemented on a version of the M.I.T. 5-D motions program, so the two theories can be compared without undue concern for the impact of numerical techniques. Admittedly, there will be discrepancies because Loukakis made his computation using only vertical motions in order to avoid a roll resonance problem. To summarize, it will be possible to make comparisons with the same theory in a different motions program (M.I.T.-NAV) and a different theory in the same motions program (M.I.T.-NTUA). The ship for these comparisons is the Mariner-type fast cargo vessel already described.

Figure 7 shows the predicted impact of forward speed on added resistance in head waves. Results are plotted for

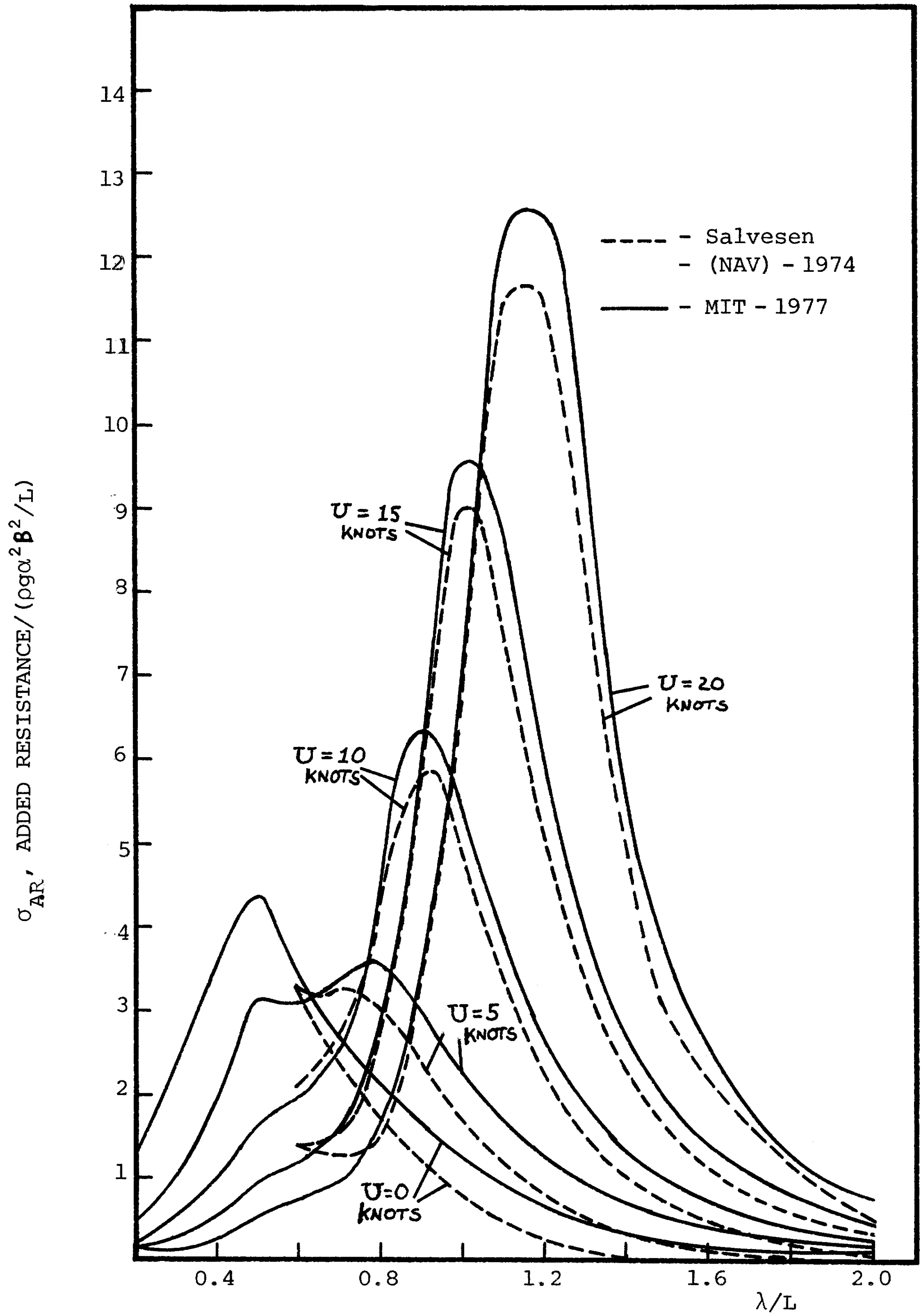


FIGURE 7: MARINER- ADDED RESISTANCE HEAD WAVES
(β = 180) VARIOUS SPEEDS

both the M.I.T. program (MIT) and the Salvesen program (NAV). The magnitude of the peak appears to increase with the speed, except at speeds near zero. The higher speeds also tend to reach their associated peak values at longer wavelengths coinciding with the heave and pitch peaks. This is very important when the graph is considered as a response operator for use in the spectral analysis of irregular sea states because most of the ocean energy is in relatively long waves. The two computational methods compare well for the shorter wavelengths before the peaks; however, some discrepancies can be seen in the medium wavelength area beyond the peak. No explanation can be offered for this since the (NAV) program details are unavailable to the author. There is also some difference in the actual peak value predicted, but it is very difficult to obtain enough data points to truly characterize the peak. Of course, for long waves, all the results go to zero.

Figures 8 and 9 show the general effect of a heading change in bow waves on the added resistance component of the second order force. Figure 8 shows Salvesen and M.I.T. calculations, and Figure 9 gives Loukakis and M.I.T. Both Figures 8 and 9 were run at a speed of fifteen knots. The graphs show how the second order force peak comes in much shorter waves as Beta goes to ninety degrees, following

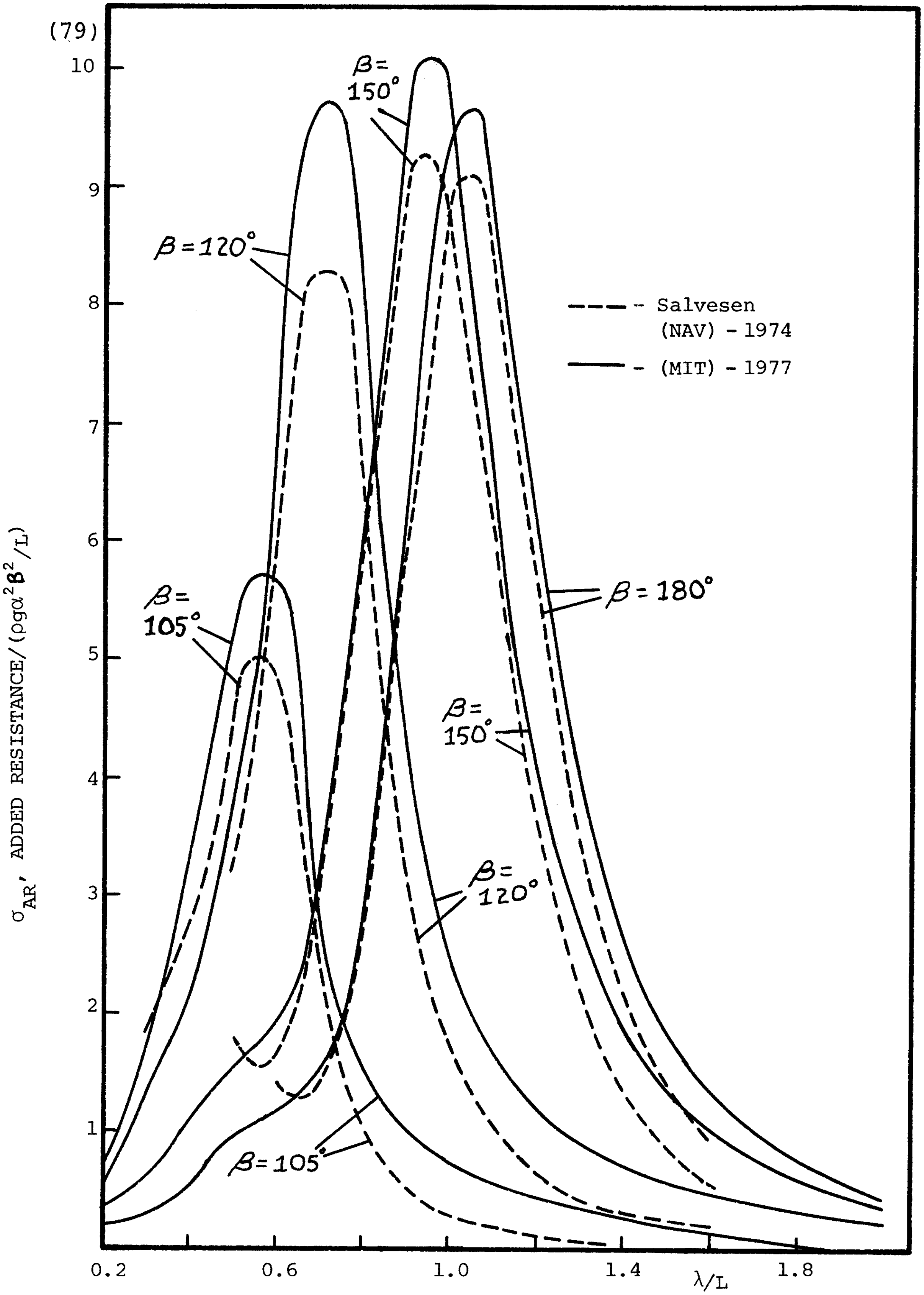


FIGURE 8: MARINER - ADDED RESISTANCE (SPEED - 15 knots) Various Headings

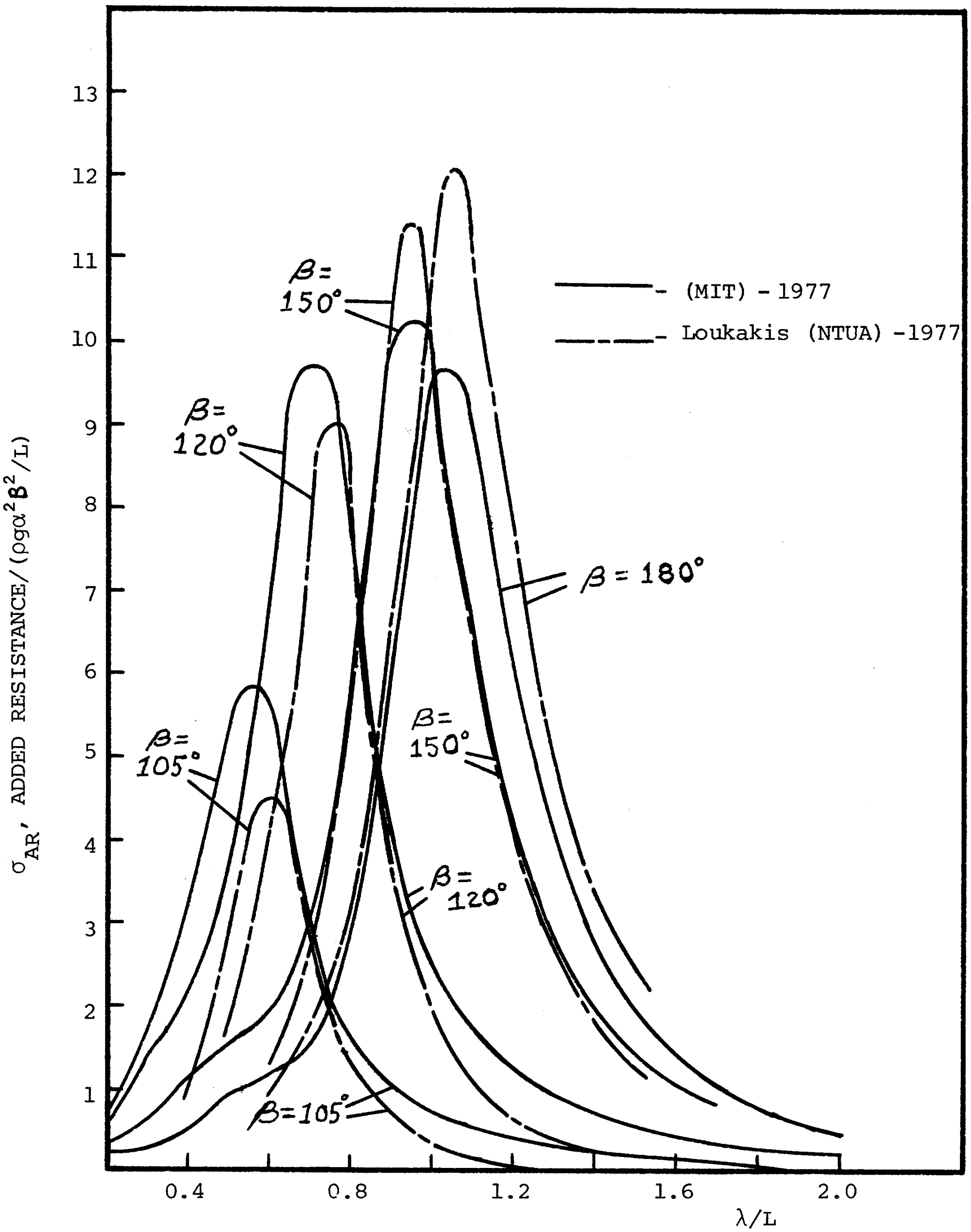


FIGURE 9: MARINER - ADDED RESISTANCE (SPEED - 15 knots); Various Headings

closely the movement of the heave motion peak. The M.I.T.-Salvesen correltaion is much the same as in Figure 7: good correspondence for low wavelengths (except for $\beta = 105°$), peak magnitude differences and M.I.T. overpredicting NAV results for medium length waves. The Loukakis program predicts much higher peaks than the M.I.T. results for near head seas, but lower peaks for near beam seas. It also gives slightly different peak locations, and noticeable differences in the medium wavelength area. It is interesting to note that the character of the three solutions is generally the same, but they are by no means the same. Also, the M.I.T. program predicts more resistance at $\beta = 150°$ than in head waves (presumably due to contributions from the three extra degrees of freedom).

Figure 10 illustrates the effect of forward speed on drift force in oblique bow waves ($\beta = 120°$). Nondimensional drift force (defined by σ_{DF} = drift force/$\rho g \alpha^2 B^2/L$) is shown as a function of wavelength over ship length. Results are shown for the M.I.T. and Salvesen schemes. Higher forward speed again increases the peak drift force, as was observed for the added resistance in Figure 7. Increasing speed also shifts the peak value to longer wavelengths. Furthermore, (from the figures in Reference 28), there is an indication that the Salvesen curves turn sharply up in

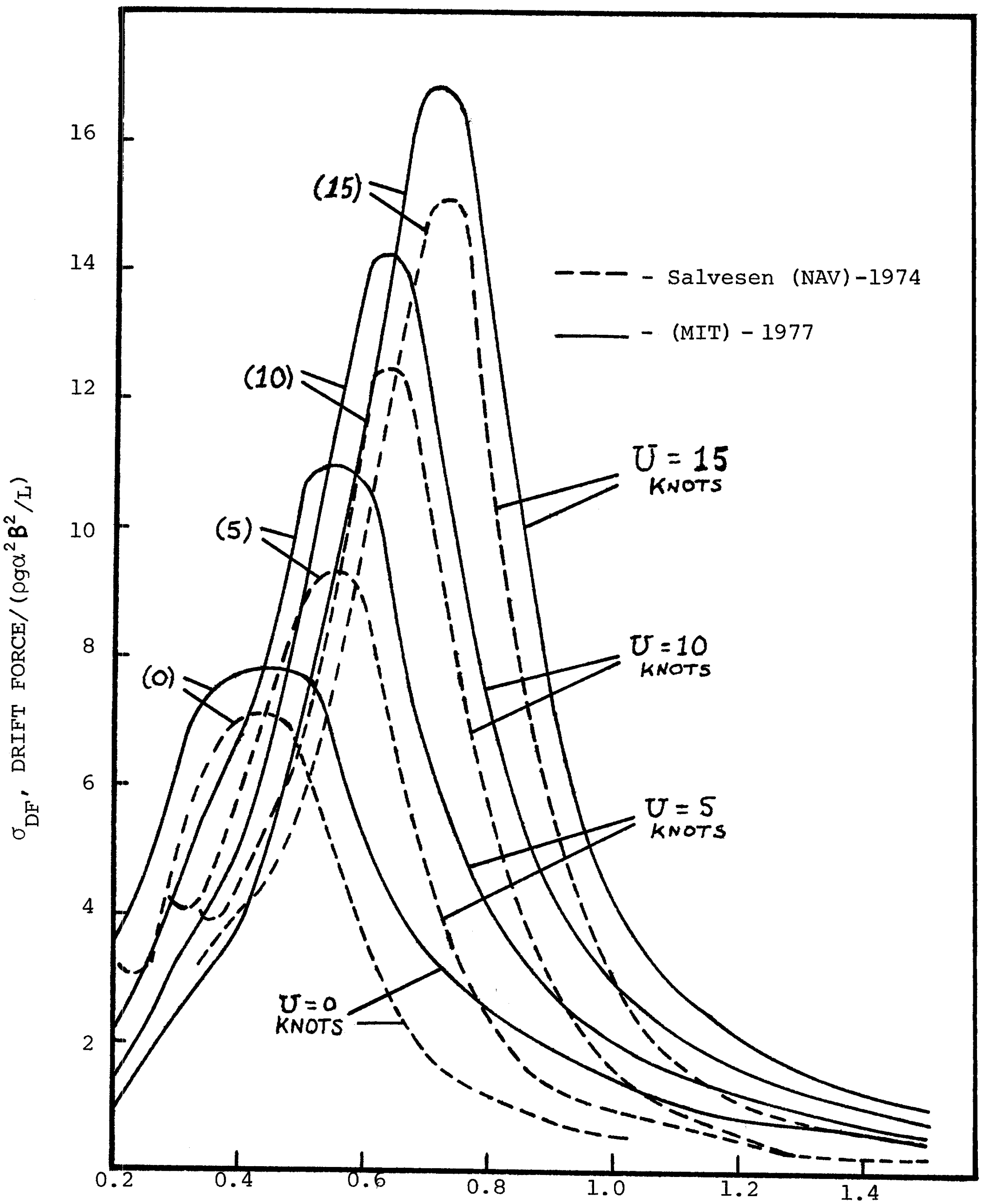


FIGURE 10: MARINER - DRIFT FORCE
$\beta = 120^\circ$; Various Speeds

the shorter waves; this does not occur in the M.I.T. calculation. The reason for this cannot be explained or justified. The usual differences between the two results in medium wavelengths show clearly. In general, it could be said that the correlation is improved by a speed increase.

It would perhaps be useful, at this point, to interject a quantitative idea concerning the real force magnitudes involved in these nondimensional numbers. For example, the peak added resistance at twenty knots (about '12' in Figure 7) in 5-foot amplitude waves, corresponds to a force of about 200,000 pounds on the real ship, a figure roughly on the order of the calm water resistance at the same speed. The peak drift force in Figure 10 at fifteen knots (about '17') would translate into almost 300,00 pounds in 5-foot amplitude waves. Of course, these numbers are for one particular highly tuned regular wave frequency. The spectral analysis to be outlined in Chapter V will lead to much lower mean values but the significance of the numbers involved can surely be appreciated.

Figures 11 and 12 present the impact on drift force of a heading angle change in bow waves at fifteen knots. The first of the two presents M.I.T. and Salvesen (NAV) results while the second gives M.I.T. and NTUA predictions. As expected from the added resistance curves of Figures 8

