Of Energy and the Economy
--Theory and Evidence for Their Functional Relationship

by

Vincent H. Chang

Ph.D., Electrical Engineering and Computer Sciences
University of California at Berkeley, 1990

Submitted to the Department of Economics
in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in Economics

at the

Massachusetts Institute of Technology

February 2007

Signature of Author..
Department of Economics
January 12, 2007

Certified by...Jan 11, 2007...............................
Jerry A. Hausman
John & Jennie S. MacDonald Professor of Economics
Thesis Advisor

Certified by..
Ernst R. Berndt
Louis B. Seley Professor of Applied Economics, Sloan School of Management
Thesis Advisor

Accepted by..
Peter Temin
Elisha Gray II Professor of Economics
Chairman, Departmental Committee on Graduate Studies

Of Energy and the Economy
--Theory and Evidence for Their Functional Relationship

by

Vincent H. Chang

Submitted to the Department of Economics on January 15, 2007
in partial fulfillment of the requirements for the Degree of
Doctor of Philosophy in Economics

ABSTRACT

This paper offers a set of explicit functional relationships that link energy and the economy. Despite the reliance on energy permeating the whole economy, no such complete relationships had been presented before. How related are energy and the economy? What role does energy play in the economic growth? Motivated to seek an explicit functional answer, I theorize the role of energy and then test it with economic models, using data for 16 OECD countries from 1980 to 2001. First, I find that energy is a cross-country representative good whose prices are equalized when converted to a reference currency. Thus, energy prices satisfy the purchasing power parity. For all but one country, the half life of the real energy exchange rate is less than a year and as low as six months, shorter than those derived by other real exchange rate measures. Second, considering energy a cross-time representative good, I obtain that a country's utility function is inversely proportional to both its income share of energy and its energy price. I also obtain an explicit, unified two-dimensional (cross countries and time) production function with energy and non-energy as the two inputs. Third, I conclude a cross-country parity relationship for income shares of energy, similar to that for energy prices. Further, I provide an intertemporal connection between the trajectory of the income share of energy and the productivity growth of the economy. Lastly, I demonstrate the tradeoffs between energy efficiency and economic wellbeing, with the energy price being the medium for the tradeoffs. One may apply the functional roles of energy offered in this paper to help frame the current global-scale issues that are energy relevant.

Thesis Advisor: Jerry A. Hausman
Title: John & Jennie S. MacDonald Professor of Economics

[blank]

BIBLIOGRAPHICAL NOTE

In addition to a PhD in economics from MIT, Vincent holds another PhD in electrical engineering and computer sciences from the University of California at Berkeley, an MPPM (MBA) from Yale School of Management, an MPA from Harvard Kennedy School of Government, and a BS in electrical engineering from National Taiwan University. He also attended the doctoral program of astrophysics at Princeton.

Vincent has one and a half decades of industry experience, including strategy, capital markets, investment management, risk management, banking, financing, marketing, engineering, startup, and product development. He has served, in a reverse chronological order, as Senior Research Associate at ExxonMobil Research, Executive Vice President at ECapital Financial, Vice President & Chairman's Advisor at General Bank, Vice President and Assistant Vice President at JP Morgan Investment Management, Management Consultant at McKinsey, and Advanced Development/Signal Processing Engineer at Acuson (Siemens Medical Ultrasound).

He has done research in a wide range of topics, covering energy, economic growth, international trade, derivative pricing, asset swaps, medical imaging, cancer therapy, and space communications.

On the teaching side, he taught electrical engineering at the University of California at Berkeley as a teaching assistant, Mandarin at the Princeton Chinese Language School as a lecturer, and computer programming at National Taiwan University as a lecturer.

[blank]

ACKNOWLEDGEMENTS

It seems fitting that I started writing the acknowledgements while waiting for a flight in the Long Beach Airport. For the last three years and four months at MIT, I have been in and out Boston's Logan International Airport for far more than 100 times. Indeed, logistics has been an extreme challenge to me, as a student, an employee, a son and the head of a household.

As a student, I am demanded to stay in Cambridge for coursework and research. The Economics Department at MIT is a no nonsense institution. As an employee, I need to make regular trips to connect with my employer near 300 miles away in New Jersey. As an eldest son in a traditional Chinese family, I am responsible for my mother, whose health has deteriorated and lives alone 8,000 miles away in Taiwan, the only place that she feels comfortable. As a father of two preteens, I ought to be there for her auditions and his little league games, all of which take place some 3,000 miles away in Southern California.

For many times past midnight, when I received phone calls from thousands of miles away, or when I was on a red-eye flight dashing back to Boston, or when I lost my sleep due to the worry over having to get up at 4 AM in order not to miss the earliest flight out of Boston (sometimes a day like that did not end until 20 some hours later in the following day), or when I still stayed in the MIT Science Library's study room, I often asked myself, "Is this worth it? How can I finish it as quickly as possible?" At the end, fortunately, I made it within three and a half years. No, it's not I, it's we. They have helped me make it. And it has taken more than a village.

I am thankful to the MIT Economics Department and its staff. More than a decade ago, I was thrilled to receive the admission to the MIT Economics PhD Program, my first choice. After deferring for two years, I attended for shorter than a semester and then left the Program for a Wall Street job, thinking I would never return. Seven years later, upon my request, the Department kindly granted me a readmission to the Program. Since then, it has been rather uneventful, thanks to Gary King, who has gone the extra mile to assist me in readmission and many other matters.

I specially thank my advisors, Professors Jerry Hausman and Ernst Berndt, for their guidance and kind consideration. Without Jerry's prompt, timely feedbacks, it would have been impossible for me to finish my degree within my planned time frame. I thank too Professor Whitney Newey for reading the final draft of the thesis. I appreciate Professor Peter Temin's patience for accepting my economics history term paper that was exactly 10 years over due. I thank the following people for their useful discussions: Chemistry Department's Professor John Deutch, Sloan School's Professor Henry Jacoby, Harvard's Professor Richard Cooper, Rensselaer Polytechnic Institute's Professor David Stern, and Lund University (Sweden)'s Professor Astrid Kander.

At MIT, I have received excellent support/treat from the following individuals: Marry Murray (Student Services), Kim Scantlebury (Economics), Rebecca Bourke (Sloan), Gretchen Slemmons (Sloan), and the lady who works in the Whitehead Institute's cafeteria. Thanks.

This thesis is the result of a bold idea initiated by two visionary gentlemen in ExxonMobil Research & Engineering. Dr. Roger Cohen kicked off the thinking of studying the transition to a new energy economy. He is a person with a horizon of 1,000 years. Dr. Eldon Priestley is a visionary, steady, firm and caring leader, who is instrumental in ensuring that I am not unnecessarily distracted. *I dedicate this thesis to both of them and wish them happy retirement.*

I appreciate very much the smooth administrative assistances from Kim Lacy and Joanne Ponak in ExxonMobil. Kim has gone beyond her normal duty by preparing my expense statements on a regular basis for the past three plus years. I thank Dr. Bhaskar Sengupta for his advice in corporate matters.

I consider the staff of the Marriott Residence Inn in Cambridge part of my family. They have sincerely looked after me. When they learned that I had passed my General Exams, they spent their own money on a special gift for me. On a daily basis, if I came back to the hotel way past midnight, they would point to their watch and kindly remind me of the time. Or if I came back a lot earlier, they would "command" me go back to the library. Their genuine caring makes me feel at home. The representatives are Dottie, Tony, Diana, Debbie, Enercinda, Vernon, Herbert, Susan, Erin, Kevin, Natalie, Jose, Sunny, Thomas, Joe, Stephanie, and many other familiar faces.

I must mention a few people who have also made my days go by pleasantly. I have learned much about the real world economics from a taxi driver who often drove me from Residence Inn to BOS as early as 4:30 AM when the sky was still dark and the street was quite. I felt warm, secure and relaxed in his cab in the early morning. I will never forget the smile of the cashier at EWR. She recognized me every single time among hundreds of thousands of passengers that passed by Continental's Terminal C. Her greetings refreshed my long, tiring day. Chatting with the Blue Sky shuttle drivers, who regularly took me from my home in Los Angeles to LAX or LGB, helped clam my emotions stirred by my children's squeezing hugs of goodbye. Heading toward the airport and staring at the sun setting against the backdrop of the Pacific Ocean, we shared life's sweet and sour.

I appreciate the services of the Starbucks chain stores and of Jet Blue, American and Continental Airlines. They are all parts of the market economy at work. I lighted up 90% of the ideas that form the base of this thesis while at the cruising altitude of 33,000 feet. I ran more than 50% of the empirical tests in the Starbucks in Massachusetts, California, New Jersey and New York. I have not taken their services for granted.

I treasure the friendships of those whom I have met in Cambridge. In the Salon Chaos Club, which I co-founded with Victor, Itsung, Mohan and Joe, we discussed and debated anything coming to our minds every other Saturday evenings. It was fun and intriguing.

To many other friends at MIT, Harvard and Boston University, I will keep your friendships in my heart.

My ultimate study of economics at the age of 40 something is not totally out of blue. Rather, it has been a winding but poetic journey.

I have grown myself as a pragmatic idealist in order to balance my life between my heart's pursuit and the obligations owing to my humble socioeconomic beginning. My pragmatic side lies in engineering study. My idealist side, which is more literatures/humanities inclined, was first nurtured by the open environment of National Taiwan University, where I received my college degree in engineering. Since then, over the course of next 20 some years, many times because of the bite of the reality, my heart's pursuit had subsided, but it has never faded away.

Thanks to the encouragement and inspiration from the people that I have met along the way, I have never stopped thinking big and asking grand questions. The names of the people that often come to my thought are those with NTU's Mainland China Study Society, NTU's history/philosophy/literature classes, NTU's Electromagnetic Waves Group, the off-campus classics lessons, numerous extracurricular activities, Princeton campus, Berkeley's bookstore, Silicon Valley's Acuson, and Manhattan's World Trade Center.

I was deeply inspired by the literatures written by the youth generation who had experienced the Cultural Revolution. Years later, accidentally (or perhaps by fate), I encountered some of the seemingly historical figures in the US.

I will not forget the kindness of Mr. and Mrs. Lee, who voluntarily financed my one-way air ticket from Taipei to New York plus some extra funds for emergence, when I first came to the US for my graduate study at Princeton. The sum of the money, though not a huge amount, was the largest I had seen up to that time. It is a debt that I will never clear from my heart.

At some point, I realized that the economics was perhaps a good balance between my pragmatism and my idealism and that it was perhaps more actionable than history or literature. I was extremely lucky to take my first macroeconomics course under Professor Janet Yellen at Berkeley. It is the encouragement of her and her husband Professor George Akerlof that made me dream I might too become a useful economist someday. Admiration of their later achievements (appointed Chair of the President's Council of Economic Advisers and awarded the Nobel Prize in Economics, respectively) ultimately tipped my year-long decision to accept ExxonMobil's offer and come back to MIT for an economics PhD.

Or, maybe all these years, I have been obsessed by the literal meaning of "economics" in the Chinese language, which, when translated into English, means something grand--"manage the world and enrich people's life."

This thesis could not have been written without the love and support of my family. My mother and mother-in-law, both widows, took turns to help care for my children during the first two years after I restarted MIT. My sister and brother in Taiwan share my responsibility as the eldest son in the family. I may never be able to repay my debt to them.

My daughter, a born Bostonian, always drawing cards claiming her dad is the best in the world, goes to bed at night wearing her Red Sox cap. My son, a born New Yorker, always looking forward to my coming home to play baseball with him, dreams of playing for the Yankees and retiring at 41 (afterwards he would pursue two other careers). She is also an Angels' fan and he is a Dodgers' fan. My life would be awfully dull without their sibling bond and rivalry.

My wife, a Yale MBA, has been my best partner and personal consultant. She has handled our two active preteens elegantly without me most of time while I am away in Boston. At times, she had chosen to sacrifice her ambitions for the sake of the family and my career. She has accompanied me graciously through many joyful and many more frustrating, struggling moments in the life's journey. When I retreat, she is my reinforcement. When I am tired, she is my support. And when I am lost, she always helps me find a way.

I dedicate this thesis to my father, who had passed away some months before I came back to MIT. He believed in his children's education, though he himself had never set foot in school for even a day (neither has my mother), due to the constraints of his youth times. Zero schooling had been his greatest regret but he had never complained. He had lived a humble, yet proud life. The ocean was his data set, with which his had tested his theory of living a life, literally and figuratively, to the extent that he could have written many of his doctoral theses in philosophy. He was an exceptionally fine fisherman. And a decent human being.

Every time when I leave a town (and I have left quite a few), I recite to myself frequently one of poems by an 11th century Chinese poet, Su Shi:
"To what can human life be likened?
Perhaps to a wild goose's footprint on snow;
The foot imprint is accidentally left,
But carefree, the bird flies east and west."

Last time in January 1997 when I left Cambridge, like the goose's footprint on snow was accidental and would melt away, I never thought I would one day come back to the town. This time, 10 years later in January 2007, as I again bid farewell to Cambridge, I am actually thinking of something otherwise. Perhaps another 10 years?

Reader's Guide:

To economize a reader's time, along with the title line in each section, I indicate in square brackets whether the reader may choose to skip that section partially or entirely. For example, together with Section 2.2.2's title line, the guide reads, **[Reader may skip Section 2.2.2 entirely].** Another example, together with Section 2.2.1's title line, the guide reads, **[Reader may go directly to the <u>Conclusion</u> part of Section 2.2.1, and skip the rest of the section]**. Should a reader choose to follow the guide to skip some parts of this paper, he/she would still be able to understand the paper's logic flow as well as its key results.

[blank]

Table of Content

Notations and Glossaries [Partial List]

BTU: British Thermal Unit, a unit of measure for heat content.

AU: The symbol for gold in the periodic table of chemistry. Depending on the content, in most places it may also be used as an abbreviation to represent the country of Australia.

i: The country index, $i = 1 - 16$ to represent 16 OECD countries: Canada (CA), United States (US), Austria (AT), Belgium (BE), Finland (FI), France (FR), Italy (IT), Netherlands (NL), Norway (NO), Sweden (SW), Switzerland (SZ), United Kingdom (UK), Australia (AU), Japan (JA), South Korea (KS), and New Zealand (NZ).

t: The time index, $t = 1 - 22$ to represent years 1980-2001.

u: A country index to represent the US.

w: A country index to represent the World. The World is defined over the 16 countries in Sections 3-5.

b: A time index to represent the base year. In most places, it is used to represent year 2000.

Y_{it}: GDP in terms of nominal national currency for country i in year t.

E_{it}: Real end-user energy consumption in terms of heat content for country i in year t. The heat content unit I choose to use is the British Thermal Unit, or BTU.

P_{it}: Price of energy to the end users in terms of nominal national currency for country i in year t. Its unit is nominal national currency per BTU.

X_{it}: Nominal market exchange rate of country i's current national currency vis-à-vis the current US dollar in year t. Its unit is current (nominal) US dollar per current (nominal) national currency.

Q_{it}: Inverse of the GDP deflator for country i in year t. Its unit is constant national currency per current national currency. The constant currency is based on that of year 2000.

Y_{it}^{*}: GDP in constant national currency for country i in year t. Its unit is constant national currency. Note $Y_{it}^{*} \equiv Q_{it} Y_{it}$.

P_{it}^{*}: Price of energy in constant national currency for country i in year t. Its unit is constant national currency per BTU. Note $P_{it}^{*} \equiv Q_{it} P_{it}$.

R_{it}: Real energy exchange rate for country i in year t.

$$R_{it} \equiv \frac{X_{wit} P_{it}}{P_{wt}} = \frac{X_{it} P_{it}}{P_{ut}} R_{ut}.$$

α_{it}: Energy income share, or, income share of energy. $\alpha_{it} = \frac{P_{it} E_{it}}{Y_{it}}$. $\alpha_{it} \in (0,1)$.

S_{it}: Energy income share indexed to the world for country i in year t.

$$S_{it} = \frac{\alpha_{it}}{\alpha_{wt}}.$$

V_{it}: RIVER, which stands for Ratio of Income Vs. Energy Revenue.

$$V_{it} \equiv \frac{Y_{it}}{P_{it}E} = \frac{1}{\alpha_{it}}.$$

CES: Constant Elasticity of Substitution.

Cross-country CES production function:

$$Y_{it} = \left[\alpha_{it} \left(A_{E,it} E_{it} \right)^{\frac{\sigma_t - 1}{\sigma_t}} + (1 - \alpha_{it}) \left(A_{N,it} N_{it} \right)^{\frac{\sigma_t - 1}{\sigma_t}} \right]^{\frac{\sigma_t}{\sigma_t - 1}}$$

Intertemporal CES production function:

$$Y_{it} = \left[\beta_{it} \left(B_{E,it} E_{it} \right)^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \beta_{it}) \left(B_{N,it} N_{it} \right)^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}$$

K_{it}: Real capital employed in country i at time t.

L_{it}: Real labor employed in country i at time t.

N_{it}: Real non-energy employed in country i at time t.

$A_{E,it}$: Energy factor augmenting technology in the cross-country production function.

$A_{N,it}$: Non-energy factor augmenting technology in the cross-country production function.

$B_{E,it}$: Energy factor augmenting technology in the cross-time production function.

$B_{N,it}$: Non-energy factor augmenting technology in the cross-time production function.

σ_t: Elasticity of substitution between energy and non-energy factors in the cross-country CES production function at time *t*, for all countries. $\sigma_t \in (0,\infty)$

σ_i: Elasticity of substitution between energy and non-energy factors in the intertemporal CES production function for country *i*, at all time. $\sigma_i \in (0,\infty)$

λ_t: $\lambda_t \equiv \dfrac{\sigma_t - 1}{\sigma_t}$. $\lambda_t \in (-\infty,1)$.

λ_i: $\lambda_i \equiv \dfrac{\sigma_i - 1}{\sigma_i}$. $\lambda_i \in (-\infty,1)$.

β_{it}: The distribution parameter which determines the relative importance of the energy factor in the intertemporal CES production function. $\beta_{it} \in (0,1)$.

M_{it}: Cross-sectional marginal product of energy. $M_{it} \equiv \dfrac{\partial Y_{it}}{\partial E_{it}} = \left(\dfrac{Y_{it}}{E_{it}}\right)^{1-\lambda_t} \alpha_{it} A_{it}^{\lambda_t}$

M_{it}: Intertemporal marginal product of energy. $\mathrm{M}_{it} \equiv \dfrac{\partial Y_{it}}{\partial E_{it}} = \left(\dfrac{Y_{it}}{E_{it}}\right)^{1-\lambda_i} \beta_{it} B_{E,it}^{\lambda_i}$

CRRA: Constant Relative Risk Aversion

$U_{it}(\cdot)$: Utility function for country i at time t. A CRRA utility is defined in this paper: $U_{it}(\cdot) = \frac{1}{1-\theta_i}\left(\frac{Y_{it}}{P_{it}E_{it}}\right)^{1-\theta_i}\delta_{it}$

$U_{it}'(\cdot)$: Marginal utility. $U_{it}'(\cdot) = \left(\frac{Y_{it}}{P_{it}E_{it}}\right)^{-\theta_i}\delta_{it}$

θ_{it}: Constant relative risk coefficient of a CRRA utility function

δ_{it}: Subject discount factor of the utility function.

H_{it}: Energy efficiency. $H_{it} \equiv \frac{Y_{it}^*}{E_{it}} = \frac{1}{\alpha_{it}}P_{it}^*$.

C_{wt}^*: The world's real marginal cost of extracting a BTU from the energy source. It also includes any energy supply shocks.

F_{wt}: The world's real marginal efficiency of converting a source BTU to an end-use BTU.

γ: Exponent of the conjectured biology-analogous one-factor production function: $Y_t^* \propto E_t^{\gamma}$.

D: The dimensions of for the energy distribution network in the conjectured biology-analogous energy economy.

1 Introduction

1.1 Motivation

It is perhaps common knowledge that energy is important to the economic growth and the welfare of the society. Most of us have gained such knowledge from our daily reading of newspapers, magazines and commentaries. In the US and perhaps throughout the developed economies, the public are particularly sensitive to the gasoline price at the pump. Additionally, there are issues of global scale, such as those of environment, geopolitical satiability, and climate change, etc., that all seem to be closely connected with energy.

Physically speaking, energy is used to transfer raw material into useful output, and to transport goods and services to every corner of the economy. One may consider the role that energy plays to today's economy might be similar to that to a biological body. Inspired by this possible similarity, I start looking into whether I could find an explicit relationship between energy and the economy, whether or not it may be analogous to the relationship between energy consumption and a biological mass.

However, I have found the existing models[1] unsatisfactory as far as my goal is concerned. The explicit functional relationships, if exist, have remained elusive to economists. It is therefore the objectives of this paper to attempt for such relationships. I wish to come up with explicit expressions that link energy and the economy, functionally and quantitatively. Should such functional relationships be found, we would be able to put the global issues such as climate change, alternative energy, etc. into a perhaps more robust framework that may lead to more fruitful discussions across industries and governments. It is with such an objective that I start the research that leads to this paper.

1.2 Questions and Scope

[1] I will review the models in the existing literatures in Sections 3-5 in this paper.

The questions I am asking are:

1. What is the cross-sectional role of energy in the international economy?
2. What is the role of energy in economic growth?
3. What energy investments do we need in order to sustain the economic growth?
4. What type of new energy economy, if necessary, is desired?
5. Would there be an optimal transition path to the new energy economy? And if yes, what might it be?

This paper will address only the first two questions. The remaining questions will be discussed in subsequent papers.

In order to attain meaningful functional forms to address the first two questions, I constrain the scope of this paper to the country level; i.e., I will not discuss industry-level relationships. Also, I treat the economy consisting of only two sectors, namely energy and non-energy. Lastly, I leave out the energy's interaction with the environment.

To ensure smooth and coherent flow of the paper, I will review the literatures only in the sections that are respectively relevant to the sections, namely, purchasing power parity literatures in Section 3, production function in Sections 4-5, and so on.

1.3 Key Findings

I find the following functional relationships between energy and the economy:

1. Energy is a cross-country representative good whose prices are equalized when converted to a reference currency and thus satisfies the purchasing power parity. The average life of real energy exchange rate is within a year, shorter than any other real exchange rate measures.
2. Energy can be an intertemporal representative good whose marginal values (defined as the multiplication of marginal utility and marginal product of energy) are equalized when converted to the same base year.

3. A country's utility function is risk-neutral and inversely proportionally to the energy income share, and carries a discount factor that depends on the energy price.
4. The production functions, both cross countries and over time, can be approximated to be of Cobb-Douglas form with energy and non-energy being the two factors, and can be unified into a single two-dimensional production function. The exponents to the energy and non-energy factors are energy income share and non-energy income share, respectively.
5. There exists a cross-sectional relationship for the energy income share, which is parallel to that of the energy purchasing power parity.
6. The energy income share's time trajectory is closely connected with the total factor productivity's growth. Because the energy income share seems ubiquitous in almost all of the relationships, I propose that it be one of the state variables that describe the status of the economy.
7. There exist tradeoffs, intratemporal and intertemporal, between energy efficiency and economic wellbeing, with the energy price being the medium for the tradeoffs. These tradeoffs should deserve policy makers' attention.

1.4 Outline of the Paper

I organize the paper as follows. Section 2 lists the data that I use for this paper. The data are from various sources including institutions and individuals. I characterize the data in terms of their stationarity for statistical inferences. Section 3 uses no-arbitrage argument to link the market exchange rates with the energy prices. It also provides empirical evidences. Section 4 explores the functional form for the cross-country production function. Section 5 hypothesizes and tests empirically the intertemporal relationships between energy and economic growth. This involves the assumption and testing of the preference function and the intertemporal production function. Section 6 combines the results from Sections 3-5 and concludes that it is the energy income share that runs through almost all the functional relationships between energy and the economy, both intratemporally and intertemporally. Section 7 extends the results further to discuss energy efficiency, the utility function, and the intratemporal tradeoff between welfare and energy efficiency. Section 8 concludes several key fundamental functional relationships that I have obtained through economic modeling and empirical tests from Sections 2-7. Section 9 concludes the paper and also discusses future work.

2 Data

2.1 Data Sources and Descriptions

I use the following set of data for the empirical tests in this paper. They and their sources are:

- GDP in constant US dollar, 135 countries, 1980-2001: US Energy Information Agency
- Energy end-use Consumption, 135 countries, 1980-2001: US Energy Information Agency
- GDP in current (nominal) national currency, 16 selected countries: Global Financial Data Corp.
- GDP in constant national currency, 16 selected countries: Global Financial Data Corp.
- GDP in current (nominal) US dollar based on the market exchange rates, 16 selected countries: Global Financial Data
- GDP deflator, 16 selected countries: calculated from the data of GDP in current (nominal) national currency and GDP in constant national currency
- Nominal exchange rate, 16 selected countries: calculated from the data of GDP in current (nominal) national currency and GDP in current (nominal) US dollar
- Energy price index in nominal national currency, 16 countries, 1980-2001: International Energy Agency
- Energy end-use consumption, over 100 countries, 1970-2001: International Energy Agency
- Swedish energy consumption & price, 1900-2000: hand-copied from Kander's thesis (2002)
- Swedish GDP, 1900-2000: provided directly by Lennart Schon, Department of Economic History, Lund University, Sweden
- Consumer price index: Global Financial Data Corp.

Because of data availability issue, I confine my universe for this paper to 16 OECD countries for the time period from 1980 to 2001. The data are all on an annual basis. Throughout this paper, I use i for the country index which takes value from 1 to I, where $I = 16$ denoting the total number for countries, to represent Canada (CA), United States (US), Austria (AT), Belgium (BE), Finland (FI), France (FR), Italy (IT), Netherlands (NL), Norway (NO), Sweden (SW), Switzerland (SZ), United Kingdom (UK), Australia (AU), Japan (JA), South Korea (KS), and New Zealand (NZ), respectively. Notice that I exclude Germany because there is no energy data for East Germany prior to its reunification with West Germany in 1989. For the time index, I use t and it goes from 1 to T to represent years from 1980 to 2001, where $T = 22$ denoting the total numbers of time periods.

To set up for later discussions in this section, I use the following notions:

Y_{it}: GDP in terms of nominal national currency for country i in year t.

E_{it}: Real end-user energy consumption in terms of heat content for country i in year t. The heat content unit I choose to use is the British Thermal Unit, or BTU.

P_{it}: Price of energy to the end users in terms of nominal national currency for country i in year t. Its unit is nominal national currency per BTU.

X_{it}: Nominal market exchange rate of country i's current national currency vis-à-vis the current US dollar in year t. Its unit is current (nominal) US dollar per current (nominal) national currency.

Q_{it}: Inverse of the GDP deflator for country i in year t. Its unit is constant national currency per current national currency. The constant currency is based on that of year 2000.

Y_{it}^*: GDP in constant national currency for country i in year t. Its unit is constant national currency. Note $Y_{it}^* \equiv Q_{it} Y_{it}$.

P_{it}^*: Price of energy in constant national currency for country i in year t. Its unit is constant national currency per BTU. Note $P_{it}^* \equiv Q_{it} P_{it}$.

R_{it}: Real energy exchange rate for country i in year t, defined in Section 3.

S_{it}: Energy income share indexed to the world for country i in year t, defined in Section 4.

u: A county index to represent the US.

b: A time index to represent the base year, which is year 2000 in Section 2.

For the complete list of the notations, see Notations and Glossaries.

2.2 Data Characteristics

2.2.1 Stationarity Tests for Time-Series Data [Reader may go directly to the Conclusion part of Section 2.2.1, and skip the rest of the section]

In order to have valid statistical inferences, a data series needs to be stationary. One way of testing stationary is unit root test. If a series is stationary, then it is an integrated of order 0, or $I(0)$, process; if its first difference is stationary, then the series is an integrated of order 1, or $I(1)$, process; and so on. The unit root tests below will allow us to determine what kind of processes our data are.

To test the unit root for the nominal GDP time series, I use following $AR(1)$ regression model, for a given *i*,

$$\ln Y_{it} = \rho \ln Y_{i,t-1} + \varepsilon_{it}, t \in [2,T], \tag{2.1}$$

where ε_{it} is i.i.d. with zero mean and constant variance. The null hypothesis is that $\rho = 1$ and the alternative $\rho < 1$. When the null is true, the *t*-statistics from the *OLS* regression is no longer *t*-distributed. To obtain the critical values of that distribution, one has to use simulation model. Table 2.1 reports the critical value for the distribution under the null. In our case, since we have only 21 samples or fewer, we have to extrapolate from the table to get the critical values. For 95% and 90%, the critical values I choose to use are -1.95 and -1.6, respectively. So if the *t*-statistics for the slope coefficient estimator, $(\hat{\rho}-1)/se(\hat{\rho})$, is less than -1.95, then we would reject the null hypothesis and conclude that $\ln Y_{it}$ does not have a unit root at the 5% level.

However, rejecting the null is insufficient to conclude that the series under test is stationary. One needs to further test whether $\rho \le -1$, because if $\rho \le -1$, then the series is also nonstationary[2]. This additional test will be null $\rho = -1$ vs. alternative $\rho > -1$. The *OLS* *t*-statistics under the null is *t*-distributed. I call this test the negative unit root test.

So if we reject the unit root test that $\rho = 1$ (vs. alternative $\rho < 1$) and also reject the negative unit test that $\rho = -1$ (vs. alternative $\rho > -1$), then $\ln Y_{it}$ is stationary.

In case the unit root test of (2.1) is not rejected, I proceed to consider testing the first difference, $\Delta_t \ln Y_{it} \equiv \ln Y_{it} - \ln Y_{i,t-1}$, using the following regression model:

$$\Delta_t \ln Y_{it} = \rho \Delta_{t-1} \ln Y_{i,t-1} + \varepsilon_{it}, t \in [3,T]. \tag{2.2}$$

[2] In this case, then process is nonstationary while oscillating, because the *AR(1)* coefficient is negative and its magnitude is greater or equal to 1. This additional test is rarely discussed in the literatures.

If we reject both $\rho = 1$ (vs. $\rho < 1$) and $\rho = -1$ (vs. $\rho > -1$), then $\Delta_t \ln Y_{it}$ is stationary.

For the series that I use in this paper, the following tables summarize their *OLS t*-statistics, $(\hat{\rho} - 1)/se(\hat{\rho})$, for the coefficient ρ. They are: Table 2.2 unit root test for level regressions (2.1), Table 2.3 unit root test for first difference regressions (2.2), and Table 2.4 negative unit root test for fist difference regressions (2.2). In these tables, because of the data availability issue[3], I use $P_{it}/P_{ib} \cdot P_{ub}/X_{ib}$ for P_{it} and $P_{it}^*/P_{ib}^* \cdot P_{ub}/X_{ib}$ for P_{it}^*. I also include in these tables two variables R_{it} and S_{it} that I will define later in Sections 3 and 4, respectively. The former stands for the real energy exchange rate, and the latter the energy income share indexed to the world.

It is apparent that from the *OLS t*-statistics in Table 2.2 for unit root test of the level regressions, we cannot reject that $\rho = 1$ for almost all the series and for almost all countries. Thus none of the data series is stationary.

However, in Table 2.3 for the unit root test of the first difference regressions, for the series of $\Delta_t \ln Y_{it}$ and $\Delta_t \ln Y_{it}^*$, we cannot reject the null. For $\Delta_t \ln E_{it}$, we reject South Korea at the 10% level and the rest at the 5% level. For $\Delta_t \ln Y_{it}/E_{it}$ we reject the null for all countries at the 5% level except for Sweden and UK, of which we reject at the 5-10% level. For the series of $\Delta_t \ln P_{it}$ and $\Delta_t \ln P_{it}^*$, we reject the null at the 5% level for all countries. For $\Delta_t \ln R_{it}$ and $\Delta_t \ln S_{it}$, we reject the null at the 5% level for all countries.

Further, in Table 2.4 for the negative unit root test of the first difference regressions, for all series that need further tests, we reject the null for all countries. Therefore we cannot reject that $\Delta_t \ln E_{it}$, $\Delta_t \ln Y_{it}/E_{it}$, $\Delta_t \ln P_{it}$, $\Delta_t \ln P_{it}^*$, $\Delta_t \ln R_{it}$ and $\Delta_t \ln S_{it}$ are stationary. This is very important as I will use these five series later in this paper for modeling and regression.

---

[3] Except for the US, I have data for only the price index, not price itself.

Notice that $\Delta_t \ln Y_{it}$ and $\Delta_t \ln Y_{it}^*$ are still nonstationary. I have tested that $\Delta_t^2 \ln Y_{it}$ and $\Delta_t^2 \ln Y_{it}^*$ are stationary, though I do not display the testing results here.

