Default and Renegotiation: Financial Structure and Incentive in 
Public-Private-Partnership Contracts
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Abstract
This research explores the functional relationship between financial structure and agent’s 
cost-reducing effort in the realm of public-private-partnership (PPP) contracting. I consider a 
canonical contracting problem where incomplete financial provisions are used to govern the 
execution of a project that involves uncertain cost. The existing literature shows that debt (as 
opposed to external equity) is the better financing alternative in terms of effort induction. I 
show, on the other hand, that internal equity (as opposed to debt) is the better financing 
alternative for effort induction when the parties are allowed to achieve ex post Pareto 
 improvement through self-enforcement, renegotiation, and replacement. Under the 
assumptions that the consumer surplus is always greater than the realized cost, and that the 
social cost of public funds (the tax rate) is greater than the private agent’s cost of capital, I 
show that:

(1) Ex ante capital structure and control rights regime jointly determine the equilibrium game 
form of the contract (the agent’s effort level and the uncertainty of cost don’t matter).

(2) The agent’s optimal cost-reducing effort is determined by a hold-up factor and a self-
 enforcement factor. The former is a strict disincentive to the agent’s effort, whereas the 
latter can be an incentive or a disincentive to the agent’s effort depending on the capital 
structure of the contract.

(3) For any given initial capital investment, the agent’s optimal cost-reducing effort decreases 
with the magnitude of debt.

(4) For any given initial capital investment, the agent’s optimal cost-reducing effort increases 
with the magnitude of a performance bond when the contract is one with no positive self-
 enforcement effect.

The theoretical result is consistent with the conventional wisdom that internal equity and 
performance bond generally enhance the agent’s devotion to a PPP contractual relationship. 
This research also contributes a (subjective) PPP contract valuation method that takes into 
account the underlying agency problem.

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# Table of Contents

List of Figures ..................................................................................................... 8  
List of Tables ........................................................................................................ 9  

Chapter 1 Introduction .......................................................................................... 11  
1.1 The Structure of a Typical PPP Project .......................................................... 11  
1.2 PPP Financial Contract and Default ............................................................... 12  
1.3 The Agent’s Cost-reducing Investment Problem .............................................. 13  
1.4 Research Questions ....................................................................................... 14  
1.5 Research Framework .................................................................................... 15  

Chapter 2 Literature Review ................................................................................. 16  
2.1 Agency Issues Underlying PPP Contracting .................................................. 16  
2.2 Mechanism Design and Optimal PPP Contract ............................................. 17  
2.3 The Relationship Between Contracting and Renegotiation ........................... 18  
2.4 PPP Financial Contracting ............................................................................ 20  

Chapter 3 The Model of PPP Financial Contracting ............................................. 22  
3-1 Model Setup ................................................................................................. 22  
3-2 Full-recourse Debt Contract ....................................................................... 25  
3.2.1 Parties’ Payoffs in the Four Scenario Outcomes ...................................... 26  
3.2.2 The Break-even Costs That Separate the Four Scenarios ....................... 31  
3.2.3 Financial Structure and Contract Type .................................................... 33  
3.2.4 Parties’ Expected Payoffs with the Two Contract Types ......................... 33  
3.2.5 Agent’s Optimal Cost-reducing Effort ..................................................... 36  
3.2.6 Financial Structure and Agent’s Effort Incentive .................................... 37  
3-3 Non-recourse Debt Contract .................................................................... 39  
3.3.1 Parties’ Payoffs in the Four Scenario Outcomes .................................... 39  
3.3.2 The Break-even Costs That Separate the Four Scenarios ...................... 45  
3.3.3 Financial Structure and Contract Type ...................................................... 47  
3.3.4 Parties’ Expected Payoffs with the Five Contract Types ......................... 47  
3.3.5 Agent’s Optimal Cost-reducing Effort ..................................................... 55  
3.3.6 Financial Structure and Agent’s Effort Incentive ................................... 58  
3-4 Numerical Example ...................................................................................... 59  
3.4.1 Full-recourse debt contract ................................................................. 59  
3.4.2 Non-recourse debt contract ................................................................. 64  

Chapter 4: Case Study: The Taiwan High-Speed Rail Project ............................. 78  
4.1 The THSR Project ......................................................................................... 78  
4.2 The Analysis of the THSR Contract .............................................................. 81  

Chapter 5 Summary and Future Research Directions ......................................... 83  
5.1 Research Summary ...................................................................................... 83  
5.2 Future Research Directions ........................................................................ 94  

Appendix ............................................................................................................. 96  

Reference ............................................................................................................ 98
List of Figures

Figure 3-1 The timeline of the Model ............................................................ 22
Figure 3-2 The parties' decision rules under a given contract \((I_0, D_0, R_0, H)\) ........ 24
Figure 3-3 The parties' payoffs with a Type F-I contract .................................. 34
Figure 3-4 The parties' payoffs with a Type F-II contract .................................. 35
Figure 3-5 The parties' payoffs with a Type N-I contract .................................. 48
Figure 3-6 The parties' payoffs with a Type N-II contract .................................. 50
Figure 3-7 The parties' payoffs with a Type N-III contract ................................. 51
Figure 3-8 The parties' payoffs with a Type N-IV contract .................................. 52
Figure 3-9 The parties' payoffs with a Type N-V contract .................................. 54
Figure 3-10 The agent's optimal effort with a F-I contract ................................. 60
Figure 3-11 The agent's expected utility with a F-I contract ................................. 60
Figure 3-12 The government's expected utility with a F-I contract ......................... 61
Figure 3-13 The agent's optimal effort with a F-II contract ................................. 62
Figure 3-14 The agent's expected utility with a F-II contract ................................. 63
Figure 3-15 The government's expected utility with a F-II contract ......................... 63
Figure 3-16 The agent's optimal effort with a N-I contract .................................. 65
Figure 3-17 The agent's expected utility with a N-I contract .................................. 66
Figure 3-18 The bank's expected utility with a N-I contract .................................. 66
Figure 3-19 The agent's optimal effort with a N-II contract .................................. 67
Figure 3-20 The agent's expected utility with a N-II contract .................................. 68
Figure 3-21 The bank's expected utility with a N-II contract .................................. 68
Figure 3-22 The agent's optimal effort with a N-III contract .................................. 69
Figure 3-23 The agent's expected utility with a N-III contract .................................. 70
Figure 3-24 The bank's expected utility with a N-III contract .................................. 70
Figure 3-25 The government's expected utility with a N-III contract ......................... 71
Figure 3-26 The agent's optimal effort with a N-IV contract .................................. 72
Figure 3-27 The agent's expected utility with a N-IV contract .................................. 73
Figure 3-28 The bank's expected utility with a N-IV contract .................................. 73
Figure 3-29 The government's expected utility with a N-IV contract ......................... 74
Figure 3-30 The agent's optimal effort with a N-V contract .................................. 75
Figure 3-31 The agent's expected utility with a N-V contract .................................. 76
Figure 3-32 The bank's expected utility with a N-V contract .................................. 76
Figure 3-33 The government's expected utility with a N-V contract ......................... 77
List of Tables

Table 3-1 Accounting sheet for the “No-reinvestment” scenario in a full recourse debt contract.................................................................................................................................26
Table 3-2 Accounting sheet for the “Self-enforcement” scenario in a full recourse debt contract ........................................................................................................................................27
Table 3-3 Accounting sheet for the “New Agent” scenario in a full recourse debt contract.................................................................................................................................28
Table 3-4 Accounting sheet for the “New Contract” scenario in a full recourse debt contract.................................................................................................................................29
Table 3-5 Accounting sheet for the “No-reinvestment” scenario in a non-recourse debt contract........................................................................................................................................40
Table 3-6 Accounting sheet for the “Self-enforcement” scenario in a non-recourse debt contract ........................................................................................................................................41
Table 3-7 Accounting sheet for the “New Agent” scenario in a non-recourse debt contract.................................................................................................................................42
Table 3-8 Accounting sheet for the “New Contract” scenario in a non-recourse debt contract.................................................................................................................................43
Table 4-1 The Planned and the Resulting Financing Sources of THSR.........................................................81
Chapter 1 Introduction

The negotiation of terms of financial contracts has long been one of the most critical elements in the negotiation of public-private-partnership (PPP) contracts. Financial contracts specify revenue, funding sources, contingent financial claims, control rights of the underlying assets and serve as a risk-sharing mechanism. The existing PPP literature is strongly focused on (1) contract design problems such as moral hazard and adverse selection, and (2) property rights allocation problems that compare efficiencies of PPP contracts and traditional procurement contracts. The relationship between financial contracts and the agent’s incentive for cost-reducing efforts is limited explored and our understanding of this issue remains unclear.

The objective of this research is to establish the functional relationship between financial structure and agent’s effort. Given that the agent’s level of effort affects the project’s risk profile and in turn determines each party’s payoff under the specified risk-sharing mechanism, this issue is of critical importance. Through analysis of the agent’s cost-reducing decision-making problem, this research contributes a (subjective) contract valuation method that takes into account the underlying agency problem.

In this section I present the basic structure of a typical PPP project, the major components of a typical PPP financial contract, and the agent’s cost-reducing decision-making problem under a given PPP financial contract.

1.1 The Structure of a Typical PPP Project

A PPP project is a public project that is financed, developed and operated by a private entity for a contracted period of time. A PPP project typically involves (1) a government who offers the concession, (2) a joint venture consortium who funds an executive entity to develop and operate the project, (3) financial institutions who provide debt funding to finance the project, and (4) independent 3rd parties who audit the project and enforce the contract.

The joint venture consortium funds the executive entity in the form of equity and is therefore the de facto owner of the project. Given the aligned interests between the joint venture consortium and the executive entity, hereafter we will use an aggregate entity of “the agent” to represent these two parties.

The basic structure of a typical PPP project is as follows:

(1) Agent Selection

A PPP project starts with an agent selection session in which each would-be agent submits to the government a detailed proposal documenting the financial structure, technology plans, and professional qualifications with respect to the desired concession. The government reviews each proposal and awards the concession to the bidder who offers the optimal proposal. The criteria for the selection of optimal proposal could be the length of concession period, the amount of cost, the amount of equity, technology, or professional qualification of the bidder depending on the characteristics of the project.
After an agent is selected, the involving parties will negotiate a contract set that normally comprises a technology contract and a financial contract. The technology contract specifies the technology that will be implemented and the technical specifications that should be met, while the financial contract specifies the magnitude of equity from the joint venture consortium, the magnitude of debt from the bank, the regulated revenue from the consumer to the agent after the project starts operating, the duration of the concession, and other contingent provisions that should be followed upon contingencies.

After the contract is signed, a trust is normally set up in the debt-issuing bank to manage the cash flow of the project. The trust serves as a mean for the parties to monitor the cash flow and to enforce the financial contract.

(2) Development

In the development stage, as the agent incurs expenses from development costs, he submits invoices to the trust for reimbursement from capital held in escrow. Interest expenses incurred during the development period are normally capitalized, thus increasing the debt outstanding to the banks. The private agent has the incentive to minimize the length of the development period, not only to avoid the additional interest cost, but also to generate revenue as early as possible.

(3) Operation and Transfer

During the operation period, all revenue generated from the project flows through the trust and is used to service the liability. Residual revenues are paid to the agent in the form of dividends. Depending on the maturity profile of the debt, the agent may receive dividends across the entire operating period, or he may not be paid before debt is fully liquidated.

At the end of the concession period, the ownership of the project reverts back to the government, and yields little or no salvage value to the agent. The contract for the concession can also be renewed upon mutual agreement.

1.2 PPP Financial Contracting and Default

A PPP financial contract typically comprises the specifications of revenue, equity, debt, contingent provisions, and the control rights regimes of the underlying asset. Note that even though the revenue is under regulation, the contract cannot fully control the agent’s revenue in some specific types of project. For example, while utility contracts often specifies a lump some revenue that the government would pay the agent for generating a certain amount of electricity, in many toll road projects the contracts simply specify the toll fee and the agent’s profit is to a substantially degree determined by the demand elasticity of the consumers.

Debt and equity constitute the financial source of the investment for the project. While equity financing can be raised through internal financing or external financing, debt is always financed through 3rd party financial institutions. The control rights of the underlying asset are normally specified with the debt contract and can appear in the following two formats:

a. Full-recourse debt: When the agent defaults, the government pays off the debt and assumes the control rights of the underlying asset.
b. Non-recourse debt: When the agent defaults, the bank forgoes the debt and assumes the control rights of the underlying asset.

There exist two conditions for a PPP agent to default: (i) In the development stage, the agent defaults if the development cost is greater than the initial capital investment\(^1\), and (ii) in the operation stage, the agent defaults if the profit generated from the project is insufficient to service the debt.\(^2\)

When default occurs, the entity that assumes the control rights of the underlying asset can always hire a new agent to complete the project. Note that if the technology required to develop the project involves proprietary technologies, then the new control right owner may incur a high transfer cost for bringing in a new agent. In such cases, it will be at the new control right owner’s interest to renegotiate with the incumbent agent such that the two parties can share the surplus generated from the cost to bring in a new agent.

When a project exhibits substantial risks of default, the bank may impose additional terms to protect their loan. For example, the bank may either set an upper bound for cost, or require that the present value of the project be above the debt value. If these conditions fail to hold, the agent will be claimed to have defaulted the contract.

Contingent claims refer to financial clauses that are commonly used by the involving parties to protect their interests in the event of default. Among all, a performance bond is a very commonly used contingent claim in PPP contracting. A performance bond is a bond given to the recipient against loss in case the term of the contract is not fulfilled. In the realm of PPP contracting, the issuer of the bond is normally the agent, while the recipient of the bond is often the government.

1.3 The Agent’s Cost-reducing Investment problem

When a project exhibits cost uncertainty and, as a result, the probability of default exists, the agent has to make his cost-reducing investment decision taking into account the possible consequences of default. For a PPP contract, there exist three scenario outcomes that can occur in the event of default:

(1) Self-enforcement: The agent decides to make up the capital shortfall with his own capital and complete the project. Reinvestment is justified if the guaranteed revenue is high, or if the agent is fear of losing the performance bond.

(2) New contract: the parties can renegotiate a new contract if they are mutually benefit from maintaining the incumbent concessionship (probably due to a high transfer cost for bringing in a new agent). The parties who agree on the new contract have to make up the capital shortfall to complete the project. In other words, it can be the government, or the agent, or the bank, or a combination of all the three of them to contribute the new investment. Any new investment made by the bank and the agent has to be justified by a

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\(^1\) The depletion of capital is not a necessary condition for default to occur. Since effort is not verifiable, default could happen when funding is still sufficient but the agent expects an adverse outcome for the project (For example, the Taiwan High Speed Rail project).

\(^2\) In this research our major concern is the defaulting in the development stage. The problem of defaulting in the operation stage is not included in our analysis.
positive net payoff. In some cases, the government compensates the agent’s reinvestment by extending the concession period.

(3) New agent: The control rights owner seizes the underlying asset and completes the project with a new agent. This scenario happens in the full-recourse debt regime when the magnitude of the performance bond is high, or when the new agent’s cost is low. It happens in the non-recourse debt regime when the guaranteed revenue is high, or when the new agent’s cost is low.

The possibility of default and the subsequent renegotiation naturally gives rise to a hold-up concern. Hold-up refers to a condition in which an agent in a contractual relationship undertakes with respect to the first best investment in the early stage if he foresees that he will not be able to enjoy the payoff from his investment at a later stage. Hold-up applies only to relationship-specific investments for such investments decrease the agent’s credible threats of no trade and in turn increases the counterpart’s bargaining payoff (under the assumption that the agent has no monopoly bargaining power).³

The agent’s cost-reducing investment problem is as follows: Under a given financial contract, the agent determines his cost-reducing effort to maximize his expected payoff (given his prior probability judgments about uncertain costs) taking into account the play out of the above three scenarios and the scenario of no default. It is clear that our focus must be on how financial structure affects the hold-up effect. This decision analysis problem will be formalized in chapter 3.

1.4 Research Questions

The objective of this research is to answer the following three questions:

(1) Whether or how the financial structure of a PPP contract determines the possible scenario outcomes that can occur?

(2) Whether or how the financial structure of a PPP contract (including capital structure, performance bond, and the control rights) affects the agent’s optimal cost-reducing investment decision?

(3) Whether or how the financial structure of a PPP contract (including both capital structure, performance bond, and the control rights) affects the parties’ expected payoff?

(4) Whether or how the technology involved in a PPP project affects the agent’s optimal cost-reducing investment decision and the parties’ expected payoffs?

It will be shown in chapter 3 that when the agent’s effort level is unverifiable, a formal model using analytical approach can only address question (1) and (3). Question (2) must be addressed with numerical methods.

³ It is generally known that when there exist relationship-specific investments, ex ante efficiency cannot be achieved without a contract. When the contract has symmetric information, the involving parties can always construct a revelation mechanism to achieve the first best investment when they have full commitment of no default.
1.5 Research Framework

The framework of this research is as follows:

(1) Justification of analytical approach: I review the current literature on PPP contracting and show specifically why analyzing default is the sensible way to establish the functional relationship between financial structure and agent’s effort incentive.

(2) Theoretical model: A theoretical model is developed to establish the functional relationship between financial structure and the agent’s effort incentive. The theoretical model consists of two parts: the first part studies full-recourse debt contracts. The second part studies non-recourse debt contracts.

(3) Numerical example: Given the cost uncertainty embedded in the model, it is not possible to derive an analytical solution for the agent’s optimal effort. As a result a numerical example is used to demonstrate the financial structure’s effect over the agent’s optimal effort and the participating parties’ expected payoffs.

(4) Case Study: I relate my theoretical result to real world practice by examining the performance of a particular PPP contract (The Taiwan High Speed Rail contract).

It is with no doubt that this research can at best address only a subset of the problems underlying PPP financial contracting. I finalize this research by summarizing the research outcome and proposing the directions for further researches.
Chapter 2 Literature Review

This chapter presents a survey on PPP contracting literature. The disciplines most relevant to our problem are regulatory economic and contract theory. I start by reviewing the mechanism design literature on regulated contract design. I then review the literature on incomplete contract theory.

2.1 The Agency Issues underlying PPP Contracting

In the realm of PPP contracting, while the government can regulate the revenue, she cannot directly control the cost. Cost overruns are common in public projects. Scherer (1964) reports that the realized costs exceed the initial cost estimates by 220% on the average in the defense procurement cases he studied. It is well recognized that the agency problem plays an important role in determining the magnitude of a cost overrun.

One agency-related explanation for cost overruns is "lock-in": In a multi-stage long-term project run by a series of short-term contracts, it may pay for the firm to sacrifice short-term profit in the early stage in order to create a lock-in relationship with the government and derive rent in the latter stage. According to Scherer (1964, p.156):

"Contractors commonly submit unrealistically low cost estimates. This practice may require accepting a low realized rate of profit on development work, but contractors have generally been willing to sacrifice initial development contracts profit for the chance of earning substantial production follow-on contract profits once they are locked together with the government in a relationship analogous to a bilateral monopoly."

Farrel and Shapiro (1989) analyze optimal contracting with a lock-in feature. They consider a buyer who incurs a setup cost and a seller who chooses an unverifiable qualitative parameter as the trading attribute. They show that when setup costs are observable, first best can be achieved even though contracts cannot enforcably specify quality. When setup costs are unobservable, long term price contracts outperform short-term contracts.

The lock-in phenomenon is termed "buying-in" when several firms are bidding in the first stage of a project and the winner of the bid is likely to become the sole contractor in the following stages. Marshall (1989), and Riordan and Sappington (1989) report that buying-in is especially common in public procurements that involve multi-stage processes.

Hold-up is clearly another agency-related source of increased project costs. Croker and Reynold (1989), Gilbert and Newbery (1988), and Salant and Woroch (1988, 1991) argue that a repetition of relationship between the government and the agent as well as the relationship between the government and other regulated agents may substitute for long term contracts and guarantee appropriate investment. This argument is based on the assumption that the government’s reputation for not expropriating the agent’s ex ante investment is critical for inducing social optimal investment in new contracts.

Following this thread, Hart and Holmstrom (1987) propose a reputation model that determines the extent of underinvestment as a function of the length of the relationship between a supplier and a buyer. Lewis (1986) shows that when the government’s net value for the project increases
over time, the agent has less incentive to keep as a low-cost operator as the continuation of the project is less in question.

Laffont and Tirole (1993) point out three other possible sources of cost overruns: The first comes from order changes. This phenomenon only happens when technology adjustments occur and involve more often upgrading than downgrading or simplification. Order changes do not cause agency problems and are less a concern in economic theory. The second source comes from the contract supervisor’s encouragement of cost underestimation in order to get support from a government or congress.

There is also a long tradition of regulated firms paying excessive prices to affiliated, unregulated companies for various inputs, including investments (see Kahn 1970, vol I, pp.28-30). For instance, the relationship between AT&T and its equipment supplier, Western Electric, which is also its subsidiary, raised these concerns. Melumad (1990), and McAfee and McMillan (1990) show that it is generally not possible to generate a socially optimal contract when the government cannot measure the payment the agent makes to his subcontractors.

2.2 Mechanism Design and Optimal PPP Contract

Traditionally, the studies on optimal PPP contract design have been rooted in the theory of mechanism design. Maskin and Moore (1999) state that the motivation of mechanism design is to understand “Whether is it possible to design a game form (also call a mechanism) whose equilibrium outcomes are assured of being optimal with respect to some given criterion of social welfare?”