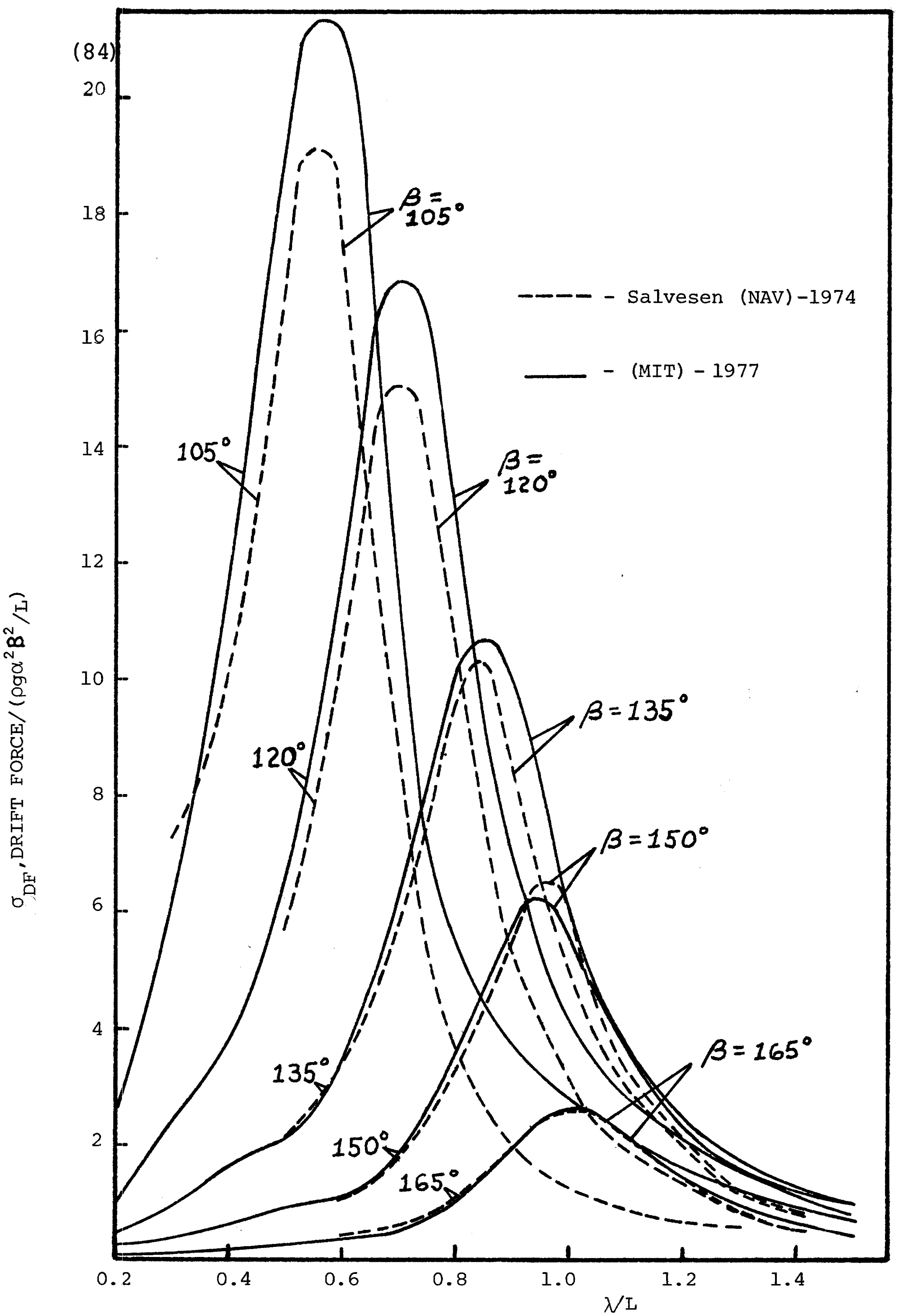


FIGURE 11: MARINER - DRIFT FORCE (SPEED - 15 knots) Various Headings

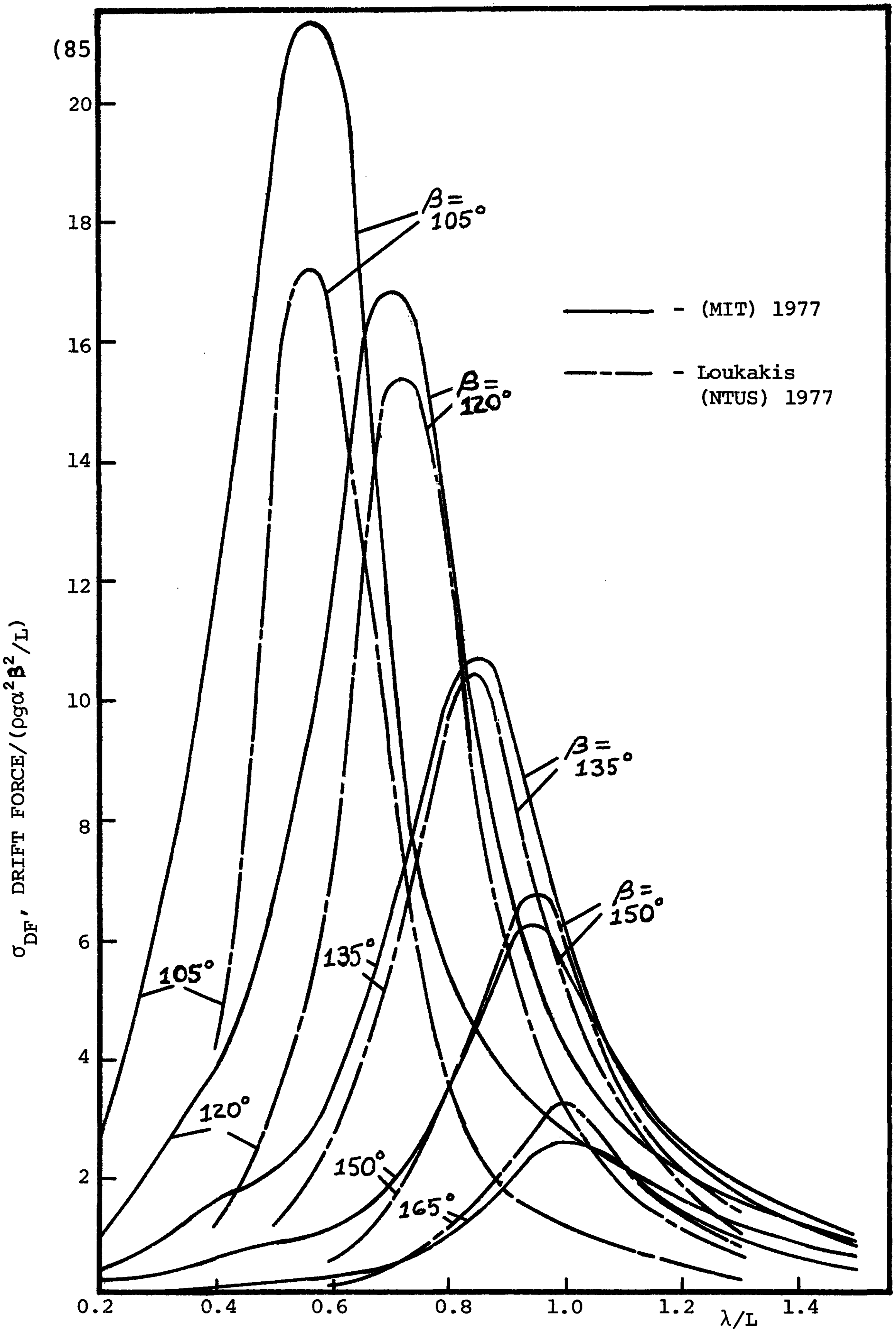


FIGURE 12: MARINER - DRIFT FORCE (SPEED-15 knots) Various Headings

and 9, the peak values occur at larger wavelengths and have decreasing magnitudes as the waves come on the bow. The M.I.T.-Salvesen comparison becomes better as the heading angle increases. The usual peak and medium wavelength differences appear except at $\beta = 150°$ where the Salvesen peak is higher. The NTUA results illustrate much the same tendencies, and generally come closer to the Salvesen computation than the M.I.T. one.

The final two graphs (Figures 13 and 14) delve into an area that is the subject of considerable controversy in ship motion theory -- following seas. Salvesen did not give any results for heading angles less than 105° in his 1974 paper, so the only available comparison is provided by Loukakis.

Figure 13 shows <u>negative</u> added resistance for a range of headings at fifteen knots. Both sets of results are fairly consistent in the longer wavelengths, showing a tendency to decrease the prediction with increasing heading angle. However, the M.I.T. results tend to wander around the axis for $\beta = 75°$ and $\beta = 60°$. This can most likely be attributed to numerics. The peak magnitudes generated by the two methods differ considerably, but anyway, the existence of any peak, at all, has not as yet been established experimentally. The container ship experiments of Journee

FIGURE 13: MARINER - NEGATIVE ADDED RESISTANCE (Speed-15 knots)
Various Headings

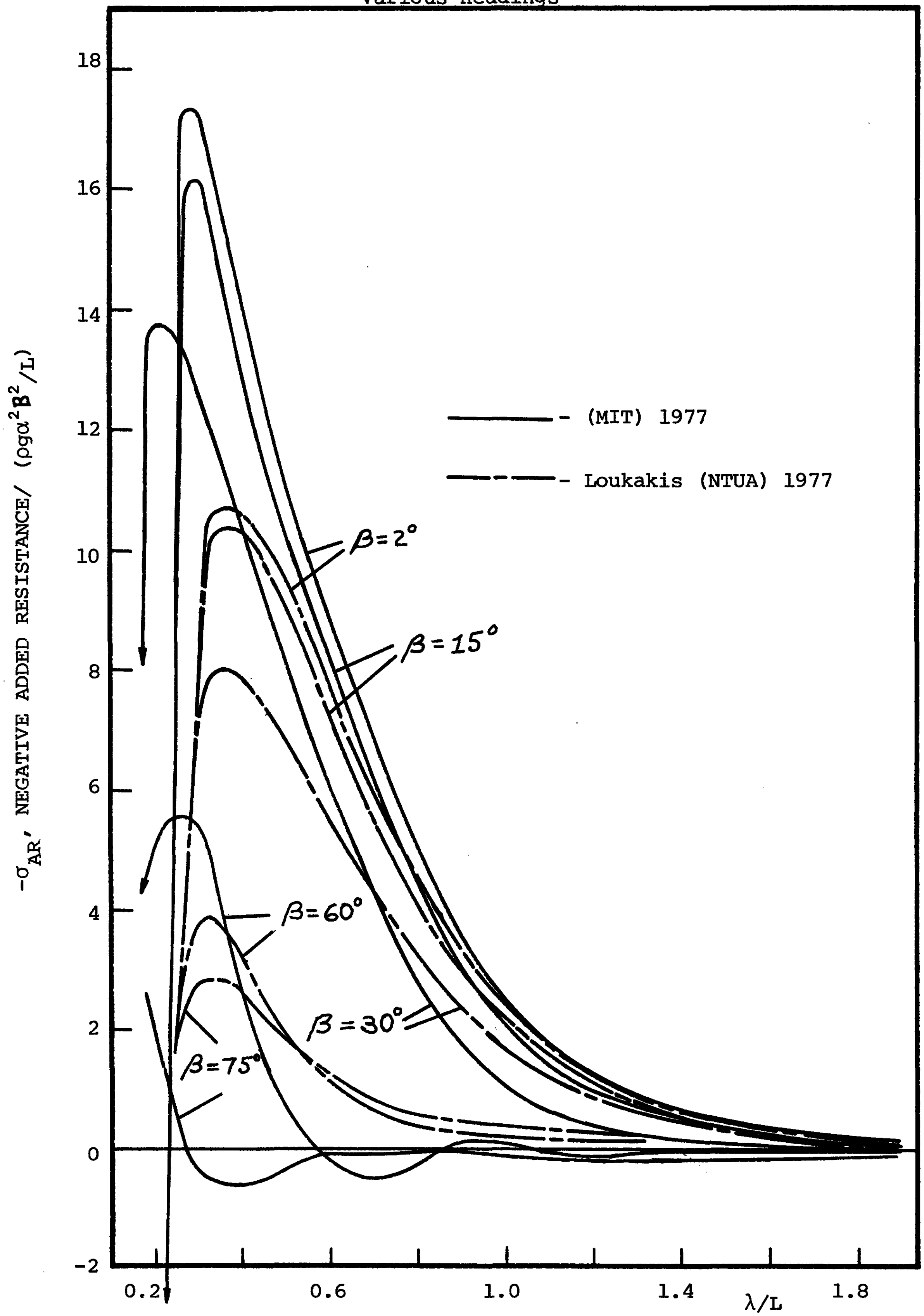
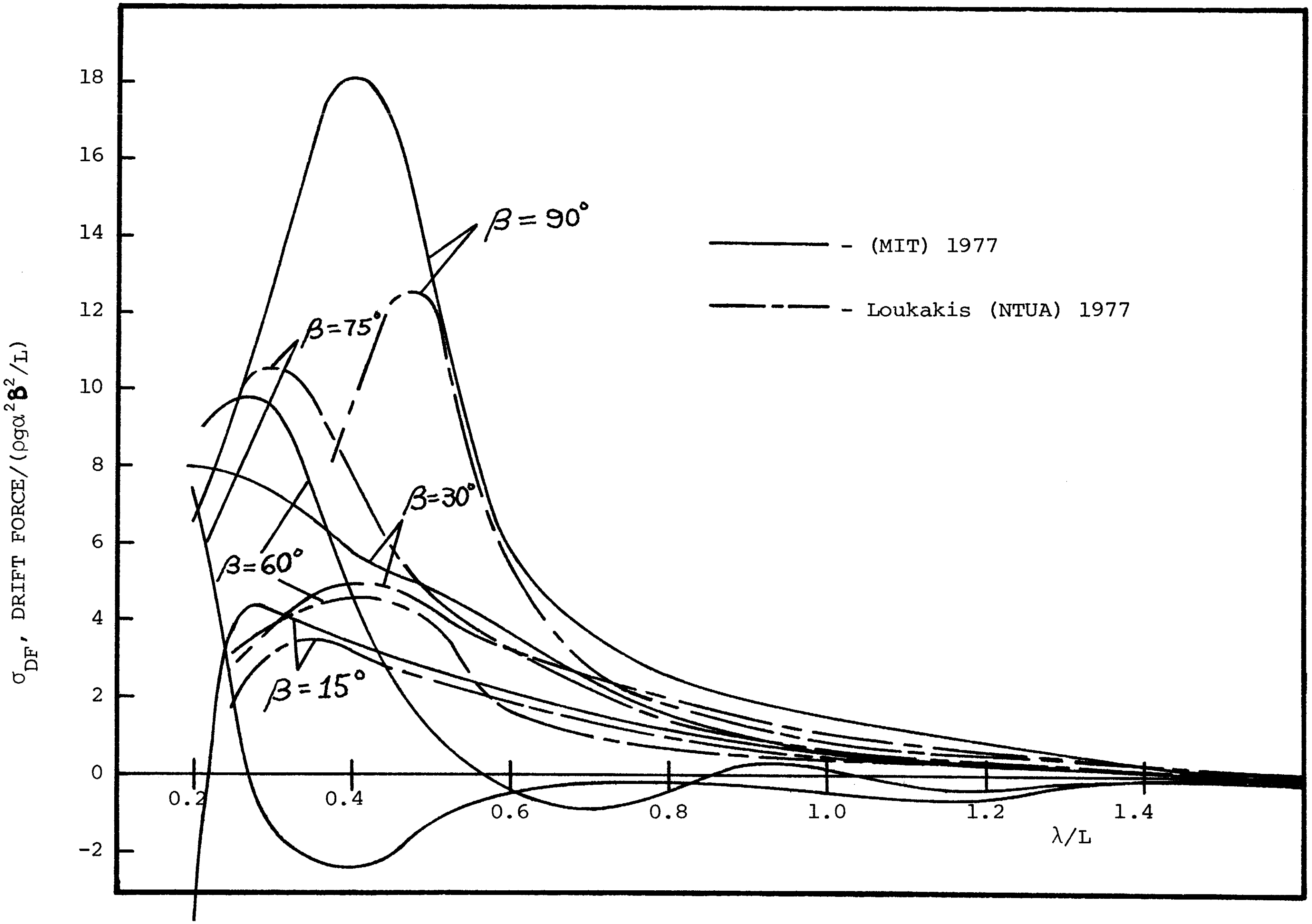


FIGURE 14: MARINER - DRIFT FORCE (SPEED - 15 knots) Various Headings



(Ref. 12) show this quite clearly since he obtained positive added resistance throughout the entire wave range in following seas ($\beta = 0°$). Loukakis attempted a computation based on the container ship and received results similar to those of Figure 13[16]. Further doubt can be placed on these peak predictions by virtue of the fact that they occur very near zero encounter frequency for the lower heading angles. As the ship passes into the negative encounter frequency region, the curves all take a large 'jump' to a positive added resistance. This jump does seem to occur in a somewhat more orderly fashion in Loukakis' work. Additional insight into the problem can be obtained by considering Figure 14 which shows positive drift force at fifteen knots for the same following seas cases and for beam seas. The results show a large amount of inconsistency in the magnitudes. There is no clear-cut decrease of force with decreasing heading angle for either set of data. All the same problems mentioned for Figure 13 are, of course, present for all the following sea headings. In addition, the beam seas case gives a rather disappointing comparison between the two theories. The M.I.T. and NTUA results do agree well in beam seas down to a wavelength/ship length ratio of 0.5; then they diverge rapidly. The experiments of Figure 4 might be used to illuminate this matter, but unfortunately they could not be extended to smaller waves.

In summary, the three theories presented compare very well with each other over the full speed range of the Mariner for heading angles greater than 105° (bow waves). In most of these cases, the M.I.T. results show a larger second-order force in medium wavelengths than the other two methods. The head seas results of Figure 3 show experimental points that fall slightly above the M.I.T. predictions in the long wave range, so this may well be an asset in the M.I.T. version of the theory. However, experimental data for any oblique bow wave case is noticeably absent, and final conclusions cannot yet be drawn for this region.

Agreement between the theories becomes noticeably poorer in the beam seas regime. Loukakis' decision to ignore the lateral motions contribution may be a factor. Also, the 'weak-scatterer' potential assumption (See Eqs. (65) and (66)) incorporated in the Salvesen theory is expected to cause problems in this area, but, again, the lack of experimental work prevents a conclusion. The beam seas experimental work that was done (Figs. 4,5,6) raised serious questions concerning the validity of any linear theory in the presence of large roll amplitudes. The Salvesen theory, in particular, is quite insensitive to the roll mode of motion. This mode makes little contribution to the seond-order force, even when its predicted amplitude is large.

On the subject of numerical computation problems, the M.I.T. version of the Salvesen theory exhibits some oscillations at fifteen knots in quartering seas (Figs. 13,14). Some of the Salvesen (NAV) results indicate a sharp upward turn in the second-order force for short bow waves (e.g., Fig. 8 or 10). This is noticeably absent in the M.I.T. computation.

In following seas, both the NTUA and M.I.T. calculations exhibit difficulties. Neither shows any real consistency, and the agreement between either theory and experiment is poor. This problem is discussed in some detail by Loukakis (Ref. 16).

He shows how the total added mass and damping tend to zero, while some of the sectional coefficients go to infinity, as the encounter frequency decreases. This decay will take on one of several forms, depending on the order of magnitude relation between the encounter frequency and the incident wave frequency. All these tendencies are artificially introduced by the strip theory assumptions. Nevertheless, the equations of motion produce 'reasonable' (but not necessarily 'accurate') predictions. This may well be due to the hydrostatic terms dominating the equations, making the damping and added mass insignificant. The linear second-order force predictions are a different matter, though. They are dependent to a large extent on the same sectional potentials that cause the problems in the motions computation.

Very near zero encounter frequency, it has been observed that several of the components of the Salvesen theory grow very large; and it is only the fact that they have opposite signs that keeps the prediction bounded. In conclusion, it is obvious that much work remains to be done on the following seas problem.

V. DESIGN ANALYSIS

An ocean seaway represents a true random process. The wave patterns are continually changing in time and space, and their complexity defies any deterministic analysis. In view of this, the results of Chapter IV for regular sinusoidal waves must be considered as only a first step in a procedure that will lead ultimately to a statistical formulation of second order wave force in the real ocean.

Conceptually, the method to be used involves obtaining a Fourier integral representation of the ocean. The actual sea surface is represented by superimposing many infinite, in theory, regular waves of different periods, random phase and infinitesimal amplitude. The end result is an amplitude or energy spectrum for the sea, depciting the energy in the ocean as a function of frequency. For a linear system, the principle of superposition can be invoked so the response to a number of input regular waves is the sum of the responses to each, individually. Therefore, if the response of the system can be obtained as a function of frequency, then this can be combined with the sea spectrum to yield a response spectrum. Under certain other probabilistic assumptions, predictions regarding, for example, the highest likely response or the probability of an event, can then be made using the response spectrum.

This theory of random excitations for linear systems has been used to good advantage to provide a design tool in the area of ship motions, and many good references on the technique are available[4]. However, it is not immediately obvious that this technique can be used for the second order force. Vassilopoulos[30] proved it rigorously using nonlinear system theory[30], but Maruo first gave an intuitive derivation[7] that is repeated below.

If the sea can be described by a long-crested (unidirectional) energy spectrum, $E(\omega)$, the following will be true for each infinitesimal frequency component, where $d\alpha$ is the infinitesimal wave amplitude.

$$E(\omega_s)\,d\omega_s \sim (d\alpha)^2 \tag{87}$$

Futhermore, if the second order force is purely proportional to wave amplitude squared for any frequency, as originally stated in the Introduction, then it follows that:

$$\Delta F/(d\alpha)^2 = f(\omega_s) \tag{88}$$

where F represents the second order force. (Note that $f(\omega)/(\rho g B^2/L)$ is the quantity plotted as a function of wavelength (frequency) in all the results of Chapter IV.) Combining (87) and (88) for a frequency increment, gives:

$$\Delta F = f(\omega_s) E(\omega_s) d\omega_s \tag{89}$$

It follows from probability theory that the integral of the second order force spectral component (89) over all possible frequencies will represent the expected mean second order force for a long period of time in the long-crested seaway,

$$\overline{F} = \int_0^\infty f(\omega_s) E(\omega_s) d\omega_s \tag{90}$$

The result (90) was developed by Maruo using a spectrum $E(\dot{\omega})$, based on the full wave amplitude (See (87)). The spectrum definition which is most commonly used in practice is based on one-half the amplitude squared. Calling this spectrum $\Phi^+_{\zeta\zeta}(\omega)$, the formula (90) becomes,

$$\overline{F} = 2\int_0^\infty f(\omega_s) \Phi^+_{\zeta\zeta}(\omega_s) d\omega_s \tag{91}$$

This is the formula given in Strom-Tejsen, et. al.[29]. Should a more rigorous derivation be desired, Vassilopoulos should be consulted.[30] If the long-crested unidirectional seas assumption (made before (87)) is relaxed, Maruo showed (and Newman[22] recently rederived rigorously) that the mean second order force in short-crested seas can be approximated in this manner,

$$\overline{F}(\theta) \cong \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \overline{F}(\beta)\cos^2\mu \, d\mu \qquad (92)$$

$$(\mu \equiv \beta - \theta)$$

In this expression, θ is the principal wind direction or primary directional source of waves (defined the same as β in Chapter III. $\overline{F}(\beta)$ is a mean force for a particular ship heading angle, developed from (91). The well-known 'cosine-squared' spreading function has been employed to distribute the effective energy in the sea placing it primarily in the waves coming from the principal wind direction. Equation (92) represents a double integral over frequency and propagation direction, but it is still, conceptually, a superposition of many small sine waves.

Returning, for now, to long-crested seas, (91) was found to be well suited to inclusion in the M.I.T. 5-D program (which already calculated many statistical quantities associated with ship motions). Response operators in the form of (88) were easily developed from the results of regular wave calculations. Then these were combined with the various sea spectra and numerically integrated in subroutine STATIS according to (91).

A separate auxiliary program that performed motions calculations in short-crested seas was already in existence. This was modified so that it could receive the second order (long-crested) mean force output from the 5-D program (according to (91). It then performs a calculation set forth in (92), yielding, finally, a predicted long-time mean added resistance and drift force in a short-crested, irregular seaway.

In order to test the feasibility of employing (91) and (92) in the design process, six wave spectra were chosen. The six are shown in Figure 15.