Conclusion

Therefore, we can conclude that for all countries, the time series data exhibit the characteristics of processes as follows: $\ln Y_t$ and $\ln Y_t^*$ are $I(2)$ processes, and $\ln E_t$, $\ln Y_t/E_t$, $\ln P_t$, $\ln P_t^*$, $\ln R_t$ and $\ln S_t$ are $I(1)$ processes.

Table 2.1: Critical value for the Dickey-Fuller test based on estimated OLS t-statistics

Sample size	Probability that t-statistics is less than entry							
	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
25	-2.66	-2.26	-1.95	-1.60	0.92	1.33	1.70	2.16
50	-2.62	-2.25	-1.95	-1.61	0.91	1.31	1.66	2.08
100	-2.60	-2.24	-1.95	-1.61	0.90	1.29	1.64	2.03
250	-2.58	-2.23	-1.95	-1.62	0.89	1.29	1.63	2.01
500	-2.58	-2.23	-1.95	-1.62	0.89	1.28	1.62	2.00

The probability shown at the head of the column is the area in the left-hand tail
Source: Hamilton (1994), p. 763.
It is originally generated by Dickey and Fuller (1976) using the Monte Carlo method.

Table 2.2: *OLS t*-statistics of unit root test for *AR(1)* time-series, level regression

Country	lnY	lnY*	lnE	lnY/E	lnP	lnP*	lnR	lnS
CA	8.77	5.53	2.42	-6.86	2.85	1.23	-3.94	1.62
US	12.96	7.67	2.02	-10.38	0.97	-1.11	-1.05	-2.95
AT	14.82	6.29	2.09	2.26	0.67	-1.25	-1.34	0.25
BE	13.52	4.95	1.87	3.31	1.08	-0.19	-3.38	-0.31
FI	6.39	3.72	1.56	-5.30	1.93	-0.72	-2.33	-1.76
FR	8.05	7.64	2.56	-9.78	1.61	-0.73	-2.05	-0.45
IT	8.58	7.92	2.40	5.59	3.50	-0.12	-1.65	-1.54
NL	14.86	5.79	1.52	-5.09	1.24	0.23	-1.07	-2.11
NO	8.74	8.44	1.85	-5.27	3.47	0.11	-1.00	-0.37
SW	8.80	4.71	0.81	-7.65	3.32	0.46	-1.28	-2.36
SZ	7.88	4.12	1.06	-3.70	0.37	-1.37	-2.99	0.48
UK	14.26	6.52	0.96	-11.27	2.98	-0.85	-0.72	1.37
AU	9.64	7.01	5.77	-7.57	2.95	0.16	-1.67	0.22
JA	5.26	4.29	2.97	2.05	-1.71	-1.91	-2.51	1.48
KS	10.57	8.19	6.74	5.32	1.37	-1.71	-1.34	-1.43
NZ	6.25	4.30	4.37	-5.88	3.02	-1.04	-1.49	0.23
Avg	9.96	6.07	2.56	-3.76	1.85	-0.55	-1.86	-0.48

The 5% critical value is -1.95 and the 10% critical value is -1.6
Reject the unit root if the *OLS t*-stat is less than the critical value

Table 2.3: *OLS t*-statistics of unit root test for *AR(1)* time-series, first-difference regression

Country	d(lnY)	d(lnY*)	d(lnE)	d(lnY/E)	d(lnP)	d(lnP*)	d(lnR)	d(lnS)
CA	-2.13	-1.85	-2.89	-2.87	-3.61	-3.29	-2.85	-3.30
US	-1.76	-1.56	-2.67	-2.64	-4.24	-3.78	-3.09	-2.99
AT	-1.07	-1.41	-4.48	-2.69	-4.37	-3.65	-5.05	-4.71
BE	-0.97	-2.05	-2.77	-2.06	-3.49	-3.37	-4.69	-4.39
FI	-1.60	-1.60	-4.08	-2.09	-4.49	-4.20	-3.67	-3.57
FR	-1.96	-1.12	-2.75	-2.67	-3.46	-3.85	-5.29	-4.84
IT	-2.74	-1.03	-3.21	-2.56	-2.87	-3.46	-4.03	-3.76
NL	-0.37	-1.47	-3.16	-2.78	-3.64	-3.77	-5.26	-5.71
NO	-1.80	-0.84	-5.29	-2.52	-3.30	-4.32	-4.99	-4.32
SW	-1.23	-2.42	-3.39	-1.77	-2.90	-4.39	-3.73	-3.73
SZ	-1.61	-2.06	-5.55	-4.23	-4.45	-3.81	-4.99	-6.84
UK	-1.15	-1.13	-5.86	-1.94	-3.52	-3.03	-2.95	-4.58
AU	-1.47	-1.88	-2.38	-2.34	-2.63	-4.10	-4.89	-3.26
JA	-1.57	-2.12	-2.98	-2.80	-2.82	-2.93	-4.02	-5.56
KS	-1.53	-1.61	-1.60	-3.29	-2.62	-1.80	-3.66	-2.28
NZ	-1.93	-2.82	-2.42	-2.76	-3.68	-5.16	-3.17	-4.71
Avg	-1.56	-1.68	-3.47	-2.63	-3.51	-3.68	-4.15	-4.28

The 5% critical value is -1.95 and 10% critical value is -1.6

Reject the unit root if the *OLS t*-stat is less than the critical value

Table 2.4: *OLS t*-statistics of negative unit root test for *AR(1)* time-series, first-difference regression

Country	d(ln(E)	d(lnY/E)	d(lnP)	d(lnP*)	d(lnR)	d(lnS)
CA	6.58	18.36	13.79	9.63	9.88	6.00
US	7.11	25.38	5.45	5.15	6.66	6.56
AT	3.82	7.98	6.22	6.03	4.09	4.25
BE	6.89	10.06	7.11	6.66	4.75	5.47
FI	4.66	10.53	5.39	4.62	5.20	5.31
FR	7.60	16.85	7.25	5.09	3.57	4.11
IT	6.13	29.20	10.76	5.80	4.95	5.00
NL	8.25	11.58	7.41	6.33	3.60	3.80
NO	3.51	7.75	9.96	4.62	3.54	4.49
SW	5.06	13.71	8.31	4.65	5.08	3.99
SZ	4.12	7.01	4.60	4.98	5.05	4.01
UK	3.64	15.96	10.01	6.77	6.21	3.74
AU	7.58	15.15	9.44	4.81	4.23	5.95
JA	6.39	9.42	7.61	6.57	4.64	3.62
KS	11.86	14.30	9.57	10.59	5.09	8.67
NZ	7.83	15.41	7.95	3.69	5.90	3.97
Avg	6.31	14.29	8.18	6.00	5.15	4.93

The 5% critical value is 1.729

Reject the negative unit root is the *OLS t*-stat is greater than the critical value

2.2.2 Stationarity Tests for Cross-Sectional Data [Reader may skip Section 2.2.2 entirely]

Although unit root test is usually used to test the stationarity of time series data, its methodology can nevertheless be applied to cross-sectional data[4].

For a given *t*, the level and first difference $AR(1)$ models are:

$$\ln Y_{it} = \rho \ln Y_{i-1,t} + \varepsilon_{it}, i \in [2, I], \tag{2.3}$$

$$\Delta_i \ln Y_{i,t} = \rho \Delta_{i-1} \ln Y_{i-1,t} + \varepsilon_{it}, i \in [3, I], \tag{2.4}$$

where $\Delta_i \ln Y_{it} \equiv \ln Y_{it} - \ln Y_{i-1,t}$.

For the series that I will use in this paper, the tables below summarize their *t*-statistics for the coefficient ρ. They are Table 2.5 unit root test for level regressions (2.3), and Table 2.6 negative unit root test for level regressions (2.4). Again, I have used $P_{it}/P_{ib} \cdot P_{ub}/X_{ib}$ for P_{it}.

In Table 2.5 for the unit root test of the level regressions, for $\ln Y_{it}$, $\ln E_{it}$, $\ln(X_{it}Y_{it}/E_{it})$, we cannot reject the null. For $\ln X_{it}$, $\ln P_{it}$, $\ln(Y_{it}/E_{it})$, we reject the null for all 22 years at the 5% level.

---

[4] The meaning is however a bit more difficult to interpret, since in my definition, from cross-sectional standpoint, for example $\ln Y_{it}$ are denominated in different national currencies. Another issue is that my ordering of the countries--by region, first North America, then Europe and Asia Pacific, and within each regions by alphabetic order of the country's name--is only one of the many possible combinations (to be exact 16 factorial, 16!). However, this issue is not a concern to me, because throughout this paper, I use only one ordering of the countries. All I need to do is ensure to obtain stationarity from the cross-sectional data based the particular ordering I use in this paper.

Further, in Table 2.6 for the negative unit root test of the level regressions, for all series that need further tests, we reject the null for all countries. Therefore we cannot reject that $\ln X_{it}$, $\ln P_{it}$, and $\ln(Y_{it}/E_{it})$ are cross-sectioanlly stationary.

Notice that $\ln Y_{it}$, $\ln E_{it}$, and $\ln(X_{it}Y_{it}/E_{it})$ are cross-sectionally nonstationary. I have tested that $\Delta_i \ln Y_{it}$, $\Delta_i \ln E_{it}$, and $\Delta_i \ln(X_{it}Y_{it}/E_{it})$ are cross-sectionally stationary, though I do not display the testing results here.

Conclusion

Therefore, we can conclude that for all 22 years, the cross-sectional data exhibit the characteristics of processes as follows: $\ln Y_i$, $\ln E_i$, and $\ln(X_iY_i/E_i)$ are cross-sectional $I(1)$ processes, and $\ln X_i$, $\ln P_i$, and $\ln(Y_i/E_i)$ are cross-sectional $I(0)$ processes.

Table 2.5: *OLS t*-statistics of unit root test for *AR(1)* cross-sectional, level regression

Year	lnY	lnE	lnXY/E	lnX	lnP	lnY/E
1980	-0.63	-0.63	-0.65	-2.24	-2.38	-3.24
1981	-0.62	-0.63	-0.61	-2.17	-2.27	-3.33
1982	-0.62	-0.62	-0.50	-2.10	-2.21	-3.40
1983	-0.61	-0.62	-0.39	-2.05	-2.19	-3.44
1984	-0.61	-0.62	-0.32	-2.00	-2.16	-3.48
1985	-0.61	-0.62	-0.33	-1.99	-2.13	-3.51
1986	-0.60	-0.61	-0.49	-2.07	-2.21	-3.53
1987	-0.59	-0.61	-0.61	-2.14	-2.22	-3.56
1988	-0.59	-0.61	-0.62	-2.16	-2.23	-3.58
1989	-0.59	-0.61	-0.51	-2.15	-2.21	-3.61
1990	-0.59	-0.60	-0.50	-2.17	-2.17	-3.65
1991	-0.59	-0.60	-0.46	-2.17	-2.17	-3.68
1992	-0.59	-0.60	-0.51	-2.15	-2.17	-3.69
1993	-0.59	-0.61	-0.61	-2.15	-2.15	-3.70
1994	-0.59	-0.60	-0.70	-2.20	-2.16	-3.71
1995	-0.59	-0.60	-0.77	-2.26	-2.15	-3.71
1996	-0.59	-0.60	-0.80	-2.22	-2.13	-3.73
1997	-0.59	-0.60	-0.74	-2.18	-2.12	-3.73
1998	-0.59	-0.60	-0.66	-2.14	-2.15	-3.71
1999	-0.59	-0.60	-0.63	-2.12	-2.14	-3.71
2000	-0.59	-0.60	-0.50	-2.06	-2.06	-3.72
2001	-0.58	-0.60	-0.49	-2.04	-2.06	-3.71
Avg	-0.60	-0.61	-0.56	-2.13	-2.17	-3.60

The 5% critical value is -1.95 and the 10% critical value is -1.6

Reject the unit root if the *OLS t*-statistics is less than the critical value

Table 2.6: *OLS t*-statistics of negative unit root test for *AR(1)* cross-sectional, level regression

Year	lnX	lnP	lnY/E
1980	6.24	5.89	4.42
1981	6.45	6.17	4.32
1982	6.66	6.32	4.22
1983	6.84	6.38	4.17
1984	6.99	6.48	4.13
1985	7.01	6.57	4.11
1986	6.74	6.31	4.10
1987	6.54	6.28	4.07
1988	6.47	6.25	4.05
1989	6.50	6.31	4.01
1990	6.44	6.42	3.95
1991	6.44	6.44	3.91
1992	6.49	6.43	3.91
1993	6.50	6.49	3.91
1994	6.36	6.48	3.89
1995	6.20	6.48	3.89
1996	6.29	6.56	3.88
1997	6.41	6.58	3.87
1998	6.52	6.50	3.88
1999	6.60	6.53	3.88
2000	6.78	6.78	3.86
2001	6.84	6.79	3.88
Avg	6.56	6.43	4.01

The 5% critical value is 1.96

Reject negative unit root if the *OLS t*-statistics is greater than the critical value

2.3 Summary

I characterize the stationarity features of both the time-series data and the cross-sectional data. Most time-series data are nonstationary in level but stationary in first difference. Some cross-sectional data exhibit the same features. However, there are other cross-sectional data that are stationary in level. To obtain valid statistical inferences, I will use only those data that are stationary for regression models.

3 Energy and Purchasing Power Parity

3.1 Background

Purchasing power parity (PPP) is a simple empirical proposition. The most common definition of PPP is that the national price levels should be equal at any given time, once converted to a common currency. It is equivalent to say that the real exchange rate for the aggregate goods and services should remain unity at any time. However, PPP does not survive empirical verification. The consensus is that the real exchange rate tends toward purchasing power parity in the long run. The speed of convergence is usually summarized by the half-life, the time necessary for half the effect of a shock to dissipate. According to Rogoff (1996), the consensus estimates for the half life is in the range of three to five years, meaning that the deviation from PPP due to a shock takes 3-5 years to get reduced to a half. Lately, Imbs *et al.* (2005) state that by accounting for the heterogeneous components of the real exchange rate, its half life may fall as low as 11 months.

Perhaps it is not so surprising that the PPP that chooses the consumer price index (CPI), the price index for a typical basket of consumption goods for a country, as the price level may not hold, even if the law of one price holds true. The law of one price states that the price of tradable goods should be same in every country. There is little reason to believe that the baskets of goods used to calculate the CPI for each country are identical across different countries. So even the law of one price holds for each good and service, the CPI PPP is unlikely to hold true.

There are other variations of PPP that choose different types of representative good. For example, The Economists magazine has chosen McDonald's fast food restaurant chain's Big Mac hamburgers as the representative good. Ong (1997) tests the Big Mac PPP empirically and finds that the Big Mac Index is accurate in tracking the exchange rates over eight out of nine years. Caetano, Moura and Da Silva (2004) do a joint test over time and reject the Big Mac PPP. They find that trade barriers can explain the departures from the Big Mac PPP.

Pokka and Pollard (1996 and 2003) maintain that the Big Mac PPP, just as other more sophisticated measures do, generally fails to hold except under special circumstances and does poorly as a predictive tool. They argue it is because Big Mac is a composite of tradable commodities and non-tradable service content, and the existence of both trade barriers and non-tradable component tend to make PPP fail. In this regard, Parsley and Wei (2004) provide a micro-foundation for the Big Mac PPP 1990-2002 by decomposing a Big Mac into tradable and non-tradable components. They include both time and country dummies in their regressions and find that the half life of a Big Mac is about 2.1 years, its tradable components 1.4 years, and its non-tradable components 3.9 years.

Whether Big Mac PPP holds or not, one may consider a piece of hamburger not a good representation. Although it is universal that McDonald's is present in many countries, a hamburger is too narrow a representative good for it does not relate to every sector or geography of the economy. Furthermore, in some countries a Big Mac may not be seen as just food but as an experience connecting to the world outside. As Pakko and Pollard (2003) point out that, for example, "MacDonald's in China attracts young urban professionals who see eating there as a way to connect with the world. For those who visit Beijing from the Chinese countryside, McDonald's is viewed as a tourist stop... These factors may be reflected in the price differences of a Big Mac around the world."

Another variation is Gold PPP. Diebold *et al.* (1991) study the behavior of real exchange rate during the gold standard era. They find that PPP holds in the long run for each currency and the typical half life of a shock to parity is approximately 3 years. Hedge and Papell (2002) study the same period and use the same data, and find the half life is 2.85-10.76 years, but is reduced to 0.61-2.54 years if regime changes are allowed in the time series data. Neither of the above two groups studies the cross-sectional relationship.

Again, whether Gold PPP holds or not, gold is rare metal and its production is concentrated in a few limited regions in the world. Like Big Max, gold is also too narrowly used in the economy to be qualified for a good representative good.

Instead of gold or a consumption bundle or a Big Mac, yet another variation is to consider energy the representative good that may satisfy the PPP. In this paper, I hypothesize an Energy Purchasing Power Parity, proposing that energy be the representative good for the PPP. I use a common physical unit to aggregate all sources of energy, oil included. As a precursor to the study of this paper, Amano and Norden (1998) document the negative correlation between term of trade and lagged oil price for the US, Japan and Germany from 1974 too 1992, and suggest that oil price shocks could be the most important factor determining real exchange rates in the long run.

3.2 Theory

3.2.1 Assumptions

Energy is used for transportation, in distributing nutrition, raw material, goods and services from one place to another. Energy is used for transformation, in changing one form of material to another form that is useful. Energy is used for transmission, in transmitting heat and information to sustain life and support economic activities.

It is a fact that energy is used by almost all sectors, manufacturing or service, across almost all regions, cities or country sides, and is needed by almost all walks of life for work or leisure. Energy is ubiquitous in the economy as much as are electromagnetic waves in the universe. So energy is universal for every country uses it, and is also broad enough for it relates to every sector of the economy.

But in order to qualify for a representative good, one may ask: is energy tradable? Not completely, if one would define energy as sources, such as oil or coal or wood, or carriers, such as electricity or hydrogen. However, energy could be considered fully tradable if one would define energy as the aggregation of final consumption in terms of heat content. The later is in fact my definition in this paper. So the energy that I am referring to throughout this paper is the aggregation of heat count from all sources and carriers that is consumed in the economy, not a particular energy sources or carriers.

There are several equivalent units for heat content one can choose from. The common ones are the British thermal unit (BTU), joules, calories, and kilowatt-hours. For their conversion schedule, see www.eia.doe.gov/basics/conversion_basics.html. In this paper, I use the BTU as the unit base for aggregating all energy forms that are consumed by the end users.[5]

Let me elaborate more about energy's tradability. When a country exports coal, it exports the BTU amount that is equal to the heat content of the coal. When it imports oil, it imports BTU. When it exports a good or service, it also exports the heat content that has been used to produce such a good or service. So it is in this sense, I assume that energy measured in terms of BTU is fully tradable.

One may argue that energy may not be a good candidate in testing the PPP because oil is usually quoted in the US dollar in the international market. The argument would be valid should all the 16 counties in my universe use oil as their only energy sources. Table 3.1 shows that in 1990, the shares of petroleum products, of which oil is only a part, range from less than 40% for the Netherlands to near 70% for South Korea. Not only do the shares of petroleum have large variability across countries, but also other energy sources and carriers. For example, natural gas shares go from 0% for Norway to 45% for the Netherlands and the electricity shares range from 13% for South Korea to 46% for Norway. So we may assert that the energy sector mix is unique in each country.

[5] Berndt (1978) argues that based on The Second Law of Thermodynamics, it ought to be the available energy or available useful work, not energy per se, that is of critical importance. However, because available energy is not measured and hence the data is unavailable, so I use BTU as a common measure for energy.

Table 3.1: Comparison of energy use mix (%) of total final consumption for 16 OECD countries in 1990

	CA	US	AT	BE	FI	FR	IT	NL	NO	SW	SZ	UK	AU	JA	KS	NZ
Coal & Coal Products	2	4	6	11	7	5	3	3	4	3	2	7	7	11	18	10
Petroleum Products	44	53	46	52	43	54	55	39	44	44	65	47	53	62	68	45
Natural Gas	27	23	15	21	4	16	26	45	0	1	8	29	15	5	1	13
Hydro	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Geothermal	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
Solar/Wind/Other	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Combustible Renewables	5	2	11	1	15	6	1	1	5	14	3	0	6	1	0	5
Electricity	22	17	18	15	22	18	16	12	46	32	21	16	19	21	13	24
Heat	0	0	3	1	8	0	0	1	0	5	1	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Source: IEA

3.2.2 Energy Purchasing Power Parity (EPPP)

3.2.2.1 Perfect No-Arbitrage for Energy

If energy is a representative good across economies, then by the argument of no arbitrage or the law of one price per BTU, at any given time *t*, the following relationship must hold:

$$X_{it}P_{it} = X_{jt}P_{jt}, \forall i, j \in [1, I] \tag{3.1}$$

where X_{it} and X_{jt} are the nominal exchange rates at time *t* for countries *i* and *j* respectively, and P_{it} and P_{jt} are the nominal energy prices at time *t* for countries *i* and *j* respectively. The unit of exchange rate is defined as the value of the home currency in terms of the reference currency. The unit of energy price is nominal home currency per BTU. So Eq. (3.1) states that the nominal exchange rates equalize the nominal energy price differentials across countries, or conversely the nominal energy prices equalize the exchange rate differentials.

Note that Eq. (3.1) asserts only a relationship in equilibrium; it is silent about causality. In fact, throughout this paper, I am only interested in exploring the functional relationships and make no attempt to claim any causal relationships.

By convention, I choose the US dollar as the reference currency. Hence $X_{ut} = 1, \forall t$, where *u* stands for the country index for the US. Then at any given time *t*, (3.1) will becomes

$$X_{it} = \frac{P_{ut}}{P_{it}}, \forall i \in [1, I]. \tag{3.2}$$

Further define the energy real exchange rate between countries i and u as the value of country i's BTU in terms of country u's BTU:

$$R_{it} \equiv \frac{X_{it}P_{it}}{P_{ut}}. \tag{3.3}$$

Suppose (3.1) holds, then

$$R_{it} = 1, \forall i \in [1, I] \tag{3.4}$$

or

$$\ln R_{it} = 0, \forall i \in [1, I] \tag{3.5}$$

This is a relationship of purchasing power parity considering energy is the representative good. I refer to this relationship as the Energy Purchasing Power Parity (EPPP) or the Absolute Energy Purchasing Power Parity.

Partly because of the limited success of empirical validation of PPP, relative PPP has been considered an alternative. Relative PPP focuses on the relationships about the rates of change over time instead of about the levels at a particular time.

So extending (3.3), the Relative EPPP holds if any of the following equivalent forms holds:

$$\frac{\frac{d}{dt}(X_{it}P_{it})}{X_{it}P_{it}} = \frac{\frac{d}{dt}P_{ut}}{P_{ut}}, \forall i \in [1, I], \tag{3.6}$$

or,

$$\frac{d}{dt}\ln X_{it}P_{it} = \frac{d}{dt}\ln P_{ut}, \forall i \in [1, I], \tag{3.7}$$

or

$$\frac{d}{dt}\ln R_{it} = 0, \forall i \in [1, I]. \tag{3.8}$$

It is straightforward to show that if Absolute EPPP holds for all t, then Relative EPPP also holds for all t. So the Absolute EPPP always implies the Relative EPPP[6]. This statement is true regardless the real exchange, R_{it}, being deterministic or stochastic over time, because PPP is a cross-sectional relationship, not an intertemporal one. Additionally, it is also obvious to see that relative EPPP does not necessarily imply the absolute EPPP.

So far, by convention, I have conveniently used the US, indexed by u, as the reference country, so $X_{ut} = 1, \forall t$. In fact, the US economy cannot be the reference economy to represent the world economy at all time. A better reference will be the "world" economy, denoted by the index of w and defined later in this section, whose exchange rate with respect to the reference economy (i.e. itself) will be always unity at all time. So modifying (3.1), we have

$$X_{wit}P_{it} = X_{wjt}P_{jt} = X_{wut}P_{ut} = X_{wwt}P_{wt} = P_{wt}, \forall i, j \in [1, I] \tag{3.9}$$

One cay verify that if

[6] It is obvious by comparing (3.5) and (3.8). We know (3.5) implies (3.8), but not the other around. Intuitively, the no arbitrage argument behind the PPP is no arbitrage for the level of combined effect of exchange rate and price and if the combined level holds true for no arbitrage, then so do their combined movement. So if we should accept Absolute PPP, we would have to accept Relative PPP, and there would be no need to test the Relative PPP.

$$X_{it} \equiv \frac{X_{wit}}{X_{wut}}, \forall i \in [1, I], \tag{3.10}$$

then all the relationships (3.1)-(3.5) for the Absolute EPPP and (3.6)-(3.8) for the Relative EPPP still hold. It is logical because all I do is simply shift the reference country between "u" and "w". For cross-sectional analysis for the Absolute EPPP, this does not matter. And if it does not affect the Absolute EPPP analysis, it does not affect the Relative EPPP analysis. In fact, in a perfect no arbitrage world, i.e. when the equalities in (3.9) hold without any errors, it does not matter regarding the choice of the reference country, whether it be "u" or "w" or any "i".

3.2.2.2 Imperfect No-Arbitrage for Energy

In an incomplete no-arbitrage world, the equalities in (3.9) can no longer hold without errors and should be replaced with approximations. One may consider the errors are due to any noises or transactions frictions in trades.

With the error terms present, (3.9) becomes

$$\frac{X_{wit}P_{it}}{R_{it}} = \frac{X_{wjt}P_{jt}}{R_{jt}} = \frac{X_{wut}P_{ut}}{R_{ut}} = \frac{X_{wwt}P_{wt}}{R_{wt}} = P_{wt}, \forall i, j \in [1, I] \tag{3.11}$$

where R_{it} denotes the measure for the deviation from the perfect no arbitrage world. If $R_{it} < 1$, we say country i's currency undervalued or its energy underpriced or both; if $R_{it} > 1$, then we say country i's currency overvalued or its energy overpriced or both. Note this kind of deviation by definition cannot be fully explored by arbitrage, because it is noisy. We should expect that the deviation is non-persistent over time.

Notice that R_{it} is the value of country i's BTU in terms of country w's BTU. This is the real energy exchange rate for country i. It must be equal to unity in the perfect no-arbitrage world. In a noisy world, however, it is not. Let me repeat its definition for clarity:

$$R_{it} \equiv \frac{X_{wit} P_{it}}{P_{wt}} \tag{3.12}$$

I then define the world's real exchange rate, R_{wt}, and the world's energy price, P_{wt}, as the geometric mean of all the countries in the universe: [7]

$$R_{wt} \equiv \left(\prod_{i=1}^{I} R_{it} \right)^{\frac{1}{I}} \equiv 1, \forall t \in [1,T] \ , \tag{3.13}$$

$$P_{wt} = \left(\prod_{i=1}^{I} \frac{X_{wit} P_{it}}{R_{it}} \right)^{\frac{1}{I}} = \left(\prod_{i=1}^{I} X_{wit} P_{it} \right)^{\frac{1}{I}}, \forall t \in [1,T]. \tag{3.14}$$

Then, the no arbitrage relationship (3.11) becomes

$$\frac{X_{it} P_{it}}{R_{it}} = \frac{X_{jt} P_{jt}}{R_{jt}} = \frac{P_{ut}}{R_{ut}} = \frac{P_{wt}}{X_{wut}}, \forall i,j \in [1,I] \tag{3.15}$$

where

$$\frac{P_{ut}}{R_{ut}} = \frac{P_{wt}}{X_{wut}} = \left(\prod_{i=1}^{I} \frac{X_{it} P_{it}}{R_{it}} \right)^{\frac{1}{I}} = \left(\prod_{i=1}^{I} X_{it} P_{it} \right)^{\frac{1}{I}}, \forall t \in [1,T] \tag{3.16}$$

Therefore we can obtain the noisy EPPP as, at any given t,

---

[7] I define the energy price and the real energy exchange rate of the "world" as the simple geometric mean of the countries in my data universe. The reason is two fold. First, conventionally in economics, we often take logarithm of interested variables before we proceed with the analysis. The geometric mean will become the arithmetic mean in a logarithmic sense. Second, that a simple arithmetic mean puts equal weighs on all countries is consistent with the *OLS* setup. We can certainly employ weighted least square (*WLS*) that would assign different weighs to different countries. The *WLS* may have the merit, because economically speaking, we do not believe that in the no-arbitrage market for the BTU, all countries have the same market power. However, I think that redefining (3.13) and (3.14) as a weighted mean will be an improvement of only the second order.

$$\frac{X_{it}P_{it}}{R_{it}} = \frac{P_{ut}}{R_{ut}}, \forall i, \text{ where } \left(\prod_{i=1}^{I} R_{it}\right)^{\frac{1}{I}} = 1 \tag{3.17}$$

and noisy Relative EPPP as, at any given *t*,

$$\frac{d}{dt}\ln\frac{X_{it}P_{it}}{R_{it}} = \frac{d}{dt}\ln\frac{P_{ut}}{R_{ut}}, \forall i, \text{ where } \left(\prod_{i=1}^{I} R_{it}\right)^{\frac{1}{I}} = 1 \tag{3.18}$$

Again, noisy EPPP implied noisy Relative EPPP, but not the other way around.

An interesting question to ask: should (3.17) hold empirically for energy, would this imply the existence of an energy standard economy?

3.3 Regression Models and Results for (Absolute) EPPP

As discussed previously, Absolute EPPP always implies Relative EPPP. So I will focus on testing first the noisy Absolute EPPP in this subsection.

3.3.1 Setup

I will focus on testing the noisy EPPP (3.17), which requires the information of X_{it} and P_{it}. However, since I do not have the data for P_{it}, I cannot directly test (3.17). Instead I have the data only for the price index, i.e. P_{it}/P_{ib}, $\forall i \in [1, I]$, where *b* stands for the reference year. Fortunately, we can derive an equivalent relationship by combining (3.17) for an arbitrary *t* and for $t = b$ and to obtain the following representations[8], one for level, and the other for first difference in a logarithmic sense. For level,

[8] This combination assumes that (3.17) holds. I propose a simple way to check the validity of (3.17) and compare energy price and CPI immediately following the Setup section in this chapter.

$$\frac{P_{it}/P_{ib}}{X_{ib}} = \frac{P_{ut}/P_{ub}}{R_{ut}/R_{ub}} \frac{1}{X_{it}} \frac{R_{it}}{R_{ib}}, \forall i \in [1, I] \tag{3.21}$$

or

$$\ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = \ln\left(\frac{P_{ut}/P_{ub}}{R_{ut}/R_{ub}}\right) + \left(-\ln X_{it}\right) + \ln\frac{R_{it}}{R_{ib}}, i \in [1, I] \tag{3.22}$$

and for first difference,

$$\frac{P_{it}/P_{ib}}{X_{ib}} \Bigg/ \frac{P_{i-1,t}/P_{i-1,b}}{X_{i-1,b}} = \left(\frac{X_{it}}{X_{i-1,t}}\right)^{-1} \cdot \left(\frac{R_{it}}{R_{ib}} \Bigg/ \frac{R_{i-1,t}}{R_{i-1,b}}\right), \forall i \in [2, I] \tag{3.23}$$

$$\Delta_i \ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = \Delta_i\left(-\ln X_{it}\right) + \Delta_i \ln\frac{R_{it}}{R_{ib}}, i \in [2, I] \tag{3.24}$$

As discussed in Section 2, $\ln X_{it}$, $\ln P_{it}$, $\Delta_i \ln X_{it}$ and $\Delta_i \ln P_{it}$ are all stationary in *i*.

Notice I have the exchange rate as the RHS variable in (3.22) and (3.24). I could have the price on the RHS. In theory, these two specifications are identical. However, for regression estimation, these two specifications may differ if we have to consider the measure error problems. Because the exchange rate data are obtained from the foreign exchange market, which is liquid and with low friction, I believe that measure errors are not a problem. As for the energy prices, according to the data source[9], the energy price data are obtained via a complicated process of calculation that uses both market data and survey information, so we cannot assume away the measure error problems. To mitigate the possible measure error problems associated with the energy prices, I thus specify in (3.22) and (3.24) energy price on the LHS and exchange rates on the RHS.