The seminal paper on mechanism design under adverse selection is Mirrlees (1971). While Mirrlees’s principal concern is optimal taxation, his theoretical framework has been carried over to wide array of economic applications. The theory was further developed by Mussa and Rosen (1978), Green and Laffont (1979), Baron and Myerson (1982), Guesnerie and Laffont (1984), and Maskin and Reily (1984) to study contracting problems.

Loeb and Magat (1979) were the first paper to use mechanism design to examine a regulated contract. They consider a special case in which there is no social cost for leaving rent to the agent. They show that it suffices to award the agent the entire net consumer surplus to induce a first best investment. This conclusion is, however, in conflict with the conventional wisdom that leaving rent to the agent reduces social welfare. Sappington (1982) introduces the notion of social cost of public funds to study regulated contract design. Baron and Myerson (1982) study the adverse selection problem and show that the socially optimal price the agent is allowed to charge the consumer exceeds its Ramsey level when the government cannot observe the agent’s cost. Chiang (2002) studies a multi-principal moral hazard problem and shows that the power of incentive for a risk-averse agent decreases by n-fold when the number of principal increases from 1 to n.

Laffont and Tirole (1993) unify the moral hazard and the adverse selection problems in a single model. They show that a socially optimal contract menu leads the agent to invest efficiently and derive a positive rent, whereas the inefficient agent under-invests and derives no rent. In fact, the ability of the efficient type agent to mimic the inefficient type agent forces the government to give up rent to the efficient type agent if the government wishes to have an active inefficient type agent.
Mechanism design literature normally considers a linear payoff contract that comprises a fixed payoff component and a performance-based payoff component. The linear contract can be replicated with a combination of the two incentive schemes commonly used in regulated contracts: price cap, and cost of services. The former scheme yields more power incentive, whereas the latter scheme yields lower power incentive.

On empirical front, Mathios and Rogers (1989) look at prices of intrastate telephone services in the United States and show that states that have adopted price-cap regulation have, on average, lower rates than those that have stuck to cost-of-service regulation.

The selection of high-powered incentives schemes by defense contractors that have favorable information is reported by Scherer (1964, p227), who writes:

“When contractors believe that cost targets will be tight (i.e. when there is a overrun bias, they bargain successfully for low-sharing proportion and high price ceiling, while when loose targets (i.e. an underrun bias) are expected, they accept a high share of overrun and (more likely) underruns and relatively low price ceiling. Or when relatively high sharing proportions are agreed upon in advance of cosr negotiations, contractors hold out for pessimistic cost targets.”

Scherer (1964) also documents that the high-power incentive schemes are correlated with better performance types.

2.3 The Relationship between Contracting and Renegotiation

Mechanism design literature has so far developed many sophisticated mechanisms that ensure Pareto equilibrium when a contract exhibits complete information. However, these finding can be challenged for two reasons:

First, in the real world most contracts are far simpler than optimal contracts derived through mechanism design. For this reason some researchers study the conditions under which the optimal complex contracts can be simple contracts. Huberman and Kahn (1988), for example, show that, given certain strategic concerns, an optimal complex contract can be replaced by a simple contract with renegotiation at a later stage.

Second, when a mechanism is adopted, an agent is presumably interested in achieving Pareto optimal outcome for each possible state of nature. However, in conditions where an out-of-equilibrium phenomenon occurs, it may not be in the parties’ best interests to stick to the original contract when there exist efficient alternatives that they mutually prefer. It is then reasonable for these agents to forgo the original contract and renegotiate to the Pareto outcome. Traditional mechanism design literature normally assigns high penalties to out-of-equilibrium strategies as a way to deter deviations, and in turn eliminate the possibility of renegotiation. This approach essentially reduces social welfare since this class of contracts should satisfy additional renegotiation-proof constraints.

It is then important to examine the issue of renegotiation. Chung (1991) and Aghion, Dewatripont, and Rey (1994) argue that the ex post bargaining game can be designed at date 1. Consider, for example, a trade that only involves seller’s investment. The first best contract can be easily achieved if the contract can endow the seller with all the bargaining power at date 2. However, for such predetermined bargaining games to work, a higher degree of verifiability is needed, so
that the court can enforce the ex post renegotiation. Schmitz (2001) argue that writing a null contract and designing a renegotiation procedure is no different from writing a complete contract.

Grossman and Hart (1986) and Hart and More (1990) propose an incomplete contract theory to study firm’s integration problems. Contrary to traditional complete contract theory which places emphasis on design of a mechanism that realizes Pareto equilibrium at each state of nature, incomplete contract theory features a contract that specifies the control rights of certain assets, with other performance-related terms being left incomplete. Given that the ex post trading decision becomes contractible through renegotiation, ex post efficiency can be achieved. The incomplete contract approach is then able to analyze agents’ ex ante investment decisions since these investment levels are determined by the incentive from the ex post payoff.

The incomplete contract approach proposed by Grossman and Hart (1986), and Hart and More (1990) is appealing for several reasons: (1) Ex post efficiency can always be achieved and the constraints that mechanism design theory imposes on ex post off-equilibrium outcomes can be removed. (2) Incomplete contract theory has implications for the economics of control rights. (3) The default option (though this outside alternative only serves as a threat point and will not necessarily be exercised) provides a link between the market and the contract.

In reality, it is practical to argue that essentially all real world contracts are incomplete, given that parties in the real world are often unable to specify a complete contingent plan to govern the execution of the contract. However, the contract featured in the incomplete contract approach is more incomplete (from traditional complete contract theorists’ view) in the sense that parties’ decisions in this type of contract are not governed by a control mechanism, but rather they are motivated by the ex post payoff from the renegotiation game (which normally lacks a directly enforcing power ex ante).

Hart and Moore (1999) elaborate the rationale for parties to sign incomplete contracts. They propose that when the number of the states is too large, it would be prohibitively expensive for the involving parties to write a complete contract. As a result, the two parties would simply write an incomplete contract. After the state of nature is realized, they will renegotiate the contract, since by then they will know the realized state of nature and hence the right good to be delivered.

However, some researchers question the theoretical robustness of the incomplete contract approach. One critical conclusion from the classical Hart and Moore’s (1986) analysis points out that a party’s incentive to invest strengthens as his control over the production asset increases. This result is derived by assuming that agents renegotiate following the Nash bargaining rule. DeMeza and Lockwood (1998) and Chiu (1998), however, point out that this result depends crucially on the exact nature of the renegotiation game; it is not certain whether or not it can be endogenously designed ex ante. They consider the so called “deal-me-out” bargaining game. In such a game, each party receives half of the gains from renegotiation, except in the case when one party receives a payoff less than his outside option payoff. Upon such condition, the worse-off party would go for his outside payoff, while the other party is the residual claimant. Under such a bargaining rule, a party’s incentive to invest may sometimes be strengthened when he loses asset ownership. The reason is simple: if party i owns the asset but his payoff from an outside trade is larger, he will default on the contract. On the other hand, party j, as the residual claimant of the total surplus, will have an incentive to invest provided that his investment does not constitute at least half of the ex post total surplus and exceeds party j’s outside trade payoff.

Schmitz (2001) criticizes the incomplete contract approach for its inability to predict an optimal contract. Let a certain allocation problem be given. If a researcher takes only contracts C_1 and C_2
into consideration and conjectures that \( C_1 \) is optimal, one cannot be sure that there is no superior contract \( C_3 \). It is therefore critical to justify why he considers only \( C_1 \) and \( C_2 \). However, if one can prove that there is no contract superior to \( C_1 \) and \( C_2 \), then he is back in the world of complete contracts, since such a proof requires the assumption of perfect rationality.

Maskin and Tirole (1999a) point out that for the incomplete contract approach to work, agents should be able to foresee the expected payoff from ex post renegotiation, so they can conduct dynamic programming to determine their optimal ex ante investment decision. This assumption endows the agent with perfect rationality. They then argue that, if agents possess perfect rationality, they should be able to construct a complete contract mechanism to govern the contract. They therefore conclude that the informal justification for contractual incompleteness based on ex ante indescribability of actions of trade is unconvincing. They further construct a series of propositions showing that as long as an agent's expected utility is quasi-linear in payoff, they can forgo considering unverifiable states of nature and design a mechanism that allows renegotiation to close the trade.

Incomplete contract proponents reject this criticism. For such a mechanism to work, it should not be renegotiated ex post. Further, Hart and Moore (1999), and Segal (1999) describe a bilateral monopoly environment with many potentially tradable goods. They show that, when the number of goods tends to infinity, the buyer and the seller cannot gain from a contractual relationship.

It is generally believed (Maskin and Tirole(1999a,b), Hart and Moore(1999), Brousseau and Glachant(2001) for example) that given the prevalence of simple contracts in the real world, it would be unreasonable to neglect incomplete contracts. Further, incomplete contracts may reside in a realm where agents possess weaker-than-perfect rationality.

2.4 PPP Financial Contracting

The capital structure problem remains a working issue for financial economists. What is the optimal capital structure? Is debt or equity is the better financial instrument for a given firm or a project? After Modigliani and Miller's (1958, 1961) two propositions showed that capital (debt/equity) structure does not affect a firm's value, the study of capital structure shifted to the agency problem underlying a given financial problem.

The financial structure of a project is determined by the set of financial contracts agreed upon by the parties involved. To analyze the agency problem underlying a given financial structure of a PPP project, it is critical to understand the nature of financial contracts. Aghion and Bolton (1992) argue that financial contracts are essentially incomplete contracts:

"In practice the difficulty in confronting this (allocation) problem arises from the inherent incompleteness of financial contracts. Most investment projects are sufficiently complex that it is impossible for the contracting parties to specify ex ante an action correspondence \( c: \Theta \rightarrow A \) determining which action ought to be taken as a function of the state of nature, \( \theta \). Even if such a correspondence \( c(\theta) \) could be specified it may be difficult to enforce ex post. Consequently, the contracting parties must find roundabout ways of implementing the most desired action-schedule, \( c(\theta) \), such as partial or total delegation of decision rights (over the future action choice) to one or the other party together with an appropriate monetary incentive scheme."
Financial contracts often leave out how agents should act under each specific circumstance unless some certain threat point (i.e. default) is met. A key feature of a financial contract is its emphasis on the allocation of property rights. Zender (1991) argues that debt serves as a mechanism for the contingent allocation of control: the debtor retains the control right of the underlying asset as long as he meets the liability repayment schedule. The creditor possesses the control right to liquidate the underlying asset when debtor fails to meet his obligation.

Aghion and Bolton (1992) show that a property right regime is analogous to a financial structure regime: (1) If it is best to give full control to the investor, the firm should issue voting equity. (2) If it is best to give full control to the entrepreneur, the firm should issue non-voting equity. (3) If it is better to adopt joint ownership, the entrepreneur and the investor should set up a trust or a partnership. (4) If it is efficient to allocate control contingent on the state of nature, the firm should issue ordinary debt, convertible debt, warrants, or convertible preferred stock.

So far, the application of incomplete contract theory to PPP financial contracting remains limited. Dewatripont and Legros (2005) use the incomplete contract framework to examine the external financing effect of PPP contracting. They show that debt works as a better financing security (as opposed to external equity) in terms of effort induction. The rational is simple: while the agent is the residual claimant of his cost-reducing effort, the agent’s ex post payoff from his cost-reducing effort is diluted with an equity contract.

Given that real financial contracts are, in the main, incomplete contracts, it is naturally difficult to structure a complete contract mechanism with pure financial provisions. In other words, when studying an agency problem by examining exclusively the ex post renegotiation game, we will not be able to utilize the revelation principle to devise a mechanism that induces socially optimal effort. Fortunately, we will show in the next chapter that in the realm of PPP contracting, financial structure actually has some monotone effects on the agent’s cost-reducing effort. This provides the information needed for PPP contract design.
Chapter 3 The Model of PPP Financial Contracting

In this chapter a model is developed to study the functional relationship between financial structure and agent’s effort incentive in PPP contracts. I focus on a simple financial structure consisting of equity and a performance bond from the agent, the debt borrowed by the agent from the bank, and an amount of regulated revenue that the agent is allowed to derive at the operation stage of the project. The control right regime is determined by the characteristic of the debt. For full-recourse debt, the government will pay back the debt and assume ownership of the project when the agent defaults. For non-recourse debt, the bank will be entitled to the ownership of the project when the agent defaults.

3.1 Model setup

I consider a project that lasts two periods and has three critical dates. At date 0, the participants of the project (the government, the bank, and the agent) sign a set of contracts specifying the financial structure of the project and the control rights regime of the underlying assets.

After the contract is signed, the agent determines a level of effort to reduce the construction cost of the project. I assume that the total construction cost is ex ante uncertain. The agent’s cost-reducing effort can reduce a specific amount of the cost but it cannot fully determine the final construction cost.

At date 1, the uncertain construction cost is realized. If the cost is lower than the initial capital investment, the incumbent agent completes the project with the existing capital. If the cost is higher than the initial capital investment, three possible scenario outcomes may happen: (1) the agent makes up the capital shortfall and finishes the project, (2) the control right owner takes over the project and hires a new agent to complete the project, and (3) the parties renegotiate a new contract under which the incumbent agent completes the project.

At date 2, the project is completed, the consumer surplus is realized and the parties derived their payoffs in accordance to the contract. The timeline of the model is shown in Figure 3-1.

![Figure 3-1 The timeline of the model](image)
Model notation and the assumptions for the notations are summarized as follows:

\( I_i^j \): Party \( i \)'s equity investment at date \( j \) (here \( i \in \{a, g\} \) where \( a \) refers to the agent, and \( g \) refers to the government; \( j \in \{0,1,2\} \)). It is assumed that at date 0 only the agent invests equity, so \( I_0 \) represents the agent’s date 0 equity investment.

\( D_j \): the debt that the agent borrows from the bank at date \( j \).

\( R_j \): the agent’s date 2 revenue that is agreed to by all parties at date \( j \).

\( X \): the uncertain construction cost. Let \( x \) denote the realized value of \( X \).

Let \( F(\cdot), f(\cdot), \) and \( (x, \infty) \) be respectively the cumulative distribution function, the probability density function, and the support of \( X \).

\( e \): the agent’s cost-reducing effort.

\( \psi(\cdot) \): the cost reduction function.

\( S \): the consumer surplus that is realized from the implementation of the project. \( S \) is assumed to be a fixed value and is known to all parties ex ante.

\( r_a \): the agent’s weighted average cost of capital. Define by \( \alpha = \frac{1}{1 + r_a} \) the agent’s discount rate.

\( r_b \): the bank’s weighted average cost of capital. Define by \( \beta = \frac{1}{1 + r_b} \) the bank’s discount rate.

\( r_g \): the government’s interest rate. Define by \( \gamma = \frac{1}{1 + r_g} \) the government’s discount rate.

\( \delta \): the social cost of public funds (that is, distortionary taxation inflicts disutility \( $(1+\delta)$ to a levy of $1 by the state).

\( f \): the incremental proportion of the new agent’s fee to complete the project. In other words, for every dollar the existing agent needs to finish the project, it costs \( (1+f) \) dollars for the control right owner to hire a new agent to finish the project.

\( A1 \) It is assumed that \( f(\cdot) \) is strictly positive in the relevant range.

\( A2 \) It is assumed that \( e \) is (1) expressible in dollars, and (2) unverifiable so uncontractible.

\( A3 \) It is assumed that the cost reduction strictly increases with \( e \) at a strictly decreasing rate:

\[ \psi'(\cdot) > 0 \quad \text{and} \quad \psi''(\cdot) < 0. \]

\( A4 \) It is assumed that \( f > 0 \).

There exist four possible scenario outcomes that can obtain. These scenario outcomes are defined as follows:

1. **No reinvestment:** The initial capital investment is sufficient to cover the construction cost \( (I_0 + D_0 \geq x - \psi(e)) \). The agent finishes the project with existing capital.
2. **Self-enforcement:** The initial capital investment is insufficient to cover the construction cost \( (I_0 + D_0 < x - \psi(e)) \). The agent makes up the capital shortfall, completes the project and derives the revenue specified in the original contract.
3. **New agent:** The initial capital investment is insufficient to cover the construction cost \( (I_0 + D_0 < x - \psi(e)) \). The control right owner takes control of the underlying asset and hires a new agent to complete the project.
4. **New contract:** The initial capital investment is insufficient to cover the construction cost \( (I_0 + D_0 < x - \psi(e)) \). The parties renegotiate a new contract under which the current agent completes the project.
Given these four possible scenario outcomes, the parties' decision rules for any given realization of $x$ can be illustrated as in Figure 3-2.

After the uncertain cost is realized, the parties can reinvest new capital greater than or equal to the capital shortfall. Moreover, the investment made by the agent can be in the form of equity, debt, or a combination of both. Without loss of generality, I make the following three basic assumptions:

(Assumption 1): In the renegotiation game, the required new investment is set equal to the capital shortfall.

(Assumption 2): The agent makes up all his reinvestment with equity.

(Assumption 3): The parties' interest rates satisfy $r_a > r_b > r_x > 0$ (or $0 < a < b < r < 1$).

To simplify our analysis, I make the following four additional assumptions:

(Assumption 4): All ex post renegotiation games are Nash bargaining games. In addition, all parties know ex ante that any ex post renegotiation game is a Nash bargaining game.

(Assumption 5): All parties have common priors about the uncertain cost $X$.

(Assumption 6): All parties are risk-neutral.
(Assumption 7): The contracting game is a one-time game (so no relationship concern).

(Assumption 8): The interest rate for the debt is the bank’s cost of capital. The interest rate for the performance bond is the agent’s cost of capital.

One of our objectives with the model is to examine the efficiency of the contract under different financial structures and control right regimes. We know that the agent’s date 0 effort \( e \) yields a date 1 cost reduction \( \psi(e) \). The net present value of the gain from this cost-reducing effort is thus \( \alpha \psi(e) - e \). The first best effort \( e^* \) is then defined as follows:

(Definition 0): The first best effort \( e^* \) is one that satisfies the first order condition: \( \psi'(e) = \frac{1}{\alpha} \).

In other words, the first best effort is the effort at which the marginal cost equals the marginal gain from the agent’s perspective.

We can now proceed with the analysis. The analysis is done in six steps:

1. Determine parties’ payoffs under the four possible scenario outcomes.
2. Analyze the properties of the break-even costs that separate the four possible scenario outcomes.
3. Determine the possible equilibrium scenario outcomes for any given contract \( (I_0, D_0, R_0) \).
4. Calculate the parties’ expected payoff given their priors on \( X \).
5. Use backward induction to determine the agent’s optimal cost-reducing effort.
6. Use comparative statics to examine whether and how the level of debt affects the agent’s optimal cost-reducing effort.

3.2 Full-recourse Debt Contract

In this section we consider a financial contract that employs a full-recourse debt control right regime. Following chapter 3-2, it is assumed that under a full-recourse debt regime, the interest rate for the debt is simply the bank’s weighted average cost of capital.

The possible scenario outcomes for a full-recourse debt contract are as follows:

(a) If the realized \( x \) exhibits \( x - \psi(e) \leq \frac{I_0}{\alpha} + \frac{D_0}{\alpha} \), the agent finishes the project with the existing capital.

(b) If the realized \( x \) exhibits \( x - \psi(e) > \frac{I_0}{\alpha} + \frac{D_0}{\alpha} \), the parties’ decisions proceed in the following sequence:

a. The agent determines whether or not to make up the remaining shortfall to complete the project.

b. If the agent does not make up the shortfall, the government and the agent may renegotiate a new contract \( (I^F_1, I^r_1, R_1) \).
c. If the government and the agent cannot reach an agreement for a new contract, the government confiscates the performance bond, pays off the debt, takes control of the underlying asset, and hires a new agent to complete the project.

The parties’ decision problem under a full-recourse debt contract can be illustrated with the tree in Figure 3-2. A full-recourse debt contract thus has the following four possible scenario outcomes: no-reinvestment, self-enforcement, new agent and new contract. We can now proceed with the analysis.

### 3.2.1 Parties’ Payoffs in the four Scenario Outcomes

In this section we calculate parties’ payoffs in the four scenarios: (1) No reinvestment, (2) Self-enforcement, (3) New Agent, and (4) New contract.

(1) No reinvestment

Table 3-1 summarizes parties’ cash flows from date 0 to date 2 in the No Reinvestment scenario. The NPV of parties’ date 0 net payoffs can be calculated accordingly.

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Debt</td>
<td>$D_0$</td>
<td>$-D_0$</td>
</tr>
<tr>
<td></td>
<td>PB</td>
<td>-$H$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Effort</td>
<td>-$e$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Construction cost</th>
<th>$-x + \Psi(e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date 2</td>
<td>Consumer surplus</td>
<td>$S$</td>
</tr>
<tr>
<td></td>
<td>Revenue</td>
<td>$R_0$</td>
</tr>
<tr>
<td></td>
<td>Debt Service</td>
<td>$-\frac{D_0}{\beta^2}$</td>
</tr>
<tr>
<td></td>
<td>PB payback</td>
<td>$H\frac{\alpha^2}{\beta^2}$</td>
</tr>
</tbody>
</table>

The agent’s date 0 net payoff = discounted revenue + debt – PB – discounted debt payment – discounted construction cost – discounted PB payback – cost-reducing effort

$$= \alpha^2 R_0 - \alpha(x - \Psi(e)) + (1 - \frac{\alpha^2}{\beta^2})D_0 - e$$

The government’s date 0 net payoff = discounted consumer surplus – discounted revenue

$$= \gamma^2 (S - R_0)$$

The bank’s date 0 net payoff = discounted debt payment – debt = 0
Since $\alpha < \gamma$, the coefficient of $H$ is strictly negative ($-\left(1 + \frac{\alpha^2}{\gamma^2}\right) < 0$). The agent loses for his high opportunity cost of capital.