Spectra 1, 2, 3, and 6 are Peirson-Moskowitz fully-developed one-parameter representations from the formula:

$$\Phi^{+}_{\zeta\zeta}(\omega_s) = \frac{A}{\omega_s^5} e^{-\frac{B}{\omega_s^4}} \tag{93}$$

where $A = .0081g^2$ and $B = 33.56/\bar{h}^2_{1/3}$. The quantity, $\bar{h}_{1/3}$, represents the significant wave height (the average height of the one-third highest observed waves). Spectra 4 and 5 were developed by use of the Bretschneider, two-parameter representation:

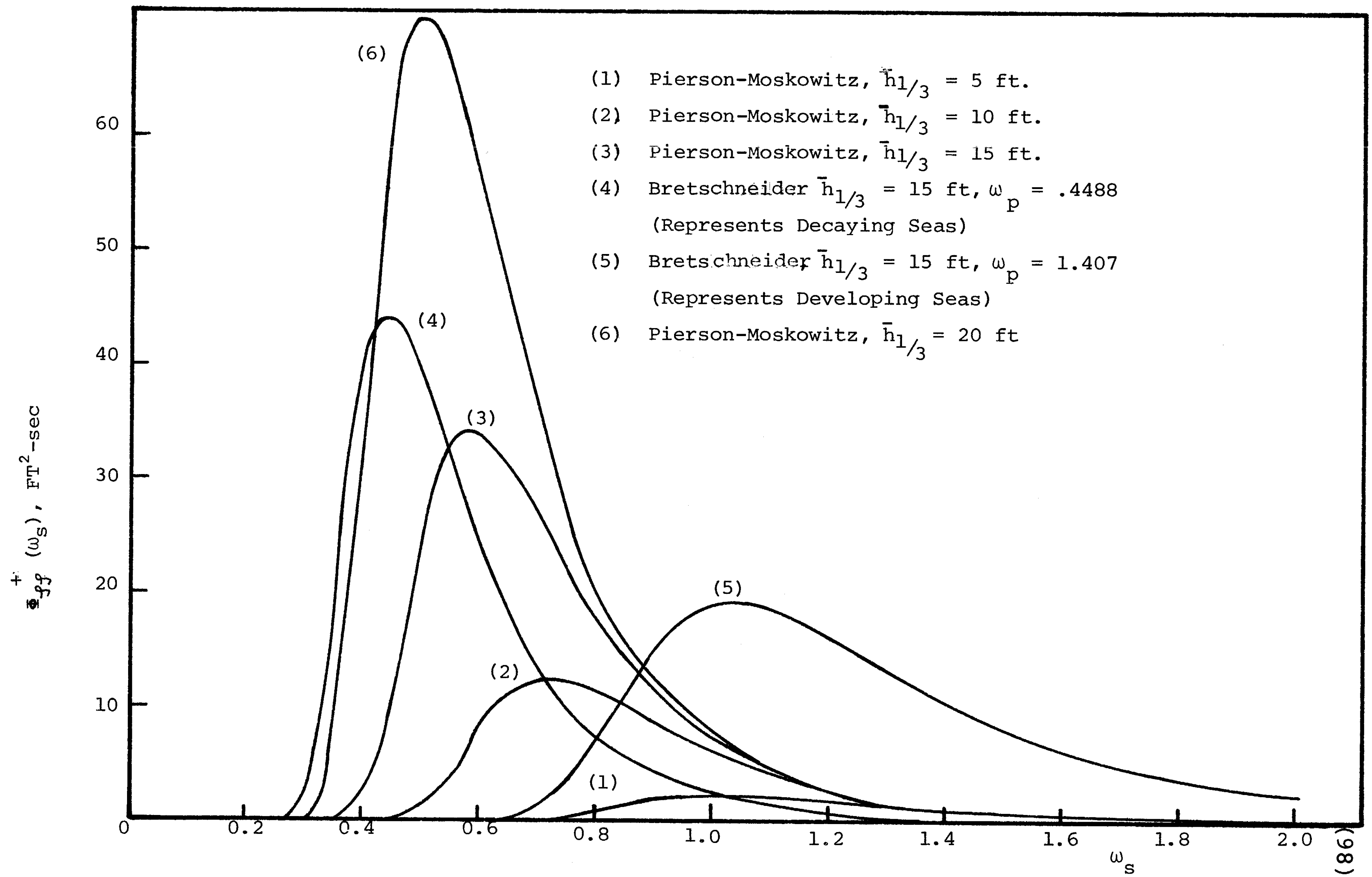


FIGURE 15: SEA SPECTRA

$$\Phi^{+}_{\zeta\zeta}(\omega_s) = \frac{5}{16}\left(\bar{h}_{1/3}\right)^2 \frac{\omega_p^4}{\omega_s} e^{-\left(\frac{5}{4}\frac{\omega_p^4}{\omega_s}\right)} \tag{94}$$

where ω_p is the desired location of the spectral peak. For spectrum 4, the peak was chosen at a much lower frequency than that generated by the Pierson-Moskowitz formula for the same $\bar{h}_{1/3}$. This corresponded to a decaying seaway, where the energy is found in longer waves. The peak for spectrum 5 was located at a much higher frequency, thereby simulating a developing sea, taking energy from the wind in the short wave region.

These six spectra were used in (91) in combination with response operators generated by the theory of Chapter III for regular waves. The operators were calculated for a full range of headings at the service speed of twenty knots. The resulting long-crested seas predictions were then used in (92) to provide information on second order forces in the corresponding short-crested seaway.

The results obtained from spectra 1, 2, 3, and 6 are shown in Figures 16 and 17 for added resistance and drift force, respectively. In these plots, the force is non-dimensionalized using significant wave height squared.

Figure 16 shows mean added resistance as a function of significant wave height for the Pierson-Moskowitz for-

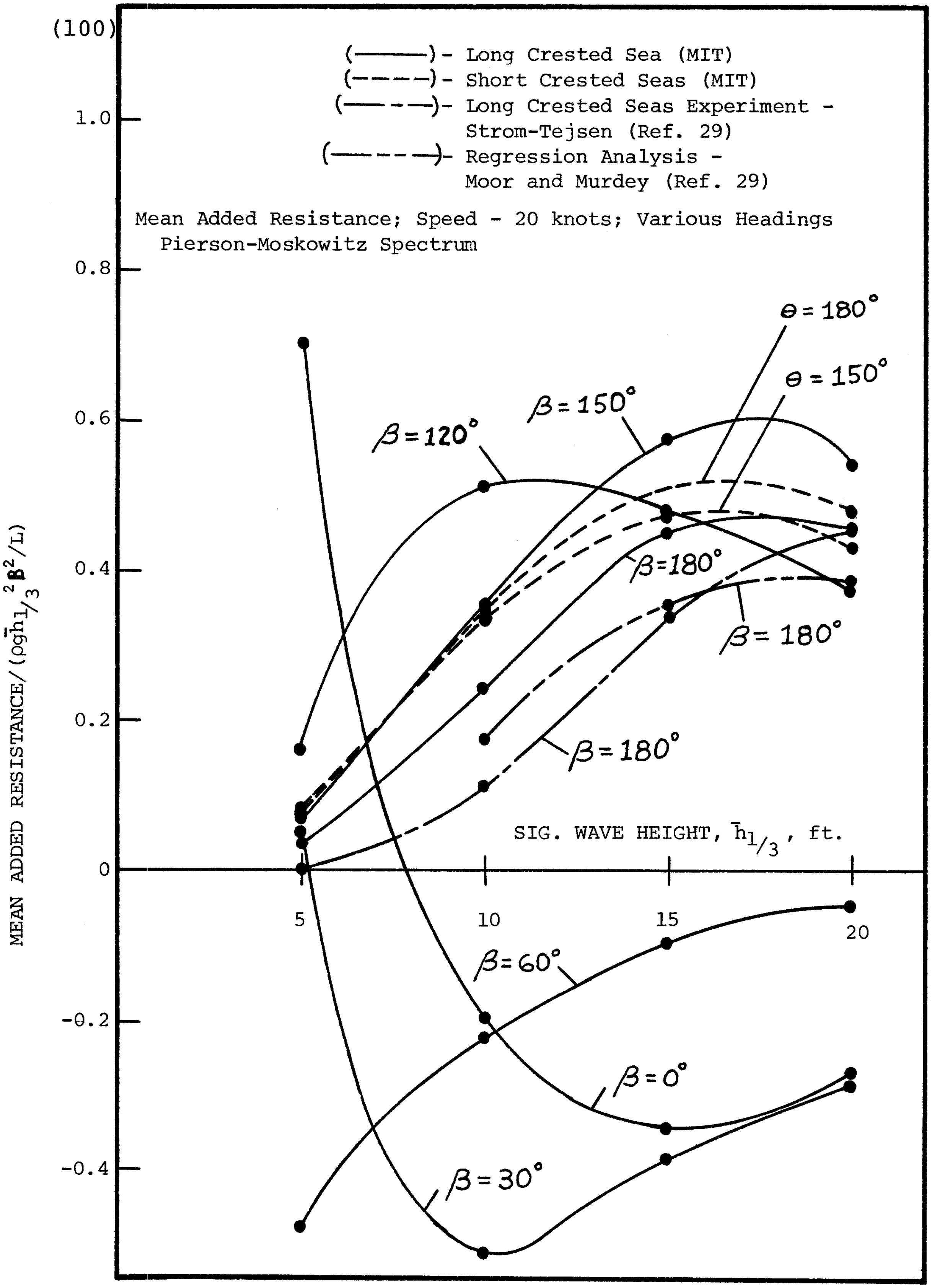


FIGURE 16: MARINER IRREGULAR SEAS RESULTS

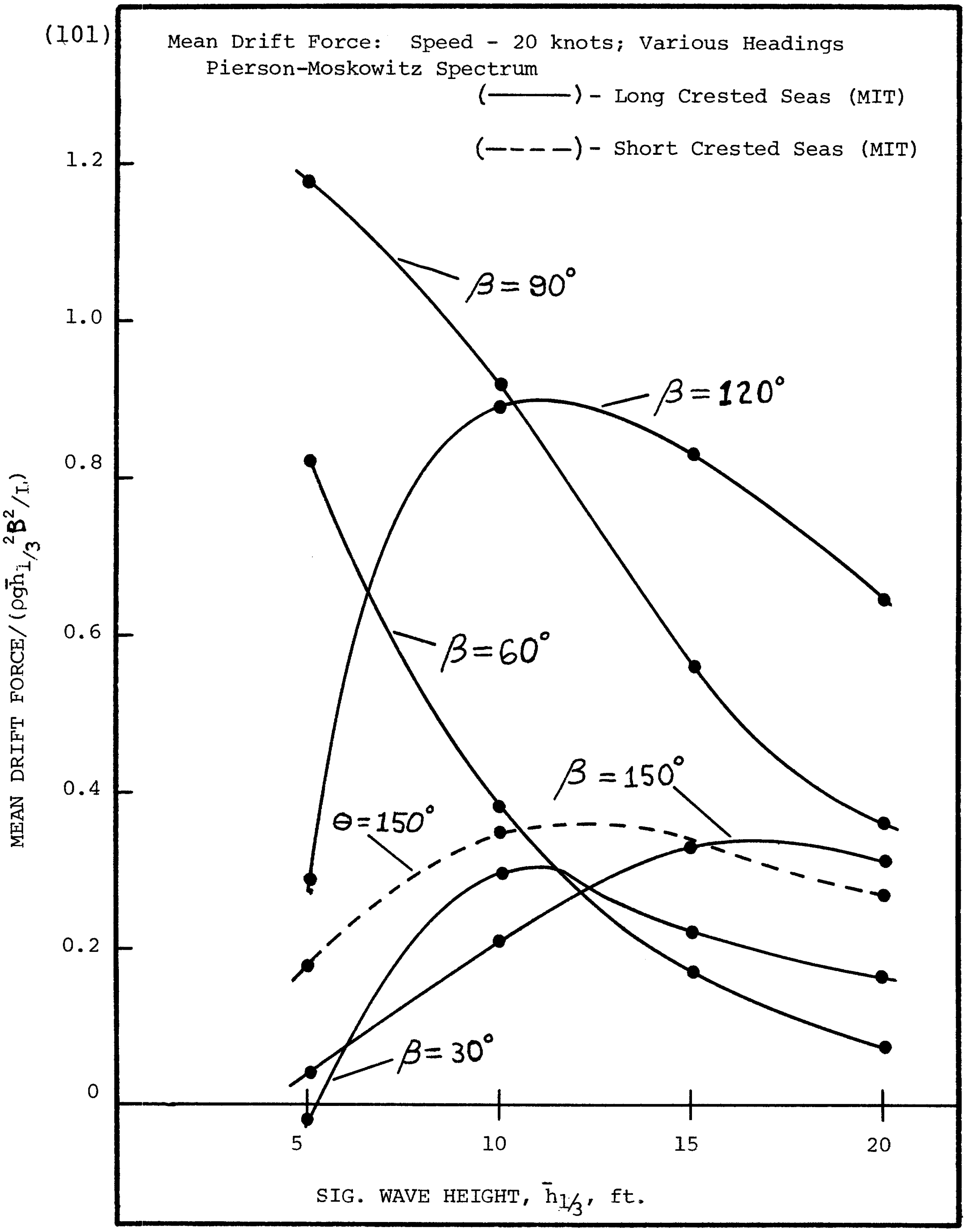


FIGURE 17: MARINER IRREGULAR SEAS RESULTS

mulation. The first important point revealed is that the added resistance in long-crested seas, for $\beta = 150°$ exceeds that of $\beta = 180°$ (head wave) throughout the entire wave height range. The reason for this must be found in the relative locations of the two response peaks and the spectral peak. In smaller waves, the $\beta = 120°$ curve outreaches both the higher beta angle predictions. The negative predictions for the following waves are created by the negative response operators (Shown in Fig. 13 for 15 knots). Since these negative response operators have not, as yet, been confirmed by experiment, any of these results for irregular following seas should be used very carefully. It should be noted that the plot for $\beta = 180°$ can be compared with the Series 60, $C_B = 0.60$ experimental and regression analysis results for irregular head seas given by Strom-Tejsen[29]. The short-crested seas calculation (92) for the 180° 'wind' direction tends to be greater than the 180° long-crested prediction. This is a result of extra energy being channeled into near head-sea headings which also have very large added resistance operators. The 150° 'wind' direction curve falls considerably below the long-crested 150° plot because, in this case, the energy is being shifted to headings with lower added resistance responses.

Figure 17 gives the mean drift force for the same heading range. The beam seas case produces the biggest drift in smaller waves, but, again as a consequence of peak positions, the $\beta = 120°$ curve dominates in higher seas. The relative amount of drift produced at $\bar{h}_{1/3} = 20'$ by the $\beta = 150°$ case is surprising since it almost matches the beam seas case. The short-crested seas calculation for a 'wind' direction of 150° overshadows its long-crested companion in smaller waves, then falls beneath in higher waves. This can be predicted from a line of reasoning similar to that given above.

In all these plots, the reader is cautioned to regard the results for beam and following seas with some suspicion in view of the results of Chapter IV. For example, the sharp dip and peak in the near beam seas experimental data (Figs. 4, 5, 6), was measured directly in the way of most of these spectral peaks. This underscores even more heavily the need for further research in this area.

Finally, Figures 18 and 19 show the effect of spectral peak location on the added resistance and drift force. Spectra 3, 4, and 5 were used as being representative of fully developed, decaying, and developing seas. For the bow waves, the decaying and fully developed seas tend to produce the most drift force or added resistance. The $\beta = 150°$ and $\beta = 120°$ long-crested added resistance cuves plot higher

FIGURE 18: MARINER IRREGULAR SEAS RESULTS

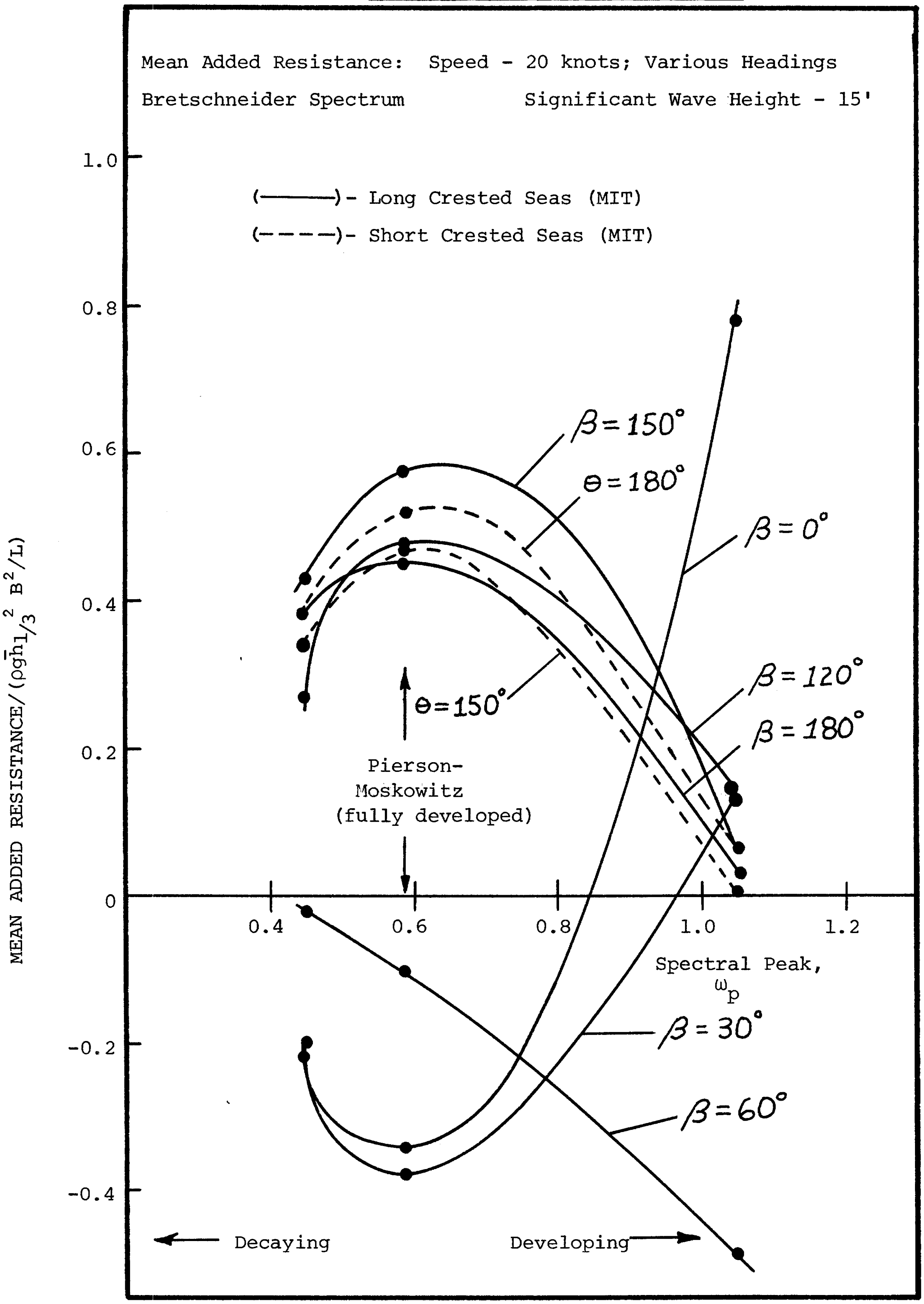
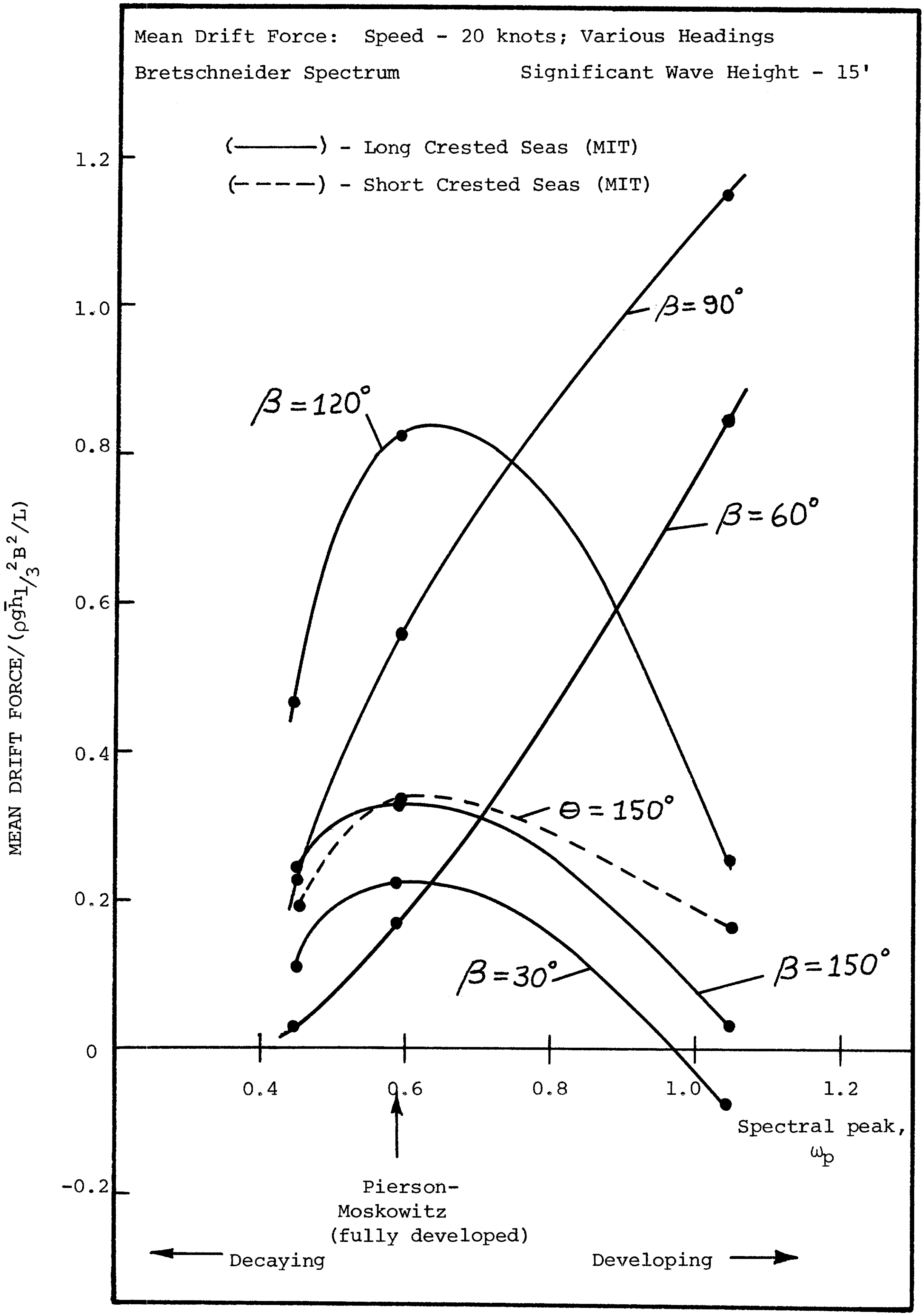


FIGURE 19: MARINER IRREGULAR SEAS RESULTS



than the head waves case through most of the range. The short-crested calculations have the same general character as their long-crested counterparts, but not the same magnitude. For the Mariner at service speed, the force maximum for bow waves seem to occur near the fully developed seas.

The drift force is dominated by the $\beta = 120°$ case for decaying and developed seas, but the beam seas case is very large in the developing sea. The short-crested seas calculation follows quite closely the comparable long-crested case.