[9] International Energy Agency

Furthermore, from regression's stand point, the specifications of (3.22) and (3.24) are preferred to their respective alternatives where all variable are normalized to the base year:

$$\ln P_{it}/P_{ib} = \ln \frac{P_{ut}/P_{ub}}{R_{ut}/R_{ub}} + \left(-\ln \frac{X_{it}}{X_{ib}} \right) + \ln \frac{R_{it}}{R_{ib}}, \forall i \in [1, I], \tag{3.25}$$

$$\Delta_i \ln(P_{it}/P_{ib}) = \Delta_i \left(-\ln \frac{X_{it}}{X_{ib}} \right) + \Delta_i \ln \frac{R_{it}}{R_{ib}}, i \in [1, I-1] \tag{3.26}$$

The reason is that in (3.25) and (3.26), as $t \to b$, both RHS and LHS will converge to 1, and thus (3.25) and (3.26) will become untestable. For a given t, the farther t is away from b, the more testable they will become because there will be more variability in both LHS and RHS variables. One way to check whether (3.17) holds before diving into full-blown tests on (3.22) and (3.24) is as follows. First, let $b = 1980$, then specify the regression models based on (3.25) and (3.26) and test the hypothesis for $t = 2001$. Second, let $b = 2001$, then specify the regression models based on (3.25) and (3.26) and test the hypothesis for $t = 1980$. Third, if we do not reject the hypotheses, then we conclude that we cannot reject the validity of (3.17) and hence we may go ahead using the specification of (3.22) and (3.24). However, if we reject either one, then we reject (3.17) and hence we cannot use the specification of (3.22) and (3.24).

3.3.2 A Simple Validation: Energy PPP vs. CPI PPP [Reader may skip Section 3.3.2 entirely]

Here, I will setup regression models base on (3.26) that will give us a simple, quick validation whether (3.17) holds. The regressions model is:

$$\Delta_i \ln(P_{it}/P_{ib}) = b_1^{FD}(t) + b_2^{FD}(t)\, \Delta_i \left(-\ln \frac{X_{it}}{X_{ib}} \right) + \Delta_i \ln \frac{R_{it}}{R_{ib}}, i \in [2, I] \tag{3.27}$$

And the null hypothesis H_0 :

for base year $b = 1980$, $b_1^{FD}(t = 2001) = 0, b_2^{FD}(t = 2001) = 1$, and

for base year $b = 2001$, $b_1^{FD}(t = 1980) = 0, b_2^{FD}(t = 1980) = 1$.

Figure 3.1 shows the results. If we choose the base year to be 1980, we cannot reject $b_1^{FD}(t) = 0$ for all *t*, and also we cannot reject $b_2^{FD}(t) = 1$ for all *t* except 1981. If we choose the base year to be 2001, we cannot reject $b_1^{FD}(t) = 0$ for all *t*, and also we cannot reject $b_2^{FD}(t) = 1$ for years 1980-1982 and 1985-1995. So, we cannot reject H_0. Therefore we cannot reject the validity of (3.17) for energy price and hence we cannot reject the specifications of (3.21)-(3.24) for energy price.

For comparison, let me also test CPI PPP with the same regression model and same null:

$$\Delta_i \ln\left(CPI_{it}/CPI_{ib}\right) = b_1^{FD}(t) + b_2^{FD}(t)\,\Delta_i\left(-\ln\frac{X_{it}}{X_{ib}}\right) + \Delta_i \ln\frac{R_{it}}{R_{ib}},\ i \in [2, I] \qquad (3.28)$$

Figure 3.2 shows the results for CPI. It is obvious that regarding the base year we choose, though we cannot reject $b_1^{FD}(t) = 0$ for all *t*, we reject $b_2^{FD}(t) = 1$ for any *t*. The joint *F* test also reject $b_1^{FD}(t) = 0$ and $b_2^{FD}(t) = 1$ for any *t*. So we reject H_0 for CPI. Therefore we reject the validity of (3.17) for CPI and hence we cannot use the specifications of (3.21)-(3.24) for CPI.

To summarize, we cannot reject the validity of (3.21)-(3.24) for the relationship between energy price and exchange rate. But we reject the validity of (3.21)-(3.24) for the relationship between CPI and exchange rate.

Given the validity for (3.21)-(3.24) for energy price, for the remaining of the paper I will choose the base year to be year 2001 through the end of Section 3.3, and then from there on use the base year to be year 2000 for the remaining of the paper.

Fig 3.1: Estimated coefficients, $\hat{b}_1(t)$ and $\hat{b}_2(t)$, and their 95% confidence bounds as functions of t, for the cross-sectional regression (3.27):

$$\Delta_i \ln P_{it} / P_{ib} = b_1(t) + b_2(t)\,\Delta_i\left(-\ln X_{it} / X_{ib}\right) + \varepsilon_{it},\ i \in [2, I]$$

Upper two panels: base year $b = 1980$,

$$H_0 : b_1(2001) = 0, b_2(2001) = 1$$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t

Lower two panels: base year $b = 2001$,

$$H_0 : b_1(1980) = 0, b_2(1980) = 1$$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t

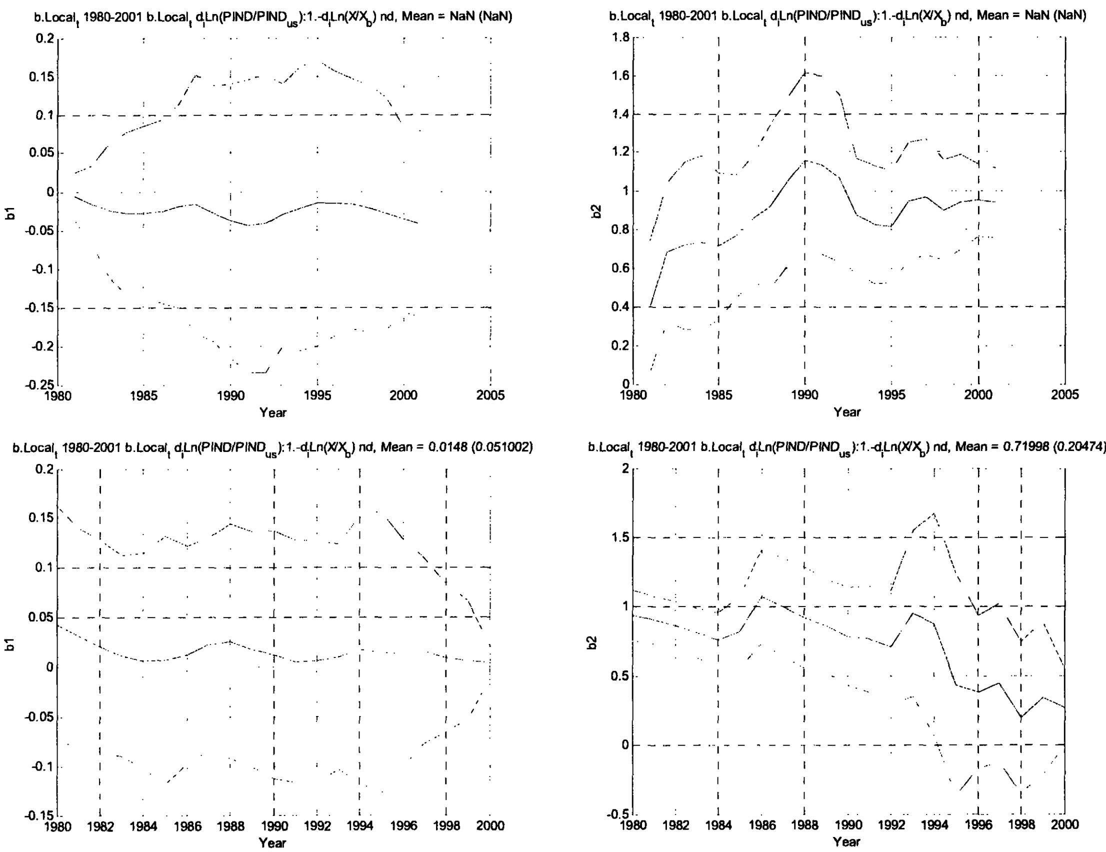


Fig 3.2: Estimated coefficients, $\hat{b}_1(t)$ and $\hat{b}_2(t)$, and their 95% confidence bounds as functions of t, for the cross-sectional regression (3.28):

$$\Delta_i \ln CPI_{it} / CPI_{ib} = b_1(t) + b_2(t)\, \Delta_i\left(-\ln X_{it} / X_{ib}\right) + \varepsilon_{it},\ i \in [2, I]$$

Upper two panels: base year $b = 1980$,

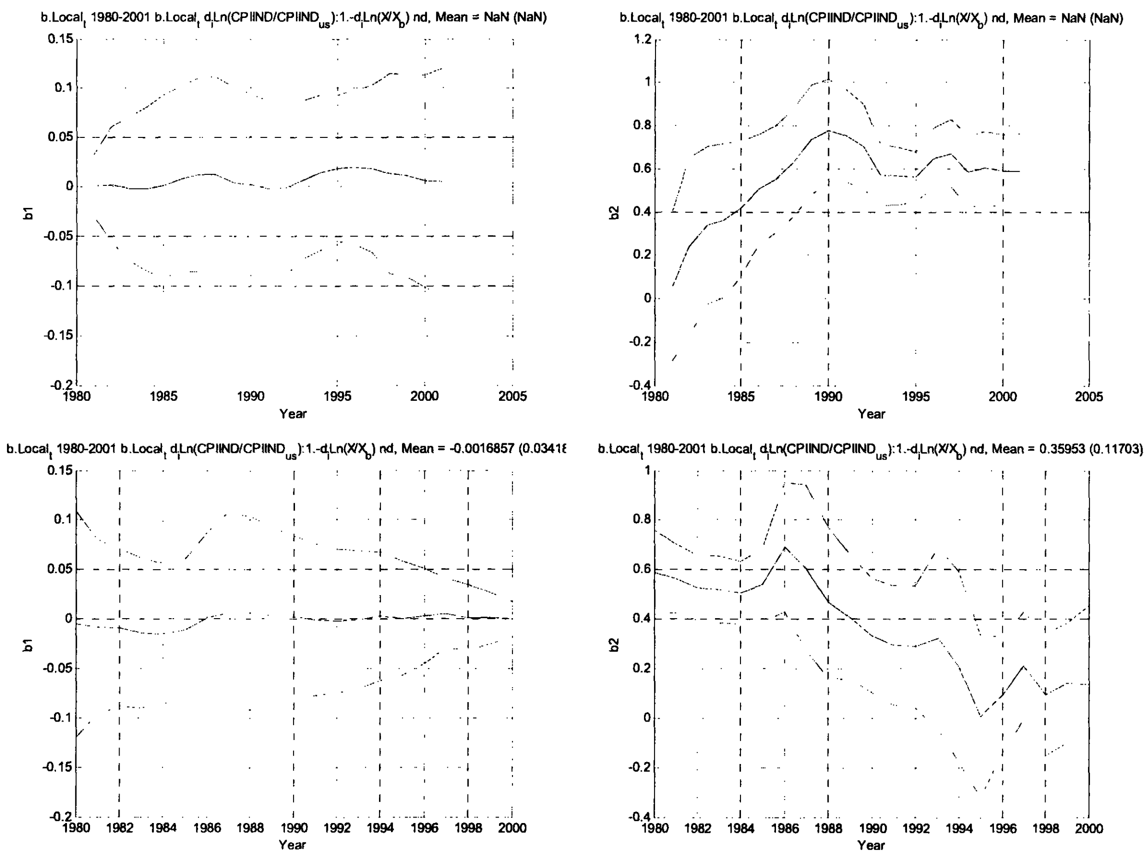
$$H_0 : b_1(2001) = 0, b_2(2001) = 1$$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t

Lower two panels: base year $b = 2001$,

$$H_0 : b_1(1980) = 0, b_2(1980) = 1$$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t



3.3.3 Period-by-Period Regressions

3.3.3.1 Period-by-Period Level Regressions

Model:

From specification of (3.22), we have the regression model: at any give $t \in [1,T]$,

$$\ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = b_1(t) + b_2(t)\left(-\ln X_{it}\right) + \ln\frac{R_{it}}{R_{ib}},\ i \in [1,I] \tag{3.31}$$

The null hypothesis for each time period: $b_2(t) = 1$, and the alternative hypothesis: $b_2(t) \neq 1$. The null for joint F-test for all time periods: $b_2(t) = 1,\ \forall t \in [1,T]$, and its corresponding alternative: $b_2(t) \neq 1,\ \exists t \in [1,T]$.

Validity of the Model:

From Section 2, we have leaned that both LHS and RHS variables are stationary across i. Notice that in setting up (3.31), I assume the RHS variable and the error term are cross-sectionally uncorrelated. To test this assumption, I employ the Hausman specification test (1978). The instrument I choose for the RHS variable, $\ln X_{it}$, is

$IV_{it} = \frac{1}{T-1}\sum_{\substack{s=1 \\ s\neq t}}^{T} \ln X_{is}$. I believe that this instrument variable for a given t, should be orthogonal to the error term in (3.31) because this instrument does not contain any sample of the same time t. I also believe that the instrument should be also correlated with the RHS variable in (3.31) since $\ln X_{it}$ is an $I(1)$ process (though not shown in Section 2) over time and therefore $\ln X_{it}$ should be intertemporally correlated.

Table 3.2 below lists the chi-square statistics for five selected years for the Hausman test using the above instrument variable. Because the test statistics are smaller than the critical values for any given time, we cannot reject the null hypothesis that the RHS variable is uncorrelated with the error term in (3.31).

Table 3.2: Hausman test for (3.31)

Year	1980	1985	1990	1995	2000
Chi2-stat	0.0018	0.4656	0.0044	0.1224	0.0001

The 5% critical value of chi square with 1 degree of freedom is 3.84
Use average over time (excluding current time) of exchange rates as instrument

Results:

Figure 3.3 consists of four graphs, representing the actual and the fitted data of the regressions (3.31) for the years of 1980, 1985, 1990, and 1995, respectively. They are all with high R^2 and slope $\hat{b}_2(t)$ being close to unity.

Figure 3.4 displays the estimated coefficients and their 95% respective confidence bounds as functions of time. The slopes are all statistically equal to 1, with high p-values (not shown in the graphs) for all *t*. It is obvious that we cannot reject the null for all *t*. The sample average across all year, $\overline{\hat{b}}_2 \equiv \frac{1}{T}\sum_{t=1}^{T}\hat{b}_2(t) = 1.0054$ (0.0169), is also statistically equal to 1, with $\overline{R^2} = 0.9965$. The *F*-statistics for the joint tests of $H_0 : b_2(t) = 1, \forall t \in [1, T]$ is 0.1391 with the 5% critical value of 1.5769. The p-value of the *F*-test is as high as 1.0000. So we cannot reject H_0.

Fig 3.3: Actual and fitted curves for different years for cross-sectional level regressions (3.31): $\ln\left(P_{it}/P_{ib}\cdot X_{ib}^{-1}\right)=b_1(t)+b_2(t)\left(-\ln X_{it}\right)+\varepsilon_{it},\ i\in[1,I]$.
From left to right and then top to down: 1980, 1985, 1990, and 1995.

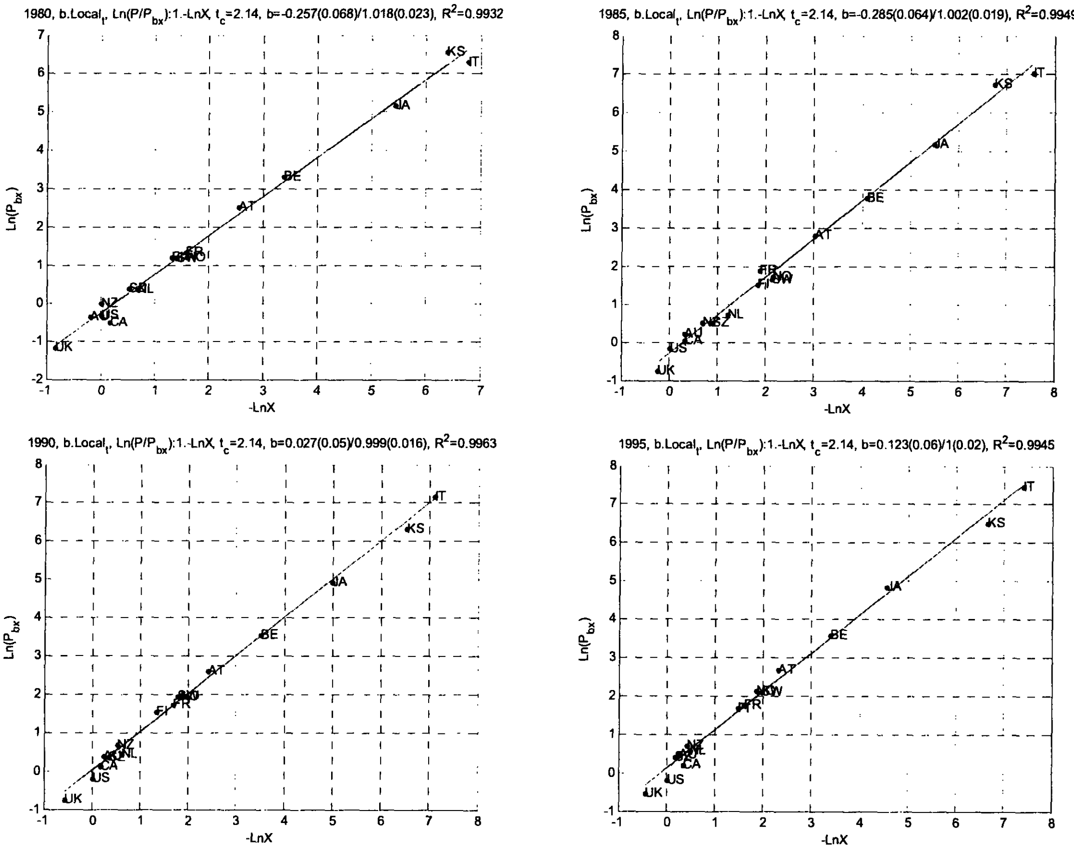
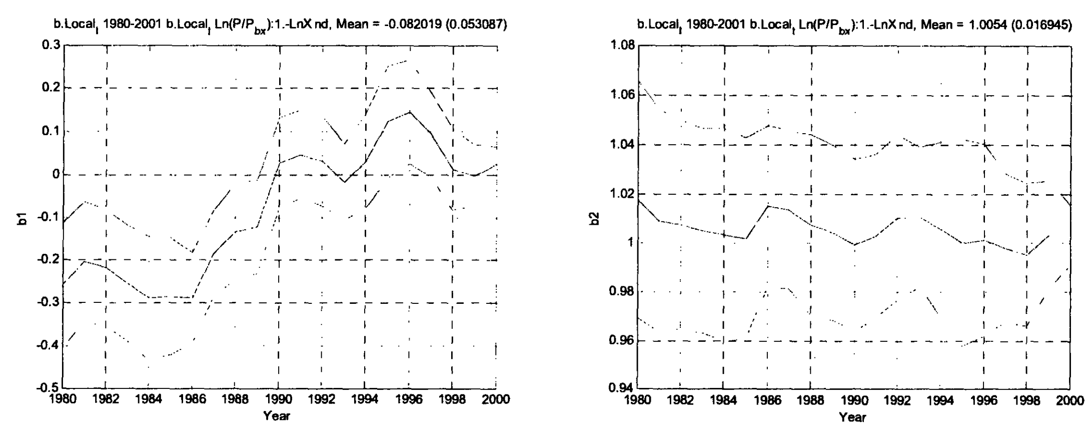


Fig 3.4: Estimated coefficients and their 95% confidence bounds as functions of t, for the cross-sectional level regression (3.31): $\ln\left(P_{it}/P_{ib}\cdot X_{ib}^{-1}\right)=b_1(t)+b_2(t)\left(-\ln X_{it}\right)+\varepsilon_{it},\ i\in[1,I]$.
$H_0: b_2(t)=1, \forall t$
Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t



3.3.3.2 Period-by-Period First Difference Regressions

Model:

From the specification from of (3.24), at any give $t \in [1, T]$,

$$\Delta_i \ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = b_1(t) + b_2(t)\,\Delta_i(-\ln X_{it}) + \Delta_i \ln\frac{R_{it}}{R_{ib}},\ i \in [2, I] \tag{3.32}$$

where the operator Δ_i is defined in Section 2. The null hypothesis for each time period: $b_1(t) = 0$ and $b_2(t) = 1$, and the alternative: $b_1(t) \neq 0$ or $b_2(t) \neq 1$. Also the null for joint F-tests for all time periods: $b_1(t) = 0$ and $b_2(t) = 1, \forall t \notin [1, T]$, and its corresponding alternative: $b_1(t) \neq 0$ or $b_2(t) \neq 1, \exists t \in [1, T]$.

Validity of the Model:

From Section 2, we have leaned that both LHS and RHS variables are stationary across *i*. Again, by way of Hausman test, we cannot reject that the RHS variable and the residual term are cross-sectionally uncorrelated.

Results:

Figure 3.5 consists of four graphs, representing the actual and the fitted data of the regressions (3.32) for the years of 1980, 1985, 1990, and 1995 respectively. They all have high R^2, intercept $\hat{b}_1(t)$ being close to 0, and slope $\hat{b}_2(t)$ being close to 1.

Figure 3.6 displays the estimated coefficients and their respective 95% confidence bounds as functions of time. The intercepts are all statistically equal to 0 and the slopes are all statistically equal to 1, with high p-values (not shown in the graphs) for all *t*. Again, it is obvious that we cannot reject the null for all *t*. The sample average across all year, $\overline{\hat{b}_1}$ =-0.0187 (0.0547) is statistically 0, and $\overline{\hat{b}_2}$ =0.9963 (0.0171) is statistically 1. Also $\overline{R^2}$ = 0.9962. Additionally, the F-statistics for the joint tests of

$H_0 : b_1(t) = 0, b_2(t) = 1, \forall t \in [1, T]$ is 0.1958 with the 95% critical value of 1.4185, making the p-value of the *F*-test be 1.0000. So we cannot reject H_0.

Fig 3.5: Actual and fitted curves for different years for cross-sectional first difference regression (3.32):

$$\Delta_i \ln\left(P_{it}/P_{ib}\cdot X_{ib}^{-1}\right)=b_1(t)+b_2(t)\,\Delta_i\left(-\ln X_{it}\right)+\varepsilon_{it},\ i\in[2,I].$$

From left to right and then top to down: 1980, 1985, 1990, and 1995.

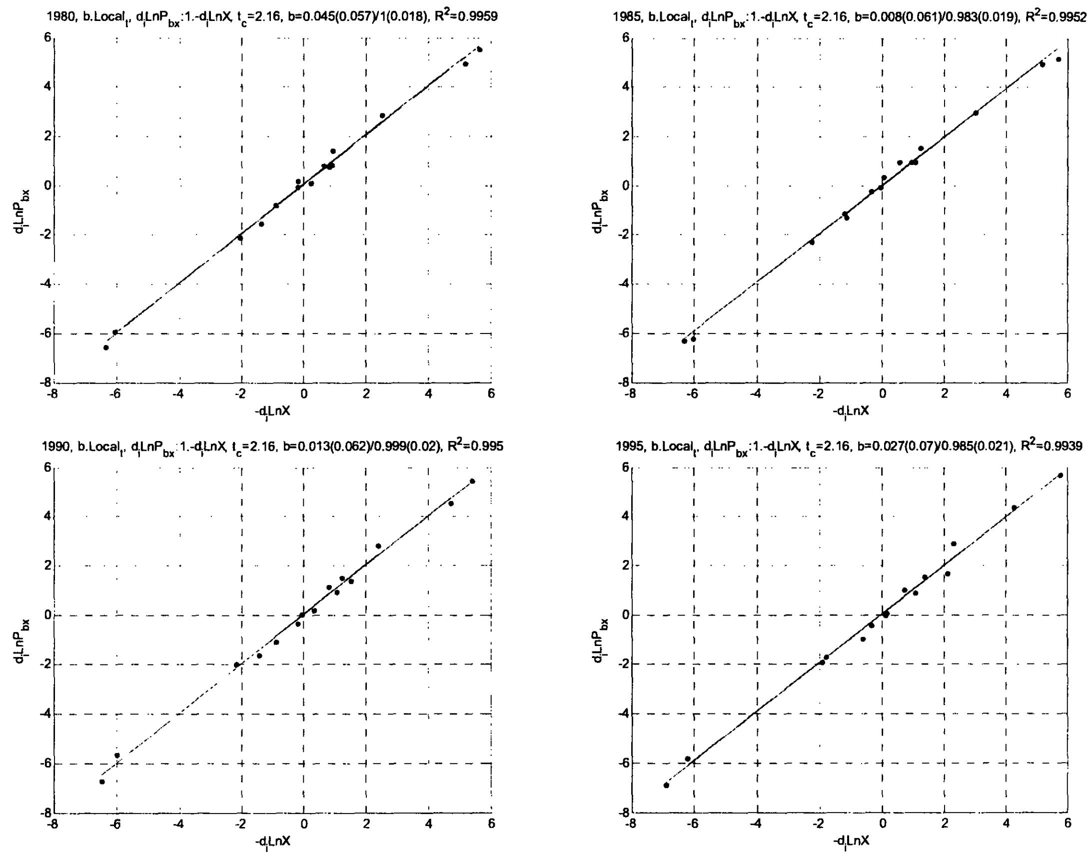
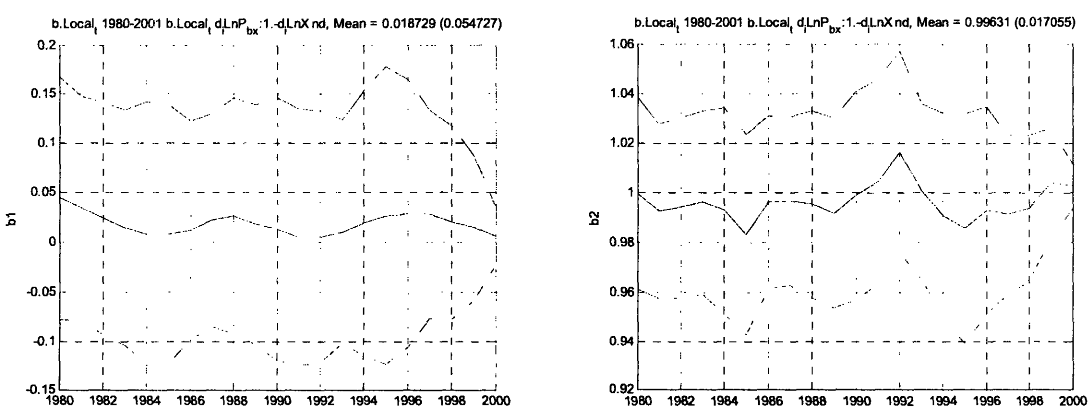


Fig 3.6: Estimated coefficients and their 95% confidence bounds as functions of t, for the cross-sectional first difference regression (3.32):. $\Delta_i \ln\left(P_{it}/P_{ib}\cdot X_{ib}^{-1}\right)=b_1(t)+b_2(t)\,\Delta_i\left(-\ln X_{it}\right)+\varepsilon_{it},\ i\in[2,I]$

$H_0 : b_1(t)=0,\ b_2(t)=1, \forall t$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t



3.3.4 Pooled Regressions

3.3.4.1 Pooled Level Regressions

Model:

In the pooled regressions, I have unique intercept (fixed effect) per time period, but have common slope for all time periods:

$$\ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = b_1(t) + b_2\ (-\ln X_{it}) + \ln\frac{R_{it}}{R_{ib}},\ i \in [1, I],\ t \in [1, T] \tag{3.33}$$

The null: $b_2 = 1$, and the alternative: $b_2 \neq 1$.

Results:

The regression results have that $\hat{b}_2$ = 1.0018 (0.0033), with the 95% critical value being 1.967, so the p-value is 0.5909. Therefore we cannot reject the null. Also, R^2= 0.9963.

3.3.4.2 Pooled First Difference Regressions

Model:

$$\Delta_i \ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = b_1 + b_2\ (-\Delta_i \ln X_{it}) + \Delta_i \ln\frac{R_{it}}{R_{ib}},\ i \in [2, I], \forall t \in [1, T] \tag{3.34}$$

The null: $b_1 = 0$ and $b_2 = 1$, and the alternative: $b_1 \neq 0$ or $b_2 \neq 1$.

Results:

The regression results have that $\hat{b}_1$ = 0.0115 (0.0108) and $\hat{b}_2$ = 0.9937 (0.0034), and the joint F-statistics = 2.2917 with the critical value being 3.0233. So the p-value is 0.1027. Therefore we do not reject the null. Also, R^2= 0.9963.

3.3.5 Alternative Setup [Reader may skip Section 3.3.5 entirely]

An alternative setup is to swap RHS and LHS variables in (3.21)-(3.24), i.e. to have the exchange rate on LHS and the energy price on the RHS. It turns out that this alternative setup yields similar results: cannot reject the EPPP and the p-values are high. The p-values are only slightly smaller than those of the setup in (3.21)-(3.24). So the measure errors on the energy price may not be a real concern.

3.4 Regression Models and Results for Relative EPPP

3.4.1 Cross-Sectional Tests [Reader may go directly to the last paragraph of Section 3.4.1 and skip the rest of this section]

The setup and models for testing the noisy Relative EPPP (3.18) are similar to those for testing noisy EPPP (3.17) that have been spelled out in Section 3.3.3. Recall that in order to use the regression models in Section 3.3.3, I have assumed that the RHS variable is orthogonal to the error term.

Similar to (3.31) for period-by period, we have the following for testing the noisy Relative EPPP, for any given $t \in [1, T-1]$,

$$\Delta_t \ln\left(\frac{P_{it}/P_{ib}}{X_{ib}}\right) = b_1(t) + b_2(t)\cdot(-\Delta_t \ln X_{it}) + \Delta_t \ln\frac{R_{it}}{R_{ib}},\ i \in [1, I] \qquad (3.35)$$

The null hypothesis for each time period: $b_2(t) = 1$, and the alternative hypothesis: $b_2(t) \neq 1$. The null for joint F-test for all time periods: $b_2(t) = 1,\ \forall t \in [1, T]$, and its corresponding alternative: $b_2(t) \neq 1, \exists t \in [1,T]$. Notice that from Section 2, we have leaned that both LHS and RHS variables are stationary across *i*.

Figure 3.7 shows the estimated coefficients and their 95% confidence bounds as functions of time for the regression specification of (3.35). It is obvious that the slopes are statistically different from 1 for all *t*. We thus reject the null. For the pooled regression, the null $b_2 = 1$ is also rejected (results not shown here). Therefore in conclusion we reject the Relative EPPP based on the setup of (3.35).

Let us try the alternative setup, where we swap the RHS and LHS variables in (3.35):

$$-\Delta_t \ln X_{it} = b_1(t) + b_2(t) \cdot \Delta_t \ln \frac{P_{it}/P_{ib}}{X_{ib}} - \Delta_t \ln \frac{R_{it}}{R_{ib}},\ i \in [1,I] \tag{3.36}$$

The null hypothesis for each time period: $b_2(t) = 1$, and the alternative hypothesis: $b_2(t) \neq 1$. The null for joint *F*-test for all time periods: $b_2(t) = 1, \forall t \in [1,T]$, and its corresponding alternative: $b_2(t) \neq 1, \exists t \in [1,T]$.

Figure 3.8 shows the estimated coefficients and their 95% confidence bounds as functions of time for the regression specification of (3.36). We shall reject at 5% level the null corresponding to the five periods of 1986-1987, 1987-1988, 1989-1990, 1990-1991, and 1999-2000, out of 20 periods. One may say this is not too bad because only 5 out 20 get rejected. For the joint test, the *F*-test for $H_0: b_2(t) = 1, \forall t \in [1,T]$ yields the *F*-statistics being 2.8304 with the critical value being 1.592; so we reject the null for the joint test. For the pooled regression, we also reject the null $b_2 = 1$ because $\hat{b}_2 = 0.6281$ (0.0864). Therefore in summary we reject we reject the Relative EPPP based on the setup of (3.36).

Recall that the regression models in (3.35)-(3.36) assume that the RHS variable is orthogonal to the error term. This orthogonality assumption is stronger than necessary. If we should not reject the null, then we would not reject the noisy Relative EPPP of (3.18). However, if we should reject the null, we still could not reject (3.18). In fact, we must not reject (3.18) because we do not reject (3.17).

Therefore we have learned that neither (3.35) nor (3.36) is the right regression specification for testing (3.18) because the orthogonality condition is violated. So we must use an instrument variable for the RHS variable in (3.35) and (3.36). I will postpone this for future work. We have also learned that by comparing the regression results from (3.35) and (3.36), the nominal exchange rate movements may be correlated with the real energy exchange rate movements more than are the energy price movements.

Fig 3.7: Estimated coefficients and their 95% confidence bounds as functions of t, for the cross-sectional regression (3.35): $\Delta_t \ln\left(P_{it}/P_{ib} \cdot X_{it}^{-1}\right) = b_1(t) + b_2(t)\cdot(-\Delta_t \ln X_{it}) + \varepsilon_{it},\ i \in [1, I]$.

