Active Noise Control in Supersonic Impinging Jets
Using Pulsed Microjets: Actuator Design, Reduced-Order Modeling

by

Jae Jeen Choi

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Abstract

In recent years, it has been demonstrated that direct microjet injection into the shear layer of the main jet disrupts the feedback loop inherent in high speed impinging jet flows, thereby significantly reduces the adverse effects. The amount of noise reduced by microjet actuation is known to be dependent on nozzle operating conditions. In this paper, two active control strategies using microjets are suggested to maintain a uniform, reliable, and optimal reduction of these tones over the entire range of operating conditions.

In the first method, a quasi-closed loop control strategy is proposed using steady microjet injection and the Proper Orthogonal Decomposition (POD) algorithm. The most energetic spatial mode of the unsteady pressure along the nozzle diameter is captured using the POD, which in turn is used to determine the distribution of microjet intensity along the nozzle exit. Preliminary experimental results from a STOVL supersonic jet facility at Mach 1.5 show that the quasi-closed loop control strategy, in some cases, provides an additional 8~10 dB reduction compared to axisymmetric injection at the desired operating conditions. The second method consists of a pulsed microjet injection, motivated by the need to further improve the noise suppression. It was observed that the pulsed microjet was able to bring about the same noise reduction as steady injection using approximately 40% of the corresponding mass flow rate of the steady microjet case. Moreover, as the duty cycle increased, the performance of pulsed injection was further enhanced and was observed to completely eliminate the impinging tones at all operating conditions.

In order to obtain an optimal performance of the actuator, a new model of the impinging jet flow field is suggested based on a collision model of two identical vortices. In addition to the colliding vortex model, a two-mode feedback model that captures both the low and high-frequency Rossiter mode was suggested to investigate the role of pulsed microjet in the feedback loop. Due to the fact that a low frequency pulsing (16.4 Hz) brought about additional reduction compared to high frequency pulsing, the
presence of low frequency mode is identified. In the context of the analytic model, the effect of pulsing is modeled using a input-shaping controller that accomplishes noise-reduction through a suitable redistribution of the acoustic excitation over the high and low frequencies.

Thesis Supervisor: Anuradha M. Annaswamy
Title: Senior Research Scientist
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Looking back on my past, I can’t not but agree that my career was not built for myself. There are so many people to guid me to the right direction in my life way. I wish to show my special thanks to all of them here.

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Chapter 1

Introduction

For many years, flow control has been an important research topic in needs of many industrial applications such as aerospace, automobile and energy. Stimulated by military needs, during the second World War, flow control techniques were investigated to make it possible to develop highly maneuverable and efficient aircraft, missile, ship and submarine. More recently, the energy crisis and severe environmental restriction also accelerated flow control schemes to enhance fuel efficiency, and improve noise reduction.

In order to achieve performance objectives such as drag reduction, separation control, noise control, and thrust recovery, several flow control methods have been proposed over the past twenty years [18]. Since drag is the key ingredient of thrust loss, drag reduction is the primary method for increasing energy efficiency. In general, drag is composed of two major components, pressure drag and surface drag. Various kind of eddy breakup devices such as riblets, compliant surfaces, wavy walls were introduced to recover pressure drag [7, 9, 27]. It is also possible to produce additional thrust recovery by delaying laminar-to-turbulence transition point because the skin friction in laminar state is as much less than that in the turbulent condition [44, 45, 15]. On the other hand, because turbulence is very effective in mixing and enhancing heat transfer, early transition is also needed when rapid mixing and effective heat transfer is required [35]. Control of separation in external flows, such as over airfoils, can lead to substantial gains in lift, increase lift-to-drag ratios, decrease buffeting and/or delay
stall, all of which enhance the system performance, while also potentially expanding the operation envelope of future. Using various actuators such as piezoelectric flaps actuator[38], acoustic excitation[57] and periodic tangential blowing, the boundary layer of airfoil can be forced to stay on the surface and ensure stable maneuvering[18]. Aside from performance efficiency, flow induced noise is regarded as one of the major problems in flow control. Compared to others, noise control is relatively young field of research. Needs to maintain stealth, overall quiet, and the associated performance metrics of reduced unsteadiness have spurred intense investigations in the flow noise control area. Flow induced noise study was firmly established using a very powerful theory by Lighthill[29], and enriched by aircraft jet noise studies. This thesis concerns a specific problem in the area of jet noise control.

In general, aircraft jet noise is produced by two noise sources; the large turbulence structures waves and the fine-scale turbulence. In the case of the Short Take off and Vertical Landing aircraft, it experiences additional discrete and high amplitude acoustic tones that are produced via a feedback process while hovering in close proximity to the ground. These feedback interactions occur thus: Instability waves are generated by the acoustic excitation of the shear layer near the nozzle exit, which then convect down and evolve into spatially coherent structures. Upon impinging on the ground, these structures generate acoustic waves, which in turn excite the shear layer at the nozzle exit, thereby closing the feedback loop (see Fig. B-1 and [2, 28]). The high amplitude impingement tones are undesirable not only due to the associated high ambient noise, but also due to the accompanied unsteady pressure loads on the ground plane and the nearby surfaces. While the high noise levels can lead to structural fatigue of the aircraft surfaces in the vicinity of the nozzles, the dynamic loads on the impingement surface can lead to an increased erosion of the landing surface as well as a dramatic lift loss during hover.

In an effort to reduce or eliminate these tones, several passive [16, 20, 40] and active control methods [2, 51, 52] have been attempted over the years to modify the feedback loop. Of these, the technique in [2] appears most promising from the point of view of efficiency, flexibility, and robustness. The method in [2] introduces
microjets along the periphery of the nozzle exit which modify the shear layer at its most receptive location thereby efficiently attenuating the impingement tones. Due to their small size, these microjets can be optimally distributed along the circumference and can also be introduced on-demand.

In [2], the microjets were injected with a steady-flow along the nozzle periphery, with the value maintained at a constant, independent of the impinging flow-field. This open-loop control strategy led, at certain heights, to the distinct tones either getting diminished or completely removed. There was an overall decline in the unsteadiness of the flow as well. Fig. B-2 shows the OASPL plot for different heights condition obtained using a 20° microjet injection with respect to the nozzle axis. It is also observed in Fig. B-2, that the magnitude of suppression is dependent to a large extent on the operating conditions and that the magnitude of reduction varies with the height of the lift plate from the ground as well as with the flow conditions. Since in practice, the operating conditions are expected to change significantly during take-off and landing, a more attractive control strategy is 'closed-loop control,' where the microjet effect is modified using suitable measurement of the impinging flow field, thereby maintaining a uniform noise reduction over a large range of operating conditions. In fact, it is well-known that for flows governed by a feedback loop, such as the present flow, screeching jets and others, the flowfield properties can change measurably even when the nominal operating conditions are the same. This is due the inherently high sensitivity of the feedback loop to very small changes in inlet and boundary conditions which can lead to changes in aeroacoustic properties (e.g. tonal frequencies and thier magnitudes, [31]). This further emphasizes the need for an adaptive control approach for such flows. In this paper, we explore such a closed-loop control strategy for reducing the impingement tones.

A traditional approach for designing a closed-loop controller is to begin with a model that describes the impinging flow-field, and carry out a model-based control design. There are, however, two difficulties in employing such an approach for the current problem. One is that the changing boundary conditions, compressibility effects, and the feedback interactions between acoustics and the shear make the modeling
significantly more complicated. The other is that the traditional feedback control paradigm typically requires the control input to be modulated at the natural frequencies of the system and mandate that the external actuator have the necessary bandwidth for operating at the natural frequencies [4]. In the problem under consideration, the impinging tones associated with the flow field are typically a few kilohertz. Given the current technology, modulating the microjets at the system frequencies while producing a microjets with significant momentum is extremely difficult, if not impossible. We note that, as discussed in [8], the development of high-frequency and high output actuators that can operate over a large range of frequencies is much needed. Although this is an active area of research, at present such actuators are not yet available in an usable platform.

To overcome these hurdles, two different control strategies were suggested in this paper. The first approach presented modulates the control input, $p$, at a slow time-scale, so that it behaves like a parameter. Here, the control input, the azimuthal distribution of microjet pressure, is chosen from ‘Proper Orthogonal Decomposition (POD)’ which calculates the most energetic spatial mode from given experimental data. If this control input is chosen judiciously, then even small and slow changes in this ‘parameter’ can lead to large changes in the process dynamics.

The POD method is a tool used to extract the most energetic modes from a set of realizations from an underlying system [21]. These modes can be used as basis functions for Galerkin projections of the model in order to reduce the solution space being considered to the smallest linear subspace that is sufficient to describe the system. The decomposition is ‘optimal’ in that the energy contained in an $N^{th}$-ordered POD base is greater than any other $N$-ordered base in a mean-squared sense. Over the years, it has been applied in several disciplines including turbulence, stochastic processes, image processing, signal analysis, data compression, process identification and control in chemical engineering, and oceanography, and has been referred to by various names including Karhunen-Loeve decomposition, principal component analysis, and singular value decomposition. In fluid mechanical systems, the POD technique has been applied in the analysis of coherent structures in turbulent flows and in obtain-
ing reduced order models to describe the dominant characteristics of the phenomena. One of the earliest studies was conducting by [6] on a fully developed pipe flow. Since then, POD models have been used to model the one-dimensional Ginzburg-Landau equation [54], the laminar-turbulent transitional flow in a flat plate boundary layer [46], pressure fluctuations surrounding a turbulent jet [5], turbulent plane mixing layer [14], velocity field for an axisymmetric jet [13], low-dimensionality of a turbulent flow near wake [33], low-dimensional leading-edge vortices in the unsteady flow past a delta wing [12], and flow over a rectangular cavity [48]. The eigenfunctions were developed using both experimental and numerical database. In this paper, we use the POD method to extract information about the mode shapes from pressure measurements which, in return, is used for control input strategy.

In the second control method, a pulsed microjet is introduced as an actuator. The rationale for doing this is that for a given mass flow rate, pulsed injection can generate larger momentum than steady continuous microjet injection, which is consequently expected to have a stronger impact on the noise reduction mechanism.

Pulsing of jet flows has been attempted in reference [25, 53, 55, 56, 62]. Wiltse and Glezer introduced an open-loop control strategy in [62] via high frequency forcing in the inertial subrange of a free shear layer on a low speed flow. They found that broadband velocity fluctuations were reduced at low frequency but increased at high frequencies. [55, 56] and [53] adopted the high frequency forcing technology for control of the cavity flows and [43] reported results applied for control of impinging tones. More recently, [25] reported reducing a resonant peak using HTFA (Hartmann Tube Fluidic Acuator), a very high speed actuator for controlling the impinging jet noise. This actuator primarily worked in a blowing-mode, required fairly large mass-flow rates, and worked over a fairly narrow range of frequencies whose selection required considerable tuning. Here, we pursue a low speed pulsing strategy which is far below the natural frequency (\( \sim 5 \) kHz) of the system. The actuator used modulates the flow at the exit of the microjet using a rotating cap. Saw-tooth structures placed in the inner race of the rotating cap block and unblock the microjet holes as the cap rotates and simulates an on-off microjet action. A similar pulsing actuator design was used
to control a free jet in reference [22]. However, as demonstrated in section 3.3.2, the design proposed here in significantly more efficient due to the location of the actuator.

Noise reduction using pulsed microjet injection is dependent on several control parameters such as duty cycle, mass flow rate, and phase difference between adjacent microjets and pulsing frequency. Among these parameters, the duty cycle and pulsing frequency play a major role in suppressing impinging tone. A change in the duty cycle from 43% to 70% was shown in ref [10] to result in an additional reduction of 6 dB. Pulsing at low frequency $\sim 16$ Hz results in an additional noise reduction of 1 $\sim$ 2 dB.

