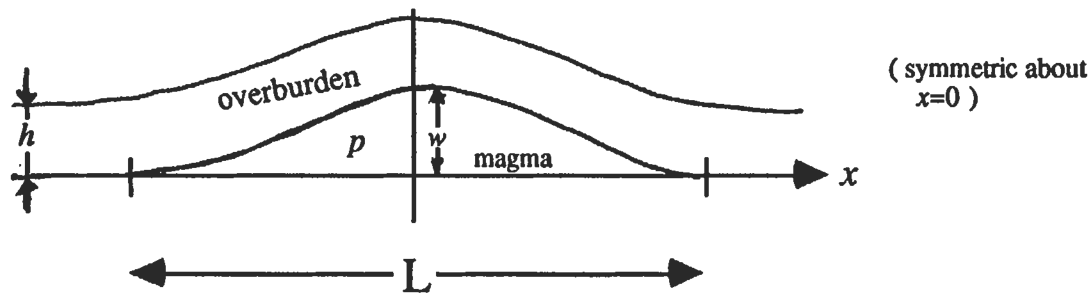
12.520 Problem Set 6

1) (20%) Treat the lithosphere as an elastic plate of thickness h, Young's modulus E, subjected to a compressive force P. Assuming that buckling will occur in the form of a sine curve, find the minimum P needed to initiate "subduction" (i.e., buckling) by this mechanism. Estimate needed parameters. (See discussion in T&S).

2) (30%) Consider the (2-D) laccolith (flat bottomed igneous intrusion) shown below. The overburden is to be treated as a thin elastic plate loaded by the uniform pressure p of the magma. The lithostatic pressure can be ignored, since it is only the excess pressure that leads to deformation.

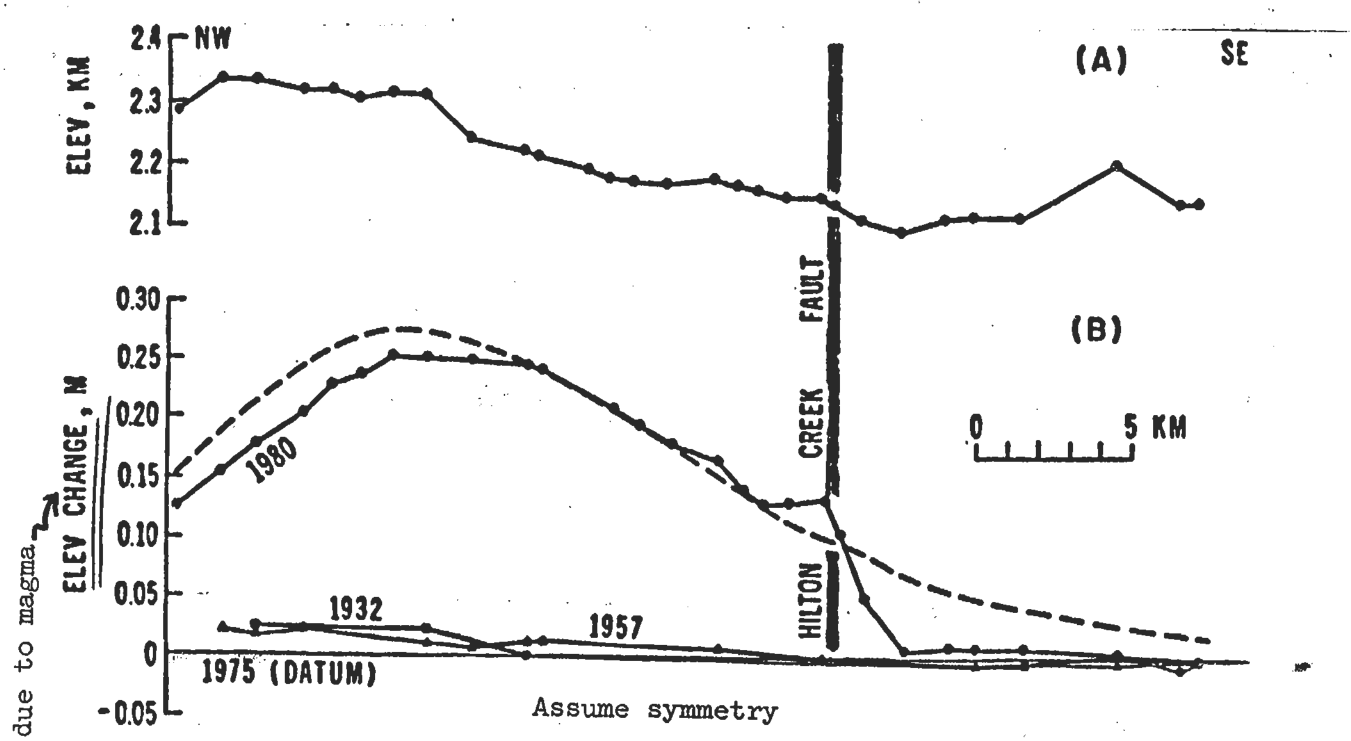
a) Calculate w, the deflection of the overburden, assuming that w and dw/dx are 0 at $x = \pm L/2$. (A polynomial expansion in x works well here.)