$H_0 : b_2(t) = 1, \forall t$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t

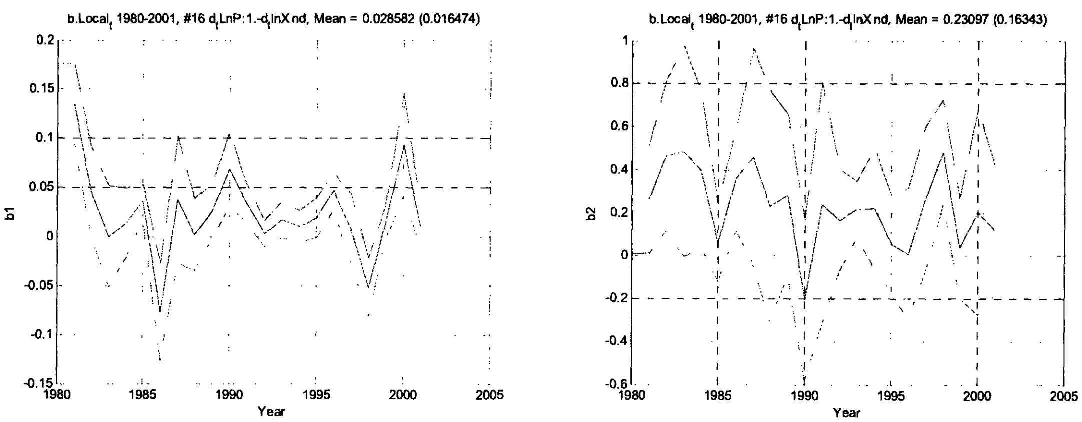
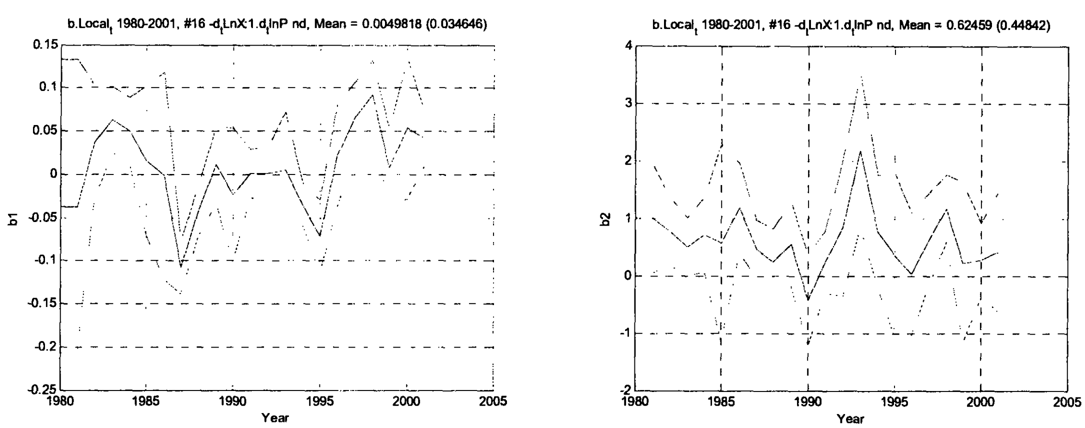


Fig 3.8: Estimated coefficients and their 95% confidence bounds as functions of t, for the cross-sectional regression (3.36): $-\Delta_t \ln X_{it} = b_1(t) + b_2(t)\cdot \Delta_t \ln\left(P_{it}/P_{ib} \cdot X_{ib}^{-1}\right) + \varepsilon_{it},\ i \in [1, I]$.

$H_0 : b_2(t) = 1, \forall t$

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t



3.5 Characteristics of Real Energy Exchange Rates

3.5.1 The Unity of Real Energy Exchange Rates

From Section 3.3, we have concluded that we cannot reject the Absolute EPPP. I.e. we cannot reject that for any given *t*, $R_{it} = 1, \forall i \in [1, I]$. Note that EPPP, absolute or relative, is a cross-sectional notion. One may ask: are we able to push our EPPP tests further to have some time-series meaning? Or, for any given *i*, is $R_{it} = 1, \forall t \in [1, T]$?

To specify a regression model, we must keep in mind that from Section 2 we have learned that R_{it} is an $I(1)$ process in time, so we have to use first difference instead of level for regression models. A simple regression model for country-by-country: for any given *i*,

$$\Delta_t \ln R_{it} / R_{ib} = b_1(i) + \varepsilon_{it}, t \in [2, T] \tag{3.41}$$

We are interested in the null: $b_1(i) = 0, \forall i \in [1, I]$.

The left panel of Figure 3.9 shows the estimated coefficients and their 95% confidence bounds as functions of time for the regression specification of (3.41). It is evident that we cannot reject the null for all the countries. The p-values (not shown) are quite high for each country.

An additional test is for pooled data:

$$\Delta_t \ln R_{it} / R_{ib} = b_1 + \varepsilon_{it}, t \in [2, T], i \in [1, I] \tag{3.42}$$

where the null: $b_1 = 0$. The test has $\hat{b}_1 = 0$ (0.0038). So we cannot reject the null.

Therefore based on the results from (3.41) and (3.42), we can conclude that for any given i, $\Delta_t \ln R_{it} = 0, \forall t \in [1,T]$, i.e. $R_{it} = R_i, \forall t \in [1,T]$, where R_i is a country-dependent constant. Therefore, combining both the cross-sectional and time-series results, we cannot reject that:

$$R_{it} = R_i, \forall i \in [1,I], \forall t \in [1,T]. \tag{3.43}$$

It is easy to verify that R_{it} obey both of the following properties, one for cross section, and the other for time series:

$$\prod_{i=1}^{I} R_{it} = 1, \forall t \in [1,T] \tag{3.44}$$

$$\left(\prod_{t=1}^{T} R_{it} \right)^{\frac{1}{T}} = R_i, \forall i \in [1,I] \tag{3.45}$$

where the value of R_i is displayed in the right panel of Figure 3.9. Notice that R_i takes the value from 0.84 for UK to 1.15 for Austria. So R_i are not exactly equal to unity for all countries as we have wished. However, for practical purpose, we may consider that $R_i \approx 1, \forall i$. I call this phenomenon the "near" parity of the energy purchasing power, or conveniently, the Energy Purchasing Power Parity, in short, the EPPP.

Given (3.44), we may tend to think that at any give time, the logarithm of the cross-sectional real energy exchange rates is randomly distributed across countries with mean zero. Also given (3.45), one may try that for any given economy, the logarithm of its real exchange rates is a random process with mean reverting around $\ln R_i$. Figure 3.10 demonstrates such properties by showing the logarithm of real energy exchange rates in both cross-sectional and cross-time dimensions. As we can see from the graphs, real energy exchange rates show no persistency across either dimension.

Fig 3.9:

Left panel: Estimated coefficients, $\hat{b}_1(i)$, and their 95% confidence bounds as functions of i, for the time-series first difference regression (3.41): $\Delta_t \ln R_{it}/R_{ib} = b_1(i) + \varepsilon_{it}, t \in [2,T]$, $H_0 : b_1(i) = 0, \forall i$,

Right panel: Country-dependent constant real energy exchange R_i vs. i

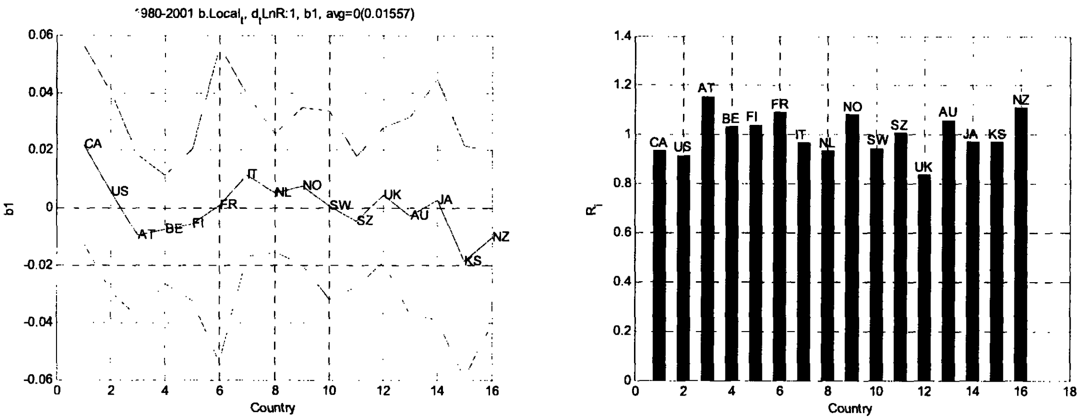
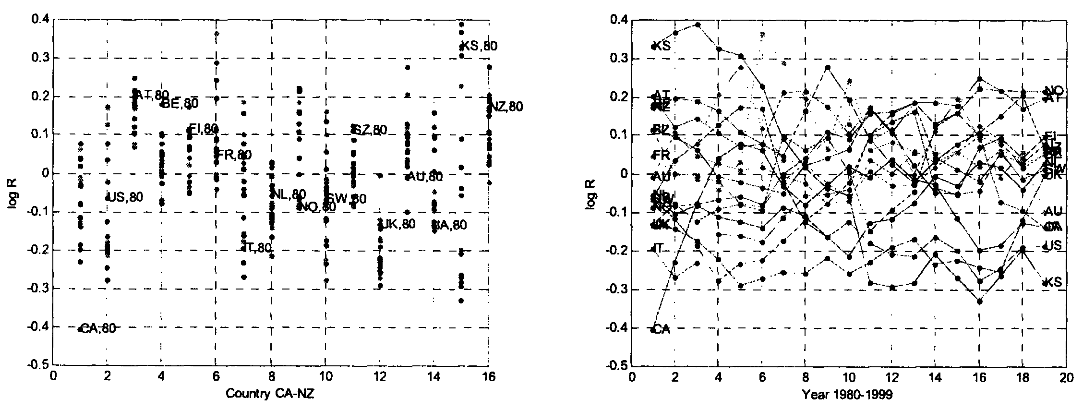


Fig 3.10: Display of the two-dimensional logarithm of real energy exchange rate $\ln R_{it}$.

Left panel: $\ln R_{it}$ vs. i

Right panel: $\ln R_{it}$ vs. t



3.5.2 Cross-Sectional Correlations and Half Life Calculations

Because of the validity of EPPP characterized by (3.13) and (3.44), $\prod_{i=1}^{I} R_{it} = 1, \forall t \in [1,T]$, the real energy exchange rates are hence cross-sectionally correlated. Therefore, any calculations for half-life should take into account such cross-sectional correlations. Why? Let me illustrate with an example below.

Suppose $I = 3$, R_{it} $\forall i \in [1, I]$ are deterministic processes, and $\ln R_{1t} = 2\varepsilon \sin(2\pi t / \tau)$, and $\ln R_{2t} = \ln R_{3t} = -\varepsilon \sin(2\pi t / \tau)$, where ε is an arbitrarily small constant and τ is a constant. One can verify that first of all, they satisfy the PPP condition. Secondly, considering that $t \in [1,T]$ and $T = n\tau$, $n \geq 1$. If n is an integer, one can find that the half life for $\ln R_{1t}$, denoted as ν_1, is infinity, i.e. $\nu_1 = \infty$; likewise, $\nu_2 = \nu_3 = \infty$. If n is not an integer but reasonably large, then $\nu_i >> 0, \forall i$.[10]

The example tells us that although PPP is satisfied, half life can still be quite large if we would leave out the cross-sectional correlations.

Therefore, I propose a "recipe" to calculate the half life correctly under PPP:

1. Test whether PPP holds. If yes, calculate the real exchange rate.
2. Use the Fourier transfer technique[11] to identify the most important components (in terms of periods or frequencies) for each country's demeaned, detrended real exchange rate. If the most important components (in terms of periods or frequencies) are shared by all or majority of the countries, then we can say that

---

[10] Given $\Delta_t \ln R_{it} = \rho_i \ln R_{i,t-1}$, half life satisfies $\exp(\rho_i \nu_i) = 1/2$, and thus $\nu_i = \ln(1/2)/\rho_i$ if $\rho_i < 0$. If $\rho_i \geq 0$, I simply define $\nu_i = \infty$. In the examples here, $\ln R_{1t} = 2\varepsilon \sin(2\pi t / \tau)$, $\Delta_t \ln R_{1t} = 4\varepsilon\pi / \tau \cdot \cos(2\pi t / \tau)$, so $\hat{\rho}_1 \approx 0$; therefore $\nu_1 \approx \infty$. Similarly, $\nu_2 \approx \infty$ and $\nu_3 \approx \infty$.

[11] See Bracewell (1999). Fourier transfer is a technique that is commonly used to transfer data between time domain and frequency domain. For time-series data, for some specific purpose, it may be more easily analyzed in the frequency domain.

these components contribute to the cross-sectional corrections among the real exchange rates across countries.

3. Take away the shared cross-sectional components, which shall include mean and trend.
4. Ensure that the PPP still holds for the residuals after removing the mean, the trend, and the cross-sectional correlation components.
5. Calculate the half life for the residuals for each country.

Now apply the recipe to our data.

<u>Step 1:</u> Test PPP. If it holds, calculate the real exchange rate.

- We know that EPPP holds.

<u>Step 2:</u> Use the Fourier transfer technique to identify the most important cross-sectional correlation components for each country.

- Figure 3.11 consists of 16 graphs, one for each country. Each graph is a spectral density of demeaned, detrended real energy exchange rate. Just eyeballing these graphs, first we can realize that shapes of the spectral density seem to look similar for all countries. Second, all countries seem to share similar periods (frequencies) for their top two lodes that have the highest density. I call the periods corresponding to these two lobes τ_1 and τ_2. Table 3.2 lists τ_1 and τ_2 for each country. The averages are $\bar{\tau}_1 = 18.10$ years and $\bar{\tau}_2 = 7.96$ years.

<u>Step 3</u>: Take away the shared cross-sectional correlation components, including mean and trend.

- We will keep the residuals from the regression below:[12]

$$\begin{aligned}\ln R_{it} &= b_{1,i} + b_{2,i} \cdot t + b_{3,i} \sin(2\pi t / \bar{\tau}_1) + b_{4,i} \cos(2\pi t / \bar{\tau}_1) \\ &\quad + b_{5,i} \sin(2\pi t / \bar{\tau}_2) + b_{6,i} \cos(2\pi t / \bar{\tau}_2) + \varepsilon_{it}, \forall t \in [1, T]\end{aligned} \tag{3.51}$$

[12] This regression is for mere curve fitting and is not an economic specification.

- Figure 3.12 consists of graphs representing the actual and the fitted data of the regressions (3.51) for all countries. They all have reasonably high R^2.

Step 4: Ensure the PPP still holds after removal of cross-sectionally correlated components.

- It is equivalent to check that the removed components satisfy the PPP too. So as long as the following holds, then we are done checking.

$$\sum_{i=1}^{I} \hat{b}_{k,i} = 0, \forall k \in [1,6] \tag{3.52}$$

- Table 3.3 shows that indeed (3.52) holds. So we can be sure that the PPP holds for both the removed components and the residual components.

Step 5: Calculate the half life on the residual components for each country.

- Let $r_{it} \equiv \ln R_{it} - \ln \hat{R}_{it}$, and $\Delta_t r_{it} = \rho_i r_{i,t-1} + \varepsilon_{it}$, then the half life, if $\rho_i < 0$:

$$v_i \equiv \ln(1/2)/\rho_i = 0.693/\rho_i \tag{3.53}$$

- Table 3.4 list the half lives of each country considering three different scenarios for the cross-sectional correlation components. The table shows us the more cross-sectional correlations are removed from the original time-series data, the shorter the half life. For example, the average half life of the 16 countries is 2.33 years (Case 1) when only the mean and trend are removed. But it is reduced to only 9.5 months (Case 3) when we remove further the $\bar{\tau}_1$ and $\bar{\tau}_2$ components.

Fig 3.11: Spectral density in the frequency domain for the demeaned, detrended logarithm of real energy exchange rate, $\ln R_{it}$, for all 16 countries. First from left to right, then from top to down are CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ.

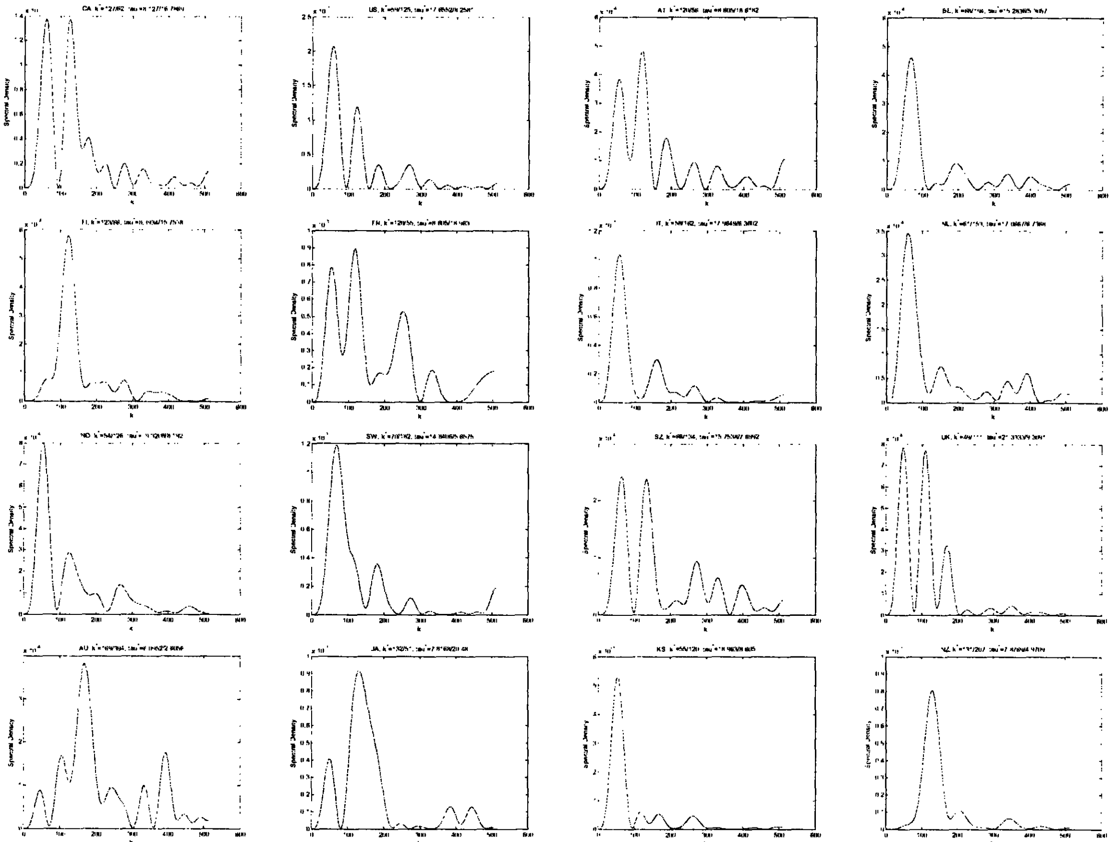


Table 3.3: Periods corresponding to the two lobes with highest spectral density

Country	tau1 (year)	tau2 (year)
CA	16.79	8.13
US	17.66	8.26
AT	18.62	8.61
BE	15.28	7.37
FI	15.75	8.39
FR	18.96	8.61
IT	17.96	6.36
NL	17.07	6.74
NO	19.32	8.19
SW	14.84	5.66
SZ	15.75	7.70
UK	21.33	9.31
AU	22.76	9.75
JA	20.48	7.82
KS	18.96	8.61
NZ	NA	7.88
Avg	18.10	7.96

Fig 3.12: Actual and fitted curves for all 16 countries for time-series regressions of (3.51):
$\ln R_{it} = b_{1,i} + b_{2,i} \cdot t + b_{3,i} \sin(2\pi t / \bar{\tau}_1) + b_{4,i} \cos(2\pi t / \bar{\tau}_1) + b_{5,i} \sin(2\pi t / \bar{\tau}_2) + b_{6,i} \cos(2\pi t / \bar{\tau}_2) + \varepsilon_{it}, \forall t \in [1, T]$,
where $\bar{\tau}_1 = 18.10$, $\bar{\tau}_2 = 7.96$ years. From left to right and then top to down: CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ.

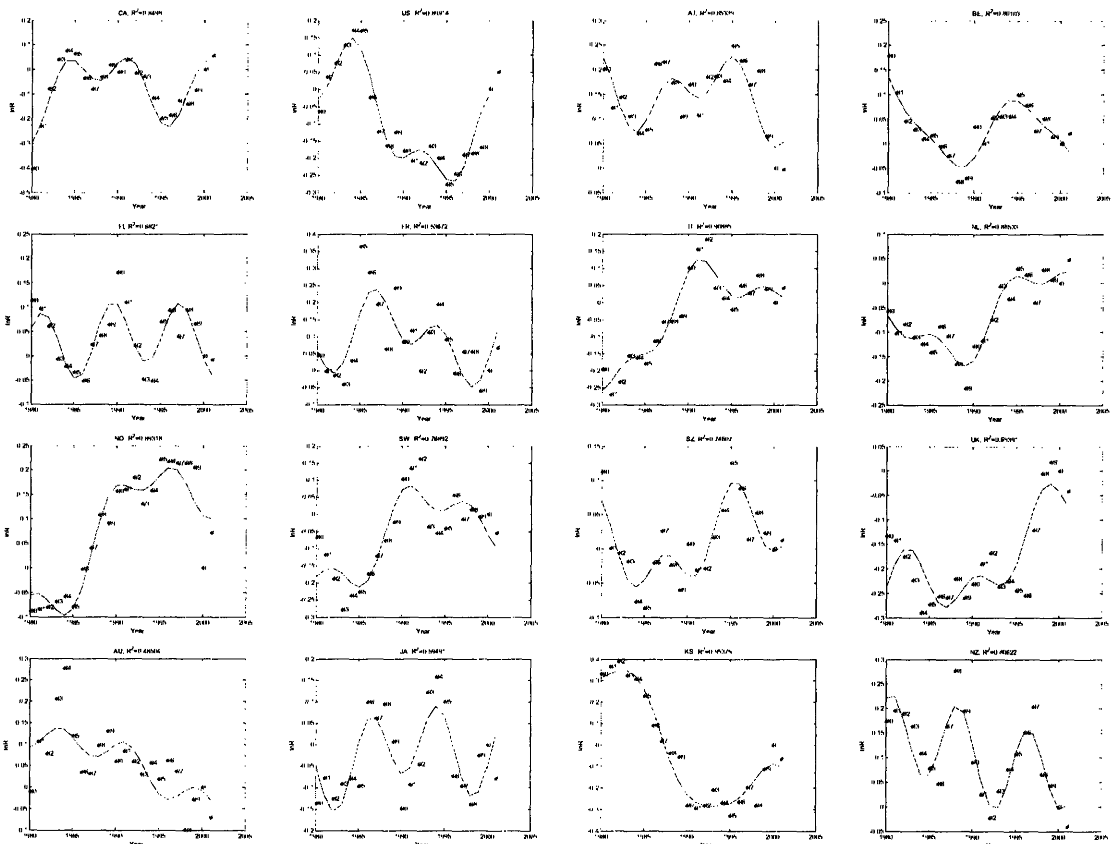


Table 3.4: The estimated coefficients of (3.51)

Country	b1	b2	b3	b4	b5	b6
CA	-0.121	0.005	0.028	-0.114	-0.047	-0.072
US	0.003	-0.009	0.128	0.002	-0.063	-0.031
AT	0.188	-0.003	-0.053	0.017	0.019	0.042
BE	0.084	-0.005	-0.028	0.077	-0.017	0.008
FI	0.031	0.001	-0.017	0.001	0.065	-0.022
FR	0.104	0.000	-0.001	-0.077	-0.032	0.066
IT	-0.172	0.013	-0.088	-0.041	0.000	-0.044
NL	-0.130	0.005	0.003	0.067	-0.026	0.018
NO	-0.057	0.013	-0.080	-0.011	0.039	-0.002
SW	-0.144	0.008	-0.099	-0.011	0.037	-0.038
SZ	-0.003	0.001	-0.017	0.048	0.010	0.037
UK	-0.287	0.008	0.048	0.040	0.019	-0.037
AU	0.152	-0.008	-0.010	-0.020	-0.012	-0.030
JA	-0.041	0.002	-0.028	-0.024	-0.043	0.076
KS	0.243	-0.026	0.198	0.055	-0.029	-0.004
NZ	0.148	-0.004	0.017	-0.009	0.080	0.033
Sum	0.000	0.000	0.000	0.000	0.000	0.000

Table 3.5: Half life of the real energy exchange rate after removing the cross-sectional correlations.
Case 1: Cross-sectional correlations are mean and trend
Case 2: Cross-sectional correlations are mean, trend, and tau1 components
Case 3: Cross-sectional correlations are mean, trend, tau1 and tau2 components

	Half Life (year)			Half Life (month)
Country	Case 1	Case 2	Case 3	Case 3
CA	1.758	1.502	0.999	12.0
US	4.937	1.422	0.798	9.6
AT	1.796	1.096	0.755	9.1
BE	1.658	0.694	0.632	7.6
FI	1.522	1.470	0.740	8.9
FR	1.165	0.841	0.648	7.8
IT	2.583	0.997	0.666	8.0
NL	1.571	0.746	0.512	6.1
NO	2.691	0.932	0.739	8.9
SW	2.386	1.363	0.987	11.8
SZ	1.161	0.831	0.580	7.0
UK	2.658	1.820	1.423	17.1
AU	0.900	0.894	0.871	10.4
JA	1.456	1.243	0.700	8.4
KS	7.357	0.855	0.784	9.4
NZ	1.678	1.633	0.803	9.6
Avg	2.330	1.146	0.790	9.5

3.6 Summary

I hypothesize an EPPP and empirically test the hypothesis. The EPPP is evidenced by high p-value.

I show that Absolute EPPP always implies Relative EPPP. So if we accept the Absolute EPPP, then there is no need to test the Relative EPPP. I illustrate that the conventional setup, which separating the exchange rate movement from the price movement, of testing the Relative EPPP would result in biased estimates because the orthogonality condition fails, since either the price movement or the exchange rate movement correlated with the real exchange rate movement. We will have to find a valid instrument to test the Relative EPPP correctly.

I find that the real energy exchange rates seem to be randomly distributed across countries at any given time, and be a mean-reversing random process over time for any given country. I also conclude that we cannot reject the unity of the real energy rate at all time and for all countries.

Finally, I propose a recipe to calculate the half life by considering the cross-sectional correlations. It turns out I am able to reduce the half life for most of the countries to be within a year, much smaller than those of other real exchange rate measures offered by the existing literatures.

Note that the EPPP is a correlation, not a causality relationship between exchange rates and energy prices. The validity of the EPPP sets the stage for all the subsequent analyses and discussions in this paper.

4 Cross-Country Production Function and Energy Income Share

4.1 Background

In the Section 3, I have shown that the EPPP holds; i.e., the exchange rates equalize the energy price differentials across countries. Logically one would expect exchange rates would equalize the nominal marginal product of energy across countries in a competitive, risk-neutral market. If such a relationship holds, it may give rise to the estimate of the cross-sectional production function.

Since the first oil crisis the early 1970s, studies have focused on energy substitution with other inputs. Berndt and Wood (1975) consider a cost function of four inputs (capital, labor, energy and material), and find that energy is and a complement to capital and a substitute to labor, using the US manufacturing time-series data.

Griffin (1976) estimates the cost function of the same fours inputs, using pooled international data from the manufacturing sector of nine industrialized countries, where the prices are converted using purchasing power parity and indexed to 1955 US dollar. Griffin finds that the elasticity of substitutions between energy and capital is statistically not different from one, and neither the elasticity of substitution between energy and labor; however, the elasticity of substitution between capital and labor is less than one. This finding means that the production is Cobb-Douglas in two inputs (energy and capital, and energy and labor), but not Cobb-Douglas in three inputs (energy, capital, and labor).

Stern and Cleveland (2004)'s survey paper highlights subsequent studies about the substitutions between energy and other production inputs, including Berndt and Wood (1979) on manufacturing sector, Kauffman and Azary-Lee (1991) on US forest product sector, and Stern (1993) on the US macroeconomy. These studies have come to various conclusions regarding whether energy and capital are substitutes or complements. "It seems that capital and energy are at best weak substitutes and possibly complements. The degree of complementarity likely varies across industries and the level of aggregation considered," they conclude.

In this paper, I will focus on the cross-country level, not the industry level. I will assume a generic two factor (energy and non-energy) cross-sectional production function and let the data dictate whether they are substitutes or complements. Also I will start from nominal quantities denominated by different currencies and assume no purchasing power parity.

4.2 Theory

4.2.1 Assumptions and Setup

Given what we have learned from Section 3, I hypothesize that because energy is a cross-country representative good, the exchange rates equalize the marginal products of energy across countries. Notice I do not require the price information to test my hypothesis. So I do not require the condition that the energy price should be proportional to the marginal product of energy in a competitive market.

Let M_{it} be the nominal marginal product of energy and assume that the foreign exchange market is risk neutral, then by the argument of no arbitrage, because energy is a representative good across countries, the following relationships[13] should hold

$$\frac{X_{wit} M_{it}}{R_{it}^{M}} = \frac{X_{wjt} M_{jt}}{R_{jt}^{M}} = \frac{X_{wut} M_{ut}}{R_{ut}^{M}} = \frac{X_{wwt} M_{wt}}{R_{wt}^{M}} = M_{wt}, \forall i, j \in [1, I] \tag{4.1}$$

[13] Equivalently, we can also start with the relationship that the energy price is proportional to the marginal product of energy in a competitive market. The reason I use the exchange rate instead of price is twofold. First, the data issue. I have the complete information for exchange rates but have only the price indexes for prices. Second, for the symmetry of my hypotheses. In this section, the currency exchange rates--the exchange rates across countries--would equalize the differentials of marginal value of energy cross countries; in next section, the GDP deflators--the exchange rates across time--would equalize the differentials of marginal value of energy across-time. Note I assume there is no marginal adjustment cost of energy in the economy, so the price of energy and the marginal product of energy are proportional to each other.

where

$$R_{it}^{M} \equiv \frac{X_{wit} M_{it}}{M_{wt}} . \tag{4.1'}$$

Following the similar rational discussed in Section 3.2.2.2, I define the world's real exchange rate for the marginal product of energy, R_{wt}^{M}, and the world's marginal product of energy, M_{wt}, as the geometric mean of all the countries in the universe:

$$R_{wt}^{M} \equiv \left(\prod_{i=1}^{I} R_{it}^{M} \right)^{\frac{1}{I}} \equiv 1, \forall t \tag{4.2}$$

$$M_{wt} = \left(\prod_{i=1}^{I} \frac{X_{wit} M_{it}}{R_{it}} \right)^{\frac{1}{I}} = \left(\prod_{i=1}^{I} X_{wit} M_{it} \right)^{\frac{1}{I}}, \forall t \in [1,T] \tag{4.3}$$

We can also write the no-arbitrage equilibrium (4.1) as:

$$X_{it} \equiv \frac{X_{wit}}{X_{wut}} = \frac{M_{ut} / R_{ut}^{M}}{M_{it} / R_{it}^{M}}, \forall i \in [1,I]. \tag{4.4}$$

Now suppose that at any given year t, the intra-temporal cross-economy production functions takes the following two-factor constant-elasticity-of-substitution (CES) form[14],

$$Y_{it} = \left[\alpha_{it} \left(A_{E,it} E_{it} \right)^{\frac{\sigma_t - 1}{\sigma_t}} + \left(1 - \alpha_{it} \right) \left(A_{N,it} N_{it} \right)^{\frac{\sigma_t - 1}{\sigma_t}} \right]^{\frac{\sigma_t}{\sigma_t - 1}} \tag{4.5}$$

where i is the country index, t is the time index, Y_{it} is the nominal GDP in national currency for country i at time t, E_{it} is the real energy consumption, N_{it} is the real non-energy employed, $A_{E,it}$ is the energy factor augmenting technology, $A_{N,it}$ is the non-

[14] See Fishelson (1978) and Acemoglu (2006).

energy factor augmenting technology, $\alpha_{it} \in (0,1)$ is a distribution parameter which determines the relative importance of the two factors of energy and non-energy, and $\sigma_t \in (0,\infty)$ is the elasticity of substitution between the two factors.[15,16]

For convenience, let

$$\lambda_t = \frac{\sigma_t - 1}{\sigma_t} \tag{4.6}$$

Notice that $\lambda_t \in (-\infty,1)$. When $\sigma_t = \infty$ or $\lambda_t = 1$, the two factors are perfect substitutes, and the production is linear, $Y = \alpha A_E E + (1-\alpha) A_N N$. When $\sigma_t = 1$ or $\lambda_t = 0$, the production is Cobb-Douglas, $Y = (A_E E)^{\alpha} (A_N N)^{1-\alpha}$. When $\sigma_t = 0$ or $\lambda_t = -\infty$, there is

---

[15] In (4.5), E and N are real quantities, α is a ratio, and any nominal notions of currencies are all embedded in Y, A_E and A_N. So (4.5) can represent different economies denominated by different currencies.