(2) Self-enforcement

Table 3-2 summarizes parties’ cash flows from date 0 to date 2 in the Self-enforcement scenario. The NPV of parties’ date 1 and date 0 net payoffs can be calculated accordingly.

Table 3-2 Accounting sheet for the “Self-enforcement” scenario in a full recourse debt contract

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>$D_0$</td>
<td>$-D_0$</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>$-H$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effort</td>
<td>$-e$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Construction cost</th>
<th>$-x + \psi(e)$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Date 2</th>
<th>Consumer surplus</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>$R_0$</td>
<td>$-R_0$</td>
</tr>
<tr>
<td>Debt Service</td>
<td>$\frac{D_0}{\beta^2}$</td>
<td>$\frac{D_0}{\beta^2}$</td>
</tr>
<tr>
<td>PB payback</td>
<td>$\frac{H}{\alpha^2}$</td>
<td></td>
</tr>
</tbody>
</table>

The agent’s date 1 net payoff = discounted revenue – reinvestment (capital shortfall) – discounted debt payment + discounted PB payback

$$= \alpha R_0 - x + \psi(e) + \frac{I_0}{\alpha} + \left(\frac{1}{\alpha} - \frac{\alpha}{\beta^2}\right)D_0 + \frac{H}{\alpha}$$

The government’s date 1 net payoff = discounted consumer surplus – discounted revenue

$$= \gamma(S - R_0)$$

The bank’s date 1 net payoff = discounted debt service = $\frac{D_0}{\beta}$

The agent’s date 0 net payoff = discounted agent’s date 1 net payoff – equity – PB – cost-reducing effort

$$= \alpha^2 R_0 - \alpha(x - \psi(e)) + \left(1 - \frac{\alpha^2}{\beta^2}\right)D_0 - e$$

The government’s date 0 net payoff = discounted consumer surplus – discounted revenue

$$= \gamma^2(S - R_0)$$

The bank’s date 0 net payoff = discounted debt payment – debt = 0
Table 3-3 Accounting sheet for the “New Agent” scenario in a full recourse debt contract

<table>
<thead>
<tr>
<th></th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date 0</td>
<td>Debt ( D_0 )</td>
<td>(- D_0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PB -H</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Effort -( e )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Date 1</td>
<td>Debt service</td>
<td>(- (1 + \delta) \frac{D_0}{\beta} )</td>
<td>( \frac{D_0}{\beta} )</td>
</tr>
<tr>
<td></td>
<td>PB transfer</td>
<td>( \frac{H}{\alpha} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>New agent’s fee</td>
<td>(- (1 + \delta)(1 + f) \left[ x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right] )</td>
<td></td>
</tr>
<tr>
<td>Date 2</td>
<td>Consumer surplus</td>
<td>( S )</td>
<td></td>
</tr>
</tbody>
</table>

The agent’s net date 1 payoff = 0
The government’s date 1 net payoff = discounted consumer surplus – new agent fee valued at the social cost – debt service valued at the social cost + PB transfer
\[ = \gamma S - (1 + \delta) \left[ (1 + f) \left( x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right) + \frac{D_0}{\beta} \right] + \frac{H}{\alpha} \]

The bank’s date 1 net payoff = discounted debt service = \( \frac{D_0}{\beta} \)
The agent’s date 0 net payoff = initial debt – cost-reducing effort – agent’s construction expenditure – PB
\[ = -I_0 - H - e \]
The government’s date 0 net payoff = discounted consumer surplus – new agent fee valued at the social cost – debt service valued at the social cost
\[ = \gamma^2 S - \gamma (1 + \delta) \left[ (1 + f) \left( x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right) + \frac{D_0}{\beta} \right] + \gamma H \]
The bank’s date 0 net payoff = discounted debt service – debt = 0

(4) New contract

Table 3-4 summarizes parties’ cash flows from date 0 to date 2 in the New Contract scenario. The NPV of parties’ date 1 and date 0 net payoffs can be calculated accordingly.
Table 3-4 Accounting sheet for the “New Contract” scenario in a full recourse debt contract

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>$D_0$</td>
<td>$-D_0$</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>-H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effort</td>
<td>$-e$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Date 1 | Government’s Investment | $I_t^g$ | $-(1+\delta)I_t^g$ |
|        | Construction cost        | $-x+\psi(e)$ |            |

| Date 2 | Consumer Surplus | $S$ |
|        | Renegotiated Revenue | $R_1$ | $-R_1$ |
|        | Debt Service       | $-D_0/\beta^2$ | $D_0/\beta^2$ |
|        | PB payback         | $H/\alpha^2$ |            |

The agent’s date 1 net payoff = discounted revenue + government reinvestment – capital shortfall – discounted debt payment + discounted PB payback

$$= \alpha R_1 - x + \psi(e) + \frac{I_0}{\alpha} + I_t^g + \left(\frac{1}{\alpha} - \frac{\alpha}{\beta^2}\right)D_0 + \frac{H}{\alpha}$$

The government’s date 1 net payoff = discounted consumer surplus – discounted revenue – government investment valued at the social cost

$$= \gamma(S - R_1) - (1+\delta)I_t^g$$

The bank’s date 1 net payoff = discounted debt service = $\frac{D_0}{\beta}$

The agent’s date 0 net payoff = discounted agent’s date 1 net payoff – equity – PB – cost-reducing effort

$$= \alpha^2 R_1 - \alpha \left[x - \psi(e) - I_t^g\right] + \left(1 - \frac{\alpha^2}{\beta^2}\right)D_0 - e$$

The government’s date 0 net payoff = discounted consumer surplus – discounted revenue – government investment valued at the social cost

$$= \gamma^2(S - R_1) - \gamma(1+\delta)I_t^g$$

The bank’s date 0 net payoff = debt - discounted debt service = 0

We next determine the values of $I_t^g$, $I_t^e$, and $R_1$ in the renegotiation game.

**Proposition 1:** The renegotiated contract specifies

$$(I_t^g, I_t^e) = \left(0, x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right)$$ if $\frac{\gamma}{\alpha} < 1 + \delta$

$$(I_t^g, I_t^e) = \left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}, 0\right)$$ if $\frac{\gamma}{\alpha} > 1 + \delta$$
Proposition 1 shows that at the Pareto optimal the parties reinvest with the agent’s funding if the cost of public funds is expensive and they reinvest with the government’s funding if the cost of public funds is cheap. Any reinvestment made by the agent must be justified by a reward that allows the agent to have a non-negative payoff after renegotiation.

Consider the following rate value: \( \delta = 0.3, r_e = 0.04, \) and \( r_a = 0.14. \) After calculation:

\[
\frac{\gamma}{\alpha} = \frac{1 + r_a}{1 + r_e} = 1.09 < 1 + \delta = 1.3.
\]

This example shows that in general the social cost of public funds is expensive (the condition \( \frac{\gamma}{\alpha} < 1 + \delta \) holds for most cases). We shall therefore proceed with our analysis under the following assumption:

(Assumption 9): The project exhibits: \( \frac{\gamma}{\alpha} < 1 + \delta. \)

Assumption 9 is critical as it determines the bargaining outcome of the renegotiation game, which in turn determines the equilibrium game form that can occur to a contract. It will be shown in the latter analysis that if the project exhibits \( \frac{\gamma}{\alpha} \geq 1 + \delta, \) the possible equilibrium game forms of a PPP contract will be different from the equilibrium game forms we get in section 3.2.3. However, I note at this point that this model is most useful for developed economies where the costs of public funds are normally high, and the government’s rational behavior is to raise revenue versus equity when default occurs.

With assumption 9, the contract specifies \( I_t^g = 0. \) Let \( R_t^* \) and \( R_t^- \) be respectively the upper bound and the lower bound of \( R_t \). The Pareto frontier of the renegotiation game is a straight line with the government and the agent bargaining over \( R_t \in [R_t^-, R_t^*]. \) We can now determine the value of renegotiated revenue \( R_t. \)

**Proposition 2:** The renegotiated contract specifies:

\[
R_t = \frac{\Delta_1}{2\alpha\gamma}(x - \psi(x) - \frac{I_0}{\alpha}) + \frac{\Delta_1}{2\alpha^2\gamma}D_0 - \frac{\alpha + \gamma}{2\alpha^2\gamma}H
\]

Where

\[
\Delta_1 = \alpha(1 + \delta)(1 + f) + \gamma
\]

\[
\Delta_2 = \frac{\alpha^2(1 + \delta)}{\beta} - \alpha(1 + \delta)(1 + f) - \gamma + \frac{\alpha^2\gamma}{\beta^2}
\]

It can be easily verified that \( R_t = \frac{1}{2}(R_t^* + R_t^-) : \) The government and the agent divide the consumer surplus equally in the renegotiation game.

Now that \( (I_t^g, I_t^a, R_t) \) are determined, we can calculate the parties’ net payoffs with the new contract at date 1 and date 0. To further simplify the notation, we let

---

4 The value of \( R_t^* \) and \( R_t^- \) are specified in Appendix 1.
\[ \Delta_1 = \alpha (1 + \delta)(1 + f) - \gamma \]
\[ \Delta_4 = \frac{\alpha^2 (1 + \delta)}{\beta} - \alpha (1 + \delta)(1 + f) + \gamma - \frac{\alpha^2 \gamma}{\beta^2} \]

The agent’s date 1 net payoff with the new contract is:
\[ U_{1\text{(New Contract)}} = \frac{\Delta_1}{2\gamma} (x - \psi(e) - \frac{I_0}{\alpha}) + \frac{\Delta_4}{2\alpha \gamma} D_0 + \frac{(\gamma - \alpha)}{2\alpha \gamma} H \tag{3-1} \]

The government’s date 1 net payoff with the new contract is:
\[ V_{1\text{(New Contract)}} = \psi - \frac{\Delta_1}{2\alpha} (x - \psi(e) - \frac{I_0}{\alpha}) - \frac{\Delta_4}{2\alpha \gamma} D_0 + \frac{(\alpha + \gamma)}{2\alpha \gamma} H \tag{3-2} \]

The agent’s date 0 net payoff with the new contract is:
\[ U_{0\text{(New Contract)}} = \frac{\alpha \Delta_1}{2\gamma} (x - \psi(e)) - \frac{\Delta_1}{2\gamma} I_0 + \frac{\Delta_4}{2\alpha \gamma} D_0 - \frac{(\alpha + \gamma)}{2\alpha \gamma} H \tag{3-3} \]

The government’s date 0 net payoff with the new contract is:
\[ V_{0\text{(New Contract)}} = \psi - \frac{\Delta_1}{2\alpha} (x - \psi(e)) - \frac{\Delta_4}{2\alpha \gamma} D_0 + \frac{(\alpha + \gamma)}{2\alpha \gamma} H \tag{3-4} \]

### 3.2.2 The Break-even Costs That Separate the Four Scenarios

We know \( \forall e \in [0, \infty) \) there exists a corresponding set of construction costs that separate the four possible scenario outcomes. The value of these break-even costs change with (1) the magnitude of the agent’s cost-reducing effort \( e \), and (2) the values of \( (I_0, D_0, R_0, H) \).

**Definition 1:** \( x_1 \) is the cost at which the initial capital investment breaks even with the construction cost:
\[ x_1 = \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha} \]

**Definition 2:** \( x_2 \) is the cost at which the agent is indifferent between self-enforcement of the existing contract or renegotiation of a new contract:
\[ x_2 = \psi(e) + \frac{1}{\alpha} I_0 - \frac{\Delta_4}{\alpha \Delta_1} D_0 + \frac{2\alpha \gamma}{\Delta_1} R_0 + \frac{(\alpha + \gamma)}{\alpha \Delta_1} H \]

**Definition 3:** \( x_3 \) is the cost at which the parties are indifferent between renegotiating a new contract or letting the government take over the project:

\[ x_3 = \psi(e) + \frac{1}{\alpha} I_0 - \frac{\Delta_4}{\alpha \Delta_1} D_0 + \frac{2\alpha \gamma}{\Delta_1} R_0 + \frac{(\alpha + \gamma)}{\alpha \Delta_1} H \]

---

\[ x_4 \] is derived by letting the agent’s self-enforcing payoff equal the agent’s new contract payoff:
\[ \alpha R_0 - x_4 + \psi(e) + \frac{I_0}{\alpha} + \left( \frac{1}{\alpha} - \frac{\alpha}{\beta} \right) D_0 + \frac{H}{\alpha} + \frac{\Delta_1}{2\gamma} (x_2 - \psi(e) - \frac{I_0}{\alpha}) + \frac{\Delta_4}{2\alpha \gamma} D_0 + \frac{(\gamma - \alpha)}{2\alpha \gamma} H \]

\[ x_3 \] is derived by letting \( R_1^* = R_0^* \):
\[ \frac{(1 + \delta)}{\gamma} \left[ (1 + f) (x_3 - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\beta}) + D_0 \right] \frac{H}{\alpha \gamma} - \frac{1}{\alpha} \left[ x_3 - \psi(e) - \frac{I_0}{\alpha} \right] \left( \frac{1}{\alpha^2} - \frac{1}{\beta^2} \right) D_0 - \frac{H}{\alpha^2} \]

31
\[ x_3 = \psi(e) + \frac{1}{\alpha} I_0 + \frac{\Delta_4}{\alpha \Delta_3} D_0 + \frac{(\alpha - \gamma)}{\alpha \Delta_3} H \]

The relationship between a realized construction cost \( x \) and the resulting scenario outcome is laid out as follows:

(1) \( x \leq x_1 \): The initial capital investment is sufficient to cover the construction cost. No reinvestment is needed to complete the project.

\( x > x_1 \): The initial capital investment is insufficient to cover the construction cost. Reinvestment is needed to complete the project.

(2) \( x > x_2 \): The agent’s self-enforcing payoff is smaller than his new contract payoff. The agent does not make up the shortfall.

\( x \leq x_2 \): The agent’s self-enforcing payoff is greater than his new contract payoff. The agent makes up the shortfall.

(3) \( x > x_3 \): The government’ maximal acceptable renegotiated revenue is greater than the agent’s minimal acceptable renegotiated revenue \( (R_1^* > R_{\tau^*}) \). The parties renegotiate a new contract.

\( x \leq x_3 \): The government’ maximal acceptable renegotiated revenue is smaller than the agent’s minimal acceptable renegotiated revenue \( (R_1^* < R_{\tau^*}) \). The government takes control of the project.

Definition 1 and 3 show that the relationship between \( x_1 \) and \( x_3 \) is exogenously determined (it depends on the values of \( \alpha, \beta, \gamma, \delta, \) and \( f \)) and cannot be changed by structuring \( (I_0, D_0, R_0, H) \). On the other hand, the relationship between \( x_1 \) and \( x_2 \) and the relationship between \( x_2 \) and \( x_3 \) are contract-specific and can be determined by structuring \( (I_0, D_0, R_0, H) \).

**Proposition 3:** Contract \( (I_0, D_0, R_0, H) \) exhibits the following properties:

1. \( x_1 > x_3 \)
2. \( x_1 = x_2 \) when \( R_0 = \Delta_5 D_0 - \frac{(\alpha + \gamma)}{2 \alpha \gamma^2} H \)
3. \( x_2 = x_3 \) when \( R_0 = \Delta_6 D_0 + \frac{[1 - (1 + \delta)(1 + f)]}{\alpha \Delta_3} H \)

where
\[ \Delta_5 = \frac{\gamma + (1 + \delta) \beta}{2 \beta^2 \gamma} \]
and
\[ \Delta_6 = \frac{(1 + \delta)(\alpha(1 + f) - \beta)}{\beta^2 \Delta_3} \]

Proposition 3-(1) holds as long as the cost of public fund is more expensive than the costs of private funds (assumption 8 holds). Proposition 3-(2) and 3-(3) show that participants of the
contract can adjust the values of $R_0, D_0$, and $H$ to determine the ordering of $x_1, x_2, x_3$. It can be easily verified that $x_1 > x_2$ if $R_0 < \Delta_4 D_0 - \frac{2\alpha}{\gamma} R_0 - \frac{\alpha + \gamma - 2\alpha y}{2\alpha^2 \gamma} H$ (alternatively $x_1 < x_2$ if $R_0 > \Delta_4 D_0 - \frac{2\alpha}{\gamma} R_0 + \frac{\alpha + \gamma - 2\alpha y}{2\alpha^2 \gamma} H$).

3.2.3 Financial Structure and Contract Type

From proposition 3-(1), there exists three possible orderings of $x_1, x_2, x_3$: $x_1 > x_2 > x_3, x_1 > x_3 > x_2$, and $x_2 > x_1 > x_3$. However, it can be shown that there exist only two contract types in equilibrium.

**Proposition 4**: When assumption 3 holds, only two types of contract exist:

1. For contracts specifying $R_0 > \Delta_4 D_0 - \frac{2\alpha}{\gamma} R_0 - \frac{\alpha + \gamma - 2\alpha y}{2\alpha^2 \gamma} H$, the possible scenario outcomes are: No investment, self-enforcement, and new contract.
2. For contracts specifying $R_0 < \Delta_4 D_0 - \frac{2\alpha}{\gamma} R_0 + \frac{\alpha + \gamma - 2\alpha y}{2\alpha^2 \gamma} H$, the possible scenario outcomes are: No investment and new contract.

3.2.4 Parties' Expected Payoffs with The Two Contract Types

Once the values of $(I_0, D_0, R_0)$ are chosen, the contract type is determined and the parties' expected payoffs can be calculated. To simplify the notation in writing the parties’ payoffs, we let:

$$B_1 = \alpha^2 R_0 + \left(1 - \frac{\alpha^2}{\beta^2}\right) D_0$$

and

$$B_2 = -\frac{\Delta_4}{2\gamma} I_0 + \frac{\Delta_4}{2\gamma} D_0 - \frac{\alpha + \gamma}{2\gamma} H$$

We now define the two contract types as follows:
(1) Type F-I contract:

A Type F-I contract is a full-recourse debt contract that has the following three equilibrium scenario outcomes: no reinvestment, self-enforcement, and new contract. A contract is a Type I contract if the contract specifies \( R_0 > \Delta_0 D_0 - \frac{(\alpha + \gamma)}{2\alpha^2\gamma} H \) (thus making \( x_2 > x_1 > x_3 \)). The parties’ payoffs with a Type F-I contract are shown in Figure 3-3.

![Figure 3-3 The parties’ payoffs with a Type F-I contract](image)

The agent’s date 0 expected payoff with a Type F-I contract is:

\[
EU_{0(\text{Type F-I})} = \left( \frac{\alpha \Delta_1}{2\gamma} \psi(e) + B_7 - B_8 \right) F(x_2(e)) - \alpha \int z_{x(e)} df(x) + \frac{\alpha \Delta_1}{2\gamma} \int z_{x(e)} df(x) - \frac{\alpha \Delta_1}{2\gamma} \psi(e) + B_8 - e \tag{3-5}
\]

The government’s date 0 expected payoff with a Type F-I contract is:

\[
EV_{0(\text{Type F-I})} = \gamma^2 S - \gamma^2 R_0 F(x_2(e)) + \frac{\gamma}{\alpha} \left( \frac{\Delta_1}{2} \psi(e) - \frac{\gamma}{\alpha} B_8 \right) \left( 1 - F(x_2(e)) \right) - \frac{\gamma \Delta_1}{2\alpha} \int z_{x(e)} df(x) \tag{3-6}
\]

The bank’s date 0 payoff is 0 with certainty. For the parties to agree on a Type I contract, the contract must satisfy both \( EU_{0(\text{Type F-I})} \geq 0 \), and \( EV_{0(\text{Type F-I})} \geq 0 \).
(2) Type F-II contract:

A Type F-II contract is a full recourse debt contract that has the following two equilibrium scenario outcomes: no reinvestment and new contract. A contract is a Type F-II contract if the contract specifies $R_0 < \Delta_x D_x - \frac{(\alpha + \gamma)}{2\alpha \gamma} H$ (thus making $x_1 > x_2 > x_3$ and $x_1 > x_3 > x_2$). The agent’s payoffs with a Type F-II contract are shown in Figure 3-4.

The agent’s date 0 expected payoff with a Type F-II contract is:

$$EU_{0(Type \text{ F-II})} = \left[ \frac{\alpha \Delta_1}{2\gamma} \psi(e) + B_x - B_y \right] F(x_1(e)) - \alpha \int_{x_1(e)}^{x_2(e)} x \, dF(x)$$

$$+ \frac{\alpha \Delta_1}{2\gamma} \int_{x_1(e)}^{x_2(e)} x \, dF(x) - \frac{\alpha \Delta_1}{2\gamma} \psi(e) + B_y - e$$

(3-7)

The government’s date 0 expected payoff with a Type F-II contract is:

$$EV_{0(Type \text{ F-II})} = \gamma^2 S - \gamma^2 R_0 F(x_1(e)) + \frac{\gamma}{\alpha} \left( \frac{\Delta_1}{2} \psi(e) - \frac{\gamma}{\alpha} B_y \right) (1 - F(x_1(e))) - \frac{\gamma \Delta_1}{2\alpha} \int_{x_1(e)}^{x_2(e)} x \, dF(x)$$

(3-8)

The bank’s date 0 payoff is 0 with certainty. For the parties to agree on a Type F-II contract, the contract must satisfy both $EU_{0(Type \text{ F-II})} \geq 0$, and $EV_{0(Type \text{ F-II})} \geq 0$. 