Moreover, to ensure the optimal performance of the pulsed microjet actuator, a closed-loop control strategy based on the analytic model of impinging tone system is proposed. The overall impinging tone model is composed of two loops: (1) A feedforward mechanism: noise generation by impingement of large scale vortical structure, (2) A feedback mechanism: formation of instability in the shear layer of the primary jet. The feedforward mechanism is modeled as a head-on collision between two vortex rings of identical strength because the main source for generating noise is the impingement of a large scale vortical structure whose intensity is proportional to the vortex strength. It is well known that unsteady motion of vortices emits acoustic waves called “vortex sound” [24]. In his classical paper in reference [42], Powell showed that an acoustic wave is excited by the vortex acceleration term $(\omega \times \mathbf{v})$. The theory of vortex sound was further expanded in references. (Möhring [34], Obermeier [39], and Kambe [23]). Of particular interest to the topic under consideration in this paper is reference [24] (Kambe and Minota, 1983) since their analytic model takes into consideration the compressibility and viscous effects. These two effects play a major role in our problem due to the fact that the flow is supersonic, and in an impinging jet, vortices formed are sufficiently close to the ground, respectively. Using this collision mechanism, we develop a new model in this paper that explains the acoustics due to impinging jets. The impinging tone frequencies are determined in a manner similar to that in reference [36], and are shown to capture the staging phenomena quite well. Then, the model is derived using the wave equation, colliding vortices of a certain
strength, and Green’s function. This model is compared with experimental results obtained in the FMRL, FSU, and is shown to match the data quite well. In particular, it is shown that the noise intensity is found to be increasingly proportional to the vortex strength.

To understand the role of pulsed microjet actuation, a two-mode lumped parameter model is proposed in this thesis that captures both the dominant impinging tone and the low-frequency mode discussed above. Here, the effect of pulsing is modeled through an input-shaping controller which suitably redistributes the acoustic excitation over the high and low frequencies. In addition, stagnation bubble formed on the impinging area is investigated as a source of the low frequency mode.

The thesis begins with an explanation of the experimental setup in section 2, followed by open-loop control test for hot/cold jet in section 3.1. In section 3, two major control strategies, the POD-based control (section 3.2) and pulsed control (section 3.3), are presented. In this section, experimental results using POD based control strategy are presented, compared with an open-loop control strategy, and the dependency of noise reduction on pulsed microjet control parameters such as duty cycle, mass flow rate, phase difference and pulsing speed are studied in detail. Finally, analytical study of impinging tone is suggested in the following section 4.2 (colliding vortex model) and 4.3 (two-mode lumped parameter model).
Chapter 2

Experimental Setup

2.1 Test Configuration and Facility

The following experiments were carried out at the supersonic STOVL jet facility of the Fluid Mechanics Research Laboratory located at the Florida State University. A schematic of experimental setup with a single impinging jet is shown in Fig. B-3. This facility is used primarily to study jet induced phenomenon on STOVL aircraft hovering in and out of the ground effect [2, 28]. A circular plate of diameter D (25.4 cm ≈ 10d) was flush mounted with the nozzle exit and, henceforth referred to as the ‘lift plate’, represents a aircraft planform. A 1 m × 1 m × 25 mm aluminum plate is mounted under the nozzle, which serves as the ground plane simulating the hovering situation by fixing it to the desired position. Further facility details can be found in [28].

The supersonic impinging jet was produced by an axisymmetric, convergent-divergent (C-D) nozzle with a design Mach number of 1.5. The throat and exit diameters \(d, d_e\) of the nozzle are 2.54 cm and 2.75 cm (see Fig. B-3 & B-4). The divergent part of the nozzle is a straight-walled conic section with a 3° divergence angle from the throat to the nozzle exit. A high-pressure blow-down compressed air facility was used to supply air to the nozzles. A high-displacement reciprocating air compressor drives the facility, which is capable of supplying air at a maximum storage pressure of 160 bars. Large storage tanks provides a total capacity of 10\(m^3\), which
makes it possible to drive the Mach 1.5 jet continuously up to 40 min.

A Validyne pressure transducer measures the stagnation pressure in the settling chamber just upstream of the nozzle. Although tests were conducted over a range of Nozzle Pressure Ratios (NPR, where NPR = stagnation pressure/ambient pressure), the results discussed in the present paper are limited to NPR = 3.7 that corresponds to an ideally expanded Mach 1.5 jet. The nozzle total pressure was maintained within ± 0.2 psi of the desired conditions.

Sixteen microjets fabricated using 400 µm diameter stainless tubes were used as actuators for active flow control. These are flush mounted circumferentially around the main jet as shown in Fig. B-4(a). While the orientation of the jets can be varied between 0 and 90°, most of the experiments reported in this paper correspond to the microjets at either 20° or 30° with respect to the nozzle axis. The supply for the microjets was provided by compressed nitrogen cylinders through a main and four secondary plenum chambers. In this manner, the supply pressures to each bank of microjets could be independently controlled. The microjets were operated over a range of NPR = 5 to 7 where the combined mass flow rate from all the microjets was less than 0.5% of the primary jet mass flux.

2.2 Pressure Measurements

Near-field noise was measured using B&K™ microphones placed approximately 25 cm away from the jet. The microphone signal was measured with an estimated uncertainty of ± 1 dB. The distribution of unsteady loads on the lift plate was measured by six high frequency response miniature Kulite™ pressure transducers (model: XCS-062), placed axisymmetrically around the nozzle periphery plate, at r/d =1.3 from the nozzle centerline (Fig. B-4). The Kulites were frequently calibrated throughout the experiments (almost every day) where the sensitivity was found to be very close to the values quoted by the vendor - between 24~26 mV/psi. The use of such transducers for measuring dynamic pressures is a standard, well-established practice for low speed and high speed flows for more than a decade [17, 60, 61]. According to
the manufacturer’s specifications, these transducers have a flat frequency response up to 20\% of their natural frequency. The transducers have a combined non-linearity and hysteresis(max) of ±0.5\%. The noise floor for these transducers was about 60 dB below the measured dynamic pressures, i.e. the signal to noise ratio was roughly 1000. Note that lift plate Kulites were only used for the steady microjet experiments. They were not used for the pulsed jet studies due to the vibration of the lift plate as a result of the pulsing hardware.

The transducer output were measured using National Instruments digital data acquisition cards (PC-MIO-16E-1 card coupled with SC 2040 Sample and Hold card) and LabViewTM software at a sampling rate of 70 kHz. The signal was low-pass filtered at 33 kHz, using Stanford Research Systems, SR640(8 pole-elliptic) Low Pass filters. These filters are phase-matched to better than ±0.75°. between channels. The phase lags between the various channels was checked by processing known signals through the entire chain of data acquisition and conditioning hardware, e.g. Filters, amplifiers and data acquisition cards. This was done using test signals over the range of frequencies of interest in the present study. The phases between the various channels matched within ±5°. Similar Kulite transducers have also been calibrated by Ukeiley (Ref: Private communication, March 2006) where the phase between different transducers was found to be within 2°. The results discussed in section 3.2.3 indicate a phase difference of about 15° which is well above the random error in phase introduced due to the hardware. 100k points were recorded for each signal. Standard statistical analysis techniques were used to obtain the spectral content and the Overall Sound Pressure Level (OASPL) from these measurements. For the spectral analysis, the Fast Fourier Transform (FFT) block size was 1024 points, with the resulting frequency resolution of 68.4 Hz. Consequently, the spectra shown in this paper represent an ensemble average of 100 ‘instantaneous’ or short-time-duration spectra with an associated random error of 10\%. The uncertainty associated with the unsteady pressure \( P_{\text{rms}} \), is ± 0.02 psi.
2.3 Particle Image Velocimetry (PIV)

The flow field and noise characteristics of impinging jet is investigated using Particle Image Velocimetry (PIV). The main feature of PIV is to record two consecutive images in quick succession. From a series of two images, the velocity field is derived using a cross-correlation algorithm. PIV system is composed of two hardware: high speed camera and laser illumination system. The images were recorded by a cross correlation CCD camera (Kodak ES 1.0 digital video camera). Its resolution is $1 \times 1$ k, and is operated in double pulsed mode. In this mode of operation, with proper synchronization with laser pulses, the camera can acquire double images at a rate of 15 images pairs per second. The image is the oil or smoke particles illuminated by a double pulsed digitally sequenced Nd:YAG laser (Spectra-Physics, 400 mJ) which is a light sheet about 1.5 mm thickness created by suitable combination of spherical cylindrical lenses. The laser sheet is placed parallel to the jet direction and intersects the centerline of the nozzle. The jet was seeded with small ($\sim 1 \mu m$) oil droplets generated using a modified Laskin nozzle. The ambient air was seeded with smoke particle ($\sim 5 \mu m$) produced by a Rosco fog generator. A schematic of the experimental arrangement of the PIV system is shown in Fig. B-5.

The double pulsed images were acquired through an Imaging Technologies ICPCI board, which resides on a single slot of the PCI bus of a personal computer. The time between pulses was optimized at $1.2 \mu s$. An image matching approach was used for the digital processing of the image pairs to produce the displacement field. To achieve velocity data with high spatial resolution, a novel processing scheme was developed in Lourenco and Krothapalli [32]. The flow field at any points is described by an analytical function using a least-squares-fitting algorithm.

$$u = ax^2 + bx + cy^2 + dy + exy + f$$  \hfill (2.1)

The marked advantage of this approach is that the velocity field is described at any point with second-order accuracy and thus computation of derivatives is accomplished with higher precision. In the absence of shock cells, the mean velocity data using PIV
is in very good agreement (± 1 %) with the exit velocity calculated from isentropic relation. Moreover, instantaneous velocity field can capture the presence of large scale vortical structures in the primary jet and the wall jet. For further technology about the PIV, the reader can refer to a reference by A.Krothapalli et al. [28].
Chapter 3

Control of Supersonic Impinging Jets

3.1 Steady Microjet Actuation

3.1.1 Open-Loop Control Strategy: Cold Jet

The microjet has been used for a noble actuator in suppressing the impinging tones for a long time. The very small size of the sensor/actuator hardware and the minimal mass flow rates requires minimal power consumption and is expected to result in negligible thrust loss of the primary jet. In contrast to the traditional passive control methods, the proposed actuator can be switched on and off strategically. Therefore, it will not degrade the operational performance of the aircraft when it is not needed. In the present experiments, microjets were made using 400 mm diameter stainless tubes. 16 microjets were located in the circumference of nozzle exit. For the open-loop control, the supply pressure of secondary plenum which determines the intensity of microjet strength was kept constant (100 psia) during the test.

In reference [2], supersonic impinging jets produce a very unsteady flow field, with high noise levels and discrete frequency acoustic tones. The instantaneous shadowgraph in Fig. B-6 (a) show the representative image for an uncontrolled - microjet off - impinging jet. We can see the distinct wave propagating up to the nozzle exit and...
bounded wave from the lift plate. A large-scale structure are also conspicuous near the ground plane. This structure is thought to play a major role in flow entrainment and consequent cause of a large amount of lift loss suffered by STOVL aircraft during the hovering mode. On the other hand, the effect of microjet is visible from the shadowgraph image in Fig. B-6(b). The image is completely changed from the uncontrolled one in some features that totally different dynamics seem to be dominant at the controlled case. The ambient air becomes free from the acoustic wave and a distinct large-scale structure in the middle of the jet disappeared.

Fig. B-7 shows the narrowband spectra of the unsteady pressure signal on the lift plate for NPR = 3.7, h/d = 4.5. The presence on multiple tones is apparent by the discrete peaks in the spectra. By activating the microjet, distinct peaks are significantly diminished or entirely eliminated. Along with the narrow band noise reduction, the broadband noise amplitude was diminished to a certain extent. Such overall amplitude reduction patterns are seen in other data captured on the ground plane and near-field acoustic measurement collected by microphone.