[16] Conventionally, we consider capital and labor are the primary production factors in the economy, while other goods such as energy and materials are intermediate inputs. So the prices paid for all inputs are seen as eventually the payments to the owners of the primary inputs. Then the economy's gross income will be split between capital and labor. In (4.5), in no contraction to the conventional capital and labor economy, I look from a different perspective at the economy, and define energy and non-energy as the factors. So in the economy specified by (4.5), the economy's gross income will be split between energy and non-energy factors. The gross energy income contains some labor income and some capital income, and so does the gross non-energy income. Vice verse, the gross capital income contains some energy income and some non-energy income, and so does the gross labor income. Mathematically, under the assumption of Cobb-Douglas production function specified by either the capital & labor pair or the energy & non-energy pair, we can have the value-added relationship: $Y = Y_K + Y_L = Y_E + Y_N$, where the gross income, Y, equals to the gross capital income, Y_K, plus the gross labor income, Y_L, and it also equals to the gross energy income, Y_E, plus the gross non-energy income, Y_E. If one prefers thinking that in (4.5), energy and non-energy are the primary inputs, while capital and labor are the intermediate inputs, I would consider it mathematically correct. Economically, we may think that we could strip the energy components from the capital income, and the remaining would be the non-energy component. We could also strip the energy component from the labor income and the remaining would be the non-energy component. Then the gross energy income would be equal to the sum of the energy incomes stripped from both capital and labor incomes. And the gross non-energy income would be equal to the non-energy incomes stripped from both the capital and labor incomes. One of the drawbacks of (4.5) is that I am unable to quantify the non-energy factor. Fortunately, for the purpose of this paper, the quantification of non-energy factor is not needed. Nevertheless, we are able to quantify the non-energy income, which is simply $Y_N = Y - Y_E$, where Y and Y_E are observable and quantifiable.

no substitution between the two factors and the production is Leontieff, $Y = \min(A_E E, A_N N)$.[17]

Given the functional form (4.5), we can obtain that

$$M_{it} \equiv \frac{\partial Y_{it}}{\partial E_{it}} = \left(\frac{Y_{it}}{E_{it}}\right)^{1-\lambda_t} \alpha_{it} A_{it}^{\lambda_t} \tag{4.7}$$

Then the no arbitrage relationship of (4.4) becomes

$$\frac{Y_{it}}{E_{it}} = \frac{M_{ut}}{R_{ut}^M} \cdot X_{it}^{\frac{-1}{1-\lambda_t}} \cdot \frac{R_{it}^M}{\alpha_{it} A_{it}^{\lambda_t}} \tag{4.8}$$

$$\frac{Y_{it}/E_{it}}{Y_{i-1,t}/E_{i-1,t}} = \left(\frac{X_{it}}{X_{i-1,t}}\right)^{\frac{-1}{1-\lambda_t}} \left(\frac{R_{it}^M}{R_{i-1,t}^M}\right) \left(\frac{\alpha_{it}}{\alpha_{i-1,t}}\right)^{-1} \left(\frac{A_{E,it}}{A_{E,i-1,t}}\right)^{-\lambda_t}, i \in [2, I] \tag{4.9}$$

4.2.2 Hypothesis

I hypothesize a constant elasticity of substitution between energy production factor and non-energy production factors; i.e. null: $\lambda_t = \lambda_0, \forall t \in [1, T]$. Notice that the no-arbitrage relationship is valid regardless whether we reject or accept the hypothesis. But if we should not reject the null, and determine the value of λ_0, then we would be able to explicitly determine the CES production function of (4.3).

4.3 Regression Models and Results

4.3.1 Setup

[17] Arrow, Cheney, Minhas and Solow (1961) have shown that linear, Cobb-Douglas, and Leontieff production functions are all special cases of the CES production function.

For level, we have

$$\ln \frac{Y_{it}}{E_{it}} = \ln \frac{M_{ut}}{R_{ut}^{M}} - \frac{1}{1-\lambda_t} \ln X_{it} + \frac{1}{1-\lambda_t} \ln \frac{R_{it}^{M}}{\alpha_{it} A_{it}^{\lambda_t}} \tag{4.11}$$

For ratio or first difference in logarithmic sense, we have

$$\Delta_i \ln \frac{Y_{it}}{E_{it}} = -\frac{1}{1-\lambda_t} \Delta_i \ln X_{it} + \frac{1}{1-\lambda_t} \Delta_i \ln \frac{R_{it}^{M}}{\alpha_{it} A_{it}^{\lambda_t}},\ i \in [2, I] \tag{4.12}$$

As discussed in Section 2, $\ln X_{it}$, $\ln Y_{it}/E_{it}$, $\Delta_i \ln X_{it}$, $\Delta_i \ln Y_{it}/E_{it}$ are all are cross-sectionally stationary.

4.3.2 Period-by-Period Regressions

4.3.2.1 Period-by-Period Level Regressions

Model:

$$\ln \frac{Y_{it}}{E_{it}} = b_1(t) + b_2(t)\,(-\ln X_{it}) + \varepsilon_{it},\ i \in [1, I] \tag{4.13}$$

The null hypothesis for each time period: $b_2(t) = (1-\lambda_0)^{-1}$. Also the null for joint F-test for all time periods: $b_2(t) = (1-\lambda_0)^{-1}$, $\forall t \in [1, T]$.

Validity of the Model:

From Section 2, we have leaned that both LHS and RHS variables are stationary across i. Also notice that I assume the RHS variable and the residual term are cross-sectionally uncorrelated. Again, I employ Hausman test to test this assumption. I use the same instrument variable as defined in Section 3.3.3. Table 4.1 below lists the chi-square statistics for five selected years. Because the test statistics are smaller than the critical values for any given time, we cannot reject the null hypothesis that the RHS variable is uncorrelated the error term in (4.13).

Table 4.1: Hausman test for (4.13)

Year	1980	1985	1990	1995	2000
Chi2-stat	0.0018	0.0072	0.0168	0.0853	0.0769

The 5% critical value of chi square with 1 degree of freedom is 3.84
Use average over time (excluding current time) of exchange rates as instrument

Results:

Figures 4.1 consist of four graphs, representing the actual and the fitted data of the regressions (4.13) for the years of 1980, 1985, 1990 and 1995 respectively. Eyeballing the graphs, I am tempting to hypothesize that $b_2(t) = 1, \forall t$ and hence $\lambda_0 = 0$. If this is true, then we would have a simple Cobb-Douglas production to work with.

Figure 4.2 displays the estimated coefficients and their 95% respective confidence bounds as functions of time. The slopes are all statistically equal to 1, with high p-values (not shown in the graphs) for all *t*. It is obvious that for all *t*, we cannot reject the null, where $\lambda_0 = 0$. Also based on the sample average across all year, $\bar{\hat{b}}_2 \equiv \frac{1}{T}\sum_{t=1}^{T}\hat{b}_2(t) = 1.0337$ (0.0413), which is statistically not different from 1, implying that $\hat{\lambda}_0 = 0.0326$ is statistically not different from 0. Also $\overline{R^2} = 0.9782$.

The F-statistics for the joint tests of $H_0 : b_2(t) = (1-\lambda_0)^{-1}, \forall t \in [1,T]$, where $\lambda_0 = 0$, is 0.7965 with the 5% critical value of 1.5769. So the p-value for the F-test is 0.73. Therefore we cannot reject the null that $H_0 : b_2(t) = (1-\lambda_0)^{-1} = 1, \forall t \in [1,T]$; i.e. we cannot reject that $\lambda_0 = 0$.

Fig 4.1: Actual and fitted curves for different years for cross-sectional level regression (4.13): $\ln Y_{it} E_{it}^{-1} = b_1(t) + b_2(t)\,(-\ln X_{it}) + \varepsilon_{it},\ i \in [1, I]$.

From left to right and then top to down: 1980, 1985, 1990, and 1995.

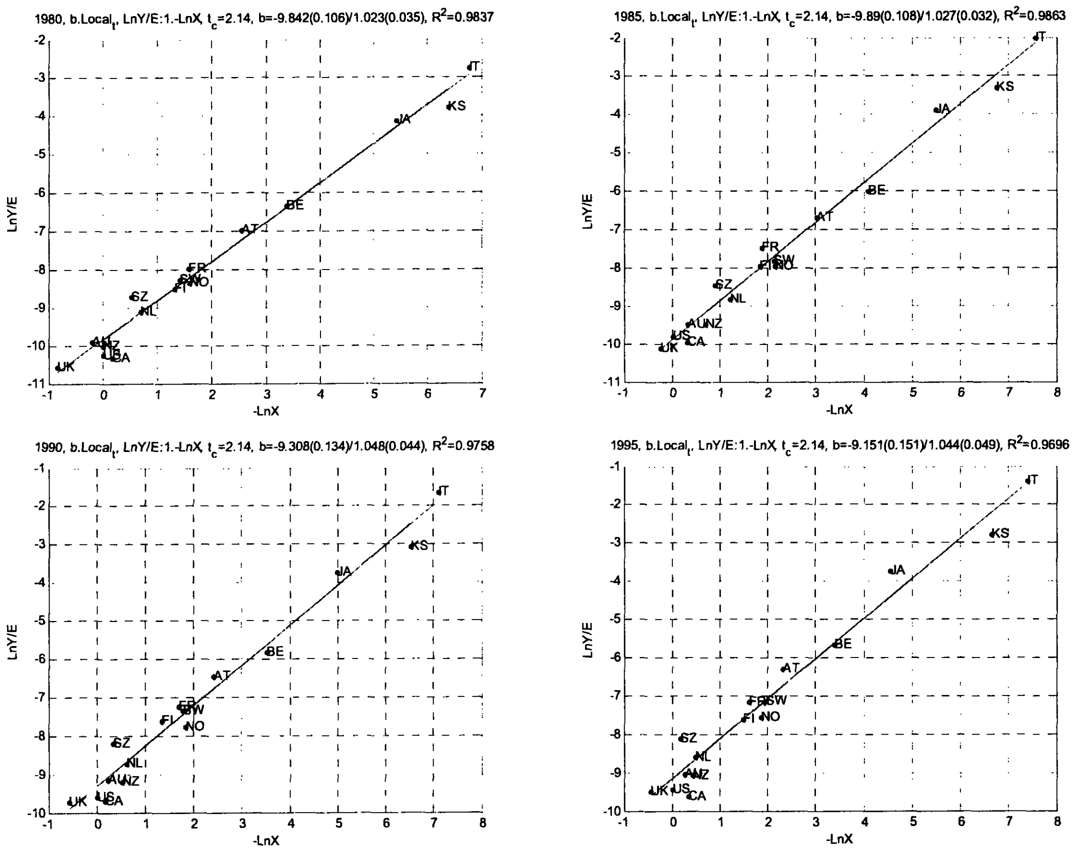
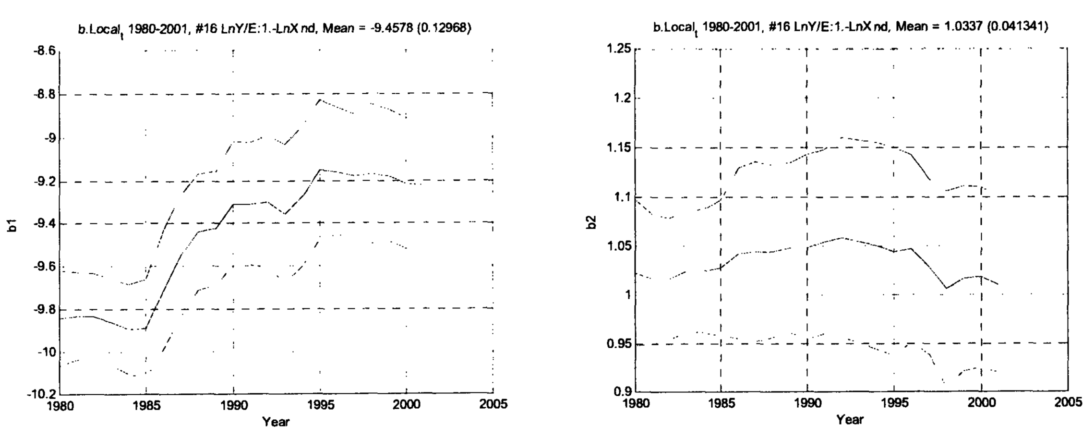


Fig 4.2: Estimated coefficients and their 95% confidence bounds as functions of t, for the cross-sectional level regression (4.13): $\ln Y_{it} E_{it}^{-1} = b_1(t) + b_2(t)\,(-\ln X_{it}) + \varepsilon_{it},\ i \in [1, I]$,

$H_0 : b_2(t) = (1 - \lambda_0)^{-1} = 1, \forall t$, where $\lambda_0 = 0$.

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t



4.3.2.2 Period-by-Period First Difference Regressions

Model:

$$\Delta_i \ln\frac{Y_{it}}{E_{it}} = b_1(t) + b_2(t)\,\Delta_i(-\ln X_{it}) + \varepsilon_{it},\; i \in [2, I] \qquad (4.14)$$

where Δ_i is as defined in Section 2. The null for each time period:

$b_1(t) = 0,\; b_2(t) = (1-\lambda_0)^{-1}$, and the null for joint F-test for all time periods:

$b_1(t) = 0,\; b_2(t) = (1-\lambda_0)^{-1},\; \forall t \in [1, T]$, where $\lambda_0 = 0$.

Validity of the Model:

From Section 2, we have leaned that both LHS and RHS variables are stationary across *i*. Again I assume the RHS variable and the error term are cross-sectionally little uncorrelated. Again, by way of Hausman test, we cannot reject that the RHS variable and the residual term are cross-sectionally uncorrelated.

Results:

Figure 5.1 shows the actual and fitted data of the regressions (5.22) for all 16 countries. Figure 4.4 displays the estimated coefficients and their 95% respective confidence bounds as functions of countries. Below is a summary:

Figure 4.4 displays the estimated coefficients and their 95% respective confidence bounds as functions of time. The intercepts are all statistically equal to 0, with high p-vales (not shown in the graphs). The slopes are all statistically equal to 1, with high p-values (not shown in the graphs) for all *t*. It is obvious that for all *t*, we cannot reject the null, where $\lambda_0 = 0$. Also based on the sample average across all years, $\overline{\hat{b}_2} = 1.0285$ (0.0411), which is statistically not different from 1, implying that $\hat{\lambda}_0$ = 0.0277 is statistically not different from 0. Also not shown in the table, $\overline{R^2} = 0.9797$.

Additionally, the F-statistics for the joint tests of $H_0 : b_1(t) = 0, b_2(t) = (1-\lambda_0)^{-1}, \forall t \in [1,T]$, where $\lambda_0 = 0$, is 0.3125 with the 5% critical value of 1.4185. So the p-value equals 1.0000. Therefore, we cannot reject the null.

Fig 4.3: Actual and fitted curves for different years for cross-sectional first difference regression (4.14):

$$\Delta_i \ln Y_{it} E_{it}^{-1} = b_1(t) + b_2(t)\, \Delta_i(-\ln X_{it}) + \varepsilon_{it},\; i \in [2, I].$$

From left to right and then top to down: 1980, 1985, 1990, and 1995.

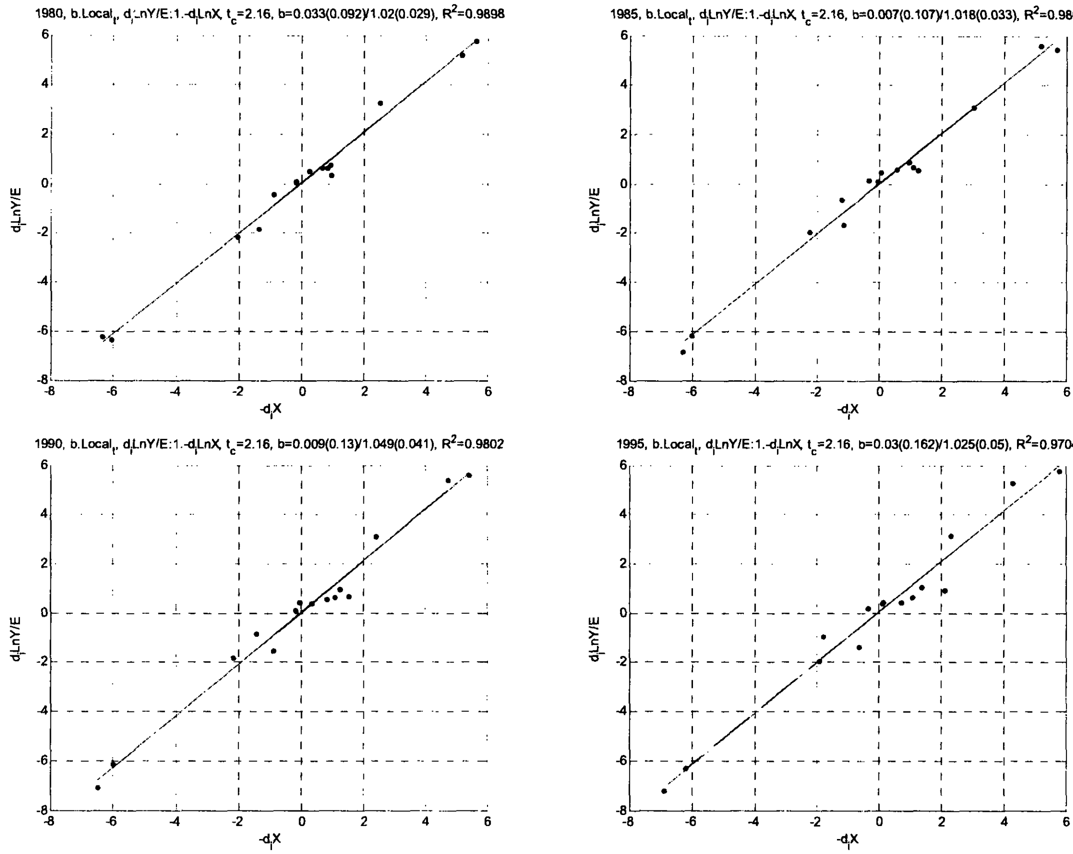
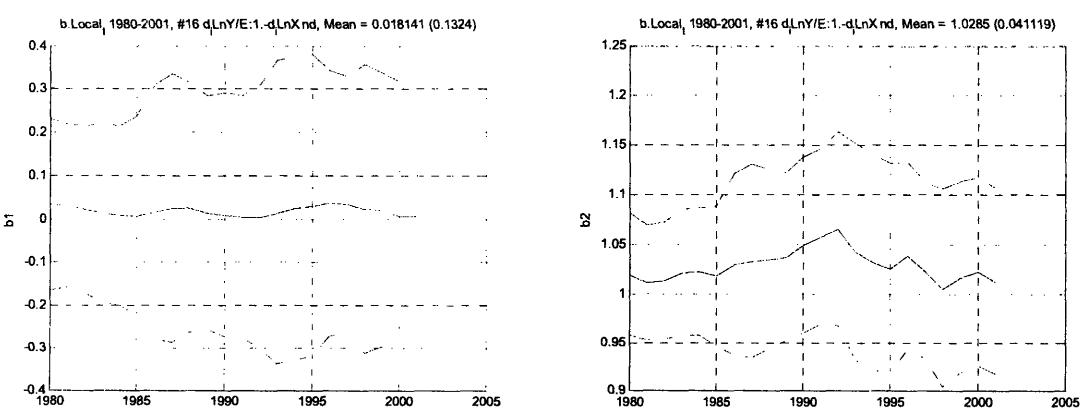


Fig 4.4: Estimated coefficients and their 95% confidence bounds as functions of t, for the cross-sectional first difference regression (4.14): $\Delta_i \ln Y_{it} E_{it}^{-1} = b_1(t) + b_2(t)\, \Delta_i(-\ln X_{it}) + \varepsilon_{it},\; i \in [2, I]$.

$H_0 : b_1(t) = 0,\; b_2(t) = (1 - \lambda_0)^{-1} = 1, \forall t$, where $\lambda_0 = 0$.

Left panel: $\hat{b}_1(t)$ vs. t, right panel: $\hat{b}_2(t)$ vs. t



4.3.3 Pooled Regressions

4.3.3.1 Pooled Level Regressions

Model:

$$\ln\frac{Y_{it}}{E_{it}} = b_1(t) + b_2 \cdot (-\ln X_{it}) + \varepsilon_{it},\ i \in [1, I], t \in [1, T] \tag{4.15}$$

The null: $b_2 = (1-\lambda_0)^{-1}$, where $\lambda_0 = 0$.

Results:

The results have $\hat{b}_2$ = 1.0333 (0.0086), which is statistically not different from 1, so $\hat{\lambda}_0$ = 0.0322 is statistically not different from 0. We do not reject the null. Also, R^2= 0.9978.

4.3.3.2 Pooled First Difference Regressions

Model:

$$\Delta_i \ln\frac{Y_{it}}{E_{it}} = b_1 + b_2 \cdot \Delta_i(-\ln X_{it}) + \varepsilon_{it},\ i \in [2, I], t \in [1, T] \tag{4.16}$$

The null hypothesis: $b_1 = 0, b_2 = 1-\lambda_0$, where $\lambda_0 = 0$.

Results:

The results have $\hat{b}_1$ =0.0182 (0.0265) and $\hat{b}_2$ = 1.0282 (0.0082), so $\hat{\lambda}_0$ = 0.02743. The *F*-test in this case rejects the null since *F* statistics of 6.1 is greater than the critical value of 3.0. Though in this testing, $\hat{\lambda}_0$ is statistically from 0, practically speaking, a value of 0.02743 is small enough so can still be considered close to 0. Also, $R^2 = 0.9794$.

4.3.4 Alternative Setup [Reader may skip Section 4.3.4 entirely]

An alternative setup is to swap RHS and LHS variables in (4.11)-(4.12), i.e. is to have the exchange rate on LHS and the ratio of GDP to energy on the RHS. It turns out that this alternative setup yields similar results: cannot reject the null and the p-values are high. The p-values are only slightly smaller than those of the setup in (4.11)-(4.12).

4.4 Discussions

4.4.1 Cross-Sectional Production Function

Based on the regression results, we have high R^2, and in most cases we do not reject the null which assumes $\lambda_0 = 0$. In some cases, though we reject the null which assume $\lambda_0 = 0$, $\hat{\lambda}_0$ is practically close to 0. In summary, $\hat{\lambda}_0 = 0.274 - 0.326$; in all cases but one (first difference, pooled regression), it is statistically 0. Given $\hat{\lambda}_0$ from the tests above, we have $\hat{\sigma}_0 = (1-\hat{\lambda}_0)^{-1} \approx 1.03$. So $\hat{\sigma}_0 = 1$ statistically, or if not, practically.

Therefore using $\sigma_t = \sigma_0 = 1, \forall t$, the CES production function of (4.5) becomes a Cobb-Douglas production function:

$$Y_{it} \approx \left(A_{E,it}E_{it}\right)^{\alpha_{it}}\left(A_{N,it}N_{it}\right)^{1-\alpha_{it}} = A_{it}E_{it}^{\ \alpha_{it}}N_{it}^{\ 1-\alpha_{it}} \tag{4.21}$$

where $A_{it} \equiv A_{E,it}^{\ \alpha_{it}} A_{N,it}^{\ 1-\alpha_{it}}$.

4.4.2 Energy Income Share

For $\lambda_t = \lambda_0 = 0, \forall t$ or $\sigma_t = \sigma_0 = 1, \forall t$, the marginal product of energy of (4.7) becomes

$$M_{it} = \alpha_{it} \frac{Y_{it}}{E_{it}} \tag{4.22}$$

So the no-arbitrage relationship of (4.4) becomes

$$X_{it} = \frac{M_{ut} / R_{ut}^M}{M_{it} / R_{it}^M} = \frac{\alpha_{ut} \dfrac{Y_{ut}}{E_{ut}} \dfrac{1}{R_{ut}^M}}{\alpha_{it} \dfrac{Y_{it}}{E_{it}} \dfrac{1}{R_{it}^M}} \tag{4.23}$$

Recall (3.17) in Section 3, we have EPPP:

$$X_{it} = \frac{P_{ut} / R_{ut}}{P_{it} / R_{it}} \tag{4.24}$$

Further, in a competitive environment[18], $P_{it} = k_t M_{it}$. Thus

$$R_{it}^M = \frac{X_{wit} M_{it}}{M_{wt}} = \frac{X_{wit} P_{it}}{P_{wt}} = R_{it} \tag{4.25}$$

Therefore, combing (4.23) -(4.25), we have, for a given *t*,

$$\frac{\alpha_{it}}{\alpha_{ut}} = \frac{\dfrac{E_{it}}{X_{it} Y_{it}} R_{it}}{\dfrac{E_{ut}}{X_{ut} Y_{ut}} R_{ut}} = \frac{\dfrac{P_{it} E_{it}}{Y_{it}}}{\dfrac{P_{ut} E_{ut}}{Y_{ut}}}, \forall i \tag{4.26}$$

---

[18] Recall that earlier in Section 4.2.1, I maintain that we do not require the assumption for competitive market to test my hypothesis. However, in order to pin down the expression for the energy income share, α_{it}, we need the competitive market assumption.

Note that $X_{ut} = 1$ by definition.

Note in (4.25), if we can also let $\alpha_{ut} \equiv \frac{P_{ut}E_{ut}}{Y_{ut}}$, then

$$\alpha_{it} = \frac{P_{it}E_{it}}{Y_{it}}, \forall i$$

(4.27)[19]

I then define the "income share of energy" of "energy income share" α_{it} as described by (4.27).

4.5 Characteristics of Energy Income Share

4.5.1 The Uniformality of Energy Income Share

We are interested in the characteristics of the income share of energy defined by (4.27). Does it behave similarly as the real energy exchange rate? Recall the logarithm of the real energy exchange is randomly distributed across countries and is a mean-reversing random process over time.

From cross-sectional regression's standpoint, α_{it} is the residual of the period-by-period cross-sectional regression models (4.11)-(4.12) when null is true. So we can define the energy income share of the "world" w as

$$\alpha_{wt} \equiv \frac{P_{wt}E_{wt}}{Y_{wt}} \equiv \left(\prod_{i=1}^{I}\alpha_{it}\right)^{\frac{1}{I}} = \left(\prod_{i=1}^{I}\frac{P_{it}E_{it}}{Y_{it}}\right)^{\frac{1}{I}}, \forall t \qquad (4.31)$$

[19] Note we can also obtain (4.27) via the route that deals with the relationship of price and marginal product of energy.

Also consequently define "the energy income share indexed to the world" as

$$S_{it} \equiv \frac{\alpha_{it}}{\alpha_{wt}}, \text{ and } \left(\prod_{i=1}^{I} S_{it}\right)^{\frac{1}{I}} = 1, \forall t \tag{4.32}$$

From time-series, we need to examine them more carefully. The question is: are α_{it} move along the similar trajectory over time for all i? To address this question, I apply Fourier transform techniques to the time-series data to obtain its spectrum in the frequency domain. If the spectra are the same across all countries, then we may conclude that indeed there is a shared time-dependent function embedded in α_{it} for all countries, so we should feel comfortable about the assertions of (4.31) and (4.32).

Figure 4.5 shows such a spectrum density for all 16 countries, which I obtain using Fourier analysis on the detrended α_{it}. Notice that all 16 countries seem to have similar shape of the spectrum and their two main lobes are located in similar ranges. All countries seem to follow the similar trajectory along time. Table 4.2 lists the periods, inverse of the frequencies that correspond to the two main lobes that have the highest spectrum density in the frequency domain. They are indeed within similar ranges.

Further, for each i, I regress the α_{it} on the common RHS variables: year index and the two periods of 17.4251 year and 8.0062 years, which are Constant, t, $\sin(2\pi t/\tau_1)$, $\cos(2\pi t/\tau_1)$, $\sin(2\pi t/\tau_2)$, and $\cos(2\pi t/\tau_2)$. Figure 4.6 displays the actual data and the fitted curve for all countries. The R^2 ranges from 0.88 to 0.99; average over countries is 0.9567. This gives an additional justification that economically, (4.31) and (4.32) are a valid representation for the "world."

It is interesting to notice that in Figure 4.6, the income share of energy, α_{it}, declines along time for all the countries. I will come back to discuss this phenomenon later in Section 6.

Fig 4.5: Spectral density in the frequency domain for the demeaned, detrended logarithm of energy income share, $\ln \alpha_{it}$, for all 16 countries. First from left to right, then from top to down are CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ.