Figure 3-4 The parties’ payoffs with a Type F-II contract
3.2.5 Agent’s optimal cost-reducing effort

By now we know that there exist only two contract types when the contract employs a full-recourse debt control right mechanism. In this sub-section we examine the effect of financial structure over agent’s optimal cost-reducing effort. We will show (1) Type I contract induces higher level of cost-reducing effort than Type II contract, and (2) For both contract types, the agent’s cost-reducing effort decreases with the magnitude of equity and increases with the magnitude of a performance bond under a given initial capital investment.

Now we calculate the agent’s optimal cost-reducing effort in Type F-I and Type F-II contracts.

(1) Type F-I contract

The agent’s optimal cost-reducing effort for a Type F-I contract satisfies the first order condition:

$$\frac{\partial E U_{U(\text{Type F-I})}}{\partial e} = \frac{\alpha}{2\gamma} \left( \Delta F(x_2(e)) - \Delta \right) \nu'(e) - 1 = 0 \quad (3-9)$$

The term $\frac{1}{2\gamma} \left( \Delta F(x_2(e)) - \Delta \right)$ represents the hold-up effect. It is the weighted sum of the agent’s share of gains in the No reinvestment, Self-enforcement, and New contract scenarios. We know $\forall x > x_2$ the agent’s gain from cost-reduction will be shared with the government in the ex post bargaining game so the agent’s gain become discounted at the $x = x_2$.

(2) Type F-II contract

The agent’s optimal cost-reducing effort for a Type F-II contract satisfies the first order condition:

$$\frac{\partial E U_{U(\text{Type F-II})}}{\partial e} = \alpha \left[ \frac{1}{2\gamma} \left( \Delta F(x_1(e)) - \Delta \right) + \left( \alpha R_0 - \alpha \Delta D_0 + \frac{\alpha + \gamma}{2\alpha\gamma} H \right) f(x_1(e)) \right] \nu'(e) - 1 = 0 \quad (3-10)$$

The first term $\frac{1}{2\gamma} \left( \Delta F(x_1(e)) - \Delta \right)$ represents the hold-up effect. It is the weighted sum of the agent’s share of gains in the No reinvestment, and New contract scenarios. We know $\forall x > x_1$ the agent’s gain from cost-reduction will be shared with the government in the ex post bargaining game so his gain is discounted.

The second term $\left( \alpha R_0 - \alpha \Delta D_0 + \frac{\alpha + \gamma}{2\alpha\gamma} H \right) f(x_1(e))$ is the marginal gain the agent gets from an adjustment in the revenue in the New Contract scenario. For $R_0 - \Delta D_0 - \frac{H}{\gamma^2}$ is negative in Type F-II contracts, the low revenue (and the low performance bond as well) essentially post as a disincentive for the agent to make efforts. In other words, the agent exerts lower level of effort for a lower revenue contract with a hope to renegotiate the revenue through ex
post bargaining. The term \( \left( \alpha R_0 - \alpha \Delta z D_0 + \frac{\alpha + \gamma}{2 \alpha \gamma} H \right) f(x_1(e)) \) represents the self-enforcement effect.

Equation (3-9) and (3-10) show that the financial structure as well as the distribution of \( x \) affect the agent's optimal choice of \( e \). Note that if the agent firmly believes that the initial capital investment is sufficient to cover the construction cost (so \( f(x_1) = 0 \), \( F(x_1) = F(x_2) = 1 \)), the agent's optimal efforts in both Type F-I and Type F-II contracts satisfy \( \psi'(e) = \frac{1}{\alpha} \). That is, the agent exerts first best efforts in both contract types.

### 3.2.6 Financial Structure and Agent's Effort Incentive

We now assume that at date 0 the parties set the initial capital investment at a fixed value \( I_0 + D_0 = \pi \), where \( \pi \) is a fixed value that satisfies \( \pi \in \pi = [D_0, S] \). The three decision variables are \( D_0, P \) and \( R_0 \). (\( I_0 \) is determined by \( I_0 = \pi - D_0 \))

With the introduction of \( \pi \), we rewrite \( x_1, x_2 \) and \( B_2 \) as follows:

\[
x_1 = \psi(e) + \frac{\pi}{\alpha} + \alpha z \Delta z \frac{2 \alpha \gamma}{\Delta z} D_0 + \frac{2 \alpha \gamma}{\Delta z} R_0 + \frac{(\alpha + \gamma)}{\alpha \Delta l} H
\]

We will now compare the agent's optimal efforts in Type F-I and Type F-II contracts under a given initial capital investment.

**Proposition 5:** For any given initial capital investment \( \pi \in \pi \), the order of the agent's optimal cost-reducing effort is \( e_{\text{type F-I}}^* > e_{\text{type F-II}}^* \).

Proposition 5 shows that for any given \( \pi \in \pi \), the agent's cost-reducing effort is always higher with a Type F-I contract than with a Type F-II contract regardless of the setting of the relationships between other contractual terms. It means that the agent exerts higher level of effort whenever the possibility of self-enforcement exists.

We now examine the effect of capital structure over the agent's optimal cost-reducing effort.

**Proposition 6:** For any given initial capital investment \( \pi \in \pi \), the agent's optimal cost-reducing effort (1) decreases with the level of debt, and (2) increases with the magnitude of the performance bond in a full-recourse debt contract.
Proposition 6 shows that for contracts that have the same values for debt, equity and revenue, a contract that employs a performance bond always induce higher level of cost-reducing effort than a contract without performance bond. Notice that the agent’s optimal cost-reducing effort is a function of revenue, performance, debt, parties’ cost of capitals, the social cost of public funds, the new agent’s cost, the probability of cost-overrun. It can observed that most parameters are predetermined before the contract is signed. After a contract is signed, whether or not the agent will exert first best effort depends substantially on the agent’s prior about $X$. 
3.3 Non-recourse Debt

In this section we consider a financial contract with a non-recourse debt control right regime. Following section 3-2, let \( r_r \) be the interest rate for the non-recourse debt and denote by
\[
\rho = \frac{1}{1 + r_r}
\]
the discount rate for the debt. We assume that under the non-recourse debt regime, the bank charges a risk premium for the debt so \( r_r > r_b \) and \( \rho < \beta \).

The parties’ decision rule under a non-recourse debt contract is as follows:

(a) If the realized \( x \) exhibits \( x - \psi(e) \leq I_0 + D_0 \), the agent finishes the project with the existing capital.

(b) If the realized \( x \) exhibits \( x - \psi(e) > I_0 + D_0 \), the parties’ decisions proceed in the following sequence:
   a. The agent determines whether or not to make up the remaining capital shortfall to complete the project.
   b. If the agent does not make up the shortfall, two renegotiations may occur:
      i. The agent and the bank may renegotiate a new fee \( P \) for the agent to pay to the bank at date 2 such that the latter will not seize the project.
      ii. The government and the agent may renegotiate over new reinvestments (\( I^*_1, I^*_2 \)) and new revenue \( R_i \) such that the agent is willing to complete the project.
   c. If no agreement between the agent and the bank can be reached, the agent defaults. Under such condition the government confiscates the performance bond, and the bank takes over the underlying asset and hires a new agent to complete the project.

The parties’ decision problem can be illustrated with the same tree depicted in Figure 3-2. A non-recourse contract thus shares with a full-recourse debt contract the same possible scenario outcomes: No-investment, self-enforcement, new agent and new contract. We can now proceed with the analysis.

3.3.1 Parties’ payoffs in the four scenarios

In this section we calculate parties’ in the four scenarios: (1) No reinvestment, (2) Self-enforcement, (3) New Agent, and (4) New contract.
(1) No reinvestment

Table 3-5 summarizes parties’ cash flows from date 0 to date 2 in the No Reinvestment scenario. The NPV of parties’ date 0 net payoffs can be calculated accordingly.

Table 3-5 Accounting sheet for the “No-reinvestment” scenario in a non-recourse debt contract

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date 0</td>
<td>Debt</td>
<td>$D_0$</td>
<td>$-D_0$</td>
</tr>
<tr>
<td>PB</td>
<td>-H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effort</td>
<td>-e</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Date 1</td>
<td>Construction cost</td>
<td>$-x + \psi(e)$</td>
<td></td>
</tr>
<tr>
<td>Date 2</td>
<td>Consumer surplus</td>
<td>Revenue</td>
<td>$R_0$</td>
</tr>
<tr>
<td></td>
<td>Debt Service</td>
<td>$-\frac{D_0}{\rho^2}$</td>
<td>$\frac{D_0}{\rho^2}$</td>
</tr>
<tr>
<td></td>
<td>PB payback</td>
<td>$\frac{H}{\alpha^2}$</td>
<td></td>
</tr>
</tbody>
</table>

The agent’s date 0 net payoff = discounted revenue + debt – PB – discounted debt payment – discounted construction cost – cost-reducing effort + discounted PB payback

$$= \alpha^2 R_0 - \alpha(x - \psi(e)) + (1 - \frac{\alpha^2}{\rho^2})D_0 - e$$

The government’s date 0 net payoff = discounted consumer surplus – discounted revenue

$$= \gamma^2 (S - R_0)$$

The bank’s date 0 net payoff = discounted debt payment – debt = $\left(\frac{\beta^2}{\rho^2} - 1\right)D_0$

Since $\rho < \beta$, the coefficient of $D_0$ is strictly positive $\left(\frac{\beta^2}{\rho^2} - 1 > 0\right)$. The bank gains from the risk premium charges to the debt.
(2) Self-enforcement

Table 3-6 summarizes parties' cash flows from date 0 to date 2 in the Self-enforcement scenario. The NPV of parties' date 1 and date 0 net payoffs can be calculated accordingly.

Table 3-6 Accounting sheet for “Self-enforcement” in a non-recourse debt contract

<table>
<thead>
<tr>
<th>Date</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date 0</td>
<td>Debt $D_0$</td>
<td>$-D_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PB $-H$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Effort $-e$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Date 1</td>
<td>Construction cost $-x + \psi(e)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Date 2</td>
<td>Consumer surplus $S$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Revenue $R_0$</td>
<td>$-R_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Debt Service $\frac{D_0}{\rho^2}$</td>
<td>$\frac{D_0}{\rho^2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PB payback $\frac{H}{\alpha^2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The agent’s date 1 net payoff = discounted revenue – reinvestment (capital shortfall) – discounted debt payment + discounted PB payback

$$= \alpha R_0 - x + \psi(e) + \frac{I_0}{\alpha} + \left( \frac{1 - \frac{\alpha}{\rho^2}}{\frac{\alpha}{\rho^2}} \right) D_0 + \frac{H}{\alpha}$$

The government’s date 1 net payoff = discounted consumer surplus – discounted revenue

$$= \gamma (S - R_0)$$

The bank’s date 1 net payoff = discounted debt service = $\frac{\beta D_0}{\rho^2}$

The agent’s date 0 net payoff = discounted agent’s date 1 net payoff – equity – PB – cost-reducing effort

$$= \alpha^2 R_0 - \alpha (x - \psi(e)) + \left( 1 - \frac{\alpha^2}{\rho^2} \right) D_0 - e$$

The government’s date 0 payoff = discounted consumer surplus – discounted revenue

$$= \gamma^2 (S - R_0)$$

The bank’s date 0 net payoff = discounted debt payment – debt = $\left( \frac{\beta^2}{\rho^2 - 1} \right) D_0$
(3) New Agent

Table 3-7 summarizes parties' cash flows from date 0 to date 2 in the New Agent scenario. The NPV of parties' date 1 and date 0 net payoffs can be calculated accordingly.

Table 3-7 Accounting sheet for “New Agent”
in a non-recourse debt contract

<table>
<thead>
<tr>
<th>Date</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date 0</td>
<td>Debt</td>
<td>$D_0$</td>
<td>$-D_0$</td>
</tr>
<tr>
<td></td>
<td>PB</td>
<td>$-H$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Effort</td>
<td>$-e$</td>
<td></td>
</tr>
<tr>
<td>Date 1</td>
<td>New agent's fee</td>
<td>$-(1 + f)\left[ x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PB transfer</td>
<td>$\frac{H}{\alpha}$</td>
<td></td>
</tr>
<tr>
<td>Date 2</td>
<td>Consumer surplus</td>
<td>$S$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Revenue</td>
<td>$-R_0$</td>
<td>$R_0$</td>
</tr>
</tbody>
</table>

The agent’s date 1 net payoff = 0
The government’s date 1 net payoff = discounted consumer surplus – discounted revenue + PB transfer

$$= \gamma(S - R_0) + \frac{H}{\alpha}$$

The bank’s date 1 net payoff = discounted revenue – new agent fee

$$= \beta R_0 - (1 + f)\left[ x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right]$$

The agent’s date 0 net payoff = initial debt – cost-reducing effort – agent’s construction expenditure - PB

$$= -I_0 - H - e$$

The government’s date 0 net payoff = discounted consumer surplus – discounted revenue + PB

$$= \gamma^2(S - R_0) + \frac{\gamma}{\alpha}H$$

The bank’s date 0 net payoff = discounted revenue – discounted new agent fee - debt

$$= \beta^2 R_0 - \beta(1 + f)\left[ x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right] - D_0$$

42
(4) New contract

Let \( P \) be the agent’s renegotiated payment to the bank at date 2. Table 3-8 summarizes parties’ cash flows from date 0 to date 2 in the New Contract scenario. The NPV of parties’ date 1 and date 0 net payoffs can be calculated accordingly.

Table 3-8 Accounting sheet for “New Contract”
in a non-recourse debt contract

<table>
<thead>
<tr>
<th>Date</th>
<th>Agent</th>
<th>Government</th>
<th>Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Debt</td>
<td>( D_0 )</td>
<td>( -D_0 )</td>
</tr>
<tr>
<td></td>
<td>PB</td>
<td>(-H)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Effort</td>
<td>(-e)</td>
<td></td>
</tr>
<tr>
<td>Date 1</td>
<td>Government's Investment</td>
<td>( I_1^g )</td>
<td>((1+\delta)I_1^g)</td>
</tr>
<tr>
<td></td>
<td>Construction cost</td>
<td>(-x+\psi(e))</td>
<td></td>
</tr>
<tr>
<td>Date 2</td>
<td>Consumer Surplus</td>
<td>( S )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Renegotiated Revenue</td>
<td>( R_1 )</td>
<td>(-R_1)</td>
</tr>
<tr>
<td></td>
<td>Debt Service</td>
<td>(-P)</td>
<td>( P)</td>
</tr>
<tr>
<td></td>
<td>PB payback</td>
<td>( H )</td>
<td>( \frac{\alpha^2}{\alpha} )</td>
</tr>
</tbody>
</table>

The agent’s date 1 net payoff = discounted revenue + government reinvestment – capital shortfall – discounted debt payment + discounted PB payback

\[
= \alpha R_1 - x + \psi(e) + \frac{I_0}{\alpha} + I_1^g + \frac{D_0}{\alpha} - \alpha P + \frac{H}{\alpha} - \alpha^2 P - e
\]

The government’s date 1 net payoff = discounted consumer surplus – discounted revenue – government investment valued at the social cost

\[
= \gamma(S - R_1) - (1 + \delta)I_1^g
\]

The bank’s date 1 net payoff = discounted debt service = \( \beta P \)

The agent’s date 0 net payoff = discounted agent’s date 1 net payoff – equity – PB – cost-reducing effort

\[
= \alpha^2 R_1 - \alpha(x - \psi(e) - I_1^g) + D_0 - \alpha^2 P - e
\]

The government’s date 0 net payoff = discounted consumer surplus – discounted revenue – government investment valued at the social cost

\[
= \gamma^2(S - R_1) - \gamma(1 + \delta)I_1^g
\]

The bank’s date 0 net payoff = discounted debt service – debt = \( \beta^2 P - D_0 \)

As stated in the beginning of this section, two renegotiation games may be held when

\[
x > \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha}
\]

occurs:

(1) The renegotiation between the agent and the bank over debt repayment \( P \).
(2) The renegotiation between the agent and the government over reinvestments and revenue \((I^g, I^a, R_1)\).

There exist interrelations between these two possible renegotiation games: The perspective of the outcome of the first bargaining game affects the BATNA of the second bargaining game, while the perspective of the outcome of the second bargaining game affects the BATNA of the first bargaining game. However, it can be shown that the second bargaining game will not occur regardless the outcome of the first bargaining game.

**Proposition 7:** Under the non-recourse debt regime, the agent’s strategy is to not renegotiate with the government over \((I^g, I^a, R_1)\) when \(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} > 0\) occurs.

Proposition 8 shows the necessary and sufficient condition for the renegotiation between the agent and the bank to occur.

**Proposition 8:** When the event \(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} > 0\) occurs:

i. The agent and the bank renegotiate over \(P\) if the realized \(x\) satisfies:

\[
\frac{(\alpha(1 + f) - \beta)}{\alpha\beta}\left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{H}{\alpha^2} \geq 0
\]

ii. When the above condition holds, the agent and the bank specify in the new contract:

\[P = R_0 - \Delta_1 \left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{H}{2\alpha^2}\]

where

\[\Delta_1 = \frac{\beta + \alpha(1 + f)}{2\alpha\beta}\]

Proposition 8 shows that the surplus from the recovering of PB is shared by the agent and the bank after renegotiation. We can now determine the parties’ payoffs with the new contract. Following proposition 19 an 20 we know there exists no new contract if the realized \(x\) satisfies

\[
\frac{(\alpha(1 + f) - \beta)}{\alpha\beta}\left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{H}{\alpha^2} < 0.
\]

On the other hand, if the realized \(x\) satisfies

\[
\frac{(\alpha(1 + f) - \beta)}{\alpha\beta}\left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{H}{\alpha^2} \geq 0:
\]

The agent’s date 1 net payoff with the new contract is:

\[
U_{1\text{\{(New\ contract)}} = (\alpha\Delta_1 - 1)\left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{H}{2\alpha}
\]

(3-11)

The banks’ date 1 net payoff with the new contract is:

\[
W_{1\text{\{(New\ Contract)}} = \beta \left[R_0 - \Delta_1 \left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{H}{2\alpha^2}\right]
\]

(3-12)

The government’s date 1 net payoff with the new contract is:
\[ V_{1(\text{New\_contract})} = \gamma (S - R_0) \]  
(3-13)

The agent’s date 0 net payoff with the new contract is:

\[ U_{0(\text{New\_contract})} = \alpha (\alpha \Delta - 1) \left( x - \psi(e) - \frac{D_0}{\alpha} \right) - H \frac{1}{2} \]  
(3-14)

The bank’s date 0 net payoff with the new contract is:

\[ W_{0(\text{New\_contract})} = \beta^2 \left[ R_0 - \Delta \left( x - \psi(e) - \frac{I_0}{\alpha} \right) \right] + \left( \frac{\beta \Delta}{\alpha} - 1 \right) D_0 + \frac{\beta^2}{2\alpha^2} H \]  
(3-15)

The government’s date 0 net payoff with the new contract is:

\[ V_{0(\text{New\_contract})} = \gamma^2 (S - R_0) \]  
(3-16)

### 3.3.2 The Break-even Costs that Separate the Four Scenarios

This section examines the properties of the break-even costs for the non-recourse debt contracts. Different from the non-PB contracts where the parties’ renegotiation decision depends exclusively on the value of \( \frac{\beta}{\alpha} - (1 + f) \), there exists a break-even cost for renegotiation to occur in PB contracts.

**Definition 1**: \( x_1 \) is the cost at which the initial capital investment breaks even with the construction cost:

\[ x_1 = \psi(e) + \frac{1}{\alpha} \left[ \frac{I_0}{\alpha} + D_0 \right] \]

**Definition 2**: \( x_2 \) is the cost at which the agent is indifferent between self-enforcement of the existing contract or renegotiation of a new contract:

\[ x_2 = \psi(e) + \frac{1}{\alpha} \left[ \frac{I_0}{\rho^2 \Delta} \right] D_0 + \frac{1}{\Delta} R_0 + \frac{1}{2\alpha^2 \Delta} H \]

**Definition 3**: \( x_3 \) is the cost at which the agent and the bank are indifferent between renegotiating a new contract or letting the bank take over the project:

\[ x_3 = \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha} + \frac{H}{2\alpha (1 - \alpha \Delta)} \]

The relationship between a realized construction cost \( x \) and the resulting scenario outcome is laid out as follows:

\[ x_2 \] is derived by letting the agent’s self-enforcing payoff equal the agent’s new contract payoff:

\[ x_2 = \psi(e) + \frac{I_0}{\alpha} + \frac{1}{\alpha} \left( \frac{I_0}{\alpha} \right) D_0 + \frac{H}{\alpha (1 - \alpha \Delta)} \left( x - \psi(e) - \frac{1}{\alpha} \right) \]

\[ x_3 \] is derived by letting \( P^* = P^e \):

\[ R_0 \frac{1}{\alpha} \left( x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right) + \frac{H}{\alpha \beta} = R_0 \frac{1 + f}{\beta} \left( x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right) \]
(1) \( x \leq x_1 \): The initial capital investment is sufficient to cover the construction cost. No reinvestment is needed to complete the project.