Fig. B-2 are the overall reduction in the unsteady pressure levels on the lift plate for NPR = 3.7. From the graph, we can easily notice that the microjet did reduce the overall sound pressure level to a certain degree under any circumstance. But, the most noteworthy fact is the reduction is non-uniform with respect to the height and is not repeatable. It is well-known that the properties of feedback loop of the uncontrolled jet, such as the amplitude and frequency of the impingement tones and the dominant instability modes in the flow, are highly sensitive to operating conditions. It is also worth noting due to the sensitivity of the feedback loop on the exact operating conditions, the effect of microjet control can vary even if the conditions are kept constant. As an example, although the height at which the microjets are minimally effective is h/d = 4.5 for the conditions, it can on occasion shift to h/d = 4 or 5 during a particular test. Hence, an efficient control scheme should be able to adapt to the changes in the local flow conditions, on-line to provide optimal control over the entire operating range.
3.1.2 Open-Loop Control Strategy: Hot Jet

Having demonstrated the efficacy of microjets in controlling the highly unsteady flowfield generated by cold impinging jets; we next examined the potential of microjet control for impinging hot jets. This brings us closer to flow conditions that occur in practical applications while allowing us to reevaluate some of the physical mechanisms developed and proposed during the study of cold impinging jets. Accordingly, the STOVL/Impinging Jet facility was modified via the addition of a 200 kW resistive heater. This allows the main jet to be operated at temperatures between 600°F and 800°F depending on the mass flow rate through the system.

Fig. B-8 shows near-field noise measurements, in terms of OASPL for hot and cold impinging jets, where the hot jet was operated at a stagnation temperature of approximately 240°F and the cold jet at about 85°F. These measurements were made using a B&K microphone placed 10 diameters from the jet centerline. A few points regarding these plots are worth noting. First, a comparison of the no control cases between the hot and cold jets shows that for all the cases (in terms of $h/d$) shown here, the overall noise levels for the hot jet are notably higher, as one would expect. Second, the elevated noise levels decay much more gradually with increasing ground plane distance - $h/d$, for the hot jet case. In fact, the noise levels for the hot impinging jet remain roughly constant up to $h/d = 8$ after which noise decreases with increasing $h/d$. In contrast, the OASPL for the cold jet begins to decrease monotonically after $h/d = 4$. This indicates that the feedback loop, which is responsible for the increased noise levels, is stronger and persists longer for hot impinging jets.

Microjet control for both these jets was implemented using sixteen, 60° microjets operated at $\sim 100$ psi. The effect of microjet control can be observed by comparing the solid lines (no control) with the dashed lines (control) in each of plots in Fig. B-8. For ease of comparison, this difference in OASPL for the hot and cold jets is shown in Fig. B-9. As seen in this plot, microjets are almost equally effective in weakening the resonance loop, and hence reducing the impinging jet noise, for both hot and cold jets up to about $h/d = 5$. However, beyond this distance, microjet control is rather
ineffective for cold jets. In contrast, microjets are significantly more effective for the hot jet up to $h/d = 9$. Recalling the fact that elevated noise levels are measured up to $h/d = 9$ for the hot jet (see Fig. B-8 (a), this behavior is expected.

3.2 Steady Microjet Actuation: POD-based Control

To maintain a uniform reduction of the unsteady pressure over a wide range of operating conditions, a quasi closed-loop control method was investigated. The details of this control strategy, the analytical basis, and the results obtained are reported in this section.

3.2.1 The Proper Orthogonal Decomposition Algorithm

As mentioned earlier, the POD method is a tool used to extract the most energetic modes from a set of realizations from an underlying system, a brief description of which is provided below. Given a flow defined on a domain $\Omega$ over a time interval $T$, the flow-field variables such as velocity, pressure, and density, can be predicted using the governing equation. To ensure better accuracy, the flow is treated as a random process with parameter of time and space. We shall denote the flow variable as the sum of orthonormal basis $a(t)$ and $\phi(x)$, or

$$u(x, t) = \sum_{n=1}^{\infty} a_n(t)\phi_n(x),$$

(3.1)

with the complexity of the model reduced by truncating the series at a suitable value. While a large number of basis functions $\phi(x)$ can be used, the simplest, yet most powerful, basis function is that which is obtained using the Karhunen-Loeve expansion (3.1) [37].
Using the Karhunen-Loeve expansion, the unsteady pressure is expressed as

\[ p(t, \theta) = \sum_{n=1}^{\infty} \sqrt{\lambda_n} a_n(t) \phi_n(\theta) \]  

(3.2)

where the temporal terms are uncorrelated and are given as

\[ a_n(t) = (\sqrt{\lambda_n})^{-1} \int_{\Omega} \phi_n(\theta)p(t, \theta)d\theta \]  

(3.3)

\[ E[a_m(t)a_n(t)] = \delta_m^n \]  

(3.4)

\[ \int_{\Omega} \phi_m(\theta)\phi_n(\theta)dx = \delta_m^n \]  

(3.5)

and the orthonormal basis functions \( \{ \phi_n \} \) are calculated from integral equations based on a covariance function \( R_p(\theta_1, \theta_2) \) as

\[ \int_{\Omega} R_p(\theta_1, \theta_2)\phi_n(\theta_2)d\theta_2 = \lambda_n\phi_n(\theta_1), \quad \theta_1, \theta_2 \in \Omega \]  

(3.6)

\[ R_p(\theta_1, \theta_2) = E[(p_{\theta_1} - \mu(\theta_1))(p_{\theta_2} - \mu(\theta_2))] \]  

(3.7)

where \( \mu(\theta_1), \mu(\theta_2) \) are mean values of variable \( p_{\theta_1}, p_{\theta_2} \), respectively and \( \delta_m^n = 0 \) (if \( m \neq n \)), 1 (if \( m = n \)). The derivation of the temporal term, the uncorrelated property and rigorous proofs can be found in Newman’s paper [37]. The spatial mode \( \phi_n(\theta_1) \) can be calculated using ‘method of snap shot’ as follows [59]. Let the \( p_n(j) \) be the pressure variable at a spatial point \( n \) at some time \( j \) where \( n = 1, 2, \cdots, N \) and \( j = 1, 2, \cdots, J \), with \( n \) much smaller than \( J \). Now the matrix \( Q \) can be expressed from singular value decomposition as

\[ Q = \begin{pmatrix}
    p^1(1) & p^1(1) & \cdots & p^1(J) \\
    p^2(1) & p^2(1) & \cdots & p^2(J) \\
    p^3(1) & p^3(1) & \cdots & p^3(J) \\
    \vdots & \vdots & \cdots & \vdots \\
    p^N(1) & p^N(1) & \cdots & p^N(J)
\end{pmatrix} = U\Sigma V^T \]
where $U(N \times l)$ and $V(J \times l)$ are unitary matrix

$$
[U]^T[U] = [I]_{l \times l}, \quad [V]^T[V] = [I]_{l \times l}
$$

and

$$
[\Sigma]_{l \times l} = \begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_l
\end{pmatrix}
$$

$$
\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \sigma_l
$$

The matrix $V$ and $\sigma$ are the eigenvector and the square-root of the eigenvalue, respectively of the correlation matrix $Q^TQ$.

$$
$$

The mode-shape can be computed by normalizing each column of the following matrix $\Phi$.

$$
\Phi \equiv QV = [U][\Sigma][V]^T[V] = [U][\Sigma] = [\phi_1, \phi_2, \ldots, \phi_l].
$$

In short, the spatial $i^{th}$ POD mode can be obtained as given below:

$$
\phi_i(x_n) = \sum_{j=1}^{l} \frac{V(j,i)Q(x_n,j)}{\sigma_i}, \quad i = 1, \ldots, l; \quad n = 1, \ldots, N
$$

### 3.2.2 POD-based Control of Impingement Tones

In order to find the POD modes of the system, the measurements of pressure at all flow points and a real time calculation scheme are needed. However, this is not feasible either experimentally or computationally due to obvious constraints. Fortunately, the
feedback loop is most sensitive to the conditions in the immediate vicinity of the jet nozzle. Therefore we analyze the flow field by focusing only to the nozzle exit. That is, we derive the control strategy using the expansion:

\[ p(\theta, t) \triangleq p_+(r = R_s, \theta, z = z_{\text{nozzle}}, t) = \sum_{i=1}^{L} X_i(t) \phi_i(\theta) \]  \hspace{1cm} (3.13)

where \( R_s \) is the radial position of the sensors on the lift plate and \( p_+ \) is the pressure outside of main jet. Note that \( \phi_i \)’s in Equation (3.13) are the spatial modes of the flow field confined to nozzle exit. Once the mode shapes are determined, we simply choose the control strategy as:

\[ p_{\mu}(\theta) = k\phi_1(\theta) \]  \hspace{1cm} (3.14)

where \( \phi_1 \) is the most energetic mode in Equation (3.13) and \( k \) is a calibration gain. The complete quasi closed-loop procedure therefore consists of collecting pressure measurements \( p(\theta, t) \), expanding them using POD modes as in Equation (3.13), determining the dominant mode \( \phi_1 \), and matching the control input — which is the microjet pressure distribution along the nozzle — to this dominant mode as in Equation (3.14), and is denoted as a 'mode-matched' control strategy.

3.2.3 Results Using POD-Based Control

The mode-matched control strategy described above was implemented at the STOVL supersonic jet facility of the Fluid Mechanics Research Laboratory, FSU (see [31] for details). Four banks of microjets were distributed around the nozzle exit, while pressure fluctuations were sensed using six \( \text{Kulite}^{\text{TM}} \) transducers flush mounted around the nozzle periphery on the lift plate, at \( r/d = 1.3 \), from the nozzle centerline (\( d \) is the nozzle throat diameter).

The control experiment was performed for a range of heights (of the nozzle above ground). At each height, in addition to the 'mode-matched' control, the open-loop
control strategy described earlier where the microjet pressure around the nozzle exit was maintained at a uniform value was also implemented. To ensure a fair comparison between the two control methods, the main nozzle was operated under the same conditions while implementing these two different control strategies. The calibration constant $k$ in Equation (3.14) was chosen such that the minimum and maximum values of the POD mode over $\theta$ correspond to 70 psia and 120 psia, respectively, which ensured maximum effectiveness of the actuator. The value of 70 psia was chosen since it was the lowest pressure at which any measurable control effect could be observed and 120 psia was chosen since it was the pressure that the steady control (with 20° inclination) effect is almost saturating. Fig. B-10 (a) shows the shape of the first mode and the suggested microjet bank pressure distribution for several heights.

Fig. B-10 (b) shows the results for the “POD-based” control strategy, which indicates a better performance compared to the open-loop controller throughout all operational conditions, with a large improvement at heights $h/d = 4, 4.5$ and 5. The reason for this increased noise reduction can be attributed to the percentage of energy contained in the dominant mode, which is used in the control strategy. Seen in the Tbl. A.1, at heights 4 to 5, the energy content of the first mode is almost 90%. In contrast, at heights 2 and 3, the energy level drops to about 50% and hence the corresponding improvement in the closed-loop strategy also drops to about half the dB-value at heights 2 and 3 compared to at heights 4 and 5.

As noted in the introduction, flows governed by a feedback loop are highly sensitive to very small changes in the local conditions, displaying different behavior under the same nominal conditions. This is illustrated in Fig. B-11 which shows the effect of control using microjets inclined at 30° with respect to the nozzle axis. A comparison of the OASPL of the uncontrolled cases in Fig. B-10 and B-11 shows that the noise characteristics in these two experiments are quite different, though major characteristics of the flow-field such as a monotonically decreasing OASPL as $h/d$ increases and the overall noise level are the same. The distinct feature of this case is that the open-loop control with 30° injection led to a large amount of noise reduction without any feedback action. The POD-based control was observed to be slightly better with
an additional 1~ 2 dB reduction over open-loop control at heights 4.5 and 6 and denoted in Fig. B-11. From the energy content view (seen Fig. B-11 and Tbl. A.2), the microjet should have reduced the noise level further but the result was not as dramatic as before.