b) The figure below shows the results of a repeat leveling survey in the Owens Valley near Mammoth Lakes. The large uplift observed in 1980 is believed to be the result of the inflation of a magma chamber.

i) If the magma chamber can be approximated as a 2-D laccolith, find its depth and the overpressure. (The dots are the data - the dashed line is a different kind of theoretical model, assuming a spherical magma chamber. Fit the data, not the model. Note: your answer may surprise you!)

ii) Is the inferred normal faulting along the Hilton Creek fault consistent with the near-surface stress predicted for the laccolith model? What about if the fault nucleated at a depth of 10 km?

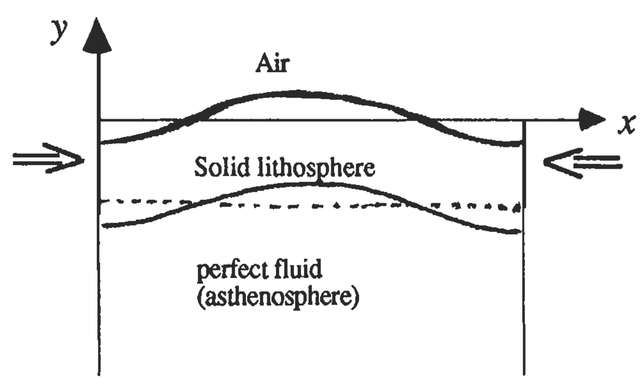


3) (30%) We will often need to solve geodynamics problems where the boundaries are not exactly aligned with the coordinate system. As an example, consider the situation sketched below, where compressive stresses in the x direction lead to a warping of the lithosphere.

a) Derive the boundary conditions, expressed in terms of τ_{xx}, τ_{xy}, τ_{yy}, etc., on the deformed boundaries near y=0 and y=h. (For this part of the problem, you might find it convenient to first define a local coordinate system favorably located with respect to the deformed boundaries, then rotate into the desired cartesian system. Assume that the amplitude of the boundary deformation is much smaller than its wavelength.)

b) Use a Taylor's series expansion to express the physical boundary conditions on the deformed surface as mathematical boundary conditions on the flat surfaces y=0 and y=h. Use your knowledge of the lithospheric (overburden) pressure gradient for the y direction. Assume that x and y derivatives of the "geodynamic" stresses are proportional to horizontal wavenumber, as for the problem of a topographic load on an elastic hall space.

c) How would the boundary conditions change if the asthenosphere were a viscous fluid, rather than a perfect (inviscid) fluid?



4) (20%) Consider the problem discussed in lecture, where a surface load due to topography of the form $\tau_{zz} = -\rho g h \cos(2\pi x/\lambda)$ loads a half space made up of a Poisson solid.

a) Calculate the horizontal and vertical displacements at the surface.

b) If GPS is capable of measuring displacements to an accuracy of 3 mm, independent of distance, what amplitude of ice sheet thickness variations could cause elastic displacements of the Earth large enough to measure for i) λ=100 km; ii) λ=1,000 km? (Look up or estimate needed parameters.)