To provide an assessment of the actual forces in the different sea states without dealing with the significant wave height, the predicted forces, in pounds, for all the points plotted in Figures 16 through 19 are given in Table I.

The maximum added resistance occurs at a heading of 150°, in 20-foot seas and its magnitude represents around three quarters of the calm water resistance (inferred from the Mariner shaft horsepower requirements[25] to be about 200,000 pounds). Before the requirements for service margins in ship propulsion are rewritten, it should be pointed out that these high waves put the linearity assumptions of the theory to a severe test. Also, there may be other factors (e.g. structural integrity, motions, etc.) that prevent operation of a 500-foot ship at service speed

(Long Crest)	Second-Order Wave Force*	Fully Developed $\bar{h}_{1/3}=5'$	Fully Developed $\bar{h}_{1/3}=10'$	Fully Developed $\bar{h}_{1/3}=15'$	Fully Developed $\bar{h}_{1/3}=20'$	Decaying $\bar{h}_{1/3}=15'$	Developing $\bar{h}_{1/3}=15'$
$\beta = 0$	AR / DF	12,290 / 0	-13,413 / 0	-53,443 / 0	-75,959 / 0	-34,498 / 0	120,663 / 0
30	AR / DF	871 / -503	-35,583 / 20,554	-59,902 / -34,585	-78,475 / 45,308	-30,904 / 17,843	20,505 / -11,838
60	AR / DF	-8,268 / 14,321	-15,227 / 26,461	-15,450 / 26,761	-12,418 / 21,509	- 2,607 / 4,516	-76,551 / 132,591
90	AR / DF	0 / 20,476	0 / 63,924	0 / 87,856	0 / 99,102	0 / 35,016	0 / 180,428
120	AR / DF	2,865 / 4,961	35,679 / 61,797	74,922 / 129,769	103,643 / 179,515	42,309 / 73,282	22,968 / 39,782
150	AR / DF	1,197 / 691	24,604 / 14,205	89,622 / 51,744	149,707 / 86,433	67,044 / 38,708	9,857 / 5,691
180	AR / DF	622 / 0	16,695 / 0	70,253 / 0	126,118 / 0	59,369 / 0	5,075 / 0
(Short-Crest) $\theta = 150$	AR / DF	1,370 / 3,119	23,345 / 24,328	73,636 / 52,699	119,818 / 74,745	53,354 / 30,915	11,118 / 26,404
180	AR / DF	1,283 / 0	23,813 / 0	80,715 / 0	134,166 / 0	60,363 / 0	10,448 / 0

*AR/DF = Added Resistance/Drift Forces in pounds (Arrangement of figures in columns to the left)

TABLE 1: ADDED RESISTANCE/DRIFT FORCE IN POUNDS FOR VARIOUS SEA STATES

in such a sea state. In a more modest sea (say $\bar{h}_{1/3} = 10'$) the maximum added resistance occurs at a heading of 120° and it represents a more modest twenty percent of the calm water resistance. The drift force reaches its maximum in the developing beam seas. A similar value is obtained in a 20-foot fully developed sea state at a heading of 120°. However, the drift force at speeds near zero is more likely to be of interest to the naval architect.

It has been demonstrated in this section that the techniques of spectral analysis can be used in combination with a regular wave theory to gain quantitative insight into the second order force acting on ships at sea. On the basis of this brief study, it is already possible to conclude that the a prior assumption that irregular head seas will represent the worst case for added resistance is unjustified. It is encouraging for the simplicity of calculation that, at least for Mariner-like ships, fully developed seas appear to yield the maximum mean added resistance responses. The problems associated with regular following seas were seen to carry naturally into the spectral analysis. These difficulties and the doubts raised by the beam seas experimental work should make designers wary of the full application of the short-crested sea analysis (92). However, the author is optimistic that (92) will be very useful in connection with the Salvesen second order force theory if added resistance predictions are desired for primarily head winds ($\theta \approx 180°$). Finally, the extreme sensitivity of the calculated mean force to the input ocean spectrum can be easily seen.

VI. CONCLUSION AND RECOMMENDATIONS

This report represents an attempt to develop a design tool for use by the naval architect in calculating the second order wave force in a real seaway on a proposed vessel. This force impacts heavily on the design in that it is a major source of extra resistance and/or side slip that must be counteracted by the power plant in a seaway. The main thrust of the effort has been to develop a reliable computational method requiring only basic information about the ship (See Appendix) as an input. Second order force predictions were desired for any heading angle of the ship at any speed in regular waves of any frequency. The present study starts by introducing and defining the problem in the context of the general ship design process used by naval architects. Then the previous theoretical work on second order forces was outlined, and it was shown how several methods of prediction have been recently developed by hydrodynamicists for use in oblique seas. One of these theories, due to Salvesen, was derived in the context of modern strip theory of ship motions.

This theory was implemented within the M.I.T. 5-D ship motions program. Calculations were performed using the *Mariner*-type cargo ship in regular waves, and a brief investigation of the character of the final equation was conducted. The next step involved comparison with existing regular head sea experimental data. Due to the lack of oblique seas experimental data, an experiment using a Mariner model for net drift force measurements in near beam seas was conducted in the M.I.T. Towing Tank. When this data was compared with the computer predictions, it was partially supportive, but it also raised some important linearity questions.

A theoretical comparison was then made using results from the same theory in a different motions program and a different theory in the same motions program. The results were encouraging for heading angles between 180° and 90°, but the predictions were shown to be subject to question in the following waves. Next, the theoretical extension of the regular wave computer results to an arbitrary long-crested or short-crested irregular seaway was outlined. This extension is crucial to the usefulness of the method as a design tool. The resulting spectral analysis technique was implemented in the M.I.T. 5-D program, and a brief design analysis of the Mariner was performed using six representative ocean spectra and a range of headings at the service speed.

The following important points can be presented as a result of this study:

1. The linear estimate of second order force used in this study represents a very useful design tool in combination with the spectral analysis.
2. It is very important that a full range of bow wave headings be considered in the spectral analysis in order to define the true maximum added resistance.
3. Further experimental work in oblique bow waves is urgently needed to provide comparisons with the available theories.
4. The problems encountered in obtaining reasonable calculated results in following seas indicate the necessity for further theoretical developments in this area. Further experimental work would also be very helpful.
5. Every effort must be made to gain an understanding of the source of the phenomenon observed in regular beam wave experiments -- particularly in view of the fact that this second order force 'jump' occurs very near most spectral peaks.
6. As a result of point 4 above, the short-crested seaway equation would be most usefully applied, at this time, for waves propagating in a direction generally opposite to the ship (head winds).

7. The second order yaw moment, which can also be calculated in an extension of the theory presented (Ref. 28), should be developed for the M.I.T. 5-D program. This should not be a high priority effort, however, since there are very few theoretical or experimental comparisons to make.

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APPENDIX A

CHANGES TO M.I.T. 5-D SEAKEEPING PROGRAM USER'S MANUAL

The program is now capable of computing mean second order force (added resistance and drift force) for any ship heading, in addition to calculating ship motions, dynamic loadings, and events. The second order force is calculated using a theory published by Salvesen (1974). It includes forces arising from wave-ship motion interaction and wave reflection. The second order force routine given here operates properly only in English unit systems with length dimensions in feet. For example, it has been tested in a system describing the ship in tons and feet; for this case, it gives output in pounds force.

All the subroutines in the original MIT 5-D seakeeping program remain in existence, although some have been modified as noted in Appendix C. RESIST has been changed from a function subprogram to a subroutine.

The input format remains unchanged. However, there is a new option associated with the integer, NADR, of card set number four. If NADR = 2, only the final totals of the added resistance and drift force are printed out in dimensional and non-dimensional form. IF NADR = 1, all eleven components involved in the Salvesen computation (two for each mode of motion and one for wave reflection) are printed out. Examples of each of these output forms are presented in Appendix B. If NADR = 0, no second order force computations are performed.

The short-crested seas auxiliary program has been modified to include a mean second order force calculation. In order to implement this change, one integer called NADR has been added to card set number one. It is written in column 40, following the standard I5 format used by the original program. If second order force data is to be read, this integer should be equal to one. The mean drift force and added resistance will be prepared for input by the 5-D (if requested) in the same way as the mean squares of the other responses, so card set #3 can still be included just as it is punched by the main program.

APPENDIX B

SAMPLE OUTPUT

B-1: Example of the short form of output for second order force in regular waves obtained by setting NADR = 2. This follows the print out of the motion amplitudes and phase angles. The totals shown represent the sum of all eleven terms of the second order force formula developed in the main text (See Eq. (86)). Added resistance is the component along the longitudinal ship axis (positive aft). Drift force is the component along the transverse ship axis (positive in the direction of wave propagation). The dimensional quantities are in pounds force provided XRHO is given in slugs/ft^3 and provided that the ship is described in English units with feet as the length dimension. As noted in Appendix A, if this restriction is not met then the second order force output will be meaningless. The nondimensional quantities have been divided by the factor: $\rho g \alpha^2 B^2/L$ where ρ is the mass density of the water, g is the gravitational acceleration, α is the wave amplitude, B is the ship beam, and L is the ship length.

MEAN ADDED RESISTANCE/DRIFT FORCE CALCULATION

MODE	SOURCE	DIMENSIONAL DRIFT FORCE	DIMENSIONAL ADDED RESIST.	NON-DIMENSIONAL DRIFT FORCE	NON-DIMENSIONAL ADDED RESIST.
	TOTALS	8106.7070	4680.4102	11.6726	6.7392

B-2: Example of the long form of output for second order force in regular waves obtained by setting NADR = 1. This follows the print out of the motion amplitudes and phase angles. Dimensional and non-dimensional quantities are as defined in B-1. Here all eleven components that are summed to give the final Salvesen second order force are shown. There are two components for each mode of motion; FR-KRL which represents the Froude-Kriloff or Havelock portion of the motion interaction, $\mathfrak{F}_j^I$ (Eq. (75) of the text), WVDIFF which represents the diffraction potential contribution of the motion interaction, $\mathfrak{F}_j^D$ (Eqs. (78) and (79) of the text). The eleventh component is DIFFR.POT.CONT., which represents the wave reflection, $\mathfrak{F}_D$ (Eqs. (80) and (85) of the text). Numerous subtotals are also given, including the contribution of each mode of motion, the total of all the $\mathfrak{F}_j^I$'s, the total of all the $\mathfrak{F}_j^D$'s, and the total of the $\mathfrak{F}_j^D$'s and the $\mathfrak{F}_j^I$'s.

MEAN ADDED RESISTANCE/DRIFT FORCE CALCULATION

MODE	SOURCE	DIMENSIONAL DRIFT FORCE	DIMENSIONAL ADDED RESIST.	NON-DIMENSIONAL DRIFT FORCE	NON-DIMENSIONAL ADDED RESIST.
	FR-KRL	-5.6201	-9.7342	-0.0081	-0.0140
SWAY	WVDIFF	27.2093	47.1279	0.0392	0.0679
	TOTALS	21.5892	37.3936	0.0311	0.0538
	FR-KRL	1276.6926	2211.2944	1.8383	3.1840
HEAVE	WVDIFF	-1076.8801	-1865.2090	-1.5506	-2.6857
	TOTALS	199.8125	346.0854	0.2877	0.4983
	FR-KRL	0.7873	1.3637	0.0011	0.0020
ROLL	WVDIFF	-1.5898	-2.7535	-0.0023	-0.0040
	TOTALS	-0.8024	-1.3899	-0.0012	-0.0020
	FR-KRL	3994.1274	6918.0234	5.7510	9.9610
PITCH	WVDIFF	-1139.3252	-1973.3669	-1.6405	-2.8414
	TOTALS	2854.8022	4944.6562	4.1105	7.1196
	FR-KRL	3.0620	5.3035	0.0044	0.0076
YAW	WVDIFF	99.2581	171.9197	0.1429	0.2475
	TOTALS	102.3200	177.2232	0.1473	0.2552

SUM OF ALL THE MODES AND THE DIFFRACTION POTENTIAL CONTRIBUTION

ALL MODES FR-KRL	5269.0430	9126.2461	7.5867	13.1406
ALL MODES WVDIFF	-2091.3271	-3622.2815	-3.0112	-5.2156
TOTALS	3177.7158	5503.9609	4.5755	7.9250
DIFFR. POT. CONT.	676.2947	1171.3755	0.9738	1.6866
TOTALS	3854.0105	6675.3359	5.5493	9.6116

<u>B-3</u>: Example of the long-crested irregular seas output giving the mean second order force in a long-crested seaway (Eq. (91)). This is given if both spectral calculations and second order force calculations are requested. The (LBS) notation shown here is only correct if the variable XRHO in the main program (See Input, Appendix D) is given in slugs per cubic foot and the ship is described in English units using feet as the length dimension. If these requirements are not met, the computation will be invalid.

The two components, ADD.RES.(x) and DRIFT(Y-AX) are always in the output. The spectral amplitudes and associated statistical quantities may be obtained (as shown here) if NSPC $\neq 0$ (See Appendix D). The integer (zero) that appears at the end of the line, "Response spectrum for..." appears because the SPIN subroutine (Appendix C) is used for all the statistical calculations. The integer has meaning only in the bending moment calculations, where it transmits the station number.

The spectral amplitudes represent the value of the integrand in Eq. (91) in the text at each spectral frequency (input by the user or given in default by the program; See Appendix D). The only statistical quantity shown that has meaning for the second order force analysis is the zeroth moment, which is the value of the integral in (91) (half the second order force). The second moment, fourth moment, and broadness factor are printed out because use is made of subroutine SPIN already in existence in the 5-D program. Of course, these additional statistical quantities are useful, when they are associated with a motion spectrum in another part of the output.

```
MEAN SECOND ORDER FORCE (LBS.)

RESPONSE SPECTRUM FOR COMPONENT OF ACC.RES.(X)  0
0TH MOMENT =  0.4481103E+05  2ND MOMENT =  0.4647184E+05  4TH MOMENT =  0.7848762E+05  BROADNESS FACTOR = 0.62126

SPECTRAL AMPLITUDE'S :    -0.378027E+01 0.611304E+02 0.101371E+04 0.529430E+04 0.586294E+04 0.261470E+05 0.642269E+05 0.104773E+06
 0.242569E+06 0.274682E+06 0.297934E+06 0.311757E+06 0.316065E+06 0.311160E+06 0.297637E+06 0.262984E+06 0.137077E+06 0.389563E+05
 0.529523E+05 0.262669E+05 0.254747E+05 0.172924E+05 0.124658E+05 0.109724E+05 0.879109E+04 0.549538E+04 0.319227E+04 0.196768E+04
 0.126207E+04 0.892378E+03 0.719916E+03 0.580470E+03 0.470122E+03 0.379709E+03 0.305550E+03 0.244373E+03 0.193556E+03 0.154312E+03
 0.123293E+03 0.961033E+02
                  COMPONENT OF ACC.RES.(X)         89622.0625

RESPONSE SPECTRUM FOR COMPONENT OF DRIFT(Y-AX)  0
0TH MOMENT =  0.2587186E+05  2ND MOMENT =  0.2683072E+05  4TH MOMENT =  0.4531491E+05  BROADNESS FACTOR = 0.62126

SPECTRAL AMPLITUDE'S :    -0.219250E+01 0.352942E+02 0.585269E+03 0.305668E+04 0.338498E+04 0.150960E+05 0.370815E+05 0.604909E+05
 0.140047E+06 0.158588E+06 0.172012E+06 0.179993E+06 0.182480E+06 0.179648E+06 0.171841E+06 0.151834E+06 0.791416E+05 0.571325E+05
 0.305720E+05 0.151652E+05 0.147078E+05 0.998375E+04 0.719711E+04 0.633493E+04 0.507554E+04 0.317276E+04 0.184306E+04 0.113604E+04
 0.728658E+03 0.515215E+03 0.415645E+03 0.335135E+03 0.271426E+03 0.219226E+03 0.176409E+03 0.141089E+03 0.111750E+03 0.890923E+02
 0.711834E+02 0.566400E+02
                  COMPONENT OF DRIFT(Y-AX)         51743.7109
```

B-4: Example of the output of auxiliary program Shortcrest giving the mean second order force in a short-crested irregular seaway (Eq. (92)). An entire set of output including case identification, motions, and second order force is obtained by specifying NADR = 1 in the short-crest input. There is no full component print out option like the one described for the main 5-D. Either NADR = 0 (and no second order force output is generated) or NADR = 1 and the output shown is generated. If NADR = 1 in Shortcrest, then the user must be sure to run data on long crested seas from the main 5-D that includes second order force computations (i.e. NADR = 1 or 2 in the corresponding main 5-D data generation run).

The (LBS) notation is correct if the variable XRHO in the main 5-D program (See Input, Appendix D) was given in slugs per cubic foot and the ship was described in English units using feet as the length dimension. If these requirements are not met, the computation will be invalid.

IRREGULAR WAVE RESULTS -- SHORT CRESTED MULTI-DIRECTIONAL SEAS   SHIP SPEED :   33.7800

SIGNIFICANT WAVE HEIGHT   15.0000
PEAK SPECTRAL FREQUENCY   0.5877
PRINCIPLE WIND DIRECTION   150.0000

MOTIONS @ ORIGIN

	RMS	H(1/3)	H(1/10)	H(1/1000)
HEAVE	3.6421	14.5682	18.5745	28.0438
PITCH	1.4433	5.7731	7.3607	11.1132
SWAY	1.0443	4.1771	5.3258	8.0409
ROLL	2.1777	8.7109	11.1064	16.7684
YAW	0.2250	0.8999	1.1473	1.7323

MEAN SECOND ORDER FORCE (LBS.)

ADDED RESISTANCE COMPONENT   73636.3125

DRIFT FORCE COMPONENT   52699.4297

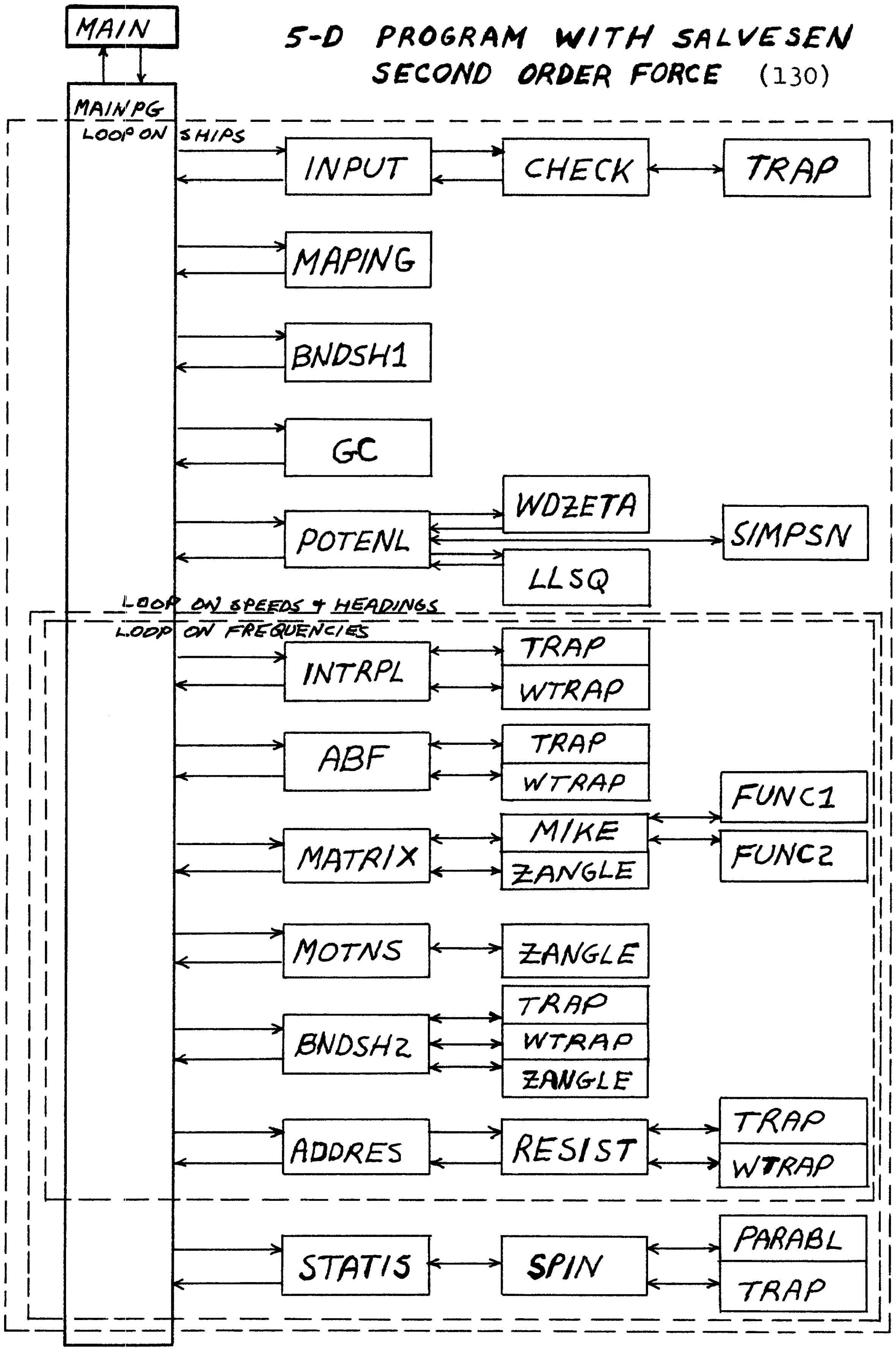
APPENDIX C

SUBROUTINES MODIFIED

The following briefly describes each subroutine modified in order to implement the Salvesen second order force computation in the MIT 5-D seakeeping program. A flow chart of this new version of the program is shown on the next page so that an understanding of the position and function of each routine is available. Further information on the routines not modified (and consequently not listed here) can be obtained from the 5-D Seakeeping Program User's Manual or Reference [3] of the main text.