A possible reason for the occasional lack of impact from the microjets is discussed below. It has been observed in [28] that the flow characteristics evolve from a helical mode to a axisymmetric mode and return to a helical mode as the nozzle to ground distance becomes larger. Fig. B-15(a) shows the phase difference measured by Kulites mounted on the lift plate at the most dominant frequency, at $h/d = 3.5$, 4.0, and 4.5. A small deviation in the phase difference between the signals implies that the flow characteristics are axisymmetric.

From Fig. B-11 and Fig. B-15(a)(b), we note that there is a correlation between the amount of noise reduction and the flow mode. It can be seen that at heights $h/d = 3.5$ and 4.5, the dominant mode is helical with the helicity being stronger at $h/d = 3.5$, while at $h/d = 4.0$, the dominant mode is axisymmetric. Correspondingly, we note that the OASPL reduction is the least at $h/d = 3.5$, maximum at $h/d = 4.0$, and medium at $h/d = 4.5$. That is, the lack of noise reduction appears to be correlated with the presence of a helical mode in the 30°-injection case. This is also consistent with the results of the 20°-injection case reported in Fig. B-10, where the dominant mode was predominantly axisymmetric at most heights. A low OASPL reduction of 1 dB occurring at $h/d = 3.0$ for the 20°-injection case could therefore be due to the fact that the component of the axisymmetric mode at this height was 55% which was less than the amount of axisymmetric mode present at other $h/d$. One could argue that the specific case of $h/d = 4.5$ with 30°-injection shown in Fig. B-11 is somewhat of an exception to the above hypothesis, which links lack of reduction to the presence of a helical mode. At this $h/d$, an OASPL reduction of 7 dB occurs despite the presence of a helical mode. This anomalous behavior could be due to the fact that the helical effect at the nozzle becomes less important as $h/d$ increases.

A final observation is a comparison between the amount of noise reduction achieved in Fig. B-10 and Fig. B-11 which corresponds to a microjet injection of 20° and 30°,
respectively. We note that larger reductions occur in the latter case, which is most likely due to the fact that with a larger injection angle, a greater penetration depth of the microjet streak into the shear-layer of the primary jet is achieved, as shown in [30]. At a given angle of injection, a larger penetration depth is always achievable by raising the mass-flux. However, increasing the mass-flux may not often be desirable due to practical constraints. Therefore, any actuation method that leads to a larger penetration depth at reduced mass-flux rates has a good chance of assuring a larger and more consistent noise reduction at all operating conditions. In the next section, we present the results of a different actuation method that has the above desirable property.

3.3 Pulsed Microjet Actuation using a Rotating Cap

The results in section 3.2 show that a ‘quasi-closed loop control’ method produced additional noise reduction compared to the open-loop control strategy. To obtain a more consistent noise reduction over a larger range of jet operating conditions, we examined a different control strategy, which consists of a technique that pulses the microjet flow. The rationale for introducing pulsing is discussed below.

For a given mass flow rate \( \dot{m} = \rho AU_{\mu,0} \), the force induced by steady microjet injection is given by the rate of momentum change in time. Using the same mass flow rate, an unsteady injection can exert more force on the primary jet shear layer of the flow than steady injection, in an average sense. Equation (3.15) described below shows that, if as an example, the unsteady flow through the microjets is represented in sinusoidal form, the additional force increase is realized by \( \rho A (B^2/2) \):

\[
\begin{align*}
F_{\mu,\text{steady}} &= \dot{m}U_{\mu,0} = \rho AU_{\mu,0}^2 \\
U_{\mu,\text{unsteady}} &= U_{\mu,0} + B \sin(\omega t) \\
F_\mu(t) &= \dot{m}U_{\mu,\text{unsteady}} = \rho A (U_{\mu,0} + B \sin(\omega t))^2
\end{align*}
\]
That is, for a given mass flow rate, a pulsed injection can generate more momentum than steady continuous microjet injection, and hence can perhaps have a stronger impact on the jet shear layer, thus disrupting the feedback mechanism more effectively and hence reducing the noise more significantly.

### 3.3.1 Modulated Microjet - High Speed Valve

The easiest way to generate pulsing is using a high speed valve. The traditional feedback control paradigm requires the control input to be modulated at the natural frequencies of the system (for example, see Rowley et al. [50]). This, in turn, mandates that the external actuator have the necessary bandwidth for operating at the natural frequencies. In the problem under consideration, the edge tones associated with the flow-field are typically of a few kilohertz. Given the current valve technology, modulating the microjets at the system frequencies is nearly impossible. It follows that the first approach, as presented above, is reduced to pulsing microjet injection at the frequency of sub harmonics of the resonance frequencies. The specification of the relevant high speed valve is listed in Tbl. A.5. Fig. B-12 shows the schematic diagram of the high speed valve assembly. Highly pressurized flow is passing through a filter, supplied to the inlet of the block and drained out of the connecting tube. The high speed valve inside the assembly block modulates the exit flow. Unsteady pressure transducer by Kulite™, located at the end of the connecting tube measures the total pressure fluctuation.

To verify the correct function of the valve, an experiment was conducted under the condition of 50% duty cycle and at 165 psi supply pressure. The result shown in Fig. B-13 indicates noticeable flow modulation issuing out of the connecting tube at the 150 Hz valve speed. The next step was to size down the diameter of the connecting tube so as to match the microjet size at the other end. Because the
length and diameter of the flow channel are major factors in determining the flow resistance, several experiments were conducted to minimize the resistance, an example of which is given here. An adaptor which decreases the channel diameter from 0.50" to 0.25" was plugged at the end of the connecting tube. Because the cross sectional area of the tube becomes narrow downstream direction, flow speed through the tube increases. However the upper limit of flow speed is bounded by the sonic limit, hence the resulting modulation of the flow speed is almost completely decayed as seen in Fig. B-14. Since the overall performance of this pulsing strategy was not satisfactory, it was not pursued any further.

3.3.2 Pulsing Using a Rotating Cap

Because flow modulation applied at the upstream end results in steady injection, Flow modulation was introduced using direct modulation at the exit of the microjet using a rotating cap (see Fig. B-16). This cap consists of several teeth which block and unblock the microjet holes as the cap rotates, simulating an on-off microjet action. The design of the lift plate is slightly changed to install the rotating cap actuator, and is composed of a small and a large lift plate (see Fig. B-17, Fig. B-18). The small lift plate is assembled along with the rotating cap at the center of the big lift plate using a bearing. A motor mounted behind the lift plate drives the rotating cap connected by a belt. Finally, the lift plate is supported by three arms attached to the holder which itself is attached to pipe supplying air to the main nozzle. Fig. B-19 shows the lift plate assembly.

The effect of the pulsed microjets through the rotating cap was quantified by spinning the motor at different speeds and measuring the unsteady total pressure at the microjet exit using a Kulite mounted in a total pressure probe configuration. These measurements were sampled at 70 kHz with a cut-off frequency of 33 kHz. In Fig. B-20(a)-(b), the pressure response with respect to time using a steady and pulsed microjet flow are shown, respectively. In Fig. B-20(c), the power spectra corresponding to the time series in Fig. B-20(b) shows that the rotating cap produces a fairly large amplitude perturbations around 300 Hz. This unsteady effect was
observed for motor speeds over a range of 0 to 2000 rpm. The speed of pulsing is determined by the motor speed, the diameter of rotating cap and number of teeth in the cap. One revolution of the cap introduces 16 pulses since the cap has 16 internal holes. Moreover, the pulley which drives the rotating cap has a smaller diameter than the rotating cap. Hence, the pulsing speed is obtained from the following relations:

\[ f_{\text{pulsing}} = 16 \times \left( \frac{D_{\text{pulley}}}{D_{\text{cap}}} \right) \frac{\text{RPM}_{\text{pulley}}}{(60)} \]  

(3.16)

where \( D_{\text{cap}} = 2.625 \text{ in}, D_{\text{pulley}} = 2 \text{ in} \). Here the resultant pulsing speed by the rotating cap was set to 121 Hz, which corresponds to a moderate motor speed of 596 rpm. At this speed, the vibrations due to the rotating mechanism are minimal and do not lead to a broadband noise increase.

In addition to providing a direct method of pulsing the microjet flow, it is also of interest to be able to vary different parameters of the pulsed flow such as amplitude, frequency, duty-cycle, and phase. This can be accomplished by varying the design parameters of the rotating cap. The pulsing amplitude is directly proportional to the supply pressure delivered to microjet chamber, while pulsing frequency is solely controlled by the rotation speed of the cap. Therefore, these two parameters can be easily and electronically varied by changing the microjet pressure and the motor speed. The duty cycle and the phase, on the other hand, depends on the design of the rotating cap and requires a mechanical design procedure. For example, if \( d_c \) is the duty cycle of pulsing, which is the ratio of the valve opening time to pulsing period, then

\[ d_c = 100 \left( \frac{N_h d_h}{\pi d} \right) \% \]  

(3.17)

where \( d \) is the main jet diameter, \( d_h \) is the diameter of the holes in the rotating cap (see Fig. B-21), and \( N_h \) is the number of holes in the rotating cap. This implies that the duty cycle(\( d_c \)) is changed by varying the number and diameter of holes of the rotating cap. If the number of holes in the rotating cap is the same as that of microjets, all the microjets pulse synchronously. To achieve a phase difference
between two adjacent microjet pulses, the number of holes in the cap was chosen to be different from that of microjets. This phase difference, \( \phi_{\text{phase}} \), can be calculated using equation (3.18)

\[
\phi_{\text{phase}} = \left( \frac{N_h}{N_m} - 1 \right) \times 360
\]

(3.18)

where \( N_m \) is the number of microjets.

We illustrate a realization of \( \phi_{\text{phase}} = 120^\circ \) in Fig. B-22 which occurs by choosing \( N_h = 4 \) and \( N_m = 3 \). This configuration produces pulsing with \( \phi_{\text{phase}} = 120^\circ \). In the impinging jet problem, 18 \( (N_h) \) holes were made in the rotating cap while 16 \( (N_m) \) microjets were installed in the lift plate. From equation (3.18), we observe that this produces a \( \phi_{\text{phase}} \) of 45°.

It should be pointed out that the swirl caused by the cap rotation itself does not significantly affect the baseline performance. This is demonstrated in Fig. B-23, where the OASPL of the uncontrolled impinging jet is compared to that while the cap was rotating without any microjet action. As can be seen in the figure, the two OASPLs are almost identical.