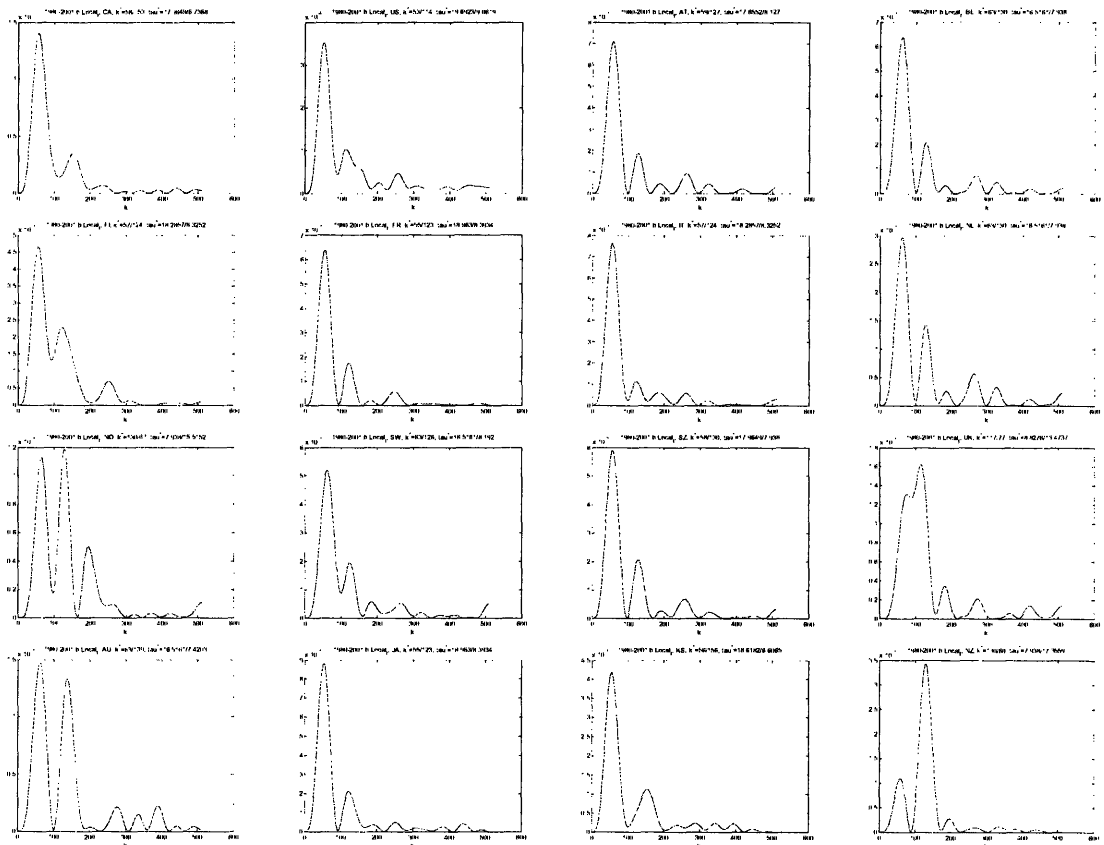


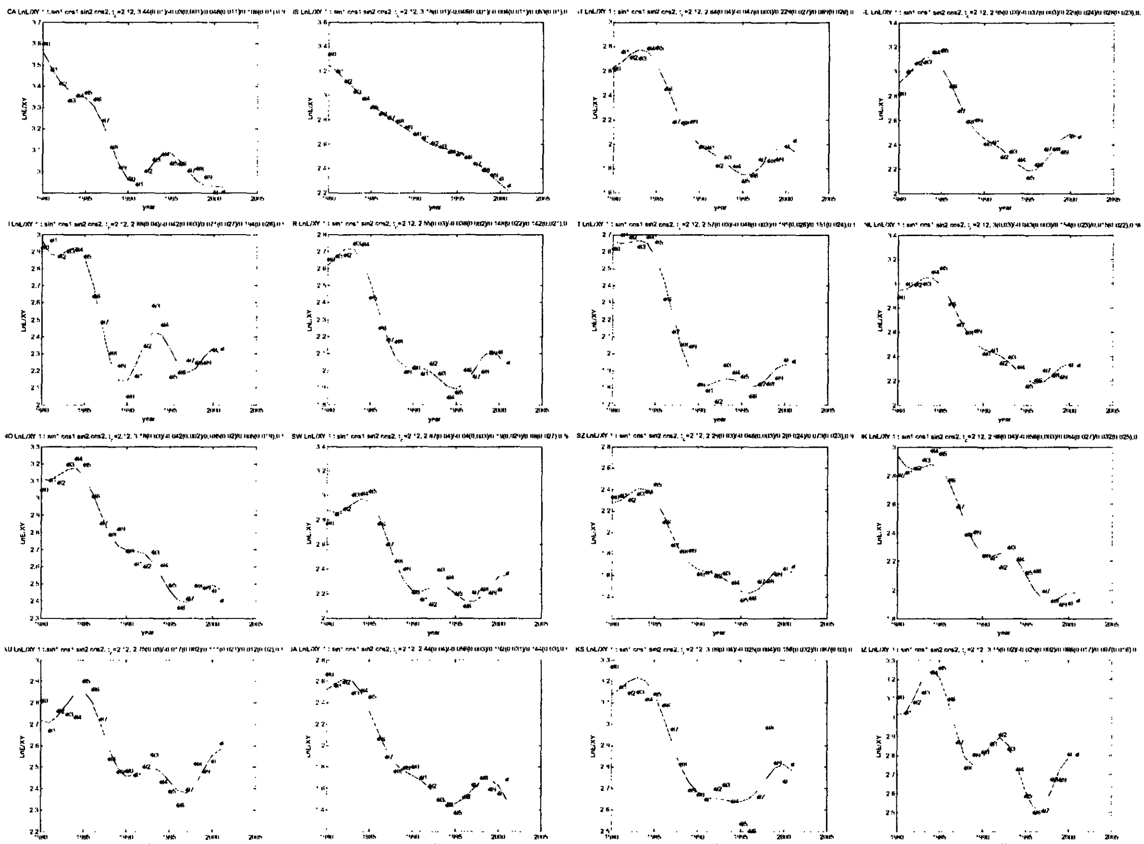
Table 4.2: Periods corresponding to the two lobes with highest spectral density

Country	tau1 (year)	tau2 (year)
CA	17.96	6.74
US	19.69	9.06
AT	17.66	8.13
BE	16.52	7.94
FI	18.29	8.33
FR	18.96	8.39
IT	18.29	8.33
NL	16.52	7.94
NO	15.52	7.94
SW	16.52	8.19
SZ	17.96	7.94
UK	13.47	8.83
AU	16.52	7.42
JA	18.96	8.39
KS	18.62	6.61
NZ	17.36	7.94
Avg	17.43	8.01

Fig 4.6: Actual and fitted curves for all 16 countries for time-series regressions:

$$\ln \alpha_{it} = b_{1,i} + b_{2,i} \cdot t + b_{3,i} \sin(2\pi t / \bar{\tau}_1) + b_{4,i} \cos(2\pi t / \bar{\tau}_1) + b_{5,i} \sin(2\pi t / \bar{\tau}_2) + b_{6,i} \cos(2\pi t / \bar{\tau}_2) + \varepsilon_{it}, \forall t \in [1, T],$$

where $\bar{\tau}_1 = 17.43, \bar{\tau}_2 = 8.01$ years. From left to right and then top to down: CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ.



4.5.2 Energy Income Share Indexed to the World

Now what about S_{it}? Like R_{it}, S_{it} too is a cross-sectional notion. But does S_{it} behave like R_{it}? One may ask: are we able to push our EPPP tests further to have some time-series meaning? Or, for any given *i*, is $S_{it} = 1, \forall t \in [1,T]$?

Following what have done in Section 3.5, I have the following regression model:[20] for any given *i*,

$$\Delta_t \ln S_{it} = b_1(i) + \varepsilon_{it}, t \in [2,T] \tag{4.41}$$

We are interested in the null: $b_1(i) = 0, \forall i \in [1,I]$.

The left panel of Figure 4.7 shows the estimated coefficients and their 95% confidence bounds as functions of time for the regression specification of (4.41). We do not reject the null for all countries at the 5% level except for Canada, for which we do not reject the null at the 3% level, meaning that intertemporally $\Delta_t \ln S_{it} \cong 0$ or $S_{it} \cong S_i$ for all *i*. Therefore, in conclusion, although S_{it} do change over time, S_{it} are persistent for almost all 16 countries, at least for the period of 1980-2001. To approximate, I would consider that for a short horizon (a few years or so), S_{it} only depends on *i* not *t*. I.e.,

$$S_{it} = S_i, \forall i \in [1,I], \forall t \in [1,T], \text{ and} \tag{4.42}$$

And consequently,

$$\alpha_{it} = \alpha_{wt} S_i, \forall i \in [1,I], \forall t \in [1,T] \tag{4.43}$$

[20] Keep in mind that from Section 2 we have learned that S_{it} is an intertemporal $I(1)$ process.

It is easy to verify that S_{it} obey both of the following properties, one for cross section, and the other for time series:

$$\prod_{i=1}^{I} S_{it} = 1, \forall t \in [1,T] \tag{4.44}$$

and

$$\left(\prod_{t=1}^{T} S_{it} \right)^{\frac{1}{T}} = S_i, \ \forall i \in [1,I] \tag{4.45}$$

The right panel of Figure 4.7 displays S_i, $\forall i$. Note that S_i are quite dispersive, from 0.5341 for Switzerland to 1.8181 for Canada.

Given (4.44), we may tend to think that at any give time, $\ln S_{it}$ is randomly distributed across countries with mean zero. Also given (4.45), we may try that for any given economy, $\ln S_{it}$ is a random process with mean reverting around $\ln S_i$. Figure 4.8 demonstrates such properties by showing $\ln S_{it}$ both cross-sectional and cross-time dimensions. As we can see from the graphs, $\ln S_{it}$ seems somewhat persistent over time.

Now we can compare the real energy exchange, R_{it}, and the world-indexed income share of energy, S_{it}. Below are the comparisons.

From economic no-arbitrage argument and regression results, we have

$$\text{Given } t, \left(\prod_{i=1}^{I} R_{it} \right)^{\frac{1}{I}} = 1 \tag{4.51}$$

$$\text{Given } t, \left(\prod_{i=1}^{I} S_{it}\right)^{\frac{1}{I}} = 1 \tag{4.52}$$

So cannot reject:

$$\text{Given } t,\ R_{it} = 1, \forall i \in [1, I] \tag{4.53}$$

$$\text{Given } t,\ S_{it} = 1, \forall i \in [1, I] \tag{4.54}$$

From time-series testing, cannot reject:

$$\text{Given } i,\ R_{it} = R_i, \forall t \in [1, T], \text{ where } R_i \equiv \left(\prod_{t=1}^{T} R_{it}\right)^{\frac{1}{T}} \neq 1, \exists i \tag{4.55}$$

$$0.8364 \leq R_i \leq 1.1538 \tag{4.56}$$

$$\text{Given } i,\ S_{it} = S_i, \forall t \in [1, T], \text{ where } S_i \equiv \left(\prod_{t=1}^{T} S_{it}\right)^{\frac{1}{T}} \neq 1, \exists i \tag{4.57}$$

$$0.5341 \leq S_i \leq 1.8181 \tag{4.58}$$

So we can conclude:

$$R_{it} = R_i, \forall i \in [1, I], \forall t \in [1, T], \text{ and } R_i \approx 1, \forall i \in [1, I] \tag{4.59}$$

$$S_{it} = S_i, \forall i \in [1, I], \forall t \in [1, T], \text{ and } S_i \neq 1, \exists i \tag{4.60}$$

Fig 4.7:

Left panel: Estimated coefficients, $\hat{b}_1(i)$, and their 95% confidence bounds as functions of i, for the time-series regression (4.41): $\Delta_t \ln S_{it} = b_1(i) + \varepsilon_{it}, t \in [2,T]$, $H_0 : b_1(i) = 0, \forall i$.

Right panel: Country-dependent constant energy income share indexed to the world S_i vs. i

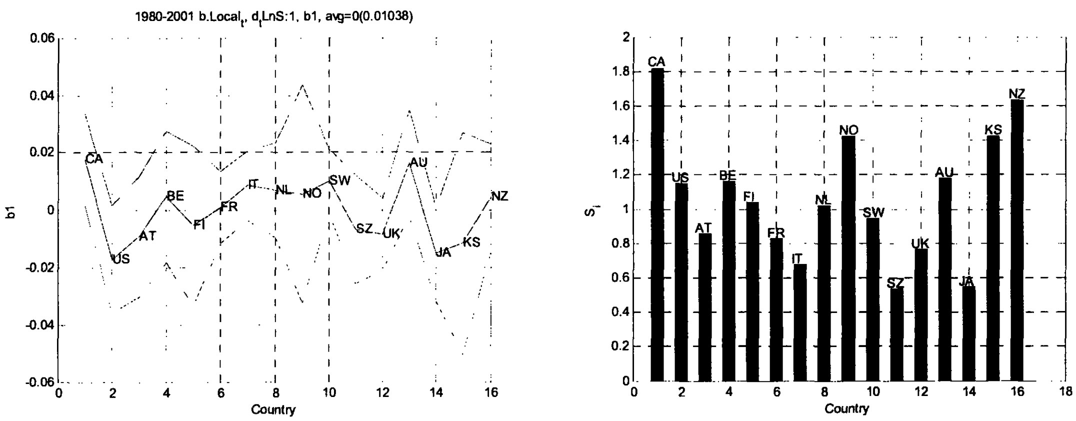
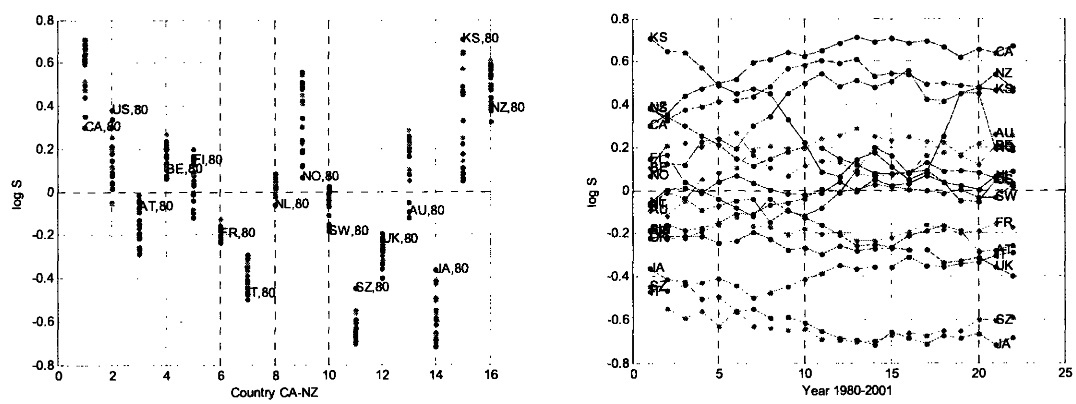


Fig 4.8: Display of the two-dimensional logarithm of energy income share indexed to the world: $\ln S_{it}$.

Left panel: $\ln S_{it}$ vs. i

Right panel: $\ln S_{it}$ vs. t



4.5.3 Cross-Sectional Correlations and Half Life Calculations [Reader may go directly to Table 4.5 of Section 4.5.3 and skip the rest of the section]

Let us apply the recipe I develop in Section 3.5 to S_{it} here.

Step 1: Test PPP. If it holds, calculate the real exchange rate.

- We really cannot claim that there exist a so-called "energy income share parity," because S_i, the country-dependent constant of energy income share index to the word, does not equal to 1 for all countries. However, loosely speaking, because $\prod_{i=1}^{I} S_{it} = 1, \forall t \in [1,T]$ holds, I still consider there exist some sort of parity, even if not as convincing as that for the real energy exchange rate. For convenient reference, I call the "parity" associated with the energy income share, "Quasi-Parity of Energy Income Share."

Step 2: Use the Fourier transfer technique to identify the most important cross-sectional correlation components for each country.

- Figure 4.9 consists of 16 graphs, one for each country. Each graph is a spectral density of demeaned, detrended $\ln S_{it}$. Table 4.2 lists τ_1 and τ_2, periods corresponding to the two lobes that I choose from Figure 4.9, for each country. The averages are $\bar{\tau}_1 = 17.08$ years, and $\bar{\tau}_2 = 9.74$ years.

Step 3: Take away the shared cross-sectional correlation components, including mean and trend.

- We will retain the residuals from the following regressions.

$$\begin{aligned} \ln S_{it} &= b_{1,i} + b_{2,i} \cdot t + b_{3,i} \sin(2\pi t / \bar{\tau}_1) + b_{4,i} \cos(2\pi t / \bar{\tau}_1) \\ &\quad + b_{5,i} \sin(2\pi t / \bar{\tau}_2) + b_{6,i} \cos(2\pi t / \bar{\tau}_2) + \varepsilon_{it}, \forall t \in [1,T] \end{aligned} \tag{4.61}$$

- Figure 4.10 consists of graphs representing the actual and the fitted data of the regressions (4.61) for all countries.

Step 4: Ensure the "parity" still holds after removal of cross-sectionally correlated components.

- We need to check the following holds.

$$\sum_{i=1}^{I} \hat{b}_{k,i} = 0, \forall k \in [1,6] \quad (4.62)$$

- Table 4.3 shows that (4.62) holds. So we can be sure that the parity holds for both the removed components and the residual components.

Step 5: Calculate the half life on the residual components for each country.

- Let $r_{it} \equiv \ln R_{it} - \ln \hat{R}_{it}$, and $\Delta_t r_{it} = \rho_i r_{i,t-1} + \varepsilon_{it}$, then the half life, if $\rho_i < 0$:

$$v_i \equiv \ln(1/2)/\rho_i \quad (4.63)$$

- Table 4.4 list the half lives of each country considering three different scenarios for the cross-sectional correlation components. The average half life is as low as 8.7 months (Case 3).

Fig 4.9: Spectral density in the frequency domain for the demeaned, detrended $\ln S_{it}$, for all 16 countries. First from left to right, then from top to down are CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ.

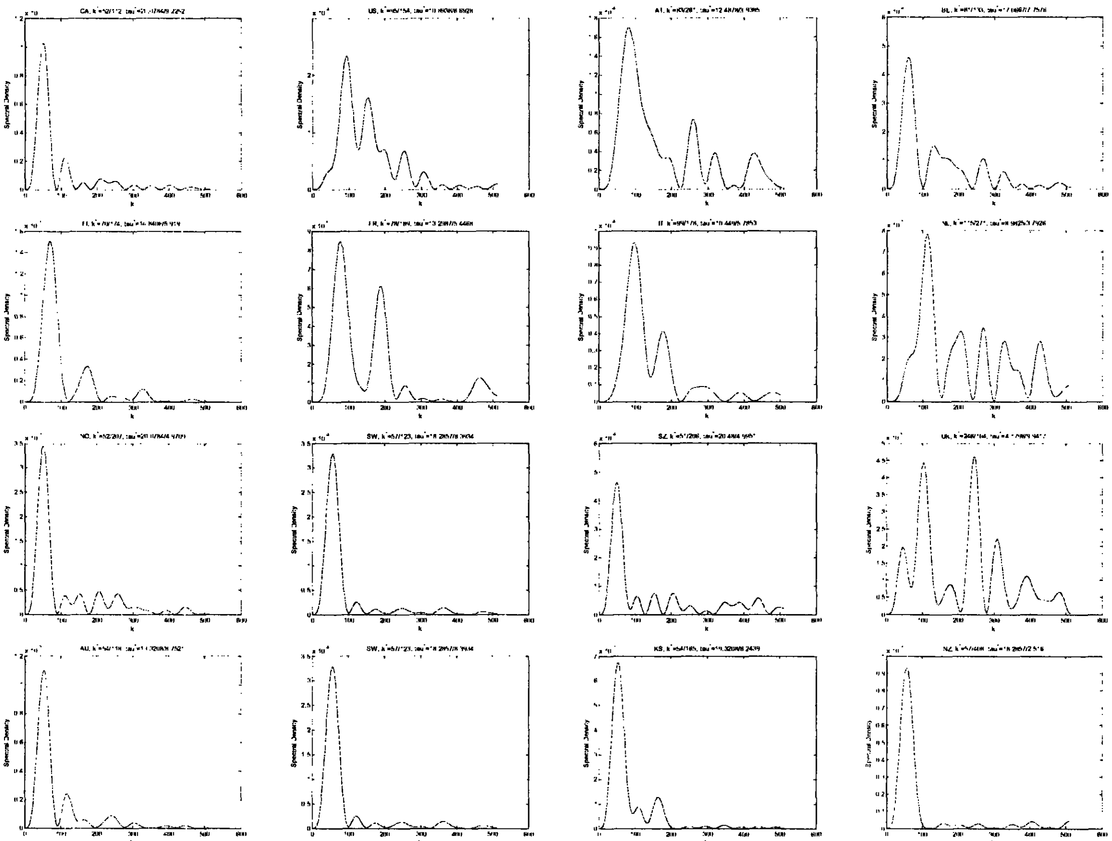


Table 4.3: The periods corresponding to the two lobes I choose from Fig. 4.9

Country	tau1	tau2
CA	20.08	9.23
US	NA	10.89
AT	NA	12.49
BE	17.07	7.76
FI	14.84	5.92
FR	NA	13.30
IT	NA	10.45
NL	NA	8.98
NO	20.08	NA
SW	18.29	8.39
SZ	20.48	5.00
UK	NA	9.94
AU	19.32	8.75
JA	20.08	9.23
KS	19.32	6.24
NZ	18.29	NA
Avg	17.08	9.74

Fig 4.10: Actual and fitted curves for all 16 countries for time-series regressions of (4.61):
$\ln S_{it} = b_{1,i} + b_{2,i} \cdot t + b_{3,i} \sin(2\pi t / \bar{\tau}_1) + b_{4,i} \cos(2\pi t / \bar{\tau}_1) + b_{5,i} \sin(2\pi t / \bar{\tau}_2) + b_{6,i} \cos(2\pi t / \bar{\tau}_2) + \varepsilon_{it}, \forall t \in [1, T]$,
where $\bar{\tau}_1 = 17.08, \bar{\tau}_2 = 9.74$ year. From left to right and then top to down: CA, US, AT, BE, FI, FR, IT, NL, NO. SW. SZ. UK. AU. JA. KS. and NZ.

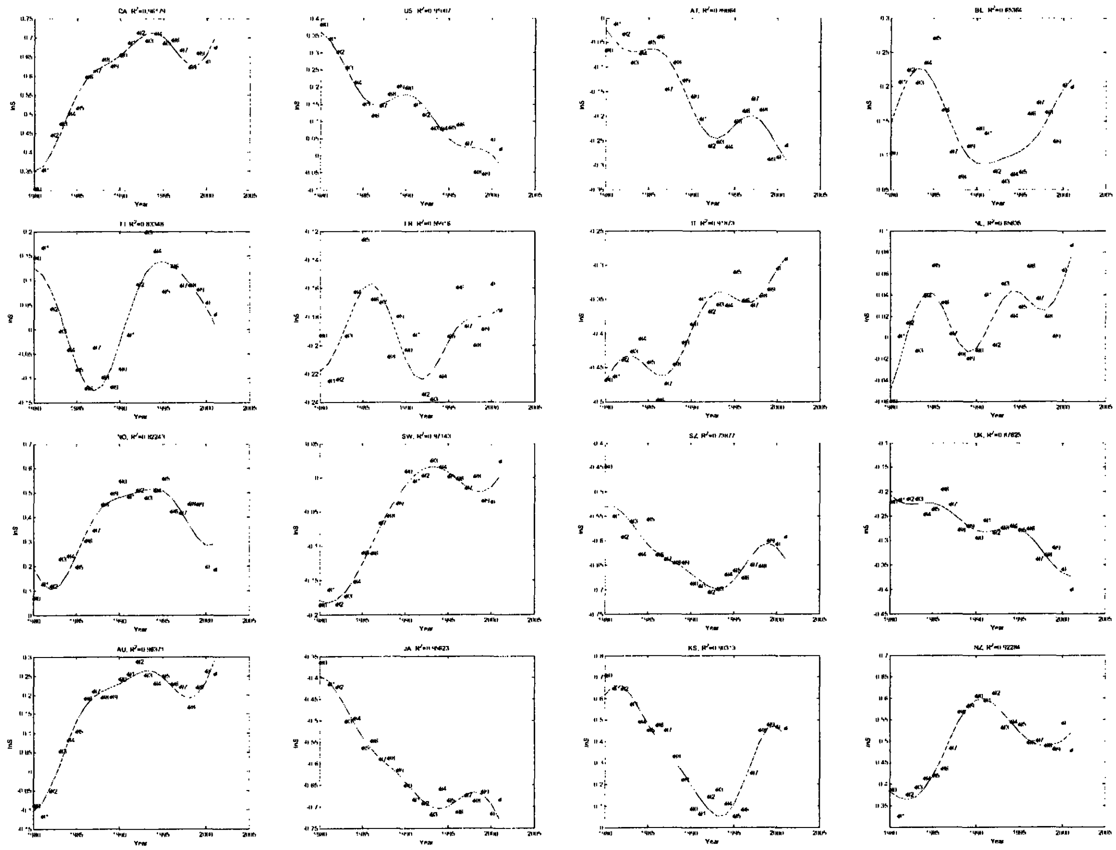


Table 4.4: The estimated coefficients of (4.61)

Country	b1	b2	b3	b4	b5	b6
CA	0.444	0.015	-0.064	-0.051	-0.025	-0.030
US	0.329	-0.017	-0.015	0.021	0.034	0.017
AT	-0.019	-0.012	0.029	-0.002	-0.027	0.018
BE	0.143	0.000	0.065	-0.011	0.010	-0.019
FI	0.014	0.001	-0.019	0.122	0.022	-0.011
FR	-0.202	0.001	0.020	-0.016	-0.015	-0.001
IT	-0.489	0.008	-0.004	0.013	0.022	-0.017
NL	-0.024	0.004	0.016	-0.010	-0.004	-0.028
NO	0.277	0.010	-0.154	-0.017	-0.054	0.012
SW	-0.154	0.009	-0.049	-0.012	-0.004	-0.003
SZ	-0.584	-0.005	0.049	0.021	0.011	0.019
UK	-0.185	-0.007	-0.010	0.005	-0.016	-0.008
AU	0.004	0.016	-0.057	-0.070	-0.022	-0.028
JA	-0.445	-0.015	0.045	0.028	0.011	0.020
KS	0.464	-0.014	0.222	0.016	0.051	0.050
NZ	0.427	0.007	-0.075	-0.038	0.005	0.010
Sum	0.000	0.000	0.000	0.000	0.000	0.000

Table 4.5: Half life of the real energy exchange rate after removing the cross-sectional correlations.
Case 1: Cross-sectional correlations are mean and trend
Case 2: Cross-sectional correlations are mean, trend, and tau1 components
Case 3: Cross-sectional correlations are mean, trend, tau1 and tau2 components

	Half Life (year)			Half Life (month)
Country	Case 1	Case 2	Case 3	Case 3
US	1.66	1.63	1.11	13.4
AT	1.26	0.97	0.63	7.5
BE	1.86	0.91	0.87	10.5
FI	2.64	1.00	0.89	10.7
FR	1.47	0.89	0.72	8.6
IT	1.60	1.30	0.78	9.4
NL	0.88	0.84	0.57	6.8
NO	5.27	1.09	0.83	10.0
SW	3.56	0.52	0.47	5.6
SZ	1.62	0.72	0.64	7.6
UK	0.97	0.87	0.61	7.3
AU	3.76	1.22	0.59	7.1
JA	2.19	0.74	0.50	6.1
KS	8.16	1.49	1.19	14.2
NZ	4.36	0.57	0.52	6.3
Avg	2.58	0.92	0.68	8.2

4.6 Summary

Using the results from Section 3 that energy is a representative cross-sectional good, I use a no-arbitrage argument to link the exchange rate with the marginal product of energy. Then I obtain an explicit functional form for the cross-sectional production function--a Cobb-Douglas form with separate factor augmenting productivity for both the energy and non-energy factors and with the exponent being the energy income share and non-energy income share respectively.

I find that the energy income share consists of two components, one is shared by all countries, and the other is a country-specific component. The country-specific component seems to behave like the real energy exchange rate in that it is randomly distributed across countries at any given time, and is a mean-reversing random process over time for any given country. I call this phenomenon the "quasi-parity" of the energy income share.

I also find that the country-specific component seems to be quite persistent over time, and is statistically equal to a constant over time. That constant is country-specific and does not necessarily equal to unity. For most countries, the half life for the country-specific component of the energy income share is lesser than a year.

5 Preference Function and Intertemporal Production Function

5.1 Background

In addition to the substitutionality between energy and other factors that I discuss in Section 4, researchers are interested in the causal relationship between energy and economic growth. For instance, Jorgenson (1984) finds that the decline of the real prices of both electricity and non-electricity energy contributes to the overall productivity growth from 1920 too 1955. Researchers also use statistical techniques for time-series data to study the Granger-causality and/or test cointegration between energy consumption and economic growth. However, as Liddle (2004) points out, there are no clear stories emerged. Some studies produce contradictory results using different time periods for the same country.[21] Other multi-country studies find full-spectrum of results, including causality from energy to growth, causality from growth to energy, no causality, bi-causality, cointegration, and no cointegration, etc.[22]

In this section, I will take a different approach from the previous literatures. Instead of focusing on analyzing the cointegration and causality relationships between energy and GDP, I will start with economic models and test the models directly. My models will represent equilibrium relationships and will be in reduced forms.

In Sections 3-4, I have shown that energy as a cross-sectional representative good that energy prices equalize the exchange rate differentials, or vice verse, and that the marginal products of energy (also the marginal values of energy in a risk-neutral market) equalize the exchange rate differentials, or vice verse. Consequently, the former gives rise to the

---

[21] For example, for the case of the US, Kraft and Kraft (1978) find Granger causality running from GDP to energy consumption for the period of 1947-1974. Later, Yu and Hwang (1984) find no causality for the period of 1947-1979. Stern (2000) finds that causality goes from energy consumption to GDP using a multivariate approach including capital and labor.

[22] For example, using a multivariate analysis including the consumer price index, Masih and Masih (1996) find neither cointegration nor causality for both Singapore and South Korea, cointegration for India and causality from energy to GDP, cointegration for Indonesia but causality from GDP to energy. The same authors, Masih and Masih (1997), find cointegration and bi-directional causality for Taiwan. Also for Taiwan, Cheng and Lai (1997) find no cointegration and the causality going from GDP to energy. However for Taiwan, Chang, Fang and Wen (2001) find causality direction going from energy to GDP.

EPPP, and the latter allow us to obtain an explicit cross-sectional production. Logically we would ask whether we are able to do the same for time-series. I.e., can energy be also an intertemporal representative good, so the marginal values of energy equalize the GDP deflator differentials, or vice verse? And then are we able to obtain an explicit preference function? An explicit intertemporal production function?

5.2 Theory

5.2.1 Assumptions and Setup

For a given country *i*, let Q_{it} be inverse of the GDP deflator normalized at base year *b*. So $Q_{ib} = 1$. One may interpret Q_{it} as the nominal value of the country *i*'s output at *t* in terms of its output at *b*; i.e. is the inverse of cumulated, combined effect of inflations plus productivity changes up to year *t* with the reference being the base year *b*. Let M_{it} be the nominal marginal product of energy. Denote marginal utility $U_{it}^{'}(\cdot)$. So then $\mathrm{M}_{it}U_{it}^{'}(\cdot)$ stands for the nominal marginal value of energy. Then, the following relationship should hold if energy is a representative/common good across time[23]:

$$Q_{it}\mathrm{M}_{it}U_{it}^{'}(\cdot) = Q_{is}\mathrm{M}_{is}U_{is}^{'}(\cdot),\ \forall t,s \in [1,T] \tag{5.1}$$

[23] Eq. (5.1) is based on the dynamic optimization argument, or analogous to the cross-sectional no arbitrage, the marginal utility-adjusted intertemporal "no arbitrage" argument. In the standard consumption-based asset pricing model (Cochrane 2001), a consumer maximizes his present expected utility subject to a dynamic budget constraint in each period. The first order condition (FOC) is: $p_t u_t^{'}(c_t) = \mathrm{E}_t[u_{t+1}^{'}(c_{t+1})p_{t+1}]$, where p_t, c_t, $u_t^{'}$ denote asset price, consumption, and marginal utility at time *t*, respectively, and E_t stands for the conditional expectation at time *t*. By the law of iterative expectation, we can replace $t+1$ in the FOC with any $s, \forall s \geq t+1$. I argue that if p_t is the price of an intertemporal representative good, then we can remove the conditional expectation in (FOC), and (FOC) will become (FOC'): $p_t u_t^{'}(c_t) = u_s^{'}(c_s)p_s, \forall t,s$. Economically, the marginal utility-weighted real price of the representative good shall be the same across time. If the utility is risk-neutral, then (FOC') will represent the intertemporal no arbitrage for the intertemporal representative good. This equation of (FOC') represents the same relationship as does (5.1) if we can consider (a) that p_t, which is the real price of the representative good, is proportionally to $Q_t\mathrm{M}_t$, which shall be the real marginal product of the representative good, and (b) that $u_t^{'}$ is the same as $U_t^{'}$.

Or taking $s = b$,

$$Q_{it} = \frac{M_{ib} U'_{ib}(\cdot)}{M_{it} U'_{it}(\cdot)}, \forall t \in [1, T]. \tag{5.2}$$

One may find that (5.1)-(5.2) are analogous to (4.1) and (4.6).

Suppose that for a given country *i*, the inter-temporal production functions takes the following two-factor constant elasticity of substitution (CES) form,

$$Y_{it} = \left[\beta_{it} \left(B_{E,it} E_{it} \right)^{\frac{\sigma_i - 1}{\sigma_i}} + \left(1 - \beta_{it}\right)\left(B_{N,it} N_{it}\right)^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}} \tag{5.3}$$

where *i* is the country index, *t* is the time index, Y_{it} is the nominal GDP in national currency for country i at time t, E_{it} is the real energy consumption, N_{it} is the real non-energy employed, $B_{E,it}$ is the energy factor augmenting technology, $B_{N,it}$ is the non-energy factor augmenting technology, $\beta_{it} \in (0,1)$ is a distribution parameter which determines the relative importance of the two factors of energy and non-energy, and $\sigma_i \in (0, \infty)$ is the elasticity of substitution between the two factors.

Again let

$$\lambda_i = \frac{\sigma_i - 1}{\sigma_i} \tag{5.4}$$

Notice that $\lambda_i \in (-\infty, 1)$. When $\sigma_i = \infty$ or $\lambda_i = 1$, the two factors are perfect substitutes, and the production is linear, $Y = \beta A_E E + (1 - \beta) A_N N$. When $\sigma_i = 1$ or $\lambda_i = 0$, the production is Cobb-Douglas, $Y = (A_E E)^{\beta} (A_N N)^{1-\beta}$. When $\sigma_i = 0$ or $\lambda_i = -\infty$, there is no substitution between the two factors and the production is Leontieff, $Y = \min(A_E E, A_N N)$.

From the functional form (5.3), one can obtain that

$$\mathrm{M}_{it} \equiv \frac{\partial Y_{it}}{\partial E_{it}} = \left(\frac{Y_{it}}{E_{it}}\right)^{1-\lambda_i} \beta_{it} B_{E,it}^{\ \lambda_i} \tag{5.5}$$

Further assume that the representative agent for a given country *i* takes the form of constant relative risk aversion (CRRA) in the ratio of income to energy expenditure, namely,

$$U_{it}(\cdot) = \frac{1}{1-\theta_i}\left(\frac{Y_{it}}{P_{it}E_{it}}\right)^{1-\theta_i} \delta_{it} \tag{5.6}$$

where θ_i is the coefficient of the CRRA utility function and δ_{it} denote the "subjective" discount factor. Note that Y/PE equals Y^*/P^*E, which I use as a proxy for the after-energy real consumption $C^* = Y^* - P^*E$. The reason is to simplify the algebra. From (5.6), we have the marginal utility,

$$U_{it}^{'}(\cdot) = \left(\frac{Y_{it}}{P_{it}E_{it}}\right)^{-\theta_i} \delta_{it} \tag{5.7}$$

Combing (5.2), (5.5) and (5.7), obtain

$$\frac{Y_{it}}{E_{it}} = \left(P_{it}^*\right)^{\frac{-1}{1-\theta_i-\lambda_i}} P_{it}^{\ \frac{1-\theta_i}{1-\theta_i-\lambda_i}} \left(\delta_{it}\beta_{it}B_{it}^{\ \lambda_i}\right)^{\frac{-1}{1-\theta_i-\lambda_i}} \left(\mathrm{M}_{ib}v_{ib}^{'}(\cdot)\right)^{\frac{1}{1-\theta_i-\lambda_i}} \tag{5.8}$$

Recall that $P_{it}^* \equiv Q_{it}P_{it}$.