NEW 5-D FLOW CHART

5-D PROGRAM WITH SALVESEN SECOND ORDER FORCE



C-1: MAIN PG - This subroutine is the actual main program for the 5-D; it calls all the required routines, loops on frequencies, heading angles, and ship speeds. Several lines were added as part of the scheme for computing the sectional quantity $\hat{h}_j(x)$ (Eq. (78d)). These new lines include numbers:

(14) A new common storage

(19) Some additional output data

(25-28) A zeroing routine

(100-102) Three new Write statements

A listing of the modified routine follows on pages 132-135.

```
      SUBROUTINE MAINPG(AB,PR,PA)
      COMPLEX F2,F3,F4,H2,H3,H4,ETA,F,WS,WC,CMPLX,HHAT2,HHAT3,HHAT4
      COMMON /PAPA/ WS,WC,SB,OMO,OM,OM2,WN,RHO,GRAV,BETAA,XI(32),DX(31)
     1,V,NSTA,NSE,MD,XLBP,ZETAA,ALFA
      COMMON /ABFH/ F2(32),F3(32),F4(32),H2(32),H3(32),H4(32)
     1,A33(32),B33(32),A22(32),B22(32),A44(32),B44(32),A24(32),B24(32)
      COMMON /VISC/ B4$(32),B44V,TON,TADR
      COMMON /MTN/ XMOT(10),YMOT(10),ZMOT(10),NMOT
      COMMON /SPCTRM/ AM(1200),BS(6400),RM(200),H13(10),OMP(10),NROMS
     1,SPCTM(10,40),SPOMS(10,40),NSOMS,NWK,II,N1,N2,M1,NSEA,L,NS,NSPC,IO
     2,NADR,RST(80),NEVT,EVENT(3,3),NN1,NN2,NN3,VCR,REVNT(120),OMEGE(40)
      COMMON /COEFFS/ ETA(6),F(6),G(6,6),A(6,6),B(6,6),C(6,6)
      COMMON /MOTHER/ UOB(16),BETA(16),OMEGA(40),NVL,NENC,NPCH,NFR,NFQ
      COMMON/ERB/HHAT2(32),HHAT3(32),HHAT4(32)
      DIMENSION EFGABC(168),I(24),AB(1),PR(1),PA(1)
      EQUIVALENCE (EFGABC(1),ETA(1))
      DATA I/'ETA ','G   ','A   ','B   ','C   ','F   ','A22 ','A33 '
     *      ,'A44 ','A24 ','B22 ','B33 ','B44 ','B24 ','F2  ','F3  '
     *      ,'F4  ','H2  ','H3  ','H4  ','B44*','HHT2','HHT3','HHT4'/
      PI=3.141593
      RAD=57.29578
      READ(5,1000) NCS
        CALL ERASE(EFGABC,168)
        DO 100 NCASE=1,NCS
      DO 5 IERB=1,32
      HHAT2(IERB)=(0.0,0.0)
      HHAT3(IERB)=(0.0,0.0)
    5 HHAT4(IERB)=(0.0,0.0)
        CALL TIMING(IPR)
        CALL INPUT
        CALL TIMING(ISR)
        TIN=(ISR-IPR)/100.
        CALL MAPING
        NBD=2/(N1+1)*N1
        IF(NMOT.NE.0.AND.IO.NE.0)
     1  WRITE(IO,2000) (XMOT(N),YMOT(N),ZMOT(N),N=1,NMOT)
```

```
      IF(N1.NE.0) CALL BNDSH1(N2,810,8100)                                MNPG0037
      WRITE(6,1002)                                                       MNPG0038
   10 CALL GC                                                             MNPG0039
      CALL TIMING(ISR)                                                    MNPG0040
      CALL POTENI(AB,BR,PA)                                               MNPG0041
      CALL TIMING(ISE)                                                    MNPG0042
      TPL=(ISE-ISR)/100.                                                  MNPG0043
      WRITE(6,2400) TIN,TPI                                               MNPG0044
        DO 90 N=1,NVL                                                     MNPG0045
        V=UOB(N)                                                          MNPG0046
        IF(IO.NE.0) WRITE(IO,2000) V                                      MNPG0047
          DO 80 M=1,NFNC                                                  MNPG0048
          CALL ERASE(EFGABC,12)                                           MNPG0049
          MD=2                                                            MNPG0050
          BETAA=BETA(M)                                                   MNPG0051
          IF(BETAA.EQ.180..OR.BETAA.EQ.0..OR.B44V.LT.0.) MD=1             MNPG0052
          II=3*MD                                                         MNPG0053
          M1=MD+1                                                         MNPG0054
          BET=BETAA/RAD                                                   MNPG0055
          SB=SIN(BET)                                                     MNPG0056
          CSB=COS(BET)                                                    MNPG0057
          IF(IO.NE.0) WRITE(IO,2000) BETAA                                MNPG0058
          CALL TIMING(IPB)                                                MNPG0059
            DO 70 I=1,NRCMS                                               MNPG0060
            OMO=OMEGA(L)                                                  MNPG0061
            WN=OMO*OMO/GRAV                                               MNPG0062
            IF(ALFA.NE.0.) ZETAA=ALFA/WN                                  MNPG0063
            XL=2.*PI/WN/XLBP                                              MNPG0064
            CK=-WN*CSB                                                    MNPG0065
            OM=OMO+V*CK                                                   MNPG0066
            OMEGE(L)=OM                                                   MNPG0067
            OM2=OM*OM                                                     MNPG0068
            WC=CMPLX(0.,CK)                                               MNPG0069
            WS=CMPLX(0.,SB*WN)                                            MNPG0070
            IF(NSEA.NE.1.AND.NBD.EQ.0) WRITE(6,1003)                      MNPG0071
            IF(NSEA.NE.1) WRITE(6,1001) NBD,V,BETAA,OMO,OM,XL,ZETAA       MNPG0072
```

```
      CALL INTRPL(AB,PR,PA)                                    MNPG0073
      CALL ABF                                                 MNPG0074
      CALL MATRIX                                              MNPG0075
      IF(NMOT+NEVT.NE.0) CALL MOTNS                            MNPG0076
      IF(N1.NE.0) CALL BNDSH2                                  MNPG0077
      IF(NADR.NE.0) CALL ADDRES                                MNPG0078
      IF(NPCH.LE.0) GO TO 65                                   MNPG0079
      WRITE(6,3000) T( 1),EIA                                  MNPG0080
      WRITE(6,3000) T( 2),G                                    MNPG0081
      WRITE(6,3000) T( 3),A                                    MNPG0082
      WRITE(6,3000) T( 4),B                                    MNPG0083
      WRITE(6,3000) T( 5),C                                    MNPG0084
      WRITE(6,3000) T( 6),F                                    MNPG0085
      WRITE(6,3001) T( 7),(A22(I),I=1,NSTA)                    MNPG0086
      WRITE(6,3001) T( 8),(A33(I),I=1,NSTA)                    MNPG0087
      WRITE(6,3001) T( 9),(A44(I),I=1,NSTA)                    MNPG0088
      WRITE(6,3001) T(10),(A24(I),I=1,NSTA)                    MNPG0089
      WRITE(6,3001) T(11),(B22(I),I=1,NSTA)                    MNPG0090
      WRITE(6,3001) T(12),(B33(I),I=1,NSTA)                    MNPG0091
      WRITE(6,3001) T(13),(B44(I),I=1,NSTA)                    MNPG0092
      WRITE(6,3001) T(14),(B24(I),I=1,NSTA)                    MNPG0093
      WRITE(6,3001) T(15),( F2(I),I=1,NSTA)                    MNPG0094
      WRITE(6,3001) T(16),( F3(I),I=1,NSTA)                    MNPG0095
      WRITE(6,3001) T(17),( F4(I),I=1,NSTA)                    MNPG0096
      WRITE(6,3001) T(18),( H2(I),I=1,NSTA)                    MNPG0097
      WRITE(6,3001) T(19),( H3(I),I=1,NSTA)                    MNPG0098
      WRITE(6,3001) T(20),( H4(I),I=1,NSTA)                    MNPG0099
  WRITE(6,3001)T(22),(HHAT2(I),I=1,NSTA)                       MNPG0100
  WRITE(6,3001)T(23),(HHAT3(I),I=1,NSTA)                       MNPG0101
  WRITE(6,3001)T(24),(HHAT4(I),I=1,NSTA)                       MNPG0102
65    IF(NPCH.NE.0)                                            MNPG0103
     *WRITE(6,3001) T(21),(B4$(I),I=1,NSTA),B44V,B(4,4)        MNPG0104
70    CONTINUE                                                 MNPG0105
     IF(NSEA.NE.0) CALL STATIS                                 MNPG0106
     CALL TIMING(IPE)                                          MNPG0107
     THD=(IPE-IPB)/100.                                        MNPG0108
```

```
         WRITE(6,2500) THD                                               MNPG0109
         IF(N1.NE.1) WRITE(6,1002)                                       MNPG0110
  80     CONTINUE                                                        MNPG0111
  90   CONTINUE                                                          MNPG0112
 100 CONTINUE                                                            MNPG0113
      RETURN                                                             MNPG0114
1000  FORMAT(I5)                                                         MNPG0115
1001  FORMAT(I1,'*** REGULAR WAVE RESULTS    SHIP SPEED',F9.4,'  HEADING MNPG0116
     1 ANGLE DEG.',F10.4,'  OMEGA',F8.4,'  OMEGAE',F8.4,'  LAMBDA/XLBP'  MNPG0117
     2,F9.4/'OZETAA =',F9.4)                                             MNPG0118
1002  FORMAT('1')                                                        MNPG0119
1003  FORMAT('0',130(1H-))                                               MNPG0120
2000  FORMAT(8F10.4)                                                     MNPG0121
2400  FORMAT('OTIMING OF INPUT DATA  :',F6.2,' SEC.'/                    MNPG0122
     *        'OTIMING OF POTENI CALC :',F6.2,' SEC.')                   MNPG0123
2500  FORMAT('OTIMING OF RESPONSES :',F5.2,' SEC.')                      MNPG0124
3000  FORMAT('0',A4,6G14.6/(5X,6G14.6))                                  MNPG0125
3001  FORMAT('0',A4,9G14.6/(5X,9G14.6))                                  MNPG0126
      END                                                                MNPG0127
```

C-2: INTRPL - This routine interpolates the hydrodynamic coefficients for the desired frequency and calculates Froude-Kriloff and sectional diffraction forces. Now it also computes the sectional quantity $\hat{h}_j(x)$, (Eq. (78d)) needed for the second order force calculation. The new lines include:

14, 15, 16, 26, 28, 30, 37, 42, 43, 44, 69, 72, 82, 84, 86, 88, 90, 92, 94, 97, 109, 110, 114, 118, 122, 125, 129.

A listing of the modified routine follows on pages 137-140.

```
      SUBROUTINE INTRPL(AB,PR,PA)
      IMPLICIT COMPLEX (C,F,H,W)
      REAL WN
      COMMON WPI(15),WH(15),FW(15),RDY(15),ADY(15),RDZ(15),ADZ(15)
     1,AYZ(15),DT(14)
      COMMON /ABFH/ F2(32),F3(32),F4(32),H2(32),H3(32),H4(32)
     1,A33(32),B33(32),A22(32),B22(32),A44(32),B44(32),A24(32),B24(32)
      COMMON /PAPA/ WS,WC,SB,CM0,CM,CM2,WN,RHO,GRAV,BETAA,XI(32),DX(31)
     1,V,NSTA,NSP,MD,XLBP,ZETAA,ALFA
      COMMON /STA/ YM(32),ZM(32),SIGMA(32)
      COMMON /MAP/ NC(32),NP,MP
      COMMON /POT/ YN(32,14),ZN(32,14),DYN(32,15),DZN(32,14),DTB(32,14)
      COMMON /INTPOL/ DELTA(41)
      COMMON/ERB/ HHAT2(32),HHAT3(32),HHAT4(32)
      DIMENSION FH(192),ADDA(256),AB(1),PR(1),PA(1),RDYY(15),RDZZ(15),
     1ADYY(15),ADZZ(15),FWRB(15),WHRB(15)
      EQUIVALENCE (F2(1),FH(1)),(A33(1),ADDA(1))
      D(P,Q,R,S)=(P-R)*(P-S)/(R-Q)/(S-Q)
      X(Y1,Y2,Y3)=D1*Y1+D2*Y2+D3*Y3
      NP1=NP-1
      NN=0
      NEW=41*(3*NP-2)
      OMA=ABS(OM)
      SQ=SQRT(OMA)
      RDZ(NP)=0.
      RDZZ(NP)=0.0
      ADZ(NP)=0.
      ADZZ(NP)=0.0
      ADY(NP)=0.
      ADYY(NP)=0.0
      AYZ(NP)=0.
        DO 200 N=1,NSTA
        NEQ=NC(N)
        IF(NEQ.GE.0) GO TO 30
        IF(NEQ.EQ.-2) GO TO 200
        WX=CEXP(WC*CMPLX(-DX(N-1),0.))
```

```
      WBBY=CEXP(WC*CMPLX(DX(N-1),0.0))                                  INTL0037
          DO 10 I=N,256,32                                              INTL0038
10        ADDA(I)=ADDA(I-1)                                             INTL0039
          DO 20 I=N,192,32                                              INTL0040
20        FH(I)=WX*FH(I-1)                                              INTL0041
      HHAT2(N)=WBBY*HHAT2(N-1)                                          INTL0042
      HHAT3(N)=WBBY*HHAT3(N-1)                                          INTL0043
      HHAT4(N)=WBBY*HHAT4(N-1)                                          INTL0044
        GO TO 200                                                       INTL0045
30      NWPN=NN*NEW                                                     INTL0046
        DL=OM2*ZM(N)/GRAV                                               INTL0047
        IF(DL.GT.5.) DL=5.                                              INTL0048
        DL1=DELTA(1)                                                    INTL0049
        DL2=DELTA(2)                                                    INTL0050
        DL3=DELTA(3)                                                    INTL0051
          DO 40 J=4,42                                                  INTL0052
          L=J                                                           INTL0053
          IF(DL.LT..5*(DL2+DL3)) GO TO 50                               INTL0054
          IF(J.EQ.42) GO TO 50                                          INTL0055
          DL1=DL2                                                       INTL0056
          DL2=DL3                                                       INTL0057
40        DL3=DELTA(J)                                                  INTL0058
50      D1=D(DL,DL1,DL2,DL3)                                            INTL0059
        D2=D(DL,DL2,DL1,DL3)                                            INTL0060
        D3=D(DL,DL3,DL2,DL1)                                            INTL0061
        O=SQ                                                            INTL0062
        NI=328*NN+L-3                                                   INTL0063
          DO 60 I=N,256,32                                              INTL0064
          O=1./O                                                        INTL0065
          ADDA(I)=X(AB(NI),AB(NI+1),AB(NI+2))*(O*SQ)                    INTL0066
60        NI=NI+41                                                      INTL0067
        WX=CEXP(WC*CMPLX(XI(N),0.))                                     INTL0068
      WBBY=CEXP((-1.0,0.0)*WC*CMPLX(XI(N),0.0))                         INTL0069
        WXG=WX*CMPLX(GRAV,0.)                                           INTL0070
        WXO=WX*CMPLX(OMO,0.)                                            INTL0071
      WXBB=WBBY*CMPLX(OMO,0.0)                                          INTL0072
```

```
      DT(1)=DTB(N,1)                                              INTL0073
        DO 70 I=1,NP1                                             INTL0074
        Y=YN(N,I)                                                 INTL0075
        Z=ZN(N,I)                                                 INTL0076
        DY=DYN(N,I)                                               INTL0077
        DZ=DZN(N,I)                                               INTL0078
        IF(NEQ.EQ.1)  DT(I)=DTB(N,I)                              INTL0079
        EX=EXP(-WN*Z)                                             INTL0080
        CX=CEXP(WS*CMPLX(Y,0.))                                   INTL0081
    CBBY=CEXP((-1.0,0.0)*WS*CMPLX(Y,0.0))                         INTL0082
        RW=EX* REAL(CX)                                           INTL0083
    RWBB=EX*REAL(CBBY)                                            INTL0084
        AW=EX*AIMAG(CX)                                           INTL0085
    AWBB=EX*AIMAG(CBBY)                                           INTL0086
        RDY(I)=RW*DY                                              INTL0087
    RDYY(I)=RWBB*DY                                               INTL0088
        ADY(I)=AW*DY                                              INTL0089
    ADYY(I)=AWBB*DY                                               INTL0090
        RDZ(I)=RW*DZ                                              INTL0091
    RDZZ(I)=RWBB*DZ                                               INTL0092
        ADZ(I)=AW*DZ                                              INTL0093
    ADZZ(I)=AWBB*DZ                                               INTL0094
 70     AYZ(I)=AW*(Y*DY+Z*DZ)                                     INTL0095
      RDY(NP)=EXP(-WN*ZM(N))*DYN(N,NP)                            INTL0096
    RDYY(NP)=EXP(-WN*ZM(N))*DYN(N,NP)                             INTL0097
        DO 160 NC=1,3                                             INTL0098
        NPP=NP-(NC/2)                                             INTL0099
        NWP=NWPN+(L-4)*NPP+((NC-1)*NP-(NC/3))*41                  INTL0100
          DO 80 I=1,NPP                                           INTL0101
          NWI=NWP+I                                               INTL0102
          PGR=X(PR(NWI),PR(NWI+NPP),PR(NWI+2*NPP))*OMA            INTL0103
          PGM=X(PA(NWI),PA(NWI+NPP),PA(NWI+2*NPP))*OMA            INTL0104
 80       WPI(I)=CMPLX(PGR,PGM)                                   INTL0105
        GO TO (100,120,140),NC                                    INTL0106
100       DO 110 I=1,NP                                           INTL0107
          FW(I)=CMPLX(-ADY(I)-SB*RDZ(I),0.)                       INTL0108
```

```
      FWRB(I)=CMPLX(-ADYY(I)+SB*BDZZ(I),0.0)                            INTL0109
      WHRB(I)=CMPLX(BDYY(I)+SB*ADZZ(I),0.0)*WPI(I)                      INTL0110
110         WH(I)=CMPLX(BDY(I)-SB*ADZ(I),0.)*WPI(I)                     INTL0111
          F3(N)=CMPLX(-2.*TRAP(DT,BDY,NP,NEQ),0.)*WXG                   INTL0112
          H3(N)=( 0.,2.)*WTRAP(DT,WH,NP,NEQ)*WXO                        INTL0113
      HHAT3(N)=(0.,2.)*WTRAP(DT,WHRB,NP,NEQ)*WXRB                       INTL0114
          GO TO 160                                                     INTL0115
120       WPI(NP)=(0.,0.)                                               INTL0116
            DO 130 I=1,NP                                               INTL0117
      WHRB(I)=FWRB(I)*WPI(I)                                            INTL0118
130         WH(I)=FW(I)*WPI(I)                                          INTL0119
          F2(N)=CMPLX(0.,-2.*TRAP(DT,ADZ,NP,NEQ))*WXG                   INTL0120
          H2(N)=(-2.,0.)*WTRAP(DT,WH,NP,NEQ)*WXO                        INTL0121
      HHAT2(N)=(2.,0.)*WTRAP(DT,WHRB,NP,NEQ)*WXRB                       INTL0122
          GO TO 160                                                     INTL0123
140         DO 150 I=1,NP                                               INTL0124
      WHRB(I)=FWRB(I)*WPI(I)                                            INTL0125
150         WH(I)=FW(I)*WPI(I)                                          INTL0126
          F4(N)=CMPLX(0.,-2.*TRAP(DT,AYZ,NP,NEQ))*WXG                   INTL0127
          H4(N)=( 2.,0.)*WTRAP(DT,WH,NP,NEQ)*WXO                        INTL0128
      HHAT4(N)=(2.,0.)*WTRAP(DT,WHRB,NP,NEQ)*WXRB                       INTL0129
160       CONTINUE                                                      INTL0130
        NN=NN+1                                                         INTL0131
200     CONTINUE                                                        INTL0132
      RETURN                                                            INTL0133
      END                                                               INTL0134
```

C-3: ADDRES - This is a new routine which computes the second order mean force on a ship in a regular wave due to all five motion components and wave reflection. This is performed according to the 1974 theory of Salvesen. ADDRES is primarily intended to handle output computing various subtotals and storing the response operators for the statistical routines.