As mentioned in section 1, a similar design to the rotating cap approach discussed above has been reported in reference [22] for suppressing jet noise. A comparison between our design and that of reference [22] is briefly illustrated in Fig. B-24. The major distinction between the two is the distance from the microjet injection point to the shear layer of main jet. In reference [22] shown in Fig. B-24(a), thirty-six microjets with a diameter of 0.5mm were used, with a mass flow rate of 2 ~ 4% that of primary jet. But the location of the microjets were at five-diameters away from the shear-layer of the main jet on the azimuthal plane. Our design, shown in Fig. B-24(b), collocates the actuator with the nozzle exit, thereby allowing the microjet flow to have an inclination angle with respect to the flow direction. In contrast, in reference [22], the microjet injection is forced to remain normal to the main flow. The lack of success reported in reference [22] may in part be due to the location of the actuators, since the penetration depth of microjet injection into the shear layer is known to play a critical role in the noise suppressing mechanism.
3.3.3 Results Using Pulsed Microjets

Using the above experimental setup, studies of the impinging jets were conducted with and without pulsing. Unlike ‘POD-based control’ case, the unsteady pressure measurements could not obtained on the lift plate due to the vibrations from the spinning cap incorporated in the small lift plate (see Figs. B-16 and B-18). Instead the noise level was measured by a microphone located at 25 cm away from the nozzle axis. The results obtained when the rotating cap was spun at a frequency of \( f = 121 \) Hz and \( \phi_{\text{phase}} = 0 \), and \( d_c = 70\% \) are shown in Fig. B-25(b). These results show that the impinging tones are completely eliminated by the pulsed microjets. In order to further understand the impact of the rotating cap and the sensitivity of the impinging flow field to the pulsing parameters such as frequency and duty-cycle, a number of parametric studies were conducted, which are summarized below:

(a) Effect of mass flow rate \((\dot{m})\): In reference [30], the steady microjet depends on several control parameters such as mass flow rate \((\dot{m})\), injection angle \((\alpha)\) and nozzle operating condition (NPR). Among these parameters, the mass flow rate \((\dot{m})\) has the most dominant impact on microjet performance. As the steady actuation shows, the performance of pulsed microjet also strongly depends on this parameter. The amount of mass flow rate through microjet is exactly proportional to the supplied pressure which calculates from the following equation:

\[
\dot{m}_{\mu,\text{steady}} = d_c \frac{0.684P_0A^*}{(RT_0)^{1/2}}
\]

where \(\dot{m}_{\mu}\): mass flow rate of pulsed microjet actuation, \(P_0\): upstream pressure, \(T_0\): upstream temperature to microjet, \(A^*\): microjet diameter (400\(\mu m\)), \(R\): specific gas constant (297\(J/kg \cdot K\)), \(d_c\): duty cycle.

For most cases, performance of steady microjet increases at higher microjet intensity because larger mass flow rate guarantees deep penetration of primary shear layer. However, once the microjet intensity becomes strong enough to penetrate shear layer completely, the amount of noise reduction saturates to its maximum [30]. Similar result is repeated by pulsed microjet actuation. Seen in Fig. B-30, the amount of
noise reduction saturates at \( \sim 0.8 \dot{m}_{100} \) regardless of other parameters, where \( \dot{m}_{100} \) is 0.228 \( \times 10^{-3} \) kg/s, which represents the reference mass flow rate for \( T_0 = 300 \)K, \( P_0 = 100 \) psig respectively.

(b) Effect of pulsing: The effect of pulsed microjet actuation is confirmed by the amount of noise reduction in the microphone data. In addition to the acoustic data, the effect is also indicated by flow visualization technique (PIV). Because the impinging tone triggers development of the large scale vortical structure, the noise reduction is expected to cause weakening of the large scale vortical structure in the controlled flow field. The impinging jet flow field controlled by the pulsed microjet is compared to the controlled flow field using same cost (the same mass flow rate with pulsed microjet actuation) of steady injection. The blowing media (nitrogen) was supplied at the rate of 0.186 \( \times 10^{-3} \) kg/s which is denoted by 0.8 \( \dot{m}_{100} \), where \( \dot{m}_{100} \) is the reference mass flow rate driven by 100 psig supply pressure. Two experiments were conducted under the same nozzle operating condition using 56% duty cycle rotating cap: (i) Pulsing with the same mass flow rate \( (0.8 \dot{m}_{100}) \) as the steady microjet to compare the pure pulsing effect. For this condition, the corresponding supply pressures are \( P_{steady} = 77 \) psig, \( P_{pulse} = 140 \) psig respectively. (ii) Pulsing under the same supply pressure as the steady \( (P_{pulse} = 77 \) psig) to see the effect of mass flow rate. Fig. B-26, B-27 represent the mean velocity field of impinging jet for \( h/d = 3.5 \) and 4.0 respectively. Seen in these figures, the mean velocity field of primary jet is almost identical under either steady or pulsed injecting condition. On the other hand, the turbulent properties are clearly distinguished for different actuation schemes shown in Fig. B-28, B-29. The turbulence intensity field is calculated using 400 PIV image pairs. In Fig. B-28, the uncontrolled flow field denoted by (a) is not very much different from that of steady or pulsing with same supply pressure. But different from (a)-(c), the pulsing with the same mass flow rate as steady injection shows dramatically elimination of wiggles in (d). These turbulence properties represent the large scale vortical structure. The impingement of the large scale vortical structure causes the discrete tone. The elimination of this structure indicates the dramatic reduction in acoustic data. The effect of pulsing is more clearly shown in the result
of height \( h/d = 3.5 \) in Fig. B-28. At \( h/d = 4.0 \), the steady microjet already controls the flow field sufficiently, which supports the selective performance of steady microjet shown in Fig. B-11.

(c) Effect of duty-cycle \((d_c)\): The effect of duty-cycle was explored and is presented in Fig. B-25(a)-(b). These were obtained for two different duty-cycles of 43\% and 70\% at a pulsing frequency of 121 Hz with 115 psia microjet supply pressure. Two points should be noted from this figure. The first is that both pulsing and steady microjet action yield about the same amount of pressure reduction, and since the supply pressures were the same, it implies that the pulsing action allows noise reduction to occur at 43\% of the mass flow rate needed for the steady case. The second is that a significantly larger reduction can be obtained from the pulsing action under certain duty cycles, which follows from Fig. B-25(b). It was in fact observed that the pulsed injection completely destroyed the distinct impinging tones at almost all \( h/d \). Moreover, this occurred at a mass flow rate that is 70\% of the steady injection case. Yet another point to note is the robustness of the pulsed actuation method. It should be noted that the amount of noise in the no-control case is a little different in the two different experiments shown in Fig. B-25(a) and (b), primarily at \( h/d = 3 \) and 4 - due to the sensitivity of impinging jet properties on slight changes in the boundary conditions. Nevertheless, the lack of reduction using steady microjet injection at the height \( h/d = 3.5 \) is consistent between the two experiments. In contrast in both cases, the pulsed injection maintains to an additional reduction of 2 dB or more at this height In the repeated test (seen in Fig. B-31), the optimal noise reduction is achieved at 56 \% duty cycle for various heights and mass flow rate conditions.

(d) Effect of frequency \((f_{pulsing})\): The rotating cap was spun over a range of frequencies from 0 to 150 Hz, corresponding results of which are shown in Fig. B-32. This shows that over this range, the amount of noise reduction is quite independent of \( f_{pulsing} \). To prevent the possible damage from friction of rotating part and make the whole assembly working in safe, we chose the pulsing frequency \( f_{pulsing} \) as 121 Hz as the primary test case.

(e) Effect of phase-difference \( \phi_{phase} \): As mentioned in section 3.2.3 and il-
illustrated in Fig. B-15, we initially anticipated that the phase difference will play an important role in noise reduction mechanism. Two experiments were conducted by changing the rotating direction of the cap to check the effect of phase lead and phase lag on noise reduction. The duty cycle was set at 50%, the supply pressure delivered to the microjet chamber was set to 115 psia, and the pulsing speed was 121 Hz. The results obtained for a phase \( \phi = 45^\circ \) and a phase \( \phi = -45^\circ \) are shown in Fig. B-33 and compared with the synchronously pulsed injection case, where the duty cycle was maintained at 43%. We note that these phase variations did not result in any noticeable improvement over the synchronous scheme and uneven noise reduction for various heights is still conspicuous. While the reason for this remains unresolved, more experiments are being planned to understand the role of phase difference in pulsed microjet actuation.
Chapter 4

A Reduced-Order Model of Supersonic Impinging Jets

To develop a physical model of acoustic field generated by impinging jet is very difficult work due to several factors such as strong coupling with highly unsteady flow. Tam [58] developed the vortex sheet model to investigate the structure of the free jet. He approved the model by showing the nonexistence of helical mode in the subsonic free jet. Motivated by the vortex sheet model, Annaswamy et al. [3] introduced the reduced order model of impinging jet. This model gave insight on predicting the most dominant frequency and its multiples, but it does not predict the trend of reducing amplitude at high frequency (see Fig. B-34) because the reduced order model holds only near the nozzle exit but the author tried to expand the model into the downstream of the impinging jet. In this paper, we suggest a modification to the model in [3] using a different approach.

4.1 Prediction of Impingement Frequencies

The most dominant frequency at a certain height was suggested by Neuwerth [36] to be
\[ f = \frac{M}{h \left( \frac{1}{C_a} + \frac{1}{C_v} \right)} \]  

(4.1)

where \( f \) is the frequency, and \( h \) is the distance between the ground and the jet nozzle. \( C_a \) and \( C_v \) are the propagation speed of vortex rings and the speed of sound, \( M \) is an integer. As we change \( h \), the value of \( M \) changes, which leads to a discontinuous jump in the frequency of the dominant peak. The integer \( M \) which is a function of \( h \) is chosen by fitting the above equation into the experimental data. Explicit prediction of \( M \) is the first step for the impinging jet model, which is carried out below.

Gharib et al. [19] postulated that there exists a universal upper bound of the 'formation number' to form a stable vortex ring by an impulsive piston motion. The formation number is defined as \( N_f = \frac{U t}{d} \) where \( U \) is the speed of the piston, and \( t \) is the time duration of the motion. \( d \) is the diameter of the piston apparatus. They found that one single vortex ring could be maintained, if \( N_f < N^* \) and \( N^* \) lies in the range of 3.6 ~ 4.5. Although the case considered in [19] is pretty different from the mechanism of vortex ring formation in an impinging jet, where a supersonic jet is perturbed by the incident acoustic wave created by the collision of the jet at the ground, the dependence of the vortex formation on the foundation number in our case seems to be similar. To use \( N^* \) in our derivation, we define the Strouhal number as follows.

\[
St = \frac{f d}{U_j} = \frac{M}{\frac{h}{d} \left( \frac{U_j}{C_a} + \frac{U_j}{C_v} \right)} = \frac{M}{Ma + U_j/C_v} \left( \frac{h}{d} \right)^{-1}
\]

(4.2)

Here, \( d \) is the exit diameter and \( U_j \) are the jet velocity. \( U_j/C_a \) is simply the Mach number of the jet (\( Ma \)). The convective velocity of the large scale vortical structure is around 50% of the main jet speed in [28], hence \( U_j/C_v \sim 2 \). Thus,

\[
St = \frac{M}{Ma + 2} \left( \frac{h}{d} \right)^{-1}
\]

(4.3)

Now, we assume that the circulation introduced by the jet within one period is
contained in one single vortex ring. Since the period $t$ is the inverse of the frequency $f$, the formation number can be written as follows.

$$N_f = \frac{U_j t}{d} = \frac{U_j}{f d} = St^{-1}$$

(4.4)

which implies that the formation number is simply the inverse of the Strouhal number under this assumption. If we extrapolate the result in [19], this leads to the following conclusion: there exists a universal lower bound on the Strouhal number for impinging tones, which we predicted in Ref [19] to 4. Hence, we set $N_f < N^* \sim 4$. Tbl. A.3 contains $N_f$ computed from $St$ for various values of $h/d$ and $M$. The Mach number of the jet was chosen to be 1.5, which is the experimental condition reported in [3]. Note that $N_f$ should be less than $N^*$ in order to contain the circulation introduced by the jet in one single vortex ring. The bold-faced number indicates the largest formation number($N_f$) satisfying this condition for each value of $h/d$. We expect that the dominant impinging tone must occur at this largest formation number, since each vortex ring contains the largest possible circulation at this condition. Tbl. A.4 contains the corresponding frequency in Hz.

The existence of the largest possible Strouhal number seems to be a valid assumption for $h/d$ between 3 and 6 from the data given in [3]. Also, the frequency predicted in Tbl. A.4 seems to be reasonably accurate for the range.