\( x > x_1 \): The initial capital investment is insufficient to cover the construction cost. Reinvestment is needed to complete the project.

(2) \( x > x_2 \): The agent’s self-enforcing payoff is smaller than his new contract payoff. The agent does not make up the shortfall.

\( x \leq x_2 \): The agent’s self-enforcing payoff is greater than his new contract payoff. The agent makes up the shortfall.

(3) If \( \frac{\beta}{\alpha} < 1 + f \):

\( x > x_3 \): The agent’s maximal acceptable renegotiated debt payment is greater than the bank’s minimal acceptable renegotiated debt payment \((P^* > P)\). The parties renegotiate a new contract.

\( x \leq x_3 \): The agent’s maximal acceptable renegotiated debt payment is smaller than the bank’s minimal acceptable renegotiated debt payment \((P^* < P)\). The bank takes control of the project.

If \( \frac{\beta}{\alpha} > 1 + f \):

\( x > x_3 \): The agent’s maximal acceptable renegotiated debt payment is smaller than the bank’s minimal acceptable renegotiated debt payment \((P^* < P)\). The bank takes control of the project.

\( x \leq x_3 \): The agent’s maximal acceptable renegotiated debt payment is greater than the bank’s minimal acceptable renegotiated debt payment \(P^* > P\). The parties renegotiate a new contract.

Note that the relationship between \( x_1 \) and \( x_2 \) and the relationship between \( x_2 \) and \( x_3 \) are contract-specific and can be determined by structuring \((I_0, D_0, R_0, H)\). The relationship between \( x_1 \) and \( x_3 \) are project-specific and cannot be determined by structuring \((I_0, D_0, R_0, H)\).

**Proposition 9:** \( x_1 \), \( x_2 \) and \( x_3 \) exhibit the following properties:

1. \( x_1 > x_3 \) if the project exhibits \( \frac{\beta}{\alpha} < 1 + f \)

2. \( x_1 < x_3 \) if the project exhibits \( \frac{\beta}{\alpha} > 1 + f \)

3. \( x_1 = x_2 \) when \( R_0 - \frac{D_0}{\rho^2} + \frac{H}{2\alpha^2} = 0 \)

4. \( x_2 = x_3 \) when \( R_0 - \frac{D_0}{\rho^2} + \frac{(1 - 2\alpha\Delta_7)H}{2\alpha^2(1 - \alpha\Delta_7)} = 0 \)
Proposition 9 shows that the participants of the contract can adjust the values of \( R_0 \), \( D_0 \) and \( H \) to determine the ordering of \( x_1, x_2, \) and \( x_3 \). It can be easily verified that (1) \( x_1 \geq x_2 \) if
\[
R_0 - \frac{D_0}{\rho^2} + \frac{H}{2\alpha^2} < 0 \quad \text{(alternatively \( x_1 < x_2 \) if \( R_0 - \frac{D_0}{\rho^2} + \frac{H}{2\alpha^2} > 0 \)), and (3) \( x_2 > x_3 \) if
\]
\[
R_0 - \frac{D_0}{\rho^2} + \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)} > 0 \quad \text{(alternatively \( x_2 < x_3 \) if \( R_0 - \frac{D_0}{\rho^2} + \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)} < 0 \)).
\]

3.3.3 Financial Structure and Contract Type

Given that the values of \( D_0 \) and \( R_0 \) determine the ordering of \( (x_1, x_2, x_3) \) and that the ordering of \( (x_1, x_2, x_3) \) determines the possible scenario outcomes that could occur at date 1, the parties can determine the possible scenario outcomes by devising the financial contract \( (I_0, D_0, R_0, H) \).

**Proposition 10:** For a contract that employs a non-recourse debt control right regime:

(a) If the project exhibits \( \frac{\beta}{\alpha} < 1 + f \):

a. For contracts specifying \( R_0 > \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \), the possible scenario outcomes are: no reinvestment, self-enforcement, and new contract.

b. For contracts specifying \( R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \), the possible scenario outcomes are: no reinvestment and new contract.

(b) If the project exhibits \( \frac{\beta}{\alpha} > 1 + f \):

a. For contracts specifying \( R_0 > \frac{D_0}{\rho^2} - \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)} \), the possible scenario outcomes are: no reinvestment, self-enforcement, and new agent.

b. For contracts specifying \( \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} < R_0 < \frac{D_0}{\rho^2} - \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)} \), the possible scenario outcomes are: no reinvestment, self-enforcement, new contract and new agent.

c. For contracts specifying \( R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \), the possible scenario outcomes are: no reinvestment, new contract, and new agent.

3.3.4 Parties' Expected Payoffs with The Five Contract Types

Once the values of \( (I_0, D_0, R_0, H) \) are chosen, the contract type is determined and the parties' expected payoffs can be calculated. To simplify the notation in writing the parties' payoffs, we let:
We now define the five possible contract types as follows:

(1) Type N-I contract:

A Type N-I contract is a non-recourse debt contract that has the following three equilibrium scenario outcomes: No reinvestment, self-enforcement, and new contract. A contract is a Type I contract if the project exhibits \( \frac{\beta}{\alpha} \leq 1 + f \) (so \( x_1 \geq x_3 \)) and when the contract specifies \( R_0 > \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \) (thus making \( x_2 > x_1 \)). The parties’ payoffs with a Type N-I contract are shown in Figure 3-5.

The agent’s date 0 expected payoff with a Type N-I contract is:
The bank’s date 0 expected payoff with a Type N-I contract is:

$$EU_{0(\text{Type N-I})} = \left( \alpha^2 \Delta \gamma \psi(e) + B_9 - B_{10} \right)f(x_2(e)) - \alpha \int_{x_2(e)}^\infty xdF(x)$$

$$+ \alpha(\alpha \Delta - 1) \int_{x_2(e)}^\infty xdF(x) - \alpha(\alpha \Delta - 1) \psi(e) + B_{10} - e$$

(3-17)

The government’s date 0 payoff with a Type N-I contract is $\gamma^2(S - R_0)$ with certainty. For the parties to agree on a Type N-I contract, the contract must satisfy:

$$EU_{0(\text{Type N-I})} \geq 0,$$

$$EW_{0(\text{Type N-I})} \geq 0,$$

and $S \geq R_0$.
(2) Type N-II contract:

A Type N-II contract is a non-recourse debt contract that has the following two equilibrium scenario outcomes: No reinvestment and new contract. A contract is a Type N-II contract if the project exhibits \( \frac{\beta}{\alpha} \leq 1 + f \) (so \( x_1 \geq x_2 \)) and when the contract specifies \( R_0 \leq \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \) (thus making \( x_1 > x_2 \)). The parties’ payoffs with a Type N-II contract are shown in Figure 3-6.

\[ \text{Agent: } B_{10} - \alpha (x - \psi(e)) - e \]
\[ \text{Bank: } \left( \frac{\beta^2}{\rho^2} - 1 \right) D_0 \]
\[ \text{Government: } \gamma^2 (S - R_0) \]

\[ \text{Agent: } B_{11} + \alpha (\Delta_\gamma - 1) (x - \psi(e)) - e \]
\[ \text{Bank: } B_{11} - \beta^2 \Delta_\gamma (x - \psi(e)) \]
\[ \text{Government: } \gamma^2 (S - R_0) \]

Figure 3-6 The parties’ payoffs with a Type N-II contract

The agent’s date 0 expected payoff with a Type N-II contract is:

\[
EU_{0(\text{Type N-II})} = \left( \alpha^2 \Delta_\gamma \psi(e) + B_{10} - B_{11} \right) F(x_1(e)) - \alpha \int_{x_1(e)}^\infty x dF(x)
\]
\[
+ \alpha (\Delta_\gamma - 1) \int_{x_1(e)}^\infty x dF(x) - \psi(e) + B_{10} - e
\]

The bank’s date 0 expected payoff with a Type N-II contract is:

\[
EW_{0(\text{Type N-II})} = \left( \left[ \frac{\beta^2}{\rho^2} - 1 \right] D_0 - \beta^2 \Delta_\gamma \psi(e) - B_{11} \right) F(x_1(e))
\]
\[
- \beta^2 \Delta_\gamma \int_{x_1(e)}^\infty x dF(x) + \beta^2 \Delta_\gamma \psi(e) + B_{11}
\]

The government’s date 0 payoff with a Type N-II contract is \( \gamma^2 (S - R_0) \) with certainty. For the parties to agree on a Type N-II contract, the contract must satisfy: \( EU_{0(\text{Type N-II})} \geq 0 \), \( EW_{0(\text{Type N-II})} \geq 0 \), and \( S \geq R_0 \).
(3) Type N-III contract:

A Type N-III contract is a non-recourse debt contract that has the following three equilibrium scenario outcomes: No reinvestment, self-enforcement, and new agent. A contract is a Type N-III contract if the project exhibits \( \frac{\beta}{\alpha} > 1 + f \) (so \( x_1 < x_3 \)) and when the contract specifies \( R_0 > \frac{D_0}{\rho^2} - \frac{(1 - 2\alpha \Delta_1)H}{2\alpha^2(1 - \alpha \Delta_1)} \) (thus making \( x_2 > x_3 \)). The parties’ payoffs with a Type N-III contract are shown in Figure 3-7.

![Diagram of Type N-III contract outcomes]

Figure 3-7 The parties' payoffs with a Type N-III contract

The agent's date 0 expected payoff with a Type N-III contract is:

\[
EU_{0(\text{Type N-III})} = [\alpha \psi(e) + I_0 + H + B_9] \int_{x_1(e)}^{x_3(e)} F(x) - I_0 - H - e
\]  

(3-21)

The bank's date 0 expected payoff with a Type N-III contract is:

\[
EW_{0(\text{Type N-III})} = -\beta(1 + f)\psi(e) + \left[\frac{\beta^2}{\rho^2} - 1\right]D_0 - B_{12} \int_{x_1(e)}^{x_3(e)} x F(x) - \psi(e) + B_{12}
\]  

(3-22)

The government's date 0 expected payoff with Type N-III contract is:
\[ EV_{0(\text{Type N-III})} = \gamma^2 (S - R_0) + \frac{\gamma}{\alpha} (1 - F(x_3(e))) H \] (3-23)

For the parties to agree on a Type N-III contract \((I_0, D_0, R_0, H)\), the contract must satisfy both \(EU_{0(\text{Type N-III})} \geq 0\), \(EW_{0(\text{Type N-III})} \geq 0\), and \(EV_{0(\text{Type N-III})} \geq 0\).

(4) Type N-IV contract:

A Type N-IV contract is a non-recourse debt contract that has the following four equilibrium scenario outcomes: No reinvestment, self-enforcement, new contract, and new agent. A contract is a Type N-IV contract if the project exhibits \(\frac{\beta}{\alpha} > 1 + f\) (so \(x_1 < x_3\)) and when the contract specifies \(\frac{D_0}{\alpha^2} - \frac{H}{2 \alpha^2} < R_0 < \frac{D_0}{\alpha^2} - \frac{(1 - 2 \alpha \Delta_1) H}{2 \alpha^2 (1 - \alpha \Delta_1)}\) (thus making \(x_3 > x_2 > x_1\)). The parties’ payoffs with a Type N-IV contract are shown in Figure 3-8.

![Figure 3-8](image)

The agent’s date 0 expected payoff with a Type N-IV contract is:
\[ EU_{\text{Type N-IV}} = \left( \alpha^2 \Delta, \psi(e) + B_0 - B_{10} \right) F(x_2(e)) + \left[ \alpha \left( 1 - \alpha \Delta \right), \psi(e) + I_0 + H + B_{10} \right] F(x_3(e)) \]
\[ - \alpha \int xH(x) + \alpha \left( \alpha \Delta - 1 \right) \int xdF(x) - I_0 - H - e \]

The bank’s date 0 expected payoff with a Type N-IV contract is:

\[ EW_{\text{Type N-IV}} = \left( - \beta^2 \Delta, \psi(e) + \left( \frac{\beta^2}{\rho^2} - 1 \right) D_0 - B_{11} \right) F(x_2(e)) + \left[ \beta \left( \beta \Delta - f - 1 \right) \psi(e) + B_{11} - B_{12} \right] F(x_3(e)) \]
\[ - \beta^2 \Delta, \int xH(x) - \beta(1 + f) \int xdF(x) + \beta(1 + f) \psi(e) + B_{12} \]

The government’s date 0 expected payoff with a Type N-IV contract is:

\[ EV_{\text{Type N-IV}} = \gamma^2 (S - R_0) + \frac{\gamma}{\alpha} (1 - F(x_3(e))) H \]

For the parties to agree on a Type N-IV contract, the contract must satisfy \( EU_{\text{Type N-IV}} \geq 0 \), \( EW_{\text{Type N-IV}} \geq 0 \), and \( EV_{\text{Type N-IV}} \geq 0 \).
(5) Type N-V contract:

A Type N-V contract is a non-recourse debt contract that has the following three equilibrium scenario outcomes: No reinvestment, new contract, and new agent. A contract is a Type N-V contract if the project exhibits \( \frac{\beta}{\alpha} > 1 + f \) (so \( x_1 < x_3 \)) and when the contract specifies \( R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \) (thus making \( x_1 > x_2 \)). The parties’ payoffs with a Type N-V contract are shown in Figure 3-9.

\[
\begin{align*}
\text{Sign } (I_0, D_0, R_0, H) & \\
\begin{array}{c}
\text{No reinvestment} \\
\end{array} & \\
\begin{array}{c}
\text{New contract} \\
\end{array} & \\
\begin{array}{c}
\text{New agent} \\
\end{array} & \\
\end{align*}
\]

\[x \leq x_1\]

\[x > x_1\]

\[x \leq x_3\]

\[x > x_3\]

Agent: \( B_0 - \alpha(x - \psi(e)) - e \)

Bank: \( \left( \frac{\beta_1^2}{\rho^2} - 1 \right) D_0 \)

Government: \( \gamma^2 (S - R_0) \)

Agent: \( B_{10} + \alpha(\alpha \Delta \gamma - 1) (x - \psi(e)) - e \)

Bank: \( B_{11} - \beta^2 \Delta \gamma (x - \psi(e)) \)

Government: \( \gamma^2 (S - R_0) \)

Agent: \( -I_0 - H - e \)

Bank: \( B_{12} - \beta(1 + f)(x - \psi(e)) \)

Government: \( \gamma^2 (S - R_0) + \frac{\gamma}{\alpha} H \)

Figure 3-9 The parties’ payoffs with a Type N-V contract

The agent’s date 0 expected payoff with a Type N-V contract is:

\[
EU_{0(\text{Type N-V})} = \left( \alpha^2 \Delta \gamma \psi(e) + B_9 - B_{10} \right) F(x_1(e)) + \left[ \alpha(1 - \alpha \Delta \gamma) \psi(e) + I_0 + H + B_{10} \right] F(x_3(e)) - \alpha \int_{x_1(e)}^{x_3(e)} x dF(x) + \alpha(\alpha \Delta \gamma - 1) \int_{x_1(e)}^{x_3(e)} x dF(x) - I_0 - H - e
\]

(3-27)

The bank’s date 0 expected payoff with a Type N-V contract is:

\[
EW_{0(\text{Type N-V})} = \left( - \beta^2 \Delta \gamma \psi(e) + \left( \frac{\beta_1^2}{\rho^2} - 1 \right) D_0 - B_{11} \right) F(x_1(e)) + \left[ \beta \beta_1^2 - f - 1 \right] \psi(e) + B_{11} - B_{12} \right] F(x_3(e)) - \beta^2 \Delta \gamma \int_{x_1(e)}^{x_3(e)} x dF(x) - \beta(1 + f) \int_{x_1(e)}^{x_3(e)} x dF(x) + \beta(1 + f) \psi(e) + B_{12}
\]

(3-28)
The government’s date 0 expected payoff with a Type N-V contract is:

\[ EV_{0\text{(Type N-V)}} = \gamma^2 (S - R_0) + \frac{\gamma}{\alpha} (1 - F(x_1(e))) H \]  

(3-29)

For parties to agree on a Type N-V contract, the contract must satisfy \( EU_{0\text{(Type N-V)}} \geq 0 \), \( EW_{0\text{(Type N-V)}} \geq 0 \), and \( EV_{0\text{(Type N-V)}} \geq 0 \).

3.3.5 Agent’s optimal cost-reducing effort

By now we know that there exist five contract types when the contract employs a non-recourse debt control right regime. In this sub-section we examine the effect of financial structure over the agent’s optimal cost-reducing effort. We will show (1) which contract type induces higher level of cost-reduction effort, and (2) whether the agent’s cost-reducing effort increases or decreases with the level of performance bond under a given initial capital investment.

Using backward induction, we calculate the agent’s optimal cost-reducing effort under each contract type by differentiating the agent’s date 0 expected utility with respect to \( e \).

(1) Type N-I contract

The agent’s optimal cost-reducing effort for a Type N-I contract satisfies the first order condition:

\[ \frac{\partial EU_{0\text{(Type N-I)}}}{\partial e} = \alpha \left[ 1 - \alpha \Delta, (1 - F(x_2(e))) \right] \nu'(e) - 1 = 0 \]  

(3-30)

The term \( (1 - \alpha \Delta, (1 - F(x_2(e))) \) represents the hold-up effect. It is the weighted marginal gain from the agent’s cost-reducing effort in the No reinvestment, Self-enforcement, and New contract scenarios. We know \( \forall x > x_2 \) the agent’s gain from cost-reduction will be shared with the government in the ex post bargaining game so his gain is discounted.

(2) Type N-II contract

The agent’s optimal cost-reducing effort for a Type N-II contract satisfies the first order condition:

\[ \frac{\partial EU_{0\text{(Type N-II)}}}{\partial e} = \alpha \left[ 1 - \alpha \Delta, (1 - F(x_1(e))) \right] + \left( \alpha R - \frac{\alpha}{\rho} D_{1/2} + \frac{H}{2\alpha} \right) f(x_1(e)) \right] \nu'(e) - 1 = 0 \]  

(3-31)

The term \( (1 - \alpha \Delta, (1 - F(x_1(e))) \) represents the hold-up effect. It is the weighted marginal gain from the agent’s cost-reducing effort in the No reinvestment and New contract scenarios. We know \( \forall x > x_1 \) the agent’s gain from cost-reduction will be shared with the government in the ex post bargaining game so his gain is discounted.

55
The second term \( \left( \alpha R_0 - \frac{\alpha}{\rho} D_0 + \frac{1}{2\alpha} H \right) f(x_i(e)) \) is the marginal gain the agent gets from an adjustment in the revenue in the New Contract scenario. For \( R_0 - \frac{1}{\rho^2} D_0 + \frac{H}{2\alpha^2} \) is negative in Type N-II contracts, the low revenue (and the low performance bond as well) essentially post as a disincentive for the agent to make efforts. In other words, the agent exerts lower level of effort for a low revenue contract with a hope to increase the revenue through ex post renegotiation.

(3) Type N-III contract

The agent’s optimal cost-reducing effort for a Type N-III contract satisfies the first order condition:

\[
\frac{\partial EU_{\text{Type N-III}}}{\partial e} = \alpha \left[ F(x_3(e)) + \left( R_0 - \frac{1}{\rho^2} D_0 + \frac{(1-2\alpha\Delta_x)}{2\alpha^2(1-\alpha\Delta_x)} H \right) f(x_3(e)) \right] \psi'(e) - 1 = 0
\]

The first term \( F(x_3(e)) \) represents the hold-up effect. It is the weighted marginal gain from the agent’s cost-reducing effort in the No reinvestment, Self-enforcement, and New agent scenarios. Obviously \( \forall x \leq x_3 \) the agent gets the full fruit of his investment in the New Agent and Self-enforcement scenarios, \( \forall x > x_3 \) the agent’s gain from cost-reduction will be simply equal to 0 in the New Agent scenario.

The second term \( \left( R_0 - \frac{1}{\rho^2} D_0 + \frac{(1-2\alpha\Delta_x)}{2\alpha^2(1-\alpha\Delta_x)} H \right) f(x_3(e)) \) is the marginal gain of cost-reducing effort that the agent gets from a New Agent scenario. Since \( R_0 - \frac{D_0}{\rho^2} + \frac{(1-2\alpha\Delta_x)H}{2\alpha^2(1-\alpha\Delta_x)} > 0 \) in a Type N-III contract, the high revenue (and the high performance bond as well) essentially post as an incentive for the agent to make efforts to deter the occurrence of the New Agent scenario.

(4) Type N-IV contract

The agent’s optimal cost-reducing effort for a Type N-IV contract satisfies the first order condition:

\[
\frac{\partial EU_{\text{Type N-IV}}}{\partial e} = \alpha \left[ F(x_3(e)) + \alpha\Delta_x (F(x_2(e)) - F(x_3(e))) \right] \psi'(e) - 1 = 0
\]

The term \( F(x_3(e)) + \alpha\Delta_x (F(x_2(e)) - F(x_3(e))) \) represents the hold-up effect. It is the weighted sum of the agent’s share of gains in the No Investment, Self-enforcement, and New Contract scenarios.
The second term $\alpha^2 \Delta_1 (F(x_1(e)) - F(x_1(e)))$ represents the hold-up effect over the marginal gain from the agent’s cost-reducing effort in the New Contract scenario. Since $F(x_1) < F(x_2)$ in a Type N-IV contract, this term imposes a disincentive for the agent to make efforts.