A listing of this new routine follows on pages 142-144.

```
      SUBROUTINE ADDRES                                                ADRS0001
      IMPLICIT COMPLEX (F,H)                                           ADRS0002
      COMPLEX WS,WC,ETA                                                ADRS0003
      REAL*8 $(6)                                                      ADRS0004
      REAL H13                                                         ADRS0005
      COMMON/PAPA/WS,WC,SB,CMO,OM,OM2,WN,RHO,GRAV,BETAA,XI(32),DX(31), ADRS0006
     1V,NSTA,NSP,MD,XLBP,ZETAA,ALFA                                    ADRS0007
      COMMON/STA/YM(32),ZM(32),SIGMA(32)                               ADRS0008
      COMMON/ABFH/F2(32),F3(32),F4(32),H2(32),H3(32),H4(32),A33(32),   ADRS0009
     1B33(32),A22(32),B22(32),A44(32),B44(32),A24(32),B24(32)          ADRS0010
      COMMON/VISC/ B4$(32),B44V,ION,TADB                               ADRS0011
      COMMON/SPCTRM/ AM(1200),BS(6400),RM(200),H13(10),OMP(10),NBOMS,  ADRS0012
     1SPCTM(10,40),SPOMS(10,40),NSOMS,NWX,TI,N1,N2,M1,NSEA,L,NS,NSPC,IO,ADRS0013
     2NADR,RST(80),NEVT,EVENT(3,3),NN1,NN2,NN3,VCR,PEVNT(120),OMEGE(40) ADRS0014
      COMMON/COEFFS/ETA(6),F(6),G(6,6),A(6,6),B(6,6),C(6,6)            ADRS0015
      COMMON/ERB/HHAT2(32),HHAT3(32),HHAT4(32)                         ADRS0016
      COMMON/INTNL/DIM(4),TRIG(4),FACX,S2,PON                          ADRS0017
      DIMENSION ADRST(2,3,4)                                           ADRS0018
      DATA RAD/57.29578/                                               ADRS0019
      DATA $/'        ',' SWAY   ',' HEAVE  ',' ROLL   ',' PITCH  ',   ADRS0020
     1' YAW    '/                                                      ADRS0021
      IRB=2                                                            ADRS0022
      JRB=6                                                            ADRS0023
      KRB=1                                                            ADRS0024
      ICNT=1                                                           ADRS0025
      DO 5 JKJ=1,2                                                     ADRS0026
      DO 5 JKP=1,3                                                     ADRS0027
      DO 5 JKR=1,4                                                     ADRS0028
    5 ADRST(JKJ,JKP,JKR)=0.0                                           ADRS0029
      Z2=ZETAA*ZETAA                                                   ADRS0030
      CB=COS(BETAA/RAD)                                                ADRS0031
      S2=SB*SB                                                         ADRS0032
      FACX=(0.0,1.0)*V/CM                                              ADRS0033
      W2=OMC*CMO/OM                                                    ADRS0034
      PON=Z2*WN*W2                                                     ADRS0035
      DIM(1)=1.0                                                       ADRS0036
```

```
      DIM(2)=1.0                                                        ADRS0037
      DIM(3)=TADR*Z2                                                    ADRS0038
      DIM(4)=DIM(3)                                                     ADRS0039
      TRIG(1)=SB                                                        ADRS0040
      IF(BETAA.EQ.180.0.OR.BETAA.EQ.0.0) TRIG(1)=0.0                    ADRS0041
      TRIG(2)=-CB                                                       ADRS0042
      IF(BETAA.EQ.90.0) TRIG(2)=0.0                                     ADRS0043
      TRIG(3)=TRIG(1)                                                   ADRS0044
      TRIG(4)=TRIG(2)                                                   ADRS0045
      IF(NADR.NE.2.AND.NSEA.NE.1) WRITE(6,999)                          ADRS0046
      IF(NSEA.NE.1)                                                     ADRS0047
     *WRITE(6,1000)                                                     ADRS0048
      IF(MD.EQ.2) GO TO 10                                              ADRS0049
      IPB=3                                                             ADRS0050
      JRP=5                                                             ADRS0051
      KRB=2                                                             ADRS0052
10    DO 30 IQK=IPB,JRB,KRB                                             ADRS0053
      CALL RESIST (ICNT,IQK,ADRST)                                      ADRS0054
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0055
     *WRITE(6,1001) (ADRST(1,1,KQK),KQK=1,4)                            ADRS0056
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0057
     *WRITE(6,1002) $(IQK),(ADRST(1,2,KQK),KQK=1,4)                     ADRS0058
      DO 20 JQK=1,4                                                     ADRS0059
20    ADRST(2,3,JQK)=ADRST(1,1,JQK)+ADRST(1,2,JQK)                      ADRS0060
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0061
     *WRITE(6,1003) (ADRST(2,3,KQK),KQK=1,4)                            ADRS0062
30    CONTINUE                                                          ADRS0063
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0064
     *WRITE(6,1004) (ADRST(2,1,KQK),KQK=1,4)                            ADRS0065
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0066
     *WRITE(6,1005) (ADRST(2,2,KQK),KQK=1,4)                            ADRS0067
      DO 40 JQK=1,4                                                     ADRS0068
40    ADRST(2,3,JQK)=ADRST(2,1,JQK)+ADRST(2,2,JQK)                      ADRS0069
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0070
     *WRITE(6,1006) (ADRST(2,3,KQK),KQK=1,4)                            ADRS0071
      IF(NADR.NE.2.AND.NSEA.NE.1)                                       ADRS0072
```

```
     *WRITE(6,1007)  (ADRST(1,3,KOK),KOK=1,4)                                      ADRS0073
      DO 50 JOK=1,4                                                                ADRS0074
50    ADRST(2,3,JOK)=ADRST(2,3,JOK)+ADRST(1,3,JOK)                                 ADRS0075
      IF(NSEA.NE.1)                                                                ADRS0076
     *WRITE(6,1006)  (ADRST(2,3,KOK),KOK=1,4)                                      ADRS0077
      IF(NADR.NE.2.AND.NSEA.NE.1)                                                  ADRS0078
     *WRITE(6,1008)                                                                ADRS0079
      IF(NSEA.EQ.0) RETURN                                                         ADRS0080
      RST(L)=ADRST(2,3,2)/Z2                                                       ADRS0081
      RST(L+NROMS)=ADRST(2,3,1)/Z2                                                 ADRS0082
      RETURN                                                                       ADRS0083
999   FORMAT('1')                                                                  ADRS0084
1000  FORMAT('0',10X,'MEAN ADDED RESISTANCE/DRIFT FORCE CALCULATION',//,           ADRS0085
     *36X,'DIMENSIONAL',19X,'NON-DIMENSIONAL',/,7X,'MODE    SOURCE',7X,            ADRS0086
     *'DRIFT FORCE   ADDED RESIST.',5X,'DRIFT FORCE   ADDED RESIST.')              ADRS0087
1001  FORMAT ('0',14X,'FR-KRL ',4(5X,F11.4))                                       ADRS0088
1002  FORMAT ('0',5X,A8,' WVDIFF ',4(5X,F11.4))                                    ADRS0089
1003  FORMAT ('0',14X,'TOTALS ',4(5X,F11.4),//,1X,90(1H-))                         ADRS0090
1004  FORMAT(1X,90(1H-),//,11X,'SUM OF ALL THE MODES AND THE DIFFRACTION           ADRS0091
     * POTENTIAL CONTRIBUTION',//,5X,'ALL MODES FR-KRL ',4(5X,F11.4))              ADRS0092
1005  FORMAT ('0',4X,'ALL MODES WVDIFF ',4(5X,F11.4))                              ADRS0093
1006  FORMAT (22X,4(5X,11(1H-)),/,15X,'TOTALS ',4(5X,F11.4))                       ADRS0094
1007  FORMAT ('0',4X,'DIFFR. POT. CONT.',4(5X,F11.4))                              ADRS0095
1008  FORMAT(1H1)                                                                  ADRS0096
      END                                                                          ADRS0097
```

C-4: RESIST - This is a new routine which is called by ADDRES. It performs the real calculation of the second order force outlined in the theory of Chapter III, and then returns the resulting values to ADDRES for manipulation.

A listing of this new routine follows on pages 146-147.

```
      SUBROUTINE RESIST (ICNT,IQK,ADRST)                                RSST0001
C                                                                       RSST0002
C        DOFS NOT WORK IN METERS--- HAS A GLICH------                   RSST0003
C                                                                       RSST0004
      IMPLICIT COMPLEX (F,H)                                            RSST0005
      COMPLEX WS,WC,ETA,CMPLX,WTRAP,CONJG                               RSST0006
      COMMON/PARA/ WS,WC,SB,CMC,OM,OM2,WN,RHO,GRAV,BETAA,XI(32),DX(31),V RSST0007
     1,NSTA,NSP,MD,XLBP,ZETAA,ALFA                                      RSST0008
      COMMON/STA/ YM(32),ZM(32),SIGMA(32)                               RSST0009
      COMMON/ABFH/F2(32),F3(32),F4(32),H2(32),H3(32),H4(32),A33(32),    RSST0010
     1B33(32),A22(32),B22(32),A44(32),B44(32),A24(32),B24(32)           RSST0011
      COMMON/VISC/ B4$(32),B44V,TON,TADR                                RSST0012
      COMMON/COEFFS/ETA(6),F(6),G(6,6),A(6,6),B(6,6),C(6,6)             RSST0013
      COMMON/ERB/HHAT2(32),HHAT3(32),HHAT4(32)                          RSST0014
      COMMON/INTNL/DIM(4),TRIG(4),FACY,S2,PQN                           RSST0015
      DIMENSION YY(32),FYY(32),ADRST(2,3,4)                             RSST0016
      FACT=(0.0,0.5)*WN*ETA(IQK)                                        RSST0017
      IF(ICNT.EQ.2) GO TO 10                                            RSST0018
      DO 8 ISK=1,NSTA                                                   RSST0019
      F2= EXP(-2.0*WN*ZM(ISK)*SIGMA(ISK))                               RSST0020
8     YY(ISK)=F2*(B33(ISK)+(S2*B22(ISK)))                               RSST0021
      CMPD=0.5*TRAP(DX,YY,NSTA,NSP)*PQN                                 RSST0022
      DO 9 ISK=1,4                                                      RSST0023
9     ADRST(1,3,ISK)=CMPD*TRIG(ISK)*TON/DIM(ISK)                        RSST0024
      ICNT=2                                                            RSST0025
 10   GO TO (11,12,13,14,15,16),IQK                                     RSST0026
 11   RETURN                                                            RSST0027
 12   FRKR=RHO*ZETAA*WTRAP(DX,F2,NSTA,NSP)                              RSST0028
      FRKR=CONJG(FRKR)*FACT                                             RSST0029
      FDF=RHO*ZETAA*WTRAP(DX,HHAT2,NSTA,NSP)*FACT                       RSST0030
      GO TO 17                                                          RSST0031
 13   FRKR=RHO*ZETAA*WTRAP(DX,F3,NSTA,NSP)                              RSST0032
      FRKR=CONJG(FRKR)*FACT                                             RSST0033
      FDF=RHO*ZETAA*WTRAP(DX,HHAT3,NSTA,NSP)*FACT                       RSST0034
      GO TO 17                                                          RSST0035
 14   FRKR=RHO*ZETAA*WTRAP(DX,F4,NSTA,NSP)                              RSST0036
```

```
      FRKR=CONJG(FRKR)*FACT                                    RSST0037
      FDF=RHO*ZETAA*WTRAP(DX,HHAT4,NSTA,NSP)*FACT              RSST0038
      GO TO 17                                                 RSST0039
15    DO 25 ISK=1,NSTA                                         RSST0040
25    FYY(ISK)=XI(ISK)*F3(ISK)                                 RSST0041
      FRKR=-1.0*RHO*ZETAA*WTRAP(DX,FYY,NSTA,NSP)               RSST0042
      FRKR=CONJG(FRKR)*FACT                                    RSST0043
      DO 35 ISK=1,NSTA                                         RSST0044
35    FYY(ISK)=(CMPLX(XI(ISK),0.0)+FACX)*HHAT3(ISK)            RSST0045
      FDF=-1.0*RHO*ZETAA*WTRAP(DX,FYY,NSTA,NSP)*FACT           RSST0046
      GO TO 17                                                 RSST0047
16    DO 26 ISK=1,NSTA                                         RSST0048
26    FYY(ISK)=XI(ISK)*F2(ISK)                                 RSST0049
      FRKR=RHO*ZETAA*WTRAP(DX,FYY,NSTA,NSP)                    RSST0050
      FRKR=CONJG(FRKR)*FACT                                    RSST0051
      DO 36 ISK=1,NSTA                                         RSST0052
36    FYY(ISK)=(CMPLX(XI(ISK),0.0)+FACX)*HHAT2(ISK)            RSST0053
      FDF=RHO*ZETAA*WTRAP(DX,FYY,NSTA,NSP)*FACT                RSST0054
17    DO 18 KD=1,4                                             RSST0055
      ADRST(1,1,KD)=REAL(FRKR)*TRIG(KD)*TON/DIM(KD)            RSST0056
      ADRST(1,2,KD)=REAL(FDF)*TRIG(KD)*TON/DIM(KD)             RSST0057
      ADRST(2,1,KD)=ADRST(2,1,KD)+ADRST(1,1,KD)                RSST0058
18    ADRST(2,2,KD)=ADRST(2,2,KD)+ADRST(1,2,KD)                RSST0059
      RETURN                                                   RSST0060
      END                                                      RSST0061
```

C-5: STATIS AND SPIN - These routines perform the statistical calculations necessary for long-crested irregular seaway response predictions. They have been modified to include the calculations of a mean second order force. The new lines in STATIS include:

22, 115-126, 135, 150.

The new lines in SPIN include:

26-31.*

*NOTE: These new lines in SPIN exist to take care of the problem of a negative value for the variable, SUM. This does not occur for motions which have R.A.O.'s which are always positive. However, it might occur for second order force (e.g. following seas where the waves will help to push the ship). Since it was desired to use SPIN for the second order force calculation (just as for all the other response calculations), it was necessary to provide a way to avoid taking the square root of a negative number and causing an error (See line 30). To summarize, these lines do not affect the positive motion R.A.O.'s and they exist only to avoid computer error messages when SPIN is called for the second order force. The quantities S, S3, S10, S1000 are not meaningful for the second order force, and are not written out.

A listing of these two modified routines follows on pages 150-156.

```
      SUBROUTINE STATIS                                                 STTS0001
      IMPLICIT REAL*8 ($)                                               STTS0002
      LOGICAL LS                                                        STTS0003
      COMPLEX WS,WC                                                     STTS0004
      COMMON SPC(40),SPO(40),DOM(39),VGB,SM(40),SM2(40),SM4(40),AJMV(2) STTS0005
     1,S(32),S3(32),S10(32),S1000(32),OMOT(5),VMOT(7,10),BNDS(5,32),ZZ  STTS0006
      COMMON /PAPA/ WS,WC,SB,OMC,OM,OM2,WN,RHO,GRAV,BETAA,XI(32),DX(31) STTS0007
     1,V,NSTA,NSP,MD,XLBE,ZETAA,ALFA                                    STTS0008
      COMMON /MTN/ XMOT(10),YMOT(10),ZMOT(10),NMOT                      STTS0009
      COMMON /SPCTRM/ AM(1200),BS(6400),RM(200),H13(10),OMP(10),NBOMS   STTS0010
     1,SPCTM(10,40),SPOMS(10,40),NSOMS,NWX,II,N1,N2,M1,NSEA,L,NS,NSPC,IO STTS0011
     2,NADR,RST(80),NEVT,EVENT(3,3),NN1,NN2,NN3,VCR,REVNT(120),OMEGE(40) STTS0012
      COMMON /MOTHER/ UCB(16),BETA(16),OMEGA(40),NVL,NENC,NPCH,NFR,NFO  STTS0013
      DIMENSION $(41),R(40),I1(7),I2(7),I3(7),LL(7),OVB(235)            STTS0014
      EQUIVALENCE (CMOT(1),OVB(1))                                      STTS0015
      DATA $/'        ','   HEAVE','   PITCH','    SWAY','    ROLL'     STTS0016
     1      ,'     YAW','ABSOLUTE','L MOTION','VELOCITY','    ACCE'     STTS0017
     2      ,'LERATION','RELATIVE','TRANSVER','SE  ACCE','LONGITUD'     STTS0018
     3      ,'INAL SHE','AR  STA.','LONGITUD','INAL  B.','M.  STA.'     STTS0019
     4      ,'     LAT','ERAL SHE','AR  STA.','   TORSI','ONAL  B.'     STTS0020
     5      ,'M.  STA.','     LAT','ERAL  B.','M.  STA.',' VERTICA'     STTS0021
     6      ,'COMPONEN','T OF ADD','T OF DEI','.RES.(X)','FT(Y-AX)'     STTS0022
     7      ,'    DECK','PROPELLE','        ',' WETNESS','R RACING'     STTS0023
     8      ,'SLAMMING'/                                                STTS0024
      DATA I1/7,1,1,12,1,1,13/,I2/30,1,10,30,1,10,14/,I3/8,9,11,8,9,11  STTS0025
     1,11/,LL/0,2,4,0,2,4,0/                                            STTS0026
      LS=.TRUE.                                                         STTS0027
      IF(IO.EQ.0) LS=.FALSE.                                            STTS0028
      IF((LS).AND.II.EQ.3) CALL ERASE(OVB,235)                          STTS0029
      RAD=57.29578                                                      STTS0030
      VGB=V/GRAV*COS(BETAA/RAD)                                         STTS0031
        DO 200 NW=1,NWX                                                 STTS0032
        ZZ=1.                                                           STTS0033
        WRITE(6,1000) V,BETAA                                           STTS0034
        WRITE(6,1001) H13(NW),OMP(NW)                                   STTS0035
        SPO(1)=SPOMS(NW,1)                                              STTS0036
```

```
      SPC(1)=SPCTM(NW,1)                                                STTS0037
        DO 10 J=2,NSOMS                                                 STTS0038
        SPO(J)=SPOMS(NW,J)                                              STTS0039
        SPC(J)=SPCTM(NW,J)                                              STTS0040
   10   DOM(J-1)=SPO(J)-SPO(J-1)                                        STTS0041
C                                                                       STTS0042
C    MOTIONS @ ORIGIN                                                   STTS0043
C                                                                       STTS0044
      K=0                                                               STTS0045
      WRITE(6,1002)                                                     STTS0046
        DO 30 I=2,II                                                    STTS0047
          DO 20 L=1,NRCMS                                               STTS0048
          K=K+1                                                         STTS0049
   20     R(L)=RM(K)                                                    STTS0050
   30   OMOT(I-1)=SPIN($(I),$(1),$(1),R,I,0)                            STTS0051
      WRITE(6,1003)($(1),$(1),$(I),S(I),S3(I),S10(I),S1000(I),I=2,II)   STTS0052
      IF(NMOT.EQ.0) GO TO 70                                            STTS0053
C                                                                       STTS0054
C    MOTIONS @ SPECIFIED PTS.                                           STTS0055
C                                                                       STTS0056
      M2=M1+4                                                           STTS0057
      WRITE(6,1004)                                                     STTS0058
        DO 60 N=1,NMOT                                                  STTS0059
        J1=M1*(N-1)                                                     STTS0060
          DO 50 M=1,M2                                                  STTS0061
          J2=(M-1)/3                                                    STTS0062
          K=NRCMS*(J1+J2)                                               STTS0063
          J2=I1(3*J2+1)                                                 STTS0064
            DO 40 L=1,NRCMS                                             STTS0065
            K=K+1                                                       STTS0066
   40       R(L)=AM(K)*OMEGR(L)**LL(M)                                  STTS0067
   50     VMOT(M,N)=SPIN($(J2),$(I2(M)),$(I3(M)),R,M,0)                 STTS0068
        WRITE(6,1005) XMOT(N),YMOT(N),ZMOT(N)                           STTS0069
   60   WRITE(6,1003)  ($(I1(M)),$(I2(M)),$(I3(M)),S(M),S3(M),S10(M)    STTS0070
     1,                S1000(M),M=1,M2)                                 STTS0071
   70 IF(N1.EQ.0) GO TO 110                                             STTS0072
```

```
C                                                                       STTS0073
C     SHEAR & BENDING MOMENTS                                           STTS0074
C                                                                       STTS0075
      K=0                                                               STTS0076
      WRITE(6,1006)                                                     STTS0077
        DO 100 N=2,II                                                   STTS0078
        J1=3*N+9                                                        STTS0079
        J2=J1+2                                                         STTS0080
          DO 90 I=N1,N2                                                 STTS0081
            DO 80 L=1,NBCMS                                             STTS0082
            K=K+1                                                       STTS0083
   80       B(L)=BS(K)                                                  STTS0084
   90     BNDS(N-1,I)=SPIN($(J1),$(J1+1),$(J2),B,I,I)                   STTS0085
        WRITE(6,1003)  ($(J),J=J1,J2)                                   STTS0086
  100   WRITE(6,1007)  (I,S(I),S3(I),S10(I),S1000(I),I=N1,N2)           STTS0087
  110 IF(NEVT.EQ.0) GO TO 150                                           STTS0088
C                                                                       STTS0089
C     EVENTS : DECK WETNESS, PROPELLER RACING, AND SLAMMING             STTS0090
C                                                                       STTS0091
      WRITE(6,1009)                                                     STTS0092
      DO 140 I=NN1,NN2,NN3                                              STTS0093
        DO 130 K=1,2                                                    STTS0094
          DO 120 L=1,NBCMS                                              STTS0095
  120     B(L)=BEVNT(L+NBCMS*(I-1))*OMEGE(L)**LI(K)                     STTS0096
        J=K+3                                                           STTS0097
  130   AJMV(K)=SPIN($(I1(J)),$(I2(J)),$(I3(J)),B,K,0)                  STTS0098
      WRITE(6,1005)  (EVENT(I,J),J=1,3)                                 STTS0099
      WRITE(6,1003)  $(12),$(30),$(8),S(1),S3(1),S10(1),S1000(1)        STTS0100
      WRITE(6,1003)  $( 1),$( 1),$(9),S(2),S3(2),S10(2),S1000(2)        STTS0101
      VV=0.                                                             STTS0102
      IF(I.EQ.3)  VV=VCB**2                                             STTS0103
      FJ=EVENT(I,3)**2                                                  STTS0104
      PP=.5*(FJ/AJMV(1)+VV/AJMV(2))                                     STTS0105
      PROB=0.                                                           STTS0106
      IF(PP.LT.175.)  PROB=EXP(-PP)                                     STTS0107
      PP=10.*RAD*PROB*SQRT(AJMV(2)/AJMV(1))                             STTS0108
```

```
140   WRITE(6,1010) $(I+35),$(I+38),FP
150 IF(NADR.EQ.0) GO TO 190
C
C     MEAN ADDED RESISTANCE
C
      ZZ=0.
      WRITE(6,1011)
      K=0
        DO 170 I=1,2
          DO 160 L=1,NROMS
          K=K+1
160       R(L)=RST(K)
        J=31+I
        M=33+I
        ADRST=2.*SPIN($(31),$(J),$(M),R,I,0)
      IF(I.EQ.1)  ADRES=ADRST
      IF(I.EQ.2)  DRIFT=ADRST
170     WRITE(6,1003) $(31),$(J),$(M),ADRST
190     IF(.NOT.IS) GO TO 200
C
C     IO OF MEAN SQUARES FOR SHORT CRESTED SEA PREDICTIONS
C
        WRITE(IO,2000) H13(NW),CMF(NW)
        WRITE(IO,2000) (CMOT(I),I=1,5)
        IF(NMOT.NE.0) WRITE(IO,2000) ((VMOT(I,J),I=1,7),J=1,NMOT)
        IF(N1.NE.0) WRITE(IO,2000) ((BNDS(I,J),I=1,5),J=N1,N2)
      IF(NADR.NE.0)   WRITE(IO,2000) ADRES,DRIFT
200     CONTINUE
      RETURN
1000 FORMAT('1IRREGULAR WAVE RESULTS -- LONG CRESTED UNIDIRECTIONAL SEA
    1S'/'0SHIP SPEED : ',F10.4,9X,'HEADING ANGLE : ',F10.4)
1001 FORMAT('0SIGNIFICANT WAVE HEIGHT ',F10.4/' PEAK SPECTRAL FREQUENCY
    1 ',F10.4/'0',T58,'RMS',T71,'H(1/3)',T85,'H(1/10)',T99,'H(1/1000)')
1002 FORMAT(' MOTIONS @ ORIGIN')
1003 FORMAT(17X,3A8,5X,4F15.4)
1004 FORMAT('0MOTIONS @ SPECIFIED PTS.')
```

```
1005 FORMAT('0X=',F10.4,',  Y=',F9.4,',  Z=',F9.4/)                STTS0145
1006 FORMAT('0DYNAMIC LOADINGS')                                    STTS0146
1007 FORMAT(38X,I2,6X,4F15.4)                                       STTS0147
1009 FORMAT('0STATISTICS OF EVENTS')                                STTS0148
1010 FORMAT('0',24X,2A8,F8.2,' OCCURANCES PER HR.')                 STTS0149
1011 FORMAT('0MEAN SECOND ORDER FORCE (LBS.)')                      STTS0150
2000 FORMAT(6F13.6)                                                 STTS0151
     END                                                            STTS0152
```

```
      FUNCTION SPIN($1,$2,$3,R,I,M)                                     SPIN0001
      REAL*8 $1,$2,$3                                                   SPIN0002
      COMMON SPC(40),SPO(40),DOM(39),VGB,SM(40),SM2(40),SM4(40),AJMV(2) SPIN0003
     1,S(32),S3(32),S10(32),S1000(32),OMOT(5),VMOT(7,10),BNDS(5,32),ZZ  SPIN0004
      COMMON /SECTRM/ AM(1200),RS(6400),RM(200),H13(10),OMP(10),NRCMS   SPIN0005
     1,SPCIM(10,40),SPOMS(10,40),NSOMS,NWX,II,N1,N2,M1,NSEA,L,NS,NSPC,IO SPIN0006
     2,NADR,RST(80),NEVT,EVENT(3,3),NN1,NN2,NN3,VCB,REVNT(120),OMEGE(40) SPIN0007
      COMMON /MOTHER/ UCB(16),BETA(16),OMEGA(40),NVL,NENC,NECH,NFR,NFQ  SPIN0008
      DIMENSION R(1)                                                    SPIN0009
      L=2                                                               SPIN0010
      OM1=OMEGA(1)                                                      SPIN0011
      OM2=OMEGA(2)                                                      SPIN0012
      OM3=OMEGA(3)                                                      SPIN0013
        DO 20 J=1,NSCMS                                                 SPIN0014
        OM=SPC(J)                                                       SPIN0015
   10   IF(OM.LT..5*(OM2+OM3)) GO TO 20                                 SPIN0016
        IF(I+1.EQ.NRCMS) GO TO 20                                       SPIN0017
        L=L+1                                                           SPIN0018
        OM1=OM2                                                         SPIN0019
        OM2=OM3                                                         SPIN0020
        OM3=OMEGA(I+1)                                                  SPIN0021
        GO TO 10                                                        SPIN0022
   20   SM(J)=PARABL(OM,OM1,OM2,OM3,R(L-1),R(L),R(L+1),ZZ)*SPC(J)       SPIN0023
      SUM=TRAP(DOM,SM,NSCMS,1)                                          SPIN0024
      SPIN=SUM                                                          SPIN0025
      SYGN=1.0                                                          SPIN0026
      IF(SUM.EQ.0.0) GO TO 25                                           SPIN0027
      SYGN=SIGN(1.0,SUM)                                                SPIN0028
      SUM=ABS(SUM)                                                      SPIN0029
   25 S(I)=SQRT(SUM)*SYGN                                               SPIN0030
      SUM=SUM*SYGN                                                      SPIN0031
      S3(I)=4.0*S(I)                                                    SPIN0032
      S10(I)=5.1*S(I)                                                   SPIN0033
      S1000(I)=7.7*S(I)                                                 SPIN0034
      IF(NSPC.EQ.0) RETURN                                              SPIN0035
        DO 30 J=1,NSCMS                                                 SPIN0036
```

```
      OM=SPO(J)*(1.-SPO(J)*VGB)                                          SPIN0037
      SM2(J)=SM(J)*OM**2                                                 SPIN0038
   30 SM4(J)=SM(J)*OM**4                                                 SPIN0039
      SUM2=TRAP(DOM,SM2,NSOMS,1)                                         SPIN0040
      SUM4=TRAP(DOM,SM4,NSOMS,1)                                         SPIN0041
      IF(SUM.EQ.0.0.OR.SUM4.EQ.0.0) GO TO 35                             SPIN0042
      S24=SUM2*SUM2/SUM/SUM4                                             SPIN0043
      IF(S24.GT.1.) S24=1.                                               SPIN0044
      EPSILN=SQRT(1.-S24)                                                SPIN0045
      GO TO 39                                                           SPIN0046
35    EPSILN=99.                                                         SPIN0047
39    WRITE(6,1000) $1,$2,$3,M,SUM,SUM2,SUM4,EPSILN,(SM(J),J=1,NSOMS)    SPIN0048
      RETURN                                                             SPIN0049
 1000 FORMAT('0RESPONSE SPECTRUM FOR ',3A8,I3/' 0TH MOMENT =',E15.7      SPIN0050
     1,'  2ND MOMENT =',E15.7,'  4TH MOMENT =',E15.7,'  BROADNESS FACTOR SPIN0051
     2 =',F8.5/'0SPECTRAL AMPLITUDE''S :',4X,8E13.6/(1X,10E13.6))        SPIN0052
      END                                                                SPIN0053
```