### 4.2 Collision of Two Identical Vortices

The highly unsteady behavior of the jet and the resulting impinging tones is governed by a feedback mechanism, between the instability waves in the jet that originate at the nozzle and grow as they propagate downstream towards the impingement surface, and the acoustic waves that are produced upon impingement which then travel upstream and excite the nascent shear layer near the nozzle exit. The main source for the noise generation of impinging jet is the impingement of the large scale vortical structure which was formed by the evolution of the instability triggered by acoustic wave near the nozzle. In this paper, we model the impingement of the large scale vortical
structure on the wall by viewing it as a head-on collision of two identical vortices. Seen in Fig. B-36(b), the acoustic field of impinging vortex is composed of two acoustic wave equations (i) a wave produced by impingement of jet travels directly toward the nozzle exit, and (ii) the other bouncing from the wall which also travels toward the nozzle. Similar to this, the acoustic field by the colliding vortices is the superposition of two wave equations produced by each vortex in Fig. B-36 (a). If we assume that the acoustic energy of bouncing wave is perfectly conserved while reflecting from the wall, the colliding model is quite analogous to the impinging tone generation mechanism.

In the reference [24], the far field acoustic equation generated by the colliding vortex is

\[
\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right)p = -\rho F(x, t)
\]  

(4.5)

where

\[
F(x, t) = \nabla \cdot L + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \nu^2 + \frac{\partial}{\partial t} \left( \frac{1}{c_p} \frac{D}{Dt} s \right) + \frac{1}{c^2} \frac{\partial}{\partial t} (v \cdot \nabla) \frac{1}{2} \nu^2 - \frac{\nu}{c^2} \frac{\partial}{\partial t} (v \nabla : e) + \frac{4}{3} \nu \nabla^2 (\nabla \cdot v)
\]  

(4.6)

where \(L = \mathbf{\omega} \times \mathbf{v}, s: \) entropy of the system.

This equation is obtained from the basic mass, momentum and energy conservation equation for a viscous fluid, where \(x = (x_1, x_2, x_3), r = |x|.\) Solving the equation using Green function, the acoustic pressure wave \(p\) is expressed as

\[
p(x, t) = \rho \frac{x_i x_j}{c^2 r^3} Q_{ij}''(t - r/c) + \rho \frac{1}{15\pi c^2 r} K''(t - r/c)
\]  

(4.7)

where

\[
Q_{ij}(t) = \frac{1}{12\pi} \int x_i (x \times \omega(x, t))_j dx
\]  

(4.8)

and

\[
K(t) = \frac{1}{2} \int v^2(x, t) dx.
\]  

(4.9)

The properties \(\rho\) and \(c\) are the density and sound speed in the undisturbed medium. The prime denotes differentiation with respect to time. The first term \(Q_{ij}(t)\) is derived as Möhring’s quadrupole, and the second term is the monopole representing
the change in the total kinetic energy. When the vortices are far from each other, 
they are inviscid and the kinetic energy ($K$) is conserved. As they come closer, the 
viscous effect is not negligible and hence neither the kinetic energy nor the vorticity 
is conserved. Taking advantage of the axisymmetric property ($Q_{ij} = Q_{ji}$) of circular 
vortex-lines, the far-field acoustic pressure of equation 4.7 is reduced to the following 
form

$$p(r, \theta, t) = \frac{\rho}{4\pi c^2} \frac{1}{r} \left( \cos^2 \theta - \frac{1}{3} \right) Q''(t - r/c) + \frac{\rho}{15\pi c^2} \frac{1}{r} K''(t - r/c),$$

(4.10)

where $r$ is the distance from the observation point to the center of vortex ring, $\theta$ is 
its polar angle from the z-axis seen in Fig. B-37 and the function $Q(t)$ is defined by

$$Q(t) = \int \int R^2 \omega dR dz,$$

(4.11)

where $z, R$ are the axial and radial coordinate respectively in the cylindrical coordi-
nate system. The factor $\omega dR dz$ in the equation (4.11) can be replaced by $d\Gamma$. Hence 
the appropriate $Q(t)$ form from $N$ discrete vortices can be written as

$$Q(t) = \sum_{i=1}^{N} R_i^2 Z_i \Gamma_i,$$

(4.12)

where $R_i, Z_i$ and $\Gamma_i$ are the radius, axial position and strength of the $i$th vortex 
respectively.

As seen in Fig. B-37, the head-on collision case is composed of two identical circular 
vortices whose common axis coincides with the z-axis with the same circulation but 
opposite direction. Incorporating the property of $R_1 = R_2 = R$, $Z_1 = -Z_2 = Z(> 0)$, 
$-\Gamma_1 = \Gamma_2 = \Gamma(> 0)$, into the equation (4.12). We obtain

$$Q(t) = -2\Gamma R^2 Z,$$

(4.13)
and the corresponding total kinetic energy $K$ is represented as follows

$$\quad K = \Gamma^2 R \left( \ln \frac{8R}{\delta} - \frac{7}{4} \right) - \Gamma^2 \Phi (Z, R). \quad (4.14)$$

The trajectory of $R$ and $Z$ can be calculated from the following equation.

$$\quad \frac{dZ}{dt} = -U_s + \frac{\Gamma}{4\pi R} \frac{\partial \Phi}{\partial R} \quad (4.15)$$
$$\quad \frac{dR}{dt} = -\frac{\Gamma}{4\pi R} \frac{\partial \Phi}{\partial Z}, \quad (4.16)$$

where $R, Z$ and $\Gamma$ are defined above 4.13 and

$$\quad \Phi(Z, R) = \Phi(R; k) = R[(2/k - k)F(k) - (2/k)E(k)] \quad (4.17)$$

with $k = R/\sqrt{R^2 + Z^2}$. The $F(k)$ and $E(k)$ in equation 4.15 are the complete elliptic integral of the first and second kinds respectively.

$$\quad F(k) = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 - k^2 \sin^2 \psi}} \quad (|k| < 1), \quad (4.18)$$
$$\quad E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \psi} d\psi \quad (|k| < 1), \quad (4.19)$$

$U_s$ in the equation (4.15) represents the self-speed of a single vortex ring without interaction

$$\quad U_s = \frac{\Gamma}{4\pi R} \left( \ln \frac{8R}{\delta} - \frac{1}{4} \right), \quad (4.20)$$

where $\delta$ is the effective size of the vortex core which is much less than the ring radius $R$. As $R$ changes, the core is assumed to satisfy the condition $R\delta^2 = R_0\delta_0^2$. Considering the viscous effect, the vortex strength $\Gamma(t)$ is no more conserved and assumed to be given by

$$\quad \Gamma(t) = \Gamma_0 erf \left( \frac{Z}{\epsilon} \right), \quad (4.21)$$
where

\[ c^2(t) = \frac{(R_0/R)}{t} \left( \alpha^2 + 4\nu \tau \right), \]

\[ \tau = \alpha \int_0^t \exp \left( 2 \int_0^t \left| \frac{Z(t)}{Z(t)} \right| dt \right) ds. \]  

(4.22)

Here \( \epsilon \) is the second core size which represents as in equation (4.21), while the first core size \( \delta \) determines the kinematic motion of the vortex center. The impingement model proposed in this paper can therefore be summarized thus: The pressure field is given by Equation (4.10) where the functions \( K \) and \( Q \) are determined by equations (4.15),(4.22).

The Fig. B-38 (a) is the pressure signal \( p_{cycle} \) plot predicted from the equation 4.10 with respect to time. This plot is based on the data simulated at the same position where microphone is located in experimental test which will be mentioned in the next section in detail. \( h/d = 4, \rho = 1.23kg/m^3 \) and with the initial condition of \( \delta_0/R_0 = 0.3 \). Alternately, the impinging tone data can be reconstructed by repeating one cycle of pressure signal seen in the Fig. B-38 (a) to the most dominant frequency \( (f) \) obtained from equation 4.1 with predicted integer \( M \) using the criterion mentioned at the previous section.

\[ p_{reconst}(t) = \sum_{n=1}^{\infty} p_{cycle} \left( t - n/f \right) \]  

(4.23)

where \( p_{cycle} \) is the data obtained from the equation 4.10, \( f \) is the most dominant peak frequency calculated from the equation 4.1, \( n \) is integer. The reconstructed data presented in the Fig. B-38(b) is simulated at \( h/d = 4.0, f = 6048 \text{ Hz} \).

As mentioned in the introduction, the impingement model proposed in our earlier work in reference [3] was accurate in its frequency prediction but poor in its amplitude prediction. The experimental data from the impinging jet shows several distinct peaks (Fig. B-34), at 6, 12, and 18 kHz. In this section, we have presented a model which attributes the noise generation to creation of vortices that collide with the ground. The simplest possible explanation for the peaks in Fig. B-34 using these
colliding vortices is to say that the high amplitude peaks at 6 kHz may be due to the impingement of slow but strong vertical structures, whereas the low amplitude peaks at 12 and 18 kHz may be caused by fast but weak vortices with the vortex intensity given by \( \omega = \nabla \times v \). However, this is not necessarily the case but is instead due to the impingement of a single vortical structure at the dominant frequency of 6 kHz. As shown by the vortex-collision model in Equation (4.10) and Fig. B-38 (a), this impingement causes a pressure time-trace that is not perfectly sinusoidal and hence can produce peaks at multiple frequencies. Indeed this is corroborated by Fig. B-39 which contains the spectral plot of the predicted pressure from the collision model.

Several points should be made in reference of Fig. B-39. First of all, the frequency content shows not only a dominant peak but also harmonics of the frequency which was expected above. Secondly, similar to the experimental result, the amplitude of each peak also tends to decrease at high frequency, which was not predicted by the previous model [3]. Third, the amplitude of the peak matches to the value obtained from experiment. However, the overall amplitude predicted is in general much less than that of experiment data, which is due to the fact that we considered only the impinging vortex as the noise generation source. In reality, the actual signal contains broadband noise which is produced by mixing noise, edge tone and etc. Once we take into consideration of these sound sources, we could obtain closer predictions to the experiment.

### 4.3 A Reduced-order Feedback Model of the Impinging Jet

In this section, we discuss the effect of low frequency pulsing and propose a reduced-order feedback model of the impinging jet that captures these effects.
4.3.1 Identification of A Low Frequency Mode

As noted in the previous section 3.3.3, the amount of noise reduction was independent of the input pulsing frequency in the range of 60 ~ 150 Hz, as shown in Fig. B-32. However, in subsequent experiments, an additional noise reduction was possible by pulsing at low frequencies around 20 Hz. In repeated trials (seen in Fig. B-40), an additional 1 ~ 2 dB reduction was always achieved by low frequency pulsing injection. This indicates the impinging jet flow has a global mode in this low frequency region. This low frequency peak can also be observed in the spectral plot of uncontrolled impinging jet noise, shown in Fig. B-41.