(4) Type N-V contract

The agent’s optimal cost-reducing effort for a Type N-V contract satisfies the first order condition:

$$
\frac{\partial \mathcal{E}_{\text{Type N-V}}}{\partial e} = \alpha \left[ \alpha R_0 - \frac{\alpha}{\rho^2} D_0 + \frac{H}{2\alpha} \right] f(x_1(e)) + \left( \alpha R_0 - \frac{\alpha}{\rho^2} D_0 + \frac{H}{2\alpha} \right) f(x_1(e)) \psi'(e) - 1 = 0
$$

(3-34)

The term $F(x_1(e)) + \alpha \Delta_1 (F(x_2(e)) - F(x_1(e)))$ represents the hold-up effect. It is the weighted sum of the agent’s share of gains in the No Investment and New Contract scenarios.

The third term $\left( \alpha R_0 - \frac{\alpha}{\rho^2} D_0 + \frac{1}{2\alpha} H \right) f(x_1(e))$ is the marginal gain the agent gets from an adjustment in the revenue in the New Contract scenario. For $R_0 - \frac{1}{\rho^2} D_0 + \frac{H}{2\alpha^2}$ is negative in Type N-V contracts, the low revenue (and the low performance bond as well) essentially post as a disincentive for the agent to make efforts. In other words, the agent exerts lower level of effort for a low revenue contract with a hope to increase the revenue through ex post renegotiation.

Equation (3-30), (3-31), (3-32), (3-33) and (3-34) show that the financial structure as well as the distribution of $x$ affect the agent’s optimal choice of $e$. Note that if the agent firmly believes that the initial capital investment is sufficient to cover the construction cost (so $f(x_1) = 0$, $F(x_1) = F(x_2) = 1$), the agent’s optimal efforts in all of the five types of contract all satisfy $\psi'(e) = \frac{1}{\alpha}$. In other words, the agent exerts first best efforts in all contract types.

It can be shown that Type N-I and Type N-II contracts exhibit the following monotone properties:

**Proposition 11:** When the project involves proprietary technology ($\frac{\beta}{\alpha} < 1 + f$), for any given initial capital investment $\pi \in \mathcal{P}$, the order of the agent’s optimal cost-reducing effort is $e^*_{\text{Type N-I}} > e^*_{\text{Type N-II}}$.

It can also be shown that Type N-III, Type N-IV and Type N-II contracts exhibit the following monotone properties:
Proposition 12: When the project involves competitive technology \( \frac{\beta}{\alpha} > 1 + f \), for any given initial capital investment \( \pi \in \pi \), the order of the agent's optimal cost-reducing effort is: \( e_{Type\,N-III}^* > e_{Type\,N-IV}^* > e_{Type\,N-V}^* \).

3.3.6 Financial Structure and Agent's Effort Incentive

We now assume that at date 0 the parties set the initial capital investment at a fixed value \( I_0 + D_0 = \pi \), where \( \pi \) satisfies \( \pi \in \pi = [D_0, S] \). The three decision variable are \( R_0, D_0 \), and \( H \) (\( I_0 \) is determined by \( I_0 = \pi - D_0 \)). With the introduction of \( \pi \), we can rewrite \( x_1, x_2, x_3, B_{10}, B_{11}, \) and \( B_{12} \) as follows:

\[
\begin{align*}
    x_1 &= \psi(e) + \frac{\pi}{\alpha} \\
    x_2 &= \psi(e) + \frac{\pi}{\alpha} - \frac{D_0}{\rho^2 \Delta_\gamma} + \frac{R_0}{\Delta_\gamma} + \frac{H}{2\alpha^2 \Delta_\gamma} \\
    x_3 &= \psi(e) + \frac{\pi}{\alpha} + \frac{H}{2\alpha(1-\alpha \Delta_\gamma)} \\
    B_{10} &= -\alpha \Delta_\gamma \pi + D_0 - \frac{H}{2} \\
    B_{11} &= \beta^2 \left( R_0 + \frac{\Delta_\gamma}{\alpha} \pi \right) - D_0 + \frac{\beta^2}{2\alpha^2} H \\
    B_{12} &= \beta^2 R_0 + \frac{\beta(1+f)}{\alpha} \pi - D_0
\end{align*}
\]

We will now compare the agent's optimal efforts in Type N-I to Type N-V contracts under a given initial capital investment.

Proposition 13: For any given initial capital investment \( \pi \in \pi \), the agent's optimal cost-reducing effort (1) decreases with the level of debt, and (2) increases with the level of the performance bond in Type N-I, N-II, N-IV, and N-V contracts.

Given that the agent's optimal cost reducing effort in Type N-III contract is a function of \( f(x_3) \), we cannot verify the effect of performance bond over the agent's cost-reducing effort without further specifying the property of \( f(x_3) \).
3.4 Numerical Example

Given the cost uncertainty embedded in our model, it is not possible to derive analytical solutions for the agent's optimal effort in each type of contract. Here we use a numerical example to demonstrate the effect of financial structure over the agent's optimal effort and the parties' expected payoffs. It can be shown that while the agent's optimal cost-reducing effort exhibits a monotone property (it generally decreases with the magnitude of debt and increases with the magnitude of the performance bond), the parties' expected payoffs do that possess these monotone properties. In most contract types the agent's expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance, whereas the control right owner's expected payoff decreases with debt and increases with the performance bond.

3.4.1 Full-recourse Debt

The parameters and input we have for the full-recourse debt contract is as follows:

\[ r_a = 0.14 \text{ so } \alpha = 0.877193 \]
\[ r_b = 0.08 \text{ so } \beta = 0.925926 \]
\[ r_g = 0.04 \text{ so } \gamma = 0.961538 \]
\[ r_r = 0.12 \text{ so } \rho = 0.892857 \]
\[ f = 0.3, \delta = 0.3, I_0 + D_0 = 80, H = 1 \sim 20 \]

Distribution (Gamma distribution): so pdf is \( f(x) = \frac{x^{a-1}e^{-\frac{x}{b}}}{b^a\Gamma(a)} \), and cdf is \( \frac{1}{b^a\Gamma(a)} \int_0^x t^{a-1}e^{-\frac{t}{b}}dt \). (use \( a=5, b=10 \))

By assumption \( \psi'(e) > 0 \) and \( \psi'(e) < 0 \) so we use \( \psi(e) = \ln e \).

(1) Type F-I contract: (condition: the contract specifies \( R_0 > \Delta_s D_0 - \frac{(\alpha + \gamma)}{2\alpha^2 \gamma} H ) \)

For \( D_0 \in [0,80], R_0 = 110 \) satisfies \( R_0 > \Delta_s D_0 - \frac{(\alpha + \gamma)}{2\alpha^2 \gamma} H \mid_{\Delta_s=80, H=0} = 105.0435 \)

Figure 3-10 shows that the agent’s optimal effort decreases with the magnitude of debt and increases with the magnitude of the performance bond (though not very sensitive).
Figure 3-10 shows that the agent’s optimal effort with a F-I contract increases with the magnitude of debt and decreases with the magnitude of the performance bond (though not very sensitive).

Figure 3-11 shows that the agent’s expected utility with a F-I contract.

Figure 3-12 shows that the government’s expected payoff decreases with the magnitude of debt and increases with the magnitude of the performance bond (though not very sensitive).
Fig 3-12 The government's expected utility with a F-I contract
(2) Type F-II contract: (condition: the contract specifies $R_0 < \Delta s D_0 - \frac{(\alpha + \gamma)}{2\alpha^2 \gamma} H$)

For $D_0 \in [70,80]$, $R_0 = 75$ satisfies, $R_0 < \Delta s D_0 - \frac{(\alpha + \gamma)}{2\alpha^2 \gamma} H \bigg|_{D_0 = 70, H = 10} = 79.504$

Figure 3-13 shows that the agent’s optimal effort decreases with the magnitude of debt and increases with the magnitude of the performance.

![Fig 3-13 The agent’s optimal effort with a F-II contract](image)
Figure 3-14 shows that the agent’s expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond.

Fig 3-14 The agent’s expected utility with a F-II contract

Figure 3-15 shows that the government’s expected payoff decreases with the magnitude of debt and increases with the magnitude of the performance bond.

Fig 3-15 The government’s expected utility with a F-II contract
3.4.2 Non-recourse Debt

Parameter and input

\[ r_a = 0.14 \text{ so } \alpha = 0.877193 \]
\[ r_b = 0.08 \text{ so } \beta = 0.925926 \]
\[ r_g = 0.04 \text{ so } \gamma = 0.961538 \]
\[ r_r = 0.12 \text{ so } \rho = 0.892857 \]
\[ f = 0.3, \delta = 0.3, I_0 + D_0 = 80, H = 1 \sim 20 \]

Distribution (Gamma distribution): so pdf is 
\[ f(x) = \frac{x^{a-1}e^{-x/b}}{b^a\Gamma(a)} \text{, and cdf is } F(a) = \frac{1}{b^a\Gamma(a)} \int_0^x t^{a-1}e^{-t/b}dt. \](use a=5, b=10)
(1) Type N-I contract: (conditions: the project exhibits \( \frac{\beta}{\alpha} \leq 1 + f \), and the contract

specifies \( R_0 > \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \))

\( f = 0.3 \) satisfies \( \frac{\beta}{\alpha} = 1.0556 < 1 + f = 1.3 \)

For \( D_0 \in [0, 80] \), \( R_0 = 100 \) satisfies \( R_0 > \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \bigg|_{D_0=80, H=0} = 98.922 \)

Figure 3-16 shows that the agent’s optimal effort decreases with the magnitude of debt and increases with the magnitude of the performance bond (though not very sensitive).

Fig 3-16 The agent’s optimal effort with a N-I contract
Figure 3-17 shows that the agent’s expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond (though not very sensitive).

![Agent's Expected Utility with a N-I contract](image1)

Fig 3-17 The agent’s expected utility with a N-I contract

Figure 3-18 shows that the bank’s expected payoff increases with the magnitude of debt and also increases with the magnitude of the performance bond (though not very sensitive).

![Bank's Expected Utility with a N-I contract](image2)

Fig 3-18 The bank’s expected utility with a N-I contract
(2) Type N-II contract: (conditions: the project exhibits $\frac{\beta}{\alpha} \leq 1 + f$, and the contract specifies $R_0 \leq \frac{D_0 - \frac{H}{2\alpha^2}}{\rho^2}$)

For $D_0 \in [70, 80]$, $R_0 = 80$ satisfies $R_0 \leq \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \bigg|_{D_0=70, H=10} = 81.31$

Figure 3-19 shows that the agent's optimal effort decreases with the magnitude of debt and increases with the magnitude of the performance bond.

![3D graph showing agent's optimal effort with a N-II contract](image)

Fig 3-19 The agent's optimal effort with a N-II contract
Figure 3-20 shows that the agent’s expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond.

![Figure 3-20 The agent’s expected utility with a N-II contract](image)

Figure 3-21 shows that the bank’s expected payoff increases with the magnitude of debt and also increases with the magnitude of the performance bond.

![Figure 3-21 The bank’s expected utility with a N-II contract](image)
(3) Type N-III contract: (conditions: the project exhibits \( \frac{\beta}{\alpha} > 1 + f \), and the contract specifies

\[
R_0 > \frac{D_0}{\rho^2} - \frac{(1 - 2\alpha \Delta_r)H}{2\alpha^2(1 - \alpha \Delta_r)}
\]

\( f = 0.03 \) satisfies \( \frac{\beta}{\alpha} = 1.0556 > 1 + f = 1.03 \) (applying to the next 2 cases as well)

For \( D_0 \in [0, 10] \), \( R_0 = 550 \) satisfies \( R_0 > \frac{D_0}{\rho^2} - \frac{(1 - 2\alpha \Delta_r)H}{2\alpha^2(1 - \alpha \Delta_r)} \) \( |D_0 = 10, H = 10| = 536.34 \)

Figure 3-22 shows that the agent's optimal effort decreases with the magnitude of debt (though not sensitive) and increases with the magnitude of the performance bond.

![3D graph showing the agent's optimal effort with a N-III contract](image-url)

Fig 3-22 The agent's optimal effort with a N-III contract
Figure 3-23 shows that the agent’s expected payoff increases with the magnitude of debt (though not sensitive) and decreases with the magnitude of the performance bond.

Fig 3-23 The agent’s expected utility with a N-III contract

Figure 3-24 shows that the bank’s expected utility increases with the magnitude of debt (though not sensitive) and also increases with the magnitude of the performance bond.

Fig 3-24 The bank’s expected utility with a N-III contract
Figure 3-25 shows that the government's expected payoff decreases with the magnitude of debt and increases with the magnitude of the performance bond.

Fig 3-25 The government’s expected utility with a N-III contract
(4) Type N-IV contract (conditions: the project exhibits $\frac{\beta}{\alpha} > 1 + f$, and the contract specifies $\frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} < R_0 < \frac{D_0}{\rho^2} - \frac{(1-2\alpha\Delta_\gamma)H}{2\alpha^2(1-\alpha\Delta_\gamma)}$)

For $D_0 \in [0,10]$, $R_0 = 100$ satisfies

$$\frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \bigg|_{\Delta_\gamma=0.0001, H=10} = 12.544 < R_0 < \frac{D_0}{\rho^2} - \frac{(1-2\alpha\Delta_\gamma)H}{2\alpha^2(1-\alpha\Delta_\gamma)} \bigg|_{\Delta_\gamma=0.0001, H=10} = 523.795$$

Figure 3-26 shows that the agent’s optimal effort decreases with the magnitude of debt (though not sensitive) and increases with the magnitude of the performance bond.

![Figure 3-26 The agent’s optimal effort with a N-IV contract](image-url)
Figure 3-27 shows that the agent’s expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond.

Figure 3-28 shows that the bank’s expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond.
Figure 3-29 shows that the government's expected payoff decreases with the magnitude of debt (though not sensitive).

Fig 3-29 The government's expected utility with a N-IV contract
(5) Type N-V contract (conditions: the project exhibits $\frac{\beta}{\alpha} > 1 + f$, and the contract specifies $R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2}$)

For $D_0 \in [70,80]$, $R_0 = 80$ satisfies $R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \big|_{D_0=70, H=10} = 81.31$

Figure 3-30 shows that the agent's optimal effort decreases with the magnitude of debt and increases with the magnitude of the performance bond.

Fig 3-30 The agent’s optimal effort with a N-V contract
Figure 3-31 shows that the agent's expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond.

\[ t^{2.5, 2.0, CLN} \]

**Fig 3-31** The agent's expected utility with a N-V contract

Figure 3-32 shows that the bank's expected payoff increases with the magnitude of debt and decreases with the magnitude of the performance bond (though not sensitive).

\[ 2.5, 2.0, CLN \]

**Fig 3-32** The bank's expected utility with a N-V contract
Figure 3-33 shows that the government’s expected payoff increases with the magnitude of debt and increases with the magnitude of the performance bond.

Fig 3-33 The government’s expected utility with a N-V contract
Chapter 4 Case Study: The Taiwan High-Speed Rail Project

In this chapter I relate theoretical results I derive from chapter 3 to real world practice by examining the financial contract of the Taiwan High Speed Rail (THSR) project. The THSR contract is a typical PPP contract with a financial structure very similar to the one I consider in my theoretical model. However, given the huge scale of the project, the financing of the project has been divided into multiple stages (as opposed to the one shot financing structure we consider in our simple model). This sequential financing structure gives rise to severe hold-up and buy-in problems.

4.1 The THSR Project

The Taiwan High-Speed Rail (THSR) project is a USD 15B high-speed rail system that provides transport service to Taiwan’s western corridor, where 94 percents of the island’s 23 million people live. THSR is a 220-mile rail system that operates at a speed of 186 mph with a passenger volume of 150,000 passengers per day. The construction of the project started on December 1999, and was scheduled to complete October 2005. As of the end of 2006, the project has not yet completed as planned.

In early 1990’s, the government of Taiwan started planning the development of a high-speed rail system in the island’s western corridor. In Sep 1997, The Taiwan High Speed Rail Consortium (THSRC), founded by five domestic companies (hereafter the parent firms), outbid the Chinese Development Consortium by promising to pay USD 3.1B to the government in the end of the railway concession. In return, THSRC is awarded a contract that includes a 35-year high-speed railway concession and a 50-year land development concession.

With the USD 15B capital investment, THSR is the largest Build-Operate-Transfer project in the world. The contract specifies that the government will finance 21% of the construction cost (USD 3.1B), whereas the remaining 79% (USD 11.99B) will be financed with private funding. Within private funding, it is specified that 69% of the private funding (USD 8.23B) will be financed with debt, whereas the remaining 31% (USD 3.76B) will be financed with equity from THSRC. Furthermore, the contract demands that the 5 parent firms of THSRC should account for at least 51% of equity, institutional investors should account for 29-34% of equity, and the remaining 12-20% equity can be raised from public offering in the local stock market. In addition, a USD 0.5B performance bond is required to guarantee the realization of the project. Due to the huge scale of the project, the contract specifies a multi-phase financing schedule.

(Stage 1) November 1997

Plan: At this stage, a capital investment of USD 9.7B (80% of total capital investment) should be in place. Among the USD 9.7B capital investment, USD 8.23B (100% of total debt) should be in debt, and the remaining USD 1.47B (40% of total equity) should be in equity.

Result:

Debt financing:
After encountering difficulties in deriving debt funding from both local and foreign banks, THSRC filed a USD 8.23B loan application to the Council for Economic Planning and Development (CEPD) by resorting to the Construction and Operation Agreement (C&OA) in the contract, which states: “The government should assist THSRC in applying for mid-term to long-term funds with the supports from CEPD.”

After renegotiation, CEPD agreed to deposit USD 6.18B of postal funds (which accounts for 75% of the USD 8.23 debt) into 25 banks, which in turn lent a 6.18B syndicate loan to THSRC with the postal funds as the mortgage. Since the syndicate debt is directly secured by the government’s postal fund, the debt contract specifies a “forced buyout and government takeover” clause that entitles the government the control rights of the project upon default. This provision makes the syndicate debt a full-recourse debt. The government also agreed that the remaining USD 2.05B debt could be raised in stage 2 (in the original plan, there is actually no stage 2).

Equity financing:

THSRC also renegotiated equity investment with the government. After renegotiation, among the USD 1.47B equity that was due to be raised at this phase, 16% of the equity funding (USD 253M) came from the government, while the remaining 84% of the equity funding (USD 1.23B) came from the 5 parent firms.

(Stage 2) December 1999

Plan: At this stage, a USD 2.05B debt (25% of total debt) was to be secured from foreign banks.

Result:

THSRC remained unable to elicit the USD 2.05B debt (25% of total debt) from foreign banks in spite of the backing of 6.18B debt (75% of total debt) secured in the first stage. The Japan bullet train system (Shinkansen), formerly a member of the Chinese Development Consortium, was outbid in the bidding phase. After THSRC experienced difficulties in securing the USD 2.05B loan, Shinkansen approached THSRC with a proposal that offered THSRC the assistance to get the USD 2.05B export loan (about 85% of the core system contract fee) from the Export-Import Bank of Japan (EIBJ, later became Japan Bank for International Cooperation), and promised to invest at least 10% of the equity of THSRC.

Shinkansen’s proposal gained them the core system contract. However, the change of the core system contractor entailed a lawsuit from the Euro Train Group, who was a member of THSRC’s joint venture and an incumbent contractor for the core system. It eventually cost THSRC USD 20M to settle this lawsuit.

After Shinkansen gained the core system contract, THSRC remained unable to derive the loans from EIBJ due to some strict terms EIBJ imposed on this project. Under such condition, THSRC again turned to CEPD for more aids on loan. After negotiation, CEPD agreed to install another USD 882M pension funds in the 25 banks as the source of another USD 882M syndicate loan. The 25 banks then approved the remaining USD 1.176B loan to THSRC without the backing of any government funds.
At this phase, THSRC completed the required debt financing by deriving 86% of the loan from the government, and 14% of the loan from the 25 local banks – a big deviation from the original plan which specifies a 75% loan from local banks, and 25% loan from foreign banks.

(Stage 3) December 2002 and onwards

Plan: THSRC was to secure the remaining equity investment of USD 2.3 B to complete the project.

Result:

Starting from December 2002, THSRC launched a sequence of public offerings for its convertible, preferred stocks. The features of this convertible preferred stock are: (1) a promised 5% yearly interest rate, but without the right to vote. (2) Without the right to get dividends. (3) Period: six year, but can be extended to 7 years and 1 month (4) After possessing this convertibles for three years, the investors can choose to convert the stocks to common stock, but they will lose the predetermined, promised 5% interest rate. It is perceived that private financial institutions, insurance companies, and government-owned institutions are prospective stockholders.