If $\lambda_i = 0,\ \forall i \in [1, I]$, i.e. the production takes the Cobb-Douglas form, then (5.8) becomes

$$\frac{Y_{it}}{P_{it}E_{it}} = \left(P_{it}^{*}\right)^{\frac{-1}{1-\theta_i}} \left(\delta_{it}\beta_{it}\right)^{\frac{-1}{1-\theta_i}} \left(\mathrm{M}_{ib}U_{ib}^{'}(\cdot)\right)^{\frac{1}{1-\theta_i}} \tag{5.9}$$

If the preference is risk neutral, i.e. $\theta_i = 0,\ \forall i \in [1, I]$, then (5.8) would become

$$\frac{Y_{it}}{E_{it}} = Q_{it}^{\frac{-1}{1-\lambda_i}} \left(\delta_{it}\beta_{it}B_{it}^{\lambda_i}\right)^{\frac{-1}{1-\sigma_i}} \mathrm{M}_{ib}^{\frac{1}{1-\lambda_i}} \tag{5.10}$$

Note that (5.10) is analogous to (4.8), with Q_{it} to X_{it}, M_{ib} to M_{ut}, and $\delta_{it}\beta_{it}$ to α_{it}.

If the production function is Cobb-Douglas and the preference is risk neutral, then (5.8) becomes

$$\frac{Y_{it}}{E_{it}} = Q_{it}^{-1}\left(\delta_{it}\beta_{it}\right)^{-1}\mathrm{M}_{ib} \tag{5.11}$$

5.2.2 Hypothesis

I hypothesize a constant elasticity of substitution between energy production factor and non-energy production factors; i.e. null: $\lambda_i = \lambda_0,\ \forall i \in [1, I]$. I also hypothesize a constant CRRA coefficient for the preference function; i.e. null: $\theta_i = \theta_0,\ \forall i \in [1, I]$. Notice that the optimization relationship (5.1) is valid regardless whether we reject both of the hypotheses. But if we should not reject the null, and determine what λ_0 and θ_0 are, then we would be able to explicitly determine the CES production function (5.3) and the preference function (5.6).

5.3 Regression Models and Results

5.3.1 Setup

Although in Section 2, I have shown that most variables are $I(1)$ time-series processes, I still have the regression models built for levels. The reason is simply for reference only--I will use the results from level regressions to compare those from first difference. I will consider only the results from the first difference regressions valid.

Below is the model for the level regressions:

$$\frac{Y_{it}}{E_{it}} = \left(P_{it}^{*}/P_{ib}^{*}\right)^{\frac{-1}{1-\theta_i-\lambda_i}} \left(P_{it}/P_{ib}\right)^{\frac{1-\theta_i}{1-\theta_i-\lambda_i}} \left(\delta_{it}\beta_{it}B_{it}^{\lambda_i}\right)^{\frac{-1}{1-\theta_i-\lambda_i}} \left(\mathrm{M}_{ib}v_{ib}^{'}(\cdot)P_{ib}^{\theta_i}\right)^{\frac{1}{1-\theta_i-\lambda_i}},\ t \in [1,T]$$

(5.21)

And the model for the first difference regressions is:

$$\frac{Y_{i,t+1}/E_{i,t+1}}{Y_{it}/E_{it}} = \left(\frac{P_{i,t+1}^{*}/P_{ib}^{*}}{P_{it}^{*}/P_{ib}^{*}}\right)^{\frac{-1}{1-\theta_i-\lambda_i}} \left(\frac{P_{i,t+1}/P_{ib}}{P_{it}/P_{ib}}\right)^{\frac{1-\theta_i}{1-\theta_i-\lambda_i}} \left(\frac{\delta_{i,t+1}\beta_{i,t+1}B_{i,t+1}^{\lambda_i}}{\delta_{it}\beta_{it}B_{it}^{\lambda_i}}\right)^{\frac{1}{1-\theta_i-\lambda_i}},\ t \in [1,T-1]$$

(5.22)

As shown in Section 2, the first differences for all the variables are $I(0)$ processes in time, and therefore are stationary.

5.3.2 Country-by-Country Regressions

5.3.2.1 Country-by-Country Level Regressions (FOR REFERENCE ONLY) [Reader may skip Section 5.3.2.1 entirely]

<u>Model:</u>

For a given *i*,

$$\ln\left(\frac{Y_{it}}{E_{it}}\right) = b_1^L(i) + b_2^L(i)\ln\left(\frac{P_{it}^*}{P_{ib}^*}\right) + b_3^L(i)\ln\left(\frac{P_{it}}{P_{ib}}\right) + b_4^L(i)\cdot t + \varepsilon_{it}^L,\ t\in[1,T] \qquad (5.23)$$

The null hypothesis: $b_2^L(i) = b_{2,0},\ \forall i \in [1,I]$ and $b_3^L(i) = b_{3,0},\ \forall i \in [1,I]$

Results:

Figure 5.1 shows the actual and fitted data of the regressions (5.22) for all 16 countries. Figure 5.2 displays the estimated coefficients and their 95% respective confidence bounds as functions of countries. Below is a summary:

- The null: $b_{2,0} = -1$, i.e. $\theta_0 + \lambda_0 = 0$.
  - Reject six countries: IT, NO, SW, JA, KS, NZ
- The null: $b_{3,0} = 1$, $\lambda_0 = 0$.
  - Reject six countries: FR, IT, NO, AU, KS, NZ
- The null $b_{2,0} = -1$ and $b_{3,0} = 1$, i.e. $\theta_0 = \lambda_0 = 0$.
  - Reject seven countries: IT, NL, NO, SW, JA, KS, NZ
- Based on the sample average across all countries,
  - $\overline{\hat{b}_2} \equiv \frac{1}{I}\sum_{t=1}^{I}\hat{b}_2(i)$ = -0.9035 (0.1876), $\overline{\hat{b}_3}$ = 0.9753 (0.2125), implying that $\hat{\theta}_0$ = -0.0794, $\hat{\lambda}_0$ = -0.0274, and hence $\hat{\sigma}_0$ = 0.9733.
  - $\overline{\hat{b}_4}$ = 0.0120 (0.0061).
  - $\overline{R^2}$ = 0.9797
- Jointly F- test for $b_2(i) = b_{2,0} = -1, \forall i$ and $b_3(i) = b_{3,0} = 1, \forall i$, i.e. $\theta_i = \theta_0 = 0, \forall i$ and $\lambda_i = \lambda_0 = 0, \forall i$
  - Reject the null, because the F-statistics of 3.5 is greater than the critical value of 1.5.

Fig 5.1: (For Reference Only) Actual and fitted curves for all 16 countries for time-series level regressions (5.23):

$$\ln\left(Y_{it}E_{it}^{-1}\right) = b_1^L(i) + b_2^L(i)\ln\left(P_{it}^{*}P_{ib}^{*-1}\right) + b_3^L(i)\ln\left(P_{it}P_{ib}^{-1}\right) + b_4^L(i)\cdot t + \varepsilon_{it}^L,\ t \in [1,T]$$

From left to right and then top to down: CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ. Horizontal axis is $-\ln P^{*} + \ln P$.

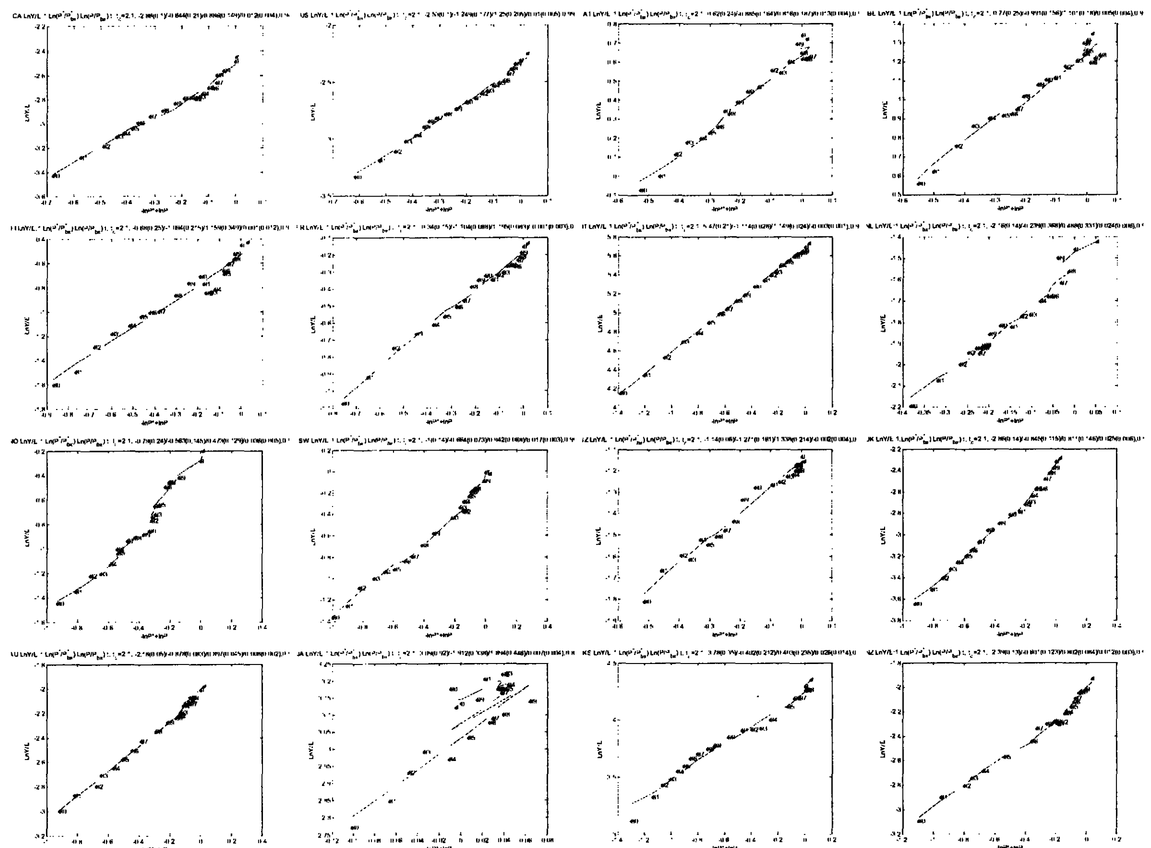


Fig 5.2: (For Reference Only) Estimated coefficients and their 95% confidence bounds as functions of i, for the time-series level regression (5.23):

$$\ln\left(Y_{it}E_{it}^{-1}\right) = b_1^L(i) + b_2^L(i)\ln\left(P_{it}^{*}P_{ib}^{*-1}\right) + b_3^L(i)\ln\left(P_{it}P_{ib}^{-1}\right) + b_4^L(i)\cdot t + \varepsilon_{it}^L,\ t\in[1,T]$$

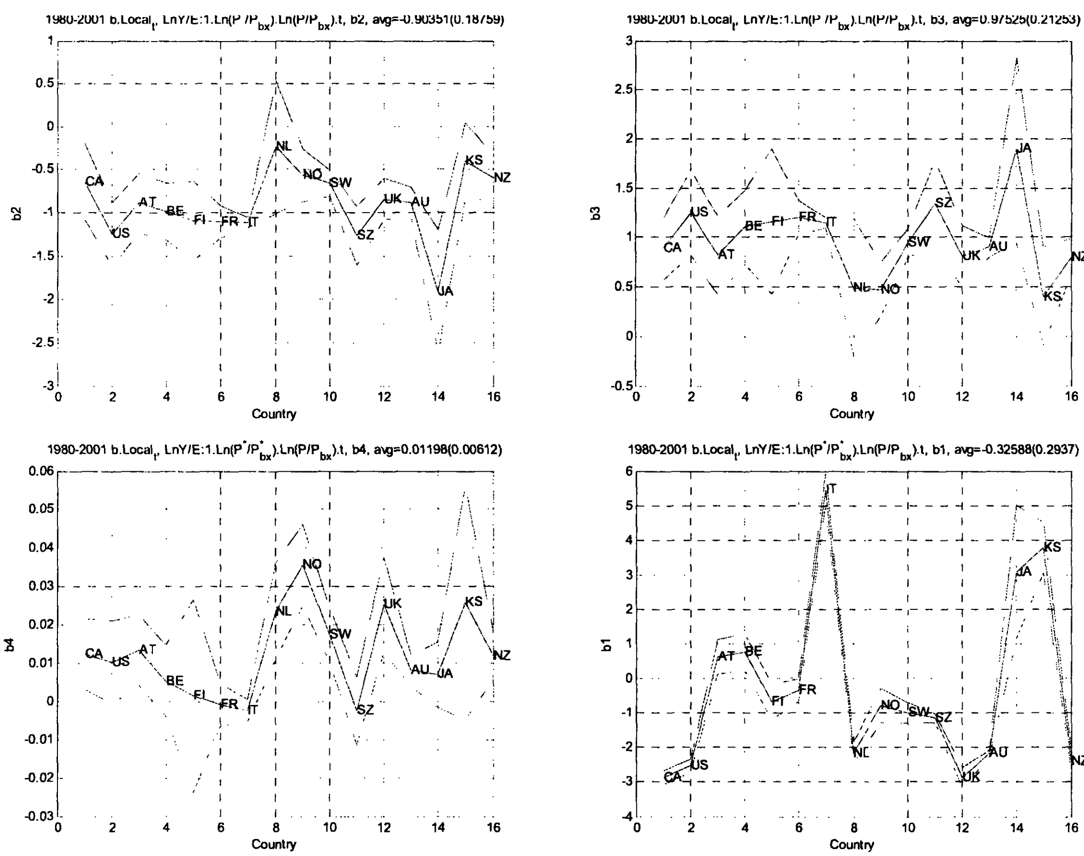
$H_0 : b_2^L(i) = -(1-\theta_0-\lambda_0)^{-1} = -1,\ b_3^L(i) = (1-\theta_0)(1-\theta_0-\lambda_0)^{-1} = 1, \forall i$, where $\theta_0 = \lambda_0 = 0$.

Upper left panel: $\hat{b}_2^L(i)$ vs. i

Upper right panel: $\hat{b}_3^L(i)$ vs. i

Lower left panel: $\hat{b}_4^L(i)$ vs. i

Upper right panel: $\hat{b}_1^L(i)$ vs. i



5.3.2.2 Country-by-Country First Difference Regressions

Model:

For a given *i*,

$$\Delta_t \ln\left(\frac{Y_{it}}{E_{it}}\right) = b_1^{FD}(i) + b_2^{FD}(i)\,\Delta_t \ln\left(\frac{P_{it}^*}{P_{ib}^*}\right) + b_3^{FD}(i)\Delta_t \ln\left(\frac{P_{it}}{P_{ib}}\right) + \varepsilon_{it}^{FD},\ t \in [2,T] \quad (5.24)$$

where Δ_t is an operator defined in Section 2.

Note:

$$b_2^{FD}(i) = \frac{-1}{1-\theta_i-\lambda_i},\ b_3^{FD}(i) = \frac{1-\theta_i}{1-\theta_i-\lambda_i} \quad (5.25)$$

So

$$\theta_i = 1 + \frac{b_3^{FD}(i)}{b_2^{FD}(i)},\ \lambda_i = (1-\theta_i)\left(1-\frac{1}{b_3^{FD}(i)}\right) \quad (5.26)$$

The null hypothesis: $b_2^{FD}(i) = b_{2,0},\ \forall i \in [1,I]$ and $b_3^{FD}(i) = b_{3,0},\ \forall i \in [1,I]$.

Validity of the Model:

In setting up (5.24), I assume the RHS variables and the error term are intertemporally uncorrelated. To test this assumption, again I employ the Hausman specification test. The instrument I choose for $\ln P_{it}$ is $IV_{it} = \frac{1}{2}(\ln P_{JA,t} + \ln P_{KS,t}), \forall i, i \neq JA, KS$, $IV_{JA,t} = \ln P_{KS,t}$, and $IV_{KS,t} = \ln P_{JA,t}$. Similarly, the instrument for $\ln P_{it}^*$ is $IV_{it}^* = \frac{1}{2}(\ln P_{JA,t}^* + \ln P_{KS,t}^*), \forall i, i \neq JA, KS$, $IV_{JA,t}^* = \ln P_{KS,t}^*$, and $IV_{KS,t}^* = \ln P_{JA,t}^*$. Why I single out JA and KS? I find that in terms of $\Delta_t \ln(P_{it}^*/P_{it})$, either JA or KS is least intertemporally correlated with the remaining countries; for some countries, JA is less correlated than KS; for the others, KS is less correlated than JA. So for a given country, should the RHS variables are corrected with the error term in (5.24), the instruments should provide better estimators than the RHS variables in (5.24).

Table 5.1 below lists the chi-square statistics for five selected years for the Hausman test using the above instrument variable. Because the test statistics are smaller than the critical values for all countries, we cannot reject the null hypothesis that the RHS variables are uncorrelated with the error term in (5.24).

Table 5.1: Hausman test for (5.24)

Country	CA	US	AT	BE	FI	FR	IT	NL	NO	SW	SZ	UK	AU	JA	KS	NZ
Chi2-stat	0.09	4.48	1.47	2.77	1.99	0.00	2.13	0.13	0.11	0.43	0.01	0.79	4.05	3.64	0.45	2.50

The 5% critical value of chi square with 2 degrees of freedom is 5.99
Use average over JA and KS of real prices as the instrument for the real prices for all countries except JA and KS
Use average over JA and KS of nominal prices as the instrument for the nominal prices for all countries except JA and KS
Use JA's real/nominal prices for the real/nominal prices for KS
Use KS's real/nominal prices for the real/nominal prices for JA

Results:

Figure 5.3 shows the actual and fitted data of the regressions (5.24) for all 16 countries. Figure 5.4 displays the estimated coefficients and their 95% respective confidence bounds as functions of countries. Below is a summary

- The null: $b_{2,0} = -1$, i.e. $\theta_0 + \lambda_0 = 0$.
  - Reject only one country (AU) at the 5% level
  - However, reject none at the 2.5% level
- The null: $b_{3,0} = 1$, $\lambda_0 = 0$.
  - Reject only one country (AU) at the 5% level
  - However, reject none at the 4% level
- The null $b_{2,0} = -1$ and $b_{3,0} = 1$, i.e. $\theta_0 = \lambda_0 = 0$.
  - Reject none at the 5% level
- Based on the sample average across all countries,

- $\overline{\hat{b}}_2 \equiv \frac{1}{I}\sum_{t=1}^{I}\hat{b}_2(i)$ = -0.8654 (0.3005), and $\overline{\hat{b}}_3$ = 0.9456 (0.2820), implying that $\hat{\theta}_0$ = -0.0928, $\hat{\lambda}_0$ = -0.0628, and hence $\hat{\sigma}_0$ = 0.9409.
- $\overline{\hat{b}}_1$ = 0.0125 (0.0063).
- $\overline{R^2}$ = 0.4749

* Jointly F- test for $b_2(i) = b_{2,0} = -1, \forall i$ and $b_3(i) = b_{3,0} = 1, \forall i$, i.e. $\theta_i = \theta_0 = 0, \forall i$ and $\lambda_i = \lambda_0 = 0, \forall i$
  - Cannot reject the null. F-statistics of 1.3560 is less than the critical value of 1.4840; p-value is 0.1019.
* Jointly F- test for $b_2(i) = b_{2,0} = -0.8228, \forall i$ and $b_3(i) = b_{3,0} = 0.9054, \forall i$, i.e. $\theta_i = \theta_0 = -0.1004, \forall i$ and $\lambda_i = \lambda_0 = -0.1150, \forall i$
  - Cannot reject the null. F-statistics of 0.9365 is less than the critical value of 1.4840; p-value equal 0.5697

<u>Conclusion</u>

We cannot reject the null in almost all tests.

Fig 5.3: Actual and fitted curves for all 16 countries for time-series first difference regressions (5.24):

$$\Delta_t \ln\left(Y_{it} E_{it}^{-1}\right) = b_1^{FD}(i) + b_2^{FD}(i)\,\Delta_t \ln\left(P_{it}^* P_{ib}^{*-1}\right) + b_3^{FD}(i)\Delta_t \ln\left(P_{it} P_{ib}^{-1}\right) + \varepsilon_{it}^{FD},\ t \in [2,T]$$

From left to right and then top to down: CA, US, AT, BE, FI, FR, IT, NL, NO, SW, SZ, UK, AU, JA, KS, and NZ. Horizontal axis is $\Delta_t(-\ln P^* + \ln P)$.

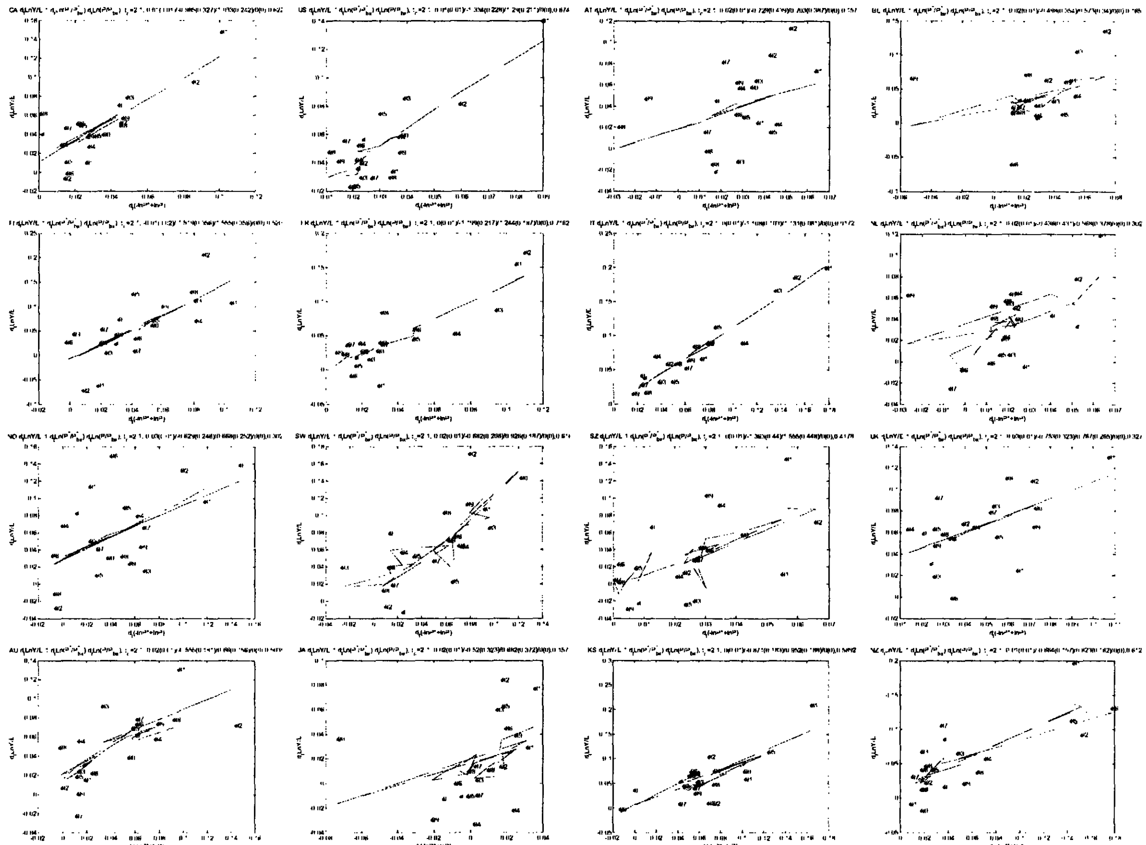


Fig 5.4: Estimated coefficients and their 95% confidence bounds as functions of i, for the level time-series regression (5.24):

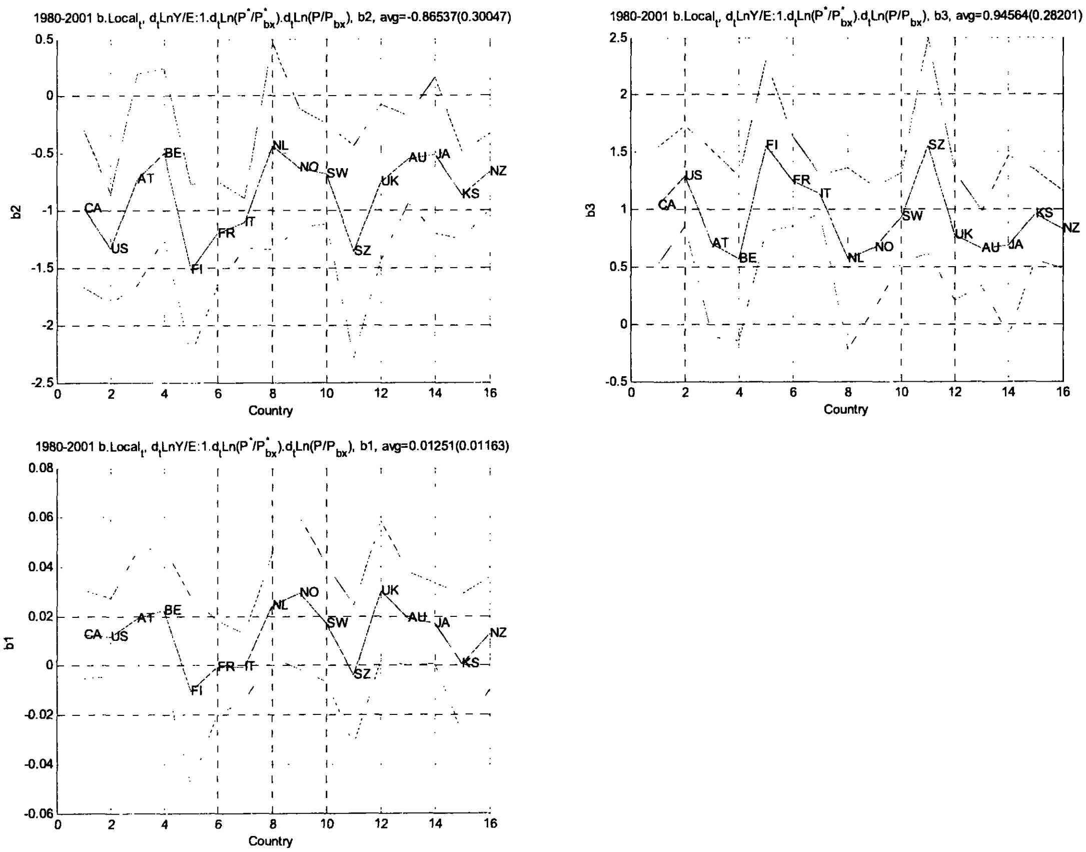
$$\Delta_t \ln\left(Y_{it}E_{it}^{-1}\right) = b_1^{FD}(i) + b_2^{FD}(i)\,\Delta_t \ln\left(P_{it}^{*}P_{ib}^{*-1}\right) + b_3^{FD}(i)\Delta_t \ln\left(P_{it}P_{ib}^{-1}\right) + \varepsilon_{it}^{FD},\ t\in[2,T]$$

$H_0 : b_2^{FD}(i) = -(1-\theta_0-\lambda_0)^{-1} = -1,\ b_3^{FD}(i) = (1-\theta_0)(1-\theta_0-\lambda_0)^{-1} = 1, \forall i$, where $\theta_0 = \lambda_0 = 0$.

Upper left panel: $\hat{b}_2^{FD}(i)$ vs. i

Upper right panel: $\hat{b}_3^{FD}(i)$ vs. i

Lower left panel: $\hat{b}_1^{FD}(i)$ vs. i



5.3.3 Pooled Regressions

5.3.3.1 Pooled First Difference Regressions

Model:

For a given i,

$$\Delta_t \ln\left(\frac{Y_{it}}{E_{it}}\right) = b_1^{FD}(i) + b_2^{FD}\Delta_t \ln\left(\frac{P_{it}^*}{P_{ib}^*}\right) + b_3^{FD}\Delta_t \ln\left(\frac{P_{it}}{P_{ib}}\right) + \varepsilon_{it}^{FD},\ i \in [1, I], t \in [1, T-1] \tag{5.28}$$

The null hypothesis: $b_2^{FD} = b_{2,0}$ and $b_3^{FD} = b_{3,0}$

Results:

- Jointly F- test for $b_2 = -1$ and $b_3 = 1$, i.e. $\theta_0 = 0$ and $\lambda_0 = 0$
    - Reject the null. F-statistics of 6.7126 is greater than the critical value of 3.0241; p-value equal 0.0014.
    - The point estimate: $\hat{b}_2$ = -0.8228 (0.0599), so $\hat{b}_3$ = 0.9054 (0.0556), implying $\hat{\theta}_0$ = -0.1004, $\hat{\lambda}_0$ = -0.1150, and hence $\hat{\sigma}_0$ = 0.8969.
    - Notice that since $\hat{b}_3$ is statistically not different from 0, so $\hat{\lambda}_0$ is statistically not different from 0, and hence $\hat{\sigma}_0$ is statistically not different from 1. Also notice that since $\hat{b}_2$ is statistically different from 0, so $\hat{\theta}_0 + \hat{\lambda}_0$ is statistically different from 0.
    - The average of estimates: $\bar{\hat{b}}_1$ = 0.0149 (0.0144).
    - R^2 = 0.7819.
- Jointly F- test for $b_2 = -0.8228$ and $b_3 = 0.9054$, i.e. $\theta_0 = -0.1004$ and $\lambda_0 = -0.1150$
    - Cannot the null. F-statistics is almost equal to 0 and is much smaller than the critical value of 3.0241; the p-value equals 1.0000.

5.3.4 Alternative Setups [Reader may skip Section 5.3.4 entirely]

There are alternatives setups to (5.24) for country-by-country regressions and to (5.28) for pooled regressions. For example, we can choose for the LHS variable from one of $\Delta\ln(Y/E)$, $\Delta\ln(Y^*/E)$, or $\Delta\ln(Y/PE)$, and can choose for the RHS variables from any pairs of $\Delta\ln P^*$, $\Delta\ln P$, or $\Delta\ln Q$. The results for the hypothesis tests for these setups are the same as those obtained from the setup of (5.24) and (5.28). We do not reject the null for most tests.

5.4 Discussions

5.4.1 Level vs. First Difference [Reader may skip Section 5.4.1 entirely]

As discussed earlier, we cannot rely on the results obtained from level regressions for statistical inferences, because levels of the interested variables are $I(1)$ processes. So we have to use the results from the first difference regressions. However, it is my wish that the results from level regressions, despite being invalid, would still be close to the valid ones from first difference regressions. If this would be the case, then it might add more validity to our economic theory spelled out in Section 5.2.

Comparing Sections 5.3.2.1 and 5.3.2.2, we can find that country-by-country level regressions tend to reject the null more countries than the country-by-country first difference regressions. Beside this difference, they seem to behave quite similar in terms of averages of the estimates as Table 5.2 shows. This may further justify our model specifications.

Table 5.2: Comparisons of the results from the level and the first difference regressions

Average across 16 countries	Level	First difference
b2	-0.9035 (0.1876)	-0.8654 (0.3005)
b3	0.9753 (0.2125)	0.9456 (0.2820)
b4, b1	0.0120 (0.0061)	0.0125 (0.0063)
theta	-0.0794	-0.0928
lambda	-0.0274	-0.0628
sigma	0.9733	0.9409

5.4.2 Preference Function

To determine the preference function, I will focus on θ_0 in this subsection. For country-by-country first difference regression, the joint F-test does not reject the null that $b_{2,0} = -1$ and $b_{3,0} = 1$, i.e. $\theta_0 = \lambda_0 = 0$ for all 16 countries. The average of the estimates for θ_0 is $\hat{\theta}_0 =$ -0.0928. In the pooled first difference regression, the null is rejected. The point estimate for θ_0 is $\hat{\theta}_0 =$ -0.1004, which practically speaking is close to 0 as far as CRRA coefficient is concerned.