C-6: AUXILIARY PROGRAM SHORTCREST

This program uses input prepared by the main 5-D program (subroutine STATIS) to calculate the mean responses in short-crested random seas according to Eq. (92) of the text. The conventional cosine-squared spreading function is used. This program has been modified to include a calculation of mean second order force as well as motion responses. The new lines include:

5, 6, 17, 58, 59, 71, 93, 148-161, 180, 181, 182.

A listing of the modified program follows on pages 158-163.

```
      COMMON /SHCEST/ OMOT(13,10,5),VMOT(13,10,10,7),BNDS(13,10,32,5)    SCRS0001
     1,XMOT(10),YMOT(10),ZMOT(10),BETA(16),V,NENC,NWX,N1,N2,NMOT,NWIND  SCRS0002
     2,H13(10),SP(34),DB(33),R(32),LM(32),THETA(36)                     SCRS0003
     3,S(32),S3(32),S10(32),S1000(32),OMP(10),N,I                       SCRS0004
     4,ADRES(13,10),DRIFT(13,10),SGNDF(32),NADR                         SCRS0005
      READ(5,1000) NENC,NWX,N1,N2,NMOT,NWIND,K,NADR                      SCRS0006
      READ(5,1001) (THETA(N),N=1,NWIND)                                  SCRS0007
      IF(NMOT.NE.0) READ(K,1001) (XMOT(N),YMOT(N),ZMOT(N),N=1,NMOT)     SCRS0008
      READ(K,1001) V                                                     SCRS0009
        DO 10 M=1,NENC                                                   SCRS0010
        READ(K,1001) BETA(M)                                             SCRS0011
          DO 10 NW=1,NWX                                                 SCRS0012
          READ(K,1002) H13(NW),OMP(NW)                                   SCRS0013
          READ(K,1002) (OMOT(M,NW,I),I=1,5)                              SCRS0014
          IF(NMOT.NE.0) READ(K,1002) ((VMOT(M,NW,J,I),I=1,7),J=1,NMOT)   SCRS0015
          IF(N1.NE.0) READ(K,1002) ((BNDS(M,NW,J,I),I=1,5),J=N1,N2)      SCRS0016
          IF(NADR.NE.0) READ(K,1002)  ADRES(M,NW),DRIFT(M,NW)            SCRS0017
   10     CONTINUE                                                       SCRS0018
       WRITE(6,1001) (THETA(N),N=1,NWIND)                                SCRS0019
       WRITE(6,1001) (BETA(N),N=1,NENC)                                  SCRS0020
      CALL SHORTC                                                        SCRS0021
   20 CONTINUE                                                           SCRS0022
      STOP                                                               SCRS0023
 1000 FORMAT(16I5)                                                       SCRS0024
 1001 FORMAT(8F10.4)                                                     SCRS0025
 1002 FORMAT(6E13.6)                                                     SCRS0026
      END                                                                SCRS0027
      SUBROUTINE SHORTC                                                  SCRS0028
      IMPLICIT REAL*8 ($)                                                SCRS0029
      COMMON /SHCEST/ OMOT(13,10,5),VMOT(13,10,10,7),BNDS(13,10,32,5)    SCRS0030
     1,XMOT(10),YMOT(10),ZMOT(10),BETA(16),V,NENC,NWX,N1,N2,NMOT,NWIND  SCRS0031
     2,H13(10),SP(34),DB(33),R(32),LM(32),THETA(36)                     SCRS0032
     3,S(32),S3(32),S10(32),S1000(32),OMP(10),N,I                       SCRS0033
     4,ADRES(13,10),DRIFT(13,10),SGNDF(32),NADR                         SCRS0034
      DIMENSION $(30),I1(7),I2(7),I3(7)                                  SCRS0035
      DATA $/'        ','   HEAVE','   PITCH','    SWAY','    ROLL'      SCRS0036
```

```
     1       ,'      YAW','ABSOLUTE','L MOTION','VELOCITY','    ACCE'     SCRS0037
     2       ,'LERATION','RELATIVE','TRANSVER','SE  ACCE','LONGITUD'     SCRS0038
     3       ,'INAL SHE','AR  STA.','LONGITUD','INAL  B.','M.  STA.'     SCRS0039
     4       ,'      LAT','ERAL SHF','AR  STA.','   TORSI','ONAL  B.'     SCRS0040
     5       ,'M.  STA.','      LAT','ERAL  B.','M.  STA.',' VERTICA'/    SCRS0041
      DATA I1/7,1,1,12,1,1,13/,I2/30,1,10,30,1,10,14/,I3/8,9,11,8,9,11   SCRS0042
     1,11/                                                               SCRS0043
      SPREDF(B)=COS((T-B)/RAD)**2                                        SCRS0044
      RAD=57.29578                                                       SCRS0045
C                                                                        SCRS0046
C     SEARCH FOR HEADING ANGLES WITHIN + OR -90. DEGREES                 SCRS0047
C                                                                        SCRS0048
      DO 200 K=1,NWIND                                                   SCRS0049
      T=THETA(K)                                                         SCRS0050
      T1=T-89.999                                                        SCRS0051
      T2=T+89.999                                                        SCRS0052
      B(1)=0.                                                            SCRS0053
      SP(1)=0.                                                           SCRS0054
      L=1                                                                SCRS0055
      B1=T-90.                                                           SCRS0056
      L1=1                                                               SCRS0057
      DO 99 IERB=1,32                                                    SCRS0058
99    SGNDF(IERB)=1.0                                                    SCRS0059
      IF(T1.GT.0.) GO TO 20                                              SCRS0060
      IF(BETA(1).EQ.0.) L1=2                                             SCRS0061
      MM=NENC+L1                                                         SCRS0062
        DO 10 M=L1,NENC                                                  SCRS0063
        B2=-BETA(MM-M)                                                   SCRS0064
        IF(B2.LT.T1) GO TO 10                                            SCRS0065
        DB(L)=B2-B1                                                      SCRS0066
        L=L+1                                                            SCRS0067
        LM(L)=MM-M                                                       SCRS0068
        SP(L)=SPREDF(B2)                                                 SCRS0069
        B1=B2                                                            SCRS0070
        SGNDF(L)= -1.0                                                   SCRS0071
10      CONTINUE                                                         SCRS0072
```

```
20      DO 30 M=1,NENC
        B2=BETA(M)
        IF(B2.LT.T1) GO TO 30
        IF(B2.GT.T2) GO TO 60
        DB(L)=B2-B1
        L=L+1
        LM(L)=M
        SP(L)=SPEEDF(B2)
        B1=B2
30      CONTINUE
      B1=B2-360.
      MM=NENC+1
      IF(BETA(NENC).EQ.180.) MM=NENC
        DO 40 M=1,NENC
        B2=-BETA(MM-M)
        IF(B2+360..GT.T2) GO TO 50
        DB(L)=B2-B1
        L=L+1
        LM(L)=MM-M
        B1=B2
        SGNDF(L)= -1.0
40      SP(L)=SPEEDF(B2)
50    B1=B1+360.
60    DB(L)=T+90.-B1
      M=L
      N=L+1
      B(N)=0.
      SP(N)=0.
      UNITY=SQRT(TRAPZD(DB,SP,N)/90.)
      WRITE(6,1010) UNITY
    WRITE(6,6000) (DB(L),L=1,M)
    WRITE(6,7000) (LM(L),L=2,M)
    WRITE(6,5000) (SP(L),L=1,N)
C
C     SEA STATE CALCULATIONS
C
```

```
      DO 200 NW=1,NWX
      WRITE(6,1000) V
      WRITE(6,1001) H13(NW),OMP(NW),T
C
C    MOTIONS @ ORIGIN
C
      WRITE(6,1002)
        DO 120 I=1,5
          DO 110 L=2,M
  110     R(L)=SP(L)*OMOT(LM(L),NW,I)
  120   CALL SPIN
      WRITE(6,1003)($(1),$(1),$(I+1),S(I),S3(I),S10(I),S1000(I),I=1,5)
      IF(NMOT.EQ.0) GO TO 160
C
C    MOTIONS @ SPECIFIED PTS.
C
      WRITE(6,1004)
        DO 150 J=1,NMOT
          DO 140 I=1,7
            DO 130 L=2,M
  130       R(L)=SP(L)*VMOT(LM(L),NW,J,I)
  140     CALL SPIN
        WRITE(6,1005) XMOT(J),YMOT(J),ZMOT(J)
  150   WRITE(6,1003) ($(I1(I)),$(I2(I)),$(I3(I)),S(I),S3(I),S10(I)
     1,                S1000(I),I=1,7)
  160 IF(N1.EQ.0) GO TO 195
C
C    SHEAR & BENDING MOMENTS
C
      WRITE(6,1006)
        DO 190 J=1,5
        J1=3*J+12
        J2=J1+2
          DO 180 I=N1,N2
            DO 170 L=2,M
  170       R(L)=SP(L)*BNDS(LM(L),NW,I,J)
```

```
 180        CALL SPIN                                                  SCRS0145
          WRITE(6,1003)  ($(I),I=J1,J2)                                SCRS0146
 190      WRITE(6,1007)  (I,S(I),S3(I),S10(I),S1000(I),I=N1,N2)        SCRS0147
195     IF(NADR.EQ.0) GO TO 200                                        SCRS0148
C                                                                      SCRS0149
C     SECOND ORDER FORCE                                               SCRS0150
C                                                                      SCRS0151
        WRITE(6,7500)                                                  SCRS0152
        DO 196 L=2,M                                                   SCRS0153
196     R(L)=SP(L)*ADRES(LM(L),NW)                                     SCRS0154
        SUM=TRAPZD(DE,R,N)/90.                                         SCRS0155
        WRITE(6,8000) SUM                                              SCRS0156
        DO 197 L=2,M                                                   SCRS0157
197     R(L)=SP(L)*DRIFT(LM(L),NW)*SGNDF(L)                            SCRS0158
         SUM=TRAPZD(DB,R,N)/90.                                        SCRS0159
        IF(T.EQ.0.0.OR.T.EQ.180.0)  SUM=0.0                            SCRS0160
        WRITE(6,9000) SUM                                              SCRS0161
 200    CONTINUE                                                       SCRS0162
      RETURN                                                           SCRS0163
 1000 FORMAT('1IRREGULAR WAVE RESULTS -- SHORT CRESTED MULTI-DIRECTIONAL SCRS0164
     1 SEAS    SHIP SPEED : ',F10.4)                                   SCRS0165
 1001 FORMAT('0SIGNIFICANT WAVE HEIGHT ',F10.4/' PEAK SPECTRAL FREQUENCY SCRS0166
     1 ',F10.4/' PRINCIPLE WIND DIRECTION',F10.4/'0',T58,'RMS',T71,'H(1/ SCRS0167
     23)',T85,'H(1/10)',T99,'H(1/1000)')                               SCRS0168
 1002 FORMAT(' MOTIONS @ ORIGIN')                                      SCRS0169
 1003 FORMAT(17X,3A8,5X,4F15.4)                                        SCRS0170
 1004 FORMAT('0MOTIONS @ PTS. OF INTEREST')                            SCRS0171
 1005 FORMAT('0X=',F10.4,',  Y=',F9.4,',  Z=',F9.4/)                   SCRS0172
 1006 FORMAT('0DYNAMIC LOADINGS')                                      SCRS0173
 1007 FORMAT(38X,I2,5X,4F15.4)                                         SCRS0174
 1010 FORMAT('1THE SPREADING FUNCTION WITH THE INPUT HEADING ANGLES INTEG SCRS0175
     *RATED TO ',F10.6)                                                SCRS0176
 5000 FORMAT('0SE :',6X,12F10.6/(' ',13F10.6))                         SCRS0177
 6000 FORMAT('0DB :',6X,12F10.4/(' ',13F10.4))                         SCRS0178
 7000 FORMAT(11X,12I10/(' ',13I10))                                    SCRS0179
7500  FORMAT('0MEAN SECOND ORDER FORCE (LBS.)')                        SCRS0180
```

```
8000  FORMAT('0    ADDED RESISTANCE COMPONENT ',F15.4)                  SCRS0181
9000  FORMAT('0    DRIFT FORCE COMPONENT ',F15.4)                       SCRS0182
      END                                                               SCRS0183
      SUBROUTINE SPIN                                                   SCRS0184
      COMMON /SHCRSI/ OMOT(13,10,5),VMOT(13,10,10,7),RNDS(13,10,32,5)   SCRS0185
     1,XMOT(10),YMOT(10),ZMOT(10),BETA(16),V,NENC,NWX,N1,N2,NMOT,NWIND  SCRS0186
     2,H13(10),SP(34),DB(33),R(32),IM(32),THETA(36)                     SCRS0187
     3,S(32),S3(32),S10(32),S1000(32),OMP(10),N,I                       SCRS0188
     4,ADRES(13,10),DRIFT(13,10),SGNDF(32),NADR                         SCRS0189
      S(I)=SQRT(TRAPZD(DB,R,N)/90.)                                     SCRS0190
      S3(I)=4.0*S(I)                                                    SCRS0191
      S10(I)=5.1*S(I)                                                   SCRS0192
      S1000(I)=7.7*S(I)                                                 SCRS0193
      RETURN                                                            SCRS0194
      END                                                               SCRS0195
      FUNCTION TRAPZD(DX,Y,N)                                           SCRS0196
      DIMENSION DX(1),Y(1)                                              SCRS0197
      TRAPZD=0.                                                         SCRS0198
      N1=N-1                                                            SCRS0199
      IF(N.LE.1) RETURN                                                 SCRS0200
      DO 10 I=1,N1                                                      SCRS0201
   10 TRAPZD=TRAPZD+DX(I)*(Y(I)+Y(I+1))                                 SCRS0202
      TRAPZD=.5*TRAPZD                                                  SCRS0203
      RETURN                                                            SCRS0204
      END                                                               SCRS0205
```

C-7: MATRIX - In Chapter IV-A of the main text, it was noted that, to obtain some of the computer results shown in Figures 4, 5, and 6, the 5-D program was restrained to roll and heave (a 2-D system). This was done by making a few changes in subroutine MATRIX. The concept behind the changes can best be illustrated by letting D_{jk} represent the term in brackets in Eq. (24); and letting the terms indicated become zero:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_{22} & 0 & \cancelto{0}{D_{24}} & 0 & \cancelto{0}{D_{26}} \\ 0 & 0 & D_{33} & 0 & \cancelto{0}{D_{35}} & 0 \\ 0 & \cancelto{0}{D_{42}} & 0 & D_{44} & 0 & \cancelto{0}{D_{46}} \\ 0 & 0 & \cancelto{0}{D_{53}} & 0 & D_{55} & 0 \\ 0 & \cancelto{0}{D_{62}} & 0 & \cancelto{0}{D_{64}} & 0 & D_{66} \end{bmatrix} \begin{bmatrix} 0 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \end{bmatrix} = \begin{bmatrix} 0 \\ \cancelto{0}{F_2} \\ F_3 \\ F_4 \\ \cancelto{0}{F_5} \\ \cancelto{0}{F_6} \end{bmatrix}$$

Giving $D_{22}\zeta_2 = 0$, $D_{33}\zeta_3 = F_3$, $D_{44}\zeta_4 = F_4$, $D_{55}\zeta_5 = 0$, $D_{66}\zeta_6 = 0$ which represent the equations for a decoupled two-degree of freedom system. Of course, this can be done for any mode or modes of motion desired.

The necessary changes to subroutine MATRIX are given next. The new lines include: 26, 27, 29, 41, 42, 44, 45.