In almost all investigations of the impinging jets, the spectral plot of the uncontrolled impinging jet noise is distinguished by a dominant peak ~ 4.6 kHz which corresponds to the impinging tone (shown in Fig. B-41.(b)). In the higher frequency range (> 4.6 kHz), the harmonics of this impinging tone appear repeatedly since the acoustic fluctuation is not a perfectly sinusoidal shape [11]. Fig. B-41 (a) which is the spectral plot of low frequency band shows a moderate peak at ~ 16 Hz. The plot (FFT size is 1024) was obtained using 20 seconds of microphone data with 2048 Hz sampling rate filtering through 8th order elliptic low pass filter with 1 kHz cutoff frequency. It is quite long period of data to identify the low frequency phenomenon, and the corresponding resolution (which is 2 Hz) is accurate enough to capture the low frequency mode. In addition to the spectral plot, several other evidences also support the existence of low frequency mode. For example, the appearance of low frequency mode depends on the FFT bin size. Fig. B-42 is the spectral plots of pressure and acoustic signals measured from several positions for different sampling rate. At the high sampling case (70 kHz), the resolution of the spectrum is too high (68.3 Hz) to capture the low frequency mode (~ 16 Hz), whereas the low frequency mode is clearly shown in the spectral plot at the high resolution result. Moreover, peaks in the low frequency region are commonly observed in other data measured from the microphone, unsteady pressure fluctuation of ground and lift plate respectively in Fig. B-44. On the other hands, we do not have the low frequency mode in the tare
conditions. The background noise is compared with the actual signal to check the possible noise induced by other electric devices such as motor and power generator. In Fig. B-39, the blue line represents the unsteady pressure signal captured from the ground Kulite while primary jet is impinging on the ground. The red line represents the pure noise collected from the same sensor (ground Kulite) without impinging jet operation. The noise includes possible interference from pulsed microjets, motor, compressor and power generator. Seen in Fig. B-43, we can confirm that there is no distinct peak at the range of 16 Hz. But, another peak is also identified at $\sim 60$ Hz, which is thought to be generated from other experimental devices. The above results prove that the low frequency peak is not due to experimental errors but a meaningful mode that should be considered for model construction. In the following section, we model both the dominant (Rossiter) mode at a high frequency, and the low-frequency mode. The possible mechanism of the low frequency mode will be investigated in the following section in detail.

4.3.2 A Reduced-Order Model
At the region near the nozzle exit, the evolution of shear layer instability $P_{shear}$ to a large scale vortical structure can be modeled using a linear stability theory with hyperbolic velocity profile of main jet. But the determination of transfer function from the shear layer instability $P_{shear}$ to impinging tone of the ground plane $P_{ground}$ based on the underlying physics is far too difficult to build as proposed in the equation 4.5 at the previous section 4.2. This is primarily due to the fact that the acoustic noise on the ground is produced by the process of the vortex annihilation while impinging on the ground [11], which is a very nonlinear phenomenon. We therefore use an alternate, reduced-order, modeling approach using input-output relationships. This model, shown in Fig. B-45, consists of a forward loop and a feedback loop. The forward loop represents the transfer function from $P_{shear}$ to $P_{ground}$ where $P_{shear}$ is pressure fluctuation of shear layer at the nozzle exit and $P_{ground}$ is the acoustic noise on the ground plane. Our goal is to retain the amplitudes at two most dominant frequencies, one being the largest peak at a Rossiter mode, and the second being a
low frequency peak, we choose the transfer function $G_{shear}(s)$ to be of the form

$$G_{shear}(s) = \sum_{i=1}^{N_{mode}} \frac{K_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}$$

(4.24)

where the $\zeta_i$ is the damping ratio, the $\omega_i$ is the natural frequency, and $K_i$ is the amplitude of the $i^{th}$ mode, respectively. $N_{mode}$ is the number of dominant modes in underlying system, which, in this case, is two. The delay $T_a = h/C_a$ represents the convective time-delay between the nozzle and the ground. The feedback loop is represented by the pure time-delay transfer function $e^{-sT_b}$ where $T_b = h/C_a$ is the time taken for the reflected acoustic wave to travel from the ground to the nozzle exit, which in turn excites the shear-layer. The transfer function $G_{shear}(s)$ represents the dominant effects of the shear-layer, and is described in more detail below. The noise input $N$ represents all other input excitation that exist at other frequencies including broad band noise effects. This results in a closed-loop transfer function of the form

$$G_{closed}(s) = \frac{G_{shear}(s)e^{-sT_b}}{1 - G_{shear}(s)e^{-s(T_a+T_b)}}$$

(4.25)

Using these two dominant modes whose natural frequencies are 16 Hz and 4.6 kHz, the model of impinging tone system was built and compared to the experimental data seen in Fig. B-46 (a). The blue lined indicated above is the microphone data measurements and the red line represent acoustic data produced by a model. The part that is not modeled, which is in the range of between 50 and 500 Hz is due to the broadband noise. Because it is a purely aeroacoustic phenomena, we can exclude its effects in consideration of the impinging tone mechanism, and is represented through the external input $N$. It should be noted that a feedback model similar to that suggested in Fig. B-45 was introduced in Rowley et al. [49] in the context of cavity tones.

### 4.3.3 Effect of Pulsed Microjet-Control

The most immediate effect of the pulsed microjets is the alteration of the pressure $P_{shear}$ in the immediate vicinity of the nozzle. The impact of these microjets is the
reduction of the peak amplitude at 4.6 kHz, as shown in Fig. B-46 (a) and (c). Such a reduction is possible only through one of two effects, one of which is via damping, while the other is via a reduction of the input excitation at this frequency. The former case requires the introduction of an input at the same frequency but with a different phase, an evidence of which was not available in the impinging jet. We therefore model the effects of the pulsed microjet-control via an alternation of the input excitation, and is represented via a transfer function $G_f(s)$ in Fig. B-47. This transfer function is chosen as

$$G_f(s) = \frac{Z_1(s)}{Z_2(s)}$$

(4.26)

where the polynomials $Z_1$ and $Z_2$ are such that they result in a redistribution of the input energy. As shown in Fig. B-46 (d) and (f), the uncontrolled system produces an input excitation which has a large amplitude at the impinging frequency of 4.6 kHz. Relatively speaking, with a pulsed microjet, the input, which is now modified due to the presence of the pulsed microjet action, has a much smaller amplitude at the same frequency. It should also be noted that the pulsed microjet increases the input amplitude at the lower frequency. However, due to the fact that the overall flow field is such that the lower frequency has a much higher damping, the pressure response at this frequency is not increased despite the increase at the input. This could be the reason for the effective noise reduction due to the pulsed microjet.

For comparison, the effect of a steady microjet is also modeled using the same transfer functions, the results of which are illustrated in Fig. B-46 (b) and (e). A comparison of Fig. B-46 (d), (e), and (f) shows two facts; first, the same input-shaping effect as in the pulsed microjet is exhibited in the steady microjet in that the input-excitation at the higher frequency is lowered and is increased at the lower frequency. However, this shaping is not as effective as in the pulsed case; compared to the pulsed case, the excitation at the higher frequency is decreased to a smaller extent. This in turn could be the reason why the corresponding noise reduction in the steady microjet case is smaller.
4.3.4 Low Frequency Mode: A Possible Mechanism

The high frequency mode corresponding to 4.6 kHz is produced by impingement of a large scale vortical structure on the ground. It is a well known mode as predicted by many researchers such as Rossiter [47], Neuwerth [36] and Powell [41] in the context of feedback mechanism. However, the presence of low frequency mode (16 Hz) has not been reported hitherto in any investigations. The question then is raised as to what the possible mechanisms are that are responsible for such a low frequency behavior.

In the cavity test by Kegerise et al. [26], a low frequency mode was reported to be present without any distinct physical noise source. For example, it is argued that two Rossiter modes $f_a, f_b$ may produce another peak which corresponds to the sum $(f_a + f_b)$ or difference $(f_a - f_b)$ of these frequencies when the nonlinear interaction between Rossiter modes are prominent. Such a nonlinear interaction may not be the mechanism of the low frequency mode in this problem because the Rossiter modes of the impinging jet under consideration are at a fairly high frequency ($f_a = 4.6$ kHz, $f_b = 6.1$ kHz) and hence both the sum and difference of the two are far greater than the low frequency mode of 16 Hz. Moreover, the spectral plots of experimental data shown in Fig. B-46 (a) ~ (c) indicates that the peaks in the higher frequency region are quite harmonic, which also implies that the nonlinear interaction between Rossiter modes may be fairly weak.

The flapping (/or helical) motion of the impinging jet column is another possible mechanism of low frequency phenomena. Flapping mode of a plane jet (/helical mode of axisymmetric jet) has a relatively slow motion compared to the feedback loop, but the exact frequency of this mode is unknown. Previous research [11] indicates that the uncontrolled flow structure is dominated by helical mode at $h/d = 3.5$, $Ma = 1.5$, which supports the helical motion as a possible noise mechanism. However, in the repeated experiments, the helical structure is found to be switched to the axisymmetric mode once microjets are activated. If the helical motion is the primary source of low frequency phenomenon, the frequency content of the controlled flow should not have the low frequency peak. In fact, low frequency mode in spectral plot
in Fig. B-46 (a) ~ (c) remains unchanged even in the presence of pulsed injection controls the impinging jet. Hence, we can conclude that flapping motion cannot be the possible mechanism of low frequency mode.

We can also attribute the low frequency phenomenon to the structural mode. The vibration of structure could interfere flow field, which, in turn, generate the specific mode recorded as low frequency tone in acoustic data. Seen in Fig. B-48, analysis using simple model calculates the resonance frequency (~ 1 kHz) which is far higher than the low frequency mode. Because this analysis is too coarse, it is not a sufficient evidence to exclude structural mode from low frequency phenomenon. To investigate the structural mode more accurately, empirical tests based on impact response were tried using Kulite sensor. In fact, the Kulite is designed to capture pressure fluctuation on the ground plane, but it is thought to be able to measure structural vibration due to its sensitivity.

Fig. B-49 shows the flow frequency content in the range of (< 10^4) Hz. However, the low frequency content for impact response is almost identical to the reference signal of the situation without any impact. The impact response for different sampling rate also show similar trend to reference signals, which implies that Kulite, the unsteady pressure transducer, may not be appropriate sensor for capturing structural vibration. For accurate measurement of structural mode, motion sensor such as accelerometer is required.

As mentioned at the beginning, the low frequency mode exists in all measurements including acoustic data, unsteady pressure signal of ground and lift. Due to this fact, we can assume that the noise source of the low frequency mode is perhaps located on the ground, and generates acoustic waves that travel toward the lift plate. Hence, a closer investigation of the flow field near the impinging region may be called for.

Using a flow visualization with Particle Image Velocimetry (PIV) technique, the normalized vorticity field was determined and is shown in Fig. B-54. At the center of impinging zone, we can clearly see a pair of vorticity field which is counter directional to that of main jet. In addition to the vorticity field (Fig. B-54), the stagnation bubble near the impinging region is identified in turbulence intensity fields (Fig. B-55, B-
This local vorticity field is produced by the stagnation bubble which is formed by the recirculating flow at the center of impinging region, therefore its presence can be easily identified in the voriticity field. In Fig. B-50, the stagnation bubble is formed by interaction of three different shocks (plate, jet and tail shock), and enclose a region of recirculating fluid with relatively low velocities [1]. Moreover, the bubble is very prominent for an underexpanded impinging jet, and very weak for an overexpanded jet. Because the nozzle was planned to be operating under ideally expanded condition, we cannot expect the strong formation of stagnation bubble. However, the bubble of ideally expanded jet experiences breathing with very low frequency, which is thought to be the low frequency mode in the frequency spectrum.

This stagnation bubble can be the possible noise source because the properties of the bubble satisfies some characteristics in acoustic measurements: (i) the pulsation of this bubble is relatively weak compared to the impingement of vortical structure and hence, the corresponding low frequency peak is much smaller than the high frequency peak, and (ii) the bubble still stays at the center while microjet actuation is applied and hence the low frequency mode doesn’t change while control is on. We therefore speculate that the periodic formation of the stagnation bubble is a possible mechanism of low frequency mode. More careful investigations need to be carried out to confirm it.
Chapter 5

Conclusion

Control of flow noise is extremely important in vehicle propulsion from the points of view of stealth, enhanced lift, and reduced unsteadiness. In this thesis, the problem of noise control of impinging jets was considered using a microjet-array installed along the periphery of the jet-nozzle. Prior investigations revealed that open loop control strategies, where the microjets were fired in a steady manner, were effective in suppressing the jet noise both in the cold and hot jet cases, but was very sensitive to nozzle operating conditions such as the height of the impinging jet from the ground. This provided the motivation for the approach suggested in this thesis, which concerns the active control of supersonic impinging jets. In order to achieve consistent noise reduction for various operating conditions, the first active control method suggested in this thesis uses the ‘Proper Orthogonal Decomposition’ methods. POD determines the most energetic mode of unsteady load near the nozzle exit along the azimuthal direction. By matching the microjet-array’s intensity proportional to this mode, it was able to suppress the dominant mode effectively, and achieve an additional $8 \sim 10$ dB reduction to the open loop control strategy for certain heights. However, the ‘mode-matched control’ strategy also demonstrated nonuniform noise reduction as the height was varied.