To promote the market's interests in THSRC's public offering, the government revised the Statute for the Encouragement of Private Participation in Major Transportation Projects (SEPP) be adding the following terms: “For those entities/people who hold the stocks of domestic major transportation projects or significant infrastructures for more than two years, these entities/people can claim 20% of their investments as deductibles from taxable income, starting from year 2.”

The company aimed to attract USD 0.54B of equity by the end of 2002. Due to the tax incentive created by the government, the convertible, preferred stock offering attracted USD 0.79B of investment by January 2003, which is USD 0.25B higher than the original target. In the subsequent public offerings, THSRC managed to raise the required USD 2.3 B equity from the local stock market.

As of May 2005, THSRC reported a forecasted cost overrun of USD 1.5B. On October 2005, two government agencies invested USD 0.2B to THSRC in the form of equity. On July 2006, THSRC secured the remaining capital by signing a syndicate loan of USD 1.3 B from 7 local banks.

As of the end of 2006, the construction cost has increased from USD 15B to USD 16.5B and the project is 1 year behind schedule. In terms of equity funding, 27% is from the 5 parent firms, 19% is from the government, and 54% is from institutional investors and the general public. In terms of debt, 86% is from the government and 14% is from the 25 banks. The initial financial plan and the resulting financial structure are shown in table 4-1.
Table 4-1 The planned and the resulting financing sources of THSR

<table>
<thead>
<tr>
<th></th>
<th>Plan</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Debt</strong> (USD 8.23B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government</td>
<td>0</td>
<td>86%</td>
</tr>
<tr>
<td>Local banks</td>
<td>75%</td>
<td>14%</td>
</tr>
<tr>
<td>Foreign Banks</td>
<td>25%</td>
<td>0</td>
</tr>
<tr>
<td><strong>Equity</strong> (USD 3.76B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government</td>
<td>0</td>
<td>18.86%</td>
</tr>
<tr>
<td>5 parent firms</td>
<td>51%</td>
<td>27%</td>
</tr>
<tr>
<td>Institutional investors and general public</td>
<td>49%</td>
<td>54%</td>
</tr>
</tbody>
</table>


4.2 Analysis of the THSR contract

I proceed with my analysis with two different approaches. In the first approach, I analyze THRC’s financial contract using my theoretical model. In the second approach, I discuss all the possible agency problems underlying this contractual relationship in a descriptive manner.

(1) Analysis based on the theoretical model

The unique feature of the THRC project is the continuous increase of the government’s financial investment in the project after each round of renegotiation. We have shown in proposition 1 that when the social cost of public fund is low, the government will make up the capital shortfall in the event of default. More specifically, the government reinvests if \( \frac{Y}{\alpha} > 1 + \delta \) (or \( \frac{1 + r_x}{1 + r_g} > 1 + \delta \)).

It is known that the tax rate in Taiwan is about 19%. The average interest rate for Taiwan government’s 10 year bond is about 2-2.5%. For the government to reinvest, it must be that that the cost of capital of THSRC exceeds 21%. This value is certainly high in the standard of corporate cost of capital. However, for a project that involves a development cost much greater than the sum of the 5 parent firm’s prior-to-THRC market value, it is highly likely that the cost of capital for this specific project has substantially changed the parent firms’ costs of capital.

The high cost of capital for THSRC may come from the high uncertainties on both the cost side and the revenue side of the underlying project. The fact that THSRC was not able to derive loans from both local and foreign banks without government-backing mortgage has shown the participants’ strong concern about the substantial amount of risks underlying this project. As a result, the interest rate for the loan that can be offered to THSRC is unacceptably high.

On the other hand, the fact that the government is willing to renegotiate the financial structure even before the project started signaled a strong buy-in effect, which leads to further distortion in subsequent renegotiation. In other words, the opportunity cost of public funds may be lower in real terms than in nominal term such that government reinvestment is justified even with a moderate cost of capital for THSRC.

(2) Other agency issues

There are other factors that may result in this buy-in phenomenon.
(i) Political factors:

The THSR project is a highly political sensitive project. The population in Taiwan is historically well distributed along the west corridor of the island. It is generally believed that the high speed rail project will help industries move jobs to rural areas. Therefore voters, especially rural area voters, are concerned about the realization of the project. The government therefore has a high political stake in the project.

(ii) Macroeconomic factors:

Taiwan’s economy plunged from an average annual growth rate of 6-7% to an annual growth rate of 3-4% starting from year 2000. A key measure the government pursues to counter economic meltdown and the rising unemployment is by increasing public expense in infrastructure development. The government agent CEPD estimates that the construction of THSR has created 480,000 jobs and may contribute 1 percentage point to annual economic growth. A termination of the project will hurt the already sluggish economy.
Chapter 5 Summary and Future Research Directions

The objective of this research is to establish the functional relationship between financial structure and agent’s effort incentive in the realm of PPP contracting. In this chapter I summarize my major findings and discuss each determinant factor’s effect over the agent’s effort incentive. I suggest how this research can be applied to PPP contract design and suggest further research directions.

5.1 Research Summary

Here are the major findings from the theoretical analysis.

(1) Financial structure and the possible scenario outcomes (contract types)

I show that ex ante capital structure and control rights regime jointly determine the possible equilibrium outcomes that can occur (the agent’s effort level and the uncertainty of cost doesn’t matter). This phenomenon is inferred from Proposition 3 and Proposition 9 which state that the ordering of the break-even costs that separate the four scenarios are determined exclusively by capital structure and control rights regime. As a result, this property also holds when the effect of the agent’s cost-reducing effort is uncertain.

(2) The agent’s investment strategy

The agent’s optimal cost-reducing effort is determined by a hold-up factor and a self-enforcement factor. The former acts as a strict disincentive to the agent’s effort, whereas the latter can be an incentive or a disincentive to the agent’s effort depending on the capital structure of the contract.

I show that the concern of held-up results in the agent’s effort being multiplying by a weight sum of the agent’s share of gain in each specific scenario outcomes. On the other hand, the self-enforcement effect is characterized by the relative value of revenue and performance bond with respect to debt. A high revenue to debt ratio (and also a high performance bond to debt ratio) creates a high power incentive, whereas a low revenue to debt ratio (and also a low performance bond to debt ratio) creates a low power incentive. The revenue and the performance bond are essentially substitutes in the formulation of the self-enforcement effect.

(3) The effect of debt

I show that for any given initial capital investment, the agent’s optimal cost-reducing effort decreases with the magnitude of debt in all full-recourse debt and non-recourse debt contract types. This phenomenon holds when there exhibits one unique solution for the agent’s optimal effort.

(4) The effect of performance bond

I show that for any given initial capital investment, the agent’s optimal cost-reducing effort increases with the magnitude of a performance bond when the contract is one with no positive self-enforcement effect. This phenomenon holds when there exhibits one unique solution for the agent’s optimal effort.
(5) The effect of technology
I show that in a full-recourse debt contract the type of technology doesn’t directly affect the possible equilibrium outcomes, instead it only affects the agent’s payoff in the renegotiation game (for technology determines the new agent’s cost, which in turn determines the government’s default status quo point).

In the non-recourse debt contract, the type of technology has a direct impact on the possible equilibrium outcomes. For projects that involve high switching-cost technologies, the contract can be Type N-I and type N-II contracts. For projects that involve low switching-cost technologies, the contract can be Type N-III, Type IV, and Type N-V contracts.

5.2 Future Research Directions
Along the course of this study I unearthed several research problems that deserve further exploration. Below is a list some that I believe are important for understanding PPP contracting.

(1) The subsidy for financially irrational projects

It is common that countries develop certain public projects for economic reasons versus financial reasons. A project is financially irrational if the consumer surplus is lower than the cost to develop the project. It is well recognized that some public projects (i.e. water treatment) are financially irrational but are critical for economic development. Projects of this type are typically financed thorough both consumer surplus and government subsidy.

In our model it is assumed that the consumer surplus is always greater than the development cost of the project. This assumption, together with Assumption 9 (which states that the social cost of public funds is expensive), removes the possibility for a government subsidy to occur. By doing so I greatly reduce the number of contract types that a PPP contract can assume. Certainly the relaxing of the financial rational assumption does not require a big extension from our existing model. But I believe it will yield important insights into the role of public funding in PPP contracting.

(2) The relationships among auction, bidding and financial contracting in the PPP realm

It is always of major interest to understand how to design a PPP contract to induce socially optimal investment. The complete contract literature offers many interesting design mechanisms to implement first best investment, but they do not to address the out-of-equilibrium condition where renegotiation can be used to achieve Pareto improvement. This research establishes the functional relationship between financial contract and agent’s effort incentive but it does not propose a method to design the optimal contract. It will therefore be of great value to provide a model that either combines the two modeling frameworks, or explain the relationship between these two issues (i.e. renegotiation and the design of the first best contract).
In the real world, as illustrated in chapter 1, the contracting process is often made up of two stages. In the first stage, a first round public bidding process is used to pre-select or narrow down the number of potential agents. A second round negotiation is then used to finalize the contract. The second round contract (which is often made up of incomplete financial provisions) is often developed based on the common agreements or announcements made in the first round bidding stage. It is then interesting to see whether this type of two-stage mechanism is practical in implementing the first best contract, or how to devise a mechanism to implement the first best contract under this two-stage contracting framework.

(3) The hierarchical agency problem of “tunneling”

Tunneling refers to a phenomenon where an agent ships out money or goods from the contracted project to an affiliated agent without been detected by the principal. The hierarchical contractual relationship in PPP projects exhibits substantial opportunities for this to occur. Tunneling is a possible source of cost overruns, which may lead to default and renegotiation.

Existing literature on sub-contracting and contract delegation has shed light on relevant issues. But so far there is no clear answer as for why in most PPP contracts the members of a joint venture consortium are almost always allowed to be the subcontractors for the executive entity without being regulated. Understanding such problem is, I believe, critical for improving the development of PPP contracts.
Appendix

Proof of Proposition 1:
We examine the properties of $I^g_t$, $I^o_t$, and $R_t$ before analyzing the Pareto frontier. Following assumption 2, the parties’ reinvestments is set equal to the capital shortfall:

$$I^g_t + I^o_t = x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}$$

The parties’ new investments ($I^g_t$, $I^o_t$) therefore satisfy:

$$I^g_t \in I^G = [0, x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}]$$
$$I^o_t \in I^A = [0, x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}]$$

The renegotiated revenue $R_t$ should satisfy two conditions (1) the government’s payoff with the renegotiated new contract is greater than her payoff with the new agent:

$$R_t \geq \left(1 + \frac{\delta}{\gamma}\right) \left(1 + f\right) \left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{1}{\beta}\frac{D_0}{\alpha} - I^g_t - \frac{H}{\alpha \gamma}$$

and (2) the agent’s payoff with the renegotiated new contract is positive:

$$R_t \geq \frac{1}{\alpha} \left(x - \psi(e) - \frac{I_0}{\alpha} - I^g_t\right) - \frac{1}{\beta^2} \frac{D_0}{\alpha^2} - \frac{H}{\alpha^2}$$

Let $R_t = \bar{R}_t = [R_{t^*}, R_{t^*}]$, it is clear that:

$$R_{t^*} = \frac{1}{\gamma} \left(1 + f\right) \left(x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}\right) + \frac{1}{\beta}\frac{D_0}{\alpha} - I^g_t - \frac{H}{\alpha \gamma}$$
$$R_{t^*} = \frac{1}{\alpha} \left(x - \psi(e) - \frac{I_0}{\alpha} - I^g_t\right) - \frac{1}{\beta^2} \frac{D_0}{\alpha^2} - \frac{H}{\alpha^2}$$

After the properties of $I^g_t$, $I^o_t$, and $R_t$ are determined, we can now analyze the Pareto frontier of the renegotiation game:

$$\max_{I^g_t \in I^G} \alpha R_t - x + \psi(e) + \frac{I_0}{\alpha} + I^g_t + \left(\frac{1}{\alpha} - \frac{\alpha}{\beta^2}\right)D_0 + \frac{H}{\alpha}$$

s.t.

$$\gamma(S - R_t) - (1 + \delta)I^g_t = v$$

Where $v$ is a fixed value belongs to the government’s feasibility utility set.

Substituting the objective function with $R_t = S - \frac{1}{\gamma}\left[(1 + \delta)I^g_t + v\right]$, the coefficient of $I^g_t$ is

$$1 - \frac{\alpha}{\gamma} (1 + \delta)$$

It then follows that if $\frac{\gamma}{\alpha} < 1 + \delta$, the agent’s utility is maximized by setting $I^g_t$ at the lower bound ($I^g_t = 0$ and therefore $I^g_t = x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}$ following the new contract’s
budget balance constraint). If \( \frac{\gamma}{\alpha} > 1 + \delta \), the agent’s utility is maximized by setting \( I_1^* \) at the upper bound (\( I_1^* = x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \) and therefore \( I_1^* = 0 \)). Proposition 1 is then proved. □

**Proof of Proposition 2:**
The government’s Best Alternative To a Negotiated Agreement (BATNA) is:

\[
V_{1(BATNA)} = \gamma S - (1 + \delta) \left[ (1 + f) \left( x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\beta} \right) + \frac{D_0}{\beta} \right] + \frac{H}{\alpha}
\]

The agent’s BATNA is: \( U_{1(BATNA)} = 0 \)
The government’s date 1 net payoff with the new contract is:

\[
V_{1(New\_contract)} = \gamma (S - R_1)
\]
The agent’s date 1 net payoff with the new contract is:

\[
U_{1(New\_contract)} = \alpha R_1 - x + \psi(e) + \frac{I_0}{\alpha} + I_1^* + \left( \frac{1}{\alpha} - \frac{\alpha}{\beta^2} \right) D_0 + \frac{H}{\alpha}
\]

Solving for the maximization program of the Nash product:

\[
\max_{R_1 \in R_1}\left( U_{1(New\_contract)} - U_{1(BATNA)} \right) V_{1(New\_contract)} - V_{1(BATNA)}
\]

Substituting the above program with \( \Delta_1 \) and \( \Delta_2 \):

\[
\max_{R_1 \in R_1} -2\alpha r_1^2 + \left\{ \Delta_1 \left( x - \psi(e) - \frac{I_0}{\alpha} \right) + \Delta_2 D_0 - \left( 1 + \frac{\gamma}{\alpha} \right) H \right\} R_1
\]

The first order condition is:

\[
-2\alpha r_1 + \left\{ \Delta_1 \left( x - \psi(e) - \frac{I_0}{\alpha} \right) + \Delta_2 D_0 - \left( 1 + \frac{\gamma}{\alpha} \right) H \right\} = 0
\]

So we have:

\[
R_1 = \frac{\Delta_1}{2\alpha r} \left( x - \psi(e) - \frac{I_0}{\alpha} \right) + \frac{\Delta_2}{2\alpha^2} D_0 - \frac{(\alpha + \gamma)H}{2\alpha^2 \gamma}
\]

Proposition 2 is then proved. □

**Proof of Proposition 3:**

(1) Definition 1 and definition 3 show that \( x_1 - x_3 = \frac{\alpha (\beta (1 + \delta) - \gamma)}{\beta^2 \Delta_3} D_0 - \frac{(\alpha - \gamma)}{\alpha \Delta_3} H \). By assumption 9 we have \( \alpha (1 + \delta) > \gamma \) so \( \Delta_3 > 0 \). By assumption \( \alpha < \beta < \gamma \) and together with assumption 9 we have \( \beta (1 + \delta) > \alpha (1 + \delta) > \gamma \) so \( \beta (1 + \delta) - \gamma > 0 \). (1) is then proved.

(2) is derived by equating \( x_1 \) in definition 1 and \( x_2 \) in definition 2.

(3) is derived by equating \( x_2 \) in definition 2 and \( x_3 \) in definition 3.

Proposition 3 is then proved. □

**Proof of Proposition 4:**
Following the decision rule specified in the beginning of this section, the equilibrium scenario outcomes under each realized cost can be analyzed as follows:

i. If the contract specifies $R_0 > \Delta_5D_0 + \frac{(y-\alpha)}{2\alpha^2\gamma}$, the ordering of the break-even costs is $x_2 > x_1 > x_3$.
   (a) If $x > x_2$, the government will not take over the project (for $x > x_1$) and the agent will not self-enforce the contract (for $x > x_2$) so they renegotiate a new contract.
   (a) If $x_2 > x > x_1$, the government will not take over the project (for $x > x_1$) but the agent will self-enforce the contract (for $x > x_2$). For the agent makes self-enforcement decision before the parties make the renegotiation decision so the agent reinforces the contract.
   (b) If $x < x_1$, the project can be completed with the initial capital investment.

ii. If the contract specifies $R_0 < \Delta_5D_0 + \frac{(y-\alpha)}{2\alpha^2\gamma}$, it can be either

   $\Delta_5D_0 + \frac{(y-\alpha)}{2\alpha^2\gamma}H > R_0 > \Delta_6D_0 + \frac{(1-(1+\delta)(1+f))}{\alpha\Delta_3}H$ or

   $\Delta_6D_0 + \frac{(1-(1+\delta)(1+f))}{\alpha\Delta_3}H > R_0$.

1. If the contract specifies $\Delta_5D_0 + \frac{(y-\alpha)}{2\alpha^2\gamma}H > R_0 > \Delta_6D_0 + \frac{(1-(1+\delta)(1+f))}{\alpha\Delta_3}H$, the ordering of the break-even costs is $x_1 > x_2 > x_3$.
   (a) If $x > x_1$, the government will not take over the project (for $x > x_1$) and the agent will not self-enforce the contract (for $x > x_2$) so they renegotiate a new contract.
   (b) If $x < x_1$, the project can be completed with the initial capital investment.

2. If the contract specifies $\Delta_6D_0 + \frac{(1-(1+\delta)(1+f))}{\alpha\Delta_3}H > R_0$, the ordering of the break-even costs is $x_1 > x_3 > x_2$.
   (a) If $x > x_1$, the government will not take over the project (for $x > x_1$) and the agent will not self-enforce the contract (for $x > x_2$) so they renegotiate a new contract.
   (b) If $x < x_1$, the project can be completed with the initial capital investment.

It turns out that when $R_0 < \Delta_5D_0 + \frac{(y-\alpha)}{2\alpha^2\gamma}H$, both

$\Delta_5D_0 + \frac{(y-\alpha)}{2\alpha^2\gamma}H > R_0 > \Delta_6D_0 + \frac{(1-(1+\delta)(1+f))}{\alpha\Delta_3}H$ and
\[
\Delta_0 D_0 + \frac{(1-(1+\delta)(1+f))}{\alpha \Delta_3} H > R_0 \text{ yield the same possible scenario outcomes: no reinvestment and new contract. Proposition 4 is therefore proved.} \]

**Proof of Proposition 5:**

The agent’s optimal effort in the Type F-I contract satisfies:

\[
\psi'(e) = \frac{1}{\Phi_6}
\]

where \( \Phi_6 = \alpha \frac{\Delta_3 F(x_2) - \Delta_3}{2\gamma} \)

The agent’s optimal effort in the Type F-II contract satisfies:

\[
\psi'(e) = \frac{1}{\Phi_7}
\]

where \( \Phi_7 = \left( \alpha^2 R_0 - \alpha^2 \Delta_3 D_0 + \frac{(\alpha + \gamma) H}{2\gamma} f(x_1) + \frac{\alpha}{2\gamma} \Delta_3 F(x_1) - \Delta_3 \right) \)

For \( \psi(\cdot) \) is a strictly increasing and concave function. The agent’s optimal effort for a Type I contract is higher than his optimal effort for a Type II contract if \( \Phi_6 > \Phi_7 \).

From proposition 16 we know that \( R_0 + \Delta_3 D_0 - \frac{(y - \alpha) H}{2\gamma} < 0 \) for a Type II contract. Also we know \( F(x_2) > F(x_1) \) for a Type I contract so it follows that \( \Phi_6 > \Phi_7 \): given any initial capital investment \( \pi \in \pi_\gamma \), the agent’s optimal cost-reducing effort with a Type F-I contract is higher than his optimal cost-reducing effort with a Type F-II contract. Proposition 5 is then proved. \( \square \)

**Proof of Proposition 6:**

From comparative statics, the sign of \( \frac{\partial e}{\partial D_0} \) is the same as the sign of \( \frac{\partial^2 EU_0}{\partial e \partial D_0} \) and also the sign of \( \frac{\partial e}{\partial H} \) is the same as the sign of \( \frac{\partial^2 EU_0}{\partial e \partial H} \) when there exists a unique solution of optimal \( e \).

(1) The effect of debt over the agent’s optimal effort:

(1-i) Type F-I contract:

\[
\frac{\partial^2 EU_{0}(Type \ F^{-I})}{\partial e \partial D_0} = -\alpha^2 \Delta_3 f(x_1) \psi'(e) < 0
\]

(1-ii) Type F-II contract:

\[
\frac{\partial^2 EU_{0}(Type \ F^{-II})}{\partial e \partial D_0} = -\alpha^2 \Delta_3 f(x_1) \psi'(e) < 0
\]

From (1-i) and (1-ii), the sign of \( \frac{\partial e}{\partial D_0} \) is strictly negative in both Type F-I and Type F-II contracts.