Therefore we may conclude that the CRRA coefficient for the utility function, θ_0, is either statistically 0, or if not, practically 0. Hence the preference function is risk neutral with respect to its argument, $Y_{it} / P_{it} E_{it}$, the proxy for real consumption.

So we obtain an explicit utility function,

$$U(\cdot) \approx \frac{Y_{it}}{P_{it} E_{it}} \delta_{it} \tag{5.31}$$

and its marginal utility

$$U'(\cdot) \approx \delta_{it} \tag{5.32}$$

Recall that δ_{it} is the "subjective" discount factor. I will discuss it in details later in this paper.

5.4.3 Intertemporal Production Function

To determine the production function, I will focus on λ_0 in this subsection. For country-by-country first difference regressions, the *t*-test does not reject the null $b_{3,0} = 1$, i.e. $\lambda_0 = 0$ for all countries except AU at the 5% level and does not reject AU at the 3% level. The joint *F*-test accept the null that $b_{2,0} = -1$ and $b_{3,0} = 1$, i.e. $\theta_0 = \lambda_0 = 0$ for all 16 countries. The average of the estimates is $\hat{\lambda}_0 =$-0.0628. In the pooled regression in Section 5.3.3.2, the null is rejected. The point estimate for λ_0 is $\hat{\lambda}_0 =$-0.1250, and hence $\hat{\sigma}_0 = 0.8969$, which for practical and simplifying purpose, I consider close to 1.

Therefore, the elasticity of substitution, σ_0, is either statistically equal to 1, or for practical and simplifying purpose, equal to 1.

So then the production function of (5.3) becomes that of Cobb-Douglas

$$Y_{it} \approx \left(B_{E,it}E_{it}\right)^{\beta_{it}}\left(B_{N,it}N_{it}\right)^{1-\alpha_{it}} = B_{it}E_{it}^{\beta_{it}}N_{it}^{1-\beta_{it}} \tag{5.33}$$

where $B_{it} \equiv B_{E,it}^{\beta_{it}}B_{N,it}^{1-\beta_{it}}$.

5.4.4 Energy Income Share and the "Subjective" Discount Factor

With $\lambda_0 = 0$, the marginal product of energy (5.5) becomes

$$\mathrm{M}_{it} = \beta_{it} \frac{Y_{it}}{E_{it}} \tag{5.34}$$

And the optimality condition (5.2) gives rise to:

$$Q_{it} = \frac{\mathrm{M}_{ib} U'_{ib}(\cdot)}{\mathrm{M}_{it} U'_{it}(\cdot)} = \frac{\delta_{ib}\beta_{ib} \dfrac{Y_{ib}}{E_{ib}}}{\delta_{it}\beta_{it} \dfrac{Y_{it}}{E_{it}}} \tag{5.35}$$

Since by definition for P^*_{it} in this paper, we have

$$Q_{it} \equiv \frac{P^*_{it}}{P_{it}} = \frac{P^*_{it} / P_{it}}{P^*_{ib} / P_{ib}} \tag{5.36}$$

Note that $P^*_{ib} \equiv Q_{ib} P_{ib} = P_{ib}$ since $Q_{ib} = 1$.

So combing (5.35) and (5.36), we obtain that for a given i,

$$\frac{\delta_{it}\beta_{it}}{\delta_{ib}\beta_{ib}} = \frac{\dfrac{E_{it}}{Q_{it} Y_{it}}}{\dfrac{E_{ib}}{Q_{ib} Y_{ib}}} = \frac{P^{*-1}_{it}}{P^{*-1}_{ib}} \frac{\dfrac{P_{it} E_{it}}{Y_{it}}}{\dfrac{P_{ib} E_{ib}}{Y_{ib}}}, \forall t \tag{5.37}$$

Further, given (4.27), then (5.37) becomes

$$\frac{\delta_{it}\beta_{it}}{\delta_{ib}\beta_{ib}} = \frac{P^{*-1}_{it}}{P^{*-1}_{ib}} \frac{\alpha_{it}}{\alpha_{ib}}, \forall t \tag{5.38}$$

I will come back to this representation in Section 6.

5.5 Summary

Setting up energy as an intertemporal representative good, whose marginal values are equalized across all time periods, I find that the intertemporal production is Cobb-Douglas, with separate factor augmenting productivity for both the energy and non-energy factors and with the exponent being the energy income share and non-energy income share respectively. I also find that a country's utility function is risk neutral and proportional to the inverse of the energy income share.

6 Energy as a Unified Representative Good

6.1 And Energy Income Share Runs Through It

Recall that in Section 4, we have found that the cross-country exchange rates equalize the marginal values of energy, which, in a risk-neutral market, are equal to the marginal products of energy, across countries. Also in Section 5, we have obtained that the cross-time exchange rates, i.e. the inverse of the GDP deflators, equalize the marginal values of energy, which are equal to the product of marginal utility and marginal product of energy, across different time periods. Given such representative good characteristics presented in Sections 4 and 5, we shall be able to combine them to have unified common good characteristics.

First of all, let us compare the marginal products of energy (4.22) and (5.34). These two representations must be the identical. Therefore, we have

$$\alpha_{it} = \beta_{it}, \forall i, \forall t \tag{6.1}$$

Secondly, compare the production functions (4.21) and (5.33). There are two different representations for the same function. Therefore, we must have

$$A_{E,it} = B_{E,it},\ A_{N,it} = B_{N,it},\text{ and } A_{it} = B_{it}, \forall i, \forall t \tag{6.2}$$

Further, compare the cross-sectional energy intensity representation in (4.27) and its intertemporal counterpart in (5.38), we shall obtain first that

$$\delta_{it} = \frac{1}{P_{it}^{*}}, \forall i, \forall t. \tag{6.3}$$

It is worth recalling that the energy cost share is defined as α_{it}:

$$\alpha_{it} = \frac{P_{it}E_{it}}{Y_{it}} \ \forall i, \forall t \tag{6.4}$$[24]

The relationships of (6.1)-(6.2) allow us to write a unified production function, one that is applicable both cross-sectionally and intertemporally:

$$Y_{it} = \left(A_{E,it}E_{it}\right)^{\alpha_{it}}\left(A_{N,it}N_{it}\right)^{1-\alpha_{it}} = A_{it}E_{it}^{\ \alpha_{it}}N_{it}^{\ 1-\alpha_{it}} \tag{6.5}$$[25]

Also the relationship of (6.3), allow us to explicitly express the preference function:

$$U_{it}(\cdot) = \frac{1}{\alpha_{it}}\frac{1}{P_{it}^{*}} \tag{6.8}$$[26]

Finally, we have obtained an explicit production function (6.5) and an explicit preference function (6.8) for all countries and for all time periods, and the income share of energy, α_{it}, runs through them. So I propose that we consider the income share of energy, α_{it}, be one of the states variables that define the status of the economy, both cross-sectional and intertemporal. Indeed we can consider energy a unified representative good.

6.2 Relationship between Income Share of Energy and Technology

---

[24] Note that $\alpha_{it} = \frac{P_{it}E_{it}}{Y_{it}} = \frac{P_{it}^{\$}E_{it}}{Y_{it}^{\$}} = \frac{P_{it}^{*}E_{it}}{Y_{it}^{*}}, \forall i, \forall t$,
where $P_{it}^{\$} \equiv X_{it}P_{it}$, $Y_{it}^{\$} \equiv X_{it}Y_{it}$, $P_{it}^{*} \equiv Q_{it}P_{it}$, and $Y_{it}^{*} \equiv Q_{it}Y_{it}$.

[25] The following are also the correct representations:
$Y_{it}^{\$} = \left(A_{E,it}^{\$}E_{it}\right)^{\alpha_{it}}\left(A_{N,it}^{\$}N_{it}\right)^{1-\alpha_{it}} = A_{it}^{\$}E_{it}^{\ \alpha_{it}}N_{it}^{\ 1-\alpha_{it}}$ or
$Y_{it}^{*} = \left(A_{E,it}^{*}E_{it}\right)^{\alpha_{it}}\left(A_{N,it}^{*}N_{it}\right)^{1-\alpha_{it}} = A_{it}^{*}E_{it}^{\ \alpha_{it}}N_{it}^{\ 1-\alpha_{it}}$.

[26] Note that $U_{it}(\cdot) = \frac{Y_{it}}{P_{it}E_{it}}\frac{1}{P_{it}^{*}} = \frac{Y_{it}^{\$}}{P_{it}^{\$}E_{it}}\frac{1}{P_{it}^{*}} = \frac{Y_{it}^{*}}{P_{it}^{*}E_{it}}\frac{1}{P_{it}^{*}} = \frac{1}{\alpha_{it}}\frac{1}{P_{it}^{*}}$.

A fact about income share of energy is that it has continued declining for almost all developed countries in the past few decades. Recall that in Fig 4.6 in Section 4, we have noticed such an interesting phenomenon. In a graph in her thesis, Kander (2004) shows that in the past 200 years, the income share of energy of the Swedish economy has continued decreasing since 1800, when it was as high as almost 100%. Why? What may have caused the income share of energy to decline along time?

Instead of providing explanations for all countries, here I will attempt to but only a generic example. Consider a representative economy, or the "world" economy,

$$Y^*_{wt} = A^*_{wt} E_{wt}{}^{\alpha_{wt}} N_{wt}{}^{1-\alpha_{wt}} \tag{6.11}$$

where $A^*_{wt} = A^*_{E,wt}{}^{\alpha_{wt}} A^*_{N,wt}{}^{1-\alpha_{wt}}$ stands for the total factor productivity, and

$$\alpha_{wt} = \frac{P^*_{wt} E_{wt}}{Y^*_{wt}} \tag{6.12}$$

represents the income share of energy.

To study its trajectory over time, we have

$$\frac{\dot{\alpha}_{wt}}{\alpha_{wt}} = -\frac{\dot{A}^*_{wt}}{A^*_{wt}} + (1-\alpha_{wt}) \cdot \left(\frac{\dot{E}_{wt}}{E_{wt}} - \frac{\dot{N}_{wt}}{N_{wt}} \right) + \frac{\dot{P}^*_{wt}}{P^*_{wt}} \tag{6.13}$$

The price of energy P^*_{wt} is determined by intersection of the demand of energy that is governed by the economy's marginal product of the energy function and of the supply of the energy that is governed by the profit maximization of the energy sector. Assuming linear technology, we have:

$$P^*_{wt} = \frac{C^*_{wt}}{F_{wt}} \tag{6.14}$$

where C^*_{wt} is the real marginal cost of extracting a BTU from the energy source, and F_{wt} stands for the real marginal efficiency of converting a source BTU to an end-use BTU.[27] Note that I consider any energy supply shocks are embedded in C^*_{wt}.

Assuming the economy is in fixed supply, i.e $E_{wt} = E_{w0}$ and $N_{wt} = N_{w0}, \forall t$, and Combing (6.13) and (6.14), we have

$$\frac{\dot{\alpha}_{wt}}{\alpha_{wt}} = -\frac{\dot{A}^*_{wt}}{A^*_{wt}} - \frac{\dot{F}_{wt}}{F_{wt}} + \frac{\dot{C}^*_{wt}}{C^*_{wt}} \tag{6.14}$$

This equation explicitly ties the trajectory of the energy intensity with the total factor productivity growth, the energy conversion efficiency growth, and the growth of marginal cost of obtaining an end-use BTU from the nature. If the third term in (6.14) is outweighed by the first two terms combined, then energy intensity drops along time. However, if technology stagnates or the marginal cost shoots up, then likely the energy intensity will increase. How energy intensity moves along time will have a profound correlation with both the energy efficiency and the economic wellbeing. I will discuss about this correlation in Section 7.

For an individual country, note that from (4.43) in Section 4, we have learned that for a not too long horizon, $\alpha_{it} \cong \alpha_{wt} S_i, \forall i, \forall t$. Thus,

$$\frac{\dot{\alpha}_{it}}{\alpha_{it}} = \frac{\dot{\alpha}_{wt}}{\alpha_{wt}}, \forall i \tag{6.15}$$

---

[27] The energy supply sector solves the profit maximization problem: $\max_E \pi = P^* E - C^* \cdot (E/F)$. So the FOC will be $0 = P^* - C^* / F$. Hence $P^* = C^* / F$.

So for the discussions for an individual country, the discussions for the world economy pertaining (6.14) applies.

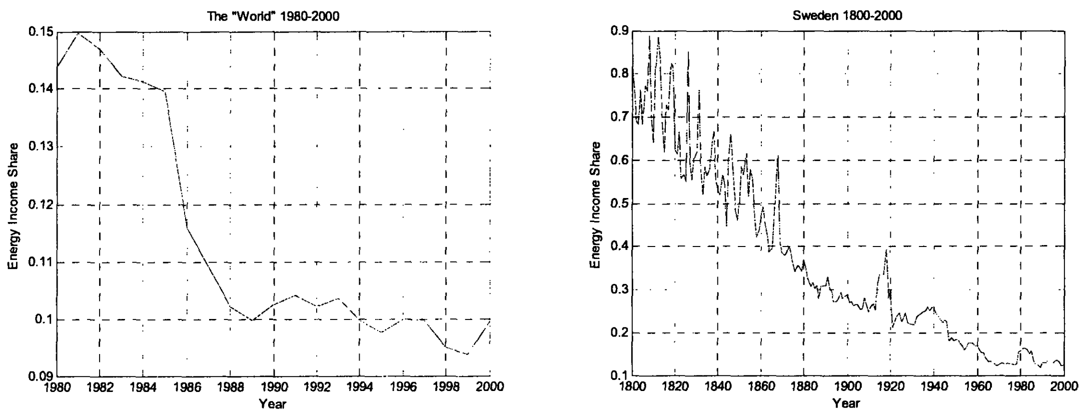
The left panel of Fire 6.1 shows the income shares of energy and non-energy for the "world" for the last 20 years from 1980 to 2000. One shall notice that the income share of the energy factor has decreased from 14-15% in the early 1980's to around 10% in the late 1980's and throughout the 1990's and early 2000's, and consequently the income share of the non-energy factor has climbed from 85-86% to about 90% in the same period. This confirms that the right hand side of (6.14) is negative on average during 1980-2001. So, on average, the combined technology effect outgrows the cost effect (including the effect of any supply shocks) during that period, so the energy income share deceases. Hypothetically, should it have been the other way around, the energy income share would have increased.

Is the declining energy income share a phenomenon to only for 1980-2001? The right panel of Figure 6.1 shows the energy income share for Sweden for the last 200 years from 1800 to 2000. Although some of data of the early 1980s may not be as reliable as the data of later days[28], it shows that the energy income share has continued declining for the last two centuries in Sweden. So the Swedish case for the last 200 years may also confirm that on average the combined technology effect outgrows the cost effect during the last 200 years in Sweden.

Recall that the energy income has a component that is shared by all countries. Given the theory governed by (6.14) along with the empirical evidence depicted by the two graphs in Figure 6.1 as well as by the 16 graphs in Figure 4.6, I hypothesize that the energy income share may have also declined over the last 200 years for most of West European and North American countries.

[28] According to the data source for Swedish energy consumption and price (Kander 2002), data of the fist half of 19th century are estimated.

Fig 6.1:
Left panel: The trajectory of the world's energy income share from 1980 to 2000 (20 years)
Right panel: The trajectory of Swedish energy income share from 1800 to 2000 (200 years)



7. Energy Efficiency and Economic Wellbeing

According to the definition of Ortiz and Sollinger (2002), energy efficiency is the relative thrift with which energy inputs are used to provide goods and services. It is the ratio of a delivered good or service to the energy consumed in the process. In this sense, we can define energy efficiency as real GDP to the total final energy consumption, mathematically expressed as below

$$H_{it} \equiv \frac{Y_{it}^*}{E_{it}} \qquad (7.1)$$

This in fact is also the inverse of energy intensity, which is commonly defined as final energy consumption to GDP.

Given (7.1), we can express energy efficiency further in term of income share of energy:

$$H_{it} \equiv \frac{Y_{it}^*}{E_{it}} = \frac{1}{\alpha_{it}} P_{it}^* \cong \frac{1}{\alpha_{wt} S_i} P_{it}^* \qquad (7.4)$$

Separately, recall that utility function in terms of income share of energy:

$$U_{it} \equiv \frac{Y_{it}}{P_{it} E_{it}} \frac{1}{P_{it}^*} = \frac{1}{\alpha_{it}} \frac{1}{P_{it}^*} \cong \frac{1}{\alpha_{wt} S_i} \frac{1}{P_{it}^*} \qquad (7.5)$$

Note the approximations in (7.4) and (7.5) hold if the horizon is not too long (a few years or so).

Then, after combing (7.4) and (7.5), we have

$$H_{it} U_{it} = \alpha_{it}^{-2} \qquad (7.6)$$

It is obvious that (7.6) explicitly expresses the inherent intratemporal tradeoff between energy efficiency and economic wellbeing. For high energy price is related to better energy efficiency, it is at the expense of the economic wellbeing. Conversely, low energy price is good for economic wellbeing, but at the expense of energy efficiency.

Moreover, expressing (7.6) in terms of growth rate, we can obtain below:

$$\frac{\dot{H}_{it}}{H_{it}}+\frac{\dot{U}_{it}}{U_{it}}=-2\frac{\dot{\alpha}_{it}}{\alpha_{it}}=-2\frac{\dot{\alpha}_{wt}}{\alpha_{wt}},\ \forall i \tag{7.7}$$

It is interesting to note that the combined growth rate of energy efficiency and utility of an individual country is exactly opposite to twice the growth rate of the world's energy income share, which is independent from any country. The relationship of (7.7) gives rise to the intertemporal tradeoff between energy efficient and economic wellbeing.

Is it possible that we eliminate the tradeoffs, intratemporal or intertemporal, and achieve both the goals of satisfactory energy efficiency and continuing improvement in economic wellbeing? Unlikely, if we believe (7.1)-(7.6) are the right representations. However, suppose we could achieve a state of very low income share of energy and could continue driving it down along time, then what (7.6) and (7.7) would tell us is that the tradeoff, though still would exist, might not be an important consideration any more. Low energy income share would offer relatively sufficient room for the intratemporal tradeoff, and continuing improving the share would allow enough flexibility for intertempotal tradeoff.

It is important to notice that the medium for tradeoff between energy efficiency (7.4) and utility (7.5) is the real energy price. This may deserve an attention to the policy makers when it comes to any policy related to energy or energy price.

To summarize, I have demonstrated that there is a tradeoff between energy efficiency and utility, with the energy price being the medium of the tradeoff. This may have important policy implication. To avoid the tradeoff, ultimately, it is the supply of affordable BTU that matters.

8 Summary of the Key Functional Relationships

It is worth summarizing the key functional relationships and their links that we have learned from the economic models and empirical tests in Sections 2 through 7.

First of all, for the validity of statistically inferences, I have tested the stationarity of the data of interest; I have found that some cross-sectional data are stationary in level and some are in first difference, and that most of the time-series data are only stationary in first difference. I use only the stationary series for empirical tests.

Second, I have concluded the validity of the Energy Purchasing Power Parity (EPPP) that the real energy exchange rates are near unity for all countries at all times, or the nominal currency exchange rates equalize the nominal energy price differentials across countries for all countries at all times. I have also calculated the half life of the real energy exchange rate and find it is smaller than any of those presented by the existing literatures. Though there have been limited evidences for the law of one price for either a typical consumption bundle or a Big Mac, there may be evidences, as offered by this paper, for the law of one price for energy.

Third, I have found that the cross-country production function is of Cobb-Douglas form with two production factors, energy and non-energy; their respective exponents are energy income share and non-energy income share. I have also demonstrated that the energy income share of each country consists of two components, one is a shared trajectory along time, and the other is unique and somewhat fixed for each country. I name such phenomenon the quasi-parity of the energy income share.

Fourth, I have approximated the intertemporal production function to be also of Cobb-Douglas format, with two production factors of energy and non-energy. After comparing it with its cross-sectional counterpart, I have then obtained a unified production function that is applicable both cross-sectionally and intertemporally.

Fifth, I have concluded that the utility function of each country is risk neutral in energy income share, which is a proxy for consumption. The utility function also contains a "subjective" discount factor that is equal to the inverse of the country's own real energy price.

Next, I have pointed out that the term of energy income share has appeared in almost all the functional relationships throughout this paper. So it may be a good candidate as a state variable of the economy. I have also demonstrated energy income share's relationship with both technology and the cost of extracting a BTU in the nature into end use. The technology includes the total factor productivity and the energy sector's conversion efficiency. The energy income share declines along time if the technology component outgrows the cost component.

Further, I have showed that there exist both intratemporal and intertemporal tradeoffs between energy efficiency and economic wellbeing, with energy price being the medium for the tradeoffs. Such tradeoffs via energy price shall deserve policy makers' close attention. Only under the state of very low energy income, may the intratemporal tradeoff not be of a major concern. And only when the energy income share continues to decline in time, may we not worry too much about the intertemporal tradeoff.

Mathematically, the above relationships can all be expressed as follows.

Energy Purchasing Power Parity (EPPP):

$$\frac{X_{it}P_{it}}{P_{wt}} = R_{it}, \forall i, \forall t, \text{ where } R_{it} \text{ satisfy}$$

$$\left(\prod_{i=1}^{I} R_{it}\right)^{\frac{1}{I}} = 1, \forall t, \text{ and } \left(\prod_{t=1}^{T} R_{it}\right)^{\frac{1}{T}} = R_i, \forall i, R_i \approx 1, \forall i$$

Quasi-Parity of Energy Income Share:

$$\frac{P_{it}E_{it}}{Y_{it}} = \alpha_{it}, \forall i, \forall t, \quad \frac{\alpha_{it}}{\alpha_{wt}} = S_{it}, \forall i, \forall t, \text{ where } S_{it} \text{ satisfy}$$

$$\left(\prod_{i=1}^{I} S_{it}\right)^{\frac{1}{I}} = 1, \forall t, \text{ and } \left(\prod_{t=1}^{T} S_{it}\right)^{\frac{1}{T}} = S_i, \forall i, \; S_i \neq 1, \exists i$$

Cobb-Douglas Production Function for a Two-Factor Economy:

$$Y_{it} = A_{it}E_{it}^{\alpha_{wt}} N_{it}^{1-\alpha_{wt}}, \forall i, \forall t \quad \text{where } A_{it} = A_{E,it}^{\alpha_{wt}} A_{N,it}^{1-\alpha_{wt}}$$

Risk-Neutral Preference with an Explicit Subjective Discount Factor:

$$U_{it}(\cdot) = \frac{1}{\alpha_{it}}\delta_{it}, \text{ and } U_{it}^{'}(\cdot) = \delta_{it}, \forall i, \forall t, \text{ where } \delta_{it} = \frac{1}{P_{it}^{*}}$$

Trajectories of Energy Income Share and Technology:

For a fixed supply economy,

$$\frac{\dot{\alpha}_{wt}}{\alpha_{wt}} \approx -\frac{\dot{A}_{wt}^{*}}{A_{wt}^{*}} - \frac{\dot{F}_{wt}}{F_{wt}} + \frac{\dot{C}_{wt}^{*}}{C_{wt}^{*}}, \forall t$$

Energy Efficiency (or Inverse of Energy Intensity):

$$H_{it} \equiv \frac{Y_{it}^{*}}{E_{it}} = \frac{1}{\alpha_{it}} P_{it}^{*}, \forall i, \forall t$$

Tradeoff between Energy Efficiency and Economic Wellbeing:

Intratemporal Tradeoff:

$$H_{it}U_{it} = \frac{1}{\alpha_{it}^2} \approx \frac{1}{S_i^2} \cdot \frac{1}{\alpha_{wt}^2}, \forall i, \forall t$$

Intertemporal Tradeoff (short horizon):

$$\frac{\dot{H}_{it}}{H_{it}} + \frac{\dot{U}_{it}}{U_{it}} = -2\frac{\dot{\alpha}_{it}}{\alpha_{it}} \approx -2\frac{\dot{\alpha}_{wt}}{\alpha_{wt}}, \forall i$$

9 Conclusion and Future Work

In the paper, I have used economic models and provided empirical evidences for the explicit functional relationships between energy and the economy. They are detailed in Section 2-7 and summarized in Section 8.

I hope that they could help us understand more about the role that energy has played in our economy. They might also help us debate and the discuss the rich dynamics between energy and the economy in relation to the current global-scale issues, such as energy discovery, energy conservation, alternative energy development, environmental considerations, and global climate change, etc. It is my hope that this paper would help build a stepping stone for more macro-level functional understanding about the relationships between energy and the economy.

There seem to be some interesting work related to this paper that can be further pursued in the future. To name a few, test the EPPP with higher frequency (quarterly) data, find an instrument variable to test the relative EPPP, offer solid micro foundation for the EPPP, treat energy income share as a state variable for asset valuation in financial economics, look for the cause of the relatively large variability of the energy income share across countries, find the functional relationships in the industry level, add environmental considerations, and integrate with the current global change research, etc.

I will plan to apply the methodology developed in this paper to the developing economies. I will also apply it to study the industrialized economics since the industrial revolution. In the future, we shall aim to have a unified, explicit production function that consists of energy, capital and labor.

They will be also interesting research in the policy and business implication. For instance, realizing that energy price is a macroeconomic instrument that offers tradeoffs between energy efficiency and economic wellbeing, a policy maker is not totally free of constraint pertaining pursing any policy that may move the energy price. Related, we may need to reexamine the net impact of energy conservation vs. exploration, in terms of efficiency and wellbeing.

Recall that in Section 1, I have posed several questions, among which this paper have addressed the first two about the functional role of energy. The remaining questions concerning how to think about optimal energy investment, energy portfolio management, alternative energy, and transition to a new energy economy will surely be the relevant and logically subsequent work.

References

Acemoglu, Daron, "Equilibrium Bias of Technology," MIT Economics Department mimeo, 2006

Amano, Robert A., Simon van Norden, "Exchange Rate and Oil Prices," *Review of International Economics*, 1998, Vol. 6, No. 4, pp. 683-694

Arrow, K. J., H. B. Chenery, B. S. Minhas, and R. M. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, 1961, Vol. 67, No. 3, pp. 474-481

Berndt, Ernst R., "Aggregate Energy, Efficiency, and Productivity Measurement," *The Annual Review of Energy*, 1978, 3:225-73

Berndt, Ernst R. and David O. Wood, "Engineering and Econometric Interpretation of Energy-Capital Complementarity," The American Economic Review, Vol. 69, No. 3 1979, pp. 342-354

Berndt, Ernst R. and David O. Wood, "Technology, Prices, and the Derived Demand for Energy," *Review of Economics and Statistics*, Aug. 1975, Vol. 57, pp. 259-268

Bracewell, Ronald N., *The Fourier Transform and its Applications*, 3rd ed., McGraw-Hill, New York, 1999.

Caetano, Sidney, Guilherme Moura, Sergio Da Silva, "Big Mac Parity, Income, and Trade," *Economics Bulletin*, Vol. 6, No. 11, 2004, pp. 1-8

Chang, Tsangyao, Wenshwo Fang, and Li-Fang Wen, "Energy Consumption, Employment, Output, and Temporal Causality: Evidence from Taiwan based on Cointegration and Error-correction Modeling Techniques," *Applied Economics*, Vol. 33, 2001, pp. 1045-1056

Cheng, Benjamin S. and Tin Wen Lai, "An Investigation of Cointegration and causality between Energy Consumption and Economic Activity in Taiwan," *Energy Economics*, Vol. 19, 1997, pp. 435-444

Cleveland, Cutler C., Robert K. Kaufmann and David I. Stern, "Aggregation and the Role of Energy in the Economy," Center for Energy and Environmental Studies Working Paper, Boston University, 2004?

Cochrane, John H., *Asset Pricing*, Princeton University Press, 2001, Chapter 1

Diebold, Francis X., Steven Husted, Mark Rush, "Real Exchange Rate under the Gold Standard," *Journal of Political Economy*, 1991, Vol. 99, No. 6, pp. 1252-1271

Fishelson, Gideon, "Elasticity of Factor Substitution in Cross-Section Production Functions," *The Review of Economics, and Statistics*, 1978, pp. 432-436

Fouquet, R. & Pearson, P.J.G., "Fiver Centuries of Energy Prices in the United Kingdom," *World Economics*, 4 (3), 1998, pp. 93-119.

Fuller, Wayne A., *Introduction to Time Series Analysis*, Wiley, New York, NY, 1976, p. 642

Griffin, James M., Paul R. Gregory, "An Intercountry Translog Model of Energy Substitution Responses," *The American Economic Review*, Vol. 66, No. 5, Dec. 1976, pp. 845-857

Hamilton, James D., *Time Series Analysis*, Princeton University Press, Princeton, NJ, 1994, p. 763

Hausman, Jerry A., "Specification Tests in Econometrics," *Econometrica*, Vol. 46, No. 6, Nov. 1978, pp. 1251-1271

Hegwood, Natalie D., and David H. Papell, "Purchasing Power Parity under the Gold Standard," *Southern Economic Journal*, 2002, Vol. 69, No. 1, pp. 72-91

Imbs, Jean, Haroon Mumtza, Mortem O. Ravn, and Rey Lelene, "PPP Strikes Back: Aggregation and the Real Exchange Rate," National Bureau of Economic Research Working Paper, Dec. 2005

Jorgenson, Dale W., "The Role of Energy in Productivity Growth," *The American Economic Review*, Vol. 74, No. 2, Papers and Proceedings of the Ninety-Sixth Annual Meeting of the American Economic Association, Mar. 1984, pp. 26-30

Kander, Astrid, *Economic Growth, Energy Consumption, and CO_2 Emissions in Sweden 1800-2000*, 2002, Almqvist & Wiksell International, Sodertalje, Sweden

Kauffman, R.K., and I. G. Azary-Lee, "A Biological Analysis of Substitution: Does Substitution Save Energy in the US Forest Products Industry?" *Proceedings of Ecological Economics: Implications for Forest Management and Practice,* 1993

Kraft, J. and A. Kraft, "On the Relationship between Energy and GNP," *Journal of Energy and Development*, Vol. 3, 1978, pp. 401-403

Liddle, Brentley, "How Tied are Energy and GDP? Revisiting Energy-GDP Cointegration and Causality for Disaggregated OECD Country Data," *European Association of Environmental and Resource Economics, 13th Annual Conference,* Budapest, Hungary, June 25-28 2004

Masih, Abul M.M. and Rumi Masih, "Energy Consumption, Real Income and Temporal Causality: Results from a Multi-country Study Based on Cointegration and Error-correction Modeling Techniques," *Energy Economics*, Vol. 18, 1996, pp. 165-183

Masih, Abul M.M. and Rumi Masih, "On the Temporal Causal Relationship between Energy Consumption, Real Income, and Prices: Some New Evidence from Asian-Energy Dependent NICs Based on A Multivariate Cointegration/Vector Error-Correction Approach," *Journal of Policy Modeling*, Vol. 19, No. 4, 1997, pp. 417-440

Ong, Li Lian, "Burgernomics: the Economics of the Big Mac Standard," *Journal of International Money and Finance*, Vol. 16, No. 6, 1997, pp. 856-878

Ortiz, David, and Jerry Stollinger, "Shaping Our Future by Reducing Energy Intensity in the U.S. Economy," *E-Vision 2002 Conference Proceedings*, Vol. 1, Rand Science and Technology Policy Institute, 2002, p. 5

Pakko, Michael R. and Patricia S. Pollard, "For Here or to Go? Purchasing Power Parity and the Big Mac," *Federal Bank of St. Louis Review,* Jan./Feb. 1996, pp. 3-21

Pakko, Michael R. and Patricia S. Pollard, "Burgernomics: A Big Mac Guide to Purchasing Power Parity," *Federal Bank of St. Louis Review,* Nov./Dec. 2003, pp. 9-28

Pollak, Robert A., Robin C. Sickles and Terrence J. Wales, "The CES-Translog: Specification and Estimation of a New Cost Function," 1984

Rogoff, Kenneth, "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, Vol. 34, 1996, pp. 647-668

Stern, David I., "A Multivariate Cointegration Analysis of the Role of Energy in the US Macroeconomy," *Energy Economics*, Vol. 22, 2000, pp. 267-283

Stern, David .I. and Cutler J. Cleveland, "Energy and Economic Growth," Working Paper, Department of Economics, Rensselaer Polytechnic Institute, 2004

Solow, Robert M., "Technical Change and the Aggregate Production Function," *The Review of Economics and Statistics*, Vol. 39, No. 3, 1957, pp. 312-320

Yu, Eden S.H. and Been-Kwei Hwang, "On the Relationship between Energy and GNP: Further Results," *Energy Economics*, Vol. 6, 1984, pp. 186-190