A listing of the routine modified for a 2-D system follows on pages 166-168.

```
      SUBROUTINE MATRIX
      IMPLICIT COMPLEX (C,D,E,F,W)
      REAL C,DX,EM,ET,ETM,MIKE,WN,EVENT,CABS
      COMPLEX ZANGLE
      COMMON D(6,6),E(6),EM(6)
      COMMON /PAPA/ WS,WC,SB,OMC,OM,OM2,WN,RHO,GRAV,BETAA,XI(32),DX(31)
     1,V,NSTA,NSE,MD,XLBE,ZETAA,ALFA
      COMMON /VISC/ B4$(32),B44V,ION,IADR
      COMMON /SPCTRM/ AM(1200),BS(6400),RM(200),H13(10),OMP(10),NBOMS
     1,SPCTM(10,40),SPOMS(10,40),NSOMS,NWX,II,N1,N2,M1,NSEA,L,NS,NSPC,IO
     2,NADR,RST(80),NEVT,EVENT(3,3),NN1,NN2,NN3,VCR,REVNT(120),OMEGE(40)
      COMMON /COEFFS/ ETA(6),F(6),G(6,6),A(6,6),B(6,6),C(6,6)
      DIMENSION ET(2,6),ETM(18),P(2),Z(2),MJ(6)
      EQUIVALENCE (E(1),ET(1,1),ETM(1)),(F2,F(2)),(F4,F(4)),(F6,F(6))
      DATA P/'    ','NON-'/,Z/'   P','HASE'/,MJ/-1,2,0,3,1,4/
      DNV(D1,D2,D3,D4,D5,D6)=(F2*(D1*D2-D3*D4)+F4*(D4*D5-D2*D6)
     1                              +F6*(D3*D6-D1*D5))/DET
      M=5-M1
      N=M-1
      I=0
C
C     LONGITUDINAL MOTIONS
C
        DO 10 J=3,5,2
        DO 10 K=3,5,2
        D(J,K)=CMPLX(-OM2*(G(J,K)+A(J,K))+C(J,K),OM*B(J,K))
      IF(J.NE.K)  D(J,K)=CMPLX(0.0,0.0)
10     CONTINUE
      F(5)=CMPLX(0.0,0.0)
      DET=D(3,3)*D(5,5)-D(3,5)*D(5,3)
      ETA(3)=(F(3)*D(5,5)-F(5)*D(3,5))/DET
      ETA(5)=(F(5)*D(3,3)-F(3)*D(5,3))/DET
      IF(M1.NE.2) GO TO 15
      CALL ERASE(ETM,18)
      GO TO 60
C
```

```
C     TRANSVERSE MOTIONS                                                  MTRX0037
C                                                                         MTRX0038
   15   DO 20 J=2,6,2                                                     MTRX0039
        DO 20 K=2,6,2                                                     MTRX0040
        D(J,K)=CMPLX(-OM2*(G(J,K)+A(J,K))+C(J,K),OM*B(J,K))               MTRX0041
      IF(J.NE.K)  D(J,K) =CMPLX(0.0,0.0)                                  MTRX0042
 20   CONTINUE                                                            MTRX0043
      F2=CMPLX(0.0,0.0)                                                   MTRX0044
      F6=CMPLX(0.0,0.0)                                                   MTRX0045
 25   B44I=B(4,4)                                                         MTRX0046
      D44=D(4,4)                                                          MTRX0047
      DT1=D(2,4)*(D(4,6)*D(6,2)-D(4,2)*D(6,6))                            MTRX0048
     1   +D(6,4)*(D(4,2)*D(2,6)-D(2,2)*D(4,6))                            MTRX0049
      DT2=          D(2,2)*D(6,6)-D(2,6)*D(6,2)                           MTRX0050
      DET=DT1+DT2*D(4,4)                                                  MTRX0051
      ETA(2)=DNV(D(4,4),D(6,6),D(4,6),D(6,4),D(2,6),D(2,4))               MTRX0052
      ETA(4)=DNV(D(4,6),D(6,2),D(4,2),D(6,6),D(2,2),D(2,6))               MTRX0053
      ETA(6)=DNV(D(4,2),D(6,4),D(4,4),D(6,2),D(2,4),D(2,2))               MTRX0054
      ET4=ETA(4)*DET                                                      MTRX0055
      GO TO 60                                                            MTRX0056
   30 N=2                                                                 MTRX0057
C                                                                         MTRX0058
C     ITERATE TO ACCOUNT FOR VISCOUS ROLL DAMPING                         MTRX0059
C                                                                         MTRX0060
      ROLL1=CABS(ETA(4))                                                  MTRX0061
        DO 40 J=1,40                                                      MTRX0062
        I=I+1                                                             MTRX0063
        B44V=MIKE(ROLL1,OM)                                               MTRX0064
        D(4,4)=D44+CMPLX(0.,OM*B44V)                                      MTRX0065
        DET=DT1+DT2*D(4,4)                                                MTRX0066
        ETA(4)=ET4/DET                                                    MTRX0067
        ROLL2=CABS(ETA(4))                                                MTRX0068
        IF(ABS((ROLL1-ROLL2)/ROLL1).LT..001) GO TO 50                     MTRX0069
   40   ROLL1=.1*ROLL1+.9*ROLL2                                           MTRX0070
   50 ETA(2)=DNV(D(4,4),D(6,6),D(4,6),D(6,4),D(2,6),D(2,4))               MTRX0071
      ETA(6)=DNV(D(4,2),D(6,4),D(4,4),D(6,2),D(2,4),D(2,2))               MTRX0072
```

```
60    DO 70 J=M,6,N
      Y=J/4
    Y=1.+Y*56.29578
      Y=(Y*WN+(1.-Y))*ZETAA
      E(J)=ZANGLE(FIA(J))
      FM(J)=FT(1,J)/Y
      E(J)=CMPLX(FT(1,J)*X,FT(2,J))
70    RM(L+MJ(J)*NROMS)=(FT(1,J)/ZETAA)**2
    IF(NSEA.EQ.1) GO TO 80
    IF(I.EQ.0) WRITE(6,1000) Z,Z,Z,Z,Z
    WRITE(6,1001) P(N/M+1),(E(J),J=2,6),(FM(J),J=2,6)
80  IF(M1.EQ.2) RETURN
    IF(I.EQ.0) GO TO 30
    IF(NSEA.NE.1) WRITE(6,1002) I
    RETURN
1000 FORMAT('0',T37,'SWAY',3X,2A4,T57,'HEAVE',2X,2A4,T77,'ROLL',3X,2A4
   1,T97,'PITCH',2X,2A4,T118,'YAW',3X,2A4)
1001 FORMAT(' ',A4,'LINEAR MOTIONS',T32,10F10.4/' NON-DIMENSIONALIZED'
   1,T22,5F20.4/)
1002 FORMAT(' VISCOUS ROLL DAMPING ATTAINED AFTER',I3,' ITERATIONS')
    END
```

APPENDIX D

LISTING OF MARINER EXAMPLE DATA

This section contains listings of data used in the MIT 5-D program to create the various graphs presented in the text. Two data decks are given. The first is based on 21 stations, and it represents the data used to make the comparisons in all the graphs except Figures 4, 5, and 6. These three graphs (for the beam seas experiment) were created using the second data deck based on 11 stations. The "towing tank" data cards have a different density, metacentric height, roll and pitch radius of gyration, and a different number of stations. These changes were made to more accurately represent the model and tank characteristics as measured. The station number was lowered to minimize expense.

An explanation of the meaning of each number is provided by reproducing the portion of the 5-D output that gives the input data (See D-3). The input shown includes a sample selection of regular wave frequencies, ship speeds, and ship heading angles. Spectral information may be added if desired as shown in D-3. Further details may be obtained from the 5-D User's Manual already mentioned. This manual should be consulted before any runs are attempted.

D-1: Data used for all graphs and calculations except Figures 4, 5, and 6 (the beam seas experiments).

```
MARINER - BETA RANGE AT 15 KNOIS
21   25   12    1    0    0   10    6    1    1    0    0    0    0    0    0
                                2
0.62028   528.      75.684    29.75     32.17     -9.6425   -2.97     4.63406
128.6340  22.9729   128.6000  .00001023.0008900     1.9899998.00001280  52919.0
        0 -9.6425   1.0
264.
237.6       9.2522  29.75     0.904887  -14.4859  64.0839
211.2      20.6762  29.75     0.771227  -13.2134  68.2646             10.0
184.8      33.6770  29.75     0.756442  -12.8616  75.2049             15.0
158.4      46.6480  29.75     0.781804  -12.3773  84.6208
132.0      58.0422  29.75     0.814983  -13.1334  94.9312
105.6      66.9672  29.75     0.857838  -13.4392  105.014
 79.2      72.6496  29.75     0.901838  -13.8361  113.877
 52.8      75.2080  29.75     0.942307  -14.1595  120.632
 26.4      75.6840  29.75     0.975657  -14.4466  125.108
           75.6840  29.75     0.982668  -14.5472  126.495
-26.4      75.6840  29.75     0.976865  -14.5055  126.296
-52.8      75.6840  29.75     0.970319  -14.3198  123.866
-79.2      75.6840  29.75     0.942816  -13.8668  119.289
-105.6     75.6246  29.75     0.870373  -13.4339  112.903
-132.0     73.7800  29.75     0.790980  -12.8558  105.304
-158.4     69.0796  29.75     0.696764  -12.1463  96.7346
-184.8     60.1842  29.75     0.601106  -11.4443  87.2348
-211.2     46.4512  29.75     0.499875  -10.8540  78.3807
-237.6     30.6722  29.75     0.353958  -9.9263   70.3269
-264.0

        1
        1
        1
        4
        4
        4
        4
        9 39.80   15.19     24.0      1.5       0.2708    0.17458   23.496
```

```
        7 40.25     10.48      24.0       1.5        0.2708     0.17458    26.4          0037
        7 40.50      9.52      24.0       1.5        0.2708     0.17458    26.4          0038
        7 40.50      9.52      24.0       1.5        0.2708     0.17458    26.4          0039
        7 40.00     13.68      24.0       1.5        0.2708     0.17458    26.4          0040
        8 38.25     14.00      24.0       1.5        0.2708     0.17458     7.392        0041
        3                                                                              0042
        3                                                                              0043
        3                                                                              0044
        3                                                                              0045
        3                                                                              0046
        3                                                                              0047
                                                                                       0048
25.335                                                                                 0049
   2.        15.        30.        60.        75.        90.        105.       120.    0050
 135.       150.       165.       180.                                                 0051
 .1         .15        .2         .25        .3         .35         .4         .45     0052
 .4375      .4612      .4891      .5052      .5229      .5427      .5648      .5899    0053
 .6187      .6522      .6918      .7395      .7988      .8750      .9783     1.1296    0054
 1.3835                                                                                0055
```

<u>D-2</u>: Data used for preparing comparisons with beam seas experiments (Figures 4, 5, and 6).

```
MARINER      10 STA FUN TOW TANK                                                                   0001
  11   16     3     1               8     6     1     1                     1     1     0          0002
                                    1                                                              0003
0.62028   528.        75.684     29.75      32.17       -9.6425    -2.97      8.00                 0004
106.0000  11.0000     106.0000   .00001023.0008675      1.9396954.00001280    52919.0              0005
         0 -9.6425    1.0                                                                          0006
264.                                                                                               0007
211.2     20.6762     29.75      0.771227   -13.2134    68.2646               10.0                 0008
158.4     46.6480     29.75      0.781804   -12.9773    84.6208                                    0009
105.6     66.9672     29.75      0.857838   -13.4392    105.014                                    0010
 52.8     75.2080     29.75      0.942307   -14.1595    120.632                                    0011
          75.6840     29.75      0.982668   -14.5472    126.495                                    0012
-52.8     75.6840     29.75      0.970319   -14.3198    123.866                                    0013
-105.6    75.6246     29.75      0.870373   -13.4339    112.903                                    0014
-158.4    69.0796     29.75      0.696764   -12.1463    96.7346                                    0015
-211.2    46.4512     29.75      0.499875   -10.8540    78.3807                                    0016
-264.0                                                                                             0017
                                                                                                   0018
         1                                                                                         0019
         4                                                                                         0020
         4                                                                                         0021
         9 39.80      15.19      24.0       1.5         0.2708     0.17458    23.496               0022
         7 40.50       9.52      24.0       1.5         0.2708     0.17458    26.4                 0023
         7 40.00      13.68      24.0       1.5         0.2708     0.17458    26.4                 0024
         3                                                                                         0025
         3                                                                                         0026
         3                                                                                         0027
                                                                                                   0028
 0.0                                                                                               0029
 90.0     105.0       75.0                                                                         0030
 2.0      1.8         1.6        1.55       1.5         1.4        1.2        1.0                  0031
 0.9      0.8         0.7        0.6        0.5         0.4        0.3        0.2                  0032
```

D-3: SAMPLE OUTPUT OF 5-D USED FOR CHECKING INPUT DATA

The variables can be described as follows:

Card 1	(not shown)	Number of cases (ships)
Card 2	NAME	Description of case run
Card 3	NSTA	Number of ship stations
	NROMS	Number of regular wave frequencies
	NENC	Number of headings
	NVL	Number of speeds
	NMOT	Number of points where motions are calculated
	NSP	Even spacing of ship stations? 0 = yes; 1 = no
	NP	Number of points used on each ship station
	MP	Number of multipoles used in hydro-dynamic potential
	NTURB	Ship with bilge keels? 1 = yes; 0 = no
	HASBK	Vertical motions only or full 5-D
	NB	Station where bending moments computed
	NBEND	Are bending moment calculations desired?
	NWT	Number of weight ordinates
	NPCH	Controls printout of various matrices
	NFQ	Conveys form of regular wave frequency input data
	NFR	Conveys form of ship speed input data

Card 4 Set	NSEA	Regular wave calculations only?
	NWX	Number of sea states
	NSOMS	Number of spectral frequencies?
	NS	Default set of spectral frequencies?
	NSPC	Controls printout of various statistical quantities
	IO	Specifies output device for input to Shortcrest
	NADR	Controls added resistance calculations
	NEVT	Controls calculation of event probabilities
Card 5 Set	CB	Block coefficient
	XLBP	Length between perpendiculars
	BEAM	Midship beam
	DRAFT	Midship draft
	GRAV	Gravitational acceleration
	XCG	Longitudinal center of gravity measured from ⊠
	VCG	Vertical center of gravity measured from the waterline
	GM	Metacentric height
Card 6 Set	RYY	Radius of gyration (Y-AX)
	RXX	Radius of gyration (X-AX)
	RZZ	Radius of gyration (Z-AX)
	XZI	Product of inertia (X-ZAX)
	RHO	Mass density in ship units
	XRHO	Mass density in $slugs/ft^3$

	NU	Kinematic viscosity
	WSURFA	Wetted surface area
Card 7 Set	UNIT	Input units: English = 0; Metric = 1
	ORIGIN	Desired origin for motions calculation
	ZETAA	Wave amplitude
	ALFA	Maximum wave slope
Card 8 Set	XI	Distance to ship station from ⊗
	YM	Sectional waterline beam
	ZM	Sectional draft
	SIGMA	Sectional area coefficient
	ZCB	Vertical sectional center of buoyancy
	GIRTH	Girth of section
	RIFLR	Rise of floor of section
	ALPH	Angle between ship side and vertical at section
Card 9 Set	IWBK	Determines type of viscous roll damping
	BKRAD	Geometric property of bilge keel
	BILRAD	Geometric property of bilge
	BKGIR	Geometric property of bilge
	BKWID	Geometric property of bilge keel
	PHI	Geometric property of bilge
	PSI	Geometric property of bilge
	LIWO	Length of bilge keel

NOTES: 1) All of card set 9 are employed in the viscous roll damping (quasi-linear) calculation of B_{44}^*.

2) If structural bending moment or motions calculations are being performed, Card sets Number 10 and 11 must be included here as described in the User's Manual.

Card 12 Set	UOB	Ship speeds, units defined by NFR
Card 13 Set	BETA	Heading angles in degrees, ascending order
Card 14 Set	OMEGA	Wave frequencies, units defined by NFQ
Card 15 Set	H13	The 1/3 highest wave heights for which sea state calculations are desired
Card 16 Set	OMP	Peak spectral frequencies for the corresponding wave heights above. If blank, fully developed seas will be used.
Card 17 Set	SPOMS	Spectral frequencies of each sea state specified (ω_s). These values can be given as input ($NS \neq 0$) or the program will supply a default set ($NS = 0$) (as was done here).
Card 18 Set	SPCTM	Spectral amplitudes for the chosen sea spectrum. This card set may be omitted, and the program will calculate Bretschneider amplitudes that correspond to the frequencies in Card Set 17. Otherwise, NS may be set equal to two and spectral amplitudes may be read in for any type of spectrum the user may require. In this case the default Bretschneider spectrum was chosen.

NOTES: 1) Card Set 19 should follow here as described in the User's Manual if the probabilities of the various events detailed there are desired.

MARINER SPECTRAL ANALYSIS

NSTA	NROMS	NFNC	NVL	NMOT	NSP	NP	MP	NTURB	HASBK	NB	NBUND	NWT	NPCH	NFQ	NPR
21	21	7	1	0	0	10	6	1	1	0	0	0	0	0	0

NSEA	NWX	NSOMS	NS	NSPC	IO	NADR	NEVT
1	6	0	0	1	7	2	0

CB	XLBP	BEAM	DRAFT	GRAV	XCG	VCG	GM
0.6203	528.0000	75.6840	29.7500	32.1700	-9.6425	-2.9700	4.6341

RYY	RXX	RZZ	XZI	RHO	XRHO	NU	WSURFA
128.6340	22.9729	128.6000	0.0000	0.0008300	1.9899998	0.0000128	52919.00

UNIT	ORIGIN	ZETAA	ALFA
0	-9.6425	1.0000	0.0

STATION I	XI(I)	YM(I)	ZM(I)	SIGMA(I)	ZCB(I)	GIRTH(I)	RIFLR(I)	ALPH(I)
1	264.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	237.6000	9.2522	29.7500	0.9049	-14.4859	64.0839	0.0	0.0
3	211.2000	20.6762	29.7500	0.7712	-13.2134	68.2646	0.0	10.0000
4	184.8000	33.6770	29.7500	0.7564	-12.8616	75.2049	0.0	15.0000
5	158.4000	46.6480	29.7500	0.7819	-12.9773	84.6265	0.0	0.0
6	132.0000	58.0422	29.7500	0.8150	-13.1334	94.9312	0.0	0.0
7	105.6000	66.9672	29.7500	0.8576	-13.4332	105.0140	0.0	0.0
8	79.2000	72.6496	29.7500	0.9018	-13.8361	113.8770	0.0	0.0
9	52.8000	75.2080	29.7500	0.9423	-14.1595	120.6320	0.0	0.0
10	26.4000	75.6840	29.7500	0.9757	-14.4466	125.1080	0.0	0.0
11	0.0	75.6840	29.7500	0.9827	-14.5472	126.4950	0.0	0.0
12	-26.4000	75.6840	29.7500	0.9769	-14.5055	126.2900	0.0	0.0
13	-52.8000	75.6840	29.7500	0.9703	-14.3198	123.6560	0.0	0.0
14	-79.2000	75.6840	29.7500	0.9423	-13.8668	119.2890	0.0	0.0
15	-105.6000	75.6246	29.7500	0.8794	-13.4339	112.9030	0.0	0.0
16	-132.0000	73.7800	29.7500	0.7910	-12.8558	105.3040	0.0	0.0
17	-158.4000	69.0796	29.7500	0.6968	-12.1463	96.7346	0.0	0.0
18	-184.8000	60.1642	29.7500	0.6011	-11.4443	87.2348	0.0	0.0
19	-211.2000	46.4512	29.7500	0.4999	-10.8540	78.3807	0.0	0.0
20	-237.6000	30.6722	29.7500	0.3540	-9.9263	70.3269	0.0	0.0
21	-264.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0

STATION I	IWBK(I)	BKRAD(I)	HILRAD(I)	BKGIR(I)	BKWID(I)	PHI(I)	PSI(I)	LIWO(I)
1	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	9	39.8000	15.1900	24.0000	1.5000	0.2708	0.1746	23.4960
10	7	40.2500	10.4800	24.0000	1.5000	0.2708	0.1746	26.4000
11	7	40.5000	9.5200	24.0000	1.5000	0.2708	0.1746	26.4000
12	7	40.5000	9.5200	24.0000	1.5000	0.2708	0.1746	26.4000
13	7	40.0000	13.6800	24.0000	1.5000	0.2708	0.1746	26.4000
14	8	38.2500	14.0000	24.0000	1.5000	0.2708	0.1746	7.3920
15	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

UOB(N)	:	33.7800							
BETA(M)	:	0.0	30.0000	60.0000	90.0000	120.0000	150.0000	180.0000	
OMEGA(L)	:	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.8700
		1.0200	1.1700	1.2400	1.3000	1.4000	1.5000	1.6000	1.7000
		1.8000	2.0000	2.2500	2.5000	3.0000			
H13(NW)	:	5.0000	10.0000	15.0000	15.0000	15.0000	20.0000		
OMF(NW)	:	0.0	0.0	0.0	0.4489	1.0472	0.0		

Information Processing Center

MANIPULATIONS IN SUBROUTINE CHECK :

FORM PARAMETERS YIELDED : CB = 0.6203 XCG = -9.6425 WGT = 21113.18

STATION PARAMETERS YIELDED : CD = 0.6190 LCB = -9.4948 WGT = 21068.21

DX(I) :	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000
	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000				
STASPA :	13.2000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000
	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	26.4000	13.2000			
OMEGA(L) :	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.8700	1.0200	1.1700	1.2400	1.3000
	1.4000	1.5000	1.6000	1.7000	1.8000	2.0000	2.2500	2.5000	3.0000	3.1416		
H13(NW) :	5.0000	10.0000	15.0000	15.0000	15.0000	20.0000						
OMF(NW) :	1.0179	0.7196	0.5877	0.4488	1.0472	0.5090						

SPECTRUM # 1

SPOMS :	0.6107	0.6616	0.7125	0.7634	0.8143	0.8550	0.8958	0.9263	0.9568	0.9772	0.9976	1.0179
	1.0383	1.0586	1.0790	1.1095	1.1401	1.1706	1.2215	1.2724	1.3233	1.3742	1.4251	1.4760
	1.5269	1.6287	1.7304	1.8322	1.9340	2.0358	2.1376	2.2394	2.3412	2.4430	2.5448	2.6466
	2.7484	2.8501	2.9519	3.0537								
SPCTM :	0.0064	0.0602	0.2504	0.6224	1.1073	1.4904	1.8090	1.9872	2.1092	2.1606	2.1897	2.1989
	2.1906	2.1670	2.1308	2.0576	1.9678	1.8673	1.6880	1.5072	1.3344	1.1748	1.0307	0.9025
	0.7896	0.6048	0.4654	0.3606	0.2816	0.2218	0.1762	0.1412	0.1140	0.0928	0.0761	0.0629
	0.0522	0.0437	0.0368	0.0311								

SPECTRUM # 2

SPOMS :	0.4319	0.4679	0.5038	0.5398	0.5758	0.6046	0.6334	0.6550	0.6766	0.6910	0.7054	0.7198
	0.7342	0.7486	0.7630	0.7846	0.8061	0.8277	0.8637	0.8997	0.9357	0.9717	1.0077	1.0437
	1.0797	1.1516	1.2236	1.2956	1.3676	1.4395	1.5115	1.5835	1.6555	1.7275	1.7994	1.8714
	1.9434	2.0154	2.0873	2.1593								
SPCTM :	0.0362	0.3403	1.4163	3.5206	6.2639	8.4311	10.2330	11.2411	11.9316	12.2219	12.3870	12.4391
	12.3916	12.2597	12.0538	11.6397	11.1317	10.5028	9.5488	8.5260	7.5486	6.6458	5.3305	5.1052
	4.4665	3.4215	2.6328	2.0393	1.5931	1.2546	0.9969	0.7987	0.6451	0.5251	0.4366	0.3556
	0.2955	0.2472	0.2080	0.1759								

SPECTRUM # 3

SPOMS :	0.3526	0.3820	0.4114	0.4409	0.4702	0.4937	0.5172	0.5348	0.5524	0.5642	0.5759	0.5877
	0.5994	0.6112	0.6230	0.6465	0.6582	0.6758	0.7052	0.7346	0.7640	0.7934	0.8228	0.8522
	0.8815	0.9403	0.9991	1.0578	1.1166	1.1754	1.2342	1.2929	1.3517	1.4105	1.4692	1.5280
	1.5868	1.6455	1.7043	1.7631								
SPCTM :	0.0396	0.9379	3.9029	9.7017	17.2612	23.2333	28.1989	30.9767	32.8797	33.6797	34.1346	34.2780
	34.1473	33.7809	33.2163	32.0752	30.6754	29.1077	26.3134	23.4949	20.8014	18.3136	16.0668	14.0661
	12.3082	9.4286	7.2551	5.6209	4.3899	3.4578	2.7471	2.2009	1.7776	1.4470	1.1866	0.9798
	0.8144	0.6812	0.5731	0.4848								

* CONTINUES THROUGH SPECTRUM # NWX —