(2) The effect of performance bond over the agent’s optimal effort:

(2-i) Type F-I contract:

\[
\frac{\partial^2 EU_{0}(Type \ F^{-I})}{\partial e \partial H} = \frac{(\alpha + \gamma) f(x_2)}{2\gamma} \psi'(e) > 0
\]

89
(2-ii) Type F-II contract: 
\[ \frac{\partial^2 EU_{\text{Type F-II}}}{\partial \varepsilon \partial H} = \frac{(\alpha + \gamma)}{2\gamma} f(x_1) \psi'(e) > 0 \]

From (2-i) and (2-ii), the sign of \( \frac{\partial \varepsilon}{\partial H} \) is strictly positive in both Type F-I and Type F-II contracts.

Combing (1) and (2) we know for any given initial capital investment \( \pi \in \pi \), the agent’s optimal cost-reducing effort decreases with the level of debt, and increases with the level of the performance bond in both Type F-I and Type F-II contracts. Proposition 6 is then proved. □

**Proof of Proposition 7:**

We know that (1) \( R_0 - \frac{H}{\alpha \gamma} \) is the government’s BATNA revenue (i.e. \( R_0 - \frac{H}{\alpha \gamma} \) is the upper bound of the agent’s renegotiated revenue), and (2) under assumption 9 the government’s reinvestment \( I^g \) is 0 after renegotiation. For every payment \( P \) the agent pays to the bank to retain the ownership of the project

\[ P \in \left[ \frac{\beta}{\alpha} R_0 - \frac{(1+f)}{\alpha} (x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}), \frac{\beta}{\alpha} S - \frac{(1+f)}{\alpha} (x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}) \right] \]

the agent’s ex post renegotiation payoff \( \alpha R_1 - x + \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha} - \alpha P \) is strictly smaller then his non-renegotiation payoff \( \alpha R_0 - x + \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha} - \alpha P \). Therefore the agent will not renegotiate with the government over \( (I^g_1, I^g_1, R_1) \).

The complete proof is shown as follows: We first examine the properties of \( I^g_1, I^a_1 \), and \( R_1 \) before analyzing the Pareto frontier. Following assumption 1, the parties’ reinvestments is set equal to the capital shortfall:

\[ I^g_1 + I^a_1 = x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \]

The parties’ new investments \((I^g_1, I^a_1)\) therefore satisfy:

\[ I^g_1 \in I^g = [0, x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}] \]

\[ I^a_1 \in I^a = [0, x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}] \]

Let \( P \in \left[ \frac{\beta}{\alpha} R_0 - \frac{(1+f)}{\alpha} (x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}), \frac{\beta}{\alpha} S - \frac{(1+f)}{\alpha} (x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}) \right] \) be the fee that the agent pays to the bank to retain the ownership of the project. The renegotiated revenue \( R_1 \) should satisfy two conditions (1) the government’s payoff with the renegotiated new contract is greater than her payoff with the new agent scenario outcome:

\[ R_1 \leq R_0 - \frac{(1+\delta)}{\gamma} I^g_1 - \frac{H}{\alpha \gamma} \]

and (2) the agent’s payoff with the renegotiated new contract is non-negative:

\[ \beta R_0 - (1+f) \left( x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \right) \] is the bank’s payoff for turning down the agent’s offer.
\[ R_i \geq \frac{1}{\alpha} \left( x - \psi(e) - \frac{I_0}{\alpha} - I_1^g - \frac{D_0}{\alpha} \right) + p - \frac{H}{\alpha^2} \]

Let \( R_i \in R_1 = [R_i, R_1^*] \), it is clear that:
\[
R_i^* = R_0 - \frac{(1+\delta)I_1^g}{\gamma} - \frac{H}{\gamma} \\
R_{i^*} = \frac{1}{\alpha} \left( x - \psi(e) - \frac{I_0}{\alpha} - I_1^g - \frac{D_0}{\alpha} \right) + p - \frac{H}{\alpha^2} \]

After the properties of \( I_1^g, I_1^s \), and \( R_i \) are determined, we can now analyze the Pareto frontier of the renegotiation game:
\[
\max_{I_1^g \in \mathcal{I}_1^g} \alpha R_i - x + \psi(e) + \frac{I_0}{\alpha} + I_1^g + \frac{D_0}{\alpha} - \alpha P + \frac{H}{\alpha} \\
\text{s.t.} \\
\gamma(S - R_i) - (1+\delta)I_1^g = \nu
\]

Where \( \nu \) is a fixed value belongs to the government’s feasibility utility set.

Substituting the objective function with \( R_i = S - \frac{1}{\gamma} \left( (1+\delta)I_1^g + \nu \right) \), the coefficient of \( I_1^g \) is
\[
1 - \frac{\alpha}{\gamma} (1+\delta) \quad \text{By assumption 9 we have} \quad \frac{\nu}{\alpha} < 1 + \delta \quad \text{so the agent’s utility is maximized \( I_1^g \) is set at}
\]
the lower bound (\( I_1^g = 0 \) and therefore \( I_1^s = x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha} \)).

The Pareto frontier of the renegotiation game is a straight line with the government and the agent bargaining over \( R_i \in [R_1^*, R_1^\ast] \). Since the bargaining game is a Nash bargaining game, \( R_i \) can be determined as follows:

The agent’s BATNA is: \( U_{i(BATNA)} = 0 \)
The government’s BATNA is:
\[
V_{i(BATNA)} = \gamma(S - R_0) + \frac{H}{\alpha}
\]
The agent’s date 1 net payoff with the new contract is:
\[
U_{i(New\_Contract)} = \alpha R_i - x + \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha} - \alpha P + \frac{H}{\alpha}
\]
The government’s date 1 net payoff with the new contract is:
\[
V_{i(New\_contract)} = \gamma(S - R_i)
\]
Solving for the maximization program of the Nash product:
\[
\max_{R_i \in R_1^*} (U_{i(New\_contract)} - U_{i(BATNA)})(V_{i(New\_contract)} - V_{i(BATNA)})
\]
The first order condition is:
\[
2\alpha R_i - \left[ \alpha R_i + \alpha P + (x - \psi(e) - \frac{I_0}{\alpha} - \frac{D_0}{\alpha}) \right] (\frac{(\alpha + \gamma)}{\gamma}) = 0
\]
So we have:
\[ R_1 = \frac{1}{2} \left[ R_0 + P + \frac{1}{\alpha} \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} \right] \frac{(\alpha + \gamma)}{\alpha \gamma} H \]

Substituting \( U_{1,\text{New\_contract}} \) with \( R_1 \), the agent’s date 1 payoff after renegotiation with the government is:

\[ U_{1,\text{New\_contract}} = \frac{1}{2} \left( \alpha(R_0 - P) - x + \psi(e) + \frac{I_0}{\alpha} + \frac{D_0}{\alpha} \right) + \frac{(\alpha - \gamma)}{\alpha \gamma} H \]

The agent’s date 1 payoff without renegotiation with the government is\(^{10}\):

\[ U_{1,\text{NRG}} = \alpha(R_0 - P) - x + \psi(e) + I_0 + D_0 + \frac{\alpha H}{\gamma^2} \geq U_{1,\text{New\_contract}} \]

The agent’s payoff with the new contract is smaller than the agent’s payoff with the existing payoff so the agent will not renegotiate with the government over \((I^a, I^g, R_i)\). Proposition 7 is then proved. \( \square \)

**Proof of Proposition 8:**

1. The renegotiated fee \( P \) should satisfy two conditions: (i) the agent’s payoff with the new contract is greater than his payoff with no agreement:

\[ P \leq R_0 - \frac{1}{\alpha} \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} \frac{H}{\alpha^2} \]

and (ii) the bank’s payoff with the new contract is greater than his payoff with the new agent:

\[ P \geq R_0 - \frac{1 + f}{\beta} \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} \frac{H}{\alpha^2} \]

The necessary and sufficient condition for a feasible \( P \) to exist is for the upper bound of \( P \) to be greater or equal to the lower bound of \( P \):

\[ \left( \frac{\alpha(1 + f) - \beta}{\alpha \beta} \right) \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} + \frac{H}{\gamma^2} \geq 0 \]

2. If \( \left( \Delta_1 - \frac{1}{\alpha} \right) \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} + \frac{H}{\gamma^2} \geq 0 \) holds, the agent’s payment to the bank \( P \) must satisfy:

\[ P \in \mathcal{P} = [P_-, P_+] = \left[ R_0 - \frac{1 + f}{\beta} \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha}, R_0 - \frac{1}{\alpha} \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} + \frac{H}{\alpha^2} \right] \]

Given the assumption that the bargaining game is a Nash bargaining game, \( P \) can be determined as follows:

The agent’s BATNA is:

\[ U_{1,\text{BATNA}} = 0 \]

The bank’s BATNA is:

\[ W_{1,\text{BATNA}} = \beta R_0 - (1 + f) \left( x - \psi(e) \right) \frac{I_0}{\alpha} \frac{D_0}{\alpha} \]

The agent’s date 1 net payoff with the new contract is:

\[ \text{For the agent to reinvest, it must be that } U_{1,\text{NRG}} \geq 0 \]

92
$$U_{1(\text{New\_Contract})} = \alpha (R_0 - P) - x + \psi(e) + \frac{1}{\alpha}(I_0 + D_0 + H)$$

The bank’s date 1 net payoff with the new contract is:
$$W_{1(\text{New\_Contract})} = \beta P$$

Solving for the maximization program of the Nash product:
$$\max_{\beta \in \mathbb{R}} \left( U_{1(\text{New\_contract})} - U_{1(\text{BATNA})} \right)$$
$$\left( W_{1(\text{New\_contract})} - W_{1(\text{BATNA})} \right)$$

The first order condition is:
$$2\alpha \beta P - 2\alpha \beta R_0 + \left( \beta + \alpha (1 + f) \right)(x - \psi(e) - I_0 - D_0) - \frac{\beta H}{\alpha} = 0$$

So we have:
$$P = R_0 - \frac{\beta + \alpha (1 + f)}{2\alpha} (x - \psi(e) - I_0 - D_0) + \frac{H}{2\alpha^2}$$

Proposition 8 is then proved. □

Proof of Proposition 9:
(1) Calculate $x_1 - x_3$ directly:
$$x_1 - x_3 = \frac{1}{2\alpha (\alpha \Delta_\gamma - 1)} H$$

Because $\alpha \Delta_\gamma - 1 = \frac{1}{2} \left( \frac{\alpha (1 + f)}{\beta} - 1 \right)$, it follows that $x_1 \geq x_3$ if $\frac{\beta}{\alpha} \leq 1 + f$, and $x_1 < x_3$ if $\frac{\beta}{\alpha} > 1 + f$.

(2) is derived by equating definition 1 with definition 2.
(3) is derived by equating definition 2 with definition 3.

Proposition 21 is then proved. □

Proof of Proposition 10:
Following the decision rule specified in the beginning of this section, the equilibrium scenario outcomes under each realized cost can be analyzed as follows:

(a) If the project exhibits $\frac{\beta}{\alpha} < 1 + f$:

(i) If the contract specifies $R_0 > \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2}$, the ordering of the break-even costs is $x_2 > x_1$.

a. If $x > x_2$, the bank will want to renegotiate a new contract (for $\frac{\beta}{\alpha} < 1 + f$ and $x > x_3$) and the agent will not self-enforce the contract (for $x > x_2$) so they renegotiate a new contract.

b. If $x_2 > x > x_1$, the bank will want to renegotiate a new contract (for $\frac{\beta}{\alpha} < 1 + f$ and $x > x_3$) but the agent will self-enforce the contract (for $x < x_2$). Because the agent makes self-enforcement decision before the parties make the renegotiation decision so the agent self-enforces the contract.

93
c. If $x < x_1$, the project can be completed with the initial capital investment so no reinvestment is needed.

(ii) If the contract specifies $R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2}$, the ordering of the break-even costs is $x_2 < x_1$.

a. If $x > x_1$, the bank will want to renegotiate a new contract (for $\frac{\beta}{\alpha} < 1+f$ and $x > x_3$) and the agent will not self-enforce the contract (for $x > x_2$) so they renegotiate a new contract.

b. If $x < x_1$, the project can be completed with the initial capital investment so no reinvestment is needed.

(b) If the project exhibits $\frac{\beta}{\alpha} > 1+f$:

(i) If the contract specifies $R_0 > \frac{D_0}{\rho^2} + \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)}$, the ordering of the break-even costs is $x_2 > x_3 > x_1$.

a. If $x > x_2$, the bank will want to take over the project (for $\frac{\beta}{\alpha} > 1+f$ and $x > x_3$) and the agent will not self-enforce the old contract (for $x > x_2$) so the bank brings in a new agent to complete the project.

b. If $x_2 > x > x_1$, the bank may want to take over the project (when $x_3 < x < x_2$) or renegotiate a new contract (when $x_1 < x < x_3$). But the agent will self-enforce the contract (for $x < x_2$). Because the agent makes self-enforcement decision before the bank makes the take-over decision so the agent self-enforces the contract.

c. If $x < x_1$, the project can be completed with the initial capital investment so no reinvestment is needed.

(ii) If the contract specifies $\frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} < R_0 < \frac{D_0}{\rho^2} - \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)}$, the ordering of the break-even costs is $x_3 > x_2 > x_1$.

a. If $x > x_3$, the bank will want to take over the project (for $\frac{\beta}{\alpha} > 1+f$ and $x > x_3$) and the agent will not self-enforce the contract (for $x > x_2$) so the bank brings in a new agent to complete the project.

b. If $x_2 < x < x_3$, the bank will want to renegotiate a new contract (for $\frac{\beta}{\alpha} > 1+f$ and $x < x_3$) and the agent will not self-enforce the old contract (for $x > x_2$) so they renegotiate a new contract.

c. If $x_1 < x < x_2$, the bank will want to renegotiate a new contract (for $\frac{\beta}{\alpha} > 1+f$ and $x < x_3$) but the agent will want to self-enforce the contract (for $x < x_2$). Because the agent makes self-enforcement decision before
the bank makes the take-over decision so the agent self-enforces the contract.

d. If \( x < x_1 \), the project can be completed with the initial capital investment so no reinvestment is needed.

(iii) If the contract specifies \( R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \), the ordering of the break-even costs is 
\[ x_3 > x_1 > x_2. \]

a. If \( x > x_3 \), the bank will want to take over the project (for \( \frac{\rho}{\alpha} > 1 + f \) and 
\( x > x_3 \)) and the agent will not self-enforce the old contract (for \( x > x_2 \)) so 
the bank brings in a new agent to complete the project.

b. If \( x_3 > x > x_1 \), the bank will want to renegotiate a new contract (for \( x < x_3 \)) 
and the agent will not self-enforce the old contract (for \( x > x_2 \)) so they 
renegotiate a new contract.

c. If \( x < x_1 \), the project can be completed with the initial capital investment 
so no reinvestment is needed.

Proposition 10 is therefore proved. □

**Proof of Proposition 11:**
The agent's optimal effort in the Type N-I contract satisfies:
\[ \psi'(e) = \frac{1}{\Phi_3} \]
where \( \Phi_3 = \alpha(1 - \alpha \Delta_7(1 - F(x_2))) \)

The agent's optimal effort in the Type N-II contract satisfies:
\[ \psi'(e) = \frac{1}{\Phi_4} \]
where \( \Phi_4 = \alpha(1 - \alpha \Delta_7(1 - F(x_1))) + \left[ \alpha^2 R_0 - \frac{\alpha^2}{\rho^2} D_0 + \frac{H}{2} \right] f(x_1) \)

For \( \psi(\cdot) \) is a strictly increasing and concave function. The agent's optimal effort for a Type N-I contract is higher than his optimal effort for a Type N-II contract if \( \Phi_3 > \Phi_4 \).

From proposition 10 we know that \( R_0 - \frac{1}{\rho^2} D_0 + \frac{H}{2\alpha^2} < 0 \) for a Type N-II contract. Also we 
know \( F(x_2) > F(x_1) \) for a Type N-I contract so it follows that \( \Phi_3 > \Phi_9 \); given any initial capital 
investment \( \pi \in \mathbb{R} \), the agent's optimal cost-reducing effort with a Type N-I contract is higher than 
his optimal cost-reducing effort with a Type N-II contract. Proposition 11 is then proved. □

**Proof of Proposition 12:**
The agent's optimal effort in the Type N-III contract satisfies:
\[ \psi'(e) = \frac{1}{\Phi_5} \]
where \( \Phi_5 = \alpha F(x_3) + \left( \alpha^2 R_0 - \frac{\alpha^2}{\rho^2} D_0 + \frac{(1-2\alpha\Delta_7)}{2(1-\alpha\Delta_7)} H \right) f(x_3) \)

The agent’s optimal effort in the Type N-IV contract satisfies:
\[
\psi'(e) = \frac{1}{\Phi_6}
\]
where \( \Phi_6 = \alpha F(x_3) + \alpha^2 \Delta_7 (F(x_2) - F(x_3)) \)

The agent’s optimal effort in the Type N-V contract satisfies:
\[
\psi'(e) = \frac{1}{\Phi_7}
\]
where \( \Phi_7 = \alpha F(x_3) + \alpha^2 \Delta_7 (F(x_1) - F(x_3)) + \left( \alpha^2 R_0 - \frac{\alpha^2}{\rho^2} D_0 + \frac{(1-2\alpha\Delta_7)}{2(1-\alpha\Delta_7)} H \right) f(x_1) \)

For \( \psi(\cdot) \) is a strictly increasing and concave function. The agent’s optimal effort has the ordering \( e_{Type N-III}^* > e_{Type N-IV}^* > e_{Type N-V}^* \) if \( \Phi_5 > \Phi_6 > \Phi_7 \).

From proposition 10 we know that \( R_0 > \frac{D_0}{\rho^2} - \frac{(1-2\alpha\Delta_7)H}{2\alpha^2(1-\alpha\Delta_7)} \) for a Type N-III contract and \( R_0 < \frac{D_0}{\rho^2} - \frac{H}{2\alpha^2} \) for a Type N-IV contract. Also we know \( F(x_2) < F(x_3) \) for Type N-IV and N-V contracts so it follows that \( \Phi_5 > \Phi_6 > \Phi_7 \); given any initial capital investment \( \pi \in \pi \), the order of the agent’s optimal effort is \( e_{Type N-III}^* > e_{Type N-IV}^* > e_{Type N-V}^* \). Proposition 12 is then proved. \( \Box \)

**Proof of Proposition 13:**

From comparative statics, the sign of \( \frac{\partial e}{\partial D_0} \) is the same as the sign of \( \frac{\partial^2 EU_0}{\partial e \partial D_0} \) and also the sign of \( \frac{\partial e}{\partial H} \) is the same as the sign of \( \frac{\partial^2 EU_0}{\partial e \partial D_0} \) when there exists a unique solution of optimal \( e \).

1. The effect of debt over the agent’s optimal effort:

   (1-i) Type N-I contract:
   \[
   \frac{\partial^2 EU_0}{\partial e \partial D_0} = -\frac{\alpha^2}{\rho^2} f(x_2) \psi'(e) < 0
   \]

   (1-ii) Type N-II contract:
   \[
   \frac{\partial^2 EU_0}{\partial e \partial D_0} = -\frac{\alpha^2}{\rho^2} f(x_1) \psi'(e) < 0
   \]

   (1-iii) Type N-III contract:
   \[
   \frac{\partial^2 EU_0}{\partial e \partial D_0} = -\frac{\alpha^2}{\rho^2} f(x_1) \psi'(e) < 0
   \]

   (1-iv) Type N-IV contract:
   \[
   \frac{\partial^2 EU_0}{\partial e \partial D_0} = -\frac{\alpha^2}{\rho^2} f(x_2) \psi'(e) < 0
   \]

   (1-v) Type N-V contract:
   \[
   \frac{\partial^2 EU_0}{\partial e \partial D_0} = -\frac{\alpha^2}{\rho^2} f(x_1) \psi'(e) < 0
   \]

As above, the sign of \( \frac{\partial e}{\partial D_0} \) is strictly positive in all non-recourse debt contracts.
(2) The effect of performance bond over the agent’s optimal effort:

(2-i) Type N-I contract: \[
\frac{\partial^2 EU_{Type \ N-I}}{\partial e \partial H} = \frac{1}{2} f(x_2) \mu'(e) > 0
\]

(2-ii) Type N-II contract: \[
\frac{\partial^2 EU_{Type \ N-II}}{\partial e \partial H} = \frac{1}{2} f(x_1) \mu'(e) > 0
\]

(2-iii) Type N-III contract: \[
\frac{\partial^2 EU_{Type \ N-III}}{\partial e \partial H} = \frac{1}{2} (f(x_2) + f(x_3)) \mu'(e) > 0
\]

(2-iv) Type N-IV contract: \[
\frac{\partial^2 EU_{Type \ N-IV}}{\partial e \partial H} = \frac{1}{2} (f(x_1) + f(x_3)) \mu'(e) > 0
\]

(2-v) Type N-V contract: \[
\frac{\partial^2 EU_{Type \ N-V}}{\partial e \partial H} = \frac{1}{2} (f(x_1) + f(x_3)) \mu'(e) > 0
\]

From (1), (2) and (3), the sign of \(\frac{\partial e}{\partial H}\) is strictly positive in Type N-I, Type N-II and Type N-IV, and Type V contracts.

Combing (1) and (2) we know for any given initial capital investment \(\pi \in \pi\), the agent’s optimal cost-reducing effort decreases with the level of debt, and increases with the level of the performance bond in all non-recourse debt contracts. Proposition 13 is then proved. □
Reference