The second control method suggested uses a pulsed microjet actuator. Under the same flow rate, the unsteady jet injection that results from such a pulsing action can provide higher momentum than steady injection. As a result, it produces a more
significant impact on the shear layer of the main jet. The pulsing was realized by way of a saw-toothed rotating cap that was incorporated in the lift plate and spinning it so that the saw-tooth periodically blocks and unblocks the microjet flow. Using this method, we were able to pulse the microjet flow up to several hundred Hz.

Systematic experiments showed that the performance of pulsed microjet depends on several parameters. Among these, it was observed that variations in the duty-cycle and pulsing frequency of the pulsing led to a maximum impact. A pulsed microjet with a duty cycle of about 43% achieved the same amount of reduction as a steady microjet with comparable plenum pressure. A pulsed microjet with a duty cycle of about 70% duty cycle led to a much higher overall reduction, and completely destroyed the distinct impinging tones at almost all heights, which is confirmed by acoustic data together with flow visualization too. Moreover, it was also found that low frequency pulsing injection at around 16Hz found to be causing additional 1 $\sim$ 2 dB reduction, which was due to the presence of low frequency mode in the impinging flow field.

In the later part of the thesis, a physical model of impinging tone system is suggested to understand the role of microjet and to search for the optimal strategy using given control parameters. As a first step, a colliding vortex model is suggested. The acoustic field produced by a colliding vortex is very similar to impinging tones since the head-on collision of two identical vortices experiences a similar annihilation process to that of impinging tones. This model successfully predicts the major characteristics of experimental result such as (i) both the dominant peak and its harmonics, (ii) decreasing amplitude at higher frequency, (iii) peak amplitude at dominant frequency.

To enable the development of real-time control, the governing equations of the colliding vortex model is not adequate due to their complexity and nonlinearity. Instead, a reduced-order model with two lumped-parameters was derived based on a low frequency mode (16 Hz) and high frequency Rossitor mode (4.6 kHz). Here, the effect of the rotating cap was included in the form of an input-shaping controller. It was shown that the pulsed microjet control accomplishes noise reduction by extracting
energy from the high frequency region to the low frequencies. A possible mechanism that causes low frequency mode is attributed to a stagnation bubble which was periodically observed in vorticity plot of flow visualization (PIV) to form at the center of the impinging region of the jet.
Appendix A

Tables

Table A.1: The energy content of the first four modes at each height (NPR=3.7, 20° injection). Experiment of the reference [31].

<table>
<thead>
<tr>
<th>h/d</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
<th>Mode4</th>
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<tbody>
<tr>
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<td>0.4615</td>
<td>0.2488</td>
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<td>0.1111</td>
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<tr>
<td>3.0</td>
<td>0.5515</td>
<td>0.2745</td>
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<tr>
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<td>0.0691</td>
<td>0.0443</td>
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<td>4.5</td>
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<td>0.8736</td>
<td>0.0757</td>
<td>0.0314</td>
<td>0.0194</td>
</tr>
</tbody>
</table>
Table A.2: The energy content of the first four modes at each height (NPR=3.7, 30° injection).

<table>
<thead>
<tr>
<th>h/d</th>
<th>Mode1</th>
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<tr>
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<td>0.0461</td>
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Table A.3: The formation number (N_f) corresponding M at each height (h/d) condition

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td></td>
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</tr>
<tr>
<td>3.0</td>
<td>10.500</td>
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<td>3.500</td>
<td>2.625</td>
<td>2.100</td>
<td>1.750</td>
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<tr>
<td>4.0</td>
<td>14.000</td>
<td>7.000</td>
<td>4.667</td>
<td>3.500</td>
<td>2.800</td>
<td>2.333</td>
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<tr>
<td>4.5</td>
<td>15.750</td>
<td>7.875</td>
<td>5.250</td>
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<td>2.625</td>
</tr>
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<td>10.500</td>
<td>7.000</td>
<td>5.250</td>
<td>4.200</td>
<td>3.500</td>
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Table A.4: The frequency (f) corresponding formation number (N_f) at each height (h/d) condition. * denotes the value closest to experimental data

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
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<td>h/d</td>
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<td></td>
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</tr>
<tr>
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<td>1916.01</td>
<td>3832.02</td>
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<td>2554.68</td>
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<td><strong>5109.36</strong></td>
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<tr>
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<td>6897.64</td>
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<tr>
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<td>2874.02</td>
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<td>4790.03</td>
<td><strong>5748.03</strong></td>
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Table A.5: Specification of high speed valve

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<th>Vendor</th>
<th>CleanAirPower</th>
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<tr>
<td>Valve</td>
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</tr>
<tr>
<td>(SP-051)</td>
<td></td>
</tr>
<tr>
<td>Speed</td>
<td>&lt; 200Hz</td>
</tr>
<tr>
<td>Q (@100psi)</td>
<td>115 SLPM</td>
</tr>
<tr>
<td>Precision</td>
<td>± 25 μ sec</td>
</tr>
<tr>
<td>Supply Pressure</td>
<td>&lt; 300 psi</td>
</tr>
<tr>
<td>Driver</td>
<td></td>
</tr>
<tr>
<td>(SD-1)</td>
<td></td>
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<tr>
<td>Response Time</td>
<td>Opening Closing</td>
</tr>
<tr>
<td></td>
<td>3 msec 2 msec</td>
</tr>
<tr>
<td>Mics.</td>
<td>1.6 Ω / 12V(DC)</td>
</tr>
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</table>
Appendix B

Figures
Figure B-1: Schematic diagram of an impinging jet and possible feedback path.

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Transducer position

(a)
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Figure B-11: Experimental result with a 30° microjet injection. Overall sound pressure levels (OASPL) for different control (NPR = 3.7).
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Figure B-17: Rotating cap design.
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42% duty cycle

Hole 74% duty cycle

Figure B-21: Modification of duty cycle by changing hole size.

Figure B-22: Concept of pulsing with phase difference (a) Configuration of lift plate and rotating cap (b) Microjet pulsing in the consecutive stages.
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Figure B-28: Axial turbulence intensity field, h/d = 3.5, (a) No control, (b) Steady injection with $\dot{m} = 0.8 \dot{m}_{100}$ and $P_{\text{supply}} = 77$ psig, (c) Pulsing at $f_{\text{pulsing}} = 121.9$ Hz, $\dot{m} = 0.45 \dot{m}_{100}$ and $P_{\text{supply}} = 77$ psig, (d) Pulsing at $f_{\text{pulsing}} = 121.9$ Hz, $\dot{m} = 0.76 \dot{m}_{100}$ and $P_{\text{supply}} = 140$ psig where $\dot{m}_{100}$ is microjet's mass flow rate under $P_{\text{supply}} = 100$ psig, $d_c = 56\%$
Figure B-29: Axial turbulence intensity field, h/d = 4.0, (a) No control, (b) Steady injection with $\dot{m} = 0.8 \dot{m}_{100}$ and $P_{\text{supply}} = 77$ psig, (c) Pulsing at $f_{\text{pulsing}} = 121.9$ Hz, $\dot{m} = 0.45 \dot{m}_{100}$ and $P_{\text{supply}} = 77$ psig, (d) Pulsing at $f_{\text{pulsing}} = 121.9$ Hz, $\dot{m} = 0.76 \dot{m}_{100}$ and $P_{\text{supply}} = 140$ psig where $\dot{m}_{100}$ is microjet’s mass flow rate under $P_{\text{supply}} = 100$ psig, $d_c = 56\%$
Figure B-30: Noise reduction for different mass flow rate $\dot{m}$, $\dot{m}_{100}$: steady mass flow rate under 100 psig supply pressure (x axis represents normalized mass flow rate $(\dot{m}/\dot{m}_{100})$, y axis denotes noise reduction in dB scale)

Figure B-31: Noise reduction for different duty cycles, $\dot{m}_{100}$: steady mass flow rate under 100 psig supply pressure (x axis represents $h/d$, y axis denotes noise reduction in dB scale)
Figure B-32: Noise reduction for different pulsing frequencies, $d_c = 43\%$ (x axis represents pulsing frequency, y axis denotes noise reduction in dB scale)
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Figure B-36: Acoustic field generated by (a) Head-on collision of two identical vortices and (b) Vortex impinging on the wall
Figure B-37: Diagram of colliding vortices

Figure B-38: The time series of the acoustic pressure (a) Produced by head-on collision of two identical vortices $p_{cycle}$, (b) Reconstructed repeating head-on collision data, $P_{reconstr}$.
Figure B-39: The frequency content of experimental result and prediction. $h/d = 4.0$

Figure B-40: Noise reduction for different pulsing frequencies, $d_c = 56\%$, $h/d = 3.5$ (x axis represents pulsing frequency, y axis denotes noise reduction in dB scale)
Figure B-41: (a) Spectral plot (low frequency region) (b) Spectral plot (High frequency region) of uncontrolled flow, $h/d = 3.5$, NPR = 3.7

Figure B-42: Spectral plot of unsteady pressure and acoustic signal measured at different sampling rate
Figure B-43: Comparison between unsteady pressure measurement on the ground plane and pure noise.

Figure B-44: Spectral plot of (a) Unsteady pressure on the ground plane (b) Unsteady pressure on the lift plate (c) Acoustic noise of microphone. (d) Acoustic noise reduction ($\Delta$dB) by pulsing actuation.
Figure B-45: Block diagram of feedback loop for uncontrolled impinging jet

Figure B-46: Comparison of experimental data with analytic model (a) No Control case (b) Steady injection case (c) Low speed pulsing (16.4 Hz). Input signal to plant (d) No Control case (e) Steady injection case (f) Low speed pulsing (16.4 Hz).
Figure B-47: Block diagram of feedback loop for controlled impinging jet by low speed pulsing
\[ F = k \delta \]

\[ f_{\text{resonance}} = \sqrt{\frac{k}{m}} \]

Figure B-48: Analytical investigation of resonance frequency

Figure B-49: Impact test
Figure B-50: Schematics of impinging region.[1]

**Axial Mean Velocity ($U$) – ($h/d = 3.5$)**

Figure B-51: Axial mean velocity, $h/d = 3.5$
Axial Mean Velocity \((U)\) – Impingement Region \((h/d = 3.5)\)

Figure B-52: Axial mean velocity in the impinging region, \(h/d = 3.5\)
Radial Mean Velocity (V) – (h/d = 3.5)

Figure B-53: Radial mean velocity, h/d = 3.5
**Vorticity** \((\omega d/U_j) - (h/d = 3.5)\)

Figure B-54: Vorticity field, \(h/d = 3.5\)
Axial Turbulence Intensity \( \left( \sqrt{\frac{u'^2}{U_1}} \right) \) – (h/d = 3.5)

Figure B-55: Axial turbulence intensity field, h/d = 3.5
Radial Turbulence Intensity ($\sqrt{\nu^2 / U}$) – (h/d = 3.5)

Figure B-56: Radial turbulence intensity field, h/d = 3.5
Turbulence Kinetic Energy \( (\sqrt{u'^2 + v'^2} / U) \) – (h/d = 3.5)

Figure B-57: Turbulent kinetic energy, h/d = 3.5
**Reynolds Stress** \( \left( \frac{u'v'}{U^2} \right) \) – \( h/d = 3.5 \)

Figure B-58: Reynolds stress, \( h/d = 3.5 \)
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